A Two-Sided Matching Model of Monitored Finance☆

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Abstract

We develop an incentive contracting model of firm formation. Entrepreneurs of private equity firms who differ in net worth are required to borrow from institutional investors in order to finance start up projects. Investors, who differ in monitoring efficiency, may choose to monitor their borrowers at a cost. Non-verifiability of both entrepreneurial effort and monitoring gives rise to double-sided moral hazard problems, and leads to market failure. Individuals with high monitoring efficiency invest in low-net worth firms following a negatively assortative matching pattern since monitoring efficiency and net worth are strategic substitutes in mitigating incentive problems. The equilibrium debt obligation of the entrepreneur and expected firm value are in general non-monotone with respect to net worth. We solve the model numerically in order to analyze the effects of changes in the distributions of monitoring efficiency and net worth on the equilibrium loan contracts.

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1. Introduction

Incentive contracts may be quite different in a market with many heterogenous investors and entrepreneurs as opposed to the contracts for an isolated investor-entrepreneur pair. In the equilibrium of a market, individual contracts are influenced by the two-sided heterogeneity via investor-entrepreneur assignment. In this paper, we aim at developing a simple two-sided matching model of incentive contracting between lenders and borrowers. Entrepreneurs who differ in net worth and investors who differ in monitoring efficiency are matched into pairs in order to accomplish projects of fixed size. Thus, in the

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equilibrium of the market, both the sorting and the payoff that accrues to each individual are determined endogenously.

The principal-agent approach to credit markets has emphasized the importance of financial market frictions originating from informational asymmetries when lenders (principals) cannot costlessly acquire information about the opportunities or actions of borrowers (agents). Asymmetric information thus generate optimal financial arrangements involving agency costs which often induce a wedge between the cost of external finance and the opportunity cost of internal finance, and increase the cost of credit faced by borrowers. As noted by some authors (e.g. Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999), the cost of the external finance premium depends negatively on a borrower's balance sheet position, i.e., the ratio of net worth to liabilities. If borrower's stake in the outcome of an investment project increases due a stronger balance sheet position, the agency cost implied by the optimal contract falls since the incentives of the borrower to deviate from the lender's interests decline. In other words, borrower's net worth plays a crucial role in ameliorating financial market frictions.

The role of monitoring in the creation of firm value in a credit relationship is also well-recognized. Monitoring by lenders helps ameliorate the entrepreneurial moral hazard problems in private equity firms, and hence more able monitors are often more valuable. In an investor-entrepreneur relationship, informed capital is assumed to posses greater monitoring ability than the outside investors (see Hölmstrom and Tirole, 1997; Repullo and Suárez, 2000). Differences in monitoring ability also stem from other sources. In this paper we assume that differences in monitoring ability stem from differences in ability to securitize loans. A typical role of banks as financial intermediaries is "liquidity transformation", i.e., the funding of illiquid loans through liquid financial instruments. Since the end of the 1980s, the financial industry saw a fundamental change in liquidity transformation through the implementation of *asset-backed securities* (ABS).[1] It is often argued that a greater ability to securitize loans, e.g. issuing ABS is associated with a lower ability of the investors to monitor their borrowers as such instruments allow the issuers of the securities to diversify loan risks at lower costs (e.g. Pennacchi, 1988; Gorton and Pennacchi, 1995).

The main objective of the present paper is to offer a unified framework to analyze the (general) equilibrium effects of changes in entrepreneurial net worth and monitoring ability of investors when entrepreneurs and investors have incentives to form firms or partnerships through endogenous matching. In particular, we consider a market where heterogeneous entrepreneurs are assigned to heterogenous institutional investors. Entrepreneurs, heterogenous with respect to net worth, lack sufficient fund, and hence require to rely on institutional investors to fund their projects. Non-verifiability of entrepreneurial effort along with limited liability give rise to a moral hazard problem in effort choice. Investors, heterogeneous with respect to monitoring ability, may choose to monitor their borrowers in order to mitigate the entrepreneurial moral hazard problem. Since monitoring activities are costly and chosen after the firms are

[1]The basic idea behind such financial instruments is to write a new claim linked to a loan or a pool of loans, and sell these claims (or security) in the capital markets. In this manner, loans become more liquid due to the securitization process, even though real projects remain illiquid. Pennacchi (1988); Gorton and Pennacchi (1995); Gorton and Metrick (2013) offer detailed discussions on loan securitization. The decision of a financial intermediary to issue an ABS depends on a number of factors, such as the cost of internal, i.e., deposit liabilities and external funds. As a result of this financial innovation, loan securitization has become quantitatively significant in U.S. capital markets, at least prior to the financial crisis of 2007-2008. For example, *loan sales* (a type of ABS) have grown from \$8 billion in 1991 to \$238.6 billion by 2006 (see Drucker and Puri, 2009). Collateralized debt obligations or CDOs (another type of ABS) outstanding amounted to \$1.1 trillion as of 2005, of which 50% of their collateral was comprised of loans (see Lucas et al., 2006).

formed, there is an additional incentive problem in monitoring. As we have discussed earlier, differences in monitoring ability may stem from various fundamentals of the credit market, but for the purpose of the current paper we would assume that a higher ability to securitize loans translates into a lower monitoring efficiency because of the moral hazard problem that may exist between the issuers of the securities (investors in our model) and their buyers. Thus in the present context, partnership formation is subject to a double-sided moral hazard problem which implies the failure in implementing the efficient market outcomes.

Since both net worth and monitoring ability influence the performance of a firm in a significant way, competition for ‘good quality’ borrowers and lenders naturally emerges in such markets. We show that high monitoring efficiency is more effective at the margin in firms run by entrepreneurs with low net worth, i.e., high-ability monitors enjoy comparative advantages over their low-ability counterparts in low-net worth firms, and hence highly efficient monitors are matched with entrepreneurs with low net worth following a negatively assortative matching pattern. In other words, monitoring efficiency and net worth are strategic substitutes in ameliorating the double-sided moral hazard problems. Because the partnerships under incentive problems are heterogeneous, the equilibrium debt obligations of the borrowers are in general non-monotone with respect to the net worth. There is a matching effect which has a negative impact on debt obligation, whereas there is an outside option effect that influences it favorably. Depending on which of the two countervailing effects is stronger the equilibrium debt obligation may increase or decrease with respect to entrepreneurial net worth. Similar non-monotone relationships hold for the equilibrium entrepreneurial effort and expected firm value.

A negative assortative matching in equilibrium implies that less efficient monitors (due to a high ability to securitize a loan) have incentives to self-select them into firms with high net worth. This prediction is consistent with some evidence from securitization in the corporate loan market. In particular, Drucker and Puri (2009) find that firms’ assets from securitized loans are 1.7 times larger than those from loans not securitized for a sample of mostly medium to large U.S. public firms.[2] In addition, Drucker and Puri (2009) find that securitized loans are mostly term loans rather than credit lines, as opposed to about 73% of non-securitized loans are credit lines, which presumably require more intensive monitoring because the borrowers have incentives to ask for a credit line when the performance has been poor.

As it is apparent from the previous discussion that in credit markets investors and entrepreneurs have incentives to form partnerships through endogenous matching, the contribution of the present paper to the literature on partnership formation is two fold. First, when the individuals may seek for alternative partners, i.e., the matching is endogenous, the model helps endogenize the outside option of each borrower as opposed to the standard agency theory where a lender-borrower relationship is treated in isolation, and the outside option of a borrower is exogenously given. The principal-agent models (e.g. Besanko and Kanatas, 1993) are amenable to determine the optimal incentive structure in an organization in the sense that such models predicts the way the aggregate surplus must be divided between the principal and the agent. A fixed outside option of the agent also pins down the payoff achievable by the principal. In a general equilibrium model such as ours, the endogenous outside option not only determines the structure of incentive pay, but also its level in each firm. We have also discussed earlier that the strategic

[2] Unfortunately, Drucker and Puri (2009) do not report statistics on firms’ net worth. However, we should expect a positive correlation between net worth and assets in their sample. On the other hand, the authors find that the average securitized loan size is 1.8 times larger than the average non-securitized loan. Benmelech et al. (2012) report that the average securitized corporate loan is roughly $522 million, an amount that may presumably require a large net worth as collateral.

substitutability between monitoring efficiency and net worth induces negatively assortative matching in equilibrium. Such result or its variants are already established in the related literature (e.g. Sattinger, 1979; Legros et al., 2010; Dam, 2011). In this paper, we exploit this particular property of equilibrium matching to show that because individuals have incentives to form endogenous partnerships, contract terms may be non-monotone with respect to entrepreneurial net worth, which would not be predicted by the standard agency theory. For example, the model of Repullo and Suárez (2000) asserts that the equilibrium debt obligation is decreasing with respect to net worth.

Second, we contribute to the literature on partnership formation (e.g. Farrell and Sctochmer, 1988) which argues that economic agents who differ in abilities will form partnerships by equally sharing the surplus if abilities are complementary. In the context of financing of small businesses, formation of partnerships are often subject to several market imperfections, among which the informational constraints play an important role. When partnerships are subject to moral hazard problems, the incentive contract for a particular match gives rise to a non-linear Pareto frontier implying that the match-surplus cannot be transferred between the principal and the agent on a one-to-one basis, and an equal sharing of surplus cannot be implemented. Thus, substitutability rather than complementarity often explain why heterogeneous individuals may form partnerships and agree to share the match output according to endogenously determined sharing rules. Under imperfect transferability of surplus the Pareto frontier associated with a particular pair is non linear which also makes it impossible to solve the model analytically. In a numerical simulation of the model we show that changes in the distributions of monitoring ability and net worth have asymmetric effects on the entrepreneurs and investors.

The present paper also contributes to the recent literature on assignment models with incentive contracts motivated by the empirical works of endogenous matching (e.g. Ackerberg and Botticini, 2002; Chiappori and Salanié, 2003; Chen, 2013). To this end, we extend Sattinger's (1979) 'differential rents' model to an environment with moral hazard in the choice of effort and monitoring.[3] Some related works are worth mentioning. In a model of occupational choice with one-sided matching, Chakraborty and Citanna (2005) show that due to endogenous sorting effects, less wealth-constrained individuals choose to take up projects in which incentive problems are more important. In a model with endogenous matching between venture capitalists and firms, Sorensen (2007) finds evidence that more experienced VCs invest in late-stage start-up companies which are more likely to go public. Unlike the present paper, Sorensen's (2007) theoretical model does not analyze incentive problems by assuming that in each partnership the players divide surplus according to a fixed sharing rule. The work of von Lilienfeld-Toal and Mookherjee (2008) considers matching between lenders and borrowers, and analyzes the distributional impacts of a change in the personal bankruptcy law. Legros, Newman, and Pejsachowicz (2010) propose a sufficient condition, called the *generalized difference condition*, under which equilibrium allocations exhibit assortative matching when the two-sided matching induces a non-transferable utility (a concave Pareto frontier) game. These authors consider an example of partnership formation between principals and agents where the former monitors the latter to mitigate the effort incentive problem. They show that when abilities are important, high-ability principals match with high-ability agents, although a positive sorting may induce loss of efficiency. Chen (2013) estimates a model of matching between banks and firms to show that, because of endogenous matching, higher loan spreads are associated with banks with

[3]Sattinger (1979) considers the determination of wage and rental rates in a competitive equilibrium framework without incentive problems, and shows that a supermodular surplus function implies a positively assortative matching between machine size and worker quality.

greater monitoring ability, and more leveraged firms are charged higher interests. Unlike the current paper, the positive sorting in Chen's (2013) model is a possible explanation for the monotonicity of the loan rate with respect to the wealth of the firm. Finally, Macho-Stadler, Pérez-Castrillo, and Porteiro (2014) analyze coexistence of long- and short-term contracts in equilibria with endogenous sorting between heterogeneous firms and workers when contracts are subject to limited liability. In their model, sorting is non-monotone in the sense that firms with the best projects use short-term contracts to lure high-ability senior workers, firms with the least profitable projects use short-term contracts to save on the cost of hiring less experienced workers, whereas intermediate firms offer long-term contracts to incentivize their workers.

