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PARITY VIOLATION IN TWO PHOTON ATOMIC TRANSITIONS*

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ABSTRACT

Detection of parity-violating weak interactions in the $1s_{\frac{1}{2}} \rightarrow 2p_{\frac{1}{2}}$ two photon transitions in hydrogen is discussed. Polarization rotation of one of the two photon beams exciting the transition is computed for general theories of the weak interaction and in particular for the $SU2 \times U1$ theory due to Weinberg and Salam.

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In this note we discuss the possibility of detecting parity-violating weak interactions in the $1s_{1/2} \rightarrow 2p_{1/2}$ two photon transitions in hydrogen atoms. Several experiments currently in progress seek such effects in high Z atoms such as Bi and Cs[1], but the complexity of these atoms makes theoretical prediction quite difficult. For hydrogen precise calculations are straightforward and so if possible it would be quite advantageous to make precise measurements of parity violations is hydrogen. Furthermore both vector and axial-vector weak nucleon currents are important in hydrogen while only the vector current is relevant in high Z atoms. Here we will determine the polarization rotation due to parity-violating neutral weak interactions (if they exist) of one of two photon beams exciting the $1s_{1/2}$ to $2p_{1/2}$ transition. Details of the computations are contained in the Appendix.

In the experiment we envision a gas of hydrogen atoms would be irradiated from opposite directions by two laser beams of equal energy photons (equal to half the $1s_{1/2} \rightarrow 2p_{1/2}$ excitation energy). One, the saturating beam, would be quite intense and either linearly polarized or unpolarized. The other, the probe beam, would be relatively weak and linearly polarized. If parity-violating effects exist, the polarization plane of the probe would rotate as it passed through the gas.

The magnitude of this rotation is related to the difference between the indices of refraction for left and right polarized light:

$$\frac{d\phi(x)}{dx} = \frac{\omega}{c} \operatorname{Re} \left(\frac{n_L - n_R}{2}\right)$$

Noting that n is related to the photon-atom forward scattering amplitude $f(\omega)$ by (N = atom density)

$$n(\omega) = 1 + 2\pi \left(\frac{c}{\omega}\right)^2 Nf(\omega)$$

we find

$$\frac{d\phi(x)}{dx} = \frac{\pi c}{\omega} N \,\mathrm{Re}\,(f_L - f_R)$$

By the optical theorem, the absorption length at resonance for the probe beam is

$$\lambda_R = \frac{1}{\sigma_{tot} N} = \frac{\omega/c}{2\pi N \,\mathrm{Im}\,(f_L + f_R)_{res}}$$

and so the polarization rotation per absorption length is

$$\delta\phi = \frac{1}{2} \frac{\mathrm{Re}\,(f_L - f_R)}{\mathrm{Im}\,(f_L + f_R)_{res}}$$

If the hydrogen atoms are initially unpolarized, a nonzero difference $f_L - f_R$ indicates parity violation. It occurs because of mixing of atomic states of opposite parity induced by the weak interactions. In particular, if parity violation exists, the $2p_{1/2}$ hyperfine levels are mixed with the $2s_{1/2}$ levels:

$$|2p^1_{1/2}\rangle \rightarrow |2p^1_{1/2}\rangle + i\,\delta_1\,|2s^1_{1/2}\rangle$$

$$|2p^3_{1/2}\rangle \rightarrow |2p^3_{1/2}\rangle + i\delta_3\,|2s^3_{1/2}\rangle$$

It is shown in the Appendix that to lowest order

$$\delta_1 = \frac{G_F m_e^2}{\alpha} Q_W (1-\lambda) \qquad 0.01617$$

$$\delta_3 = \frac{G_F m_e^2}{\alpha} Q_W \left(1 + \frac{\lambda}{3}\right) \qquad 0.01440$$

Here G_F is the phenomenological weak interaction coupling constant $\left(G_F \simeq 10^{-5} M_p^{-2}\right)$ and λ is the strength of the nuclear axial-vector coupling relative to the nuclear vector current coupling. Also, Q_W depends on the particular theory of weak interactions assumed. For the Weinberg-Salam SU2 $\times$ U1 theory, λ is $\sim$1.25 and

and Q_W is

$$Q_W = 1 - 4 \sin^2 \theta_W$$

where $\sin^2 \theta_W \simeq 0.35 \pm .05$ is suggested by neutrino experiments.

