Relationships between affective constructs and mathematics achievement: A modeling approach

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Abstract

The objective of this study is to investigate the relationship of preservice teachers’ affective characteristics using structural equation modelling to examine the relative contributions of latent factors, and to find the strength of their impacts to mathematics achievement. Based on the presage-process-product model of student learning (Biggs, 1987), 410 first-year students in the certificate education course at Hong Kong Institute of Education are used to build a hypothesized model to investigate how preservice teachers’ mathematics self-concept, mathematics teaching self-efficacy, beliefs and attitudes towards mathematics and mathematics education, influence cognitive activities in the learning process and subsequently mathematics achievement outcomes. The result shows that mathematics teaching self-efficacy acts as the mediator of affective characteristics, learning approaches and mathematics achievement. The finding proves useful to educators in the research areas of affective domain of mathematics education. It is hopeful that the proposed model will become a useful reference for further investigations on affective domain of mathematics education.

1. Introduction

Research is indicating that there are many factors influencing academic achievement. A review of literature on mathematics self-concept, mathematics self-efficacy, learning approaches, beliefs toward mathematics and attitudes toward mathematics education has shown that these variables are closely related to mathematics achievement outcomes respectively (Wong, 1995; Drew & Watkins, 1998). However, the inter-relationships of these variables and their causal effects on each other are still unclear. In Hong Kong, there has been a rapid expansion in higher education in the past decade. An important issue that has accompanied such an expansion was academic quality in both teaching and learning. In this respect, preservice teachers play a very important role to improve the quality of teaching and learning. Much recent research has focused on the various aspects of learning of Hong Kong tertiary students and investigated the factors such as causal attributions, self-concept, attitude, belief and learning approaches that affect academic achievement (Hau & Salili, 1990; Kember & Gow, 1991; Watkins & Biggs, 2001). However, there are very few studies on pre-service teachers’ affective characteristics in Hong Kong.

Beliefs, self-concept, attitudes and self-efficacy are all interrelated and are likely to affect academic achievement (Relich & Way, 1994; Philippou, 1998; Thompson, 1992). It is argued that in the area of mathematics education, these factors form a complex network that brings about changes in mathematics achievement. Therefore, the focus of this study is to clarify the directions and magnitude of the relationships between these variables among preservice teachers of primary mathematics. In doing so, a path model is created to explain the relationships between factors affecting pre-service primary teachers’ mathematics self-concept, mathematics self-efficacy, learning approaches, beliefs and attitudes toward mathematics education in Hong Kong.

2. Research Significance

There is a widely accepted belief that one’s perception of teaching and learning mathematics is influenced by one’s conception of mathematics. Hersh (1986) said that one’s conceptions of what mathematics is, affects one’s conception of how it should be presented. In this respect, the beliefs and attitudes of pre-service teachers are of particular significance and importance because of their potential influence on their future pupils’ beliefs and attitudes and turn out the achievement. Philippou (1998) suggested that positive teacher attitudes contribute to the formation of positive pupil attitudes. Haladyna, Shaughnessy and Shaughnessy (1983) provided support for this claim. Their findings showed that teacher quality, which included teachers’ attitudes, was a causal factor for students’ attitude towards mathematics. If their argument is accepted, then it is highly undesirable for those who have unfavourable feelings about mathematics to teach mathematics in school. Thus, it seems reasonable to assess the relationships of preservice teachers’ beliefs, self-concept, attitudes and self-efficacy, with respect to mathematics and mathematics education which can serve as an indicator to evaluate the effectiveness of the programmes. Therefore, the study is practically significant to the programme designers to cater for the affective characteristics of students by some intervention programmes, and it is also policy significant, in long term, to set up appropriate selection criteria for admission to the course and revise the curriculum of courses of mathematics at Hong Kong Institute of Education, so as to improve the quality of learning mathematics for pre-service primary
Furthermore, the study is theoretically significant to find out the latent constructs influencing how the pre-service primary teachers value mathematics and mathematics education, and to what extent the affective characteristics influence their learning approaches, and in turn, the effect on achievement in mathematics and mathematics education. In particular, the model proposed is able to explain the interrelationship of beliefs, attitudes, mathematics self-concept, mathematics self-efficacy, learning approaches and mathematics achievement. The strength of the factors and the relations within and between the factors of the affective characteristics can also be found through structural equation modelling. Moreover, a clearer picture of how students learn and a better knowledge in the affective characteristics of the learners would no doubt help in improving the quality of student learning. As a result, the findings of the study may form the basis for future intervention programmes which aim at improving students’ mathematics achievement.