2. The model

2.1. Investor-entrepreneur partnerships

The economy, which lasts for three dates $t = 0, 1, 2$, consists of two classes of agents: a continuum $I = [0, 1]$ of heterogeneous risk-neutral investors or lenders, and a continuum $J = [0, 1]$ of heterogeneous risk-neutral entrepreneurs or borrowers. The sets I and J are endowed with Lebesgue measures.

Each entrepreneur with initial wealth $w \in \mathscr{W} = [0, 1]$, which is his 'type', owns a start-up project of fixed size 1. Therefore, w also represents the net worth of an entrepreneur.[4] At date 0 each entrepreneur $j \in J$ is assigned a net worth level $\omega(j) \in \mathscr{W}$ via the type assignment function $\omega : J \to \mathscr{W}$. We assume $\omega(\cdot)$ to be non-decreasing on J, and hence measurable. The type assignment function ω gives rise to the distribution of net worth $F(w)$ which is the fraction of entrepreneurs with net worth less than w. We assume without loss of generality that there is at least one entrepreneur associated with each level of net worth, i.e., the set $\{j \in J \mid w = \omega(j)\}$ is non empty for each $w \in \mathscr{W}$. This implies that the corresponding density of net worth $f(w) > 0$ for all $w \in \mathscr{W}$, i.e., $F(w)$ is strictly increasing on $\mathscr{W}$.

Since no entrepreneur has initial wealth greater than 1, they are required to borrow at least $1 - w$ from the market to start their projects. Each started project yields a stochastic but verifiable cash flow $\tilde{y} \in \{0, Q\}$ with $Q > 1$. The event of success, i.e., $\tilde{y} = Q$ occurs with probability θ, where θ is the non-verifiable effort exerted by the entrepreneur. We assume that all entrepreneurs have an identical increasing and quadratic effort cost function apiece, which is given by:

$$D(\theta) = \frac{\theta^2}{2c}, \qquad \text{with } \ c > 0.$$

Non-verifiability of entrepreneurial effort gives rise to moral hazard problem in effort choice, which can be mitigated by costly monitoring by the investor. Following Besanko and Kanatas (1993), we assume that the level of monitoring by a lender is positively correlated with the difference between *monitoring* and *non-monitoring* efforts. If the financier of a project monitors her borrower, then she can oblige the entrepreneur to exert a stipulated level of effort θ, which is referred to as the monitoring effort. On the other hand, let θ_0 denote the effort exerted by a borrower if he is not monitored, which we call the no-monitoring effort. The difference $\theta - \theta_0$ thus represents the level of monitoring by an investor. If more resources are spent in monitoring activities, higher would be the effort exerted relative to the non

[4]The net worth of an entrepreneur with initial wealth w and project size K is defined as w/K.

monitoring effort level. We assume that the cost incurred by an investor for monitoring is given by:

$$M(\theta-\theta_0)=\begin{cases}\frac{(\theta-\theta_0)^2}{2m} & \text{if } \theta>\theta_0,\\ 0 & \text{if } \theta\leq\theta_0.\end{cases}$$

The parameter $m\in\mathscr{A}=[0,c/2]$ measures the monitoring efficiency or ability of an investor, which is referred to as her 'type'. The higher the m, the greater is the monitoring ability as an investor with higher m entails lower cost for an additional unit of monitoring. We assume $m\leq c/2$ which puts a lower bound to the marginal cost of monitoring, and guarantees the second-order conditions. At date 0, each investor i is assigned a monitoring efficiency level $\mu(i)\in\mathscr{A}$ via the type assignment function $\mu:I\rightarrow\mathscr{A}$. Similar to the case with the entrepreneurs, we assume $\mu(\cdot)$ to be non-decreasing on I and the set $\{i\in I\,|\,m=\mu(i)\}$ to be non empty for each $m\in\mathscr{A}$ so that the distribution $G(m)$ of monitoring efficiency is strictly increasing on $\mathscr{A}$, i.e., the corresponding density of monitoring efficiency $g(m)>0$ for all $m\in\mathscr{A}$.

The project of an entrepreneur j can be started at $t=1$ only if a *partnership* or *firm* $(i,\,j)$ between the entrepreneur and some investor i is formed in which the investor agrees to invest $1-\omega(j)$. We treat the firm formation as a matching problem where an investor is assigned to an entrepreneur via a matching rule ξ. We also assume that an investor can invest in only one firm, and no entrepreneur may seek financing from more than one investor. Formally,

Definition 1 (Investor-entrepreneur matching) *A one-to-one matching is an assignment rule $\xi:I\cup J\rightarrow I\cup J$ such that* (a) $\xi(i)\in J\cup\{i\}$ *for each investor $i\in I$;* (b) $\xi(j)\in I\cup\{j\}$ *for each entrepreneur $j\in J$; and* (c) $i=\xi(j)$ *if and only if $j=\xi(i)$ for all $(i,\,j)\in I\times J$.*

Parts (a) and (b) of the above definition imply that either an individual is matched with another individual of the other side of the market, or she/he remains unmatched, whereas part (c) asserts the fact that investor-entrepreneur matching is one-to-one. The matching function ξ that assigns one investor to one entrepreneur gives rise to the following matching correspondence $\Lambda:\mathscr{W}\rightarrow 2^{\mathscr{A}}$ between types:

$$\Lambda(w)=\{\mu(i)\text{ for }i\in I\,|\text{ there is }j\in J,w=\omega(j)\text{ and }i=\xi(j)\}.$$

A *selection* of the matching correspondence Λ is a function $\lambda:\mathscr{W}\rightarrow\mathscr{A}$ which assigns to each net worth $w\in\mathscr{W}$ a monitoring ability $m=\lambda(w)\in\mathscr{A}$.

Definition 2 (Negatively assortative matching) *An investor-entrepreneur matching is negatively assortative (NAM) if the matching correspondence Λ is strongly decreasing, i.e., each selection λ of Λ is a strictly decreasing function on $\mathscr{W}$.*

A negatively assortative matching implies that if $w>w'$, $\lambda(w)\in\Lambda(w)$ and $\lambda(w')\in\Lambda(w')$, then $\lambda(w)<\lambda(w')$ for each selection λ of Λ. Given a partnership $(i,\,j)$, we call this *a firm of type* (m,w) if $m=\mu(i)$ and $w=\omega(j)$.

Finally at date 2, a loan contract associated with each firm $(i,\,j)$ is executed as follows. A loan contract for an arbitrary firm is a state-contingent interest payment $R(\tilde{y})$ for $\tilde{y}\in\{0,Q\}$ to the investor. In general a typical loan contract consists of three elements: the outside equity B, which is the amount lent by the investor, the inside equity E which is the participation of the entrepreneur toward the total project cost, and the state-contingent transfer $R(\tilde{y})$ to the investor. It is easy to show that full equity participation by both the investor and the entrepreneur, i.e., $B = 1 - w$ and $E = w$ is optimal.[5] Thus, lower net worth w corresponds to higher *leverage ratio* B/E and vice-versa. After the contract is signed, the entrepreneur/borrower exerts effort θ and the investor/lender decides on the monitoring level $\theta - \theta_0$. Then, the true value of the random variable $\tilde{y}$ is realized, and the corresponding agreed upon payments are made.

2.2. *The optimal loan contract in an arbitrary firm*

We assume that each firm is subject to the limited liability of the borrower. Thus, the optimal loan contract is a standard *debt contract* of the form $R(\tilde{y}) = \min\{R, 0\}$, i.e., the entrepreneur is able to meet his debt obligation R only if the project succeeds, i.e., $\tilde{y} = Q$.[6] Let $\gamma = (R, \theta, \theta - \theta_0)$ be the vector of debt obligation, effort and monitoring level. Consider now a given firm or partnership (i, j) of type (m, w), in short "a type (m, w) firm or partnership". For such a firm, the optimal debt obligation, entrepreneurial effort and monitoring levels solve the following maximization problem:

$$\max_{\{R,\theta,\theta_0\}} \quad V(R, \theta, \theta_0) := \frac{\theta R}{r_f} - (1-w) - \frac{(\theta - \theta_0)^2}{2m}$$

$$\text{subject to} \quad U(R, \theta) := \frac{\theta(Q-R)}{r_f} - w - \frac{\theta^2}{2c} \geq u, \tag{PC}$$

$$\theta_0 = \text{argmax}_{\hat{\theta}} \left\{ \frac{\hat{\theta}(Q-R)}{r_f} - w - \frac{\hat{\theta}^2}{2c} \right\} = \frac{c(Q-R)}{r_f} := \theta_0(R), \tag{ICE}$$

$$\theta = \text{argmax}_{\hat{\theta}} \left\{ \frac{\hat{\theta} R}{r_f} - (1-w) - \frac{(\hat{\theta} - \theta_0)^2}{2m} \right\} = \theta_0(R) + \frac{mR}{r_f} := \theta(R), \tag{ICI}$$

$$0 \leq R \leq Q, \tag{LL}$$

where $V(R, \theta, \theta_0)$ and $U(R, \theta)$ are the expected payoffs of the investor and the entrepreneur, respectively. The gross payoffs of the investor and the entrepreneur are the present values of their expected incomes discounted by the risk-free interest factor $r_f \geq 1$. Constraint (PC) is the borrower's participation constraint which asserts that a contract must guarantee at least his outside option $u \geq 0$. Constraints (ICE) and (ICI) are the Nash incentive compatibility constraints which imply that both the entrepreneur and the investor would choose effort and monitoring level optimally. Finally, the constraint (LL) is the limited liability constraint which guarantees non-negative incomes to the lender and the borrower, i.e., $R \geq 0$ and $Q - R \geq 0$, respectively. The following lemma, which will be used for the characterization of the market equilibrium in Section 3, describes the optimal loan contract when the borrower's participation constraint binds.[7]

Lemma 1 *Let $R^* := R(m, w, u)$ be the optimal debt obligation, $\theta^* := \theta(m, w, u)$ be the optimal entrepreneurial effort, $\theta^* - \theta_0^* := \theta(m, w, u) - \theta_0(m, w, u)$ be the optimal monitoring, and $P^* := P(m, w, u)$*

[5] Proof of this assertion is available upon request to the authors.

[6] Under double-sided moral hazard, a non-debt contract, e.g. "revenue-sharing" is in general optimal. But when the revenue of the project in the event of failure is zero, debt and share contracts are indistinguishable.

[7] In general, for low values of u the participation constraint of the borrower does not bind under limited liability, i.e., he may earn efficiency wage. We omit the analysis of this case since it will be shown that in the market equilibrium the participation constraint of each entrepreneur must be binding.

be the expected value of the firm for an arbitrary type (m, w) partnership when the entrepreneur's participation constraint binds under limited liability.

(a) *The optimal debt obligation is given by:*

$$c^2(Q-R^*)^2 - m^2R^{*2} = 2cr_f^2(w+u) \tag{1}$$

with $0 < R^* < Q$.

