A Graph Model for a Basic Process Algebra for Hybrid Systems

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1 Motivation

The motivation behind this work is to be able to add more functionality to the existing theory of "Process Algebra for hybrid Systems"(ACP_{hs}^{srt})[5]. A hybrid process is one in which continuous state changes of the environment variables are combined with the discrete actions of the process. ACP_{hs}^{srt} provides the operators and semantics with the help of which we can describe the environment dependent (discrete and continuous) behaviour of a hybrid system. It is an extension of process algebra with propositional signals (ACP_{ps}) from [1] and process algebra with standard relative timing (ACP^{srt}) from [2].

Recently formal description of hybrid systems has attracted a lot of interest especially in the field of embedded system software as many embedded systems are in fact hybrid in nature. As a consequence, a number of hybrid process algebraic and hybrid automata theories have been developed. Examples are Hybrid Automata [6], Hybrid I/O automata [7], HyPA [10], Hybrid Chi [11], ϕ-Calculus [9], and hybrid CSP [8].

In this report, we propose a graph model for BPA_{hs}^{srt}, the basic process algebra for hybrid systems (i.e. process algebra without parallelism and communication, see [5]). Our aim is to correctly specify the "signal hiding" operator. The signal hiding operator hides *some* effects of a given environment variable from the behaviour of a hybrid system. This makes it easier to study a desired property of the system regarding its discrete actions and delays. We give below some examples of the phenomena of signal hiding.

Consider a Candy machine that asks its customer to enter his or her name. If the name starts with a vowel (i.e. 'a','e','i','o','u','y'), it gives its customer a *bounty*, otherwise if the name starts with a consonant, it gives a *snickers*. The BPA_{hs}^{srt} specification of the candy machine will be as follows:

$$\begin{aligned}\texttt{Candy\_machine} \;=\;& \texttt{enter\_name}\cdot \\ & ((\texttt{ first letter is a vowel}) :\rightarrow \texttt{give\_bounty} + \\ & (\texttt{first letter is a consonant}) :\rightarrow \texttt{give\_snickers})\end{aligned}$$

On hiding the environment variable *name*, the `Candy_machine` will behave as the process,

$$\texttt{enter\_name} \cdot (\texttt{give\_bounty} + \texttt{give\_snickers});$$

and its not visible anymore how the choice between bounty and snickers is determined.

Similarly consider a process with an arbitrary delay, where the duration of the delay is controlled by some environment variable. We name this process as P and the concerned variable as l.

$$P = (l = 0) \,{}^\wedge\!\!\blacktriangle\, (\dot{l} = 0.5) \curvearrowright_l \sigma^*_{\mathsf{rel}}((l = 5) :\rightarrow \tilde{\tilde{a}}).$$

At the start of the process, the value of l is zero (symbolized by $(l = 0)\,{}^\wedge\!\!\blacktriangle$). The variable continuously evolves according to the equation $\dot{l} = 0.5$ (represented by $(\dot{l} = 0.5) \curvearrowright_l$). P can delay for any time $t \in [0, \infty)$ (represented by σ^*_{rel}). At the end of the delay, if the value of l is 5, action a is performed (symbolized by $(l = 5) :\rightarrow \tilde{\tilde{a}}$).

According to the operational semantics of BPA^{srt}_{hs}, we conclude that process P can either perform action a after 10 time units or delay indefinitely. When we hide the variable l from P, the process becomes equivalent to $\sigma^{10}_{\mathsf{rel}}(\tilde{\tilde{a}}) + \delta$, that can also perform action $\tilde{\tilde{a}}$ after 10 time units or delay indefinitely. We see that after signal hiding, we retain the information of the duration of the delay.

The structure of the report is as follows. First we explain the concept of signal hiding in ACP^{srt}_{hs} and give its axioms and first operational semantic rules. We describe the problem in the original operational semantic rules with the help of an example of a thermostat. We also point out the difficulties faced while designing operational semantic rules for the signal hiding operator (as defined in ACP^{srt}_{hs}). We propose a graph model for basic process algebra for hybrid systems as a solution to these problems. In Section 5, we give a detailed description of our graph model. Later, we add signal hiding and integration to our graph model and define the notion of bisimulation in it. We come back to the example of a thermostat to assert our claim that the problems faced in the earlier semantic model for signal hiding do not appear in the graph model. In Section 9, we give soundness proofs for some BPA^{srt}_{hs} axioms in our model.

We include the axioms and operational semantic rules of BPA^{srt}_{hs} (from [5]) in the appendix.

2 Signal Hiding

The concept of signal hiding was first introduced in [1]. While developing ACP^{srt}_{hs}, signal hiding was initially included in the theory (see [3]) but was later removed (Signal hiding is not included in [5]). The reason for its removal was that, under the operational semantics given in [3], the signal hiding operator did not preserve bisimulation, i.e. two bisimilar processes did not always remain bisimilar after applying signal hiding with respect to a certain environment variable (see Section 4 for more details). It was realized that a more detailed semantics of BPA^{srt}_{hs} is required for correctly specifying the signal hiding operator (see [5], section 3.3). The work on signal hiding was further developed than given in [3] but was not published. The axioms and operational semantic rules of signal hiding have been taken from [3], and the lifting rules (described later in this section) are taken from its (unpublished) later version.

We define the concept of signal hiding as follows:

Let P be a process and v be an environment variable. The process $v \triangle P$ denotes the signal hiding of P with respect to v. The behaviour of $v \triangle P$ is the same as that of P except that its behaviour no longer depends on the values of v and its derivative $\dot{v}$. That roughly means that at any stage, the behaviours that are possible in a given state, become possible after hiding, in all those states as well that differ only in their values of v and $\dot{v}$ from the given state. (A state assigns values to the environment variables (see section 5)). It is to be noted that while applying signal hiding, we want to put an additional constraint of not loosing the effect of the hidden variable on the duration of delays in P.

The signal hiding operator, $\triangle$, is also extended to state and transition propositions. A state proposition is a proposition concerning the state of a process. Let V be the set of all environment variables and $\dot{V}$ be the set consisting of their derivatives. Then a state proposition ψ is a proposition on variables from set $V \cup \dot{V}$. A transition proposition is a proposition regarding the states of a process immediately before and after an action or delay. We introduce two new sets ${}^{\bullet}V$ and $V^{\bullet}$, for denoting values of variables (and their derivatives), immediately before and after an action or delay. A transition proposition χ is a proposition on variables from ${}^{\bullet}V \cup V^{\bullet}$.

If ψ is a state proposition, then the signal hiding of ψ with respect to a variable $v \in V$, written $v\triangle\psi$, is the proposition ψ with the dependence of its satisfaction on the values of v and $\dot{v}$ removed. That is if ψ holds in a state called α, then $v\triangle\psi$ holds in α, and in every state that differs from α only in its values of v and $\dot{v}$. Similarly, let χ be a transition proposition. Then $v\triangle\chi$ is the transition proposition χ with dependence of its satisfaction on the values of ${}^{\bullet}v, {}^{\bullet}\dot{v}, v^{\bullet}, \dot{v}^{\bullet}$ removed.

2.1 Structural operational semantic rules for signal hiding

Here we give the operational semantic rules for the signal hiding operator as defined in [3].

For any states α and α', we write $\alpha =_v \alpha'$, to indicate that α and α' may differ from each other only in the values of variables v and $\dot{v}$. For two state evolutions ρ and ρ' over the interval $[0, r]$, we write $\rho =_v \rho'$ to indicate that at any instant during $[0, r]$, ρ may differ from ρ' only in the values of v and $\dot{v}$.

Table 1: Operational Semantic rules for signal hiding ($a \in A, r > 0$)

$\dfrac{\langle x,\alpha\rangle \xrightarrow{a} \langle x',\alpha'\rangle}{\langle v\triangle x,\alpha^*\rangle \xrightarrow{a} \langle v\triangle x',\alpha'^*\rangle}$	$\alpha^* =_v \alpha, \alpha'^* =_v \alpha'$	*Rule*1
$\dfrac{\langle x,\alpha\rangle \xrightarrow{a} \langle \surd,\alpha'\rangle}{\langle v\triangle x,\alpha^*\rangle \xrightarrow{a} \langle \surd,\alpha'^*\rangle}$	$\alpha^* =_v \alpha, \alpha'^* =_v \alpha'$	*Rule*2
$\dfrac{\langle x,\alpha\rangle \xmapsto{r,\rho} \langle x',\alpha'\rangle}{\langle v\triangle x,\alpha^*\rangle \xmapsto{r,\rho^*} \langle v\triangle x',\alpha'^*\rangle}$	$\alpha^* =_v \alpha, \alpha'^* =_v \alpha', \rho^* =_v \rho$	*Rule*3
$\dfrac{\alpha \in [\mathsf{s}(x)]}{\alpha^* \in [\mathsf{s}(v\triangle x)]}$	$\alpha^* =_v \alpha$	*Rule*4

These rules are defined on pairs of BPA^{srt}_{hs} terms and states, called *configurations*.

Four different kinds of transition relations are defined: (where t, t' are BPA_{hs}^{srt}-terms, α, α' represent states of the environment variables, a is an action, r is a non-zero time duration and ρ, also called a state evolution, gives states of the environment variables in the duration $[0, r]$.)

1. $\langle t, \alpha \rangle \xrightarrow{a} \langle t', \alpha' \rangle$: known as an *action step* represents that in state α, t is capable of first performing action a and then proceeding as process t' in state α';

2. $\langle t, \alpha \rangle \xrightarrow{a} \langle \surd, \alpha' \rangle$: known as a *termination step* represents that in state α, t is capable of first performing action a and then terminating in state α';

3. $\langle t, \alpha \rangle \xmapsto{r,\rho} \langle t', \alpha' \rangle$: known as a *time step* represents that in state α, t is capable of first idling for r time units during which the state evolves according to ρ and then proceeding as t' in state α';

4. $\alpha \in [\mathsf{s}(t)]$: known as the *signal relation* indicates that in state α, the signal emitted by process t holds.

The signal relation needs more explanation. In ACP_{ps} [1], the state of a process is made visible to some extent with the help of state propositions. In ACP_{ps}, a process may require that a certain proposition regarding the environment variables must hold for the process to behave as defined. Such a proposition is called the signal emitted by a process. The rules defining satisfaction of signal relations are given in the appendix in table 9.

In [12], congruence formats of operational semantic rules for different kinds of bisimilulations are defined. Two kinds of bisimulations are defined in BPA_{hs}^{srt}. One is simply called a bisimulation and the other is known as "Interference compatible bisimulation", abbreviated as *ic-bisimulation.* A bisimulation relates two configurations (a configuration is an ordered pair of a BPA_{hs}^{srt} process term and a state) that have the same states, if their behaviours match according to the operational semantic rules of BPA_{hs}^{srt}. An ic-bisimulation relates two process terms if their behaviours match when compared in all states and this property is reflected in all the subsequent pairs of terms obtained as a result of matching transitions. (For definitions of bisimulation and ic-bisimulation see the appendix). The concept of bisimulation is the same as the definition of state-based bisimilarity in [12] and the concept of ic-bisimulation matches that of stateless bisimilarity.

Ic-bisimulation is a more restrictive notion of equivalence than desired in a model for hybrid processes. BPA_{hs}^{srt} axioms HST5, HST14 and lifting rules of BPA_{hs}^{srt} $HSELR2$ and $HSELR3$ are not sound under ic-bisimulation.

These axioms are essential for deriving many useful results about hybrid processes, therefore in BPA_{hs}^{srt} bisimulation is a preferred equivalence over ic-bisimulation.

The congruence format for stateless bisimilarity defined in [12] is known as process-tyft format, which is given as follows:

Let $(\Sigma_p, \Sigma_d, L, D(Rel))$ be a transition system specification. A deduction rule in $D(Rel)$ is in process-tyft format if it is of the form

$$\frac{\{(t_i, u_i) \xrightarrow{l_i}_{r_i} (y_i, u_i')|i \in I\}}{(f(x_0, \ldots, x_{n-1}), u) \xrightarrow{l}_r (t', u')} \quad ,$$

where I is a set of indices, $r \in Rel$, $l \in L$, $f \in \Sigma_p$ is a process function of arity n, the variables $x_0, \ldots, x_{n-1}$ and y_i $(i \in I)$ are all distinct variables from V_p, and, for all $i \in I$: $r_i \in Rel$, $l_i \in L$, $t_i, t' \in T(\Sigma_p)$ and $u, u', u_i, u_i' \in T(\Sigma_d)$.

We name the set of process variables appearing in the left-hand-side of the conclusion X_p and in the right-hand-side of the premises Y_p. The two sets X_p and Y_p are obviously disjoint following the requirements of the format.

Looking at the semantic rules Rule 1 to Rule 3, in Table 1, we realize that they are in process-tyft format. Thus for ic-bisimulation or stateless bisimilarity, the signal hiding operator is a congruence in the current semantics.

For state-based bisimulation, more restrictions on the operational semantic rules besides the process-tyft standard apply in order for bisimulation to be a congruence. The resulting format is known as $sfsb$ (for standard format for state-based bisimilarity).

A deduction rule is in the sfsb format if it is in process-tyft format and satisfies the following data-dependency constraints:

1. If a variable $x \in X_p$ appears in t', then $u' = u$;
2. If a variable $y_i \in Y_p$ appears in t', then $u' = u_i'$;
3. If a variable $x \in X_p$ appears in some t_i, then $u_i = u$;
4. If a variable $y_i \in Y_p$ appears in some t_j $(j \in I)$, then $u_j = u_i'$.

Looking at the semantic rules Rule 1 to Rule 3, in Table 1, we see that data-dependency constraint 2 and 3 are being violated. Thus congruence of bisimulation for signal hiding cannot be proved on the basis of the format. Later on, we give a counter example to show that bisimilarity actually is not a congruence for signal hiding.

2.2 Axioms of signal hiding

The axioms concerning the signal hiding operator as defined in [3] are given in Table 2.

We briefly describe the symbols of BPA_{hs}^{srt}, given in table 2, so that the user

Table 2: Axioms for signal hiding ($a \in A_\delta$)

$v \triangle \bot = \bot$	$HSH1$
$v \triangle (\psi \mathbin{{}^\wedge\!\!\blacktriangle} \tilde{\tilde{\delta}}) = (v \triangle \psi) \mathbin{{}^\wedge\!\!\blacktriangle} \tilde{\tilde{\delta}}$	$HSH2$
$v \triangle (\psi \mathbin{{}^\wedge\!\!\blacktriangle} (\chi \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}})) = (v \triangle \psi) \mathbin{{}^\wedge\!\!\blacktriangle} ((v \triangle ({}^\bullet\psi \wedge \chi)) \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}})$	$HSH3$
$v \triangle (\psi \mathbin{{}^\wedge\!\!\blacktriangle} (\chi \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}} \cdot x)) = (v \triangle \psi) \mathbin{{}^\wedge\!\!\blacktriangle} ((v \triangle ({}^\bullet\psi \wedge \chi \wedge s_\rho(x)^\bullet)) \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}} \cdot (v \triangle x))$	$HSH4$
$v \triangle (x + y) = v \triangle (s_\rho(x+y) \mathbin{{}^\wedge\!\!\blacktriangle} x) + v \triangle (s_\rho(x+y) \mathbin{{}^\wedge\!\!\blacktriangle} y)$	$HSH5$
$v \triangle (v' \triangle x) = v' \triangle (v \triangle x)$	$HSH6$
$v \triangle (\int_{u \in U} F(u)) = \int_{u \in U} (v \triangle (s_\rho(\int_{u \in U} F(u)) \mathbin{{}^\wedge\!\!\blacktriangle} F(u)))$	$HSH7$

v is a variable, ψ is a state proposition, χ is a transition proposition and A_δ stands for the set of all actions including the deadlock process (δ).

can develop some understanding of the axioms.

$\bot$	Non-existence process A process that emits a signal that cannot hold in any state
$\tilde{\tilde{a}}$	Undelayable action An action that performs instantaneously without delay
$\tilde{\tilde{\delta}}$	Undelayable Deadlock Process A process that deadlocks without delay
$\psi \mathbin{{}^\wedge\!\!\blacktriangle} x$	Process x emits signal ψ
$\chi \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}}$	Action $\tilde{\tilde{a}}$ is performed in such a way that χ is true e.g. $(v^\bullet = {}^\bullet v - 1) \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}}$ means that after doing action $\tilde{\tilde{a}}$, the value of v decrements. $v^\bullet$ stands for value of v after doing $\tilde{\tilde{a}}$ ${}^\bullet v$ is the value of v before doing $\tilde{\tilde{a}}$
$\int_{u \in U} F(u)$	An alternative composition between process expressions $F(u)$, for all values of variable u in U
$s_\rho(x)$	The root signal of x $s_\rho(x)$ is a state proposition. It is the signal emitted by x
$s_\rho(\int_{u \in U} F(u))$	$\bigwedge_{u \in U} s_\rho(F(u))$
${}^\bullet\psi$	${}^\bullet\psi$ is a transition proposition, whereas ψ is a state proposition. When written as ${}^\bullet\psi \mathbin{\ulcorner\!\blacktriangledown} (x)$, it means process x can only start doing action or delays, if the system is in a state where ψ is true. If ψ is false, ${}^\bullet\psi \mathbin{\ulcorner\!\blacktriangledown} (x) = \tilde{\tilde{\delta}}$. For $\psi = (v = 3)$, we have ${}^\bullet\psi = ({}^\bullet v = 3)$
$\psi^\bullet$	$\psi^\bullet$ is a transition proposition. When written as $\psi^\bullet \mathbin{\ulcorner\!\blacktriangledown} (\tilde{\tilde{a}})$, it means action $\tilde{\tilde{a}}$ is performed in such a way that after performing a, ψ becomes true. For $\psi = (v = 3)$, we have $\psi^\bullet = (v^\bullet = 3)$

Roughly it can be said that the axioms show that the signal hiding operator can be distributed over process terms and propositions. At the right hand side of axioms $HSH3$ and $HSH4$, hiding is applied to the conjunction of ${}^{\bullet}\psi, \chi$ and $s_{\rho}(x)^{\bullet}$. This is because dependencies may exist between ${}^{\bullet}\psi, \chi$ and $s_{\rho}(x)^{\bullet}$, because of variables v and $\dot{v}$. After signal hiding, we want to retain the effect of these dependencies in the process $v\triangle(\psi \,{}^{\wedge}\!\!\blacktriangle\, (\chi \ulcorner\!\!\blacktriangledown\, \tilde{\tilde{a}} \cdot x))$.

For example, let ψ be $(v > 3)$, χ be $(v^{\bullet} = {}^{\bullet}v + 1)$ and $s_{\rho}(x)$ be $(v < 3)$. Then the proposition,

$$ {}^{\bullet}\psi \wedge \chi \wedge s_{\rho}(x)^{\bullet} = ({}^{\bullet}v > 3) \wedge (v^{\bullet} = {}^{\bullet}v + 1) \wedge (v^{\bullet} < 3) $$

can never be satisfied. Thus in process $\psi \,{}^{\wedge}\!\!\blacktriangle\, (\chi \ulcorner\!\!\blacktriangledown\, \tilde{\tilde{a}} \cdot ((v < 3) \,{}^{\wedge}\!\!\blacktriangle\, \tilde{\tilde{b}}))$, action $\tilde{\tilde{a}}$ can never be performed. While applying signal hiding, if we hide v in individual propositions then we get $true \,{}^{\wedge}\!\!\blacktriangle\, (true \ulcorner\!\!\blacktriangledown\, \tilde{\tilde{a}} \cdot (true \,{}^{\wedge}\!\!\blacktriangle\, \tilde{\tilde{b}}))$. By taking the conjunction of ${}^{\bullet}\psi, \chi$ and $s_{\rho}(x)^{\bullet}$, and applying signal hiding on ${}^{\bullet}\psi \wedge \chi \wedge s_{\rho}(x)^{\bullet}$, we keep the effect of this dependency. By axiom $HSH4$, (ψ and χ are as defined above),

$$ \begin{aligned} v\triangle(\psi \,{}^{\wedge}\!\!\blacktriangle\, (\chi \ulcorner\!\!\blacktriangledown\, \tilde{\tilde{a}} \cdot ((v < 3) \,{}^{\wedge}\!\!\blacktriangle\, \tilde{\tilde{b}}))) &= true \,{}^{\wedge}\!\!\blacktriangle\, (v\triangle(false) \ulcorner\!\!\blacktriangledown\, \tilde{\tilde{a}} \cdot (true \,{}^{\wedge}\!\!\blacktriangle\, \tilde{\tilde{b}})) \\ &= true \,{}^{\wedge}\!\!\blacktriangle\, (false \ulcorner\!\!\blacktriangledown\, \tilde{\tilde{a}} \cdot (true \,{}^{\wedge}\!\!\blacktriangle\, \tilde{\tilde{b}})) \\ &= true \,{}^{\wedge}\!\!\blacktriangle\, \tilde{\tilde{\delta}}. \end{aligned} $$

A proposition ψ after signal hiding is true if and only if there exists an assignment of variables v and $\dot{v}$ to real numbers for which ψ is true. i.e., let $\psi[r, r'/v, \dot{v}]$ denote the proposition with v and $\dot{v}$ replaced by some real values r and r' respectively, then

$$ v\triangle\psi \Leftrightarrow \exists r, r' \in \mathbb{R} \bullet \psi[r, r'/v, \dot{v}]. $$

2.3 Lifting Rules of signal hiding

In addition to axioms, there are *lifting rules* in BPA^{srt}_{hs}. According to these rules, results of real analysis on environment variables can be incorporated in equations about process terms. In a later version of [3], two lifting rules are defined for the signal hiding operator, which are given in Table 3.

These lifting rules indicate when signal hiding operator can be distributed over propositions and other operators in a delayable process term. The two rules cater for different continuity requirements of the variable to be hidden in the delay interval. The rule $HSHLR1$ is for the case when signal hiding with respect to a variable v, is applied to a process that requires v to be infinitely often continuously differentiable in the delay interval. Whereas rule $HSHLR2$ is for the case when signal hiding is applied to a process, that requires v only to be piecewise infinitely often continuously differentiable during the delay.

We explain here different terms used in the rules. MT is the name given to a mathematical theory used for doing real analysis on environment or state variables in BPA^{srt}_{hs}. In MT, each state variable is interpreted as a real valued function of an interval I in $\mathbb{R}^{\geq}$, that is infinitely often piecewise continuously differentiable in I.

Table 3: Lifting rules for signal hiding ($r > 0$)

$$\frac{\begin{array}{l}\mathbb{V} \cup \{v\} \subseteq C^{\infty}[0,r], \psi'' \leftrightarrow s_{\rho}(x) \vdash_{MT} \\ \quad \exists f \in C^{\infty}[0,r] \quad \bullet((v\triangle\psi)(0) \rightarrow \psi^{v}_{f}(0)) \wedge ((v\triangle\psi')(0) \rightarrow \psi'^{v}_{f}(0)) \wedge \\ \qquad\qquad\qquad\qquad (\forall t \in [0,r] \bullet (v\triangle\phi)(t) \rightarrow \phi^{v}_{f}(t)) \wedge ((v\triangle\psi'')(r) \rightarrow \psi''^{v}_{f}(r))\end{array}}{\begin{array}{l}v\triangle(\psi \mathbin{{}^{\wedge\!\blacktriangle}} (\psi' :\rightarrow (\phi \curvearrowright_{\mathbb{V}\cup\{v\}} \sigma^{r}_{\mathsf{rel}}(\nu_{rel}(x))))) = \\ (v\triangle(\psi \wedge (\psi' \rightarrow \phi))) \mathbin{{}^{\wedge\!\blacktriangle}} (v\triangle(\psi \wedge \psi' \wedge \phi)) :\rightarrow ((v\triangle\phi) \curvearrowright_{\mathbb{V}\setminus\{v\}} \sigma^{r}_{\mathsf{rel}}(v\triangle\nu_{rel}(x)))\end{array}} \; HSHLR1$$

$$\frac{\begin{array}{l}\mathbb{V}\setminus\{v\} \subseteq C^{\infty}[0,r], \psi'' \leftrightarrow s_{\rho}(x) \vdash_{MT} \\ \quad \exists f \in PC^{\infty}[0,r] \quad \bullet((v\triangle\psi)(0) \rightarrow \psi^{v}_{f}(0)) \wedge ((v\triangle\psi')(0) \rightarrow \psi'^{v}_{f}(0)) \wedge \\ \qquad\qquad\qquad\qquad (\forall t \in [0,r] \bullet (v\triangle\phi)(t) \rightarrow \phi^{v}_{f}(t)) \wedge ((v\triangle\psi'')(r) \rightarrow \psi''^{v}_{f}(r))\end{array}}{\begin{array}{l}v\triangle(\psi \mathbin{{}^{\wedge\!\blacktriangle}} (\psi' :\rightarrow (\phi \curvearrowright_{\mathbb{V}\setminus\{v\}} \sigma^{r}_{\mathsf{rel}}(\nu_{rel}(x))))) = \\ (v\triangle(\psi \wedge (\psi' \rightarrow \phi))) \mathbin{{}^{\wedge\!\blacktriangle}} (v\triangle(\psi \wedge \psi' \wedge \phi)) :\rightarrow ((v\triangle\phi) \curvearrowright_{\mathbb{V}\setminus\{v\}} \sigma^{r}_{\mathsf{rel}}(v\triangle\nu_{rel}(x)))\end{array}} \; HSHLR2$$

ψ, ψ' and ψ'' are state propositions. A state proposition, for example ψ, after signal hiding is true if and only if there exists an assignment of variables v and $\dot{v}$ to real numbers for which ψ is true. i.e., let $\psi[r, r'/v, \dot{v}]$ denote the proposition with v and $\dot{v}$ replaced by some real values r and r' respectively, then

$$v\triangle\psi \Leftrightarrow \exists r, r' \in \mathbb{R} \bullet \psi[r, r'/v, \dot{v}].$$

In the lifting rules, we use ψ^{v}_{f} as an abbreviation for $\psi[f, \dot{f}/v, \dot{v}]$, where f is a real valued function of an interval in $\mathbb{R}^{\geq}$. We write $f \in C^{\infty}[0,r]$, to indicate that f (and $\dot{f}$) are infinitely often continuously differentiable in the interval $[0,r]$. We write $f \in PC^{\infty}[0,r]$, to indicate that function f (and $\dot{f}$) are infinitely often piecewise continuously differentiable in the interval $[0,r]$.

For example, in $HSHLR1$, the statement

$$\exists f \in C^{\infty}[0,r] \bullet ((v\triangle\psi)(0) \rightarrow \psi^{v}_{f}(0)),$$

means that at the start of the process, proposition ψ after hiding can only be true, if there exists a real valued function f that is infinitely often continuously differentiable in $[0,r]$, and ψ holds when v and $\dot{v}$ in it are replaced by $f(0)$ and $\dot{f}(0)$.

In the conclusion of $HSHLR1$, signal hiding with respect to variable v, is applied to a process term $\psi \mathbin{{}^{\wedge\!\blacktriangle}} (\psi' :\rightarrow (\phi \curvearrowright_{\mathbb{V}\cup\{v\}} \sigma^{r}_{\mathsf{rel}}(\nu_{rel}(x))$. ψ is the signal emitted by the process. ψ' is the proposition of the conditional guarding the term $\phi \curvearrowright_{\mathbb{V}} \sigma^{r}_{\mathsf{rel}}(\nu_{rel}(x))$, where $\phi \curvearrowright_{\mathbb{V}} \sigma^{r}_{\mathsf{rel}}(\nu_{rel}(x))$ is the delaying part of the process. In BPA^{srt}_{hs}, $\curvearrowright$ is known as the signal evolution operator. It assigns a state proposition and a set of environment variables to the delay interval. The evolution operator requires that $\phi \curvearrowright_{\mathbb{V}} \sigma^{r}_{\mathsf{rel}}(\nu_{rel}(x))$ can only delay, under the conditions that state proposition ϕ holds during the complete duration of the delay and variables in the set $\mathbb{V} \cup \{v\}$, remain infinitely often continuously differentiable.