3. The research framework

The literature discussed leads to the formation of the following research framework. Integrating the affective characteristics and learning-approach characteristics, the interrelationships could be initially hypothesized, and the conceptual framework could be formulated in terms of a path model showing the effect of affective characteristics and learning approaches on mathematics and mathematics education achievement. Thus, based literature of McLeod’s (1992) affective categories, Biggs’ (1987a) 3P model, and Randhawa Beamer, and Lundberg’s (1993) structural model, it appears to have eight interrelated theoretic constructs as presage variables, process variables and product variables for the research framework as shown in Figure 1.

Figure 1. The hypothesized framework of interrelationships among theoretic constructs in affective domain of mathematics education

The framework assumes that pre-service teachers’ five theoretic constructs: beliefs (about mathematics and its teaching and teaching context); mathematics self-efficacy; mathematics self-concept (which includes attribution); attitudes towards mathematics education; and learning approach are interrelated. The affective constructs (presage variables) influence the learning process variables of surface approach, deep approach and achieving approach. These process variables subsequently influence the product variable of mathematics achievement. It is argued that these factors form a complex network that brings about changes in mathematics achievement. Hence the focus of the research framework is the establishment of the causal relationships between these affective variables in the area of mathematics education using structural equation modeling for the pre-service primary teachers in Hong Kong.

4. Design and method

In theory, a structural equation model contains both a measurement and structural regression component. The measurement component of the structural equation model consists of a factor model with multiple indicators for the latent variable constructs while the regression component of the structural equation model allows the regression of the exogenous latent variables on endogenous latent variables. In a structural equation model the measurement and the regression components are estimated simultaneously. SEM is based on multiple indicators and appears to provide a better alternative for assessing different aspects of stability by examining different patterns of factor structure invariance under each level of hierarchy of invariance. Thus SEM appears to be an appropriate research strategy for affective and cognitive characteristics and worthy of further delineation. The pilot study and the literature indicated that mathematics-teaching self-efficacy could mediate from mathematics beliefs, mathematics teaching and attitudes and mathematics self-concept to learning approaches that in turn influence mathematics and mathematics-education achievement. The aim of the main study is to gather data on sufficient number of pre-service teachers to investigate the relationships among the affective and learning-approach characteristics and confirm the mediation role of mathematics-teaching self-efficacy. The instruments, procedure and analysis are described in the following sections.
5. **Instrument**

The instrument consisted of a questionnaire and the grades in the first semester mathematics and mathematics-education courses (Introduction to Mathematics and Primary Mathematics Curriculum and Teaching). The questionnaire contained the sections on personal background and factors influencing pre-service teachers’ feelings and confidence as in the pilot study, plus seven instruments to measure the seven latent variables respectively: mathematics beliefs, mathematics self-concept, mathematics-teaching attitudes, mathematics-teaching self-efficacy, surface learning approach, deep learning approach and achieving learning approach. The eighth latent variable (i.e., mathematics and mathematics-education achievements) was measured through the grades in the first semester mathematics and mathematics-education courses. Each latent variable was measured through observed indicators in terms of items in the questionnaire. All the instruments were designed using Likert scale. The ordered categories are simply scored with successive integers and a pre-service teacher’s response is taken as the sum of the scores of all statements of the instruments. According to Likert (1932), this approach could be popular because it is simple and it focuses directly on a person’s attitudes.

The measurement of the variables under this study was briefly described as follows.

First, the mathematics beliefs for pre-service teacher was measured by the 15 item-sub-scale of the “Mathematics as a process” instrument used in the second international mathematics study (Burstein, 1993). The reliability coefficient alpha of this instrument in this study reported by Burstein (1993) was .77. This scale measures the extent of one’s beliefs about the nature of mathematics. The pre-service teachers were required to show to what extent they agreed or disagreed with each of the statements according to a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree).