(b) *The optimal entrepreneurial effort and monitoring level are respectively given by:*

$$\theta^* = \theta(R^*) = \frac{c(Q-R^*)+mR^*}{r_f}, \tag{2}$$

$$\theta^* - \theta_0^* = \theta(R^*) - \theta_0(R^*) = \frac{mR^*}{r_f}. \tag{3}$$

(c) *The expected value of the firm is given by:*

$$P^* = P(R^*) = \frac{\theta(R^*)(Q-R^*)}{r_f} = \frac{c(Q-R^*)^2 + mR^*(Q-R^*)}{r_f^2}. \tag{4}$$

(d) *Finally, the maximum expected payoff for the investor is given by:*

$$\begin{aligned}\phi(m, w, u) &= \max_{\{R,\theta\}} \{V(R, \theta, \theta_0) \mid U(R, \theta) = u\} = V(R^*, \theta^*, \theta_0^*)\\ &= \frac{\theta(R^*)R^*}{r_f} - (1-w) - \frac{[\theta(R^*) - \theta_0(R^*)]^2}{2m}.\end{aligned} \tag{5}$$

We omit the proof of the above standard result. Since corresponding to the return $\tilde{y} = 0$ both the investor and the entrepreneur obtains no incomes, i.e., the limited liability constraints in this case are binding, the first-best effort, which maximizes the total net surplus of a match, cannot be implemented.[8] In the above lemma, equation (1) is simply the binding participation constraint of the entrepreneur after substituting for $\theta = \theta(R)$ evaluated at the optimum. Condition (2) is derived from the incentive compatibility constraint (ICI). The optimal monitoring level is determined from both the incentive compatibility constraints. The expected value of the firm is given by the expected present value of the entrepreneur's net income $Q - R$.[9] The optimal debt obligation R^* of the entrepreneur serves as an instrument to achieve the right balance between the entrepreneurial effort and monitoring incentives in the sense that higher R provides greater incentives to the investor to exert higher monitoring effort, but undermines the incentives for entrepreneurial effort.

[8] The first-best effort is given by:

$$\theta^{fb} = \text{argmax}_\theta \left\{S(\theta) := \frac{\theta Q}{r_f} - \frac{\theta^2}{2c} - 1\right\} = \min\left\{\frac{cQ}{r_f}, 1\right\}.$$

We assume $cQ/r_f < 1$ in order to have $\theta^{fb} < 1$, and $cQ^2/2r_f^2 > 1$ so that the project is viable at the first-best level, i.e., $S(\theta^{fb}) > 0$. Notice that a necessary condition for the above two inequalities to hold simultaneously is that $Q > 2r_f$.

[9] The firms under consideration are private equity firms. The expected value of such a firm is the analog of stock price of a publicly-traded company.

Finally, $\phi(m, w, u)$ is the Pareto frontier associated with a type (m, w) firm when the borrower obtains exactly his outside option u. Note that if two firms (i, j) and (i', j') with $i \neq i'$ and $j \neq j'$ are of the same type (m, w), then the optimal loan contract, entrepreneurial effort, monitoring level, expected firm value, and the Pareto frontier will be identical for both partnerships, which are described in Lemma 1. The following lemma states some useful comparative statics results.

Lemma 2 *In any type (m, w) partnership in which the entrepreneur receives his outside option u,*

(a) *optimal debt obligation $R(m, w, u)$ is monotonically decreasing in monitoring efficiency m, net worth w and the entrepreneur's outside option u;*

(b) *Optimal effort $\theta(m, w, u)$ is monotonically increasing in monitoring efficiency m, net worth w and the entrepreneur's outside option u. Optimal monitoring $\theta(m, w, u) - \theta_0(m, w, u)$, on the other hand, is monotonically increasing in monitoring efficiency m, and monotonically decreasing in net worth w and the entrepreneur's outside option u;*

(c) *Expected firm value $P(m, w, u)$ is monotonically increasing in monitoring efficiency m, net worth w and the entrepreneur's outside option u.*

(d) *Finally, the Pareto frontier $\phi(m, w, u)$ is monotonically increasing in monitoring efficiency m and net worth w, and monotonically decreasing and strictly concave in the entrepreneur's outside option u.*

The above lemma is fairly intuitive. Higher values of m correspond to lower marginal cost of monitoring, and hence the investor is required to be compensated less at the margin. Therefore, the optimal debt obligation is lower. Higher net worth implies that the entrepreneur must be compensated more (higher $Q - R$) in order to incentivize him to exert an additional amount of effort, and consequently, R must be lower. Finally, higher values of u imply greater bargaining power of the entrepreneur which in turn implies lower debt obligation. As far as the optimal monitoring level and entrepreneurial effort are concerned, greater monitoring efficiency implies that the lender can increase monitoring effort at a lower cost. Therefore, both the level of monitoring and entrepreneurial effort increase with m. Increased net worth implies that the borrower is easier to incentivize to exert an additional amount of effort. Therefore, monitoring level decreases and effort increases with w. Higher outside option means greater marginal compensation for the entrepreneur. Thus, he chooses higher effort level, and less monitoring is required. Part (c) of the above lemma asserts the favorable impact of the monitoring efficiency of the investor, the net worth and outside option of the entrepreneur on value creation as they enhance the firm's expected value.

Note also that the Pareto frontier $\phi(m, w, u)$ for a type (m, w) firm is strictly increasing in m and w, and strictly decreasing and concave in the outside option u of the borrower. It is well-known that in the absence of incentive problems the Pareto frontier is a straight line with slope equal to -1, i.e., it can be expressed as $\phi(m, w, u) = \Phi(m, w) - u$ where $\Phi(m, w)$ is the aggregate surplus of a type (m, w) firm. In other words, utility is perfectly transferable (TU) in a partnership between an investor and an entrepreneur since the aggregate surplus is given. Under double-sided moral hazard, utility cannot be transferred on a one-to-one basis since the size of the pie crucially depends on how the pie is divided between the investor and the entrepreneur. This gives rise to a concave Pareto frontier. Such imperfect transferability will be the crux of our analysis of the market equilibrium with endogenous matching described in the following section.

3. The Market Equilibrium

3.1. Equilibrium partnerships

In this section, we analyze the set of equilibrium allocations.[10] We first analyze how the equilibrium payoffs of the investors and entrepreneurs are determined, and discuss some important properties. An allocation for the economy is a matching rule ξ, and the corresponding vectors expected payoffs $\boldsymbol{v}$ and $\boldsymbol{u}$ where $v(\mu(i)) \in \boldsymbol{v}$ represents the type-dependent payoff of each investor i, and $u(\omega(j)) \in \boldsymbol{u}$ is the type-dependent payoff of each entrepreneur j. Within any partnership (i, j), the investor is assumed to posses all the bargaining power and makes a take-it-or-leave-it offer to the entrepreneur taking into account his outside option. Therefore in an equilibrium, each investor i would choose an entrepreneur in order to maximize her expected payoff $\phi(\mu(i), \omega(j), u(\omega(j)))$. In other words,

Definition 3 (equilibrium allocation) An allocation $(\xi, \boldsymbol{v}, \boldsymbol{u})$ is a Walrasian equilibrium allocation for the investor-entrepreneur economy if the following conditions are satisfied:

(a) Given $u(\omega(j)) \in \boldsymbol{u}$ for $j \in J$,

$$\xi(i) = \operatorname{argmax}_j \phi(\mu(i), \omega(j), u(\omega(j))), \tag{6}$$

$$v(\mu(i)) = \max_j \phi(\mu(i), \omega(j), u(\omega(j))), \tag{7}$$

for each $i \in I$.

(b) Let $\mathscr{B}$ be the collection of Lebesgue-measurable sets of I and J, and $l^* : \mathscr{B} \to \mathbb{R}_+$ be the corresponding Lebesgue measure. Then $l^*(J') = l^*(I')$ for each $J' \in \mathscr{B}$ with $I' = \xi(J') \in \mathscr{B}$.

Treat $\boldsymbol{u}$ as the Walrasian price vector for the borrowers. Part (a) of the above definition asserts that each investor i of type $\mu(i)$ chooses a borrower j of a given type $\omega(j)$ in order to maximize her expected payoff, taking the price vector as given. Part (b) is a measure-consistency requirement, the standard 'demand-supply equality' condition of a Walrasian equilibrium with a continuum of individuals. In other words, if a subset J' of entrepreneurs are matched with a subset I' of investors, then I' and J' cannot have different Lebesgue measures.

3.2. Equilibrium payoffs and negatively assortative matching

Note first that equilibrium exhibits the 'equal-treatment-of-equals' property. Consider any two entrepreneurs j and j' such that $\omega(j) = \omega(j') = w$ with $j \neq j'$. Then, it must be the case that $u(\omega(j)) = u(\omega(j'))$. To see this, let there be an investor i who is matched with j. If $u(\omega(j)) > u(\omega(j'))$, then i would be strictly better-off by forming a partnership with j' since the Pareto frontier ϕ is strictly decreasing in u, and hence such a matching cannot be part of an equilibrium allocation. Denote by $u(w)$ the common utility level of two identical entrepreneurs j and j'. By the same logic, it must also be the case that in an equilibrium allocation $v(\mu(i)) = v(\mu(i')) = v(m)$ if $\mu(i) = \mu(i') = m$ with $i \neq i'$. Therefore, for an investor i with type $m = \mu(i)$, maximizing her payoff over the set of entrepreneurs J is equivalent to maximizing her payoff over the set of net worths $\mathscr{W}$, i.e., each type m investor solves

$$\max_w \phi(m, w, u(w)), \tag{8}$$

[10] Kaneko (1982), and Legros et al. (2010) prove the existence of an equilibrium allocation for this class of assignment models.

Next, the outside option of each type w entrepreneur is the maximum payoff he could obtain by switching to alternative matches, and hence it is endogenous. We first argue that in every type (m, w) firm the participation constraint of the entrepreneur must bind in a Walrasian equilibrium. Suppose an entrepreneur of type w is offered $u(w)$ in an equilibrium allocation. Since there is a continuum of types, one can find an identical investor who would also offer $u(w)$ to the same borrower, and hence $u(w)$ actually becomes his outside option. Thus, any payoff strictly above than the borrower's outside option cannot be an equilibrium payoff.[11] Therefore, if in an equilibrium allocation $(\xi, \boldsymbol{v}, \boldsymbol{u})$ we have $i = \xi(j)$ with $m = \mu(i)$ and $w = \omega(j)$, then the Pareto frontier associated with this partnership must be given by $\phi(m, w, u(w))$.

Definition 3(a) implies that each investor would choose an entrepreneur of a given type in order to maximize her expected payoff. Thus, the first-order condition of the maximization problem (8) of each type m investor implies that

$$u'(w) = -\frac{\phi_2(m, w, u(w))}{\phi_3(m, w, u(w))} \quad \text{for } m = \lambda(w) \in \Lambda(w). \tag{9}$$

From Lemma 2(d) we have $\phi_1, \phi_2 > 0$ and $\phi_3 < 0$, and hence $u'(w) > 0$, i.e., $\omega(j) > \omega(j')$ implies that entrepreneur j obtains strictly higher payoff than j'. The equilibrium payoff function $u(w)$ is determined by solving the above differential equation, which is given by:

$$u(w) = u(0) + \int_0^w \left[-\frac{\phi_2(\lambda(x), x, u(x))}{\phi_3(\lambda(x), x, u(x))} \right] dx, \tag{10}$$

where $u(0) \geq 0$ is the equilibrium payoff that accrues to the entrepreneurs with net worth 0.

Next, we analyze the matching pattern in an equilibrium allocation. Consider a matching correspondence $\Lambda(w)$ and any selection of it, $\lambda(w)$. The sign of $\lambda'(w)$ is determined by the second-order condition associated with the maximization problem (8) of each type m investor, which is given by:

$$[\phi_{21}(m, w, u(w)) + \phi_{31}(m, w, u(w))u'(w)]\lambda'(w) > 0, \text{ for } m = \lambda(w). \tag{11}$$

We show that $\phi_{21}(m, w, u(w)) = \phi_{31}(m, w, u(w)) < 0$, and hence the above second-order condition is satisfied only if the equilibrium matching is negatively assortative, i.e., $\lambda'(w) < 0$ for any $\lambda(w) \in \Lambda(w)$.