σ^r_{rel} indicates a delay interval of r time units in the setting of relative timing. Process term $(\phi \curvearrowright_{\mathbb{V}\cup\{v\}} \sigma^r_{\mathsf{rel}}(\nu_{rel}(x))$, evolves into $\nu_{rel}(x)$, (where x is any BPA^{srt}_{hs} process), after delaying for r time units. The process term $\nu_{rel}(x)$ is a process term that behaves as x, but with all initial delays removed from x. Thus $\nu_{rel}(x)$ can only perform actions in the beginning. Because of the continuity requirements of the variable v in the delay interval, the lifting rules are applicable to processes with definite delays.

ψ'' is the name given to the root signal of a process x, i.e., to $s_\rho(x)$. The root signal of a process is the signal emitted by it. For signal relations, see section 2.1 and Table 9. The root signal of $\nu_{rel}(x)$ is true in a state if and only if the root signal of x is true in that state.

$$\alpha \in [\mathsf{s}(x)] \iff \alpha \in [\mathsf{s}(\nu_{rel}(x))]$$

The statement

$$\begin{array}{l}\psi'' \leftrightarrow s_\rho(x) \vdash_{MT} \\ \exists f \in C^\infty[0,r] \bullet ((v\triangle\psi'')(r) \rightarrow \psi''^{v}_{f}(r)),\end{array}$$

means that after a delay of r time units, the root signal of x after signal hiding, is true if there exists a real valued function f that is infinitely often continuously differentiable in $[0,r]$, and root signal of x holds when v and $\dot{v}$ in it are replaced by $f(r)$ and $\dot{f}(r)$.

In $HSHLR2$, signal hiding with respect to variable v, is applied to the process $\psi \curvearrowright (\psi' :\rightarrow (\phi \curvearrowright_{\mathbb{V}\cup\{v\}} \sigma^r_{\mathsf{rel}}(\nu_{rel}(x))$. The variable set $\mathbb{V}$, that must remain infinitely often continuously differentiable in interval $[0,r]$, excludes variable v. Therefore, in the premise of $HSHLR2$, the continuity requirements on function f require it to be piecewise infinitely often continuously differentiable in $[0,r]$.

The conclusions to the two rules indicate that after signal hiding, we want to retain the effect of any dependencies because of variables v and $\dot{v}$ among propositions ψ, ψ' and ϕ. That is why for example, the conditional proposition after signal hiding is $v\triangle(\psi \wedge \psi' \wedge \phi)$. After signal hiding, the two processes in the rules, delay in $[0,r]$ according to proposition $v\triangle\phi$, i.e. without regard to the trajectory followed by v. Also in $HSHLR1$, after hiding, v is not required to be infinitely often continuously differentiable during the delay.

3 An Example: Thermostat

We give here the example of a thermostat given in [5].

A thermostat controls the heating of a room. Initially the temperature is 18° and the heating is on. The temperature (denoted by T) rises according to the equation $\dot{T} = -T + 22$. When it reaches 20°, heating is turned off. The temperature of the room then falls according to the equation $\dot{T} = -T + 17$. When it reaches 18°, the heating is again turned on and the process repeats itself.

A BPA_{hs}^{srt} specification describing the thermostat is given below:

$$
\begin{array}{ll}
Th & = (T = 18) \curvearrowright (Th^{on}), \\
Th^{on} & = (18 \leq T \leq 20 \ \wedge \ \dot{T} = -T + 22) \curvearrowright_{T} \\
 & \sigma^{*}_{\mathsf{rel}}((T = 20) :\rightarrow (T^{\bullet} = {}^{\bullet}T) \curvearrowright \widetilde{\widetilde{\mathtt{toff}}} \cdot Th^{off}), \\
Th^{off} & = (18 \leq T \leq 20 \ \wedge \ \dot{T} = -T + 17) \curvearrowright_{T} \\
 & \sigma^{*}_{\mathsf{rel}}((T = 18) :\rightarrow (T^{\bullet} = {}^{\bullet}T) \curvearrowright \widetilde{\widetilde{\mathtt{ton}}} \cdot Th^{on});
\end{array}
$$

where $\sigma^{*}_{\mathsf{rel}}(x)$ is an abbreviation for $\int_{u \in [0,\infty)} \sigma^{u}_{\mathsf{rel}}(x)$, i.e., an alternative composition of $\sigma^{u}_{\mathsf{rel}}(x)$ for all $u \in [0, \infty)$. $\sigma^{*}_{\mathsf{rel}}$ represents an indefinite delay (including a delay of zero duration) before a process.

$(T^{\bullet} = {}^{\bullet}T)$ is required here to ensure that the temperature remains constant when the actions $\widetilde{\widetilde{\mathtt{ton}}}$ and $\widetilde{\widetilde{\mathtt{toff}}}$ are performed. T under the curved arrow, in $(18 \leq T \leq 20 \ \wedge \ \dot{T} = -T + 22) \curvearrowright_{T}$ and $(18 \leq T \leq 20 \ \wedge \ \dot{T} = -T + 17) \curvearrowright_{T}$, indicates that the temperature and its derivative $\dot{T}$, are infinitely often continuously differentiable during the delays.

Now consider another process definition Th' that defines the same thermostat as follows,

$$
\begin{array}{ll}
Th' & = (T = 18) \curvearrowright (Th'^{on}), \\
Th'^{on} & = (18 \leq T \leq 20 \ \wedge \ \dot{T} = -T + 22) \curvearrowright_{T} \\
 & \sigma^{ln2}_{\mathsf{rel}}((T^{\bullet} = {}^{\bullet}T) \curvearrowright \widetilde{\widetilde{\mathtt{toff}}} \cdot Th'^{off}), \\
Th'^{off} & = (18 \leq T \leq 20 \ \wedge \ \dot{T} = -T + 17) \curvearrowright_{T} \\
 & \sigma^{ln3}_{\mathsf{rel}}((T^{\bullet} = {}^{\bullet}T) \curvearrowright \widetilde{\widetilde{\mathtt{ton}}} \cdot Th'^{on}).
\end{array}
$$

We compare the behaviours of Th and Th' in the operational semantic rules of BPA_{hs}^{srt}.

There are two kinds of bisimulations in BPA_{hs}^{srt}. One is simply called a bisimulation and the other is known as "Interference compatible bisimulation", abbreviated as *ic-bisimulation*. A bisimulation relates two configurations (a configuration is an ordered pair of a BPA_{hs}^{srt} process term and a state) that have the same states, if their behaviours match according to the operational semantic rules of BPA_{hs}^{srt}. An ic-bisimulation relates two process terms if their behaviours match when compared in all states and this property is reflected in all the subsequent pairs of terms obtained as a result of matching transitions. (See the appendix for the definitions of bisimulation and ic-bisimulation).

Under the operational semantic rules of BPA_{hs}^{srt}, $\langle Th, 18\rangle$ is bisimilar to $\langle Th', 18\rangle$, denoted by $\langle Th, 18\rangle \leftrightarrows \langle Th', 18\rangle$, but Th is not ic-bisimilar to Th', denoted by $Th \not\cong Th'$. The operational semantic rules of BPA_{hs}^{srt} and the proof that $\langle Th, 18\rangle \leftrightarrows \langle Th', 18\rangle$ are given in the appendix. Here we give a proof that $Th \not\cong Th'$.

Let $(18 \leq T \leq 20 \ \wedge \ \dot{T} = -T + 22)$ be denoted by *up*. We rewrite Th^{on} and

Th'^{on} as,

$$
\begin{array}{llll}
Th & = (T = 18) \curvearrowright\!\!\!\!\blacktriangle\, Th^{on}, & & \\
Th^{on} & = up \curvearrowright_T Th^{on}_0, & Th' & = (T = 18) \curvearrowright\!\!\!\!\blacktriangle\, Th'^{on}, \\
Th^{on}_t & = \int_{u \in [t,\infty)} \sigma_{\mathsf{rel}}^{u-t}(Th^{\rightarrow}), & Th'^{on} & = up \curvearrowright_T Th'^{on}_0, \\
Th^{\rightarrow} & = (T = 20) \mathrel{:\!\rightarrow} & Th'^{on}_t & = \sigma_{\mathsf{rel}}^{ln2-t}(Th'^{\rightarrow}), \\
 & (T^{\bullet} = {}^{\bullet}T) \curvearrowright \widetilde{\widetilde{\widetilde{\mathtt{toff}}}} \cdot Th^{off} & Th'^{\rightarrow} & = (T^{\bullet} = {}^{\bullet}T) \curvearrowright \widetilde{\widetilde{\widetilde{\mathtt{toff}}}} \cdot Th'^{off}
\end{array}
$$

where $t \in [0, \infty]$ and $q \in [0, ln2]$.

In Th^{on}_t and Th'^{on}_t, t indicates how much time is spent since the start of the process. Note for any $t \in \mathbb{R}^{\geq}$, $Th^{on}_t = \int_{u \in [0,\infty)} \sigma^u_{\mathsf{rel}}(Th^{\rightarrow}) = \sigma^*_{\mathsf{rel}}(Th^{\rightarrow})$.

Th has an indefinite delay, which is controlled by the proposition up and the conditional $T = 20$. Whereas Th' has a definite time delay of $ln2$ time units. According to the operational semantics of BPA^{srt}_{hs}, starting from 18° and evolving according to the same proposition up, the two thermostats reach temperature 20° at time $ln2$, at which both Th^{on} and Th'^{on} perform action $\widetilde{\widetilde{\widetilde{toff}}}$.

Consider some instant say $ln(4/3) \in (0, ln2)$. Starting from 18° and evolving according to the same proposition up, the two thermostats reach temperature 19° at time $ln(4/3)$,

Consider process terms $up \curvearrowright_T Th^{on}_{ln4/3}$ and $up \curvearrowright_T Th'^{on}_{ln4/3}$.

$$
\begin{array}{ll}
up \curvearrowright_T Th^{on}_{ln4/3} = & up \curvearrowright_T \int_{u \in [ln4/3,\infty)} \sigma_{\mathsf{rel}}^{u-ln4/3}(Th^{\rightarrow}) \\
 & = up \curvearrowright_T \sigma^*_{\mathsf{rel}}(Th^{\rightarrow}) \\
\text{and} & \\
up \curvearrowright_T Th'^{on}_{ln4/3} = & up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-(ln4/3)}(Th'^{\rightarrow}) \\
 & = up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln3/2}(Th'^{\rightarrow}).
\end{array}
$$

At temperature 19, $up \curvearrowright_T Th^{on}_{ln4/3}$ will wait the same amount of time as $up \curvearrowright_T Th'^{on}_{ln4/3}$, i.e., $ln(3/2)$ time units, before performing action `toff`. (See the proof of $\langle Th, 18\rangle \leftrightarrows \langle Th', 18\rangle$).

But $up \curvearrowright_T Th^{on}_{ln4/3}$ behaves differently than $up \curvearrowright_T Th'^{on}_{ln4/3}$ at states other than 19. For example at temperature 20°, $up \curvearrowright_T Th^{on}_{ln4/3}$ can perform action $\widetilde{\widetilde{\widetilde{toff}}}$, whereas $up \curvearrowright_T Th'^{on}_{ln4/3}$ cannot.

From rules {24,3,18,27,16,1} of table 8 and rule 1 of table 10,

$$
\begin{array}{rl}
 & \langle up \curvearrowright_T Th^{on}_{ln4/3}, 20\rangle \xrightarrow{toff} \langle up \curvearrowright_T Th^{off}, 20\rangle, \\
\text{where} & \langle up \curvearrowright_T Th'^{on}_{ln4/3}, 20\rangle \not\xrightarrow{toff};
\end{array}
$$

Thus process Th and Th' are not ic-bisimilar.

4 Problems in Signal hiding

4.1 Bisimulation is not preserved by signal hiding

As briefly described in section 2.1, the problem with signal hiding is that under the operational semantics given in [3], bisimulation is not preserved by signal hiding, i.e. two BPA_{hs}^{srt} processes that are bisimilar before, do not always remain bisimilar after the application of signal hiding.

Consider process Th^{on}. Action $\widetilde{\widetilde{toff}}$ is performed by Th^{on} when the temperature reaches 20°. After signal hiding, (see rules 1 & 2 in table 1), $\widetilde{\widetilde{toff}}$ can be performed at any temperature. Whereas in process Th'^{on}, there is a definite time delay of $ln2$ units and the performance of $\widetilde{\widetilde{toff}}$ does not depend on the temperature. After signal hiding, Th'^{on} still performs action $\widetilde{\widetilde{toff}}$ at time $ln2$, whereas Th can perform $\widetilde{\widetilde{toff}}$ (after hiding) at any time. Thus after hiding the temperature, Th and Th' behave differently.

After hiding $\langle T \triangle Th, * \rangle$ behaves as $\langle Th'''', * \rangle$, where $*$ indicates a state with an arbitrary value of the temperature and,

$$\begin{array}{rcl} Th'''' & = & \int_{u \in (0,\infty)} \sigma_{\mathsf{rel}}^{u}(\widetilde{\widetilde{\mathtt{toff}}}) \cdot \sigma_{\mathsf{rel}}^{*}(\widetilde{\widetilde{\mathtt{ton}}}) \cdot Th''' \\ Th''' & = & \sigma_{\mathsf{rel}}^{*}(\widetilde{\widetilde{\mathtt{toff}}}) \cdot \sigma_{\mathsf{rel}}^{*}(\widetilde{\widetilde{\mathtt{ton}}}) \cdot Th'''; \end{array}$$

and $\langle T \triangle Th', * \rangle$ behaves as $\langle Th'', * \rangle$, where,

$$Th'' = \sigma_{\mathsf{rel}}^{ln2}(\widetilde{\widetilde{\mathtt{toff}}}) \cdot \sigma_{\mathsf{rel}}^{ln3}(\widetilde{\widetilde{\mathtt{ton}}}) \cdot Th''.$$

$$\langle T \triangle Th, * \rangle \not\leftrightarrow \langle T \triangle Th', * \rangle$$

(Refer to the appendix for a detailed proof). Thus bisimulation no longer remains a congruence for signal hiding in BPA_{hs}^{srt}.

4.2 An attempt at developing new semantic rules for signal hiding

We made several attempts to develop an operational semantics that preserves bisimulation with signal hiding operator. But we could not come up with a simple set of SOS rules that correctly specifies the behaviour of signal hiding operator as defined in [3]. One such attempt of defining a desired set of SOS rules is described below.

We add a *timer*, (a nonnegative real number), to the configurations in BPA_{hs}^{srt} semantics; i.e. our configurations in the new semantic rules consist of three components, a process term, a state and a timer. The timer accumulates time while a process is idling and resets to zero when an action is performed. The motivation is to keep a record of the instant at which an action is performed, if an action is performed after a time delay. In case two actions are performed consecutively without any delay in between, then the timer of the second action will not exceed zero.

Table 4: Operational Semantic Rules for Signal hiding (timers added to configurations)

$\dfrac{\langle x,\alpha,t\rangle \xrightarrow{a} \langle x',\alpha',0\rangle}{\langle v\triangle x,\alpha^*,t\rangle \xrightarrow{a} \langle v\triangle x',\alpha'^*,0\rangle}$	$\alpha^* =_v \alpha, \alpha'^* =_v \alpha'$	$Rule1$
$\dfrac{\langle x,\alpha,t\rangle \xrightarrow{a} \langle \surd,\alpha',0\rangle}{\langle v\triangle x,\alpha^*,t\rangle \xrightarrow{a} \langle \surd,\alpha'^*,0\rangle}$	$\alpha^* =_v \alpha, \alpha'^* =_v \alpha'$	$Rule2$
$\dfrac{\langle x,\alpha,t\rangle \xmapsto{r,\rho} \langle x',\alpha',t+r\rangle}{\langle v\triangle x,\alpha^*,t\rangle \xmapsto{r,\rho^*} \langle v\triangle x',\alpha'^*,t+r\rangle}$	$\alpha^* =_v \alpha, \alpha'^* =_v \alpha', \rho^* =_v \rho$	$Rule3$
$\dfrac{\alpha \in [\mathsf{s}(x)]}{\alpha^* \in [\mathsf{s}(v\triangle x)]}$	$\alpha^* =_v \alpha$	$Rule4$

The transition rules of signal hiding in this semantics are in table 4.

Rule 1 shows the reset of the timer to zero at the execution of an action. Similarly for Rule 2. In Rule 3, the timer evolves from t to $t+r$ by the execution of a time step of duration r. The transition rules for other operators for BPA^{srt}_{hs} are constructed on the same principle, i.e. time accumulates when a process is waiting and resets when an action is performed. The rules for signal relations remain the same as given in table 9.

Consider processes Th and Th' again. The timer in the new operational semantics records the time from the start of the process until $\widetilde{\widetilde{\widetilde{toff}}}$ is performed. This information is used when temperature is hidden to know after how long $T\triangle Th$ should perform $\widetilde{\widetilde{\widetilde{toff}}}$. By this scheme we hoped to preserve bisimilarity of $T\triangle Th$ and $T\triangle Th'$. But this idea does not work as explained below.

The signal emitted by Th is only true in state 18 (by rule 8 table 9). By rule 4 table 4,

$$18 \in [\mathsf{s}(Th)] \implies * \in [\mathsf{s}(T\triangle Th)]$$

where $*$ is a state with an arbitrary value of temperature, i.e.,

$$* =_T 18 =_T 19 =_T 20.$$

Let ρ be a state evolution on the interval $[0, ln(4/3)]$ that keeps T and $\dot{T}$ infinitely often continuously differentiable and satisfies proposition up. Let $\rho(0)(T) = 18$ and $\rho(ln(4/3))(T) = 19$. Let ρ' be another state evolution such that $\rho =_T \rho'$. We can derive,

$$\langle Th, 18, 0\rangle \xmapsto{ln(4/3),\rho} \langle up \curvearrowright_T Th^{on}_{ln(4/3)}, 19, ln(4/3)\rangle.$$

From rule 3 of table 4, we can derive:

$$\langle T\triangle Th, *, 0\rangle \xmapsto{ln(4/3),\rho'} \langle T\triangle up \curvearrowright_T Th^{on}_{ln4/3}, *, ln(4/3)\rangle$$

The signal of $up \curvearrowright_T Th^{on}_{ln(4/3)}$ is true in states with temperature in the range $[18, 20]$. Also note, (from the rules given in Table 4), that the process $\langle T\triangle up \curvearrowright_T Th^{on}_{ln(4/3)}, *, ln(4/3)\rangle$ can behave as the set of processes $\{\langle up \curvearrowright_T Th^{on}_{ln4/3}, \mathbb{T}, ln(4/3)\rangle \mid \mathbb{T} \in [18, 20]\}$. i.e. the process $up \curvearrowright_T Th^{on}_{ln(4/3)}$ in any state with $T \in [18, 20]$ and with timer $ln(4/3)$.

A few possible behaviours of $\langle T\triangle Th^{on}_{ln4/3}, *, ln(4/3)\rangle$ are as follows: Let ρ_2 and ρ_2' be state evolutions on interval $[0, ln(3/2)]$ with $\rho_2(0)(T) = 19$, $\rho_2(ln(3/2))(T) = 20$ and $\rho_2 =_T \rho_2'$. Let ρ_2 satisfy proposition up while keeping T and $\dot{T}$ infinitely often continuously differentiable in $[0, ln(3/2)]$.

By Rule 3,

$$\langle up \curvearrowright_T Th^{on}_{ln4/3}, 19, ln(4/3)\rangle \xmapsto{ln(3/2),\rho_2} \langle up \curvearrowright_T Th^{on}_{ln2}, 20, ln2\rangle$$

$$\Longrightarrow \quad \langle T\triangle up \curvearrowright_T Th^{on}_{ln4/3}, *, ln(4/3)\rangle \xmapsto{ln(3/2),\rho_2'} \langle T\triangle up \curvearrowright_T Th^{on}_{ln2}, *, ln2\rangle;$$

By Rule 2, $\langle up \curvearrowright_T Th^{on}_{ln4/3}, 20, ln(4/3)\rangle \xrightarrow{\mathtt{toff}} \langle up \curvearrowright_T Th^{off}, 20, 0\rangle$

$$\Longrightarrow \quad \langle T\triangle up \curvearrowright_T Th^{on}_{ln4/3}, *, ln(4/3)\rangle \xrightarrow{\mathtt{toff}} \langle T\triangle up \curvearrowright_T Th^{off}, 20, 0\rangle.$$

The second derivation indicates that $T\triangle Th$ can also perform action $\widetilde{\widetilde{toff}}$ before $ln2$ time units. It can also perform $\widetilde{\widetilde{toff}}$ after $ln2$ time units, if we consider the behaviour of $\langle up \curvearrowright_T Th^{on}_{ln(4/3)}, 18, ln(4/3)\rangle$. Whereas $T\triangle Th'$ behaves as $Th'' = \sigma^{ln2}_{\mathsf{rel}}(\widetilde{\widetilde{\mathtt{toff}}}) \cdot \sigma^{ln3}_{\mathsf{rel}}(\widetilde{\widetilde{\mathtt{ton}}}) \cdot Th''$, under these operational semantic rules also. Thus bisimilarity is still not preserved over signal hiding.

The reason being that, while deriving a transition for a process of the form $\langle v\triangle x, \alpha', t\rangle$, (where v is any variable and $t > 0$), we may have lots of options (configurations) that can be used as a source to the premise of the rule being applied (see table 4). Some of these configurations may not actually be derivable from the initial process, for example starting from $\langle up \curvearrowright_T Th, 18, 0\rangle$, $\langle up \curvearrowright_T Th^{on}_{ln4/3}, 20, ln(4/3)\rangle$ cannot be derived from the rules. But we have no means to verify it while deriving a transition for $\langle T\triangle up \curvearrowright_T Th^{on}_{ln4/3}, *, ln(4/3)\rangle$.

In structural operational semantics, at all times our point of reference is the current configuration and the set of transition rules. As we perform actions and delays, our current configuration evolves and the information found in process terms regarding previous actions and evolutions is lost. Therefore a configuration with a process term, state and timer cannot check itself whether it is a *derivable* configuration or not. The problem with this solution led us to the idea of having a graph semantics for BPA^{srt}_{hs}. In a graph, we retain the information of the initial configuration and hence can figure out at all times whether starting from an initial configuration a given configuration is reachable or not.

4.3 Proposed Solution to the problem

We propose to solve this problem by presenting a graph model of BPA^{srt}_{hs}. The nodes of the graph, called *configurations*, consist of a process term, state and a timer. The nodes are connected with edges that correspond to action, termination and time steps as defined in the operational semantics of [5]. The timer accumulates time while idling and resets when an action is performed. The traversal of our graph begins with a special set of nodes called "initial configurations". We define a notion of *reachability* of configurations depending on what configurations in our graph are traversed when starting from an initial configuration. Finally we remove configurations in our graphs that are not reachable by any initial configuration.

Processes are represented by graphs and operations on processes are defined as operations on graphs.

5 The Graph Model for BPA^{hs}_{srt}

We define a graph model for BPA^{srt}_{hs}. Several definitions required by the graph model are given below:

We assume a predetermined set V of environment variables and a predetermined set A of actions. $\dot{V} = \{\dot{v} \mid v \in V\}$ denotes the set of derivatives of all variables $v \in V$. We define a function of the type $V \cup \dot{V} \to \mathbb{R}$, called state, which assigns real values to variables $v \in V \cup \dot{V}$. The set of all possible states is denoted by S, i.e. $S = V \cup \dot{V} \to \mathbb{R}$.

A function of the type $[0, r] \to (V \to \mathbb{R})$ gives the evolution of all variables $v \in V$ in a duration $[0, r]$, $r \in \mathbb{R}^{>}$. Let ρ be a function of type $[0, r] \to (V \to \mathbb{R})$. Then for every $v \in V$, we write ρ_v for the function $\rho_v : [0, r] \to \mathbb{R}$, defined by $\rho_v(t) = \rho(t)(v)$. We call ρ a *state evolution*, if for all $v \in V$, ρ_v is piecewise infinitely often continuously differentiable in $[0, r)$. If ρ is a state evolution, we say that ρ is smooth for a subset $\mathbb{V}$ of V, if ρ_v is infinitely often continuously differentiable in $[0, r]$ for all $v \in \mathbb{V}$.

We denote the set of all state evolutions $\rho : [0, r] \to (V \to \mathbb{R})$ defined on the interval $[0, r]$, by ϵ_r. We denote the set of all state evolutions by ϵ,

$$\epsilon = \bigcup_{0<r<\infty} \epsilon_r.$$

For a given state evolution $\rho : [0, r] \to (V \to \mathbb{R})$ and a given instant $t \in [0, r]$, there is a unique state α^ρ_t that *agrees* with ρ at time t, i.e. for all $v \in V$:

$$\alpha^\rho_t(v) = \rho_v(t) \quad \text{and } \alpha^\rho_t(\dot{v}) = \dot{\rho_v}(t).$$

A state proposition ϕ is satisfied by a state evolution $\rho \in \epsilon_r$ if and only if all the states corresponding to $\rho(t)$ for $t \in [0, r]$ satisfy ϕ, i.e.,

$$\rho \models \phi \text{ iff } \alpha^\rho_t \models \phi, \quad \text{for all } t \in [0, r].$$

In addition if ρ is also smooth for $\mathbb{V}$ a subset of V, we write,

$$\rho \models_{\mathbb{V}} \phi \text{ or } \alpha^\rho_0 \xmapsto{r,\rho} \alpha^\rho_r \models_{\mathbb{V}} \phi.$$

We denote the set of all BPA_{hs}^{srt} process terms by P, where,

$$
\begin{aligned}
P := \quad & \tilde{\tilde{a}} \mid \tilde{\tilde{\delta}} \mid \bot \mid \nu_{rel}(P) \mid P \cdot P \mid P + P \mid \sigma^{0}_{\mathsf{rel}}(P) \\
& \mid \sigma^{r}_{\mathsf{rel}}(P) \mid \psi \mathbin{{}^{\wedge}\!\blacktriangle} P \mid \psi :\rightarrow P \mid \chi \mathbin{\ulcorner\!\blacktriangledown} P \mid \phi \curvearrowright_{\mathbb{V}} P;
\end{aligned}
$$

where ψ and ϕ are state propositions, χ is a transition proposition, $r > 0$ and $\mathbb{V}$ is a subset of V.

We denote the set of all pairs of durations and state evolutions on them by D and call them *delays*, i.e.,

$$D = \{(r, \rho) \mid r \in \mathbb{R}^{>} \ \wedge \ \rho \in \epsilon_r\}.$$

5.1 Transition System

5.1.1 Definition

We define a transition system for hybrid processes TS as a five-tuple, i.e.,

$$TS = (C, \rightarrow, \longmapsto, I, F),$$

where:

1. $C \subseteq (P \cup \{\surd\} \times S \times \mathbb{R}^{\geq})$ is a set of *configurations.*
2. $\rightarrow \subseteq C \times A \times C$ is the set of all *action transitions* .

   If $(c_1, a, c_2) \in \rightarrow$, then we write $c_1 \xrightarrow{a} c_2$.

   If $c_1 \xrightarrow{a} c_2$, then the third component of c_2 is reset to 0, (i.e. $c_2 = (p, \alpha, 0)$, for some $p \in P \cup \{\surd\}$, $\alpha \in S$), as we are dealing with relative time.
3. $\longmapsto \subseteq C \times D \times C$ is the set of all *time transitions.*

   If $(c_1, (r, \rho), c_2) \in \longmapsto$, then we write $c_1 \overset{r,\rho}{\longmapsto} c_2$.

   - $(p_1, \alpha_1, t_1) \overset{r,\rho}{\longmapsto} (p_2, \alpha_2, t_2) \Rightarrow t_2 = t_1 + r$
4. $I \subseteq C$ is the set of *initial configurations* given for a process.