Second, mathematics self-concept was measured by eight items in the “Self-concept about mathematics” that is based on Marsh’s (1990) model of academic self-concept specifically designed for college and university samples. The reliability of the mathematics self-concept scale was reliable as the reported coefficient alpha was .88. In this study pre-service teachers were required to show to what extent they agreed or disagreed with each of the statements according to a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree).

Third, mathematics attitudes towards teaching were measured by nine items in the “Attitudes towards mathematics teaching” scale developed by Relich, Way and Martin (1994) who modified Fennema and Sherman’s (1976) instrument of attitudes towards mathematics. This sub-scale measured pre-service teachers’ attitudes towards teaching mathematics. The reliability estimates of attitudes towards teaching mathematics were reliable as the reported coefficient alpha was .92. The items required Likert scale responses on an eight-point continuum from 1 (definitely false) to 8 (definitely true).

Fourth, mathematics teaching self-efficacy was measured by twenty-three items in “Mathematics teacher efficacy belief instrument” (MTEBI) developed by Enochs (1996). The instrument consisted of two subscales: Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE). PMTE is a subscale that describes a personal belief in one’s ability to cope with teaching mathematics or a belief, in the case of pre-service teachers, that they are be able to effectively teach mathematics in practice. MTOE is a subscale to describe a belief that effective teaching of mathematics will result in effective learning. It refers to a belief that the teacher can overcome extrinsic factors like family, interest etc. The reliability coefficient alpha of PMTE was .86 while the reliability coefficient alpha of MTOE was .60 as reported by Grice (1998). Pre-service teachers were required to show to what extent they agreed or disagreed with each of the statements according to a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree).

Fifth, surface, deep and achieving learning approaches from the “Study Process Questionnaire” (SPQ) developed by Biggs (1987, 1992) was used to measure the learning approaches of pre-service teachers in this study. This instrument consisted of forty-two items on a five-point scale and was divided into six sub-scales: surface motive, surface strategy, deep motive, deep strategy, achieving motive and achieving strategy. Biggs (1992) reported that the reliability coefficient alpha of the SPQ were .59 for surface motive, .62 for deep motive, .77 for achieving motive, .61 for surface strategy, .68 for deep strategy and .74 for achieving strategy. Moreover SPQ has been validated in Hong Kong since 1992. In addition the norms of SPQ were provided for different groupings of academic departments separately for first year, second year and higher year university students at University of Hong Kong. In this study pre-service teachers were required to show to what extent they agreed or disagreed with each of the statements according to a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree).

Finally, mathematics and mathematics education achievement were respectively measured by the results of two mathematics-oriented courses for the pre-service teachers in Hong Kong Institute of Education. One of the courses was a methods course on primary mathematics curriculum and teaching while the other course focused on academic study related to concepts in mathematics. The grades were obtained through the student information
system at The Hong Kong Institute of Education.

6. Procedure

410 pre-service teachers completed the questionnaire that measured their mathematics beliefs, mathematics self-concept, mathematics teaching attitudes, mathematics teaching self-efficacy, learning approaches and mathematics and mathematics education achievement. Pre-service teachers were first asked to complete the beliefs statements about mathematics and mathematics self-concept. Then attitudes towards mathematics teaching, mathematics teaching self-efficacy scale and learning approaches scales were completed. All instruments were administered at the end of class period to ensure the high rate of response. The pre-service teachers completed and returned the questionnaire during the class. In addition, students’ grades obtained in Introduction to Mathematics and Primary Mathematics Curriculum & Teaching were retrieved from the Institute’s Student Information System at the end of the semester.

7. Analysis

The quantitative data of the 410 respondents were obtained from a survey questionnaire. Descriptive statistics by SPSS for Windows 10.0 produced the means and standard errors for the scales of constructs, mathematics beliefs, mathematics self-concept, mathematics teaching attitudes, mathematics teaching self-efficacy, surface approach, deep approach, achieving approach and mathematics education achievement respectively. After performing descriptive statistical analysis, reliability analysis was carried out for all the scales of the instruments. The reliability was measured by the internal consistency coefficient and assessed by calculating the coefficient alpha. Finally structural equation procedures were carried out to investigate the relationships among the latent constructs and indicator variables.

