

Financial Market Segmentation, Stock Market Volatility and the Role of Monetary Policy

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Abstract

We explore the role of monetary policy in a world of segmented financial markets, where the agents who trade stocks encounter financial income risk although the rest do not. In such an economy, we ask the question of how the monetary authority operates when it aims to maximize total welfare. We find that optimal monetary policy has the novel role of sharing the financial market risk traders face, among all agents in the economy. This finding holds for any concave utility function and is not sensitive to the degree of market segmentation. When risk is shared perfectly in this way, consumption is equalized between the two groups, and agents, if given the choice, would be indifferent between participating or not in the financial markets. We also explore the implications that this policy has for the volatility of stock prices and of inflation, when compared to the policies of constant money supply, inflation targeting and nominal interest rate pegging. We find that optimal monetary policy is not necessarily associated either with minimal stock price volatility or with minimal inflation volatility.

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1 Introduction

Should stock market changes be a concern of monetary policy? This widespread concern among central bankers, was addressed by Alan Greenspan in his famous

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December 1996 talk:

“But how do we know when irrational exuberance has unduly escalated asset values [...]? And how do we factor that assessment into monetary policy? [...] But we should not underestimate [...] the complexity of the interactions of asset markets and the economy. Thus, evaluating shifts in balance sheets generally, and in asset prices particularly, must be an integral part of the development of monetary policy.”

This paper attempts to develop a simple model to study the implications of an optimal, welfare maximizing monetary policy in the presence of a stock market. In addition it addresses the question of whether this policy entails lower stock price volatility when compared with other, widely used monetary policy rules.

In order to do that, we employed the finding of limited financial market participation (see Mankiw and Zeldes (1991), Vissing-Jørgensen (2002), Guiso, Haliassos and Jappelli (2002)). In our simple cash-in-advance model, the stock market provides its traders with a risky stream of total dividends, *à la* Lucas tree, which is shared among them according to the amount of shares they hold. The other group of agents residing in this model economy, the non-traders, does not participate in the financial markets; it has no real income risk but is however, subject to inflation risk. In addition, as is usually assumed in the limited participation literature (following Grossman and Weiss (1983), Rotemberg (1984)), financial market participants are directly influenced by monetary policy changes, although non participants are affected only indirectly through price adjustments. Specifically, only the traders receive positive transfers whenever money supply expands, although they get taxed whenever it contracts.

In such an environment, a monetary authority that assigns the population weight to each group of agents can resolve the market failure of limited participation by perfectly sharing the financial risk among traders and non-traders, maximizing in this way total welfare. Hence, optimal monetary policy equalizes the consumption of the two groups. In particular, optimal monetary policy becomes expansionary whenever dividend income is lower than expected, subsidizing traders with a positive transfer. Such a policy increases the price of the good, dismaying non-traders whose consumption decreases. On the other hand, whenever dividend income is higher than expected, monetary policy contracts, taxes traders and takes away part of their increased income. The good becomes more affordable and as an effect, non-traders' realize higher consumption. This result, as straightforward as it seems, is new in the literature and assigns to monetary policy the role of risk-sharing between heterogeneous agents. And it becomes more interesting as the heterogeneity here concerns stock market participation.

Furthermore, we compute the stock price volatility implied by the optimal monetary policy and compare it with the volatility implied by the constant money supply, inflation targeting and interest rate pegging policy rules. We find that the optimal monetary policy does not necessarily produce lower stock price volatility than the other policy rules and the outcome of the comparison depends on parameter values. The same results hold when we compare the inflation volatility these rules generate, except from the inflation targeting rule which by definition associates with minimum inflation volatility. Overall, this paper suggests that in the presence of a segmented stock market, there is a

new role for monetary policy which is to share financial market risk among all agents in the economy, so they consume the same amount, and is not associated either with minimal stock price variance or with minimal inflation variance.

Our work relates to various strands of the literature, in particular the financial limited participation literature which explores stock price volatility but ignores monetary policy issues, the empirical literature on the effects of monetary policy on the stock market and vice versa, and the limited participation literature which explores the liquidity effect. With respect to the financial limited participation literature, Allen and Gale (1994)'s work has been an important motivation for this paper. They assess stock price volatility to be the effect of low participation in financial markets in a model with agents who differ in their liquidity preferences. While the amount of cash available to these agents plays an important role in the model, monetary policy is in a sense ignored. In addition, Chien, Coll and Lustig (2007), Guo (2000) and Guvenen and Kuruscu (2006) explore stock price volatility in models with limited participation and idiosyncratic shocks and/or heterogenous preferences. In relation to this literature, we abstract from the issues of endogenous participation, idiosyncratic shocks and heterogenous preferences, to focus on the role of monetary policy in generating stock price volatility.

Concerning the empirical literature, Rigobon and Sack (2003) explore how the federal funds rate reacts to changes in the S&P500 using an improved identification technique. Bernanke and Kuttner (2005) and Rigobon and Sack (2004) study the exact reverse relationship, i.e. the stock market response to monetary policy shocks, and find that stock prices decrease as a response to monetary policy tightening. In addition, Bernanke and Gertler [2000, 2001] find that a kind of inflation targeting policy minimizes stock price volatility. These papers are related to this one to the extent that they raise arguments concerning the interplay between the stock market and monetary policy, while the specific methods and models used are unrelated.

A more related work is that of Bilbiie (2005), who among other questions, attempts to specify optimal monetary policy in a limited participation New Keynesian model. While he focuses on assigning relative weights on output and inflation in an interest rate Taylor-type of rule, we use a very different model to focus on the risk sharing role of optimal monetary policy. In addition, he does not address the issue of how stock price volatility is affected by monetary policy. Challe and Giannitsarou (2007) develop a full participation asset pricing New Keynesian model in order to match the empirical documentations of Bernanke and Kuttner (2005) and Rigobon and Sack (2004), i.e., the stock price response to monetary policy shocks. We do not attempt to match the data; we rather explore how various policy assumptions affect the stock market.

Theoretical research has previously utilized the well documented limited participation insight in order to capture the liquidity effect, as in Alvarez, Lucas and Weber (2001), forms of non-neutrality of money as in Williamson (2005) and Williamson (2006) and a positive inflation target as in Antinolfi, Azariadis and Bullard (2007). While the model we use and the character of the limited participation assumption we employ is very similar to these models, we attempt to answer very different questions.

We proceed as follows: The next section introduces the model economy

and studies the competitive equilibrium and asset prices. Section 3 describes the implications and role of the optimal, welfare maximizing monetary policy. Section 4 examines the stock price and inflation volatility for various policy rules and finishes with a subsection discussing the main points. Section 5 concludes.

2 The Model Economy

2.1 Environment

The model economy consists of the consumption good market and three asset markets: nominal bonds, stocks and money market. The bond and stock markets are segmented, so that from a continuum of infinitely lived households of measure one, only $\lambda \in (0, 1)$ participates in these markets while $1 - \lambda$ does not. The stock market is introduced in a way similar to the Lucas (1978) model. Participating agents receive a share of the stochastic dividend tree according to the amount of stocks they hold. The bonds are introduced for examining the asset pricing of the model and do not affect the agents' behavior.

All agents have identical preferences and seek to maximize their lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad (1)$$

where $0 < \beta < 1$.

The fraction $1 - \lambda$ of the population which does not participate in the financial markets, or the non-traders, receive every period a fixed real endowment y^N of the non-storable consumption good. The fraction λ of the population which participates in financial markets, or the traders, receive every period a fixed real endowment y^T and a share of the stochastic real total dividend ε_t . We assume, to be precise, that there is a firm which receives endowment ε_t at period t and distributes all this amount as dividends to its stock holders. The total dividend ε_t is random and defined as follows:

$$\varepsilon_t = \bar{\varepsilon} + \eta_t, \quad (2)$$

where η_t is an *iid* shock with mean zero and variance σ_{ε}^2 , and $\bar{\varepsilon} > 0$ is the mean of the total dividend shock. At period t each trader buys z_{t+1} share of the firm, so $z_{t+1}\varepsilon_t$ is interpreted as the real dividend each trader receives that period. Consequently, traders have a risky component in their income, while non-traders collect only the fixed endowment y^N . To make the analysis more interesting and explore various aspects of monetary policy, it is assumed that the mean income of the traders equals that of the non-traders i.e.

$$y^T + \frac{\bar{\varepsilon}}{\lambda} = y^N = \bar{y}. \quad (3)$$

Then the mean total dividend $\bar{\varepsilon} = \lambda(\bar{y} - y^T)$ depends on λ , so that the mean per trader dividend $\frac{\bar{\varepsilon}}{\lambda} = (\bar{y} - y^T)$ does not.