   Any initial configuration is of the form $i = (p, \alpha, 0)$. If $i_1 = (p_1, \alpha_1, 0)$ and $i_2 = (p_2, \alpha_2, 0) \in I$, then $p_1 = p_2 = p$, i.e. the process term is same. Term $p \in P$ (p cannot be $\surd$) is the process expression of TS, denoted by $\mathtt{expr}(TS)$.
5. $F \subseteq C$ is the set of *final states.*

   If $f \in F$, then f is of the form $(\surd, \alpha, 0)$ for some $\alpha \in S$, as a final state can only be entered by an action transition.

5.1.2 Discussion of the definition

In [5], a structural operational semantics for BPA_{hs}^{srt} is given. The process graph induced by this operational semantics has process nodes consisting of process terms and states. The process nodes are denoted by $\langle p, \alpha \rangle$, for some $p \in P$ and $\alpha \in S$. We add timing information to this model. The configurations in our transition systems have a label for a *timer* $\in \mathbb{R}^{\geq}$ in addition to process terms and states. The timer indicates the time elapsed since the last action. When an action is performed (indicated by action transitions) it is reset. It accumulates during idling (indicated by time transitions).

In an action transition $c_1 \xrightarrow{a} c_2$, c_1 is called the *source*, a is called the *action label* and c_2 is called the *target*. In a time transition $c_1 \xmapsto{r,\rho} c_2$, c_1 is called the *source*, r is called the *time duration*, ρ is called the *state evolution* and c_2 is called the *target*. The action and time transitions are said to *originate* from their sources and *end* in their targets

$\surd$ stands for the *termination* symbol and denotes successful termination of a process in BPA_{hs}^{srt}. There is no concept of an *empty process* in BPA_{hs}^{srt} (an empty process is a process that terminates without idling or performing an action), so we require that the process term of an initial configuration cannot be equal to the termination symbol. It should not be too difficult to add a successful termination process ϵ to the current set-up, but we leave this for future work for now. On the other hand, in place of a process term, a final configuration always has $\surd$. Final configurations cannot be sources to any action or time transitions. Furthermore, a final configuration can only be a target to action transitions and not to a time transition. This is in accordance with the assumption of BPA_{hs}^{srt}, that an idling process can only terminate by performing an action.

The way we form a transition system from a BPA_{hs}^{srt} term is such that the process terms of all initial configurations are equal to that BPA_{hs}^{srt} term. We call the common process term of all initial configurations as *process expression* of the transition system. The states of the initial configurations of a transition system represent the set of states in which the signal emitted by the process expression is true.

We want our process model to be consistent with the work done in [5]. This means that when we remove the time labels from all configurations, the transition system of a process becomes exactly the same as the process graph induced by the semantics of [5].

5.2 Garbage Collection on Transition Systems

We define here a notion of reachability on a transition system which is quite similar to the reachability relation defined in [4] (except that in [4] the transition systems do not have time transitions).

Let TS be a transition system given by,

$$TS = (C, \rightarrow, \longmapsto, I, F).$$

We define a *reachability* relation $\twoheadrightarrow \subseteq C \times C$, as the smallest relation, such that for all configurations $c_0, c_1, c_2 \in C$, action $a \in A$ and delay $(r, \rho) \in D$,

- $c_0 \twoheadrightarrow c_0$;
- If $c_0 \xrightarrow{a} c_1$ and $c_1 \twoheadrightarrow c_2$, then $c_0 \twoheadrightarrow c_2$;
- If $c_0 \xmapsto{r,\rho} c_1$ and $c_1 \twoheadrightarrow c_2$, then $c_0 \twoheadrightarrow c_2$.

If $c_0 \twoheadrightarrow c_1$ we say c_1 is *reachable* from c_0. $\texttt{Reach}(c_0)$ denotes the set of all configurations reachable from c_0, i.e.,

$$\texttt{Reach}(c_0) = \{c \in C \mid c_0 \twoheadrightarrow c\}.$$

For a set of configurations $C' \subseteq C$, $\texttt{Reach}(C')$ denotes the set of all configurations that are reachable from any member of C', i.e.,

$$\texttt{Reach}(C') = \{c \in C \mid \exists c' \in C' \bullet c' \twoheadrightarrow c\}.$$

As we apply different operators on a transition system, we add new configurations and transitions to it. During this process some configurations and transitions originating from them may become *unreachable* (i.e. the configurations are no longer reachable from an initial configuration) in the new transition system. Unreachable configurations and transitions have no effect on the behaviour of a transition system. So we can safely remove them. This is called *garbage collection*.

We write $\texttt{reach}(TS)$ for TS with all the unreachable configurations and transitions removed from it, i.e.,

$$\texttt{reach}(TS) = (C', \rightarrow', \longmapsto', I, F);$$

where,

- $C' = \texttt{Reach}(I)$;
- $\rightarrow' = \{c_0 \xrightarrow{a} c_1 \mid c_0 \in \texttt{Reach}(I)\}$; and
- $\longmapsto' = \{c_0 \xmapsto{r,\rho} c_1 \mid c_0 \in \texttt{Reach}(I)\}$.

5.3 Undelayable Actions

An action may change the values of variables arbitrarily. We associate with every action $a \in A$ a transition system:

$$TS(\tilde{\tilde{a}}) = (C, \rightarrow, \emptyset, I, F),$$

where,

1. $C = C_1 \cup C_1'$,
   $C_1 = \{(\tilde{\tilde{a}}, \alpha, 0) \mid \alpha \in S\}$ and $C_1' = \{(\surd, \alpha, 0) \mid \alpha \in S\}$;
2. $\rightarrow = \{(c_1, a, c_1') \mid c_1 \in C_1 \wedge c_1' \in C_1'\}$;
3. $I = C_1$; and
4. $F = C_1'$.

5.4 Non-existence

We propose to describe the non-existent process $\perp$ by a transition system with an empty configuration-set, an empty set of action transitions and an empty set of time transitions, i.e.,

$$TS(\perp) = (\emptyset, \emptyset, \emptyset, \emptyset, \emptyset).$$

We define the process expression of $TS(\perp)$ to be $\perp$, i.e.,

$$\texttt{expr}(TS(\perp)) = \perp$$

5.5 Undelayable Deadlock

The transition system corresponding to $\tilde{\tilde{\delta}}$ has no action transitions, no time transitions and no final or terminating configurations, i.e.,

$$TS(\tilde{\tilde{\delta}}) = (C, \emptyset, \emptyset, I, \emptyset),$$

with

- $C = \{(\tilde{\tilde{\delta}}, \alpha, 0) \mid \alpha \in S\}$; and
- $I = (\{(\tilde{\tilde{\delta}}, \alpha, 0) \mid \alpha \in S\} =)C$.

5.6 Signal Emission

The operation of signal emission takes two operands, one a *state proposition* and the other a transition system. A state proposition is a proposition on the variables in $V \cup \dot{V}$. For a formal definition of a state proposition see [5]. The signal emission operator acts on the set of initial configurations of a transition system. The configurations whose states do not satisfy the given state proposition are removed from the set of initial configurations, while the process terms of other initial configurations are modified to include the state proposition as an emitted signal.

Let $c = (p, \alpha, t)$ be a configuration and ψ be a state proposition.

We denote α satisfies ψ by $\alpha \models \psi$, and write,

$$c \models \psi \text{ iff } \alpha \models \psi.$$

We write $\psi \,\hat{}\, (p, \alpha, t)$ for $(\psi \,\hat{}\, p, \alpha, t)$.

Let TS be a transition system, i.e.,

$$TS = (C, \rightarrow, \longmapsto, I, F).$$

We define $\psi \,\hat{}\, TS$ as,

$$\psi \,\hat{}\, TS = (C', \rightarrow', \longmapsto', I', F),$$

where,

1. The set of initial configurations I' is defined as,

$$I' = \{\psi \mathbin{\hat{\blacktriangle}} i \mid i \in I \ \wedge \ i \models \psi\};$$

2. the set of all configurations C' is defined as,

$$C' = C \cup I';$$

3. the set of action transitions $\rightarrow'$ is defined as,

$$\rightarrow' = \rightarrow \cup \rightarrow'',$$

   where $\rightarrow''$ is the set of all action transitions originating from members of I', i.e.,

$$\rightarrow'' = \{(\psi \mathbin{\hat{\blacktriangle}} i, a, c) \mid i \in I, c \in C, a \in A, i \models \psi \ \wedge \ i \xrightarrow{a} c\};$$

4. $\longmapsto'$ is the set of all time transitions, i.e.,

$$\longmapsto' = \longmapsto \cup \longmapsto'',$$

   where $\longmapsto''$ is the set of all time transitions originating from members of I', i.e.,

$$\longmapsto'' = \{(\psi \mathbin{\hat{\blacktriangle}} i, (r,\rho), c) \mid i \in I, c \in C, (r,\rho) \in D, i \models \psi \ \wedge \ i \xmapsto{r,\rho} c\}.$$

   This means that in a time transition ψ is only required to hold in the source of the transition and not along the trajectory or in the target.

5.7 Relative Undelayable Timeout

The operation of relative undelayable timeout on a process P prevents it from idling in the start. If there are no options to a process in the start other than idling, then applying the relative undelayable timeout operator on P, denoted by $\nu_{rel}(P)$, results in undelayable deadlock. ν_{rel} has no effect on the signal of a process. It distributes over the signal emission, signal transition, signal evolution and conditional operators without affecting the signal propositions.

The relative undelayable timeout operator takes a transition system as an operand.

We write $\nu_{rel}(p, \alpha, t)$ for $(\nu_{rel}(p), \alpha, t)$, where (p, α, t) is a configuration.

Let TS be a transition system, i.e.,

$$TS = (C, \rightarrow, \longmapsto, I, F).$$

We define $\nu_{rel}(TS)$ as

$$\nu_{rel}(TS) = (C', \rightarrow', \longmapsto, I', F),$$

where

1. I', the set of initial configurations is defined as,

$$I' = \{\nu_{rel}(i) \mid i \in I\};$$

2. C', the set of all configurations is defined as,

$$C' = C \cup I';$$

3. the set of all action transitions $\rightarrow'$ is defined as,

$$\rightarrow' = \rightarrow \cup \rightarrow'',$$

   where $\rightarrow''$ is the set of all action transitions that originate from members of I', i.e.,

$$\rightarrow'' = \{(\nu_{rel}(i), a, c) \mid i \in I, c \in C, a \in A \wedge i \xrightarrow{a} c\}.$$

5.8 Conditional Proceeding

This operator takes two operands, i.e., one a state proposition and the other a transition system. As mentioned in [5], a process P proceeding conditionally on a state proposition ψ, behaves like P if ψ is true, else it behaves like undelayable deadlock.

The conditional proceeding operator while acting on a transition system modifies the process terms of its initial configurations and adds more initial configurations to it with states that do not satisfy the state proposition. Configurations that do not satisfy the state proposition have no action or time transitions originating from them.

We write $\psi :\rightarrow (p, \alpha, t)$ for $(\psi :\rightarrow p, \alpha, t)$.

Let TS be a transition system, i.e.,

$$TS = (C, \rightarrow, \longmapsto, I, F).$$

Let **p** be the process expression of TS.

We define $\psi :\rightarrow TS$ as,

$$\psi :\rightarrow TS = (C', \rightarrow', \longmapsto', I', F),$$

where,

1. I' is the set of initial configurations, i.e.,

$$\begin{aligned} I' \quad = \quad & \{\psi :\rightarrow i \mid i \in I\} \\ & \cup \{\psi :\rightarrow (\mathbf{p}, \alpha, 0) \mid \alpha \in S \wedge \alpha \not\models \psi\}; \end{aligned}$$

2. C' is the set of all configurations, i.e.,

$$C' = C \cup I';$$

3. the set of all action transitions $\rightarrow'$ is,

$$\rightarrow' = \rightarrow \cup \rightarrow'',$$

where $\rightarrow''$ is the set of all action transitions originating from the members of I', i.e.,

$$\rightarrow'' = \{(\psi :\rightarrow i, a, c) \mid i \in I, c \in C, a \in A \wedge i \models \psi \wedge i \xrightarrow{a} c\}; \text{ and}$$

4. the set of all time transitions $\longmapsto'$ is,

$$\longmapsto' = \longmapsto \cup \longmapsto'',$$

where $\longmapsto''$ is the set of all time transitions with their sources in the new initial configuration set, i.e.,

$$\begin{aligned} \longmapsto'' \quad = \quad & \{(\psi :\rightarrow i, (r,\rho), c) \mid i \in I, \\ & c \in C, (r,\rho) \in D, i \models \psi \wedge i \overset{r,\rho}{\longmapsto} c\}. \end{aligned}$$

ψ is only evaluated in the source of the time transition and not along the trajectory or in the target.

5.9 Signal transition

The signal transition operator takes a *transition proposition* and a transition system as operands. A transition proposition is a proposition regarding the state of a configuration immediately before and after an action transition. For a complete definition of the transition proposition see [5].

For each variable $v \in V \cup \dot{V}$, we introduce two new variables ${}^\bullet v$ and $v^\bullet$, denoting the values of the variable v immediately before and after an action transition. We write ${}^\bullet V$ for $\{{}^\bullet v \mid v \in V \cup \dot{V}\}$ and $V^\bullet$ for $\{v^\bullet \mid v \in V \cup \dot{V}\}$. A transition proposition is a proposition on the variables belonging to the set ${}^\bullet V \cup V^\bullet$. Furthermore if ψ is a state proposition, then ${}^\bullet\psi$ and $\psi^\bullet$are transition propositions. In a given transition system, the proposition ${}^\bullet\psi$ is satisfied by those action and time transitions that originate from configurations whose states satisfy ψ. Whereas, the proposition $\psi^\bullet$ is satisfied by those action transitions of the transition system, that end in configurations whose states satisfy ψ.

Similarly, if χ is a transition proposition, then ${}^\circ\chi$ and χ° are state propositions. ${}^\circ\chi$ is that part of χ that deals with the source configuration of an action or time transition, i.e., ${}^\circ\chi$ describes a property about the state of the source of a transition. Whereas χ° is that part of χ that assigns a property to state of the target configuration of an action transition. ${}^\circ\chi$ holds in a configuration from which an action/time transition satisfying χ can originate. χ° holds in a configuration which can be a target to an action transition satisfying χ.

The signal transition operator operating on a transition system TS with transition proposition χ, modifies the process terms of initial configurations to indicate that the first transitions will take place only in accordance with χ. It adds more configurations in the initial set of configurations that have states that do not satisfy the proposition ${}^{\circ}\chi$. Such initial configurations have no action or time transitions originating from them. Action and time transitions originating from initial configurations that do not satisfy the state proposition ${}^{\circ}\chi$ are removed from the transition system. Furthermore an action transition originating from an initial configuration satisfying the state proposition ${}^{\circ}\chi$ must end in a configuration satisfying the state proposition χ°. Otherwise it is removed from the transition system.

Let α_1, α_2 be two states and χ be a transition proposition. Then,

$$\alpha_1 \xrightarrow{a} \alpha_2 \models \chi \text{ means } \alpha_1 \models {}^{\circ}\chi \ \wedge \ \alpha_2 \models \chi^{\circ}.$$

For any configuration $c = (p, \alpha, t)$, we write:

$$c \models {}^{\circ}\chi \text{ iff } \alpha \models {}^{\circ}\chi, \text{ and } c \models \chi^{\circ} \text{ iff } \alpha \models \chi^{\circ}.$$

We use the abbreviation $\chi \ulcorner\!\!\blacktriangledown c$ for $(\chi \ulcorner\!\!\blacktriangledown p, \alpha, t)$. We use the notation $\mathtt{state}(c)$ to denote the state α of c.

Let TS be a transition system,i.e.,

$$TS = (C, \rightarrow, \longmapsto, I, F),$$

Let $\mathbf{p}$ be the process expression of TS.

We define $\chi \ulcorner\!\!\blacktriangledown TS$ as,

$$\chi \ulcorner\!\!\blacktriangledown TS = (C', \rightarrow', \longmapsto', I', F),$$

where,

1. I' is the set of initial configurations, i.e.,

$$\begin{aligned} I' \;=\; & \{\chi \ulcorner\!\!\blacktriangledown i \mid i \in I\} \\ & \cup\{\chi \ulcorner\!\!\blacktriangledown (\mathbf{p}, \alpha, 0) \mid \alpha \in S \ \wedge \ \alpha \not\models {}^{\circ}\chi\}; \end{aligned}$$

2. C' is the set of all configurations, i.e.,

$$C' = C \cup I';$$

3. $\rightarrow'$ is the set of all action transitions, i.e.,

$$\rightarrow' = \rightarrow \cup \rightarrow'',$$

where $\rightarrow''$ is the set of all action transitions originating from the members of I', i.e.,

$$\begin{aligned} \rightarrow'' \;=\; & \{(\chi \ulcorner\!\!\blacktriangledown i, a, c) \mid i \in I, c \in C, a \in A, i \xrightarrow{a} c \wedge \\ & \mathtt{state}(i) \xrightarrow{a} \mathtt{state}(c) \models \chi\}; \end{aligned} \quad \text{and}$$

4. $\longmapsto'$ is the set of all time transitions, i.e.,

$$\longmapsto' = \longmapsto \cup \longmapsto'',$$

where $\longmapsto''$ is the set of all time transitions with their sources in the new initial configuration set, i.e.,

$$\longmapsto'' = \{(\chi \urcorner i, (r, \rho), c) \mid i \in I, c \in C, (r, \rho) \in D, i \models {}^{\circ}\chi \wedge i \overset{r,\rho}{\longmapsto} c\}.$$

5.10 Sequential Composition

The sequential composition operator takes two transition systems as operands. When we *sequentially compose* two transitions systems, the resulting system does not terminate with the termination of the first transition system but continues as the second. We achieve this behaviour by replacing the final configurations in the first transition system by the initial configurations of the second. In case the second transition system denotes a non-existent process, the sequential composition of two transition systems behaves as the first one but deadlocks before terminating. In that case there is no action transition that originates from a configuration of the first transition system and ends in an initial configuration of the other.

Let $c = (p, \alpha, t)$ be a configuration. We use the notation $\texttt{state}(c)$ to denote the state α of c. Let p_2 be a process term. We use the abbreviation $c \cdot p_2$ for the configuration $(p \cdot p_2, \alpha, t)$.

Let TS_1 and TS_2 be two transition systems, i.e.,

$$TS_1 = (C_1, \rightarrow_1, \longmapsto_1, I_1, F_1); \text{and}$$
$$TS_2 = (C_2, , \rightarrow_2, \longmapsto_2, I_2, F_2).$$

Let $\mathbf{p}_2$ be the process expression of TS_2.

We define the sequential composition of TS_1 and TS_2, denoted by $TS_1 \cdot TS_2$ as:

$$TS_1 \cdot TS_2 = (C_{12}, \rightarrow_{12}, \longmapsto_{12}, I_{12}, F_2),$$

where,

1. I_{12} is formed by appending the process terms of configurations in I_1 by $\mathbf{p}_2$, i.e.,

$$I_{12} = \{i \cdot \mathbf{p}_2 \mid i \in I_1\};$$

2. C_{12}, the set of all configurations, is defined as

$$C_{12} = C' \cup C_2,$$

where C' denotes the set of configurations of C_1 with their process terms appended by the process expression of TS_2, i.e.,

$$C' = \{c \cdot \mathbf{p}_2 \mid c \in C_1 \backslash F_1\};$$

3. the set of all action transitions $\rightarrow_{12}$ is,

$$\rightarrow_{12} = \rightarrow' \cup \rightarrow_2 \cup \rightarrow'',$$

where, $\rightarrow'$ is the set of all action transitions originating and ending in members of C', i.e.,

$$\rightarrow' \;=\; \{(c \cdot \mathbf{p}_2, a, c' \cdot \mathbf{p}_2) \mid c, c' \in C_1 \backslash F_1, a \in A \,\wedge\, c \xrightarrow{a}_1 c'\};$$

and $\rightarrow''$ is the set of all action transitions originating from members of C' and ending in members of I_2, i.e.,

$$\begin{aligned}\rightarrow'' \;=\; & \{(c \cdot \mathbf{p}_2, a, i_2) \mid c \in C_1, a \in A, i_2 \in I_2, \\ & \exists\, f \in F_1 \bullet \mathtt{state}(f) = \mathtt{state}(i_2) \,\wedge\, c \xrightarrow{a}_1 f\};\end{aligned}$$

4. the set of all time transitions $\longmapsto_{12}$ is,

$$\longmapsto_{12} = \longmapsto' \cup \longmapsto_2,$$

where, $\longmapsto'$ is the set of all time transitions originating and ending in members of C', i.e.,

$$\begin{aligned}\longmapsto' \;=\; & \{(c \cdot \mathbf{p}_2, (r, \rho), c' \cdot \mathbf{p}_2) \mid c, c' \in C_1 \backslash F_1, (r, \rho) \in D, \\ & \wedge\, c \xmapsto{r,\rho}_1 c'\}.\end{aligned}$$

5.11 Alternative Composition

The alternative composition of two transition systems exhibits a choice between them. The resulting transition system behaves either as one transition system or as the other. But some options of behaviour that were present in the operand transition systems may get lost in their alternative composition. The alternative composition operator operates on the set of initial configurations of its operands. Two initial configurations, one from each transition system, are composed alternatively and included in the initial configuration set of the new transition system only if their states are same. Thus behaviour possible from an initial configuration of one transition system that has no match (according to state) in the (initial configuration set of) the other operand is lost in their alternative composition. Consider two initial configurations i_1 and i_2 with equal states, of two transition systems that are to be alternatively composed. If i_1 is a source to a time transition with a certain duration and state evolution, and i_2 is also a source to a time transition with the same duration and state evolution, then the targets of these time transitions will also be *composed alternatively*.

Let $c = (p, \alpha, t)$ be a configuration. We again use the notation $\mathtt{state}(c)$ for the state α of c and the notation $\mathtt{time}(c)$ for the time label t of c. Let $c_1 = (p_1, \alpha_1, t_1)$ and $c_2 = (p_2, \alpha_2, t_2)$ be two configurations. If $\alpha_1 = \alpha_2$ and $t_1 = t_2$, then we write $c_1 + c_2$ for the configuration $(p_1 + p_2, \alpha_1, t_1)$. For any $t \in \mathbb{R}^{>}$, we write $c \not\xmapsto{t}$ to denote that for any $\rho \in \epsilon_t$, there is no configuration c' such that $c \xmapsto{t,\rho} c'$.

Let TS_1 and TS_2 be two transition systems, where,

$$TS_1 = (C_1, \rightarrow_1, \longmapsto_1, I_1, F_1); \text{and}$$
$$TS_2 = (C_2, \rightarrow_2, \longmapsto_2, I_2, F_2).$$

Their alternative composition is denoted by $TS_1 + TS_2$, i.e.,

$$TS_1 + TS_2 = (C_{1+2}, \rightarrow_{1+2}, \longmapsto_{1+2}, I_{1+2}, F_{1+2}),$$

where,

1. the set I_{1+2} consists of alternatively composed configurations of I_1 and I_2 with equal states, i.e.,

$$I_{1+2} = \{i_1 + i_2 \mid i_1 \in I_1 \ \wedge \ i_2 \in I_2 \ \wedge \ \mathtt{state}(i_1) = \mathtt{state}(i_2)\};$$

2. the set of all configurations C_{1+2} is defined as

$$C_{1+2} = C_1 \cup C_2 \cup I_{1+2} \cup C',$$

where C' denotes the set of alternatively composed configurations of $C_1 \backslash I_1$ and $C_2 \backslash I_2$, i.e.,

$$\begin{aligned} C' \quad = \quad & \{c_1 + c_2 \mid c_1 \in C_1, c_2 \in C_2 \ \wedge \ \exists\, i_1 + i_2 \in I_{1+2}, \\ & \exists\,(r, \rho) \in D \bullet i_1 \overset{r,\rho}{\longmapsto} c_1 \ \wedge \ i_2 \overset{r,\rho}{\longmapsto} c_2\}; \end{aligned}$$

3. $\rightarrow_{1+2}$ is the set of all action transitions. It is defined as,

$$\rightarrow_{1+2} = \rightarrow_1 \cup \rightarrow_2 \cup \rightarrow' \cup \rightarrow'',$$

where $\rightarrow'$ denotes the set of all action transitions originating from members of $I_{1+2} \cup C'$ and ending in members of C_1, i.e.,

$$\begin{aligned} \rightarrow' \quad = \quad & \{(c_1 + c_2, a, c') \mid c_1 + c_2 \in C' \cup I_{1+2}, \\ & a \in A, c' \in C_1 \ \wedge \ c_1 \overset{a}{\rightarrow}_1 c'\}; \end{aligned}$$

and $\rightarrow''$ denotes the set of all action transitions originating from members of $I_{1+2} \cup C'$ and ending in members of C_2, i.e.,

$$\begin{aligned} \rightarrow'' \quad = \quad & \{(c_1 + c_2, a, c') \mid c_1 + c_2 \in C' \cup I_{1+2}, \\ & a \in A, c' \in C_2 \ \wedge \ c_2 \overset{a}{\rightarrow}_2 c'\}; \end{aligned}$$

4. The set of all time transitions $\longmapsto_{1+2}$ is defined as

$$\longmapsto_{1+2} = \longmapsto_1 \cup \longmapsto_2 \cup \longmapsto' \cup \longmapsto'' \cup \longmapsto'_{1+2},$$

where $\longmapsto'$ denotes the set of all time transitions originating from some configuration $c_1 + c_2$ in $I_{1+2} \cup C'$, with $c_1 \in C_1$ & $c_2 \in C_2$ & $c_2 \overset{t}{\not\longmapsto}$, for any $t \in \mathbb{R}^{>}$, and ending in a member of C_1. We define $\longmapsto'$ as

$$\begin{aligned} \longmapsto' \quad = \quad & \{(c_1 + c_2, (r, \rho), c') \mid c_1 + c_2 \in C' \cup I_{1+2}, \\ & (r, \rho) \in D, c' \in C_1, c_1 \overset{r,\rho}{\longmapsto}_1 c' \ \wedge \ c_2 \overset{r}{\not\longmapsto}_2)\}; \end{aligned}$$

$\longmapsto''$ denotes the set of all time transitions originating from a configuration in $I_{1+2} \cup C'$, and ending in a member of C_2, i.e.,

$$\begin{aligned}\longmapsto'' \quad = \quad & \{(c_1 + c_2, (r,\rho), c') \mid c_1 + c_2 \in C' \cup I_{1+2}, \\ & (r,\rho) \in D, c' \in C_2, c_2 \overset{r,\rho}{\longmapsto}_2 c' \ \wedge \ c_1 \overset{r}{\not\longmapsto}_1)\};\end{aligned}$$

and $\longmapsto'_{1+2}$ denotes the set of all time transitions originating from and ending in members of $I_{1+2} \cup C'$, i.e.,

$$\begin{aligned}\longmapsto'_{1+2} \quad = \quad & \{(c_1 + c_2, (r,\rho), c'_1 + c'_2) \mid c_1 + c_2, c'_1 + c'_2 \in C' \cup I_{1+2}, \\ & (r,\rho) \in D, c_1 \overset{r,\rho}{\longmapsto}_1 c'_1 \ \wedge \ c_2 \overset{r,\rho}{\longmapsto}_2 c'_2\};\end{aligned}$$

5. The set of final configurations F_{1+2} is the union of F_1 and F_2, i.e.,

$$F_{1+2} = F_1 \cup F_2.$$

5.12 Relative delay

Let TS be a transition system given by

$$TS = (C, \rightarrow, \longmapsto, I, F).$$

Let the process expression of TS be $\mathbf{p}$.