Proposition 1 *In an equilibrium allocation $(\xi, \boldsymbol{v}, \boldsymbol{u})$, matching is negatively assortative, i.e., more efficient monitors invest in firms owned by entrepreneurs with lower net worth.*

Under double-sided moral hazard both monitoring ability and net worth play roles in the creation of surplus. Notice that the aggregate surplus of a type (m, w) firm is given by:

$$\Phi(m, w, u(w)) = \phi(m, w, u(w)) + u(w),$$

and hence $\Phi_{21}(m, w, u(w)) = \Phi_{31}(m, w, u(w)) < 0$. This implies that more efficient monitors are more effective at the margin in the projects with lower net worth. In other words, net worth and monitoring efficiency are strategic substitutes. There are two types of such substitutability, not only the types are

[11] For an entrepreneur, a slack individual rationality constraint is a partial equilibrium phenomenon where only a single investor-entrepreneur pair is considered which cannot occur in the investor-entrepreneur market. In other words, in a Walrasian equilibrium there is no additional surplus to bargain over, which is similar to the "no surplus" condition of Ostroy (1984).

substitutes in producing the total surplus, i.e., $\Phi_{21} < 0$, but they also are substitutes in transferring surplus from one party to the other, i.e., $\Phi_{31} < 0$. Under imperfect transferability, for lenders with greater monitoring efficiency it is easier [at the margin] to transfer surplus to the borrowers with low net worth. Therefore, it is optimal to assign low net worth projects to more efficient monitors following a negatively assortative matching pattern.

Suppose that in an equilibrium allocation a type w entrepreneur is matched with a type m investor. An immediate consequence of Definition 3(b) and NAM is that

$$G(m) = 1 - F(w) \implies m = G^{-1}(1 - F(w)) \equiv \lambda(w), \tag{12}$$

where $\lambda(w)$ is any selection of the equilibrium matching correspondence. It is also immediate to show that the selection is unique.

Corollary 1 *Let $\lambda(w)$ be a selection of the equilibrium matching correspondence $\Lambda(w)$. The selection is unique.*

The above result follows from the fact that the type distribution functions are strictly increasing. To see this, let $\Lambda(w)$ consists of at least two distinct points m' and m'' for any $w \in \mathscr{W}$. Then from condition (12) it must be the case that $G(m') = G(m'')$ which contradicts the fact that $G(m)$ is strictly increasing on $\mathscr{A}$. From (12) it also follows that

$$\lambda'(w) = -\frac{f(w)}{g(\mu(w))}.$$

Let us call $m = \lambda(w)$ the equilibrium "matching graph". Note that the density functions $g(\cdot)$ and $f(\cdot)$ are local measures of the dispersions of the corresponding distributions. Therefore around any equilibrium selection of the matching correspondence $m = \lambda(w)$, $g(\lambda(w)) > (<) f(w)$ implies that more (less) probability mass is concentrated around $m = \lambda(w)$ than around w. In other words, the relative dispersion of the type distributions determines the slope of the matching graph.

3.3. Equilibrium debt obligation, entrepreneurial effort, and expected firm value

Recent empirical literature on incentive contracts claims that endogenous principal-agent matching is an important determinant of optimal contracts in the principal-agent relationships. Ackerberg and Botticini (2002) argue that in order to study the effects of observed principal and agent characteristics on optimal contracts, empirical models typically regress contract choice on these parameters. They show that when there are incentives whereby principals of given types end up hiring agents of particular types, the estimated coefficients of a simple regression on the observed characteristics may be misleading. To understand this point in the current context, suppose that there are two types of entrepreneurs (high- and low-net worth), and two types of investors (high- and low-monitoring ability). Standard agency models [which treats an investor-entrepreneur partnership in isolation] would predict that lower entrepreneurial effort and higher debt obligation must be associated with low net worth (see Lemma 2). Since NAM implies that borrowers with low net worth are matched with more efficient lenders, and since effort is increasing and debt obligation is decreasing in monitoring efficiency, through an endogenous matching higher effort and lower repayment obligation will be associated with low net worth. Therefore, the outcome of an assignment model will offer predictions about the behavior of entrepreneurial effort and debt obligation with respect to net worth which are exactly opposite to what would have been predicted by the standard agency theory.

The principal objective of this subsection is to analyze the behavior of the equilibrium debt obligation, entrepreneurial effort and expected firm value with respect to net worth. Since higher net worth implies less stringent moral hazard in effort choice, a natural question is to ask whether, under double-sided moral hazard, the equilibrium debt obligation, entrepreneurial effort, and expected firm value are monotone functions of net worth. Since monitoring efficiency in equilibrium is a function $\lambda(w)$ of net worth, the equilibrium transfer, entrepreneurial effort and expected firm value can also be expressed as functions of w, which are respectively given by:

$$\begin{aligned} R(w) &:= R(\lambda(w), w, u(w)), \\ \theta(w) &:= \theta(\lambda(w), w, u(w)), \\ P(w) &:= P(\lambda(w), w, u(w)). \end{aligned}$$

Thus, the behavior of the above three equilibrium variables with respect to net worth can be decomposed into two countervailing effects: a *matching effect* and an *outside option effect*. To understand this, consider for example the derivative of the equilibrium debt obligation function:

$$R'(w) = \underbrace{\frac{\partial R}{\partial m}\lambda'(w)}_{\text{matching effect}} + \underbrace{\frac{\partial R}{\partial u}[1+u'(w)]}_{\text{outside option effect}}. \tag{13}$$

The second term of the above expression is valid since $\partial R/\partial w = \partial R/\partial u$. Since the debt obligation decreases with monitoring efficiency and $\lambda'(w) < 0$, the first effect has a positive impact on equilibrium R. The second effect, on the other hand, is negative since R decreases with the entrepreneur's outside option u and the marginal income $u'(w)$ of the borrower is strictly positive. Thus the relationship between equilibrium transfer and net worth is, in general, non-monotone. Similar non-monotone relations are true for equilibrium entrepreneurial effort $\theta(w)$ and expected firm value $P(w)$. It is worth noting that the matching effect has negative impacts on the equilibrium entrepreneurial effort and expected firm value since both are increasing in m (see Lemma 2), whereas, the outside option effect is positive for $\theta(w)$ and $P(w)$ since both are increasing in u. The monotonicity of equilibrium variables with respect to w thus will depend on which of the two countervailing effects dominates. Therefore,

Proposition 2 *Let $R(w)$, $\theta(w)$ and $P(w)$ be the equilibrium debt obligation, entrepreneurial effort and expected firm value as functions of net worth, which are in general non-monotone with respect to net worth of the entrepreneurs.*

It would be interesting to see under what conditions the equilibrium variables such as debt obligation, entrepreneurial effort, and expected firm value are monotone with respect to net worth. Consider the two limiting cases. First, suppose the lenders in the economy are almost identical, i.e., $m \to m^0$, but the borrowers are sufficiently heterogenous. Then $\lambda'(w) \to 0$ (a very flat matching graph), but $u'(w) > 0$. Then from equation (13) it follows that $R'(w)$ is strictly negative, i.e., entrepreneurs with higher net worth pay lower interest. Also in this case, both $\theta'(w) > 0$ and $P'(w) > 0$. Hence, entrepreneurs with higher net worth exert higher effort, and creates greater firm value. Next consider the other limiting case when the entrepreneurs become almost homogeneous, i.e., $w \to w^0$, but the investors remain sufficiently heterogeneous. In this case, $\lambda'(w) \to -\infty$ (an almost vertical matching graph), and $u'(w) \to 0$. Since $\partial R/\partial m < 0$, it follows from equation (13) that $R'(w) > 0$. Hence, higher debt obligations are associated with higher net worth. In this case it also happens that $\theta'(w) < 0$ and $P'(w) < 0$, i.e., borrowers with low net worth exert greater effort, and create higher firm value. In the first case when the investors are almost identical, the impact of monitoring efficiency (the matching effect) becomes insignificant, and we obtain the usual behavior of the contracting variables with respect to net worth as would have been predicted by the standard agency theory. When, on the other hand, the entrepreneurs become homogeneous, the impact of monitoring efficiency on loan contract becomes significant. Hence, less efficient monitors who are matched with entrepreneurs with high net worth receive greater transfers since it is more difficult to incentivize them to choose a greater monitoring level. Consequently, such monitors induce lower entrepreneurial effort, and create lower firm value.

Results similar to Proposition 1 are already known in the literature on endogenous matching under two-sided heterogeneity when utility may or may not be perfectly transferable. At the beginning of this subsection we have discussed that the predictions of a model like ours regarding the incentive contracts with multiple principals and agents may not conform to those predicted by the standard agency theory where a principal-agent pair is treated in isolation. In this paper we exploit the property of the equilibrium matching, namely NAM to show that contract terms are in general non-monotone with respect to the fundamentals of the model. It is worth mentioning that the endogenous outside option plays a crucial role behind Proposition 2 which is purely a general equilibrium phenomenon.

4. Numerical analysis

4.1. Comparative statics

In order to study the behavior of an equilibrium variable such as $R(\lambda(w), w, u(w))$ with respect to the parameters of the model, one first requires to solve the differential equation in (9), which reduces to:

$$u'(w) = \frac{[c^2 - c\lambda(w) + (\lambda(w))^2]R(\lambda(w), w, u(w))}{c[cQ - (2c - \lambda(w))R(\lambda(w), w, u(w))]} \equiv \mathscr{U}(w, u(w); c, Q, r_f).$$

Under imperfectly transferable utility, the above ordinary differential equation does not have an analytical solution.[12] Instead, we solve the model using numerical methods. In particular, the matching graph $m = \lambda(w)$ is substituted into the first order condition (9) for given densities $g(m)$ and $f(w)$ to get the above differential equation. The resulting expression defines a differential equation for utility u in terms of net worth w, which is solved numerically. The solution yields equilibrium values u and w. Equilibrium monitoring efficiency may in turn be recovered from the equilibrium matching graph $m = \lambda(w)$. Next, we allow for changes in the distributions of types in order to analyze their effects on the equilibrium contract variables.

Moreover, it is also difficult to determine the matching and outside option effects in order to carry out any meaningful comparative static analysis. We therefore resort to a numerical simulation of the model in order to examine the effects of alternative distributions of both monitoring efficiency and net worth on several variables of interest. For this purpose, we assume that both monitoring efficiency and net worth

[12] Sattinger (1980) analyzes solution methods for such equations for marginal income determination under various type distributions.

follow beta distributions, which are respectively given by:

$$G(m) = \int_0^m t^{\alpha_m - 1}(1-t)^{\beta_m - 1} dt \quad \text{with} \;\; \alpha_m, \beta_m > 0,$$
$$F(w) = \int_0^w t^{\alpha_w - 1}(1-t)^{\beta_w - 1} dt \quad \text{with} \;\; \alpha_w, \beta_w > 0.$$

There are two principal reasons for choosing beta distributions. First, it requires to define bounded supports for monitoring efficiency, m, and net worth, w, which are consistent with the model's assumptions that $\mathscr{A} = [0, c/2]$ and $\mathscr{W} = [0, 1]$. Second, as we have seen from the theoretical analysis in the previous section that heterogeneities of the distributions of m and w are crucial to determine the relative importance of the matching effect on the equilibrium variables, a beta distribution is flexible enough to consider alternative specifications for the relative heterogeneity between investors and entrepreneurs. For example, if $\alpha_k = \beta_k = 1$ for $k = m, w$, the beta distributions reduce to uniform distributions.