This is a cash-in-advance model, where agents can use only cash when entering the financial and goods markets. Credit is assumed away, introducing the following cash-in-advance constraints:

For traders:

$$m_t^T + q_t z_t + b_t + \tau_t \geq p_t c_t^T + q_t z_{t+1} + s_t b_{t+1}. \quad (4)$$

For non-traders:

$$m_t^N \geq p_t c_t^N, \quad (5)$$

where p_t is the price of the consumption good, q_t is the price of the share and s_t is the price of the nominal bond which pays one unit of money next period.

Traders and non-traders enter period t with available money holdings m_t^T and m_t^N respectively. Traders also collect a monetary transfer τ_t from the monetary authority.¹ This assumption captures the direct effect that monetary policy has on the financial markets' participants. In addition, agents receive real endowments y^T and y^N respectively, while traders receive part of the real total dividend, ε_t , too. In order for money to have value in equilibrium, we assume that no agent consumes his own endowment and dividends, but sells them instead for cash in the goods market. Each household uses the proceeds to buy, in the same market, the consumption good from other households.

The financial markets open first (while the goods market remains closed), where at period t traders can sell the b_t amount of bonds and z_t amount of stocks they bought at $t - 1$. Note that z_t is defined in terms of the number of titles each trader holds and can be sold at the price q_t , so traders receive $q_t z_t$ dollars for holding z_t stocks titles for a period. In contrast, bonds are bought at period $t - 1$ at the price $s_{t-1} < 1$ and pay back one unit of money at period t . Using their money holdings m_t^T , the money from selling their z_t stocks, the returns from holding b_t bonds and the money transfer τ_t , traders can decide how many new bonds and stocks titles to buy. After the financial markets close the goods market opens. Households consist of a shopper-seller pair, where the role of the shopper is to get the cash left after the transactions in the financial markets are over, and to buy consumption good from the other agents. The seller gets the real endowment y^N if he is a non-trader or the real endowment y^T and part of the real dividends ε_t if he is a trader, and sells them for cash. After the operations in the goods market are over, the seller and the shopper meet, consume the good the shopper purchased and keep the cash the seller received as their money holdings for the next period. Thus, endowments and dividends distributed at period t do not finance consumption at period t and do not appear as terms at the cash-in-advance constraints (4) and (5). They are used instead as money holdings for entering period $t + 1$. The budget constraints are given below:

For traders:

$$m_t^T + q_t z_t + b_t + \tau_t + p_t z_{t+1} \varepsilon_t + p_t y^T \geq m_{t+1}^T + p_t c_t^T + q_t z_{t+1} + s_t b_{t+1}, \quad (6)$$

where $d_t = z_{t+1} \varepsilon_t$ are the real dividend payments distributed at period t (but available to use at $t + 1$).²

¹This practice is equivalent to open market operations, which affect directly the financial market participants.

²We can alternatively have an authority that taxes only the traders, can change the money supply, while keeps the real debt bounded.

For non-traders:

$$m_t^N + p_t y^N \geq m_{t+1}^N + p_t c_t^N. \quad (7)$$

Because assets markets operate before the goods market open, holding money after the financial markets close bears positive opportunity cost when the return for bonds or stocks is positive. Only the amount of money required for purchasing the desired amount of consumption good is held and the equilibrium is constructed with binding cash-in-advance constraints. Later we examine the conditions which guarantee that. The implications for the budget constraints are:

For traders:

$$p_t z_{t+1} \varepsilon_t + p_t y^T = m_{t+1}^T. \quad (8)$$

For non-traders:

$$p_t y^N = m_{t+1}^N. \quad (9)$$

The above equations reveal that the cash balances with which the agents begin period $t + 1$ match the fraction of their wealth that the cash-in-advance constraints prevented them from using at period t . These are, the proceeds from selling the real endowments at the goods market and for the case of traders, cashing out the real dividends distributed at period t .

2.2 Competitive Equilibrium and Asset Pricing

This section examines the competitive equilibrium and the asset pricing of the model. The four market clearing conditions in this economy are as follows:

For the goods market to clear, total real output is completely consumed by traders and non-traders at every period, as this is a non-storable good.

$$Y_t = \varepsilon_t + \lambda y^T + (1 - \lambda) y^N = \lambda c_t^T + (1 - \lambda) c_t^N.$$

Using equation (3) which states that the mean income of the two groups is the same, it turns out that the goods market clearing condition becomes:

$$Y_t = \bar{y} + \varepsilon_t - \bar{\varepsilon} = \lambda c_t^T + (1 - \lambda) c_t^N. \quad (10)$$

For the stock market to clear, the sum of all shares each trader holds should equal the total share of the stochastic total dividend distributed as shares. We assume that all the stochastic total dividend is distributed:

$$\lambda z_{t+1} = 1 \Rightarrow z_{t+1} = \frac{1}{\lambda}. \quad (11)$$

For the bond market to clear, the sum of all real bonds each trader holds should equal the total supply of them, which is zero:

$$\lambda b_t = 0. \quad (12)$$

For the money market to clear, the total money holdings of traders and non-traders should equal the total amount of money supplied in the economy, \bar{M}_t . The money market clearing condition is as follows:

$$\lambda m_{t+1}^T + (1 - \lambda) m_{t+1}^N = \bar{M}_t = \bar{M}_{t-1} (1 + \mu_t) = \lambda \tau_t + \bar{M}_{t-1}, \quad (13)$$

where μ_t denotes money growth from time $t - 1$ to time t . The extra money supplied at time t are distributed as transfers to the λ traders.

Substituting the bond, stock and money market clearing conditions in the cash-in-advance equations (4) and (5) we get the straightforward cash-in-advance restrictions:

For traders:

$$m_t^T + \tau_t \geq p_t c_t^T.$$

For non-traders:

$$m_t^N \geq p_t c_t^N,$$

which show that consumption expenditure cannot exceed the monetary resources agents began the period with, plus, for the case of the traders, the monetary transfer. This is true because at any given period, agents do not consume their own endowments and dividend income, but have to sell these resources and finally use them as money balances for the next period.

Furthermore, from the goods market clearing condition (10) and the cash-in-advance constraints (4) and (5) holding with equality, the following condition is implied:

$$p_t Y_t = \lambda q_t (z_t - z_{t+1}) + \bar{M}_t + \lambda (b_t - s_t b_{t+1}).$$

Applying the stock, bond, and money market clearing conditions (11), (12) and (13) in the above equation, we get a version of the quantity equation where total output is the sum of the deterministic part $\lambda y^T + (1 - \lambda) y^N = \bar{y} - \bar{\varepsilon}$, and the stochastic part ε_t , and velocity equals one:

$$p_t = \frac{\bar{M}_t}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}. \quad (14)$$

While in Alvarez, Lucas and Weber (2001) the instability between prices and money stems from velocity shocks, in this model an important role is played by the stochastic part of output distributed as dividends to the stock market participants. In particular, an increase in the total real dividend shock the participants receive puts downward pressure on prices while a decrease in the real dividend shock increases the price level.

Equilibrium consumption for the two groups of agents is calculated as follows. Combining the non-traders binding cash-in-advance constraint (5) with equation (9) we find the expression below:

$$p_{t-1} y^N = p_t c_t^N.$$

Substituting for the consumption good price from equation (14), it turns out that the non-traders consumption is given by the following equation:

$$c_t^N = \frac{p_{t-1}}{p_t} \bar{y} = \bar{y} \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{1}{1 + \mu_t}. \quad (15)$$

Using the above equation and the market clearing condition for the goods market, given by equation (10), it turns out that the traders' consumption can be written as follows:

$$c_t^T = \frac{p_{t-1}}{p_t} \left(\bar{y} + \frac{1}{\lambda} (\varepsilon_{t-1} - \bar{\varepsilon}) \right) + \frac{\tau_t}{p_t} =$$

$$= \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\lambda} \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t)}. \quad (16)$$

Note that when there is full participation in the financial markets, consumption for each trader equals total output:

$$c_t^T = \bar{y} + (\varepsilon_t - \bar{\varepsilon}) = Y_t,$$

and monetary policy becomes neutral. That is, expansionary monetary policy increases prices but also transfers, which in this case are distributed equally to all agents. These two effects cancel out without affecting consumption. However, in a limited participation economy monetary authority affects prices as before but distributes transfers only to the traders. In this case, as equations (15) and (16) reveal, monetary policy exhibits distributional effects, controlling the amount consumed by each type of agent. In an expansion, monetary policy creates an inflation tax for all households, but distributes monetary transfers only to the traders, increasing their consumption and decreasing that of non-traders. On the other hand, whenever monetary policy contracts, consumption becomes cheaper for both types of agents but only the traders are taxed, so their consumption shrinks while non-traders' consumption increases.