5.12.1 Adding relative delay of $r > 0$ time units

To explain the operation of relative delay on TS, we make use of the following functions and abbreviations:

- *Time-step reachability*, $\mapsto\!\!\!\rightarrow \subseteq C \times C$, is the smallest relation, such that for any $c_0, c_1, c_2 \in C$ and $(r,\rho) \in D$,

  – $c_0 \mapsto\!\!\!\rightarrow c_0$.
  – If $c_0 \overset{r,\rho}{\longmapsto} c_1$ and $c_1 \mapsto\!\!\!\rightarrow c_2$, then $c_0 \mapsto\!\!\!\rightarrow c_2$.

  If $c_0 \mapsto\!\!\!\rightarrow c_1$, we say c_1 is *time-step reachable* from c_0. $\texttt{tReach}(c_0)$ denotes the set of all configurations that are time-step reachable from c_0, i.e.,

  $$\texttt{tReach}(c_0) = \{c \in C \mid c_0 \mapsto\!\!\!\rightarrow c\}.$$

  For a set of configurations $C' \subseteq C$, $\texttt{tReach}(C')$ denotes the set of all configurations that are time-step reachable from any member of C', i.e.,

  $$\texttt{tReach}(C') = \{c \in C \mid \exists\, c' \in C' \bullet c' \mapsto\!\!\!\rightarrow c\}.$$

- We define a function $\texttt{incr\_timer}\colon C \times \mathbb{R}^{>} \rightarrow C$, that takes a configuration (p, α, t) and a time duration s and returns the configuration $(p, \alpha, t + s)$ with its time label incremented by s.

- If $p \in P$ and $r, s \in \mathbb{R}^{\geq}$ such that $s \leq r$, then $\texttt{after\_time}(\sigma^r_{\mathsf{rel}}(p), s)$ denotes the process term $\sigma^{r-s}_{\mathsf{rel}}(p)$, after the passage of s time units.

- We define a function $\texttt{pr\_exp} : C \rightarrow P$. It takes a configuration and returns its process term.

- For some $s, t \in \mathbb{R}^{>}$, $\rho_s \in \epsilon_s$ and $\rho_t \in \epsilon_t$, if $\rho_s(s) = \rho_t(0)$, then we define $\rho_s.\rho_t$ as
$$(\rho_s \cdot \rho_t)(r) = \begin{cases} \rho_s(r) & \text{if } 0 \leq r \leq s \\ \rho_t(r-s) & \text{if } s < r \leq s+t. \end{cases}$$

- A time transition ending at a certain configuration is *composed sequentially* with a time transition originating from the same configuration. Let $(c, (s, \rho_s), c')$ and $(c', (t, \rho_t), c'')$ be two time transitions. Note that
$$\rho_s(s) = \rho_t(0) = \texttt{state}(c').$$
$(c, (s, \rho_s), c') \cdot (c', (t, \rho_t), c'')$ stands for the time transition $(c, (s+t, \rho), c'')$, where $\rho = \rho_s \cdot \rho_t \in \epsilon_{s+t}$.

- Let $\alpha_1, \alpha_2 \in S, r \in \mathbb{R}^{>}$ and $\rho \in \epsilon_r$. When we write
$$\alpha_1 \overset{r,\rho}{\longmapsto} \alpha_2 \text{ we mean that } \alpha_1 = \alpha_0^\rho \wedge \alpha_2 = \alpha_s^\rho.$$

Let TS be a transition system. The transition system obtained after adding delay of $r \in \mathbb{R}^{>}$ time units to TS is denoted by $\sigma^r_{\mathsf{rel}}(TS)$. We precede the initial configurations of TS with configurations that can idle for any $t \in (0, r]$ time units. We add time transitions between these new configurations (i.e. time transitions that originate and end in the new configurations), and time transitions that originate from the new and end in the initial configurations of TS. As the initial configurations of TS become targets of time transitions of duration $\leq r$, therefore their time labels have to be incremented. Consequently, time labels of configurations that are time-step reachable from initial configurations are also incremented. Due to the property of *additivity of time* of σ^r_{rel} operator, new time transitions that end in an initial configuration of TS are sequentially composed with time transitions that originate from that configuration.

We define $\sigma^r_{\mathsf{rel}}(TS)$ as
$$\sigma^r_{\mathsf{rel}}(TS) = (C', \rightarrow', \longmapsto', I', F),$$
where

1. I' contains an initial configuration for each $\alpha \in S$ (as the signal of a process term (see table 6 in [5]) that can idle for $r \in \mathbb{R}^{>}$ time units is true in all states), i.e.,
$$I' = \{(\sigma^r_{\mathsf{rel}}(\mathbf{p}), \alpha, 0) \mid \alpha \in S\};$$

2. the set of all configurations C' is defined as

$$C' = C \cup C_{\texttt{new}} \cup C_r \cup I_r,$$

where $C_{\texttt{new}}$ is the set of new configurations that precede members of I

$$C_{\texttt{new}} \quad = \quad \{(\sigma^R_{\textsf{rel}}(\mathbf{p}), \alpha, r - R) \mid R \in \mathbb{R}^{>}, 0 < R \leq r, \alpha \in S\};$$

C_r denotes the set of configurations that are time-step reachable from I with their time labels incremented by r:

$$C_r \quad = \quad \{\texttt{incr\_timer}(c, r) \mid c \in \texttt{tReach}(I)\};$$

and I_r is the set of initial configurations of I with incremented labels and process terms preceded by $\sigma^0_{\textsf{rel}}$, i.e.,

$$I_r = \{(\sigma^0_{\textsf{rel}}(\mathbf{p}), \alpha, r) \mid (\mathbf{p}, \alpha, 0) \in I\};$$

3. the set of all action transitions $\rightarrow'$ is

$$\rightarrow' = \rightarrow \cup \rightarrow_{C_r} \cup \rightarrow_{I_r},$$

where $\rightarrow_{C_r}$ is the set of action transitions originating from members of C_r

$$\begin{aligned} \rightarrow_{C_r} \quad = \quad & \{(\texttt{incr\_timer}(c, r), a, c') \mid c \in \texttt{tReach}(I), c' \in C, \\ & a \in A, c \xrightarrow{a} c'\}; \end{aligned}$$

and $\rightarrow_{I_r}$ is the set of action transitions originating from members of I_r

$$\rightarrow_{I_r} \quad = \quad \{((\sigma^0_{\textsf{rel}}(\mathbf{p}), \alpha, r), a, c) \mid (\mathbf{p}, \alpha, 0) \in I, c \in C, a \in A, (\mathbf{p}, \alpha, 0) \xrightarrow{a} c\};$$

4. the set of all time transitions $\longmapsto'$ is

$$\longmapsto' = \longmapsto \cup \longmapsto_{C_{\texttt{new}}} \cup \longmapsto_{C_r} \cup \longmapsto_{I_r} \cup \longmapsto'',$$

where $\longmapsto_{C_{\texttt{new}}}$ is the set of all time transitions originating in members of $C_{\texttt{new}}$ and ending in members of $I_r \cup C_{\texttt{new}}$

$$\begin{aligned} \longmapsto_{C_{\texttt{new}}} \quad = \quad & \{(c, (s, \rho), c') \mid c \in C_{\texttt{new}}, c' \in I_r \cup C_{\texttt{new}}, (s, \rho) \in D, \\ & \texttt{time}(c') = \texttt{time}(c) + s, \texttt{state}(c) \overset{s,\rho}{\longmapsto} \texttt{state}(c') \wedge \\ & \texttt{pr\_exp}(c') = \texttt{after\_time}(\texttt{pr\_exp}(c), s)\}; \end{aligned}$$

$\longmapsto_{C_r}$ is the set of all time transitions originating and ending in members of C_r

$$\begin{aligned} \longmapsto_{C_r} \quad = \quad & \{(\texttt{incr\_timer}(c, r), (s, \rho), \texttt{incr\_timer}(c', r)) \mid c, c' \in \texttt{tReach}(I), \\ & (s, \rho) \in D \ \wedge \ c \overset{s,\rho}{\longmapsto} c'\}; \end{aligned}$$

$\longmapsto_{I_r}$ is the set of all time transitions originating from members of I_r

$$\begin{aligned} \longmapsto_{I_r} \quad = \quad & \{((\sigma^0_{\textsf{rel}}(\mathbf{p}), \alpha, r), (s, \rho), \texttt{incr\_timer}(c, r)) \mid (\mathbf{p}, \alpha, 0) \in I, c \in C, \\ & (s, \rho) \in D \ \wedge \ (\mathbf{p}, \alpha, 0) \overset{s,\rho}{\longmapsto} c\}; \end{aligned}$$

and $\longmapsto''$ is the set consisting of sequential composition of time transitions ending in members of I_r and originating from members of I_r

$$\begin{aligned} \longmapsto'' \quad = \quad & \{(c, (s + t, \rho_s \cdot \rho_t), c') \mid (s, \rho_s), (t, \rho_t) \in D, c \in C_{\texttt{new}}, c' \in C_r, \\ & \exists\, i \in I_r \bullet c \overset{s,\rho_s}{\longmapsto}_{C_{\texttt{new}}} i_r \ \wedge \ i_r \overset{t,\rho_t}{\longmapsto}_{I_r} c'\}. \end{aligned}$$

5.12.2 Adding relative delay of zero time units

Adding a relative delay of zero time units has no effect on the behaviour of a process. Consider a process term $p \in P$. The signal of the process p with a delay of $r \in \mathbb{R}^{>}$, is true in all states, where as the signal of the process p with a delay of zero time units is only true in states where the signal of p is true. Therefore while adding relative delay of zero time units to a transition system, configurations with states other than those already present among the initial configuration set are not added. The process terms in the initial configurations are prefixed by σ^0_{rel}.

Let $c = (p, \alpha, t)$ be a configuration and let $s \in \mathbb{R}^{\geq}$. We use the abbreviation $\sigma^s_{\mathsf{rel}}(c)$ to denote $(\sigma^s_{\mathsf{rel}}(p), \alpha, t)$.

The transition system obtained by applying a relative delay of zero time units to TS is

$$\sigma^0_{\mathsf{rel}}(TS) = (C', \rightarrow' \longmapsto', I', F),$$

where,

1. $I' = \{(\sigma^0_{\mathsf{rel}}(\mathbf{p}), \alpha, 0) \mid (\mathbf{p}, \alpha, 0) \in I\}$;
2. $C' = C \cup I'$;
3. $\rightarrow' = \rightarrow \cup \rightarrow''$,

   where $\rightarrow'' = \{(\sigma^0_{\mathsf{rel}}(i), a, c) \mid i \in I, a \in A, c \in C \wedge i \xrightarrow{a} c\}$.

   Note that $\sigma^0_{\mathsf{rel}}(i)$, where $i \in I$, denotes a configuration from the new initial configuration set I';
4. $\longmapsto' = \longmapsto \cup \longmapsto''$,

   where $\longmapsto'' = \{(\sigma^0_{\mathsf{rel}}(i), (r, \rho), c) \mid i \in I, (r, \rho) \in D, c \in C \wedge i \overset{r,\rho}{\longmapsto} c\}$.

   Here also $\sigma^0_{\mathsf{rel}}(i)$ denotes a configuration from the new initial configuration set I'.

5.13 Signal evolution

In BPA^{srt}_{hs}, the signal evolution operator (denoted by $\curvearrowright$), when applied to a process restricts the evolution of environment variables to a given state proposition during an initial delay of the process. Besides a process term and a state proposition, the signal evolution operator takes a subset of environment variables as an argument. This set of variables signifies that the variables in the set must evolve without discontinuities during an initial delay of the process term. When signal evolution is applied to a process that cannot idle initially, then the result is simply a process that emits the given state proposition as its signal, i.e. $\phi \curvearrowright_V P$, where P cannot idle is simply $\phi \wedge P$.

Let $c = (p, \alpha, t)$ be a configuration. We use the notation $\phi \curvearrowright_{\mathbb{V}} c$ for $(\phi \curvearrowright_{\mathbb{V}} p, \alpha, t)$.

A state proposition ϕ is satisfied by a state evolution $\rho \in \epsilon_r$, written as $\rho \models \phi$, if all the states corresponding to $\rho(t)$ for $t \in [0, r]$ satisfy ϕ, i.e.,

$$\alpha_t^\rho \models \phi, \quad \text{for all } t \in [0, r].$$

Let $\alpha_1, \alpha_2 \in S, (r, \rho) \in D$. Then we write,

$$\alpha_1 \overset{r,\rho}{\longmapsto} \alpha_2 \quad \models_{\mathbb{V}} \phi \text{ if} \\ \alpha_1 = \alpha_0^\rho, \alpha_2 = \alpha_r^\rho, \rho \models \phi \text{ and } \rho \text{ is smooth for } \mathbb{V}.$$

Let TS be a transition system, i.e.,

$$TS = (C, \rightarrow, \longmapsto, I, F).$$

A transition system in evolution according to ϕ and smooth for variables $\mathbb{V}$, is written as $\phi \curvearrowright_{\mathbb{V}} (TS)$. It is defined as

$$\phi \curvearrowright_{\mathbb{V}} (TS) = (C', \rightarrow', \longmapsto', I', F'),$$

where

1. I' is the set of initial configurations, i.e.,

$$I' = \{\phi \curvearrowright_{\mathbb{V}} i \mid i \in I \ \wedge \ i \models \phi\};$$

2. The set of all configurations C' is defined as

$$C' = C \cup C'',$$

   where

$$C'' = \{\phi \curvearrowright_{\mathbb{V}} c \mid c \in \texttt{tReach}(I) \ \wedge \ c \models \phi\}.$$

   For any set C of configurations, $\texttt{tReach}(C)$ is the same as defined in section 5.12;

3. $\rightarrow'$ is the set of all action transitions, i.e.,

$$\rightarrow' = \rightarrow \cup \rightarrow'',$$

   where $\rightarrow''$ is the set of all action transitions originating from members of C''

$$\rightarrow'' \quad = \quad \{(\phi \curvearrowright_{\mathbb{V}} c, a, c') \mid c \in \texttt{tReach}(I), c' \in C, c \models \phi, c \xrightarrow{a} c''\};$$

4. $\longmapsto'$ is the set of all time transitions, i.e.,

$$\longmapsto' = \longmapsto \cup \longmapsto'',$$

   where $\longmapsto''$ is the set of time transitions originating and ending in members of C'', i.e.,

$$\longmapsto'' \quad = \quad \{(\phi \curvearrowright_{\mathbb{V}} c, (r, \rho), \phi \curvearrowright_{\mathbb{V}} c') \mid c, c' \in \texttt{tReach}(I), \\ c \overset{r,\rho}{\longmapsto} c', \texttt{state}(c) \overset{r,\rho}{\longmapsto} \texttt{state}(c') \models_{\mathbb{V}} \phi\}.$$

6 Extending the graph model for BPA_{hs}^{srt}

We add process terms with signal hiding operator and integration operator to the set P.

$$\begin{aligned} P := \; & \tilde{\tilde{a}} \mid \tilde{\tilde{\delta}} \mid \bot \mid \nu_{rel}(P) \mid P \cdot P \mid P + P \mid \sigma^0_{\mathsf{rel}}(P) \\ & \mid \sigma^r_{\mathsf{rel}}(P) \mid \psi \curlywedge P \mid \psi :\to P \mid \chi \lrcorner P \mid \phi \curvearrowright_{\mathbb{V}} P \\ & \mid v \triangle P \mid \int_{u \in U} F(u); \end{aligned}$$

where ψ and ϕ are state propositions, χ is a transition proposition, $r > 0$, $\mathbb{V}$ is a subset of V, v is an environment variable, U is a set of non-negative real numbers and F is a process expression containing a real valued variable u.

6.1 Signal Hiding

Unlike most other operators that modify the set of initial configurations only, the signal hiding operator modifies the whole transition system. The signal hiding operator takes two operands, one a transition system and the other an environment variable. It returns a transition system that behaves independent of the value of the given variable. Thus we *hide* the effect of the given variable on the transition system. Below are some definitions that will help us in formally describing signal hiding.

For all $v \in V$, we define a binary relation $=_v$ between two states α and α', when they differ from each other only in their values of v and $\dot{v}$, i.e.,

$$\alpha =_v \alpha' \text{ iff } \alpha(v') = \alpha'(v') \text{ for all } v' \in (V \cup \dot{V}) \backslash \{v, \dot{v}\}.$$

We extend this relation to configurations having same process terms and time labels. i.e. for any two configurations (p, α, t) and (p, α', t) we have,

$$(p, \alpha, t) =_v (p, \alpha', t) \text{ iff } \alpha =_v \alpha'.$$

Similarly, for all $v \in V$, we define a binary relation $=_v$ between two state evolutions, $\rho_1 \& \rho_2 \in \epsilon_r$, $r \in \mathbb{R}^{>}$, if and only if ρ may differ from ρ' only in the evolution of the variables v and $\dot{v}$. i.e.,

$$\rho =_v \rho' \text{ iff } \alpha_t^\rho =_v \alpha_t^{\rho'} \text{ for all t} \in [0, r].$$

Let (p, α, t) be a configuration and v be a variable. We write $v \triangle (p, \alpha, t)$ for a configuration $(v \triangle p, \alpha, t)$. When we applying signal hiding of variable v on a transition system, we add $v \triangle c'$ for any configuration c present in the transition system, such that $c' =_v c$. $v \triangle c'$ behaves the same as c, except regarding the transition and evolution of variable v.

Let TS be a transition system. Signal hiding is applied to a transition system after removing unreachable configurations and transitions from it. This is done by taking reach of a transition system (see section 5.2).

Let $\texttt{reach}(TS)$ be given by,

$$\texttt{reach}(TS) = (C, \to, \longmapsto, I, F).$$

We define $v\triangle(\text{reach}(TS))$, the transition system obtained after applying signal hiding of variable v on reach(TS) as follows:

$$v\triangle(\texttt{reach}(TS)) = (C', \rightarrow', \longmapsto', I', F'),$$

where,

1. $C' = \{v\triangle c' \mid \exists\, c \in C \bullet c =_v c'\}$.
2. $\rightarrow' = \{(v\triangle c_1', a, v\triangle c_2') \mid \exists\, c_1, c_2 \in C, a \in A \bullet c_1 =_v c_1' \wedge c_2 =_v c_2' \wedge c_1 \xrightarrow{a} c_2\}$;
3. $\longmapsto' = \{(v\triangle c_1', (r, \rho'), v\triangle c_2') \mid (r, \rho') \in D, \exists\, \rho \in \epsilon_r, \exists\, c_1, c_2 \in C \bullet c_1 =_v c_1' \wedge c_2 =_v c_2' \wedge \rho' =_v \rho \wedge c_1 \overset{r,\rho}{\longmapsto} c_2\}$;
4. $I' = \{v\triangle i' \mid \exists\, i \in I \bullet i =_v i'\}$;
5. $F' = \{v\triangle f' \mid \exists\, f \in F \bullet f =_v f'\}$;
6. Signal hiding is applied to a transition system after removing unreachable configurations and transitions originating from them (see section 5.2).

Removing unreachable configurations and transitions from a transition system is critical before apply signal hiding to it. Consider the following scenario. Let c_0, c_1 be configurations of TS, where c_1 is a reachable configuration and c_0 is unreachable. Also let $c_0 =_v c_1$. In $v\triangle(TS)$, we add a configuration $v\triangle c'$, where $c' =_v c_1 =_v c_0$. Also note if $c_1 \in \mathit{Reach}(I)$ before hiding, then $v\triangle c' \in \mathit{Reach}(I')$. According to the definition above, $v\triangle c'$ will behave both as c_0 and c_1. Thus not removing an unreachable configuration and transition originating from it may add unintended behaviour in a transition system after signal hiding.

The signal hiding operator is also extended to state propositions and transition propositions. Let ψ be a state proposition then $v\triangle\psi$ is the proposition ψ with the effect of variable v and $\dot{v}$ on its satisfaction hidden. If ψ is satisfied in a state α, then $v\triangle\psi$ is satisfied by all states that differ from α only in their values of v and $\dot{v}$. Formally, it is defined that a proposition ψ after signal hiding is true only if there exists an assignment of variables v and $\dot{v}$ to real numbers for which ψ is true. i.e., let $\psi[r, r'/v, \dot{v}]$ denote the proposition with v and $\dot{v}$ replaced by some real values r and r' respectively, then

$$v\triangle\psi \Leftrightarrow \exists r, r' \in \mathbb{R} \bullet \psi[r, r'/v, \dot{v}].$$

Similarly, a transition proposition with v hidden is defined as,

$$v\triangle\chi \Leftrightarrow \exists r, r', s, s' \in \mathbb{R} \bullet \chi[r, r', s, s'/{}^\bullet v, {}^\bullet\dot{v}, v^\bullet, \dot{v}^\bullet].$$

6.2 Integration

In order to cover processes that can perform an action at all points in a certain interval of time, we add integration to the graph model of BPA^{srt}_{hs}.

Let $\mathbf{F}$ be an expression possibly containing a real valued variable u. Let r be a nonnegative real number. Then $\mathbf{F}[u/r]$ denotes the process $\mathbf{F}$ with all the instances of u replaced by r. Let U be a subset of $\mathbb{R}^{\geq}$. The process $\int_{u\in U}\mathbf{F}(u)$ behaves as one of the processes $\mathbf{F}[u/r]$, for some $r \in U$. Thus $\int_{u\in U}\mathbf{F}(u)$ represents an alternative choice among a number of processes; this choice may even be infinite.

For $u \in U$, let $TS(\mathbf{F}(u))$ denote the transition system corresponding to the expression $\mathbf{F}(u)$ and let $TS(\int_{u\in U}\mathbf{F}(u))$ denote the transition system corresponding to expression $\int_{u\in U}\mathbf{F}(u)$.

We give a definition of $TS(\int_{u\in U}\mathbf{F}(u))$ based on our intuition about its behaviour obtained from the operational semantics of integration given in Table 10. $TS(\int_{u\in U}\mathbf{F}(u))$ behaves as the alternative composition of all transition systems, $TS(\mathbf{F}(u)$, for all $u \in U$. There is added complexity in the idling behaviour of $TS(\int_{u\in U}\mathbf{F}(u))$. When $\int_{u\in U}F(u)$ waits for a duration $r \in \mathbb{R}^{>}$, the interval U may need to be partitioned in subintervals as $\mathbf{F}(u)$ may evolve into a different process term for each partition of U. We illustrate the partition of U, in expression $\int_{u\in U}(F)(u)$, as time passes by the following example:

Consider the process $\int_{u\in[0,6]}\sigma^{u}_{\mathsf{rel}}((l = 18) :\rightarrow (\dot{l} = 0.5) \curvearrowright_{l} \sigma^{4}_{\mathsf{rel}}(\tilde{\tilde{a}}))$ in state $l = 18$. Let $(l = 18) :\rightarrow (\dot{l} = 0.5) \curvearrowright_{l} \sigma^{4}_{\mathsf{rel}}(\tilde{\tilde{a}})$ be denoted by $\mathbf{p}$. Let $\rho \in \epsilon_1$ such that $\rho \models (\dot{l} = 0.5)$ and ρ is smooth for l and $\dot{l}$.

We study the effect of idling for 1 time unit on process $\langle\int_{u\in[0,6]}\sigma^{u}_{\mathsf{rel}}(\mathbf{p}), 18\rangle$, deriving transitions according to the operational semantics in Table 10.

For different partitions of $[0,6]$, $\langle\int_{u\in[0,6]}\sigma^{u}_{\mathsf{rel}}(\mathbf{p}), 18\rangle$, takes different forms as it evolves. Consider partitions $[0,0], (0,1)$ and $[1,6]$ of $[0,6]$.

For $u \in [0,0]$

$\langle\sigma^{0}_{\mathsf{rel}}(\mathbf{p}), 18\rangle \xmapsto{1,\rho} \langle(\dot{l} = 0.5) \curvearrowright_{l} \sigma^{3}_{\mathsf{rel}}(\tilde{\tilde{a}}), 18.5\rangle$ — From rule {4,26} Table 5

For $u \in (0,1)$, take $u = 0.5$.

$\langle\sigma^{0.5}_{\mathsf{rel}}(\mathbf{p}), 18\rangle \not\xmapsto{1}$ — After 0.5 time units have passed, $\langle\sigma^{0.5}_{\mathsf{rel}}(\mathbf{p}), 18\rangle \xmapsto{0.5,\rho} \langle\mathbf{p}, 18.25\rangle$ and $\mathbf{p}$ with $l > 18$ cannot wait

For all $u \in (0,1)$,
$\langle\sigma^{u}_{\mathsf{rel}}(\mathbf{p}), 18\rangle$ cannot wait for more than u time units and u is less than 1. Therefore,
$\langle\int_{u\in(0,1)}\sigma^{u}_{\mathsf{rel}}(\mathbf{p}), 18\rangle \not\xmapsto{1}$

Now for all $u \in [1,6]$,
$\langle\int_{u\in[1,6]}\sigma^{u}_{\mathsf{rel}}(\mathbf{p}), 18\rangle \xmapsto{1,\rho} \langle\int_{u\in[1,6]}\sigma^{u-1}_{\mathsf{rel}}(\mathbf{p}), 18.5\rangle$ — From Rule 3 Table 8

Thus we have,

$$\langle \int_{u\in[0,6]}\sigma^u_{\mathsf{rel}}(\mathbf{p}), 18\rangle \xmapsto{1,\rho} \langle(\dot{l} = 0.5) \curvearrowright_l \sigma^3_{\mathsf{rel}}(\tilde{\tilde{a}}) + \int_{u\in[1,6]}\sigma^{u-1}_{\mathsf{rel}}(\mathbf{p}), 18.5\rangle$$

For a deeper insight into partition formation as a process with integration operator evolves, we refer the reader to chapter 4 of [2].

These partitions account for the complexity in the definition of the configuration and time transition sets of $\int_{u\in U} TS(\mathbf{F}(u))$.

Let for all $u \in U$, $TS(\mathbf{F}(u))$ be given by,

$$TS(\mathbf{F}(u)) = (C_u, \rightarrow_u, \longmapsto_u, I_u, F_u);$$

For all $u \in U$, $\mathbf{F}(u)$ is the process expression of $TS(\mathbf{F}(u))$ and $\int_{u\in U}\mathbf{F}(u)$ is the process expression of $TS(\int_{u\in U}\mathbf{F}(u))$.

And let $TS(\int_{u\in U}\mathbf{F}(u)$ be given by,

$$TS(\int_{u\in U}\mathbf{F}(u)) = (C, \rightarrow, \longmapsto, I, F);$$

where,

1. The set of initial configurations I is defined as,

$$I \;\;=\;\; \{(\int_{u\in U}\mathbf{F}(u), \alpha, 0) \mid \forall u \in U \bullet (\mathbf{F}(u), \alpha, 0) \in I_u\};$$

2. The set of all configurations C is defined as,

$$C = \bigcup_{u\in U} C_u \cup I \cup C';$$

where C' is the set of configurations obtained by idling of an initial configuration in I, i.e.,

$$\begin{aligned} C' \;\;=\;\; & \{(\int_{u\in U_1} F_1(u) + \cdots + \int_{u\in U_n} F_n(u), \alpha', r) \mid \\ & n \in \mathbf{N}, \{U_1, \ldots, U_n\} \text{ partition of } U\backslash U_{n+1}, U_{n+1} \subset U, \\ & \alpha' \in S, r \in \mathbb{R}^{>} \wedge \exists\alpha \in S \bullet ((\int_{u\in U}\mathbf{F}(u), \alpha, 0)) \in I, \\ & \exists\rho \in \epsilon_r \bullet \forall i \in \{1, 2, \ldots, n\} \bullet (\forall u \in U_i \bullet ((F_i(u), \alpha', r) \in C_u \\ & \wedge(\mathbf{F}(u), \alpha, 0) \xmapsto{r,\rho}_u (F_i(u), \alpha', r))) \\ & \wedge (\forall u \in U_{n+1} \bullet (\mathbf{F}(u), \alpha, 0) \not\xmapsto{r}_u)\}, \end{aligned}$$

where $\mathbf{N}$ denotes the set of natural numbers.