Constructing a model

The model constructed in this study is by a well-known framework of Jöreskog & Sörbom (1993) by three matrix equations. The first equation is as follows:

$$\eta = \beta \eta + \Gamma \xi + \zeta$$

where $\eta$ is a vector of endogenous latent constructs such as mathematics teaching self-efficacy and mathematics and mathematics education achievement, $\beta$ and $\Gamma$ are matrices of structural coefficients, $\xi$ is a vector of exogenous constructs and the covariance matrix among those exogenous constructs constitutes $\Phi$. The last term $\zeta$ is a vector of “errors” in the conceptual model and the covariance matrix among these errors constitutes $\Psi$. The second equation is:

$$y = \lambda_y \eta + \varepsilon$$

where $y$ is a vector of observed endogenous indicators such as grades in the mathematics methods course and the mathematics content course, $\lambda_y$ is a matrix of structural coefficients for $y$, $\varepsilon$ is a vector of “errors” in the measurement model and the covariance matrix among these errors constitutes $\Theta_\varepsilon$. The third equation is:

$$x = \lambda_x \xi + \delta$$

where $x$ is a vector of observed exogenous indicators such as mathematics beliefs and mathematics teaching beliefs, $\lambda_x$ is a matrix of structural coefficients for $x$, $\xi$ is a vector of exogenous latent constructs such as mathematics beliefs, mathematics self-concept and mathematics attitudes, $\delta$ is a vector of “errors” in the measurement model and the covariance matrix among those exogenous constructs constitutes $\Theta_\delta$.

The above three matrix equations were used in the actual data analysis stage and can be converted into diagrammatic representation.

Estimating parameters of the model from data

The structural equation model developed in the study consists of a measurement model and a structural model where the parameters are estimated from data collected. The purpose of a measurement model is to describe how well the observed indicators serve as a measurement instrument for the latent variables. Measurement accuracy is important to measure the latent characteristics of affective constructs and learning approaches. The measurement models in this study were formulated by confirmatory factor analysis procedures. The estimation procedures define the above three equations and eight matrices.

Testing the goodness-of-fit of the model

LISREL 8.30 provides a number of goodness-of-fit measures to judge how well a proposed model fits the data.
obtained in the study: goodness-of-fit index (GFI), root mean square error of approximation (RMSEA), adjusted goodness-of-fit index (AGFI), comparative fit index (CFI) and chi-square index. A value of .90 or greater in GFI, AGFI and CFI is commonly recommended in the judgment of the proposed model fits the data. However Hart, Conn, Carter and Wearing (1993) also pointed out that measures of goodness-of-fit such as GFI, AGFI and CFI are sensitive to the number of items in the model. They suggested that values of GFI, AGFI or CFI close to .80 or above could be considered acceptable for models having more than thirty items. Their recommendations are used in the analysis of the present research study.

The data from the questionnaire and the pre-service teachers’ grades were analysed using the statistical software SPSS for Windows and LISREL 8.30. Part I of the questionnaire contained personal data. The data from Parts II to V of the questionnaire were initially analysed for internal consistency reliability using SCALE from SPSS for Windows. Then, for the SEM, Parts II to V of the questionnaire and the grades were considered as seven latent variables: (1) mathematics beliefs, (2) mathematics self-concept, (3) mathematics-teaching attitudes, (4) mathematics teaching self-efficacy, (5) surface learning approach, (6) deep learning approach and (7) achievements in mathematics and mathematics-education.

In this study, circles represent the latent variables and the observed indicator variables are represented by in rectangular boxes. For the structural equation modelling, the affective and learning-approach characteristics and the achievements are treated as latent variables. Although as latent variables cannot be directly measured, the affective characteristics, learning-approach characteristics and achievement can be measured indirectly through the related observed indicator variables. As a result a clear picture of the relationships of the affective characteristics, learning approaches, mathematics and mathematics education achievements can be found using the SEM technique.