Also, notice how, in the case of limited participation dividend income affects the consumption of the two groups. The equilibrium consumption equations reveal that an increase in the current total dividends distributed, ε_t , implies lower price for the good in period t , increasing consumption for both traders and non-traders. On the other hand, an increase in total real dividends distributed last period ε_{t-1} , decreases the price of the good at period $t - 1$ and thus the value of the good carried in the form of money balances from period $t - 1$ to period t . Assuming that monetary policy does not react to such a shock, that $1 + \mu_t \geq 0$ and that the dividend shocks are independent across time, consumption in period t decreases for non-participants while for participants the change is positive:

$$\frac{\partial c_t^N}{\partial \varepsilon_{t-1}} = -\frac{(\bar{y} + (\varepsilon_t - \bar{\varepsilon}))\bar{y}}{(1 + \mu_t)(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))^2} < 0,$$

and

$$\frac{\partial c_t^T}{\partial \varepsilon_{t-1}} = \frac{(\bar{y} + (\varepsilon_t - \bar{\varepsilon}))\bar{y}(1 - \lambda)}{\lambda(1 + \mu_t)(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))^2} > 0.$$

This is because, except from the indirect price effect, hearing all agents, traders actually receive part of the ε_{t-1} shock as a dividend and this effect turns out to be stronger. As explained earlier, the cash-in-advance constraints allow ε_{t-1} to be observed at period $t - 1$ but is available for consumption only at period t . Because of the limited participation assumption, the increase in dividends is higher than the decrease in prices and thus the traders are better off. Also note that the increase in traders consumption is $\frac{1-\lambda}{\lambda}$ higher than the decrease in non-traders consumption. This is because the increase in traders consumption depends on the participation rate, as the more participating agents, the smaller the share of the ε_{t-1} shock each of them receives. Then the increase in traders consumption is negatively affected by the participation rate.

While the above analysis did not require solving the maximization problem, the analysis of the asset prices requires such a procedure. In particular the traders' utility maximization problem needs to be solved, subject to the cash-in-advance and budget constraints.

It turns out that the bond price in terms of currency is determined as follows:

$$\beta E_t \frac{u'(c_{t+1}^T)}{p_{t+1}} = \frac{u'(c_t^T)}{p_t} s_t. \quad (17)$$

Equation (17) describes the pricing of the nominal bond: the utility increase traders expect to receive at period $t+1$, when the bond matures and pays back, equals the foregone utility they suffer from buying the nominal bond at period t . In addition, this equation reveals a Fisher effect. Defining the nominal rate as $r_t \equiv \frac{1}{s_t} - 1$, the real rate as $r_t^r \equiv \frac{p_t}{s_t p_{t+1}} - 1$ and inflation as $\pi_t \equiv \frac{p_{t+1}}{p_t} - 1$, we have that $r_t = r_t^r \frac{p_{t+1}}{p_t} + \pi_{t+1}$ which gives approximately the Fisher effect.

Note also that for the cash-in-advance constraint to bind, the multiplier associated with the cash-in-advance constraint should be strictly positive, implying that $s_t < 1$ and then the nominal rate is strictly positive.

In addition, the first order conditions imply the following expression for the stock price:

$$\beta E_t \frac{u'(c_{t+1}^T)}{p_{t+1}} (q_{t+1} + p_t \varepsilon_t) = \frac{u'(c_t^T)}{p_t} q_t, \quad (18)$$

which evaluates that the discounted marginal utility expected at period $t+1$, when the dividends are paid and the stock can be re-traded, equals the foregone utility at time t incurred from purchasing the stock.

We will return to the stock price and its volatility later, when we will apply various policy specification in equation (18) and compare across them.

3 Optimal Monetary Policy

In this section we study the implications of optimal monetary policy. The assumption adopted is that the monetary authority sets the money supply growth rate in an attempt to maximize total welfare.

The monetary authority is assumed to assign equal weight to each agent, so there is λ weight assigned to the group of traders and $1 - \lambda$ to the group of non-traders. The maximization problem is as follows:

$$\text{Max}_{\mu_t} V_t = \text{Max}_{\mu_t} \sum_{t=0}^{\infty} \beta^t (\lambda u(c_t^T) + (1 - \lambda) u(c_t^N)). \quad (19)$$

The first order conditions imply the following:

$$\lambda \frac{\partial u(c_t^T)}{\partial c_t^T} \frac{\partial c_t^T}{\partial \mu_t} + (1 - \lambda) \frac{\partial u(c_t^N)}{\partial c_t^N} \frac{\partial c_t^N}{\partial \mu_t} = 0. \quad (20)$$

From equations (15) and (16) which determine consumption in equilibrium, we can calculate the derivative of consumption with respect to money growth:

$$\frac{\partial c_t^N}{\partial \mu_t} = - \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{\bar{y}}{(1 + \mu_t)^2}$$

and

$$\frac{\partial c_t^T}{\partial \mu_t} = \frac{1 - \lambda}{\lambda} \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{\bar{y}}{(1 + \mu_t)^2}.$$

Note that non-traders' consumption decreases whenever monetary policy expands while traders consumption increases. This is because expansionary monetary policy increases current prices, affecting negatively all agents. The traders however, receive the monetary transfer and the positive effect is stronger. This is because of the limited participation assumption that prevents monetary authority to share its transfers to all agents but shares it instead only among the participating agents. Also note, as for the case of a change in ε_{t-1} , the increase in traders' consumption is $\frac{1-\lambda}{\lambda}$ higher than the decrease in non-traders consumption. This is because the increase on traders consumption depends on the participation rate. The higher the participation rate, the smaller the share of the monetary expansion each of the traders receive. An increase in traders consumption is negatively affected by the participation rate.

Substituting the above equations into equation (20) we find the following:

$$\frac{\frac{\partial u(c_t^N)}{\partial c_t^N}}{\frac{\partial u(c_t^T)}{\partial c_t^T}} = 1. \quad (21)$$

Optimal monetary policy will attempt to equate the marginal utility of consumption for the two types of agents. Then for any concave utility specification it turns out that optimal monetary policy equates consumption for the two groups:

$$\frac{c_t^T}{c_t^N} = 1.$$

This implies that optimal money growth is as given below:

$$\mu_t^{opt} = \frac{-(\varepsilon_{t-1} - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})}$$

or,

$$1 + \mu_t^{opt} = \frac{\bar{y}}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})}. \quad (22)$$

The above expression reveals a new role for monetary policy, that of sharing the financial income risk among all agents in the economy. Limited participation in the stock market exposes only a fraction of the population to dividend income risk, although all agents are subject to inflation risk. Monetary policy, through its distributional effects, acquired because of the limited participation assumption, has the ability to undo this market failure, and share the dividend risk among all agents. When total real dividend is lower than the total mean dividend $\bar{\varepsilon}$, optimal monetary policy expands, increasing money supply by the fraction of extra total dividends to total output. This is because whenever traders receive low dividend payments, their consumption decreases. Optimal monetary policy expands and increases traders consumption by distributing higher transfers to them. As monetary policy expands, current prices increase, reducing non-traders consumption. On the other hand, whenever total dividends are higher than the total mean dividend, offering high consumption to

traders, then optimal monetary policy contracts, taxing traders, so that prices decrease causing non-traders' consumption to rise. Also note, that because of the cash-in-advance constraint, traders at time t consume the previous periods' dividend. Then, optimal policy at time t reacts to total dividend distributed at time $t - 1$.

The above result does not depend on the utility specification, as long as a concave utility function is used. In addition, it is not sensitive to the fixed endowment assumption. In the case of random mean endowments \bar{y} , optimal monetary policy would still share the extra risk that participants hold, between the two types of agents and increase in this way total welfare.