As we have mentioned earlier, the magnitude of the matching effect depends on the slope of the matching graph $\lambda'(w)$. This slope is determined exclusively by the ratio of the density functions $g(m)$ and $f(w)$. For example, if both net worth and monitoring efficiency are uniformly distributed, the slope of the matching function is $-c/2$. However, this is not the case in general since net worth and monitoring efficiency follow beta distributions, with their skewness determined by parameters α_k and β_k for $k = m, w$. Thus, the magnitude of the matching effect depends crucially on the parameters of the beta distributions. Consequently, the shapes of the equilibrium functions $R(w)$, $\theta(w)$ and $P(w)$ also depend on whether the matching effect is larger than the outside option effect, for which the density functions $g(m)$ and $f(w)$ play a crucial role in the analysis.

To solve for the Walrasian equilibrium numerically, the following parameter values are assumed. First, the parameter c for the disutility of entrepreneurial effort is set at 2, and hence $\mathscr{A} = [0, 1]$. If successful, each project yields a cash flow $Q = 2.5$, and zero otherwise. Finally, the risk-free interest factor r_f is set at 1.05 so that this abides by the restriction $Q > 2r_f$. In the following simulations, the parameters of the beta distributions, α_k and β_k for $k = m, w$ are varied in order to modify the relative heterogeneity among the investors and entrepreneurs.

In the first set of exercises (henceforth "Exercise 1"), we assume a uniform distribution for net worth so that $\alpha_w = \beta_w = 1$. At the same time, β_m is set at 1, and the values of α_m are gradually increased starting from 1. From the properties of the beta distribution, this implies that monitoring efficiency becomes more concentrated at the top, i.e., the distribution becomes more (negatively) skewed. As a result, the expected value of m increases and its variance falls, i.e., a distribution corresponding to higher α_m first order stochastically dominates the one associated with lower α_m.

The top panel of Figure 1 presents the equilibrium matching graph $\lambda(w)$ under Exercise 1 corresponding to alternative values of α_m. When $\alpha_m = 1$, both m and w are uniformly distributed, and hence $\lambda'(w) = -1$, which is intuitive since the dispersions of monitoring efficiency and net worth are the same. Since there is NAM and the distribution of net worth does not change, an increase in α_m implies that low-net worth entrepreneurs are relatively more heterogeneous relative to highly-efficient monitors. In terms of the density functions, $g(m)$ is larger at the bottom of the distribution of net worth with $f(w)$ unchanged. This implies that $\lambda'(w)$ must be small in absolute value corresponding to low values of net worth. As we move to the right of the distribution of net worth, borrowers become less heterogeneous relative to lenders, so $\lambda'(w)$ gradually increases in absolute value implying a stronger matching effect.

[Insert Figure 1]

In the bottom panel of Figure 1, we draw the equilibrium payoff $u(w)$ of the borrowers which is a strictly increasing function. As monitoring efficiency becomes more concentrated at the top of the distribution (α_m increases from 1), as shown in the figure, $u(w)$ shifts downward. When α_m increases, less efficient lenders, who are matched with high-net worth borrowers, are now short in supply. This implies that competition for resources is exacerbated among these entrepreneurs which weakens their bargaining power. This explains why the spread between any two $u(w)$ curves widens for values of w close to 1, which implies a weaker outside option effect.

Figure 2 illustrates how the equilibrium repayment obligation $R(w)$, entrepreneurial effort $\theta(w)$, and expected firms value $P(w)$ are affected under Exercise 1. Given the concavity of the equilibrium matching graph in Figure 1, the greater the value of net worth, the stronger is the matching effect relative to the outside option effect. As illustrated at the top panel of Figure 2, the equilibrium debt obligation is decreasing in net worth, suggesting that the matching effect is weaker than the outside option effect for all the values of w and α_m. Compared with the case where $\alpha_m = 1$, an increase in α_m only tilts the $R(w)$ curve slightly. In contrast, the equilibrium effort function $\theta(w)$ changes its shape drastically when α_m increases as it is shown in the middle panel of Figure 2. Consider first the case where $\alpha_m = 1$. The equilibrium effort function is slightly U-shaped implying that the outside option effect is stronger than the matching effect for high values of net worth. If $\alpha_m = 5$, the matching effect is relatively weak for low values of w. As a result, the equilibrium effort function is now positively sloped. However, the opposite occurs if the values of net worth are high enough, and thus the effort function slopes downward. Similar result holds under $\alpha_m = 10$. Finally, the bottom panel of Figure 2 shows that the firm's expected value is positively sloped if $\alpha_m = 1$. It also shifts upward as α_m rises, given that the borrowers have to exert higher effort. If α_m is greater than 1, the fall in effort described above for high values of w also explains the fall in the firm's expected value .

[Insert Figure 2]

The next set of exercises (henceforth "Exercise 2") assumes that the monitoring ability is uniformly distributed, and allows for changes in the distribution of net worth. In particular, the parameter α_w is set to 1, and β_w is increased starting from 1.[13] From the properties of the beta distribution, an increase in the values of β_w implies that a greater mass of borrowers gets concentrated at the bottom, i.e., the distribution becomes more positively skewed. In addition, the mean and variance of net worth fall, and the distribution corresponding to a lower value of β_w first-order stochastically dominates the one corresponding to a higher β_w.

The changes in the equilibrium matching graph under Exercise 2 are presented at the top panel of Figure 3. As a reference, we consider the case where net worth is uniformly distributed among borrowers ($\beta_w = 1$) in which case the matching graph is a straight line with slope -1. Given that the variance of net worth falls if β_w increases, lenders become more heterogeneous relative to borrowers for low values of w since the distribution of monitoring ability does not change. A larger mass of borrowers thus increases the slope of the matching function (in absolute value) at low values of w. The opposite occurs for high values of net worth, and thus the matching graph gradually becomes horizontal. Overall, the matching effect becomes stronger relative to the outside option effect corresponding to low values of net worth for a given value of β_w. Also, the matching graph becomes more convex if β_w rises further.

[13] Qualitatively similar results may be obtained if β_w is fixed instead and α_w is decreased.

[Insert Figure 3]

The bottom panel of Figure 3 depicts the equilibrium borrower utility as a function of w. Consider the case where $\beta_w = 5$. For high values of net worth, lenders are relatively more homogeneous which implies an increase in competition for the allocation of credit. This explains why the gap between the two $u(w)$ curves [corresponding to $\beta_w = 1$ and $\beta_w = 5$] widens for high values of w. This effect is even stronger if the mass of borrowers is more concentrated at the bottom of the distribution, i. e., if β_w increases even further.

The effects of an increase in β_w on the equilibrium debt obligation, entrepreneurial effort, and the expected firm value under Exercise 2 are presented in Figure 4. Consider first the case of the equilibrium transfer function $R(w)$. As described in Figure 3, the matching effect is relatively stronger for low levels of w for a given value of β_w. If $\beta_w = 5$, $R(w)$ remains negatively sloped. However, the equilibrium debt obligation increases relative to the case of uniform distribution for low values of net worth, whereas for high values of w the equilibrium debt obligation is now lower than that corresponding to the uniform distribution. If $\beta_w = 10$, the matching effect is now stronger than the outside option effect, and thus the transfer function is positively sloped over some range of w. For high values of net worth the matching effect is relatively weaker so that the equilibrium debt obligation function is again negatively sloped. As shown in the middle and bottom panels of Figure 4, the opposite pattern occurs for the equilibrium effort and expected firm value.

[Insert Figure 4]

The final set of exercises ("Exercise 3") examines how the previous analyses are affected when neither monitoring efficiency nor net worth is uniformly distributed. In particular, we set $\alpha_w = \beta_m = 1$ and $\beta_w = 10$, whereas the shape parameter α_m is varied. This parameterization means that larger mass of net worth is concentrated at the bottom, whereas larger mass of monitoring ability is concentrated at the top of the distribution if $\alpha_m > 1$.

The changes in the variables of interest are presented in Figures 5 and 6. The intuitions behind the results obtained in Exercise 3 are very similar to the previous ones, and hence we avoid a detailed discussion. If α_m is increased to 10, the distribution of monitoring efficiency is just a mirror image of that of net worth. Given that the equilibrium matching is negatively assortative, the ratio of densities $f(w)/g(m)$ is equal to 1. Therefore, the matching graph is a straight line with slope -1 even though neither m nor w is uniformly distributed.

[Insert Figure 5]

If $\alpha_m = 10$, the debt obligation, effort, and expected firm value schedules are the same as the ones obtained under the assumption that both m and w are uniformly distributed (Figure 2 with $\alpha_m = 1$, and Figure 4 with $\beta_w = 1$). This last example suggests that if the distribution of net worth is a mirror image of that of monitoring ability, the relative heterogeneity between entrepreneurs and lenders is constant, and the results are equivalent to those obtained under uniform distributions. For all the other cases, we should expect a non-constant slope $\lambda'(w)$ and correspondingly, a non-constant matching effect.

[Insert Figure 6]

To summarize, the above exercises illustrate that the matching effect depends crucially on how net worth and monitoring efficiency are distributed. At the same time, the way how these characteristics are distributed has direct implications for how total surplus is shared among entrepreneurs and investors. Therefore, the changes in the distributions of monitoring ability and net worth affects significantly the equilibrium variables namely, the entrepreneur's debt obligation, entrepreneurial effort, and expected firm value.

4.2. Implications

Changes in the distributions of types have important testable implications for the equilibrium debt obligation and expected firm value which may be derived from the above set of comparative static exercises. Note that the equilibrium debt obligation yields an equilibrium interest premium in the sense that firms with a lower net worth must in general pay a higher interest rate. Since the risk-free interest factor r_f is constant, an equilibrium interest spread between the rate paid by the entrepreneur and the risk-free rate may be found for each net worth level. Accordingly, the interest rate spread in general will have a positive correlation with the entrepreneur's leverage.

Consider first a decrease in the average monitoring efficiency. One way of motivating such a scenario is a general securitization of loans across lenders. In the absence of securitization, a lender fully internalizes the costs and benefits of her monitoring activities. However if a lender has the option of securitizing the loan in order to diversify loan risk, she will have lower incentives to monitor.[14] This conforms to recent evidences (see Mian and Sufi, 2009; Keyes et al., 2010) suggesting that securitization had an adverse effect on the ex-ante screening effort of the issuers. In terms of the density function $g(m)$, a decrease in the average monitoring ability is captured by a fall in α_m.[15]

In Figures 2 and 6, a decrease in the average monitoring efficiency leads to an increase in the equilibrium debt obligation that must be made by low-net worth entrepreneurs, and a fall in such transfers for high-net worth borrowers. This is a consequence of NAM. A fall in average monitoring ability affects relatively more the lenders with high monitoring ability because of an accompanied change in the skewness of the distribution. Since these lenders are matched with borrowers with low net worth, in order to incentivize these lenders to monitor more intensively they must receive higher transfers. As discussed above, the equilibrium debt obligation may in fact be non-monotone in net worth if the distribution of w is positively skewed (see Figure 6). In Figure 6, 65% of the firms are concentrated at the net worth level

[14] Arguably, securitization brings a series of benefits to credit markets, such as an improvement in risk sharing and a decrease in the banks' costs of capital. However, securitization also raises the possibility of adverse selection (an incentive to securitize low-quality loans), and moral hazard (loans that may be sold are not appropriately screened, or securitized loans are not subsequently monitored as a result of risk diversification). In fact, securitization has been blamed for encouraging risky lending and for being partially responsible of the recent financial crisis (e.g. Blinder, 2007; Stiglitz, 2007).