8. Results

Initial structural equation model
The strategy for model building is to add, based on the pilot study and the review of the literature, the possible paths between constructs and to progressively delete the paths that are statistically insignificant or contradicted by the literature. Following the SEM procedures an initial model is formed by LISREL8.30, by linking all paths from exogenous constructs to endogenous constructs. The linkages are based on the correlations found to exist among the affective constructs and learning approach and achievement in the literature. The following sections only report the statistically significant results of the structural equation modelling (SEM) in the main study. The next chapter is devoted to a detailed discussion of the results including the process of SEM.

As shown in Figure 2 in the initial model there are direct paths from mathematics beliefs, mathematics self-concept and mathematics teaching attitudes to mathematics teaching self-efficacy, where mathematics teaching self-efficacy act as a mediator of these three affective constructs to a surface approach and a deep approach that subsequently influence achievement. The initial model also shows the standardised path coefficients for the model of affective and learning-approach characteristics. For example there is a direct path leading from mathematics self-concept to achievement with path coefficient of .46. There is also a direct path leading from mathematics self-concept to achievement with path coefficient of .46. Moreover there is a positive path coefficient of .16 leading from the deep approach to achievement and a negative path with path coefficient of -.17 leading from the surface approach to achievement. The size and the direction of influence are consistent with the literature (Drew & Watkins, 1998) and the results of the pilot study.

However it is interesting to find that the path coefficient from mathematics teaching self-efficacy to achievement is -.37 and the path coefficient from mathematics concept to mathematics teaching self-efficacy is -.46. These imply that mathematics teaching self-efficacy exerts a significant and negative influence on achievement. However these figures seem to contradict the hypotheses of a positive relationship between mathematics teaching self-efficacy and achievement. Similarly mathematics self-concept is also found to have a negative influence on mathematics teaching self-efficacy. However these estimates of path coefficients are also contradictory to most findings in the literature such as Bandura (1993, 1997) and Pajare’s (1996) findings. Therefore although the initial model is convergent in the sense that the LISREL 8.30 program can produce a statistically feasible solution, some path coefficients were not consistent with the findings in the literature. As a result these paths had to be removed and a competing model was introduced in order to achieve a more reasonable model consistent with the literature.
Figure 2  An initial model of affective and cognitive characteristics

The data for this initial model were found to fit the model with goodness-of-fit index (GFI) of .94, adjusted goodness-of-fit index (ACFI) of .93 and comparative-fit-index (CFI) of .93 and root mean square error of approximation (RMSEA) less than .05. It should be noted that a goodness-of-index exceeding .90 is considered to be statistically acceptable (Schumacker & Lomax, 1996). As a result the initial model was statistically acceptable. However owing to the existence of some insignificant paths, the initial model was improved by deleting the insignificant paths. In fact in the initial model, some paths indicated the opposite direction between constructs compared to findings in the literature. For example mathematics teaching self-efficacy is negatively influenced by mathematics self-concept and mathematics teaching attitudes is positively related to a surface approach to learning which are contradictory to the literature (Abu-Hilal, 2000; Drew & Watkins, 1998; Pajares, 1996). Hence the initial model can be treated as a starting point of investigation of the relationships among affective and learning-approach constructs in the study.

The competing model
After having removed the paths that were unacceptable in practice, a new model has been proposed with improved acceptability. However as explained in the design of SEM, models should be over-identified so that a lot of feasible models can be proposed from which the best model can be selected to be the final model. In this way the final model should be consistent with substantial theory in the literature. Hence a series of feasible models termed as “competing models” were evaluated.
According to parsimony principle a model should be as simple as possible. In practice there are always some paths that can be removed. However some paths have to remain either due to technical constraints of SEM or substantial theory in the literature. In competing model, mathematics teaching self-efficacy influences achievement through a deep approach and a surface approach to learning. It is consistent with the literature that mathematics teaching self-efficacy has a strong positive impact (with path coefficient .53) on a deep approach and weak negative impact (with path coefficient -.13) on a surface approach to learning (Drew & Watkins, 1998). Moreover the model also indicates that mathematics belief exerts a positive impact on deep approach to learning. Hence the model claims that pre-service teachers’ beliefs about mathematics influence their approach to learning and the more the positive beliefs they hold the higher the probability that they would choose a deep approach to learning.