Following optimal monetary policy, consumption of traders and non traders each period is given below:

$$c_t^{N*} = c_t^{T*} = Y_t = \bar{y} + (\varepsilon_t - \bar{\varepsilon}).$$

We see that optimal monetary policy shares the risk perfectly between the two groups. In addition, although we are using an exogenous participation framework, optimal monetary policy makes agents indifferent between participating in the financial markets or not. Precisely, non-traders would vote for any policy with $\mu_t > \mu_t^{opt}$ as such a policy would increase their consumption relative to what they consume under the optimal monetary policy regime. On the contrary, traders would vote for any policy with $\mu_t < \mu_t^{opt}$ for the same reason. Optimal monetary policy equates consumption for the two groups and makes agents indifferent between participating or not in the financial markets.

Finally, note that contrary to Bilbiie (2005), optimal monetary policy in this model does not depend on the participation rate. This is true because monetary authority has direct effects only on the same group of agents that faces the dividends risk. High traders' dividends are offset by contractionary policy and low traders' dividends are enhanced by expansionary policy. Both total dividend and money injected are shared among traders, so that the participation rate does not affect monetary policy's decisions. However, for monetary policy to have any real effects, the limited participation assumption is necessary. Hence, as soon as financial markets are segmented, no matter to what extent, monetary policy can operate in such a way that everybody shares and consumes the total output and becomes indifferent between participating or not in the financial markets.

4 Stock Price and Inflation Volatility

4.1 Stock Price Volatility

In this section we compute the stock price volatility for four policy rules: optimal, constant money supply, inflation targeting and nominal interest rate pegging. We ask the question whether optimal monetary policy implies lower stock price volatility when compared with the other policy rules and we find that this is not necessarily the case.

For the analysis that follows we need to specify a utility representation in order to compute explicitly the stock price and its variance. In this section we

use the logarithmic utility function, which is replaced by a generalized CRRA function in a later section. We first calculate the real stock price as $\hat{q}_t = \frac{q_t}{p_t}$, using equation (18). The recursive solution of this expression, assuming that there are no bubbles so that the transversality condition holds, is as follows:

$$\hat{q}_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{c_t^T}{c_{t+j}^T} \frac{p_t}{p_{t+j}} \varepsilon_{t+j-1}.$$

That is, we can write the stock price as the expected discounted value of all future dividends. After substituting in that expression prices and trader's consumption given by equations (14) and (16), we find the expression below for the stock price:

$$\hat{q}_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t)} \frac{(\bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon}))(1 + \mu_{t+j})}{(\varepsilon_{t+j-1} - \bar{\varepsilon})(1 + \mu_{t+j}) + \bar{y}(\lambda + \mu_{t+j})} \frac{\varepsilon_{t+j-1}}{\prod_{s=1}^j (1 + \mu_{t+s})}. \quad (23)$$

In the examples that follow we compute the variance of the stock price for a choice of policy rules. To do that we could either substitute in (23) the policy rule we wish to examine and then compute the variance of the stock price under that rule, or we could compute the variance of (23) and then substitute for μ , the policy rule we wish to consider. We are taking the first approach, seeing that in order to calculate the variance of the stock we first need to linearize around the mean total dividend shock. And to do that we need to know how monetary policy reacts to dividend changes. Then, substituting the policy rule under consideration into (23) and then linearize around the mean total dividend value seems a more plausible approach to follow.

4.1.1 Optimal Monetary Policy

We compute the stock price under the assumption that monetary policy is conducted optimally:

$$\hat{q}_t^{opt} = E_t \sum_{j=1}^{\infty} \beta^j \frac{\varepsilon_{t+j-1}}{\bar{y}} \prod_{s=1}^j (\bar{y} + (\varepsilon_{t+s-1} - \bar{\varepsilon})).$$

When the shocks hitting total dividends are independently and identically distributed (*iid*), then by linearizing around their mean value $\bar{\varepsilon} = \lambda(\bar{y} - y^T)$ we find that the stock price and its unconditional variance for the optimal monetary policy rule, are as follows:

$$\hat{q}_t^{opt} \simeq \frac{\beta \lambda (\bar{y} - y^T)}{1 - \beta} + \frac{\beta}{1 - \beta} \frac{\bar{y}(1 - \beta) + \lambda(\bar{y} - y^T)}{\bar{y}} \eta_t, \quad (24)$$

$$Var(\hat{q}_t^{opt}) = \frac{\beta^2 \sigma_{\varepsilon}^2}{\bar{y}^2 (1 - \beta)^2} [\bar{y}(1 - \beta) + \lambda(\bar{y} - y^T)]^2.$$

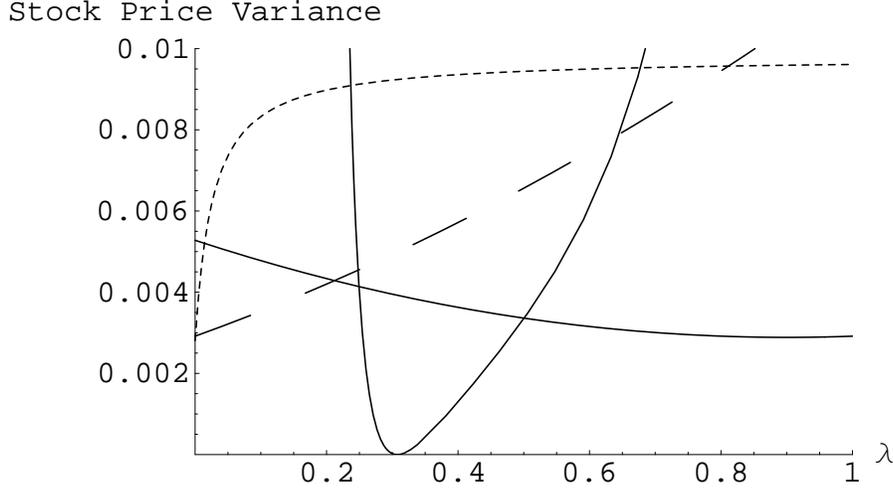


Figure 1: Stock price volatility of optimal (dashed line), constant money supply (downward sloping solid line), inflation targeting (dotted line) and interest rate peg (hump shaped solid line) policy rules.

where as discussed earlier, σ_{ε}^2 is the variance of the total dividend and η_t is defined in equation (2).

Higher dividend volatility in (24), translates to higher stock price volatility, which is true since the current dividend shock, $\eta_t = \varepsilon_t - \bar{\varepsilon}$, and the stock price are positively correlated. Intuitively, a positive shock in current dividends would make traders want to consume more today but also in the future, prompting them to buy more assets. However, when optimal monetary policy is followed, traders are induced to consume the total output because $c_t^{T, \mu=opt} = Y_t = \bar{y} + (\varepsilon_t - \bar{\varepsilon})$. Hence the price of the stock has to increase.

Furthermore, under optimal monetary policy, agents' consumption is not affected by the financial markets' participation rate because it equals aggregate output, as we show in the previous section. However, higher participation increases the mean total dividend, which in turns positively affects the stock price and its variance. This happens because, having higher dividends, traders buy more assets in order to smooth consumption over time. Yet, they are induced to consume the aggregate output, so there is pressure for the stock price to increase.

In the following sections we compute the stock price and its variance under various policy rules and compare them to that produced under the optimal monetary policy rule.

Parameter	Symbol	Value
Mean income	\bar{y}	1
Trader's endowment	y^T	0.9
Total dividend variation	σ_ε	0.06
Discount factor	β	0.9

Table 1: Parameter Values

4.1.2 Constant Money Supply Policy

We now turn our analysis to the zero money growth policy, which by setting $\mu_i = 0$ for every i in equation (23), implies that:

$$\hat{q}_t^{\mu=0} = E_t \sum_{j=1}^{\infty} \beta^j \frac{\lambda \bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})} \frac{\bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon})}{\lambda \bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon})} \varepsilon_{t+j-1}.$$

Assuming *iid* dividend shocks and by linearizing around their mean, we get the linearized stock price and its variance:

$$\begin{aligned} \hat{q}_t^{\mu=0} \simeq & \frac{\beta}{1-\beta} \lambda (\bar{y} - y^T) + \frac{\beta}{1-\beta} \frac{(1-\lambda)(\bar{y} - y^T)}{\bar{y}} \eta_{t-1} \\ & + \frac{\beta}{\bar{y}} (\lambda (\bar{y} - y^T) + y^T) \eta_t \end{aligned}$$

$$Var(\hat{q}_t^{\mu=0}) = \frac{\beta^2 \sigma_\varepsilon^2}{\bar{y}^2 (1-\beta)^2} [(1-\lambda)^2 (\bar{y} - y^T)^2 + (1-\beta)^2 (\lambda (\bar{y} - y^T) + y^T)^2]. \quad (25)$$

Here we see that higher dividend volatility translates to higher stock price volatility, through its effect on the volatility of consumption and prices. However, the effect of the financial market segmentation on the stock price and its variance is not straightforward. Under the constant money supply rule, consumption depends on the degree of segmentation, since $c_t^{T, \mu=0} = \frac{p_{t-1}}{p_t} (\bar{y} + \frac{1}{\lambda} (\varepsilon_{t-1} - \bar{\varepsilon}))$. Higher participation induces a fall in consumption whenever $\varepsilon_{t-1} - \bar{\varepsilon} = \eta_{t-1} > 0$ as the traders have to share the high dividends with more agents. But this is true for both current and future consumption. That is, whenever the dividend shock affecting consumption in the next period, i.e. $\eta_t = \varepsilon_t - \bar{\varepsilon}$, is positive and substantially higher than a positive shock affecting current consumption, i.e. $\eta_{t-1} = \varepsilon_{t-1} - \bar{\varepsilon} > 0$, then a higher degree of market segmentation decreases future consumption more than the current one, urges traders to buy more assets and hence raises the stock price. The effects of participation on the stock price volatility become ambiguous and depend on the parameter values.