The amplitude for double photon forward elastic scattering has two parts because of parity mixing. Near resonance,

$$f(\omega) = \langle 1s\gamma\gamma | T \frac{(|2p_{1/2}\rangle + i\,\delta|2s_{1/2}\rangle)(\langle 2p_{1/2}| - i\delta\langle 2s_{1/2}|)}{\Delta E - i\Gamma/2} T | 1s\gamma\gamma\rangle$$

$$= \frac{|T(1s\rightarrow 2p)|^2}{\Delta E - i\Gamma/2} - \frac{2\,\mathrm{Im}\left[\delta T^*(1s\rightarrow 2s)\; T(1s\rightarrow 2p)\right]}{\Delta E - i\Gamma/2}$$

where Γ is the width of the 2p state and $\Delta E = E_{2p} - E_{1s} - 2\omega$. If the atoms are initially unpolarized it is easily shown that

$$\frac{f_L + f_R}{2} = \frac{|T_L(1s\rightarrow 2p)|^2}{\Delta E - i\Gamma/2}$$

$$\frac{f_L - f_R}{2} = -2\,\frac{\mathrm{Im}\left[\delta T^*(1s\rightarrow 2s)\; T(1s\rightarrow 2p)\right]}{\Delta E - i\Gamma/2}$$

Thus the polarization rotation per (resonance) absorption length of the probe is

$$\delta\phi = -\frac{\Delta E\,\Gamma}{\Delta E^2 + \frac{\Gamma^2}{4}}\;\frac{1}{2}\;\frac{\mathrm{Im}\left[\delta T_L^*(1s\rightarrow 2s)\; T_L(1s\rightarrow 2p)\right]}{|T_L(1s\rightarrow 2p)|^2}$$

The relative values of $T(1s\rightarrow 2s)$ and $T(1s\rightarrow 2p)$ are computed in the Appendix. The final results are

$$\delta\phi = -\frac{\Delta E\,\Gamma}{\Delta E^2 + \frac{\Gamma^2}{4}} \left(\frac{G_F\, m_e^2}{\alpha^2}\, Q_W(1-\lambda)\right) \quad 0.0277$$

$$= \frac{\Delta E\;\Gamma}{\Delta E^2 + \frac{\Gamma^2}{4}}\; 0.154 \times 10^{-9} \qquad \text{for } SU2 \times U1$$

for the $1s\frac{1}{1/2} \rightarrow 2p\frac{1}{1/2}$ transition, and

$$\delta\phi = -\frac{\Delta E\,\Gamma}{\Delta E^2 + \frac{\Gamma^2}{4}} \left(\frac{G_F\, m_e^2}{\alpha^2} Q_W \left(1 + \frac{\lambda}{3}\right)\right) \quad 0.0247$$

$$= \frac{\Delta E\,\Gamma}{\Delta E^2 + \frac{\Gamma^2}{4}} \; 0.780 \times 10^{-9} \qquad \text{for SU2} \times \text{U1}$$

for the $1s\frac{3}{1/2} \rightarrow 2p\frac{3}{1/2}$ transition.

We note that in general Q_W is roughly proportional to the number of nucleons per atom. Therefore the effects predicted above may be approximately doubled by using deuterium in place of hydrogen. Furthermore comparison of results from hydrogen with those from deuterium can yield information about the hadronic structure of the weak currents.

It is clear that at present this experiment is quite difficult, the major problem being to obtain an absorption length for the $1s \rightarrow 2p$ transition ($\simeq 2.2 \times 10^5$ absorption lengths for the $1s \rightarrow 2s$ transition). However the possibility of an unambiguous theoretical treatment makes it very attractive. We emphasize that the observation of parity violation in the hydrogen atom provides a sensitive measurement of the sign and magnitude of the electron-proton weak neutral interaction.

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APPENDIX

A. Parity Mixing

A general potential induced by parity violating weak interactions in hydrogen has the form[1]:

$$V_W = \frac{G_F}{4\sqrt{2}\, m_e} Q_W \left[\left\{ \vec{\sigma}_e \cdot \vec{p},\, \delta^3(\vec{r}) \right\} + \lambda \left\{ \vec{\sigma}_p \cdot \vec{p},\, \delta^3(\vec{r}) \right\} \right.$$

$$\left. - i\lambda \left\{ \vec{\sigma}_p \cdot \overrightarrow{\sigma_e \times p},\, \delta^3(r) \right\} \right]$$

where Q_W and λ are model dependent. The last term does not contribute to the polarization rotation because of its phase and consequently it has been ignored throughout this paper. Terms proportional to λ result from the interaction of the leptonic vector current with the nuclear axial-vector current while the remaining term comes from the axial-vector leptonic current and the nuclear vector current. The potential V_W induces mixing of $p_{1/2}$ and $s_{1/2}$ states in the hydrogen atom

$$|2p_{1/2}\rangle \rightarrow |2p_{1/2}\rangle + i\delta |2s_{1/2}\rangle$$

where by lowest order perturbation theory

$$i\delta = \frac{\langle 2s_{1/2}| V_W |2p_{1/2}\rangle}{E_{2s_{1/2}} - E_{2p_{1/2}}}$$