The final model
The final model should satisfy the following criteria: first, it must be consistent with substantial theory in the literature; second, the goodness-of-fit should be improved upon compared with the competing models; third, the final model should be based on and consistent with the results in the pilot study. The following model satisfies the above criteria and thus it was selected as the final model. The model confirms that affective and learning-approach constructs are interrelated and the following associated hypotheses hold except H3.

H1: There is a statistically significant positive influence of mathematics beliefs on mathematics teaching self-efficacy.
H2: There is a statistically significant positive influence of mathematics self-concept on mathematics teaching self-efficacy.
H3: There is statistically significant positive influence of mathematics teaching attitudes on mathematics teaching self-efficacy.
H4: There is a statistically significant negative influence of mathematics self-concept on surface approach.
H5: There is a statistically significant negative influence of mathematics teaching self-efficacy on surface approach.
H6: There is a statistically significant positive influence of mathematics teaching self-efficacy on deep approach.
H7: There is a statistically significant positive direct influence of mathematics teaching self-efficacy on achievement.
H8: There is a statistically significant negative influence of surface approach on achievement.
H9: There is a statistically significant positive influence of deep approach on achievement.

A significant relationship between two variables can be investigated further and the influence of one variable on another variable can be quantified with the path coefficients. The significance is based on the existence of correlations between the variables and the path coefficients that are greater than .10 and the p-value are less than .05.
As shown in Figure 4, the model claims that a pre-service teacher with a high self-concept exerts a negative impact on a surface approach to learning. Moreover the model asserts that the higher the pre-service teachers’ mathematics self-concept the higher would be their mathematics teaching self-efficacy as there is a strong positively direct effect of strength of .83 from mathematics self-concept on mathematics teaching self-efficacy. Furthermore the model also indicates that the pre-service teachers with positive beliefs about mathematics tend to have high mathematics teaching self-efficacy as the path coefficient from mathematics beliefs to mathematics teaching self-efficacy is positive to the magnitude strength of .28. Moreover the model claims that mathematics teaching self-efficacy directly influences achievement, as there is also a direct path of strength .33 linking mathematics teaching self-efficacy and achievement. However the model also claims that mathematics teaching self-efficacy indirectly influences achievement through deep approach and surface approach to learning because mathematics teaching self-efficacy exerts a positive impact of strength of .60 on deep approach and .26 on surface approach to learning. Hence it claims that pre-service teachers’ mathematics teaching self-efficacy influences their approach to learning and the higher the efficacy they held, the higher the likelihood they would choose a deep approach to learning. This result informs practice that teacher-training educators should put more emphasis on enhancing students’ self-efficacy and encouraging deep approaches to learning.

Statistically all the above models are feasible as the LISREL program converges with all parameter estimates falling into the acceptable. However only the final model fits the data well in terms of the parsimony principle, goodness-of-fit indices and literature support. The result of correlations among constructs, the direct and indirect effects of influences of independent variables on dependent variables and the goodness-of-fit indices for the final model are reported in the following section.

Correlations among affective and cognitive constructs for final model

Table 1 shows the correlations among for the latent exogenous and endogenous constructs computed by LISREL 8.30. The exogenous constructs include mathematics belief, mathematics self-concept, mathematics teaching attitudes while the endogenous constructs include mathematics teaching self-efficacy, surface approach, deep approach and achievement in mathematics and mathematics education. Statistics literature has indicated that the usual Pearson correlation of coefficient is unable to measure the relationships between latent variables. Moreover the Pearson correlation coefficients are suitable for the interval scale data only. However in the present study, the data type is for Likert-type scaled ordinal data. Hence according to Schumacker and Lomax (1996) polychoric correlations are computed because they are designed to evaluate the relationships between ordinal variables in the present study.