Comparing equations (24) and (25) we see that it is not obvious which of the two policies produces higher stock price volatility. The result would depend again on the parameter values. For illustrating the result that optimal monetary policy is not necessarily associated with low stock price volatility we use the following example: Leaving the participation rate λ free and for $\bar{y} = 1$,

$y^T = 0.9$, $\beta = 0.9$ and $\sigma_\varepsilon = 0.06$ we see at figure (1) that there is a critical value for the participation rate in the horizontal axes, below which optimal monetary policy produces less volatility than the constant money supply policy, and above which optimal monetary policy generates higher volatility. For convenience, we summarize the parameter values in table (1).

4.1.3 Inflation Targeting Policy

We now explore the inflation targeting policy and its implication for stock price volatility. Inflation for any monetary policy rule μ_t is given below:

$$\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{(1 + \mu_t)(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})} - 1, \quad (26)$$

and for inflation target $\pi_t = \bar{\pi}$ the corresponding monetary policy action is:

$$1 + \mu_t^{\bar{\pi}} = (\bar{\pi} + 1) \frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon})}. \quad (27)$$

The above equation implies that for $\bar{\pi} + 1 > 0$, which is true for positive prices, whenever current dividends are low compared to the previous period, inflation increases. The monetary authority, aiming at attaining its inflation target contracts, taxes the traders and reduces inflation back to its target level. Equation (23) for $\mu_t = \mu_t^{\bar{\pi}}$ becomes:

$$\hat{q}_t^{\mu_t^{\bar{\pi}}} = E_t \sum_{j=1}^{\infty} \beta^j \frac{(\bar{\pi} + 1)(\bar{y} + (\varepsilon_t - \bar{\varepsilon})) - \bar{y}(1 - \lambda)}{(\bar{\pi} + 1)(\bar{y} + (\varepsilon_{t+j} - \bar{\varepsilon})) - \bar{y}(1 - \lambda)} \frac{\varepsilon_{t+j-1}}{(\bar{\pi} + 1)^j}.$$

Linearizing around the mean of the *iid* dividend shocks we get the linearized stock price and its variance for the inflation targeting monetary policy rule:

$$\begin{aligned} \hat{q}_t^{\bar{\pi}} &\simeq \frac{\beta\lambda(\bar{y} - y^T)}{\bar{\pi} + 1 - \beta} + \frac{\beta\eta_t}{(\bar{\pi} + 1 - \beta)(\bar{\pi} + \lambda)(\bar{\pi} + 1)\bar{y}} ((\bar{\pi} + \lambda)\bar{y}(\bar{\pi} + 1 - \beta) + \\ &\quad + \lambda(\bar{y} - y^T)(\bar{\pi} + 1)^2), \\ \text{Var}(\hat{q}_t^{\bar{\pi}}) &= \frac{\beta^2\sigma_\varepsilon^2}{(\bar{\pi} + 1 - \beta)^2(\bar{\pi} + \lambda)^2(\bar{\pi} + 1)^2\bar{y}^2} \\ &\quad ((\bar{\pi} + \lambda)\bar{y}(\bar{\pi} + 1 - \beta) + \lambda(\bar{y} - y^T)(\bar{\pi} + 1)^2)^2. \end{aligned} \quad (28)$$

An increase in the variance of the dividend shock increases the volatility of the asset prices. The effect of the degree of segmentation though is not clear, as it has ambiguous effects on consumption, which is calculated below for the inflation targeting rule:

$$c_t^{T, \mu=\bar{\pi}} = \frac{1}{\bar{\pi} + 1} (\bar{y} + \frac{1}{\lambda}(\varepsilon_{t-1} - \bar{\varepsilon})) + \frac{(\bar{\pi} + 1)(\bar{y} + (\varepsilon_t - \bar{\varepsilon})) - (\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))}{\lambda(\bar{\pi} + 1)}.$$

Similarly to the constant money supply rule, whenever $\eta_{t-1} = \varepsilon_{t-1} - \bar{\varepsilon} > 0$, increased participation decreases current consumption because there are more traders to share the high dividends. This effect can be seen at the first part of

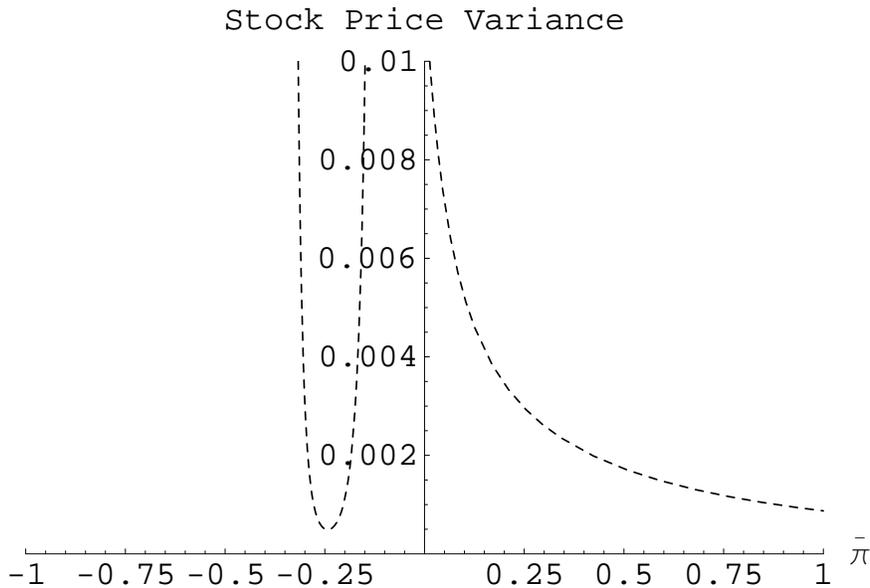


Figure 2: Stock price volatility of inflation targeting policy

the traders' consumption equation. However, here we have an additional effect through the monetary transfer, which was absent from the constant money supply policy and shows up at the second part of the consumption equation. Specifically, whenever $\eta_{t-1} > 0$, inflation targeting monetary policy contracts. Then, traders prefer to have many agents in their group, so they share the tax. The total effect of higher degree of segmentation on consumption and as a result also on the stock price and its variance, would depend on the magnitude of the shocks and the specific inflation target the monetary authority sets.

Additionally, depending on the inflation target the monetary authority chooses, we can compare the inflation targeting policy with the optimal and the constant money supply policies. For example, for $\bar{\pi} = 0$, the inflation targeting policy implies higher stock price volatility from both the two alternative policies. Especially when compared with the optimal policy, the zero inflation targeting policy produces higher stock price volatility because of the limited participation assumption. When there is full participation in the financial markets these two policies generate the same amount of stock price variance. For limited participation however, optimal policy is associated with lower volatility. Hence, in this specific case of zero inflation targeting, optimal monetary policy is indeed associated with lower stock price volatility.

As a second example of inflation targeting policy we use the policy implied by setting inflation equal to $\bar{\pi} = 2\%$. Using the same parameter values as before, summarized in table (1), we now compute the stock price variance for the 2% inflation targeting policy. We see at figure (1) that in this case, choosing

the policy that produces the least stock price variability depends crucially on the financial markets participation rate and on the other parameters values.