3. The set of all action transitions $\rightarrow$ is,

$$\rightarrow = \bigcup_{u\in U} \rightarrow_u \cup \rightarrow' \cup \rightarrow'';$$

where $\rightarrow'$ denote the set of action transitions from initial configurations, i.e.,

$$\begin{aligned} \rightarrow' \;\;=\;\; & \{(((\int_{u\in U}\mathbf{F}(u), \alpha, 0), a, c) \mid \\ & (\int_{u\in U}\mathbf{F}(u), \alpha, 0) \in I, a \in A, \exists\, u \in U \bullet (c \in C_u \wedge (\mathbf{F}(u), \alpha, 0) \xrightarrow{a}_u c)\}; \end{aligned}$$

and $\rightarrow''$ denote the set of action transitions originating from members of C', i.e.,

$$\begin{array}{rcl} \rightarrow'' & = & \{((\int_{u\in U_1} F_1(u) + \cdots + \int_{u\in U_n} F_n(u), \alpha, t), a, c) \mid \\ & & (\int_{u\in U_1} F_1(u) + \int_{u\in U_2} F_2(u) + \cdots + \int_{u\in U_n} F_n(u), \alpha, t) \in C', \\ & & a \in A,\ \exists\, i \in \{1, 2, \ldots, n\}, \exists u \in U_i \bullet (c \in C_u\ \wedge\ (F_i(u), \alpha, t) \xrightarrow{a}_u c)\}; \end{array}$$

4. The set of all time transitions $\longmapsto$ is,

$$\longmapsto = \bigcup_{u\in U} \longmapsto_u \cup \longmapsto' \cup \longmapsto'';$$

where $\longmapsto'$ is the set of all time transitions originating from the initial configurations and ending in the members of C', i.e.,

$$\begin{array}{rcl} \longmapsto' & = & \{((\int_{u\in U} \mathbf{F}(u), \alpha, 0), (r, \rho), (\int_{u\in U_1} F_1(u) + \cdots + \int_{u\in U_n} F_n(u), \alpha', r)) \mid \\ & & (\int_{u\in U} \mathbf{F}(u), \alpha, 0) \in I, (r, \rho) \in D, \\ & & (\int_{u\in U_1} F_1(u) + \cdots + \int_{u\in U_n} F_n(u), \alpha', r) \in C', \\ & & \forall i \in \{1, 2, \ldots, n\} \bullet \forall u \in U_i \bullet ((\mathbf{F}(u), \alpha, 0) \overset{r,\rho}{\longmapsto}_u (F_i(u), \alpha', r))\}; \end{array}$$

$\longmapsto''$ denotes the set of time transitions originating and ending in members of C'. We *construct* the target configuration from the source configuration by specifying how different processes with different subintervals in the source configuration idle and evolve.

$$\begin{array}{rcl} \longmapsto'' & = & \{((\int_{u\in U_1} F_1(u) + \cdots + \int_{u\in U_n} F_n(u), \alpha, t), (r, \rho), \\ & & (\int_{u\in U_1^1} F_1^1(u) + \cdots + \int_{u\in U_1^k} F_1^k(u) + \\ & & \int_{u\in U_2^1} F_2^1(u) + \cdots + \int_{u\in U_2^q} F_2^q(u) + \\ & & \ldots \\ & & + \int_{u\in U_n^1} F_n^1(u) + \cdots + \int_{u\in U_n^m} F_n^m(u), \alpha', t + r)) \mid \\ & & (\int_{u\in U_1} F_1(u) + \cdots + \int_{u\in U_n} F_n(u), \alpha, t) \in C', (r, \rho) \in D, \\ & & (\int_{u\in U_1^1} F_1^1(u) + \cdots + \int_{u\in U_1^k} F_1^k(u) + \\ & & \ldots \\ & & + \int_{u\in U_n^1} F_n^1(u) + \cdots + \int_{u\in U_n^m} F_n^m(u), \alpha', t + r) \in C', \\ & & k, q, \ldots, m \in N, \\ & & \{U_1^1, \ldots, U_1^k\} \text{ is a partition of } U_1 \backslash U_1^{k+1}, U_1^{k+1} \subseteq U_1, \\ & & \{U_2^1, \ldots, U_2^q\} \text{ is a partition of } U_2 \backslash U_2^{q+1}, U_2^{q+1} \subseteq U_2, \\ & & \ldots \\ & & \{U_n^1, \ldots, U_n^m\} \text{ is a partition of } U_n \backslash U_n^{m+1}, U_n^{m+1} \subseteq U_n, \\ & & \exists\, \rho \in \epsilon_r \bullet (\forall u \in U_1^1 \bullet ((F_1(u), \alpha, t) \overset{r,\rho}{\longmapsto}_u (F_1^1(u), \alpha', t + r)), \ldots \\ & & \forall u \in U_1^k \bullet ((F_1(u), \alpha, t) \overset{r,\rho}{\longmapsto}_u (F_1^k(u), \alpha', t + r)), \\ & & \ldots \\ & & \forall u \in U_n^1 \bullet ((F_n(u), \alpha, t) \overset{r,\rho}{\longmapsto}_u (F_n^1(u), \alpha', t + r)), \ldots \\ & & \forall u \in U_n^m \bullet ((F_n(u), \alpha, t) \overset{r,\rho}{\longmapsto}_u (F_n^m(u), \alpha', t + r))), \\ & & \forall u \in U_1^{k+1}((F_1(u), \alpha, t) \overset{r}{\not\longmapsto}_u), \forall u \in U_2^{q+1}((F_2(u), \alpha, t) \overset{r}{\not\longmapsto}_u) \ldots \\ & & \forall u \in U_n^{m+1}((F_n(u), \alpha, t) \overset{r}{\not\longmapsto}_u)\}; \end{array}$$

5. The set of final configurations F is,

$$F = \bigcup_{u \in U} F_u.$$

7 Bisimulation in the graph model

We want to define bisimulation between two transition systems. First we give a notion of bisimulation between two configurations.

Let $\mathbb{TS}$ denote the set of all transition systems. Let TS and TS' be two transition systems, i.e.,

$$\begin{array}{rcl} TS & = & (C, \rightarrow, \longmapsto, I, F) \text{ and} \\ TS' & = & (C', \rightarrow', \longmapsto', I', F'). \end{array}$$

A bisimulation $B \subseteq C \times C'$, is a binary relation such that, for all $c \in C$ and $c' \in C'$, if $B(c, c')$, then:

1. the states and time labels of c and c' are equal, i.e.,

$$\texttt{state}(c) = \texttt{state}(c') \wedge \texttt{time}(c) = \texttt{time}(c');$$

2. for $c'' \in C$ and $a \in A$, if $c \xrightarrow{a} c''$, then $\exists\, c''' \in C'$, such that $c' \xrightarrow{a}{}' c'''$ and $B(c'', c''')$.

   Vice versa, if $c' \xrightarrow{a}{}' c'''$, for some $c''' \in C'$ and $a \in A$, then $\exists\, c'' \in C$, such that $c \xrightarrow{a} c''$ and $B(c'', c''')$;

3. for $c'' \in C$ and $(r, \rho) \in D$, if $c \overset{r,\rho}{\longmapsto} c''$, then $\exists\, c''' \in C'$, such that $c' \overset{r,\rho}{\longmapsto}{}' c'''$ and $B(c'', c''')$.

   Vice versa, if $c' \overset{r,\rho}{\longmapsto}{}' c'''$, for some $c''' \in C'$ and $(r, \rho) \in D$, then $\exists\, c'' \in C$, such that $c \overset{r,\rho}{\longmapsto} c''$ and $B(c'', c''')$; and

4. if $c \in F$, then $c' \in F'$.

   Vice versa, if $c' \in F'$, then $c \in F$.

A bisimulation between two transition systems, $\mathbb{B} \subseteq \mathbb{TS} \times \mathbb{TS}$, is a symmetric binary relation, such that, if $\mathbb{B}(TS, TS')$, then,

> for every $i \in I$, there exists an $i' \in I'$, such that $B(i, i')$.
> Vice versa, for every $i' \in I'$, there exists an $i \in I$, such that $B(i, i')$.

If $\mathbb{B}(TS, TS')$, then TS and TS' are said to be *bisimilar* to each other, denoted by $TS \leftrightarrows TS'$.

8 Thermostat Again

We revisit the thermostat example given in section 3.

$$\begin{aligned}
Th &= (T = 18) \wedge Th^{on},\\
Th^{on} &= up \curvearrowright_T Th_0^{on},\\
Th_t^{on} &= \int_{u\in[t,\infty)} \sigma_{\mathsf{rel}}^{u-t}(Th^{\rightarrow}),\\
Th^{\rightarrow} &= (T = 20) :\rightarrow\\
&\quad (T^{\bullet} = {}^{\bullet}T) \curvearrowright \widetilde{\widetilde{\mathtt{toff}}} \cdot Th^{off},\\
Th^{off} &= down \curvearrowright_T Th_0^{off},\\
Th_{t'}^{off} &= \int_{u\in[t',\infty)} \sigma_{\mathsf{rel}}^{u-t'}(Th^{\leftarrow}),\\
Th^{\leftarrow} &= (T = 18) :\rightarrow\\
&\quad (T^{\bullet} = {}^{\bullet}T) \curvearrowright \widetilde{\widetilde{\mathtt{ton}}} \cdot Th^{on}.
\end{aligned}$$

$$\begin{aligned}
Th' &= (T = 18) \wedge Th'^{on},\\
Th'^{on} &= up \curvearrowright_T Th_0'^{on},\\
Th_t'^{on} &= \sigma_{\mathsf{rel}}^{ln2-t}(Th'^{\rightarrow}),\\
Th'^{\rightarrow} &= (T^{\bullet} = {}^{\bullet}T) \curvearrowright \widetilde{\widetilde{\mathtt{toff}}} \cdot Th'^{off},\\
Th'^{off} &= down \curvearrowright_T Th_0'^{off},\\
Th_{t'}'^{off} &= \sigma_{\mathsf{rel}}^{ln3-t'}(Th'^{\leftarrow}),\\
Th'^{\leftarrow} &= (T^{\bullet} = {}^{\bullet}T) \curvearrowright \widetilde{\widetilde{\mathtt{ton}}} \cdot Th'^{on}
\end{aligned}$$

where $t \in [0, ln2]$ and $t' \in [0, ln3]$.

We write *up* for proposition $(18 \leq T \leq 20 \wedge \dot{T} = -T + 22)$, and *down* for proposition $(18 \leq T \leq 20 \wedge \dot{T} = -T + 17)$.

Th_t^{on} and $Th_t'^{on}$ show processes Th^{on} and Th'^{on} after t time units respectively. Th'^{on} has a definite delay of $ln2$ time units. But Th^{on} has an indefinite delay. Starting from 18° and delaying according to *up*, the temperature reaches 20° in $ln2$ time units. According to the operational semantic rules (see rule 26 table 8), keeping the temperature infinitely often continuously differentiable and *up* true, delaying more than $ln2$ time units is not possible for Th^{on}. Therefore, we define t in interval $[0, ln2]$. $Th_t^{on} = \int_{u\in[t,\infty)} \sigma_{\mathsf{rel}}^{u-t}(Th^{\rightarrow})$ indicates that after t time units t is subtracted from the delay duration u.

Similarly, $Th_{t'}^{off}$ and $Th_{t'}'^{off}$ show processes Th^{off} and Th'^{off} after t' time units respectively. Starting from 20°, keeping *down* true and the temperature infinitely often continuously differentiable, delaying for more than $ln3$ time units is not possible for Th^{off}. Therefore, $t' \in [0, ln3]$.

Starting at $T = 18°$, evolving according to $\dot{T} = -T + 22$, the temperature at any time r is given by $(22e^r - 4)/e^r$. In the following subsections, we denote a state with $T(r) = (22e^r - 4)/e^r$ and $\dot{T}(r) = -T(r) + 22$ by α_r.

Similarly, starting at $T = 20°$, evolving according to $\dot{T} = -T + 17$, the temperature at any time r is given by $(17e^r + 3)/e^r$. We denote a state at any instant r, with $T(r) = (17e^r + 3)/e^r$ and $\dot{T}(r) = -T(r) + 17$ by α'_r.

We sometimes write a state at any instant r as, $(T(r), \dot{T}(r))$.

First we give representations of Th and Th' in the graph model proposed in this paper. Then we prove that the two graphs are bisimilar. We apply signal hiding on graphs of Th and Th' and prove that the graphs obtained after signal hiding with respect to T are also bisimilar. Thus we have been successfully able to define an operational semantics for signal hiding that preserves bisimulation.

8.1 Graph of Th

Let $TS(Th)$ be the graph of the process Th,

$$TS(Th) = (C_{th}, \rightarrow_{th}, \longmapsto_{th}, I_{th}, \emptyset).$$

- There is only one initial configuration with temperature 18,

$$I_{th} = \{(Th, (18, 4), 0)\};$$

- the initial configuration evolves (while idling), into the following set of configurations:

$$C'_{th} = \{(up \curvearrowright_T (\textstyle\int_{u \in [r,\infty)} \sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow})), \alpha_r, r) \mid r \in [0, ln2]\}.$$

After performing action $\widetilde{\widetilde{toff}}$, $(up \curvearrowright_T (\int_{u \in [ln2,\infty)} \sigma_{\mathsf{rel}}^{u-ln2}(Th^{\rightarrow})), (20, 2), ln2)$, becomes $(Th^{off}, (20, -3), 0)$.

While idling, Th^{off} evolves into the following processes:

$$C''_{th} = \{(down \curvearrowright_T (\textstyle\int_{u \in [r',\infty)} \sigma_{\mathsf{rel}}^{u-r'}(Th^{\leftarrow})), \alpha'_{r'}, r') \mid r' \in [0, ln3]\}.$$

The set of all configurations, C_{th}, is defined as,

$$C_{th} = C'_{th} \cup C''_{th} \cup I_{th};$$

- Th^{on} performs action $\widetilde{\widetilde{toff}}$ at temperature $T = 20$. The temperature reaches 20 in time $ln2$. Th^{off} performs action $\widetilde{\widetilde{ton}}$ at temperature $T = 18$. The temperature reaches 18 in time $ln3$.

The set of action transitions consists of the following transitions:

$$\{(up \curvearrowright_T (\textstyle\int_{u \in [ln2,\infty)} \sigma_{\mathsf{rel}}^{u-ln2}(Th^{\rightarrow})), (20, 2), ln2) \xrightarrow{toff}_{th} (Th^{off}, (20, -3), 0),$$
$$(down \curvearrowright_T (\textstyle\int_{u \in [ln3,\infty)} \sigma_{\mathsf{rel}}^{u-ln3}(Th^{\leftarrow})), (18, -1), ln3) \xrightarrow{ton}_{th} (Th^{on}, (18, 4), 0)\};$$

- Time transitions originating from $(Th, (18, 4), 0)$ are,

$$\{(Th, (18, 4), 0) \xmapsto{t, \rho_t}_{th} (up \curvearrowright_T (\textstyle\int_{u \in [t,\infty)} \sigma_{\mathsf{rel}}^{u-t}(Th^{\rightarrow})), \alpha_t, t)$$
$$\mid t \in (0, ln2], (t, \rho_t) \in D, \rho_t \models up \wedge \ \rho_t \text{ is smooth for } T\};$$

Time transitions among members of C'_{th} are,

$$\{(up \curvearrowright_T (\textstyle\int_{u \in [r,\infty)} \sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow})), \alpha_r, r) \xmapsto{t, \rho_t}_{th}$$
$$(up \curvearrowright_T (\textstyle\int_{u \in [r+t,\infty)} \sigma_{\mathsf{rel}}^{u-(r+t)}(Th^{\rightarrow})), \alpha_{t+r}, t + r) \mid$$
$$(t, \rho_t) \in D, \rho_t \models up \wedge \rho_t \text{ is smooth for T},$$
$$r \in [0, ln2), t \in (0, ln2], r + t \in (0, ln2]\};$$

Time transitions among members of C''_{th} are,

$$\{(down \curvearrowright_T (\int_{u\in[r',\infty)} \sigma_{\mathsf{rel}}^{u-r'}(Th^{\leftarrow})), \alpha'_{r'}, r') \xrightarrow{t,\rho_t}_{th}$$
$$(down \curvearrowright_T (\int_{u\in[r'+t,\infty)} \sigma_{\mathsf{rel}}^{u-(r'+t)}(Th^{\leftarrow})), \alpha'_{t+r'}, t+r') \mid$$
$$(t,\rho_t) \in D, \rho_t \models down \wedge \rho_t \text{ is smooth for T},$$
$$r' \in [0, ln3), t \in (0, ln3], r'+t \in (0, ln3]\};$$

- Process Th is an infinite process with recursion and there are no final configurations.

8.2 Graph of Th'

Let $TS(Th')$ be the graph of the process Th',

$$TS(Th') = (C_{th'}, \rightarrow_{th'}, \longmapsto_{th'}, I_{th'}, \emptyset).$$

- Like in the case of Th there is only one initial configuration,

$$I_{th'} = \{(Th', (18,4), 0)\};$$

- $(Th', (18,4), 0)$ can wait for upto $ln2$ time units and evolve into the following set of configurations:

$$C'_{th'} = \{(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-r}(Th'^{\rightarrow}), \alpha_r, r) \mid r \in [0, ln2]\}.$$

  After performing action $\widetilde{\widetilde{\widetilde{toff}}}$, $(up \curvearrowright_T \sigma_{\mathsf{rel}}^{0}(Th'^{\rightarrow}), (20,2), ln2)$, becomes $(Th'^{off}, (20,-3), 0)$.

  While idling, Th'^{off} evolves into the following processes:

$$C''_{th'} = \{(down \curvearrowright_T \sigma_{\mathsf{rel}}^{ln3-r'}(Th'^{\leftarrow}), \alpha'_{r'}, r') \mid r' \in [0, ln3]\}.$$

  The set of all configurations, $C_{th'}$, is defined as,

$$C_{th'} = C'_{th'} \cup C''_{th'} \cup I_{th'};$$

- After waiting for $ln2$ time units, Th'^{on} performs action $\widetilde{\widetilde{\widetilde{toff}}}$. The temperature at that time is $20°$. Th'^{off} waits for $ln3$ time units before performing action $\widetilde{\widetilde{\widetilde{ton}}}$. Temperature at that time is $18°$.

  The set of action transitions is as follows:

$$\{(up \curvearrowright_T \sigma_{\mathsf{rel}}^{0}(Th'^{\rightarrow}), (20,2), ln2) \xrightarrow{toff}_{th'} (Th'^{off}, (20,-3), 0),$$
$$(down \curvearrowright_T \sigma_{\mathsf{rel}}^{0}(Th'^{\leftarrow}), (18,-1), ln3) \xrightarrow{ton}_{th'} (Th'^{on}, (18,4), 0)\};$$

- the time transitions originating from $(Th', (18,4), 0)$ are,

$$\{(Th', (18,4), 0) \xmapsto{t,\rho_t}_{th'} (up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-t}(Th'^{\rightarrow}), \alpha_t, t) \mid \\ t \in (0, ln2], (t, \rho_t) \in D, \rho_t \models up \wedge \rho_t \text{ is smooth for } T\}.$$

Time transitions among members of $C'_{th'}$ are,

$$\{(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-r}(Th'^{\rightarrow}), \alpha_r, r) \xmapsto{t,\rho_t}_{th'} \\ (up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-(r+t)}(Th'^{\rightarrow}), \alpha_{t+r}, t+r) \mid \\ (t, \rho_t) \in D, \rho_t \models up \wedge \rho_t \text{ is smooth for T}, \\ r \in [0, ln2), t \in (0, ln2], r+t \in (0, ln2]\}.$$

Time transitions among members of $C''_{th'}$ are,

$$\{(down \curvearrowright_T \sigma_{\mathsf{rel}}^{ln3-r'}(Th'^{\leftarrow}), \alpha'_{r'}, r') \xmapsto{t,\rho_t}_{th'} \\ (down \curvearrowright_T \sigma_{\mathsf{rel}}^{ln3-(r'+t)}(Th'^{\leftarrow}), \alpha'_{t+r'}, t+r') \mid \\ (t, \rho_t) \in D, \rho_t \models down \wedge \rho_t \text{ is smooth for T}, \\ r' \in [0, ln3), t \in (0, ln3], r'+t \in (0, ln3]\};$$

- Process Th' is an infinite process with recursion and there are no final configurations.

8.3 Proof : $TS(Th)$ is bisimilar to $TS(Th')$

We define a binary relation $\mathbb{R}$ on configurations of C_{th} and $C_{th'}$ as follows:

$$\begin{aligned} \mathbb{R} \;=\; & \{((Th, (18,4), 0), (Th', (18,4), 0)), \\ & ((up \curvearrowright_T \textstyle\int_{u \in [r,\infty)} \sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow}), \alpha_r, r), (up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-r}(Th'^{\rightarrow}), \alpha_r, r)), \\ & ((down \curvearrowright_T \textstyle\int_{u \in [r',\infty)} \sigma_{\mathsf{rel}}^{u-r'}(Th^{\leftarrow}), \alpha'_{r'}, r'), \\ & (down \curvearrowright_T \sigma_{\mathsf{rel}}^{ln3-r'}(Th'^{\leftarrow}), \alpha'_{r'}, r')) \\ & \mid r \in [0, ln2], r' \in [0, ln3]\}. \end{aligned}$$

We prove that $\mathbb{R}$ is a bisimulation relation. For all pairs $(c, c') \in \mathbb{R}$, where $c \in C_{th}$ and $c' \in C_{th'}$, we prove that the pair (c, c') fulfills the four conditions mentioned in section 7. We concentrate only on Th^{on} and Th'^{on}. The proof that $((down \curvearrowright_T \int_{u \in [r',\infty)} \sigma_{\mathsf{rel}}^{u-r'}(Th^{\leftarrow}), \alpha'_{r'}, r'), (down \curvearrowright_T \sigma_{\mathsf{rel}}^{ln3-r'}(Th'^{\leftarrow}), \alpha'_{r'}, r'))$ fulfills the bisimulation conditions is left to the reader.

1. The states and time labels of c and c' for all pairs $(c, c') \in \mathbb{R}$ are the same.
2. While evolving as up, $\widetilde{\widetilde{toff}}$ is the only action that Th^{on} and Th'^{on} can

perform at time $ln2$.

$(Th, (18,4), 0) \not\xrightarrow{toff}_{th}$
$(Th', (18,4), 0) \not\xrightarrow{toff}_{th'}$

For all $r \in [0, ln2)$,
$(up \curvearrowright_T (\int_{u\in[r,\infty)} \sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow})), \alpha_r, r) \not\xrightarrow{toff}_{th}$
$(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-r}(Th'^{\rightarrow}), \alpha_r, r) \not\xrightarrow{toff}_{th'}$

At $r = ln2$,
$(up \curvearrowright_T (\int_{u\in[r,\infty)} \sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow})), (20,2), ln2) \xrightarrow{toff}_{th} (Th^{off}, (20,-3), 0)$
$(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-r}(Th'^{\rightarrow}), (20,2), ln2) \xrightarrow{toff}_{th'} (Th'^{off}, (20,-3), 0)$ and $((Th^{off}, (20,-3), 0), (Th'^{off}, (20,-3), 0))$ is in $\mathbb{R}$.

3. For all $t \in (0, ln2]$, let $(t, \rho_t) \in D$, where $\rho_t \models up$ and ρ_t is smooth for T, we can derive,

   $(Th, (18,4), 0) \xmapsto{t,\rho_t}_{th} (up \curvearrowright_T (\int_{u\in[t,\infty)} \sigma_{\mathsf{rel}}^{u-t}(Th^{\rightarrow})), \alpha_t, t)$
   $(Th', (18,4), 0) \xmapsto{t,\rho_t}_{th'} (up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-t}(Th'^{\rightarrow}), \alpha_t, t)$ and,
   $((up \curvearrowright_T (\int_{u\in[t,\infty)} \sigma_{\mathsf{rel}}^{u-t}(Th^{\rightarrow})), \alpha_t, t), (up \curvearrowright_T (\sigma_{\mathsf{rel}}^{ln2-t}(Th'^{\rightarrow})), \alpha_t, t))$ is in $\mathbb{R}$.

   For $r \in [0, ln2), t \in (0, ln2], r + t \in (0, ln2]$ let $(t, \rho_t) \in D$, where $\rho_t \models up$ and ρ_t is smooth for T, then we can derive,
   $(up \curvearrowright_T \int_{u\in[r,\infty)} \sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow}), \alpha_r, r) \xmapsto{t,\rho_t}_{th}$
   $(up \curvearrowright_T \int_{u\in[r+t,\infty)} \sigma_{\mathsf{rel}}^{u-(r+t)}(Th^{\rightarrow}), \alpha_{t+r}, t+r)$

   $(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-r}(Th'^{\rightarrow}), \alpha_r, r) \xmapsto{t,\rho_t}_{th'} (up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-(r+t)}(Th'^{\rightarrow}), \alpha_{t+r}, t+r)$

4. No configuration in $\mathbb{R}$ is a final configuration.

Thus $\mathbb{R}$ is a bisimulation relation that shows that $TS(Th)$ and $TS(Th')$ are bisimilar.

8.4 Graph of $T\triangle(Th)$

Signal hiding is applied to a graph of a process after removing unreachable configurations and transitions from it. $TS(Th)$ and $TS(Th')$, have no unreachable configurations and transitions, therefore:

$$TS(Th) = \text{reach}(TS(Th)) \text{ and } TS(Th') = \text{reach}(TS(Th'))$$

Thus we can apply signal hiding with respect to T on $TS(Th)$ and $TS(Th')$ respectively.

We define α, α' to be states with arbitrary values of T and $\dot{T}$. i.e.,

$$\alpha =_T \alpha' =_T (18, 4) =_T (19, 3) =_T (20, -3) \text{ etc.}$$

Let $T\triangle(TS(Th))$ be the graph obtained after applying signal hiding with respect to T on $TS(Th)$.

We define $T\triangle(TS(Th))$ as,

$$T\triangle(TS(Th)) = (C_{T\triangle th}, \rightarrow_{T\triangle th}, \longmapsto_{T\triangle th}, I_{T\triangle th}, \emptyset);$$

where,

- $$I_{T\triangle th} = \{(T\triangle Th, \alpha, 0) \mid \alpha =_T (18, 4)\};$$

- the initial configurations evolve into the following set of configurations while idling:

  $$C'_{T\triangle th} = \{(T\triangle(up \curvearrowright_T \textstyle\int_{u\in[r,\infty)} \sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow})), \alpha, r) \mid r \in [0, ln2], \alpha =_T \alpha_r\};$$

  Any of the configurations, $(T\triangle(up \curvearrowright_T \int_{u\in[ln2,\infty)} \sigma_{\mathsf{rel}}^{u-ln2}(Th^{\rightarrow})), \alpha, ln2)$, with timer equal to $ln2$, can perform action $\widetilde{\widetilde{toff}}$ and become one of the configurations of the form, $(T\triangle(down \curvearrowright_T \int_{u\in[0,\infty)} \sigma_{\mathsf{rel}}^{u}(Th^{\leftarrow})), \alpha', 0)$. $T\triangle Th^{off}$ evolves into the following processes:

  $$C''_{T\triangle th} = \{(T\triangle(down \curvearrowright_T \textstyle\int_{u\in[r',\infty)} \sigma_{\mathsf{rel}}^{u-r'}(Th^{\leftarrow})), \alpha, r') \mid r' \in [0, ln3]\}$$

  The set of all configurations, $C_{T\triangle th}$, is defined as,

  $$C_{T\triangle th} = C'_{T\triangle th} \cup C''_{T\triangle th} \cup I_{T\triangle th};$$

- the set of action transitions consists of transitions with two action labels, i.e., $\widetilde{\widetilde{toff}}$ and $\widetilde{\widetilde{ton}}$. For action $\widetilde{\widetilde{toff}}$, consider configuration $T\triangle c$, where

  $$c =_T (up \curvearrowright_T \textstyle\int_{u\in[ln2,\infty)} \sigma_{\mathsf{rel}}^{u-ln2}(Th^{\rightarrow}), (20, 2), ln2).$$

  $T\triangle c$ can perform action $\widetilde{\widetilde{toff}}$ and become a configuration $T\triangle c'$, where $c' =_T (Th^{off}, (20, -3), 0)$.