It can be seen that in Table 1, mathematics teaching self-efficacy is highly positively correlated with mathematics beliefs \( (r = .43) \), mathematics self-concept \( (r = .82) \) and mathematics teaching attitudes \( (r = .70) \). On the other hand it is negatively related to surface approach \( (r = .02) \) but positively correlated with deep approach \( (r = .60) \) and positively correlated with achievement \( (r = .41) \). Hence the results show that mathematics teaching self-efficacy takes the mediator role with the constructs under the study. It is these correlations that lead to the influences of the affective constructs on leaning approaches and achievement in mathematics and mathematics education.
Table 1  Polychoric Correlation Among Exogenous and Endogenous Constructs

<table>
<thead>
<tr>
<th>Latent constructs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Mathematics beliefs</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Mathematics self-concept</td>
<td>.21*</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Maths teaching attitudes</td>
<td>.70*</td>
<td>-.11</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Maths teaching self-efficacy</td>
<td>.43*</td>
<td>.82*</td>
<td>.70*</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Surface approach</td>
<td>.04</td>
<td>-.13</td>
<td>-.11</td>
<td>-.02</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Deep approach</td>
<td>.26*</td>
<td>.49*</td>
<td>.42*</td>
<td>.60*</td>
<td>-.01</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7. Achievement</td>
<td>.18</td>
<td>.34*</td>
<td>.29*</td>
<td>.41*</td>
<td>-.12</td>
<td>.33*</td>
<td>1</td>
</tr>
</tbody>
</table>

p < .05

Based on the results of the Polychoric Correlation among Exogenous and Endogenous Constructs, standardised solutions for the effect of exogenous constructs (independent variables) on endogenous constructs (dependent variables) can be made by LISREL to build the final model. It reflects a more complete picture of how mathematics beliefs, mathematics self-concept and mathematics teaching attitudes influence mathematics teaching self-efficacy that in turn influence the learning approach and subsequently achievement. It confirms that mathematics teaching self-efficacy has a significant, positive and direct contribution to surface approach (.26), deep approach (.60) and achievement (.33). It is noted that efficacy has also a positive indirect effect (.08) on achievement through surface and deep approach to learning and as a result has a total effect of .41 on achievement. On the other hand surface approach has a negligible negative effect (-.01) on achievement which implies that there may still be some pre-service teachers using surface approaches such as rote-learning strategy in their study. It also confirms that students with deep approach to learning would tend to have better achievement. In particular mathematics self-concept has a positive and indirect contribution to surface approach (.21), deep approach (.50) and achievement (.34). Thus mathematics self-concept indirectly influences achievement through mathematics teaching self-efficacy as shown in the final model.

The goodness-of-fit index for the final model is presented in Table 2. The chi-square value is 69.9 with degree of freedom 287 indicating that the hypothesis that the fitted residuals are different from zero can be rejected. The results also indicate that the data fit the final model very well and this is reflected by the root mean square error approximation less than .05. The goodness-of-fit index, adjusted goodness-of-fit index and comparative goodness-of-fit indices exceed or close to .90 provide further support for the final model and its explanation of the relationships between affective, learning-approach characteristics and achievement.

Table 2  Goodness of Fit Indices for the Final Model

<table>
<thead>
<tr>
<th>Measures of goodness-of-fit</th>
<th>Goodness-of-fit values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-square (Satorra-Bentler Scaled) with degree of freedom</td>
<td>69.9</td>
</tr>
<tr>
<td>Root mean square error of approximation (RMSEA)</td>
<td>.00</td>
</tr>
<tr>
<td>Goodness-of-fit index (GFI)</td>
<td>.91</td>
</tr>
<tr>
<td>Adjusted goodness-of-fit index (AGFI)</td>
<td>.90</td>
</tr>
<tr>
<td>Comparative-fit-index (CFI)</td>
<td>.89</td>
</tr>
</tbody>
</table>

9. Concluding remark

SEM starts from a conceptually derived model specifying the relationships among a set of variables. Theory based on the literature and pilot studies provides the centrepiece for structural equation methodologies designed for use with substantive interests in understanding complex patterns of interrelationships among variables. Cause and effect in the structural equation model are totally dependent on the way in which the relationships are specified and the results at best indicate to plausibility about the relationships. However the strength and conviction with which the researcher can assume causation between two variables lies not in the analytical methods chosen but in the theoretical justification provided to support the analysis. In conclusion, preservice teachers’ mathematics self-concept, mathematics teaching self-efficacy, beliefs and attitudes towards mathematics and mathematics education, influence cognitive activities in the learning process and subsequently mathematics achievement outcomes. The result also shows that mathematics teaching self-efficacy acts as the mediator of affective characteristics, learning approaches and mathematics achievement. The finding proves useful to educators in the research areas of affective domain of mathematics education.

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