Using the same parameter values and for $\lambda = 35\%$ we see in figure (2) how stock price volatility varies with the inflation target. It seems that the stock price varies less under the inflation targeting policy when the target is high, implying that an inflation targeting monetary authority which cares about stock price volatility, may be setting higher inflation targets. For these values of the parameters, the optimal policy produces volatility equal to $Var(\hat{q}_t^{opt}) = 0.0053$ and the constant money supply, $Var(\hat{q}_t^{\mu=0}) = 0.00378$. Hence the stock price varies less under the inflation targeting policy when the target is very high, while for lower inflation targets the optimal monetary policy rule and especially the constant money supply policy, generates less volatility than the inflation targeting policy.

4.1.4 Nominal Interest Rate Peg Policy

The last example of policy rule we consider is that of the nominal interest rate peg. Following up with equation (17) and by substituting the traders' equilibrium consumption and the price level, the bond price becomes:

$$s_t = \beta \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t)} (\bar{y} + (\varepsilon_t - \bar{\varepsilon})) E_t \frac{1}{(\varepsilon_t - \bar{\varepsilon})(1 + \mu_{t+1}) + \bar{y}(\lambda + \mu_{t+1})}.$$

Linearizing the expression inside the expectation around $\mu_{t+1} = \bar{\mu} = 0$ and solving for the nominal interest rate $r_t = \frac{1}{s_t} - 1$, it turns out that the nominal interest rate is:

$$r_t + 1 = \frac{1}{\beta} \frac{(1 + \mu_t)(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})} \frac{(\lambda \bar{y} + (\varepsilon_t - \bar{\varepsilon}))^2}{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)} \frac{1}{\lambda \bar{y} + (\varepsilon_t - \bar{\varepsilon}) - (\bar{y} + (\varepsilon_t - \bar{\varepsilon})) E_t(\mu_{t+1})}. \quad (29)$$

Before we explore the stock price implications of the nominal rate peg policy, we first examine the liquidity effect of the model. Differentiating the above expression with respect to current money growth we see that whenever $E_t(\mu_{t+1}) < \frac{\lambda \bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}$ then $\frac{\partial r_t}{\partial \mu_t} < 0$. Furthermore, this derivative becomes zero when there is full participation in the financial markets, i.e whenever $\lambda = 1$, signifying the liquidity effect. In addition, for limited participation and for $\mu_t + 1 > 0$ it turns out that the nominal interest rate increases with a rise in the expected money supply growth, i.e $\frac{\partial r_t}{\partial E_t \mu_{t+1}} > 0$.

To evaluate the stock price volatility we compute the money supply policy implied by pegging the nominal interest rate at the level \bar{r} , as follows:

$$\mu_t^{\bar{r}} + 1 = \frac{(\bar{r} + 1)\beta(\bar{y} + (\varepsilon_t - \bar{\varepsilon}))\bar{y}(1 - \lambda)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))[(\bar{r} + 1)\beta(\bar{y} + (\varepsilon_t - \bar{\varepsilon})) - (\lambda \bar{y} + \varepsilon_t - \bar{\varepsilon})]}. \quad (30)$$

The above equation reveals that whenever the dividend shock consumed at the current period is high, i.e $\eta_{t-1} > 0$, the traders are prompt to buy more assets, the price of the bond rises and the nominal interest rate decreases. The

monetary authority which aims at keeping the nominal interest rate at a specific level would then contract, tax the traders, reduce their consumption and bring in this way the nominal interest rate back to its target. In addition, whenever $\eta_t > 0$ future consumption is expected to rise. Traders tend to buy fewer assets today, forcing the price of the bond to fall and the nominal interest rate to rise. To keep the rate at its target, monetary authorities react by increasing money supply.

The stock price equation (23) for $\mu = \mu^{\bar{r}}$ becomes:

$$\hat{q}_t^{\bar{r}} = E_t \sum_{j=1}^{\infty} \beta^j \frac{\lambda \bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})} \frac{\bar{y} + (\varepsilon_{t+j} - \bar{\varepsilon})}{\lambda \bar{y} + (\varepsilon_{t+j} - \bar{\varepsilon})} \frac{\varepsilon_{t+j-1}}{((\bar{r} + 1)\beta \bar{y}(1 - \lambda))^j}$$

$$\prod_{s=1}^j \frac{(\bar{y} + (\varepsilon_{t+s-1} - \bar{\varepsilon}))[(\bar{r} + 1)\beta(\bar{y} + (\varepsilon_{t+s} - \bar{\varepsilon})) - (\lambda \bar{y} + (\varepsilon_{t+s} - \bar{\varepsilon}))]}{\bar{y} + (\varepsilon_{t+s} - \bar{\varepsilon})}.$$

Linearizing as usual around $\bar{\varepsilon} = \lambda(\bar{y} - y^T)$ we find the stock price and its variance for the interest rate pegging policy are:

$$\hat{q}_t^{\bar{r}} \simeq \lambda(\bar{y} - y^T) \frac{(\bar{r} + 1)\beta - \lambda}{(\bar{r} + 1)(1 - \lambda) - (\bar{r} + 1)\beta + \lambda} + ((\bar{r} + 1)\beta - \lambda)\eta_t$$

$$\frac{(\bar{y} - y^T)(\bar{r} + 1)(1 - \lambda) + \bar{y}((\bar{r} + 1)(1 - \lambda) - (\bar{r} + 1)\beta + \lambda)}{\bar{y}(\bar{r} + 1)(1 - \lambda)((\bar{r} + 1)(1 - \lambda) - (\bar{r} + 1)\beta + \lambda)}$$

and

$$Var(\hat{q}_t^{\bar{r}}) = \frac{((\bar{r} + 1)\beta - \lambda)^2 \sigma_{\varepsilon}^2}{[(\bar{y}(\bar{r} + 1)(1 - \lambda)((\bar{r} + 1)(1 - \lambda) - (\bar{r} + 1)\beta + \lambda)]^2}$$

$$[(\bar{y} - y^T)(\bar{r} + 1)(1 - \lambda) + \bar{y}((\bar{r} + 1)(1 - \lambda) - (\bar{r} + 1)\beta + \lambda)]^2. \quad (31)$$

The monetary authority' choice of interest rate peg, combined with the parameters values will determine whether or not this policy creates higher stock price volatility than the other policies considered. We first examine the example of pegging the equilibrium rate, \bar{r} , derived by setting $\mu = 0$ and $\varepsilon_t = \bar{\varepsilon}$ in equation (30):

$$\bar{r} + 1 = \frac{1}{\beta}.$$

It turns out that the stock price variance when pegging the equilibrium rate is equal to this produced by targeting zero inflation:

$$Var(\hat{q}_t^{\bar{r}}) = Var(\hat{q}_t^{\bar{\pi}=0}),$$

which in turn is higher than the stock price variance produced under the optimal monetary policy specification, as we explained in the previous section.

As a second example we consider a monetary authority which pegs a rate of $\bar{r} + 1 = \frac{2}{\beta}$. Using for the rest of the parameters the same values as before, summarized in table (1), we see in figure (1) that, depending on the parameter values, a different policy produces the least stock price volatility.

In addition, as we did with the inflation targeting policy, we set a value for the degree of financial market participation and see how the stock price

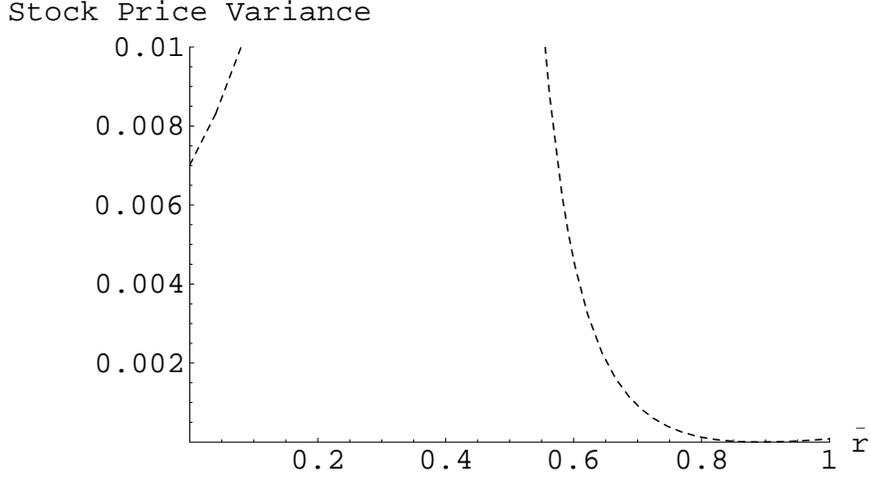


Figure 3: Stock price volatility of nominal interest rate pegging policy

variability changes with different nominal rate pegs. We set, as before, $\lambda = 0.35$ and depict this example in figure (3). When monetary authority pegs the nominal interest rate at low value, stock price volatility is very high, much higher than what the optimal policy, $Var(\hat{q}_t^{opt}) = 0.0053$, or the constant money supply $Var(\hat{q}_t^{\mu=0}) = 0.00378$, rules produce. Thus, the exact rate picked by the monetary authority greatly influences the stock price variability generated.