  For action ton, consider a configuration $T\triangle c''$, such that

  $$c'' =_T (down \curvearrowright_T (\textstyle\int_{u\in[ln3,\infty)} \sigma_{\mathsf{rel}}^{u-ln3}(Th^{\leftarrow})), (18, -1), ln3).$$

  $T\triangle c''$ can perform action $\widetilde{\widetilde{ton}}$ and become a configuration $T\triangle c'''$, where $c''' =_T (Th^{on}, (18, 4), 0)$.

  The set of actions $\rightarrow_{T\triangle th}$ consists of the following transitions:

  $$\{(T\triangle(up \curvearrowright_T \textstyle\int_{u\in[ln2,\infty)} \sigma_{\mathsf{rel}}^{u-ln2}(Th^{\rightarrow})), \alpha, ln2) \xrightarrow{toff}_{T\triangle th} (T\triangle Th^{off}, \alpha', 0)\}$$
  $$\cup$$
  $$\{(T\triangle(down \curvearrowright_T \textstyle\int_{u\in[ln3,\infty)} \sigma_{\mathsf{rel}}^{u-ln3}(Th^{\leftarrow})), \alpha, ln3) \xrightarrow{ton}_{T\triangle th} (T\triangle Th^{on}, \alpha', 0)\};$$

- time transitions originating from initial configurations are,

$$\{(T\triangle Th, \alpha, 0) \overset{t,\rho_t^*}{\longmapsto}_{T\triangle th} (T\triangle(up \curvearrowright_T \int_{u\in[t,\infty)} \sigma_{\textsf{rel}}^{u-t}(Th^{\rightarrow})), \alpha', t) \mid t \in (0, ln2],$$
$$(t, \rho_t^*) \in D,\ \exists\, \rho_t \in \epsilon_t \bullet (\rho_t \models up\ \wedge\ \rho_t \text{ is smooth for T } \wedge\ \rho_t^* =_T \rho_t)\}.$$

Time transitions among members of $C'_{T\triangle th}$ are,

$$\{(T\triangle(up \curvearrowright_T \int_{u\in[r,\infty)} \sigma_{\textsf{rel}}^{u-r}(Th^{\rightarrow})), \alpha, r) \overset{t,\rho_t^*}{\longmapsto}_{T\triangle th}$$
$$(T\triangle(up \curvearrowright_T \int_{u\in[r+t,\infty)} \sigma_{\textsf{rel}}^{u-(r+t)}(Th^{\rightarrow})), \alpha', t+r) \mid$$
$$(t, \rho_t^*) \in D,\ \exists\, \rho_t \in \epsilon_t \bullet (\rho_t \models up\ \wedge\ \rho_t \text{ is smooth for T}$$
$$\wedge\ \rho_t^* =_T \rho_t), r \in [0, ln2), t \in (0, ln2], r+t \in (0, ln2]\}.$$

Time transitions among members of $C''_{T\triangle th}$ are,

$$\{(T\triangle(down \curvearrowright_T \int_{u\in[r',\infty)} \sigma_{\textsf{rel}}^{u-r'}(Th^{\leftarrow})), \alpha, r') \overset{t,\rho_t^*}{\longmapsto}_{T\triangle th}$$
$$(T\triangle(down \curvearrowright_T \int_{u\in[r'+t,\infty)} \sigma_{\textsf{rel}}^{u-(r'+t)}(Th^{\leftarrow})), \alpha', r'+t) \mid$$
$$(t, \rho_t^*) \in D,\ \exists\, \rho_t \in \epsilon_t \bullet (\rho_t \models down\ \wedge\ \rho_t \text{ is smooth for T}$$
$$\wedge\ \rho_t^* =_T \rho_t), r' \in [0, ln3), t \in (0, ln3], r'+t \in (0, ln3]\}.$$

8.5 Graph of $T\triangle(Th')$

Let $T\triangle(TS(Th'))$ be the graph obtained by applying signal hiding with respect to T on $TS(Th')$. α, α' denote states with arbitrary values of T and $\dot{T}$ and $\alpha =_T \alpha'$.

We define $T\triangle TS(Th')$ as,

$$T\triangle(TS(Th')) = (C_{T\triangle th'}, \rightarrow_{T\triangle th'}, \longmapsto_{T\triangle th'}, I_{T\triangle th'}, \emptyset);$$

where,

- $$I_{T\triangle th'} = \{(T\triangle Th', \alpha, 0) \mid \alpha =_T (18, 4)\};$$

- the initial configurations evolve into the following set of configurations while idling:

$$C'_{T\triangle th'} = \{(T\triangle(up \curvearrowright_T \sigma_{\textsf{rel}}^{ln2-r}(Th'^{\rightarrow})), \alpha, r) \mid r \in [0, ln2], \alpha =_T \alpha_r\}.$$

Any of the configurations, $(T\triangle(up \curvearrowright_T \sigma_{\textsf{rel}}^{0}(Th'^{\rightarrow})), \alpha, ln2)$, can perform action $\widetilde{\widetilde{toff}}$ and become one of the configurations of the form, $(T\triangle(down \curvearrowright_T \sigma_{\textsf{rel}}^{ln3}(Th'^{\leftarrow})), \alpha', 0)$. $T\triangle Th'^{off}$ evolves into the following processes:

$$C''_{T\triangle th'} = \{(T\triangle(down \curvearrowright_T \sigma_{\textsf{rel}}^{ln3-r'}(Th'^{\leftarrow})), \alpha, r') \mid r' \in [0, ln3]\}.$$

The set of all configurations, $C_{T\triangle th'}$, is defined as,

$$C_{T\triangle th'} = C'_{T\triangle th'} \cup C''_{T\triangle th'} \cup I_{T\triangle th'};$$

- the set of action transitions consists of transitions with two action labels, i.e., $\widetilde{\widetilde{\widetilde{toff}}}$ and $\widetilde{\widetilde{\widetilde{ton}}}$. For action $\widetilde{\widetilde{\widetilde{toff}}}$, consider a configuration $T\triangle c$, where

  $$c =_T (up \curvearrowright_T \sigma^0_{\mathsf{rel}}(Th'^{\rightarrow}), (20, 2), ln2).$$

  $T\triangle c$ with timer equal to $ln2$ can perform action $\widetilde{\widetilde{\widetilde{toff}}}$, and become a configuration $T\triangle c'$, where $c' =_T (Th'^{off}, (20, -3), 0)$.

  Whereas for action $\widetilde{\widetilde{\widetilde{ton}}}$, consider a configuration $T\triangle c''$, such that

  $$c'' =_T (down \curvearrowright_T \sigma^0_{\mathsf{rel}}(Th'^{\leftarrow}), (18, -1), ln3).$$

  $T\triangle c''$ can perform action $\widetilde{\widetilde{\widetilde{ton}}}$ and become a configuration $T\triangle c'''$, where $c''' =_T (Th'^{on}, (18, 4), 0)$.

  The set of actions $\rightarrow_{T\triangle th'}$ consists of the following transitions:

  $$\{(T\triangle(up \curvearrowright_T \sigma^0_{\mathsf{rel}}(Th'^{\rightarrow})), \alpha, ln2) \xrightarrow{toff}_{T\triangle th'} (T\triangle Th'^{off}, \alpha', 0)\}$$
  $$\cup$$
  $$\{(T\triangle(down \curvearrowright_T \sigma^0_{\mathsf{rel}}(Th'^{\leftarrow})), \alpha, ln3) \xrightarrow{ton}_{T\triangle th'} (T\triangle Th'^{on}, \alpha', 0)\};$$

- time transitions originating from initial configurations are,

  $$\{(T\triangle Th', \alpha, 0) \xmapsto{t,\rho^*_t}_{T\triangle th'} (T\triangle(up \curvearrowright_T \sigma^{ln2-t}_{\mathsf{rel}}(Th'^{\rightarrow})), \alpha', t)$$
  $$\mid t \in (0, ln2], (t, \rho^*_t) \in D, \exists\, \rho_t \in \epsilon_t \bullet (\rho_t \models up \wedge \rho_t \text{ is smooth for T})$$
  $$\wedge\, \rho^*_t =_T \rho_t\}.$$

  Time transitions among members of $C'_{T\triangle th'}$ are,

  $$\{(T\triangle(up \curvearrowright_T \sigma^{ln2-r}_{\mathsf{rel}}(Th'^{\rightarrow})), \alpha, r) \xmapsto{t,\rho^*_t}_{T\triangle th'}$$
  $$(T\triangle(up \curvearrowright_T \sigma^{ln2-(r+t)}_{\mathsf{rel}}(Th'^{\rightarrow})), \alpha', t + r) \mid$$
  $$(t, \rho^*_t) \in D, \exists\, \rho_t \in \epsilon_t \bullet (\rho_t \models up \wedge \rho_t \text{ is smooth for T}$$
  $$\wedge\, \rho^*_t =_T \rho_t), r \in [0, ln2), t \in (0, ln2], r + t \in (0, ln2]\}.$$

  Time transitions among members of $C''_{T\triangle th'}$ are,

  $$\{(T\triangle(down \curvearrowright_T \sigma^{ln3-r'}_{\mathsf{rel}}(Th'^{\leftarrow})), \alpha, r') \xmapsto{t,\rho^*_t}_{T\triangle th'}$$
  $$(T\triangle(down \curvearrowright_T \sigma^{ln3-(r'+t)}_{\mathsf{rel}}(Th'^{\leftarrow})), \alpha', r' + t) \mid$$
  $$(t, \rho^*_t) \in D, \exists\, \rho_t \in \epsilon_t \bullet (\rho_t \models down \wedge \rho_t \text{ is smooth for T}$$
  $$\wedge\, \rho^*_t =_T \rho_t), r' \in [0, ln3), t \in (0, ln3], r' + t \in (0, ln3]\}.$$

8.6 Proof: $T\triangle(TS(Th))$ is bisimilar to $T\triangle(TS(Th'))$

We define a binary relation $\mathbb{R}'$ on configurations of $C_{T\triangle th}$ and $C_{T\triangle th'}$. The pairs of configurations in $\mathbb{R}'$ have same states. We use α and α' to denote states with arbitrary values of temperature and its derivative.

$$\begin{array}{rcl}\mathbb{R}' & = & \{((T\triangle Th,\alpha,0),(T\triangle Th',\alpha,0)),\\ & & ((T\triangle(up \curvearrowright_T \int_{u\in[r,\infty)}\sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow})),\alpha,r),(T\triangle(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-r}(Th'^{\rightarrow})),\alpha,r)),\\ & & ((T\triangle(down \curvearrowright_T \int_{u\in[r',\infty)}\sigma_{\mathsf{rel}}^{u-r'}(Th^{\leftarrow})),\alpha,r'),\\ & & (T\triangle(down \curvearrowright_T \sigma_{\mathsf{rel}}^{ln3-r'}(Th'^{\leftarrow})),\alpha,r')),\\ & & \mid r\in[0,ln2],r'\in[0,ln3]\}.\end{array}$$

We prove that $\mathbb{R}'$ is a bisimulation relation. Here we give the proof that $(T\triangle(Th^{on}),\alpha,r)$ and $(T\triangle(Th'^{on}),\alpha,r)$ are bisimilar. The proof that $((T\triangle(down \curvearrowright_T \int_{u\in[r',\infty)}\sigma_{\mathsf{rel}}^{u-r'}(Th^{\leftarrow})),\alpha,r'),(T\triangle(down \curvearrowright_T \sigma_{\mathsf{rel}}^{ln3-r'}(Th'^{\leftarrow})),\alpha,r'))$ fulfills the bisimulation conditions is left to the reader.

1. The states and time labels of c and c' for all pairs $(c,c')\in\mathbb{R}'$ are the same, where $c\in C_{T\triangle th}$ and $c'\in C_{T\triangle th'}$.

2. $\widetilde{\widetilde{toff}}$ is the action that $T\triangle Th^{on}$ and $T\triangle Th'^{on}$ will perform at time $ln2$.

   $(T\triangle Th,\alpha,0) \stackrel{toff}{\not\longrightarrow}_{T\triangle th}$
   $(T\triangle Th',\alpha,0) \stackrel{toff}{\not\longrightarrow}_{T\triangle th'}$

   For all $r\in[0,ln2)$,
   $(T\triangle(up \curvearrowright_T (\int_{u\in[r,\infty)}\sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow}))),\alpha,r) \stackrel{toff}{\not\longrightarrow}_{T\triangle th}$
   $(T\triangle(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-r}(Th'^{\rightarrow}),\alpha,r)) \stackrel{toff}{\not\longrightarrow}_{T\triangle th'}$

   For $r=ln2$,
   $(T\triangle(up \curvearrowright_T (\int_{u\in[r,\infty)}\sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow})),\alpha,ln2)) \stackrel{toff}{\longrightarrow}_{T\triangle th} (T\triangle Th^{off},\alpha',0)$
   $(T\triangle(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-r}(Th'^{\rightarrow})),\alpha,ln2) \stackrel{toff}{\longrightarrow}_{T\triangle th'} (T\triangle Th'^{off},\alpha',0)$ and $((T\triangle Th^{off},\alpha,0),(T\triangle Th'^{off},\alpha,0))$ is in $\mathbb{R}'$.

3. For all $t\in(0,ln2]$, let $(t,\rho_t)\in D$, where $\rho_t\models up$ and ρ_t is smooth for T.

Let $\rho_t^* \in \epsilon_t$ and $\rho_t^* =_T \rho_t$.

$(T\triangle Th, \alpha, 0) \xmapsto{t,\rho_t^*}_{T\triangle th} (T\triangle(up \curvearrowright_T \int_{u\in[t,\infty)} \sigma_{\mathsf{rel}}^{u-t}(Th^{\rightarrow})), \alpha', t)$
$(T\triangle Th', \alpha, 0) \xmapsto{t,\rho_t^*}_{T\triangle th'} (T\triangle(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-t}(Th'^{\rightarrow})), \alpha', t)$.
And,
$((T\triangle(up \curvearrowright_T \int_{u\in[t,\infty)} \sigma_{\mathsf{rel}}^{u-t}(Th^{\rightarrow})), \alpha', t), (T\triangle(up \curvearrowright_T (\sigma_{\mathsf{rel}}^{ln2-t}(Th'^{\rightarrow})), \alpha', t))$
is in $\mathbb{R}'$.

For $r \in [0, ln2), t \in (0, ln2], r + t \in (0, ln2]$, let $(t, \rho_t) \in D$, where $\rho_t \models up$ and ρ_t is smooth for T. Let $\rho_t^* \in \epsilon_t$ and $\rho_t^* =_T \rho_t$.
$(T\triangle(up \curvearrowright_T \int_{u\in[r,\infty)} \sigma_{\mathsf{rel}}^{u-r}(Th^{\rightarrow})), \alpha, r) \xmapsto{t,\rho_t^*}_{T\triangle th}$
$(T\triangle(up \curvearrowright_T \int_{u\in[r+t,\infty)} \sigma_{\mathsf{rel}}^{u-(r+t)}(Th^{\rightarrow})), \alpha', t + r)$

$(T\triangle(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-r}(Th'^{\rightarrow})), \alpha, r) \xmapsto{t,\rho_t^*}_{T\triangle th'}$
$(T\triangle(up \curvearrowright_T \sigma_{\mathsf{rel}}^{ln2-(r+t)}(Th'^{\rightarrow})), \alpha', t + r)$

4. No configuration in $\mathbb{R}'$ is a final configuration.

Thus $\mathbb{R}'$ is a bisimulation relation that shows that $T\triangle(TS(Th))$ and $T\triangle(TS(Th'))$ are bisimilar.

9 Soundness of Axioms of BPA_{hs}^{srt}

We prove the soundness of the axioms of BPA_{hs}^{srt}, by proving that their interpretations in the paradigm of transition systems (given in this report) hold.

1. **GC1**

   $T :\rightarrow x = x$

   Interpretation:

   $T :\rightarrow TS \leftrightarrows TS$.

   Let TS be given by

   $$TS \;=\; (C, \rightarrow, \longmapsto, I, F) \text{ and;}$$

   $T :\rightarrow TS$ is given by

   $$T :\rightarrow TS \;=\; (C', \rightarrow', \longmapsto', I', F), \text{ as defined in sec 5.8.}$$

   Let $\mathbf{p}$ be the process expression of TS. $R_1 \subseteq I \times I'$ is a binary relation on initial configurations of TS and $T :\rightarrow TS$

   $$R_1 = \{(i, T :\rightarrow i) \mid i \in I\}; \text{ and}$$

$R \subseteq C \times C'$, is a binary relation on configurations of TS and $T :\rightarrow TS$

$$R = R_1 \cup \{(c, c) \mid c \in C\}.$$

Note that if $i \in I$ then $T :\rightarrow i \in I'$. Also $C' \supseteq C$. So if $c \in C$, then $c \in C'$.

We prove that R is a bisimulation relation. For this we have to prove that for every pair $(c, c') \in R$, (c, c') satisfies the conditions in section 7. For a pair (c, c), it is easy to see that all conditions of a bisimulation relation are met. We prove it for pairs $(i, T :\rightarrow i)$, where $i \in I$ as follows(the reader may consult section 5.8 to verify statements):

(a) The notation $T :\rightarrow i$ doesn't do anything to the state or time label of i. It only changes the process term of i from $\mathbf{p}$ to $T{:\rightarrow}\mathbf{p}$.

(b) $T :\rightarrow i \xrightarrow{a}{}' c$, whenever $i \xrightarrow{a} c$, where $i \in I, a \in A, c \in C$. Therefore, $T :\rightarrow i \xrightarrow{a}{}' c$ if and only if $i \xrightarrow{a} c$. And (c, c) satisfies the conditions for bisimulation.

(c) Similarly $T :\rightarrow i \overset{r,\rho}{\longmapsto}{}' c$, whenever $i \overset{r,\rho}{\longmapsto} c$, where $i \in I, (r, \rho) \in D, c \in C$. Therefore, $T :\rightarrow i \overset{r,\rho}{\longmapsto}{}' c \iff i \overset{r,\rho}{\longmapsto} c$, and (c, c) satisfy the bisimulation conditions.

(d) Neither i nor $T :\rightarrow i$ is a final configuration.

The relation R indicates that $TS \leftrightarroweq T :\rightarrow TS$, as,

For every $i \in I$, there exists an $T :\rightarrow i \in I'$(and viceversa), such that $B(i, T :\rightarrow i)$.

2. **GC2SR**

$F :\rightarrow x = \tilde{\tilde{\delta}}$

Interpretation:

$F :\rightarrow TS \leftrightarroweq TS(\tilde{\tilde{\delta}})$.

Let TS be given by

$$TS \quad = \quad (C, \rightarrow, \longmapsto, I, F);$$

$F :\rightarrow TS$ be given by

$$F :\rightarrow TS \quad = \quad (C', \rightarrow', \longmapsto' I', F), \text{ as defined in sec 5.8;}$$

$TS(\tilde{\tilde{\delta}})$ be given by

$$TS(\tilde{\tilde{\delta}}) = (C'', \emptyset, \emptyset, I'', \emptyset), \text{ as defined in sec 5.5.}$$

Let $\mathbf{p}$ be the process expression of TS. R is binary relation on initial configurations of $TS(\tilde{\tilde{\delta}})$ and $F :\rightarrow TS$, i.e.,

$$R = \{((\tilde{\tilde{\delta}}, \alpha, 0), (F :\rightarrow \mathbf{p}, \alpha, 0)) \mid \alpha \in S\}.$$

We prove that R is a bisimulation relation.

Let $\alpha \in S$.

(a) The states and time labels of $(\tilde{\tilde{\delta}}, \alpha, 0)$ and $(F :\to \mathbf{p}, \alpha, 0)$ are the same.

(b) $(\tilde{\tilde{\delta}}, \alpha, 0)$ is not a source to any action transition.

For, $(F :\to \mathbf{p}, \alpha, 0) \xrightarrow{a}{}' c$, where $a \in A, c \in C$(c belongs to C' as well), we must have $(\mathbf{p}, \alpha, 0) \xrightarrow{a} c \wedge \alpha \models F$. Now F can never be true in any state. Therefore $(F :\to \mathbf{p}, \alpha, 0)$ is not a source to any action transitions.

(c) $(\tilde{\tilde{\delta}}, \alpha, 0)$ is not a source to any time transition.

For, $(F :\to \mathbf{p}, \alpha, 0)$ to be a source to a time transition, it must be the case that $\alpha \models F$, which can never hold. Therefore $(F :\to \mathbf{p}, \alpha, 0)$ is not a source to any time transitions.

(d) Neither $(\tilde{\tilde{\delta}}, \alpha, 0)$, nor$(F :\to \mathbf{p}, \alpha, 0)$ is a final configuration.

R indicates that $TS(\tilde{\tilde{\delta}}) \leftrightarrows F :\to TS$.

3. **GC5**

$\psi :\to (x \cdot y) = (\psi :\to x) \cdot y$

Interpretation:

$\psi :\to (TS_1 \cdot TS_2) = (\psi :\to TS_1) \cdot TS_2$

Let TS_1 and TS_2 be defined as,

$$\begin{aligned} TS_1 &= (C_1, \to_1, \longmapsto_1, I_1, F_1) \text{ and;} \\ TS_2 &= (C_2, I_2, \to_2, \longmapsto_2, I_2, F_2). \end{aligned}$$

$TS_1 \cdot TS_2$ is given by,

$$TS_1 \cdot TS_2 \quad = \quad (C_{12}, I_{12}, \to_{12}, \longmapsto_{12}, F_2), \text{ as defined in sec 5.10.}$$

Let the process expressions of TS_1 and TS_2 be $\mathbf{p}_1$ and $\mathbf{p}_2$ respectively.

First we write $\psi :\to TS_1 \cdot TS_2$.

Let $\psi :\to TS_1 \cdot TS_2$ be given by,

$$\psi :\to TS_1 \cdot TS_2 = (C', I', \to', \longmapsto', F_2), \text{ as defined in sec 5.8.}$$

Below we define I' and the sets of action and time transitions originating from members of I'.

(a) $I' = \{\psi :\to i_{12} \mid i_{12} \in I_{12}\} \cup \{\psi :\to (\mathbf{p}_1 \cdot \mathbf{p}_2, \alpha, 0) \mid \alpha \in S \wedge \alpha \not\models \psi\}$;
I' can also be written as

$$\begin{aligned} I' &= \{\psi :\to (\mathbf{p}_1 \cdot \mathbf{p}_2, \alpha, 0) \mid \alpha \in S \wedge (\alpha \not\models \psi \vee \\ &\quad (\mathbf{p}_1 {\cdot} \mathbf{p}_2, \alpha, 0) \in I_{12})\} \\ &= \{\psi :\to ((\mathbf{p}_1, \alpha, 0) \cdot \mathbf{p}_2) \mid \alpha \in S \wedge (\alpha \not\models \psi \\ &\quad \vee (\mathbf{p}_1, \alpha, 0) \in I_1)\}; \end{aligned}$$

(b) The set of action transitions originating from members of I', denoted by $\rightarrow''$, is,

$$\begin{aligned}\rightarrow'' \;=\; & \{(\psi :\rightarrow i_{12}, a, c) \mid i_{12} \in I_{12}, a \in A, c \in C_{12}, i_{12} \models \psi \\ & \wedge i_{12} \xrightarrow{a}_{12} c\};\end{aligned}$$

$\rightarrow''$ can also be written as

$$\begin{aligned}\rightarrow'' \;=\; & \{(\psi :\rightarrow i_{12}, a, i_2) \mid i_{12} \in I_{12}, a \in A, i_2 \in I_2, \\ & i_{12} \models \psi \wedge i_{12} \xrightarrow{a}_{12} i_2\} \\ & \cup\{(\psi :\rightarrow i_{12}, a, c_1 \cdot \mathbf{p}_2) \mid i_{12} \in I_{12}, a \in A, c_1 \in C_1 \backslash F_1, \\ & i_{12} \models \psi \wedge i_{12} \xrightarrow{a}_{12} c_1 \cdot \mathbf{p}_2\} \\ =\; & \{(\psi :\rightarrow (i_1 \cdot \mathbf{p}_2), a, i_2) \mid i_1 \in I_1, a \in A, i_2 \in I_2, i_1 \models \psi, \\ & \exists\, f \in F_1 \bullet \mathtt{state}(f) = \mathtt{state}(i_2) \wedge i_1 \xrightarrow{a}_1 f\} \\ & \cup\{(\psi :\rightarrow (i_1 \cdot \mathbf{p}_2), a, c_1 \cdot \mathbf{p}_2) \mid i_1 \in I_1, a \in A, c_1 \in C_1 \backslash F_1, \\ & i_1 \models \psi \wedge i_1 \xrightarrow{a}_1 c_1\};\end{aligned}$$

(c) The set of time transitions originating from members of I' is,

$$\begin{aligned}\longmapsto'' \;=\; & \{(\psi :\rightarrow i_{12}, (r, \rho), c) \mid i_{12} \in I_{12}, (r, \rho) \in D, c \in C_{12}, \\ & i_{12} \models \psi \;\wedge\; i_{12} \xmapsto{r,\rho} c\};\end{aligned}$$

$\longmapsto''$ can also be written as,

$$\begin{aligned}\longmapsto'' \;=\; & \{(\psi :\rightarrow (i_1 \cdot \mathbf{p}_2), (r, \rho), c_1 \cdot \mathbf{p}_2) \mid i_1 \in I_1, (r, \rho) \in D, \\ & c_1 \in C_1 \backslash F_1, i_1 \models \psi \;\wedge\; i_1 \xmapsto{r,\rho} c_1\};\end{aligned}$$

Now we simplify $(\psi :\rightarrow TS_1) \cdot TS_2$.
Let $\psi :\rightarrow TS_1$ be given by,

$$\psi :\rightarrow TS_1 \;=\; (C_1', I_1', \rightarrow_1', \longmapsto_1', F_2), \text{ as defined in sec 5.8.}$$

Let $(\psi :\rightarrow TS_1) \cdot TS_2$ be given by,

$$(\psi :\rightarrow TS_1) \cdot TS_2 \;=\; (C_{12}', I_{12}', \rightarrow_{12}', \longmapsto_{12}', F_2), \text{ as defined in sec 5.10.}$$

Below we define I_{12}' and the sets of action and time transitions originating from members of I_{12}'.