It is trivial to show that

$$\langle 2s_{1/2}| \left\{ \vec{\sigma}\cdot\vec{p},\, \delta^3(\vec{r}) \right\} |2p_{1/2}\rangle = -i\psi^{+}_{2s_{1/2}} \vec{\sigma}\cdot \left(\vec{\nabla}\, \psi_{2p_{1/2}} \right) \Big|_{\vec{r}=0}$$

from which it follows that

$$\langle 2s^1_{1/2}|V_W|2p^1_{1/2}\rangle = i\sqrt{\frac{3}{2}}\, \frac{G_F \alpha^4 m_e^3}{64} (1-\lambda)$$

$$<2s^3_{1/2}|V_W|2p^3_{1/2}> = i\sqrt{\frac{3}{2}}\,\frac{G_F\,\alpha^4\,m_e^3}{64}\left(1+\frac{\lambda}{3}\right)$$

The energy difference in δ is just the Lamb shift together with hyperfine level shifts:

$$E(2s^1_{1/2}) - E(2p^1_{1/2}) = m\alpha^5 \quad 0.37678$$

$$E(2s^3_{1/2}) - E(2p^3_{3/2}) = m\alpha^5 \quad 0.42310$$

These expressions may be combined to give δ_1 and δ_3 as quoted above.

We note that the nuclear spin terms in V_W also induce a small mixing between the $2p^3_{3/2}$ levels and the $2s^3_{1/2}$ levels. This does not result in a polarization rotation for unpolarized atoms because $T_L\left(1s^3_{1/2} \rightarrow 2p^3_{3/2}\right)$ goes to zero when averaged over initial spins.

B. $\underline{1s_{1/2} \rightarrow 2p_{1/2}}$ Transition Rate

Apart from some irrelevant factors, the amplitude for a general two photon atomic transition is to lowest order:

$$T(i\rightarrow f) = \sum_j \frac{<f|\hat{0}'|j><j|\hat{0}|i>}{E_i - E_j + \omega}$$

where $\omega = \frac{1}{2}(E_f - E_i)$ is the energy of each photon and $\hat{0}$, $\hat{0}'$ represent the one photon operators for E1, M1 and E2 transitions

$$<\beta|E1|\alpha> = i m_e\,\omega_{\beta\alpha} <\beta|\vec{\epsilon}\cdot\vec{r}\,|\alpha>$$

$$<\beta|E2|\alpha> = -\frac{m_e}{2}\,\omega_{\beta\alpha} <\beta|\vec{k}\cdot\vec{r}\;\vec{r}\cdot\vec{\epsilon}\,|\alpha>$$

$$<\beta|M1|\alpha> = \frac{i}{2} <\beta|\vec{k}\times\vec{\epsilon}\cdot\left(\vec{j}+\frac{\vec{\sigma}}{2}\right)|\alpha>$$

Here $\vec{k}$ is the photon's momentum and $\vec{\epsilon}$ its polarization. We assume the probe beam to be coming in the positive z direction with either left or right handed polarization. The saturating beam comes in the opposite direction and is assumed linearly polarized in the x direction.

Normally the $1s_{1/2} \rightarrow 2p_{1/2}$ transition can occur as an E1E2 transition via intermediate d states, or as an E1M1 transition via either of the $2p_{1/2}$ or $2p_{3/2}$ states. Here, because the photons have equal and opposite momenta, the E1E2 transitions do not contribute. The remaining E1M1 amplitude is

$$T_L(1s_{1/2} \rightarrow 2p_{1/2}) = i\alpha\ m\ \frac{\sqrt{2}}{16} R_{10}^{21} = i\alpha\ m \quad 0.1140$$

$$= T_R(1s_{1/2} \rightarrow 2p_{1/2})$$

where R_{10}^{21} is as defined by Bethe and Salpeter[2] (in atomic units):

$$R_{10}^{21} = \int_0^\infty r^2\, dr\, R_{21}(r)\, r\, R_{10}(r)$$

$R_{n\ell}$ being the radial part of the hydrogen wave function.

C. $\underline{1s_{1/2} \rightarrow 2s_{1/2}}$ Transition Rate

This transition is predominantly an E1E1 transition. The intermediate states are p states. The amplitude is

$$T_L(1s_{1/2} \rightarrow 2s_{1/2}) = \frac{2\sqrt{2}}{3\alpha^2} \sum_n \omega_{n2}\, \omega_{1n} \frac{R_{n1}^{20} R_{10}^{n1}}{\omega_{1n} + \omega_{2n}}$$

$$= -\frac{2\sqrt{2}}{3\alpha^2}\, \omega^2 \sum_n \frac{R_{n1}^{20} R_{10}^{n1}}{\omega_{n1} + \omega_{n2}}$$

$$= -T_R(1s_{1/2} \rightarrow 2s_{1/2})$$

The second expression is equivalent to the first[3] and more convenient for computation. An integration over continuum p states is implicit. The sums were performed numerically using both of the above expressions and the matrix elements $R_{n\ell}^{n'\ell'}$ in Ref. 2. The transition amplitude found is

$$T_L(1s_{1/2} \rightarrow 2s_{1/2}) = m_e \quad 0.3905 \quad .$$

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