4.2 Inflation Volatility

In this section we examine the inflation volatility that the optimal, constant money supply, inflation targeting and interest rate pegging policy rules imply. The general expression for inflation is given by equation (26), in which we substitute the relevant policy rule to get:

$$\pi_t^{opt} = \frac{-(\varepsilon_t - \bar{\varepsilon})}{y + (\varepsilon_t - \bar{\varepsilon})}, \quad (32)$$

$$\pi_t^{\mu=0} = \frac{-(\varepsilon_t - \varepsilon_{t-1})}{y + (\varepsilon_t - \bar{\varepsilon})}, \quad (33)$$

$$\pi_t^{\bar{\pi}} = 0, \quad (34)$$

$$\pi_t^{\bar{r}} = \frac{-((\bar{r} + 1)\beta - 1)(\lambda\bar{y} + (\varepsilon_t - \bar{\varepsilon}))}{(\bar{r} + 1)\beta(\bar{y} + (\varepsilon_t - \bar{\varepsilon})) - (\lambda\bar{y} + (\varepsilon_t - \bar{\varepsilon}))}. \quad (35)$$

Of course inflation is at its lowest whenever the exact inflation targeting policy is employed. Moreover, comparing the first two equations we see that optimal monetary policy implies lower inflation than the constant money supply policy rule whenever $\varepsilon_{t-1} > \bar{\varepsilon}$. That is, every time the total dividend is above

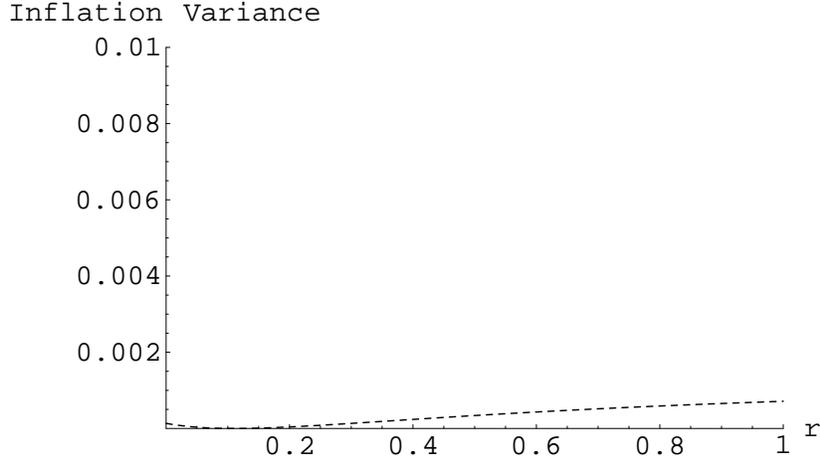


Figure 4: Inflation volatility of nominal interest rate peg policy

average, optimal monetary policy generates lower inflation than the constant money supply policy. This is true since the constant money supply policy, contrary to the optimal policy, does not react to dividend changes. As a result, all these changes are transferred into prices. When the previous period's total dividend is high, the previous period's prices are low and then, given today's total dividend, inflation is high. On the other hand, optimal policy completely compensates for the effects of the previous period's dividend changes, so there is no effect on inflation. In total, when the previous period's total dividend is high, inflation is higher under the constant money supply policy rule than under the optimal policy rule.

In addition, the nominal interest rate pegging policy generates minimal, equal to zero, inflation if the monetary authority pegs the rate at its equilibrium level, $\bar{r} + 1 = \frac{1}{\beta}$. However, if we wish to compare the inflation produced under other pegs, with that produced by the optimal and the constant money supply policy rules, the outcome will depend on the actual choice of rate, how large the dividends shocks are and on the other parameter values.

To compute the inflation variance implied by the policies above, we first linearize the inflation equations around the mean dividend value $\bar{\varepsilon}$ and then calculate the variance as follows:

$$Var(\pi_t^{opt}) = \frac{\sigma_{\varepsilon}^2}{\bar{y}^2}, \quad (36)$$

$$Var(\pi_t^{\mu=0}) = \frac{2\sigma_\varepsilon^2}{\bar{y}^2}, \quad (37)$$

$$Var(\pi_t^{\bar{r}}) = 0, \quad (38)$$

$$Var(\pi_t^{\bar{r}}) = \sigma_\varepsilon^2 \frac{((\bar{y} + 1)\beta - 1)^2 (\bar{r} + 1)^2 \beta^2 (1 - \lambda)^2}{((\bar{r} + 1)\beta - \lambda)^4 \bar{y}^2}. \quad (39)$$

The above analysis implies that inflation variance is minimized under the inflation targeting policy, while the optimal policy is always associated with less inflation volatility than the constant money supply policy. This is again because under the optimal policy the dividend shocks from the previous period are offset, while under constant money supply they are not. These shocks increase the volatility of inflation. On the other hand, the comparison with the interest rate pegging policy rule is not straightforward. When the monetary authority pegs the rate at its equilibrium level $\bar{r} + 1 = \frac{1}{\beta}$, then inflation volatility is zero, but for any other choice of rate the result would depend on the parameter values.

Using the same values as in the previous section, and for $\lambda = 35\%$ we see in figure (4) how inflation volatility changes depending on the choice of interest rate peg. For these values of the parameters, given in table (1), the optimal policy produces volatility equal to $Var(\pi_t^{opt}) = 0.0036$ while the constant money supply $Var(\pi_t^{\mu=0}) = 0.0072$. In this case it seems that the interest rate pegging policy, for a rate less than one, is associated with lower inflation volatility compared to the optimal and the constant money supply policies.

4.3 Risk Aversion

In this section we use the more general constant relative risk aversion utility function $u(c_t) = \frac{c_t^{1-a}}{1-a}$ to examine the effects of risk aversion on the stock price and its volatility. Using the stock pricing equation (18) and solving for the recursive form of the real stock price, $\hat{q}_t = \frac{q_t}{p_t}$, assuming as before that the transversality condition holds, we obtain the following expression for the stock price:

$$\hat{q}_t = E_t \sum_{j=1}^{\infty} \beta^j \left(\frac{c_t^T}{c_{t+j}^T} \right)^a \frac{p_t}{p_{t+j}} \varepsilon_{t+j-1},$$

and by substituting in the above expression the price of the good and trader's consumption given by equations (14) and (16), stock price becomes:

$$\hat{q}_t = E_t \sum_{j=1}^{\infty} \beta^j \left(\frac{\bar{y} + (\varepsilon_t - \bar{\varepsilon})}{\bar{y} + (\varepsilon_{t+j} - \bar{\varepsilon})} \frac{(\varepsilon_{t-1} - \bar{\varepsilon})(1 + \mu_t) + \bar{y}(\lambda + \mu_t)}{(\bar{y} + (\varepsilon_{t-1} - \bar{\varepsilon}))(1 + \mu_t)} \right)^a \left(\frac{(\bar{y} + (\varepsilon_{t+j-1} - \bar{\varepsilon}))(1 + \mu_{t+j})}{(\varepsilon_{t+j-1} - \bar{\varepsilon})(1 + \mu_{t+j}) + \bar{y}(\lambda + \mu_{t+j})} \right)^a \frac{\bar{y} + (\varepsilon_{t+j} - \bar{\varepsilon})}{\bar{y} + (\varepsilon_t - \bar{\varepsilon})} \frac{\varepsilon_{t+j-1}}{\prod_{s=1}^j (1 + \mu_{t+s})}. \quad (40)$$

Similarly to the previous section, we calculate the linearized version for the price of the stock and its variance for various policy rules.