(a) $I_{12}' = \{i \cdot \mathbf{p}_2 \mid i \in I_1'\}$.
I' can also be written as,

$$\begin{aligned}I_{12}' \;=\; & \{(\psi :\rightarrow \mathbf{p}_1, \alpha, 0) \cdot \mathbf{p}_2 \mid \alpha \in S \;\wedge\; (\alpha \not\models \psi \vee \\ & (\mathbf{p}_1, \alpha, 0) \in I_1)\}.\end{aligned}$$

(b) The set of all action transitions originating from members of I'_{12}, denoted by $\rightarrow''_{12}$, is given by,

$$\begin{array}{rcl}\rightarrow''_{12} & = & \{(i \cdot \mathbf{p}_2, a, c \cdot \mathbf{p}_2) \mid i \in I'_1, a \in A, c \in C'_1 \backslash F_1, \wedge i \xrightarrow{a}{}'_1 c\} \\ & & \cup\{(i \cdot \mathbf{p}_2, a, i_2) \mid i \in I'_1, a \in A, i_2 \in I_2, \\ & & \exists f \in F_1 \bullet \mathtt{state}(f) = \mathtt{state}(i_2) \wedge i \xrightarrow{a}{}'_1 f\};\end{array}$$

$\rightarrow''_{12}$ can also be written as,

$$\begin{array}{rcl}\rightarrow''_{12} & = & \{((\psi :\rightarrow i_1) \cdot \mathbf{p}_2, a, c \cdot \mathbf{p}_2) \mid i_1 \in I_1, a \in A, c \in C_1 \backslash F_1, \\ & & i_1 \models \psi \wedge i_1 \xrightarrow{a}_1 c\} \\ & & \cup\{((\psi :\rightarrow i_1) \cdot \mathbf{p}_2, a, i_2) \mid i_1 \in I_1, a \in A, i_2 \in I_2, i_1 \models \psi, \\ & & \exists f \in F_1 \bullet \mathtt{state}(f) = \mathtt{state}(i_2) \wedge i_1 \xrightarrow{a}_1 f\};\end{array}$$

(c) The set of all time transitions originating from members of I'_{12}, denoted by $\longmapsto''$, is,

$$\longmapsto'' \quad = \quad \{(i \cdot \mathbf{p}_2, (r, \rho), c \cdot \mathbf{p}_2) \mid i \in I'_1, c \in C'_1 \backslash F_1 \wedge i \xmapsto{r,\rho}{}'_1 c\};$$

$\longmapsto''$ can also be written as,

$$\begin{array}{rcl}\longmapsto'' & = & \{((\psi :\rightarrow i_1) \cdot \mathbf{p}_2, (r, \rho), c_1 \cdot \mathbf{p}_2) \mid i_1 \in I_1, c_1 \in C_1 \backslash F_1, \\ & & i_1 \models \psi \wedge i \xmapsto{r,\rho}_1 c\};\end{array}$$

A binary relation on configurations of $\psi :\rightarrow TS_1 \cdot TS_2$ and $(\psi :\rightarrow TS_1) \cdot TS_2$ is given as,

$$\begin{array}{rcl}R & = & \{(\psi :\rightarrow ((\mathbf{p}_1, \alpha, 0) \cdot \mathbf{p}_2), (\psi :\rightarrow (\mathbf{p}, \alpha, 0)) \cdot \mathbf{p}_2) \mid \alpha \in S \wedge \\ & & (\alpha \not\models \psi \vee (\mathbf{p}_1, \alpha, 0) \in I_1)\} \\ & \cup & \{(c_2, c_2) \mid c_2 \in C_2\} \\ & \cup & \{(c_1 \cdot \mathbf{p}_2, c_1 \cdot \mathbf{p}_2) \mid c_1 \in I_1\}.\end{array}$$

R is a bisimulation relation and proves that

$$\psi :\rightarrow TS_1 \cdot TS_2 \leftrightarrows (\psi :\rightarrow TS_1) \cdot TS_2.$$

4. **GC6**

$(\psi \wedge \psi') :\rightarrow x = \psi :\rightarrow (\psi' :\rightarrow x)$

Interpretation:

$(\psi \wedge \psi') :\rightarrow TS = \psi :\rightarrow (\psi' :\rightarrow TS)$.

Let TS be given by

$$TS = (C, \rightarrow, \longmapsto, I, F).$$

The binary relation on configurations of $\psi :\rightarrow (\psi' :\rightarrow TS)$ and $(\psi \wedge \psi') :\rightarrow TS$, that proves bisimulation between two transition systems is ,

$$\begin{array}{rcl}R & = & \{(\psi :\rightarrow (\psi' :\rightarrow (\mathbf{p}, \alpha, 0)), (\psi \wedge \psi') :\rightarrow \mathbf{p}, \alpha, 0) \mid \\ & & \alpha \in S \wedge ((p, \alpha, 0) \in I \vee \alpha \not\models (\psi \wedge \psi'))\} \\ & & \cup\{(c, c) \mid c \in C\}.\end{array}$$

5. **GC7**

$(\psi \vee \psi') :\rightarrow x = \psi :\rightarrow x + \psi' :\rightarrow x$

Interpretation: $(\psi \vee \psi') :\rightarrow TS = \psi :\rightarrow TS + \psi' :\rightarrow TS$.

Let TS be given by

$$TS = (C, \rightarrow, \longmapsto, I, F);$$

$(\psi \vee \psi') :\rightarrow TS$ be given by

$$(\psi \vee \psi') :\rightarrow TS = (C', I', \rightarrow', \longmapsto', F), \text{ as defined in sec 5.8;}$$

$\psi :\rightarrow TS$ be given by

$$\psi :\rightarrow TS = (C_1, I_1, \rightarrow_1, \longmapsto_1, F), \text{ as defined in sec 5.8;}$$

$\psi' :\rightarrow TS$ be given by

$$\psi' :\rightarrow TS = (C_2, I_2, \rightarrow_2, \longmapsto_2, F), \text{ as defined in sec 5.8;}$$

$\psi :\rightarrow TS + \psi' :\rightarrow TS$ be given by

$$\begin{aligned}\psi :\rightarrow TS + \psi' :\rightarrow TS \quad = \quad & (C_{1+2}, I_{1+2}, \rightarrow_{1+2}, \longmapsto_{1+2}, F_{1+2}), \\ & \text{as defined in sec 5.11.}\end{aligned}$$

Let $\mathbf{p}$ be the process expression of TS.

First we simplify $\psi :\rightarrow TS + \psi' :\rightarrow TS$. Below we give the definitions of I_{1+2} and the sets of action and time transitions originating from members of I_{1+2}.

(a) $I_{1+2} = \{i_1 + i_2 \mid i_1 \in I_1, i_2 \in I_2 \wedge \texttt{state}(i_1) = \texttt{state}(i_2)\}$.
I_{1+2} can also be written as

$$\begin{aligned} I_{1+2} \quad = \quad & \{\psi :\rightarrow i + \psi' :\rightarrow i \mid i \in I\} \\ & \cup\{(\psi :\rightarrow \mathbf{p} + \psi' :\rightarrow \mathbf{p}, \alpha, 0) \mid \alpha \in S \wedge \alpha \not\models \psi \\ & \wedge \alpha \not\models \psi'\} \\ = \quad & \{(\psi :\rightarrow \mathbf{p} + \psi' :\rightarrow \mathbf{p}, \alpha, 0) \mid \alpha \in S \wedge ((p, \alpha, 0) \in I \\ & \vee \alpha \not\models (\psi \vee \psi'))\}; \end{aligned}$$

(b) The set of action transition originating from members of I_{1+2}, denoted by $\rightarrow'_{1+2}$, is given as

$$\begin{aligned} \rightarrow'_{1+2} \quad = \quad & \{(i_1 + i_2, a, c) \mid i_1 \in I_1, i_2 \in I_2, a \in A, c \in C_1 \\ & \wedge i_1 \xrightarrow{a}_1 c\} \\ & \cup\{(i_1 + i_2, a, c) \mid i_1 \in I_1, i_2 \in I_2, a \in A, c \in C_2 \\ & \wedge i_1 \xrightarrow{a}_2 c\}; \end{aligned}$$

$\rightarrow'_{1+2}$ can also written as,

$$\begin{aligned}
\rightarrow'_{1+2} &= \{(\psi :\rightarrow i + \psi' :\rightarrow i, a, c) \mid i \in I, a \in A, c \in C, \\
& \quad i \models \psi \wedge i \xrightarrow{a} c\} \\
& \quad \cup \{(\psi :\rightarrow i + \psi' :\rightarrow i, a, c) \mid i \in I, a \in A, c \in C, \\
& \quad i \models \psi' \wedge i \xrightarrow{a} c\} \\
&= \{(\psi :\rightarrow i + \psi' :\rightarrow i, a, c) \mid i \in I, a \in A, c \in C, \\
& \quad i \models (\psi \vee \psi') \wedge i \xrightarrow{a} c\};
\end{aligned}$$

(c) The set of all time transition originating from members of I_{1+2}, denoted by $\longmapsto_{1+2}$, is

$$\begin{aligned}
\longmapsto_{1+2} &= \{(\psi :\rightarrow i + \psi' :\rightarrow i, (r, \rho), c) \mid i \in I, (r, \rho) \in D, c \in C, \\
& \quad i \models (\psi \vee \psi') \wedge i \xmapsto{r,\rho} c\} \\
& \quad \cup \{(\psi :\rightarrow i + \psi' :\rightarrow i, (r, \rho), c + c) \mid i \in I, (r, \rho) \in D, c \in C, \\
& \quad i \models (\psi \wedge \psi') \wedge i \xmapsto{r,\rho} c\}
\end{aligned}$$

The definition of $(\psi \vee \psi') :\rightarrow TS$ can easily be obtained from the definition of $\psi :\rightarrow TS$. See section 5.8.

A binary relation on initial configurations of $(\psi \vee \psi') :\rightarrow TS$ and $\psi :\rightarrow TS + \psi' :\rightarrow TS$ is

$$\begin{aligned}
R_1 &= \{(\psi :\rightarrow (\mathbf{p}, \alpha, 0) + \psi' :\rightarrow (\mathbf{p}, \alpha, 0), (\psi \vee \psi') :\rightarrow (\mathbf{p}, \alpha, 0)) \mid \\
& \quad \alpha \in S \wedge ((\mathbf{p}, \alpha, 0) \in I \vee \alpha \not\models (\psi \vee \psi'))\}.
\end{aligned}$$

Let

$$\begin{aligned}
R &= R_1 \cup \{(c, c) \mid c \in C\} \\
& \quad \cup \{(c + c, c) \mid c \in C\}.
\end{aligned}$$

We prove that R is a bisimulation relation. $c + c \leftrightarrows c$ will be proved while proving the soundness of axiom $A3$ later on in the paper. Proving that $c \leftrightarrows c$ is trivial, so we give the proof for pairs of configurations in R_1 only.

(a) The states and time labels of configurations in a pair are the same.

(b) $((\psi \vee \psi') :\rightarrow i, a, c)$, can be an action transition in $\rightarrow'$, where $i \in I, a \in A, c \in C$, if and only if

$$i \xrightarrow{a} c \wedge i \models (\psi \vee \psi').$$

A corresponding initial configuration in I_{1+2}, with $i \in I$, is $\psi :\rightarrow i + \psi' :\rightarrow i$. The conditions for $\psi :\rightarrow i + \psi' :\rightarrow i$ to be a source to an action transition in $\rightarrow_{1+2}$, with action label $a \in A$, and target $c \in C$, are the same.

Similarly,

$$\psi :\rightarrow i + \psi' :\rightarrow i \xrightarrow{a}_{1+2} c,$$

where $c \in C$ and $a \in A$, is possible only if $i \models (\psi \vee \psi') \wedge i \xrightarrow{a} c$, which are the necessary and sufficient conditions for $((\psi \vee \psi') :\rightarrow i, a, c)$ to be an action transition in $\rightarrow'$.

(c) $((\psi \vee \psi') :\rightarrow i, (r, \rho), c)$ can be a time transition in $\longmapsto'$, with $i \in I, (r, \rho) \in D, c \in C$, if and only if

$$i \xmapsto{r,\rho} c \wedge i \models (\psi \vee \psi'),$$

which are same as the conditions for $(\psi :\rightarrow i + \psi' :\rightarrow i, (r, \rho), c)$ to be a time transition in $\longmapsto_{1+2}$. Same argument applies vice versa.

For every initial configuration $(\psi \vee \psi') :\rightarrow (\mathbf{p}, \alpha, 0)$ in I', where $\alpha \in S \wedge ((\mathbf{p}, \alpha, 0) \in I \vee \alpha \not\models (\psi \vee \psi'))$, there is an initial configuration $(\psi :\rightarrow (\mathbf{p}, \alpha, 0) + \psi' :\rightarrow (\mathbf{p}, \alpha, 0)$ in I_{1+2}, such that two are bisimilar to each other and vice versa.

Therefore, R is a bisimulation relation and proves that

$$(\psi \vee \psi') :\rightarrow TS \leftrightarrows \psi :\rightarrow TS + \psi' :\rightarrow TS.$$

6. **ST3**

$\sigma^r_{\mathsf{rel}}(x) + \sigma^r_{\mathsf{rel}}(y) = \sigma^r_{\mathsf{rel}}(x + y)$, where $r > 0$

Interpretation

$\sigma^r_{\mathsf{rel}}(TS_1) + \sigma^r_{\mathsf{rel}}(TS_2) = \sigma^r_{\mathsf{rel}}(TS_1 + TS_2)$

Let TS_1 be given by,

$$(C_1, \rightarrow_1, \longmapsto_1, I_1, F_1);$$

and TS_2 be given by,

$$(C_2, \rightarrow_2, \longmapsto_2, I_2, F_2).$$

Let $\mathbf{p}_1$ and $\mathbf{p}_2$ be the process expressions of TS_1 and TS_2 respectively.

For definition of $\sigma^r_{\mathsf{rel}}(TS)$ see section 5.12 and for definition of $TS_1 + TS_2$, refer to section 5.11.

We define a binary relation R', on configurations of $\sigma^r_{\mathsf{rel}}(TS_1) + \sigma^r_{\mathsf{rel}}(TS_2)$ and $\sigma^r_{\mathsf{rel}}(TS_1 + TS_2)$ as follows:

$$R' = R_1 \cup R_2 \cup R_3 \cup R_4 \cup R_5;$$

where,

$$R_1 = \{((\sigma^R_{\mathsf{rel}}(\mathbf{p}_1) + \sigma^R_{\mathsf{rel}}(\mathbf{p}_2), \alpha, r - R), (\sigma^R_{\mathsf{rel}}(\mathbf{p}_1 + \mathbf{p}_2), \alpha, r - R)) \mid \alpha \in S, 0 < R \leq r\};$$

$$R_2 = \{((\sigma^0_{\mathsf{rel}}(\mathbf{p}_1) + \sigma^0_{\mathsf{rel}}(\mathbf{p}_2), \alpha, r), (\sigma^0_{\mathsf{rel}}(\mathbf{p}_1 + \mathbf{p}_2), \alpha, r)) \mid \alpha \in S, (\mathbf{p}_1, \alpha, 0) \in I_1, (\mathbf{p}_2, \alpha, 0) \in I_2\};$$

$$R_3 = \{(\mathtt{incr\_timer}(c_1, r) + \mathtt{incr\_timer}(c_2, r)), (\mathtt{incr\_timer}(c_1 + c_2, r)) \mid c_1 \in \mathtt{tReach}(I_1), c_2 \in \mathtt{tReach}(I_2), \exists\, (i_1 \in I_1, i_2 \in I_2, (s, \rho) \in D) \bullet \mathtt{state}(i_1) = \mathtt{state}(i_2) \wedge i_1 \xmapsto{s,\rho}_1 c_1 \wedge i_2 \xmapsto{s,\rho}_2 c_2\};$$

$$R_4 = \{(\mathtt{incr\_timer}(c, r), \mathtt{incr\_timer}(c, r)) \mid \exists\, (i_1 \in I_1, i_2 \in I_2, (s, \rho) \in D) \bullet \mathtt{state}(i_1) = \mathtt{state}(i_2) \wedge ((c \in C_1 \wedge i_1 \xmapsto{s,\rho}_1 c_1 \wedge i_2 \not\xmapsto{s}_2) \vee (c \in C_2 \wedge i_2 \xmapsto{s,\rho}_2 c \wedge i_1 \not\xmapsto{s}_1))\};$$

$$R_5 = \{((p', \alpha, 0), (p', \alpha, 0)) \mid (p', \alpha, 0) \in C_1 \ \vee\ (p', \alpha, 0) \in C_2\}.$$

It can be observed that R' is a bisimulation relation and it proves that $\sigma^r_{\mathsf{rel}}(TS_1) + \sigma^r_{\mathsf{rel}}(TS_2)$ is bisimilar to $\sigma^r_{\mathsf{rel}}(TS_1 + TS_2)$.

7. **SRU2**

$\nu_{rel}(\sigma^r_{\mathsf{rel}}(x)) = \tilde{\tilde{\delta}}$, where $r > 0$

Interpretation

$\nu_{rel}(\sigma^r_{\mathsf{rel}}(TS)) = TS(\tilde{\tilde{\delta}})$.

Let TS be given by,

$$(C, \rightarrow, \longmapsto, I, F);$$

Let $\sigma^r_{\mathsf{rel}}(TS)$ be given by,

$$(C', \rightarrow', \longmapsto', I', F);$$

and let $\nu_{rel}(\sigma^r_{\mathsf{rel}}(TS))$ be given by,

$$(C'', \rightarrow'', \longmapsto', I'', F).$$

Let $\mathbf{p}$ be the process expression of TS.

Then I'' is defined as,

$$I'' = \{(\nu_{rel}(\sigma^r_{\mathsf{rel}}(\mathbf{p}), \alpha, 0)) \mid \alpha \in S\}.$$

There are no action transitions originating from members of I', (see section 5.12), therefore there are no action transitions originating from I''. No new time transitions are added in $\nu_{rel}(\sigma^r_{\mathsf{rel}}(TS))$. Therefore, no time transitions originate from members of I'' (see section 5.7).

A binary relation R on members of I'' and initial configurations of $TS(\tilde{\tilde{\delta}})$ is as follows:

$$R = \{((\nu_{rel}(\sigma^{r}_{\mathsf{rel}}(\mathbf{p})), \alpha, 0), (\tilde{\tilde{\delta}}, \alpha, 0)) \mid \alpha \in S\}.$$

R shows that $\nu_{rel}(\sigma^{r}_{\mathsf{rel}}(TS)) \leftrightarrows TS(\tilde{\tilde{\delta}})$.

8. **HSH4**

$v\triangle(\psi \,{}^{\wedge\!\blacktriangle}\, (\chi \,\ulcorner^{\!\blacktriangledown}\, \tilde{\tilde{a}} \cdot x)) = (v\triangle\psi) \,{}^{\wedge\!\blacktriangle}\, ((v\triangle({}^{\bullet}\psi \,\wedge\, \chi \,\wedge\, s_{\rho}(x)^{\bullet})) \,\ulcorner^{\!\blacktriangledown}\, \tilde{\tilde{a}} \cdot (v\triangle x));$

In this axiom signal hiding is applied to a process x, which is preceeded by an action. The action must take place such that it satisfies the transition proposition χ. The state proposition ψ denotes the signal of the whole process to which signal hiding with respect to variable v is applied.

On the right hand side of the axiom is the process $v\triangle x$ preceeded by an action. The action is accompanied by a transition proposition, i.e. signal hiding with respect to variable v applied to a conjunction of ${}^{\bullet}\psi$, χ and $s_{\rho}(x)^{\bullet}$). The signal emitted by the process on the right hand side is $v\triangle\psi$.

Interpretation:

$$v\triangle(\psi \,{}^{\wedge\!\blacktriangle}\, (\chi \,\ulcorner^{\!\blacktriangledown}\, TS(\tilde{\tilde{a}}) \cdot TS)) = (v\triangle\psi) \,{}^{\wedge\!\blacktriangle}\, ((v\triangle({}^{\bullet}\psi \,\wedge\, \chi \,\wedge\, s_{\rho}(\mathbf{p})^{\bullet})) \,\ulcorner^{\!\blacktriangledown}\, TS(\tilde{\tilde{a}}) \cdot (v\triangle TS))$$

where TS is a transition system whose process expression is $\mathbf{p}$ and $TS(\tilde{\tilde{a}})$ denotes the transition system corresponding to action $\tilde{\tilde{a}}$.

We obtain a graph model of the transition system corresponding to the right hand side of the axiom as follows:

Let $\texttt{reach}(TS) = TS$. This supposition frees us from the need to denote $\texttt{reach}(TS)$ by a separate notation. Let TS be given by,

$$TS = (C_p, \rightarrow_p, \longmapsto_p, I_p, F_p).$$

Let $v\triangle TS$ be the transition system obtained after hiding variable v, given by,

$$v\triangle TS = (C_{v\triangle\mathbf{p}}, \rightarrow_{v\triangle\mathbf{p}}, \longmapsto_{v\triangle\mathbf{p}}, I_{v\triangle\mathbf{p}}, F_{v\triangle\mathbf{p}}).$$

The process expression of $v\triangle TS$ is $v\triangle\mathbf{p}$. See section 6.1 for details.

We give here the sequential composition of $(v\triangle({}^{\bullet}\psi \wedge \chi \wedge s_{\rho}(\mathbf{p})^{\bullet})) \,\ulcorner^{\!\blacktriangledown}\, TS(\tilde{\tilde{a}})$ and $v\triangle TS$. Let it be denoted by TS' and given by,

$$TS' = (C', \rightarrow', \longmapsto', I', F');$$

where,

- $I' \;=\; \{((v\triangle({}^{\bullet}\psi \,\wedge\, \chi \,\wedge\, s_{\rho}(\mathbf{p})^{\bullet})) \,\ulcorner^{\!\blacktriangledown}\, \tilde{\tilde{a}} \cdot (v\triangle\mathbf{p}), \alpha, 0) \mid \alpha \in S\}$;
- $C' \;=\; C_{v\triangle\mathbf{p}} \cup I'$;

- $\rightarrow' \; = \; \rightarrow_{v\triangle\mathbf{p}} \cup \rightarrow''$, where,
  $\rightarrow'' \; = \; \{(i', a, i_{v\triangle\mathbf{p}}) \mid i' \in I', a \in A, i_{v\triangle\mathbf{p}} \in I_{v\triangle\mathbf{p}}, i' \models v\triangle(\psi \wedge {}^{\circ}\chi) \wedge i_{v\triangle\mathbf{p}} \models v\triangle(\chi^{\circ} \wedge s_{\rho}(\mathbf{p}))\}$;
- $\longmapsto' \; = \; \longmapsto_{v\triangle\mathbf{p}}$;
- $F' \; = \; F_{v\triangle\mathbf{p}}$.

Let TS'' be $(v\triangle\psi) \curvearrowright TS'$, i.e.,

$$TS'' = (v\triangle\psi) \curvearrowright ((v\triangle({}^{\bullet}\psi \;\wedge\; \chi \;\wedge\; s_{\rho}(\mathbf{p})^{\bullet})) \ulcorner TS(\tilde{\tilde{a}}) \cdot (v\triangle TS)).$$

Let

$$TS'' = (C'', \rightarrow'', \longmapsto', I'', F');$$

Only the set of configurations and action transitions of TS' are modified. These would be,

- $I'' \; = \; \{((v\triangle\psi) \curvearrowright ((v\triangle({}^{\bullet}\psi \;\wedge\; \chi \;\wedge\; s_{\rho}(\mathbf{p})^{\bullet})) \ulcorner \tilde{\tilde{a}} \cdot (v\triangle\mathbf{p})), \alpha, 0) \mid \alpha \models v\triangle\psi\}$;
- $C'' \; = \; C_{v\triangle\mathbf{p}} \cup I''$;
- $\rightarrow'' \; = \; \rightarrow_{v\triangle\mathbf{p}} \cup \rightarrow'''$; , where,
  $\rightarrow''' \; = \; \{(i'', a, i_{v\triangle\mathbf{p}}) \mid i'' \in I'', a \in A, i_{v\triangle\mathbf{p}} \in I_{v\triangle\mathbf{p}}, i'' \models v\triangle(\psi \wedge {}^{\circ}\chi) \;\wedge\; i_{v\triangle\mathbf{p}} \models v\triangle(\chi^{\circ} \wedge s_{\rho}(\mathbf{p}))\}$.

Now we come to the left hand side of the interpretation of axiom $HSH4$. Let $\psi \curvearrowright ((\chi \ulcorner TS(\tilde{\tilde{a}})) \cdot TS)$ be given by,

$$(C^{\circ}, \rightarrow^{\circ}, \longmapsto^{\circ}, I^{\circ}, F^{\circ});$$

where,

- $I^{\circ} \; = \; \{(\psi \curvearrowright (\chi \ulcorner \tilde{\tilde{a}} \cdot \mathbf{p}), \alpha, 0) \mid \alpha \models \psi\}$;
- $C^{\circ} \; = \; I^{\circ} \cup C_{p}$;
- $\rightarrow^{\circ} \; = \; \rightarrow_{p} \cup \rightarrow''$, where,
  $\rightarrow'' \; = \; \{(i^{\circ}, a, i_{p}) \mid i^{\circ} \in I^{\circ}, a \in A, i_{p} \in I_{p}, i^{\circ} \models (\psi \wedge {}^{\circ}\chi) \wedge i_{p} \models \chi^{\circ}\}$;
- $\longmapsto' \; = \; \longmapsto_{p}$;
- $F' \; = \; F_{p}$.

Let the transition system obtained by applying signal hiding with respect to variable v to the $\texttt{reach}(\psi \curvearrowright (\chi \ulcorner TS(\tilde{\tilde{a}}) \cdot TS))$ be given by,

$$TS'^{\circ} = (C'^{\circ}, \rightarrow'^{\circ}, \longmapsto'^{\circ}, I'^{\circ}, F'^{\circ});$$

(See section 6.1 for its details.)

We define a binary relation $\mathbb{R}$ on the initial configurations of TS'' and TS'° as follows:

$$\begin{aligned}\mathbb{R} \;=\; & \{(((v\triangle\psi) \mathbin{\hat{\blacktriangle}} ((v\triangle({}^{\bullet}\psi \;\wedge\; \chi \;\wedge\; s_{\rho}(\mathbf{p})^{\bullet})) \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}} \cdot (v\triangle\mathbf{p})), \alpha, 0),\\ & (v\triangle(\psi \mathbin{\hat{\blacktriangle}} (\chi \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}} \cdot \mathbf{p}), \alpha, 0))),\\ & ((v\triangle\mathbf{p}, \alpha', 0), (v\triangle\mathbf{p}, \alpha', 0))\\ & \mid \alpha, \alpha' \in S, \alpha \models v\triangle\psi \;\wedge\; \alpha' \models (s_{\rho}(\mathbf{p}) \wedge \chi^{\circ})\}.\end{aligned}$$

We prove that $\mathbb{R}$ is a bisimulation relation. We prove that the pair $(((v\triangle\psi) \mathbin{\hat{\blacktriangle}} ((v\triangle({}^{\bullet}\psi \;\wedge\; \chi \;\wedge\; s_{\rho}(\mathbf{p})^{\bullet})) \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}} \cdot (v\triangle\mathbf{p})), \alpha, 0), (v\triangle(\psi \mathbin{\hat{\blacktriangle}} (\chi \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}} \cdot \mathbf{p}), \alpha, 0)))$ satisfies the bisimulation conditions.

(a) The states and time labels are the same.

(b) $(v\triangle(\psi \mathbin{\hat{\blacktriangle}} (\chi \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}} \cdot \mathbf{p})), \alpha, 0)$ can do an action a, and become $(v\triangle\mathbf{p}, \alpha', 0)$, if

$$(\psi \mathbin{\hat{\blacktriangle}} (\chi \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}} \cdot \mathbf{p}), \alpha^{*}, 0) \xrightarrow{a} (\mathbf{p}, \alpha'^{*}, 0)$$

and $\alpha =_{v} \alpha^{*}$ and $\alpha' =_{v} \alpha'^{*}$.

From the definitions of signal transition (section 5.9) and signal emission (section 5.6), we have,

$$\begin{aligned}(\psi \mathbin{\hat{\blacktriangle}} (\chi \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}} \cdot \mathbf{p}), \alpha^{*}, 0) \xrightarrow{a} \;& (\mathbf{p}, \alpha'^{*}, 0) \;\Longrightarrow\\ & \alpha^{*} \models (\psi \wedge {}^{\circ}\chi) \wedge \alpha'^{*} \models (s_{\rho}(\mathbf{p}) \wedge \chi^{\circ})\end{aligned}$$

As $\alpha =_{v} \alpha^{*}$ and $\alpha' =_{v} \alpha'^{*}$, therefore

$$\alpha \models v\triangle(\psi \wedge {}^{\circ}\chi) \text{ and } \alpha' \models v\triangle(\chi^{\circ} \wedge s_{\rho}(\mathbf{p})).$$

$((v\triangle\psi) \mathbin{\hat{\blacktriangle}} ((v\triangle({}^{\bullet}\psi \wedge \chi \wedge s_{\rho}(\mathbf{p})^{\bullet})) \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}} \cdot (v\triangle\mathbf{p})), \alpha, 0)$ can do an action a and become $(v\triangle\mathbf{p}, \alpha', 0)$, only if

$$\alpha \models (v\triangle(\psi \wedge {}^{\circ}\chi)) \;\wedge\; \alpha' \models (v\triangle((\chi^{\circ}) \wedge s_{\rho}(\mathbf{p}))).$$

(c) None of the pair can wait.