[15] The analysis of the incentive problems associated with loan securitization is beyond the scope of the present paper. Sometimes securitization requires each lender to invest in several firms instead of a single firm. Our assumption of a one-to-one matching rules out such form of debt contracts. Nevertheless, a simple one-to-many model where each lender invests in many firms can be derived from the one-to-one matching model if the monitoring cost function of a lender is separable across her borrowers. In this case all contractual relationships of the same lender can be treated as many identical partnerships of the same type, and all our results hold under this generalization. At the same time, the model would also need to include a different class of investors demanding securitized debt. As we have argued that securitization of loans leads to an additional moral hazard problem between the "loan originators" (the lenders in our model) and the buyers of such instruments, greater ability to securitize loans imply lower monitoring intensity captured by lower values of the parameter m. From this perspective, the monitoring technology in our model may be reinterpreted as a reduced-form function.

between 0 and 0.1. Therefore, the observed increase in the debt obligation affects a large proportion of the borrowers. Thus, an increase in securitization may be associated with a widening in the interest rate spread for highly-leveraged entrepreneurs, and at the same time, with a fall in the spread for high-net worth entrepreneurs.

For a net worth level sufficiently close to 1, the numerical results in Figures 2 and 6 also suggest that a lender with a higher monitoring ability would charge a higher loan rate to her borrower. This result is consistent with the empirical findings of Chen (2013) who estimates the loan spread equation under endogenous matching between banks and firms using data on bank lending to large businesses in the U.S. He concludes that, conditional on borrowers' characteristics, banks with greater monitoring ability charge a higher loan rate spread.[16] Following Diamond (1984), Chen (2013) interprets this finding as evidence that monitoring services by lenders are valued so that borrowers are willing to pay a premium for their services. Our model offers an alternative interpretation: an increase in the average monitoring efficiency through a higher α_m leads to an increase in the heterogeneity of investors relative to high-net worth entrepreneurs. Since such borrowers are matched with investors with low monitoring ability following a NAM, and less-efficient monitors are now more scarce, high-net worth borrowers must face a higher competition for informed capital, which is in turn reflected by a higher debt obligation in equilibrium.

The effect of a fall in the average monitoring efficiency on the expected value of the firm may also be observed in Figures 2 and 6. Such a fall leads to a decrease in the expected firm value for low levels of net worth. For the case of firms with low leverage, the effect depends on how net worth is distributed. If the distribution of net worth is positively skewed compared with the uniform distribution, the expected value of high-net worth firms may even rise if the average monitoring efficiency falls. Overall, the numerical exercises suggest that an increase in securitization inducing financial intermediaries to decrease monitoring would end up affecting highly leveraged entrepreneurs the most, not only through higher interest payments but also through a lower expected value of their firms.

The second set of implications relate to how changes in the mean net worth affect the variables of interest. To motivate this case, one may think of a situation where an economic contraction at the aggregate level leads to a fall in the average net worth. This effect is captured by an increase in the parameter β_w in Figure 4. One may observe that a fall in the average net worth has heterogeneous effects on the debt obligation across entrepreneurs. In particular, as a consequence of a decrease in the mean net worth highly leveraged entrepreneurs end up paying a higher interest, whereas relatively well-endowed entrepreneurs decrease their transfers. In practice, a lower debt obligation by well-endowed entrepreneurs is consistent with the fact that these firms may typically have a wider set of options to finance their projects, even under an economic contraction. In contrast, firms with a low net worth typically face more restrictive choices for financing, and thus may face a steeper competition for informed capital.

Figure 4 also illustrates that a negative shock to net worth in the form of a fall in its mean has an asymmetric effect on the expected value of the firm. For low-net worth entrepreneurs, the expected value of their firms falls as a result of the shock, whereas the expected firm value is enhanced for high-net worth entrepreneurs. Consistent with the previous interpretation, these exercises suggest that an economic

[16]To proxy for bank's monitoring ability, Chen (2013) uses the ratio of salaries and benefits to total operating expenses. This is consistent with the idea that monitoring activities are labor-intensive, and that compensation to a bank's staff may reflect their performance in monitoring activities.

contraction leading to a fall in the mean net worth has asymmetric effects across firms. Low-net worth firms are adversely affected through higher debt obligation and a lower expected firm value. Given that the opposite effect is found for high-net worth firms, it means that such a shock may exacerbate inequality across firms. Interestingly, the prediction that an economic contraction at the aggregate level may have an asymmetric effect across firms is consistent with the broad empirical findings reported by Gertler and Gilchrist (1994); Bernanke et al. (1996) for the U.S. manufacturing sector.[17]

The implications derived from a negative shock to the mean net worth are also related to the literature on "financial accelerator" (e.g. Bernanke and Gertler, 1989; Bernanke et al., 1999). In such models there exists agency problems between borrowers and lenders, and a higher value of net worth reduces the agency costs of financing real investments. A negative *collateral shock* increases the costs of external finance even if banks' willingness to supply loans [for a given quantity of collateral] is unchanged. Therefore, the increase in agency costs induces firms to cut investment. Through this mechanism, shocks to collateral amplify business cycle fluctuations. In our model, a negative collateral shock simultaneously increases interest payments and deteriorates the expected value of the firm for highly leveraged entrepreneurs. This mechanism may potentially amplify business cycle fluctuations as in the case of financial accelerator. However, the crucial difference between Bernanke and Gertler (1989) and our model is that the firms are formed endogenously via endogenous lender-borrower matching which allows us to characterize a continuum of equilibrium debt obligations, each associated with one firm. This allows us to show that a change in the mean net worth, which is a consequence of a change in its distribution, affects the entrepreneurs asymmetrically as a fall in mean net worth affects adversely the firms with low net worth, but is favorable to the borrowers with high net worth. Consequently, our model suggests that a *financial accelerator* mechanism would only apply to highly leveraged entrepreneurs, i.e., firms with small net worth.[18]

5. Conclusions

Incentive contracts may be quite different in a market with many heterogenous investors and entrepreneurs as opposed to the contracts for an isolated investor-entrepreneur pair. In the equilibrium of a market, individual contracts are influenced by the two-sided heterogeneity via investor-entrepreneur assignment. In this paper, we have developed a simple two-sided matching model of incentive contracting between lenders and borrowers. Entrepreneurs who differ in net worth and investors who differ in monitoring efficiency are matched into pairs in order to accomplish projects of fixed size. In the equilibrium of the market, both the sorting and the payoff that accrues to each individual are determined endogenously. More efficient monitors finance entrepreneurs with lower net worth following a negatively assortative matching pattern since monitoring efficiency and net worth are strategic substitutes in ameliorating the incentive problems faced by a particular match. The terms of loan contracts are in general non-monotone

[17] There is relatively a large literature supporting the idea that credit flows away from borrowers with high agency costs during recessions, which is the so-called "flight to quality" effect. Unfortunately the present model is not flexible enough to evaluate this effect appropriately since credit is always lent in equilibrium. For a discussion on the flight to quality effect, see Bernanke et al. (1996).

[18] Bernanke et al. (1996) argue that information-based models of lender-borrower relationships have a better fit with reality in those cases where the prospective borrower is a small or medium-sized firm. In contrast, the mapping from theory to reality is less direct for the case of large, publicly-held firms. Evidence from the U.S. manufacturing firms reported by Gertler and Gilchrist (1994) suggests that small firms rely proportionally more on information-intensive financing compared to large firms.

with respect to net worth.

For the analysis of a stylized model, we have employed a number of simplifying assumptions. First, a more ambitious model would consider many-to-many matching among the investors and entrepreneurs. When a lender is allowed to invest in more than one firm, additional complications arise because the monitoring cost function is in general not additively separable. Thus, the non-zero interaction terms induce externalities across matches. On the other hand, allowing an entrepreneur to borrow from more than one source may imply inability of the lenders to write binding exclusive contracts. Non-exclusivity may also lead to an externality across matches. Second, in our model the first-best contracts may not be implemented due to informational asymmetries. In particular, the market failure stems from the fact that, in the presence of limited liability, low net worth borrowers cannot be expected to exert high effort, as they cannot be forced to share losses with the lenders in the event of failure. An important assumption in the paper is the fact that the relationship between an investor and an entrepreneur lasts only for one period. Possibly, such a relationship usually involves dynamic considerations too, which in turn implies some degree of relaxation on the limited liability constraint, and the conclusions of the current paper may alter. In a dynamic model, when there are possibilities of wealth accumulation, the income distributions of an economy are, in general, endogenous. The literature on two-sided matching (e.g. Shapley and Shubik, 1971) has mostly been silent in the context of dynamic bilateral relationships. At this juncture, the paper by Mookherjee and Ray (2002) is worth mentioning, which considers a dynamic model of lending relationships where lenders and borrowers are randomly matched into pairs. They analyze a model of equilibrium short period credit contracts assuming that the bargaining power is exogenously distributed between the lenders and the borrowers. When lenders have all the bargaining power, less wealthy borrowers have no incentive to save and poverty traps emerge. On the other hand, if the borrowers have all the bargaining power, income inequality reduces due to strong incentives for savings. One significant difference between our model and that of Mookherjee and Ray (2002) is that, in the current model, the bargaining power is distributed endogenously among the principals and agents because the outside option of each individual is endogenous. The above mentioned extensions of the current model would be an interesting research agenda for the future.

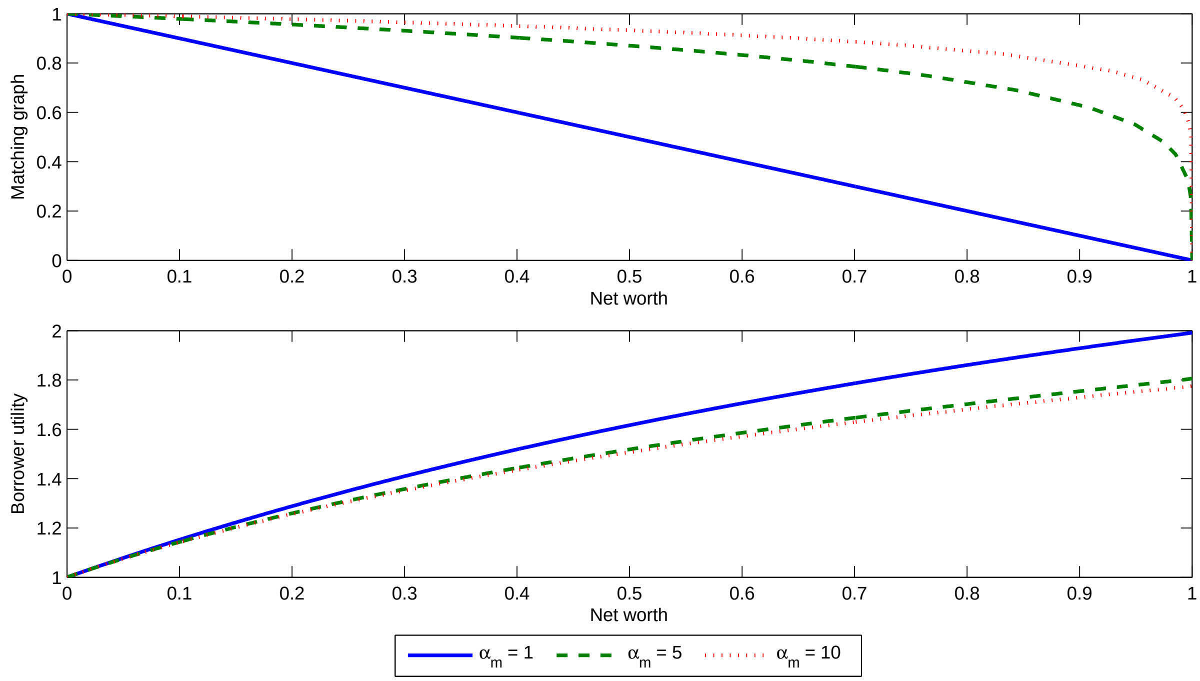

Figure 1: Effect of changes in α_m on $\lambda(w)$ and $u(w)$ when net worth is uniformly distributed