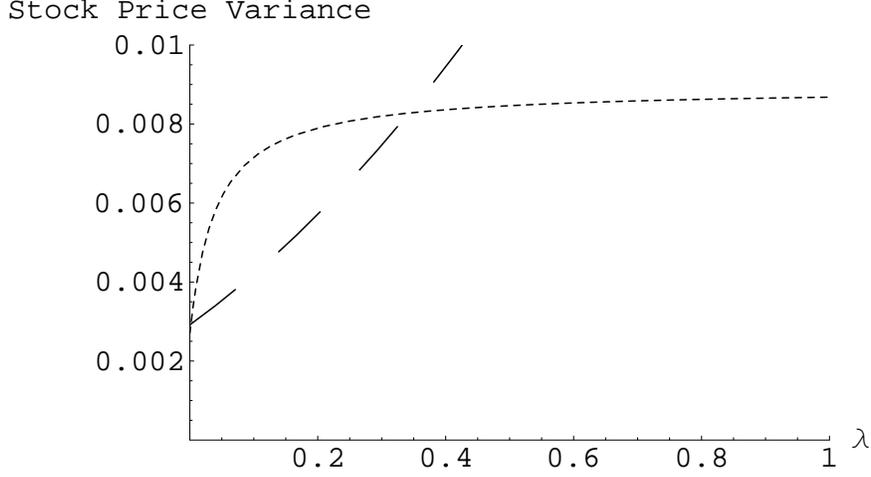


Figure 5: Stock price volatility of optimal (dashed line) and inflation targeting (dotted line) policy rules. The constant money supply policy generates much higher volatility to be presented in the graph

For the optimal monetary policy, stock price and its volatility are:

$$\hat{q}_t^{opt} = \frac{\beta\lambda(\bar{y} - y^T)}{1 - \beta} + \frac{\beta}{1 - \beta} \frac{\bar{y}(1 - \beta) + a\lambda(\bar{y} - y^T)}{\bar{y}} \eta_t,$$

$$Var(\hat{q}_t^{opt}) = \frac{\beta^2 \sigma_\varepsilon^2}{\bar{y}^2 (1 - \beta)^2} [\bar{y}(1 - \beta) + a\lambda(\bar{y} - y^T)]^2, \quad (41)$$

where an increase in the risk aversion parameter increases the variance of the stock. This is because for the optimal monetary policy rule, consumption at period $t + j$ depends only on the shocks η_{t+j} , which in expectation equal their mean, i.e., zero, making the linearized stock price variant only because of the current dividend shocks. Hence, whenever there is a current dividend shock, a more risk averse trader reacts more severely in terms of his willingness to purchase stocks, compared to a less risk averse trader. This is reflected at the expressions for the stock price and its variance.

For the constant money supply policy, stock price and its volatility are:

$$\hat{q}_t^{\mu=0} \simeq \frac{\beta}{1 - \beta} \lambda(\bar{y} - y^T) + a \frac{\beta}{1 - \beta} \frac{(1 - \lambda)(\bar{y} - y^T)}{\bar{y}} \eta_{t-1}$$

$$+ \frac{\beta}{(1 - \beta)\bar{y}} (\lambda(\bar{y} - y^T)(2a - 1 - a\beta) + (\bar{y} - a(\bar{y} - y^T)(1 - \beta))) \eta_t,$$

$$Var(\hat{q}_t^{\mu=0}) = \frac{\beta^2 \sigma_\varepsilon^2}{\bar{y}^2 (1 - \beta)^2} [a^2 (1 - \lambda)^2 (\bar{y} - y^T)^2 +$$

$$(\lambda(\bar{y} - y^T)(2a - 1 - a\beta) + (\bar{y} - a(\bar{y} - y^T)(1 - \beta)))^2], \quad (42)$$

where an increase in risk aversion has no clear effect on the stock price variance. For the inflation targeting policy, stock price and its volatility are:

$$\begin{aligned} \hat{q}_t^{\bar{\pi}} &= \frac{\beta\lambda(\bar{y} - y^T)}{\bar{\pi} + 1 - \beta} + \frac{\beta}{(\bar{\pi} + 1 - \beta)(\bar{\pi} + \lambda)(\bar{\pi} + 1)\bar{y}} ((\bar{\pi} + \lambda)\bar{y}(\bar{\pi} + 1 - \beta) + \\ &\quad + a\lambda(\bar{y} - y^T)(\bar{\pi} + 1)^2)\eta_t, \\ Var(\hat{q}_t^{\bar{\pi}}) &= \frac{\beta^2\sigma_\varepsilon^2}{(\bar{\pi} + 1 - \beta)^2(\bar{\pi} + \lambda)^2(\bar{\pi} + 1)^2\bar{y}^2} \\ &\quad ((\bar{\pi} + \lambda)\bar{y}(\bar{\pi} + 1 - \beta) + a\lambda(\bar{y} - y^T)(\bar{\pi} + 1)^2)^2, \end{aligned} \quad (43)$$

where an increase in the risk aversion parameter, increases the variance of the stock price.

We do not compute the stock price volatility for the case of interest rate pegging because this calculation would require ex ante additional assumptions about the correlation of monetary policy's reaction to dividends shocks. Figure (5) shows how the stock price variance for the optimal monetary policy and the inflation targeting rule changes with the degree of financial markets segmentation. The volatility of constant money supply policy is much higher to be represented in this graph. A risk aversion parameter of $a = 2$ is used, while the rest of the parameter values are kept as before, summarized in table (1). We see, once more, that there is no conclusive evidence concerning which policy minimizes stock price volatility.

4.4 Discussion

We conclude this section concerning stock price and inflation volatility by discussing some important points derived from the analysis above. First, it is clear that the optimal monetary policy does not necessarily associate with lower stock price or inflation volatility when compared with the rest of the policy rules considered. The outcome of the comparisons depends on the parameter values.

There are, however, some definite conclusions. The optimal monetary policy delivers lower stock price volatility than the inflation targeting policy when the target is set to zero, and lower stock price volatility than the interest rate pegging policy, when the peg is set to its equilibrium level. However, optimal monetary policy generates higher inflation volatility when compared to these two specific policies. Thus, it seems, there is a trade off between inflation volatility and stock price volatility when we compare optimal monetary policy with the policies of zero inflation targeting and equilibrium rate pegging.

In addition, we find that the optimal monetary policy always produces lower inflation volatility than the constant money supply policy rule. Also, for the parameter values used, summarized in table (1), it seems that the nominal interest rate pegging policy gives much less inflation volatility than these two policies. Nevertheless, this result cannot be generalized and depends on the choice of the rate peg and the other parameter values.

Furthermore, we see in figure (1) that increased participation does not necessarily imply lower stock price volatility. This observation is in contrast to Allen and Gale (1994) who, using a model with individual shocks, argue that

high variability in stock prices is encouraged by low stock market participation. When monetary policy actions are taken into account, the effect of the increased financial market participation on stock price volatility may become more complicated.

Moreover, comparing our assessment about the stock price volatility various policies produce to Bernanke and Gertler's (2001) results, we argue that limited participation is important in computing stock price volatility, and ignoring this issue might affect the conclusions reached. However, we employ a very different model than the one Bernanke and Gertler (2001) do.

Finally, we explore the variability of the stock prices for a general, constant relative risk aversion utility function, and we observe that there is still no definite answer about which policy produces minimal stock price volatility. The outcome of the comparisons still depends on the parameter values.

In conclusion, it seems that a monetary authority wishing to maximize total welfare does not have the role of minimizing the volatility either of the stock price or of inflation.

5 Conclusions

In a segmented financial market model, a novel role arises for the monetary policy: that of sharing the financial market risk the financial market participants face among all agents in the economy, maximizing in this way total welfare. Such a policy does not necessarily imply lower stock price volatility or inflation volatility when compared to other policy rules, indicating that, in our model, minimizing these volatilities is not the focus of optimal monetary policy.

In our setting, optimal monetary policy perfectly shares the dividend risk between traders and non-traders. Whenever dividend income is low, monetary authority distributes monetary transfers to the traders. Such a response increases prices and lowers non-traders' consumption. On the other hand, whenever dividend income is high, monetary policy contracts and taxes traders; as a result, prices decrease and non-traders are benefited. This policy equalizes consumption of the two groups and agents, if given the choice, would be indifferent between participating or not in the financial markets. These results hold for any concave utility function and are not sensitive to the degree of market segmentation.

We also examine what the optimal monetary policy implies for the stock price volatility and inflation volatility and compare these implications with that of others, commonly used monetary policy rules. Specifically, we consider the constant money supply, inflation targeting and nominal interest rate pegging policy rules and conclude that the optimal monetary policy does not imply lower stock price volatility or inflation volatility than these policies. The outcome of these comparisons depends on the model's parameter values.

In conclusion, this work suggests a new role for the optimal monetary policy, that of sharing financial market risk between agents who encounter this risk and agents who do not, equalizing in this way their consumption. This policy does not necessarily imply minimal stock price volatility or inflation volatility.

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