

# Difficult Problems Having Easy Solutions

Oscar Bolina\*

Department of Mathematics  
University of California, Davis  
Davis, CA 95616-8633 USA

## Abstract

*We discuss how a class of difficult kinematic problems can play an important role in an introductory course in stimulating students' reasoning on more complex physical situations. The problems presented here have an elementary analysis once certain symmetry features of the motion are revealed. We also explore some unexpected directions these problems lead us.*

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There is a class of kinematical problems whose general analysis lies beyond the level of undergraduate students but which nonetheless can play an important role in introductory courses. This happens because the problems involve certain symmetry features that allow for an easy solution once the symmetry is revealed.

What makes these problems worth mentioning here is that they are particularly useful in stimulating students' reasoning on richer physical situations.

Consider this typical example.

*A boat crosses a river of width  $l$  with velocity of constant magnitude  $u$  always aimed toward a point  $S$  on the opposite shore directly across its starting position. If the river also runs with uniform velocity  $u$ , how far downstream from  $S$  does the boat reach the opposite shore?*

Fig. 1 depicts the situation when the boat is at point  $B$  in its path  $BV$  toward the opposite shore. Its velocity vector along  $BS$  makes an angle  $\theta$  with  $RS$ . The river velocity is represented along  $TB$ .

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\*Supported by FAPESP under grant 97/14430-2. **E-mail:** bolina@math.ucdavis.edu

The (*easy*) solution comes from the observation that in a time interval  $\Delta t$  the distance  $BS$  of the boat to the point  $S$  on the opposite shore diminishes by  $(u - u \cos \theta)\Delta t$ , while, at the same time, its distance  $TB$  to  $T$  increases by the same amount, thus keeping the sum  $BS + TB$  constant throughout the trip along  $RV$ . Since it is clear that at the starting position  $BS = l$  and  $TB = 0$ , while at the final position  $BS = BT = d$ , which is the distance downstream, we conclude that  $d = l/2$  (See [1] for further examples).

The importance of this example lies in its far-reaching scope. For instance, many problems in central force motion (planetary motion) can be formulated in similar terms, and the same kind of analysis applies as well. Note the similarity we obtain if we turn the boat of the example into a comet describing a parabolic orbit around a sun  $S$  in the focus, as in Fig. 2. In this case, it is the component velocities along  $TB$  and normal to  $SB$  that are constants throughout the orbit (Compare with Fig. 1). As a result we learn that now the sum of the distances  $SB + BP$  is constant along the comet's orbit. This fact defines the parabolic orbit and, with  $SB = r$  and  $BP = r \cos \theta$ , yields its equation in polar coordinates as  $r(1 + \cos \theta) = \text{const.}$

We can go still further to realize that a parabolic orbit is unstable under the influence of possible nearby planets, and may be easily converted into an ellipse or an hyperbola, according to whether the velocity of the comet decreases or increases as a result of the planetary perturbation [2]. When this happens, the pattern of constant velocity components,  $u$  parallel to a fixed direction (as  $BT$ ), and  $v$  normal to the radius vector  $BS$ , is preserved in the new orbit. The difference is that these two velocities will not be equal to each other any more, and the ratio  $u/v$  will determine the shape of the orbit (though not its size), being an ellipse when  $u/v < 1$ , and a hyperbola when  $u/v > 1$ . We leave it to the reader to explore other features of planetary motion opened up by our example.

## References

- [1] A.S. Ramsey, *Dynamics*, Cambridge University Press (1951) chapter 3.
- [2] *The New Space Encyclopaedia*, E. P. Dutton & Co., INC. N.Y. (1960) Entry: Comet.

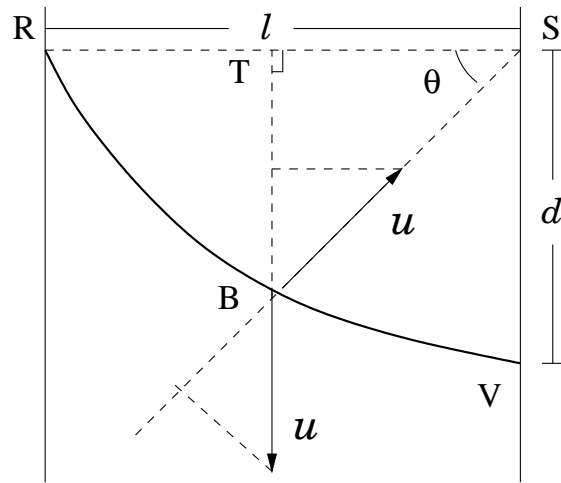


Figure 1: In this motion  $SB + TB$  is constant.

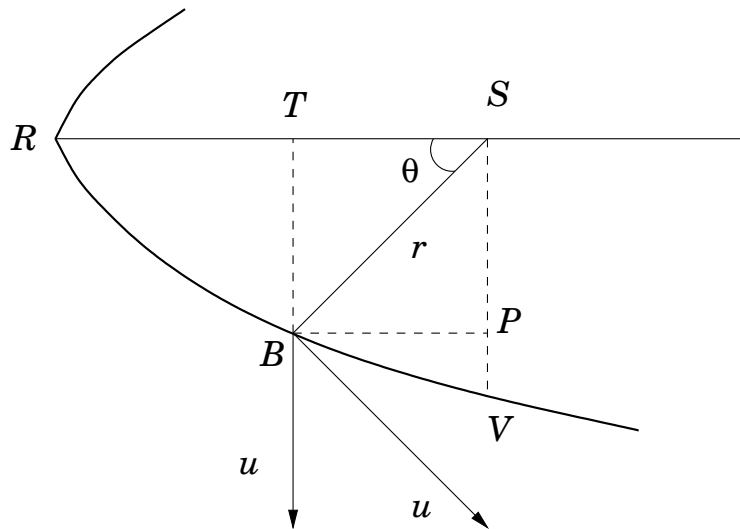


Figure 2: For the parabola,  $SB + BP$  is constant.