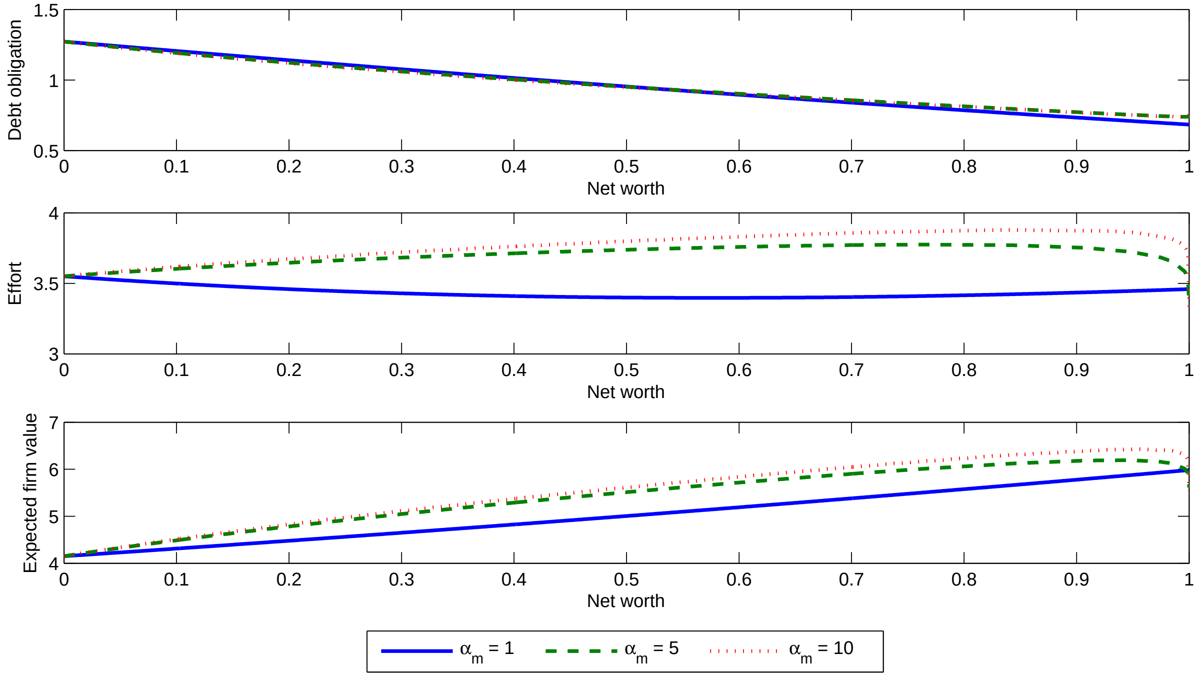

Figure 2: Effect of changes in α_m on $R(w)$, $\theta(w)$ and $P(w)$ when net worth is uniformly distributed

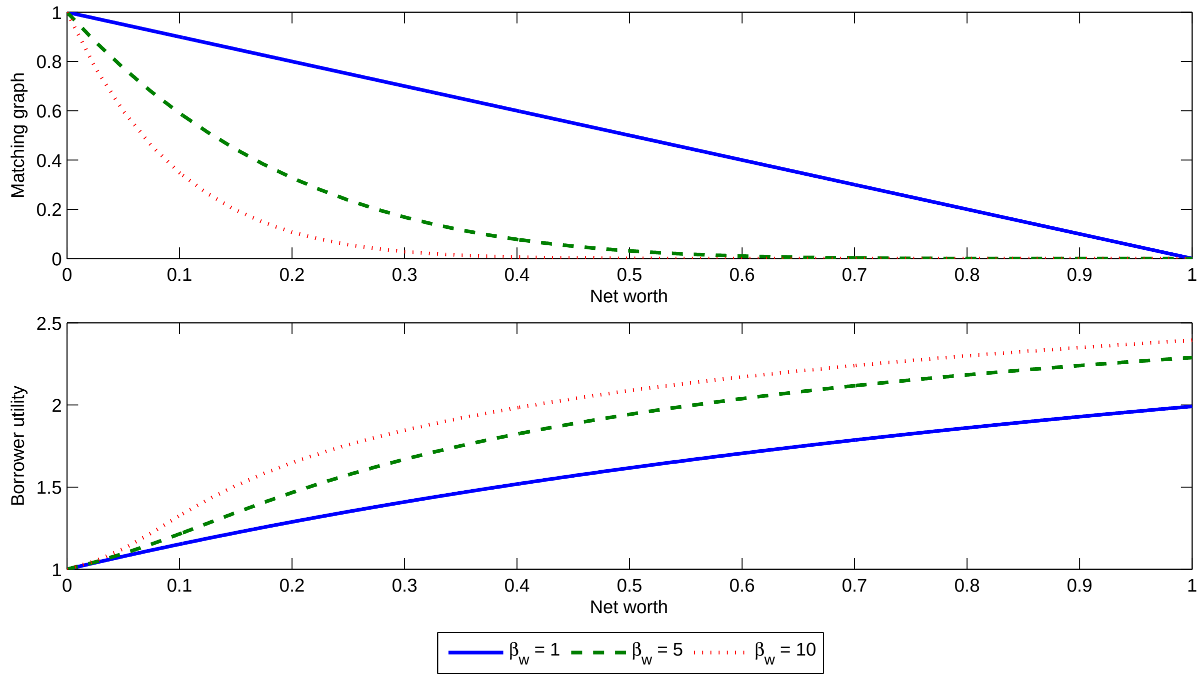

Figure 3: Effect of changes in β_w on $\lambda(w)$ and $u(w)$ when monitoring ability is uniformly distributed

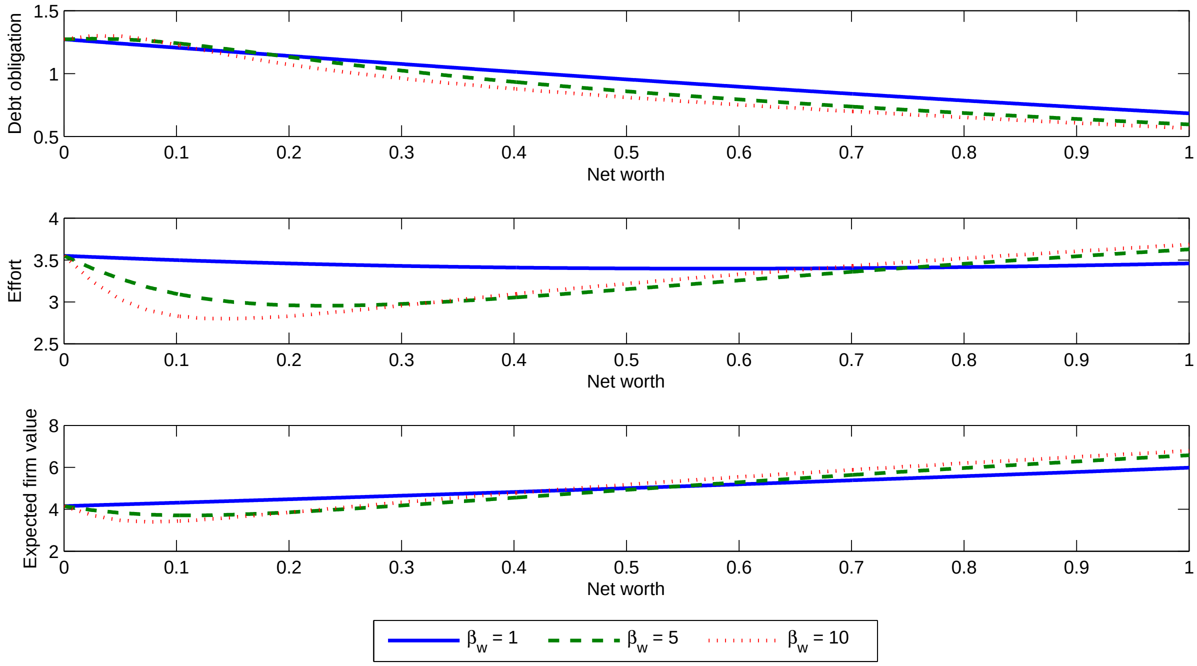

Figure 4: Effect of changes in β_w on $R(w)$, $\theta(w)$ and $P(w)$ when monitoring ability is uniformly distributed

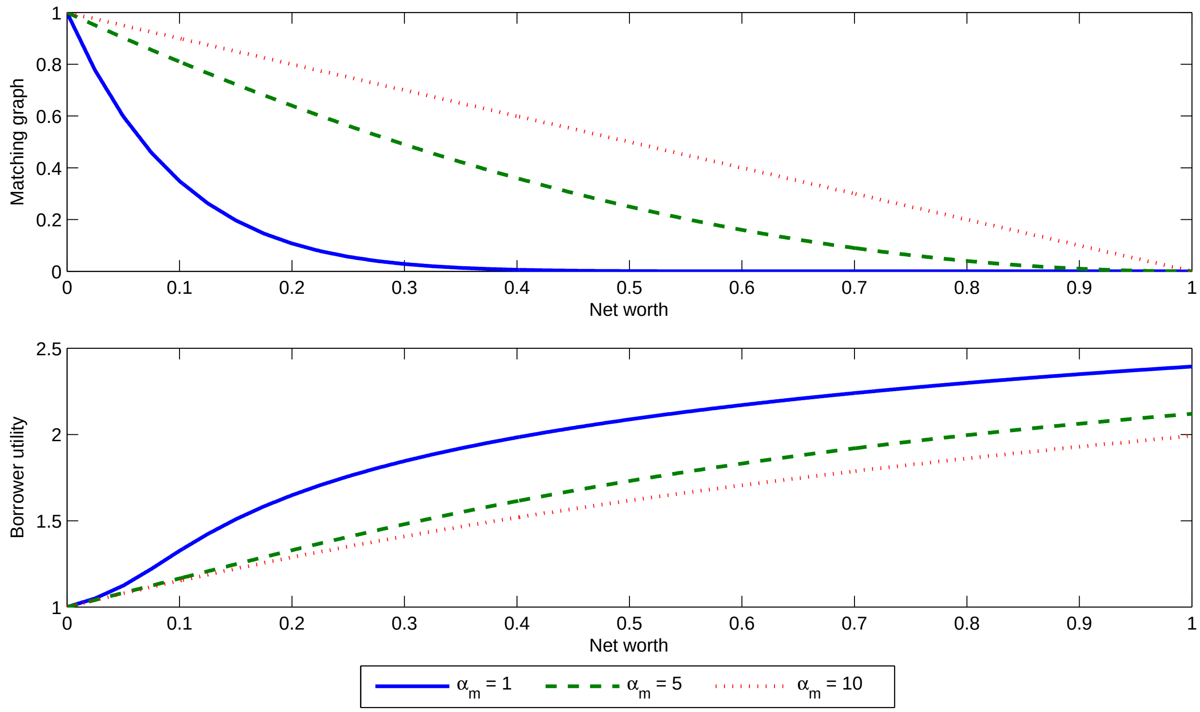

Figure 5: Effect of changes in α_m on $\lambda(w)$ and $u(w)$ when net worth follows beta distribution

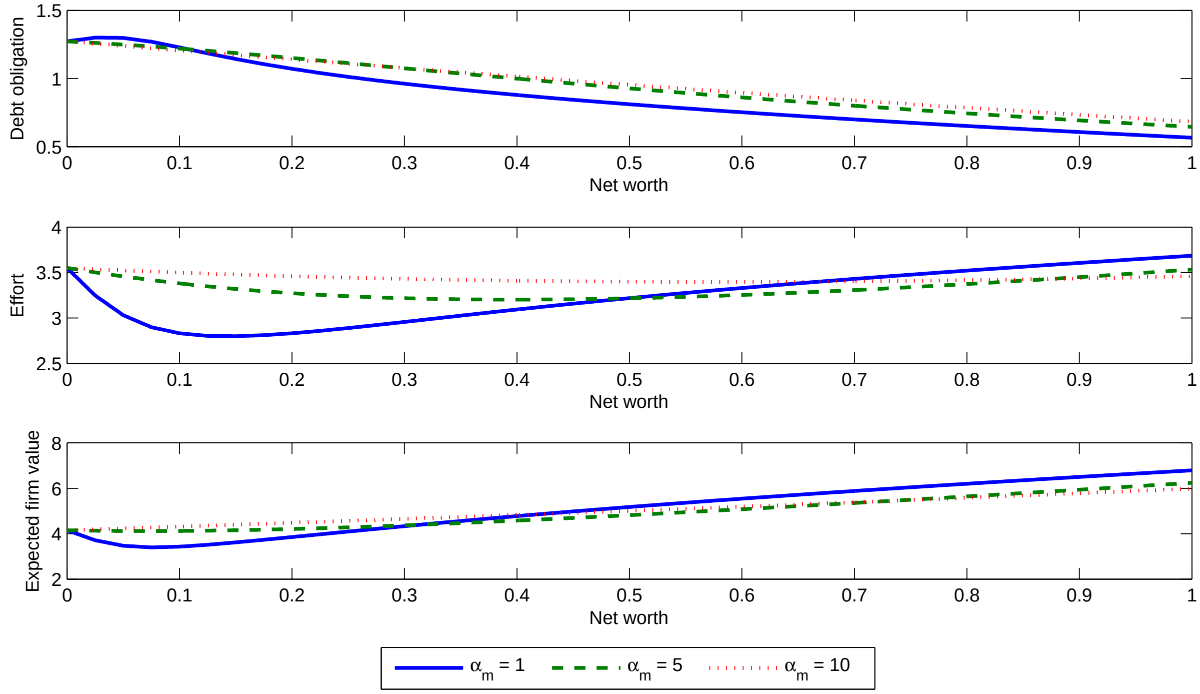

Figure 6: Effect of changes in α_m on $R(w)$, $\theta(w)$ and $P(w)$ when net worth follows beta distribution

Appendix

Proof of Lemma 2

(a) Differentiating equation (1) with respect to m we get

$$\frac{\partial R^*}{\partial m} = -\frac{mR^{*2}}{c^2(Q-R^*)+m^2R^*} < 0,$$

and hence R^* is monotonically decreasing in monitoring efficiency m. Differentiation of (1) with respect to w and u yields

$$\frac{\partial R^*}{\partial w} = \frac{\partial R^*}{\partial u} = -\frac{cr_f^2}{c^2(Q-R^*)+m^2R^*} < 0.$$

Therefore, R^* is monotonically decreasing in net worth and the entrepreneur's outside option.

(b) Since

$$\theta^* = \frac{c(Q-R^*)+mR^*}{r_f},$$

it follows that

$$\frac{\partial \theta^*}{\partial m} = \frac{1}{r_f}\left[R^* - (c-m)\frac{\partial R^*}{\partial m}\right].$$

Given that $c > m$ and $\partial R^*/\partial m < 0$, the above expression is clearly strictly positive. On the other hand, differentiating θ^* with respect to w and u we get

$$\frac{\partial \theta^*}{\partial w} = \frac{\partial \theta^*}{\partial u} = -\frac{c-m}{r_f}\frac{\partial R^*}{\partial u} > 0.$$

Define by $\tilde{\theta}(m, w, u) := \theta(m, w, u) - \theta_0(m, w, u)$ the optimal monitoring for a type (m, w) firm. Recall that $\tilde{\theta} = mR^*/r_f$ from which it follows that

$$\frac{\partial \tilde{\theta}}{\partial m} = \frac{c^2R^*(Q-R^*)}{r_f[c^2(Q-R^*)+m^2R^*]} > 0,$$
$$\frac{\partial \tilde{\theta}}{\partial w} = \frac{\partial \tilde{\theta}}{\partial u} = \frac{m}{r_f}\frac{\partial R^*}{\partial u} < 0.$$

(c) The expected firm value is given by:

$$P^* = \frac{\theta^*(Q-R^*)}{r_f}.$$

Therefore,

$$\frac{\partial P^*}{\partial m} = \frac{1}{r_f}\left[\underbrace{(Q-R^*)\frac{\partial \theta^*}{\partial m}}_{+} - \underbrace{\theta^*\frac{\partial R^*}{\partial m}}_{-}\right] > 0,$$

$$\frac{\partial P^*}{\partial w} = \frac{1}{r_f}\left[\underbrace{(Q-R^*)\frac{\partial \theta^*}{\partial w}}_{+} - \underbrace{\theta^*\frac{\partial R^*}{\partial w}}_{-}\right] > 0,$$

$$\frac{\partial P^*}{\partial u} = \frac{1}{r_f}\left[\underbrace{(Q-R^*)\frac{\partial \theta^*}{\partial u}}_{+} - \underbrace{\theta^*\frac{\partial R^*}{\partial u}}_{-}\right] > 0.$$

(d) Substituting for $\theta(R)$ and $\theta_0(R)$ from the incentive compatibility constraints of the entrepreneur and the investor into the expressions of $V(R,\theta,\theta_0)$ and $U(R,\theta)$, the investor's maximization problem reduces to:

$$\begin{aligned}
\max_{R} \quad & V(R) := \frac{1}{r_f^2}\left[cR(Q-R)+\frac{mR^2}{2}\right]-(1-w) \\
\text{subject to} \quad & U(R) := \frac{1}{2r_f^2}\left[c(Q-R)^2-\frac{m^2R^2}{c}\right]-w=u, && \text{(PC}'\text{)} \\
& R \geq 0, && \text{(LLI)} \\
& Q-R \geq 0. && \text{(LLE)}
\end{aligned}$$

The Lagrangean is given by:

$$\begin{aligned}
\mathscr{L}(R,\nu,\nu_I,\nu_E) = {} & \frac{1}{r_f^2}\left[cR(Q-R)+\frac{mR^2}{2}\right]-(1-w) \\
& +\nu\left\{\frac{1}{2r_f^2}\left[c(Q-R)^2-\frac{m^2R^2}{c}\right]-w-u\right\}+\nu_I R+\nu_E(Q-R).
\end{aligned}$$

Notice that $V(0) = -(1-w) < 0$, and hence the investor is better of by not entering into a contractual agreement with the entrepreneur. Therefor, $R > 0$ implying $\nu_I = 0$. On the other hand,

$$U(Q) = -\left[\frac{m^2Q^2}{2cr_f^2}+w\right] < 0 \leq u.$$

Therefore, the participation constraint of the entrepreneur is not satisfied at $R = Q$. Thus, $R < Q$ which implies that $\nu_E = 0$. The first order condition with respect to R yields

$$\nu = \frac{cQ-(2c-m)R}{cQ-\frac{(c^2-m^2)R}{c}}. \tag{14}$$

Under the binding participation constraint, $\nu > 0$. On the other hand,

$$cQ-(2c-m)R < cQ - \frac{(c^2-m^2)R}{c} \iff c(c-m)+m^2 > 0,$$

and hence $\nu < 1$. By the Envelope theorem we have

$$\phi_1(m, w, u) = \frac{\partial \mathscr{L}}{\partial m} = \frac{mR^2}{cr_f^2}\left[\frac{c}{2m} - \nu\right],$$
$$\phi_2(m, w, u) = \frac{\partial \mathscr{L}}{\partial w} = 1 - \nu > 0,$$
$$\phi_3(m, w, u) = \frac{\partial \mathscr{L}}{\partial u} = -\nu < 0.$$

Given that $c/2m > 1$ and $\nu < 1$, the first expression is strictly positive, i.e., $\phi_1 > 0$. To show the strict concavity of the Pareto frontier, it follows from (14) that

$$\frac{\partial \nu}{\partial u} = -\frac{c^2 Q[c(c-m)+m^2]}{[c^2 Q - (c^2 - m^2)R]^2}\frac{\partial R}{\partial u} > 0.$$

Therefore, $\phi_{33} = -\partial \nu / \partial u < 0$, and hence the Pareto frontier is strictly concave in u.

Proof of Proposition 1

The second-order condition of the maximization problem of each type m investor is given by:

$$\frac{d^2[\phi(m, w, u(w))]}{dw^2} \leq 0$$
$$\Longrightarrow [\phi_{22}(m, w, u(w)) + \phi_{23}(m, w, u(w))u'(w)] \tag{15}$$
$$+ [\phi_{32}(m, w, u(w)) + \phi_{33}(m, w, u(w))u'(w)]u'(w)$$
$$+ \phi_3(m, w, u(w))u''(w) \leq 0, \text{ for } m = \lambda(w). \tag{16}$$

Differentiating (9) at $m = \lambda(w)$ one gets

$$u''(w) = -\frac{1}{\phi_3^2}[\phi_3(\phi_{21}\lambda'(w) + \phi_{22} + \phi_{23}u'(w)) - \phi_2(\phi_{31}\lambda'(w) + \phi_{32} + \phi_{33}u'(w))] \tag{17}$$

By substituting the expressions for $u'(w)$ and $u''(w)$ in (15), the inequality reduces to

$$[\phi_{21}(m, w, u(w)) + \phi_{31}(m, w, u(w))u'(w)]\lambda'(w) \geq 0. \tag{18}$$

We first show that $\phi_{21} < 0$ and $\phi_{31} < 0$. Note that $\phi_{21} = \phi_{31} = -\partial \nu / \partial m$. The optimal R and ν are simultaneously determined by the following two first order conditions:

$$c[c(Q-2R) + mR] = \nu[c^2(Q-R) + m^2 R],$$
$$c^2(Q-R)^2 - m^2 R^2 = 2cr_f^2(w+u).$$

Differentiating the above system we get

$$\begin{bmatrix} -\{c(2c-m) - (c^2 - m^2)\nu\} & -\{c^2(Q-R) + m^2 R\} \\ -\{c^2(Q-R) + m^2 R\} & 0 \end{bmatrix}\begin{bmatrix} \partial R / \partial m \\ \partial \nu / \partial m \end{bmatrix} = \begin{bmatrix} -(c - 2m\nu)R \\ mR^2 \end{bmatrix}$$

Therefore,

$$\frac{\partial \nu}{\partial m} = \frac{[\{c(2c-m) - (c^2 - m^2)\nu\}mR + (c - 2m\nu)(c^2(Q-R) + m^2 R)]R}{[c^2(Q-R) + m^2 R]^2}$$

Notice that $c(2c-m)-(c^2-m^2)\nu > 0$ since $\nu < 1$ and $c(2c-m)-(c^2-m^2) = c(c-m)+m^2 > 0$. Also, $c-2m\nu > 0$ as $c > 2m$ and $\nu < 1$. Therefore, the numerator of the above fraction is strictly positive implying that $\phi_{21} = \phi_{31} = -\partial\nu/\partial m < 0$. These in turn imply that $\phi_{21}(m, w, u(w)) + \phi_{31}(m, w, u(w))u'(w) < 0$ since $u'(w) > 0$. Therefore the second order condition (18) holds only if $\lambda'(w) < 0$, i.e., the equilibrium matching is negatively assortative. This completes the proof of the proposition.

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