(d) None of the pair is a final configuration.

Therefore $\mathbb{R}$ is a bisimulation relation and proves that

$$\begin{aligned}v\triangle(\psi \mathbin{\hat{\blacktriangle}} (\chi \mathbin{\ulcorner\!\blacktriangledown} \;\; & TS(\tilde{a}) \cdot TS)) \leftrightarrows\\ & (v\triangle\psi) \mathbin{\hat{\blacktriangle}} ((v\triangle({}^{\bullet}\psi \;\wedge\; \chi \;\wedge\; s_{\rho}(\mathbf{p})^{\bullet})) \mathbin{\ulcorner\!\blacktriangledown} TS(\tilde{a}) \cdot (v\triangle TS))\end{aligned}$$

9. **HSH3**

$v\triangle(\psi \mathbin{\hat{\blacktriangle}} (\chi \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}})) = (v\triangle\psi) \mathbin{\hat{\blacktriangle}} ((v\triangle({}^{\bullet}\psi \;\wedge\; \chi)) \mathbin{\ulcorner\!\blacktriangledown} \tilde{\tilde{a}})$;

Interpretation:

$$\begin{aligned}v\triangle(\psi \mathbin{\hat{\blacktriangle}} (\chi \mathbin{\ulcorner\!\blacktriangledown} \;\; & TS(\tilde{\tilde{a}})) =\\ & (v\triangle\psi) \mathbin{\hat{\blacktriangle}} ((v\triangle({}^{\bullet}\psi \;\wedge\; \chi)) \mathbin{\ulcorner\!\blacktriangledown} TS(\tilde{\tilde{a}})).\end{aligned}$$

Let $TS = \psi \mathbin{\hat{\blacktriangle}} (\chi \mathbin{\ulcorner\!\blacktriangledown} TS(\tilde{\tilde{a}}))$ and let TS be given by,

$$TS = (C, \rightarrow, \longmapsto, I, F),$$

where,

- $I \;=\; \{(\psi \frown (\chi \ulcorner \tilde{\tilde{a}}), \alpha, 0) \mid \alpha \in S \wedge \alpha \models \psi\}$;
- $F \;=\; \{(\surd, \alpha, 0) \mid \alpha \models \chi^\circ\}$;

  (We include the condition that a final configuration satisfies χ° to get rid of unreachable states and make $TS = \texttt{reach}(TS)$.)
- $C \;=\; I \cup F$;
- $\rightarrow \;=\; \{(i, a, f) \mid i \in I, a \in A, f \in F \bullet i \models {}^\circ\chi$
  $\wedge f \models \chi^\circ\}$;
- $\longmapsto \;=\; \emptyset$.

Applying signal hiding to TS with respect to variable v,

$$v\triangle(\psi \frown (\chi \ulcorner TS(\tilde{\tilde{a}}))) = (C', \rightarrow', \emptyset, I', F');$$

where,

- $I \;=\; \{((v\triangle(\psi \frown (\chi \ulcorner \tilde{\tilde{a}})), \alpha^*, 0) \mid \exists i \in I \bullet$
  $\texttt{state}(i) = \alpha \wedge \alpha =_v \alpha^*\}$;
- $F' \;=\; \{(\surd, \alpha^*, 0) \mid \exists f \in F \bullet \texttt{state}(f) = \alpha \wedge \alpha =_v \alpha^*\}$;
- $C' \;=\; I' \cup F'$;
- $\rightarrow' \;=\; \{(i', a, f') \mid i' \in I', f' \in F' \bullet \exists i \in I, \exists f \in F \bullet$
  $i =_v i' \wedge f =_v f' \wedge i \xrightarrow{a} f\}$.

Now we come to the right hand side of the axiom.

Let TS'' be $(v\triangle\psi) \frown (v\triangle({}^\bullet\psi \wedge \chi) \ulcorner TS(\tilde{\tilde{a}}))$. Let it be given by,

$$TS'' = (C'', \rightarrow'', \emptyset, I'', F'');$$

- $I'' \;=\; \{((v\triangle\psi) \frown (v\triangle({}^\bullet\psi \;\wedge\; \chi) \ulcorner \tilde{\tilde{a}}), \alpha, 0) \mid$
  $\alpha \models v\triangle\psi\}$;
- $F'' \;=\; \{(\surd, \alpha, 0)\}$;
- $C'' \;=\; I'' \cup F''$;
- $\rightarrow'' \;=\; \{(i'', a, f'') \mid i'' \in I'', a \in A, f'' \in F''$
  $i'' \models v\triangle{}^\circ\chi \wedge f'' \models v\triangle\chi^\circ\}$.

We define a binary relation $\mathbb{R}$ on the initial configurations of TS'' and $v\triangle TS$ as follows:

$$\begin{aligned}\mathbb{R} \;=\; &\{((((v\triangle\psi) \frown ((v\triangle({}^\bullet\psi \;\wedge\; \chi)) \ulcorner \tilde{\tilde{a}}), \alpha, 0),\\ &(v\triangle(\psi \frown (\chi \ulcorner \tilde{\tilde{a}})))), ((\surd, \alpha', 0), (\surd, \alpha', 0))\\ &\mid \alpha, \alpha' \in S, \alpha \models v\triangle\psi \;\wedge\; \alpha' \models \chi^\circ)\}.\end{aligned}$$

We prove that $\mathbb{R}$ is a bisimulation relation. Proving $(\surd, \alpha', 0) \leftrightarrows (\surd, \alpha', 0)$ is trivial. We prove that the pair $(((v\triangle\psi) \frown (v\triangle({}^\bullet\psi \wedge \chi)) \ulcorner \tilde{\tilde{a}}), \alpha, 0), (v\triangle(\psi \frown (\chi \ulcorner \tilde{\tilde{a}}), \alpha, 0)))$ satisfies the bisimulation conditions.

(a) The states and time labels are the same.

(b) $(v\triangle(\psi \,{}^{\wedge}\!\blacktriangle\, (\chi \,\ulcorner\!\blacktriangledown\, \tilde{\tilde{a}})), \alpha, 0)$ can do an action a, and terminate in a state α', if,

$$(\psi \,{}^{\wedge}\!\blacktriangle\, (\chi \,\ulcorner\!\blacktriangledown\, \tilde{\tilde{a}}), \alpha^*, 0) \xrightarrow{a} (\surd, \alpha'^*, 0)$$

and $\alpha =_v \alpha^*$ and $\alpha' =_v \alpha'^*$.

From the definitions of signal transition (section 5.9) and signal emission (section 5.6), we have,

$$(\psi \,{}^{\wedge}\!\blacktriangle\, (\chi \,\ulcorner\!\blacktriangledown\, \tilde{\tilde{a}}), \alpha^*, 0) \xrightarrow{a} (\surd, \alpha'^*, 0) \implies \alpha^* \models (\psi \wedge {}^{\circ}\chi) \wedge \alpha'^* \models \chi^{\circ}$$

As $\alpha =_v \alpha^*$ and $\alpha' =_v \alpha'^*$, therefore

$$\alpha \models v\triangle(\psi \wedge {}^{\circ}\chi) \text{ and } \alpha' \models v\triangle\chi^{\circ}.$$

$((v\triangle\psi) \,{}^{\wedge}\!\blacktriangle\, ((v\triangle({}^{\bullet}\psi \wedge \chi)) \,\ulcorner\!\blacktriangledown\, \tilde{\tilde{a}}), \alpha, 0)$ can do an action a and terminate in state α', if

$$\alpha \models (v\triangle(\psi \wedge {}^{\circ}\chi)) \,\wedge\, \alpha' \models v\triangle(\chi^{\circ}).$$

(c) None of the pair can wait.

(d) None of the pair is a final configuration.

Therefore $\mathbb{R}$ is a bisimulation relation and proves that

$$v\triangle(\psi \,{}^{\wedge}\!\blacktriangle\, (\chi \,\ulcorner\!\blacktriangledown\, TS(\tilde{\tilde{a}}))) \leftrightarrows (v\triangle\psi) \,{}^{\wedge}\!\blacktriangle\, ((v\triangle({}^{\bullet}\psi \wedge \chi)) \,\ulcorner\!\blacktriangledown\, TS(\tilde{\tilde{a}}))$$

10 Conclusion

In this paper, we have proposed a graph model for a basic process algebra for hybrid systems (BPA^{srt}_{hs}). We present hybrid process terms as transition systems and define operations on process terms as operations on their corresponding transition systems. In addition to the operators of BPA^{srt}_{hs}, we have also defined integration and signal hiding on transition systems. These transition systems provide a semantics which is detailed enough for correctly specifying the signal hiding operator in BPA^{srt}_{hs}.

Soundness proofs of some BPA^{srt}_{hs} axioms are also included in order to show that the graph model presented here is indeed a model of BPA^{srt}_{hs}.

This graph model can also be extended to include the parallel composition of hybrid process terms.

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11 Appendix

We denote the set of all closed BPA_{hs}^{srt} terms by $\mathcal{C}(P)$. Configurations in the setting of BPA_{hs}^{srt} are pairs of closed BPA_{hs}^{srt} terms and states. We denote $(\mathcal{C}(P) \times S)$ by C_{BPA}.

11.1 Axioms for BPA_{hs}^{srt}

Table 5: Axioms of $BPA_{\perp}^{srt}(a \in A_{\delta}, p, q \geq 0, r > 0)$

$x + y = y + x$	$A1$	$\sigma_{\mathsf{rel}}^{0}(x) = x$	$SRT1$
$(x + y) + z = x + (y + z)$	$A2$	$\sigma_{\mathsf{rel}}^{p}(\sigma_{\mathsf{rel}}^{q}(x)) = \sigma_{\mathsf{rel}}^{p+q}(x)$	$SRT2$
$x + x = x$	$A3$	$\sigma_{\mathsf{rel}}^{p}(x) + \sigma_{\mathsf{rel}}^{p}(y) = \sigma_{\mathsf{rel}}^{p}(x + y)$	$SRT3$
$(x + y) \cdot z = x \cdot z + y \cdot z$	$A4$	$(x \cdot y) \cdot z = x \cdot (y \cdot z)$	$A5$
$x + \tilde{\tilde{\delta}} = x$	$A6SR$	$\nu_{rel}(\tilde{\tilde{a}}) = \tilde{\tilde{a}}$	$SRU1$
$\tilde{\tilde{\delta}} \cdot x = \tilde{\tilde{\delta}}$	$A7SR$	$\nu_{rel}(\sigma_{\mathsf{rel}}^{r}(x)) = \tilde{\tilde{\delta}}$	$SRU2$
		$\nu_{rel}(x + y) = \nu_{rel}(x) + \nu_{rel}(y)$	$SRU3$
$x + \perp = \perp$	$NE1$	$\nu_{rel}(x \cdot y) = \nu_{rel}(x) \cdot y$	$SRU4$
$\perp \cdot x = \perp$	$NE2$		
$\tilde{\tilde{a}} \cdot \perp = \tilde{\tilde{\delta}}$	$NE3SR$	$\nu_{rel}(\perp) = \perp$	$NESRU$

Table 6: Axioms of BPA_{ps}^{srt}

$T :\rightarrow x = x$	$GC1$	$T \curlywedge x = x$	$SE1$
$F :\rightarrow x = \tilde{\tilde{\delta}}$	$GC2SR$	$F \curlywedge x = \perp$	$SE2$
$\psi :\rightarrow \tilde{\tilde{\delta}} = \tilde{\tilde{\delta}}$	$GC3SR$		
$\psi :\rightarrow (x + y) = \psi :\rightarrow x + \psi :\rightarrow y$	$GC4$	$\psi \curlywedge x + y = \psi \curlywedge (x + y)$	$SE3$
$\psi :\rightarrow x \cdot y = (\psi :\rightarrow x) \cdot y$	$GC5$	$(\psi \curlywedge x) \cdot y = \psi \curlywedge x \cdot y$	$SE4$
$\psi :\rightarrow (\psi' :\rightarrow x) = (\psi \wedge \psi') :\rightarrow x$	$GC6$	$\psi \curlywedge (\psi' \curlywedge x) = (\psi \wedge \psi') \curlywedge x$	$SE5$
$(\psi \vee \psi') :\rightarrow x = \psi :\rightarrow x + \psi' :\rightarrow x$	$GC7$		
		$\psi \curlywedge (\psi :\rightarrow x) = \psi \curlywedge x$	$SE6$
		$\psi :\rightarrow (\psi' \curlywedge x) = (\psi \rightarrow \psi') \curlywedge (\psi :\rightarrow x)$	$SE7$
$\nu_{rel}(\psi :\rightarrow x) = \psi :\rightarrow \nu_{rel}(x)$	$PSSRU1$	$\nu_{rel}(\psi \curlywedge x) = \psi \curlywedge \nu_{rel}(x)$	$PSSRU2$

Table 7: Additional axioms for $BPA^{srt}_{hs}(a \in A, r > 0, \mathbb{V}, \mathbb{V}' \subseteq V)$

$T \ulcorner\!\!\blacktriangledown_{\emptyset}\, x = x$	$HSE1$
$F \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, x = \bot$	$HSE2$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, \tilde{\tilde{\delta}} = \phi \,{}^{\wedge}\!\!\blacktriangle\, \tilde{\tilde{\delta}}$	$HSE3$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, \tilde{\tilde{a}} = \phi \,{}^{\wedge}\!\!\blacktriangle\, \tilde{\tilde{a}}$	$HSE4$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, \tilde{\tilde{a}} \cdot x = \phi \,{}^{\wedge}\!\!\blacktriangle\, \tilde{\tilde{a}} \cdot x$	$HSE5$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, \sigma^r_{\mathsf{rel}}(x) = \phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, (\phi \,{}^{\wedge}\!\!\blacktriangle\, \sigma^r_{\mathsf{rel}}(\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, x)$	$HSE6$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, (x + y) = \phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, x + \phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, y$	$HSE7$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, x \cdot y = (\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, x) \cdot y$	$HSE8$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, (\psi :\rightarrow x) = \phi \,{}^{\wedge}\!\!\blacktriangle\, (\psi :\rightarrow (\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, x))$	$HSE9$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, (\psi \,{}^{\wedge}\!\!\blacktriangle\, x) = \psi \,{}^{\wedge}\!\!\blacktriangle\, (\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, x)$	$HSE10$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, (\phi' \ulcorner\!\!\blacktriangledown_{\mathbb{V}'}\, x) = (\phi \wedge \phi') \ulcorner\!\!\blacktriangledown_{\mathbb{V} \cup \mathbb{V}'}\, x$	$HSE11$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, (\chi \ulcorner\!\blacktriangledown\, \tilde{\tilde{a}}) = \phi \,{}^{\wedge}\!\!\blacktriangle\, (\chi \ulcorner\!\blacktriangledown\, \tilde{\tilde{a}})$	$HSE12$
$\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, \sigma^r_{\mathsf{rel}}(x) + \phi' \ulcorner\!\!\blacktriangledown_{\mathbb{V}'}\, (\sigma^r_{\mathsf{rel}}(\nu_{rel}(y))) =$ $\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, (\sigma^r_{\mathsf{rel}}(x) + \phi' \ulcorner\!\!\blacktriangledown_{\mathbb{V}'}\, \sigma^r_{\mathsf{rel}}(\nu_{rel}(y)))$	$HSE13$
$T \ulcorner\!\blacktriangledown\, x = x$	$HST1$
$F \ulcorner\!\blacktriangledown\, x = \tilde{\tilde{\delta}}$	$HST2$
$\chi \ulcorner\!\blacktriangledown\, \tilde{\tilde{\delta}} = \tilde{\tilde{\delta}}$	$HST3$
$\chi \ulcorner\!\blacktriangledown\, \tilde{\tilde{a}} = \chi \ulcorner\!\blacktriangledown\, ({}^{\circ}\chi :\rightarrow \tilde{\tilde{a}})$	$HST4$
$\chi \ulcorner\!\blacktriangledown\, \tilde{\tilde{a}} \cdot x = \chi \ulcorner\!\blacktriangledown\, ({}^{\circ}\chi :\rightarrow \tilde{\tilde{a}} \cdot (\chi^{\circ} \,{}^{\wedge}\!\!\blacktriangle\, x))$	$HST5$
$\chi \ulcorner\!\blacktriangledown\, \sigma^r_{\mathsf{rel}}(x) = {}^{\circ}\chi :\rightarrow \sigma^r_{\mathsf{rel}}(x)$	$HST6$
$\chi \ulcorner\!\blacktriangledown\, (x + y) = \chi \ulcorner\!\blacktriangledown\, x + \chi \ulcorner\!\blacktriangledown\, y$	$HST7$
$\chi \ulcorner\!\blacktriangledown\, x \cdot y = (\chi \ulcorner\!\blacktriangledown\, x) \cdot y$	$HST8$
$\chi \ulcorner\!\blacktriangledown\, (\psi :\rightarrow x) = \psi :\rightarrow (\chi \ulcorner\!\blacktriangledown\, x)$	$HST9$
$\chi \ulcorner\!\blacktriangledown\, (\psi \,{}^{\wedge}\!\!\blacktriangle\, x) = ({}^{\circ}\chi \rightarrow \psi) \,{}^{\wedge}\!\!\blacktriangle\, (\chi \ulcorner\!\blacktriangledown\, x)$	$HST10$
$\chi \ulcorner\!\blacktriangledown\, (\chi' \ulcorner\!\blacktriangledown\, x) = (\chi \wedge \chi') \ulcorner\!\blacktriangledown\, x$	$HST11$
$\chi \ulcorner\!\blacktriangledown\, (\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, \sigma^r_{\mathsf{rel}}(x)) = {}^{\circ}\chi :\rightarrow (\phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, \sigma^r_{\mathsf{rel}}(x))$	$HST12$
$\psi :\rightarrow \tilde{\tilde{a}} = {}^{\bullet}\psi \ulcorner\!\blacktriangledown\, \tilde{\tilde{a}}$	$HST13$
$\tilde{\tilde{a}} \cdot (\psi \,{}^{\wedge}\!\!\blacktriangle\, x) = \psi^{\bullet} \ulcorner\!\blacktriangledown\, \tilde{\tilde{a}} \cdot x$	$HST14$
$\nu_{rel}(\phi \ulcorner\!\!\blacktriangledown_{V}\, x) = \phi \ulcorner\!\!\blacktriangledown_{\mathbb{V}}\, \nu_{rel}(x)$	$HSSRU1$
$\nu_{rel}(\chi \ulcorner\!\blacktriangledown\, x) = \chi \ulcorner\!\blacktriangledown\, \nu_{rel}(x)$	$HSSRU2$

11.2 Operational Semantics of BPA^{srt}_{hs}

We have for all closed terms t and t', for all $\alpha, \alpha' : V \cup \dot{V} \rightarrow \mathbb{R}$, $a \in A$, $r, s \in \mathbb{R}^{>}$ and $\rho \in \epsilon_r, \rho' \in \epsilon_{r+s}$ the following transition rules:

Table 8: Rules for operational semantics of BPA_{hs}^{srt} $(a \in A, r, s > 0)$

1 $$\frac{}{\langle \tilde{\tilde{a}}, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle}$$

2 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle}{\langle \sigma_{\mathsf{rel}}^{0}(x), \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle}$$

3 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle}{\langle \sigma_{\mathsf{rel}}^{0}(x), \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle}$$

4 $$\frac{\langle x, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x', \alpha'\rangle}{\langle \sigma_{\mathsf{rel}}^{0}(x), \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x', \alpha'\rangle}$$

5 $$\frac{}{\langle \sigma_{\mathsf{rel}}^{r+s}(x), \alpha\rangle \overset{r,\rho}{\longmapsto} \langle \sigma_{\mathsf{rel}}^{r}(x), \alpha'\rangle}$$

6 $$\frac{\alpha' \in [\mathsf{s}(x)]}{\langle \sigma_{\mathsf{rel}}^{r}(x), \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x, \alpha'\rangle}$$

7 $$\frac{\langle x, \alpha'\rangle \overset{s,\rho' \trianglerighteq r}{\longmapsto} \langle x', \alpha''\rangle}{\langle \sigma_{\mathsf{rel}}^{r}(x), \alpha\rangle \overset{r+s,\rho'}{\longmapsto} \langle x', \alpha''\rangle}$$

8 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle, \alpha \in [\mathsf{s}(y)]}{\langle x + y, \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle}$$

9 $$\frac{\langle y, \alpha\rangle \xrightarrow{a} \langle y', \alpha'\rangle, \alpha \in [\mathsf{s}(x)]}{\langle x + y, \alpha\rangle \xrightarrow{a} \langle y', \alpha'\rangle}$$

10 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle, \alpha \in [\mathsf{s}(y)]}{\langle x + y, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle}$$

11 $$\frac{\langle y, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle, \alpha \in [\mathsf{s}(x)]}{\langle x + y, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle}$$

12 $$\frac{\langle x, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x', \alpha'\rangle, \langle y, \alpha\rangle \overset{r}{\not\longmapsto}, \alpha \in [\mathsf{s}(y)]}{\langle x + y, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x', \alpha'\rangle}$$

13 $$\frac{\langle y, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle y', \alpha'\rangle, \alpha \in [\mathsf{s}(x)], \langle x, \alpha\rangle \overset{r}{\not\longmapsto}}{\langle x + y, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle y', \alpha'\rangle}$$

14 $$\frac{\langle x, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x', \alpha'\rangle, \langle y, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle y', \alpha'\rangle}{\langle x + y, \alpha\rangle, \overset{r,\rho}{\longmapsto} \langle x' + y', \alpha'\rangle}$$

15 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle}{\langle x \cdot y, \alpha\rangle \xrightarrow{a} \langle x' \cdot y, \alpha'\rangle}$$

16 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle, \alpha' \in [\mathsf{s}(y)]}{\langle x \cdot y, \alpha\rangle \xrightarrow{a} \langle y, \alpha'\rangle}$$

17 $$\frac{\langle x, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x', \alpha'\rangle}{\langle x \cdot y, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x' \cdot y, \alpha'\rangle}$$

18 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle}{\langle \psi :\rightarrow x, \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle} \quad \alpha \models \psi$$

19 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle}{\langle \psi :\rightarrow x, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle} \quad \alpha \models \psi$$

20 $$\frac{\langle x, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x', \alpha'\rangle}{\langle \psi :\rightarrow x, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x', \alpha'\rangle} \quad \alpha \models \psi$$

21 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle}{\langle \psi \mathbin{{}^{\wedge}\!\blacktriangle} x, \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle} \quad \alpha \models \psi$$

22 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle}{\langle \psi \mathbin{{}^{\wedge}\!\blacktriangle} x, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle} \quad \alpha \models \psi$$

23 $$\frac{\langle x, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x', \alpha'\rangle}{\langle \psi \mathbin{{}^{\wedge}\!\blacktriangle} x, \alpha\rangle \overset{r,\rho}{\longmapsto} \langle x', \alpha'\rangle} \quad \alpha \models \psi$$

24 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle}{\langle \phi \curvearrowright_V x, \alpha\rangle \xrightarrow{a} \langle x', \alpha'\rangle} \quad \alpha \models \phi$$

25 $$\frac{\langle x, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle}{\langle \phi \curvearrowright_V x, \alpha\rangle \xrightarrow{a} \langle \surd, \alpha'\rangle} \quad \alpha \models \phi$$

Table 8 Continued on next Page

$$26\ \frac{\langle x, \alpha \rangle \xmapsto{r,\rho} \langle x', \alpha' \rangle}{\langle \phi \mathrel{\ulcorner\!\!\blacktriangledown_V} x, \alpha \rangle \xmapsto{r,\rho} \langle \phi \mathrel{\ulcorner\!\!\blacktriangledown_V} x', \alpha' \rangle}\ \alpha \xmapsto{r,\rho} \alpha' \models_V \phi$$

$$27\ \frac{\langle x, \alpha \rangle \xrightarrow{a} \langle x', \alpha' \rangle}{\langle \chi \mathrel{\ulcorner\!\!\blacktriangledown} x, \alpha \rangle \xrightarrow{a} \langle x', \alpha' \rangle}\ \alpha \rightarrow \alpha' \models \chi \qquad 28\ \frac{\langle x, \alpha \rangle \xrightarrow{a} \langle \surd, \alpha' \rangle}{\langle \chi \mathrel{\ulcorner\!\!\blacktriangledown} x, \alpha \rangle \xrightarrow{a} \langle \surd, \alpha' \rangle}\ \alpha \rightarrow \alpha' \models \chi$$

$$29\ \frac{\langle x, \alpha \rangle \xmapsto{r,\rho} \langle x', \alpha' \rangle}{\langle \chi \mathrel{\ulcorner\!\!\blacktriangledown} x, \alpha \rangle \xmapsto{r,\rho} \langle x', \alpha' \rangle}\ \alpha \models {}^\circ\chi$$

$$30\ \frac{\langle x, \alpha \rangle \xrightarrow{a} \langle x', \alpha' \rangle}{\langle \nu_{rel}(x), \alpha \rangle \xrightarrow{a} \langle x', \alpha' \rangle} \qquad 31\ \frac{\langle x, \alpha \rangle \xrightarrow{a} \langle \surd, \alpha' \rangle}{\langle \nu_{rel}(x), \alpha \rangle \xrightarrow{a} \langle \surd, \alpha' \rangle}$$

Note that the following implications holds for the operational semantics given in [5]:

$$\langle t, \alpha \rangle \xrightarrow{a} \langle t', \alpha' \rangle \text{ or } \langle t, \alpha \rangle \xrightarrow{a} \langle \surd, \alpha' \rangle \text{ or } \langle t, \alpha \rangle \xmapsto{r,\rho} \langle t', \alpha' \rangle \text{ implies } \alpha \in [\mathsf{s}(t)]$$
$$\langle t, \alpha \rangle \xrightarrow{a} \langle t', \alpha' \rangle \text{ or } \langle t, \alpha \rangle \xmapsto{r,\rho} \langle t', \alpha' \rangle \text{ implies } \alpha' \in [\mathsf{s}(t')]$$

Table 9: Rules for $\alpha \in [\mathsf{s}(\_)]$ $(a \in A_\delta, r > 0)$

$$1\ \frac{}{\alpha \in [\mathsf{s}(\tilde{\tilde{a}})]} \qquad 2\ \frac{\alpha \in [\mathsf{s}(x)]}{\alpha \in [\mathsf{s}(\sigma^0_{\mathsf{rel}}(x))]} \qquad 3\ \frac{}{\alpha \in [\mathsf{s}(\sigma^r_{\mathsf{rel}}(x))]} \qquad 4\ \frac{\alpha \in [\mathsf{s}(x)], \alpha \in [\mathsf{s}(y)]}{\alpha \in [\mathsf{s}(x + y)]} \qquad 5\ \frac{\alpha \in [\mathsf{s}(x)]}{\alpha \in [\mathsf{s}(x \cdot y)]}$$

$$6\ \frac{\alpha \in [\mathsf{s}(x)]}{\alpha \in [\mathsf{s}(\psi :\rightarrow x)]} \qquad 7\ \frac{}{\alpha \in [\mathsf{s}(\psi :\rightarrow x)]}\ \alpha \not\models \psi \qquad 8\ \frac{\alpha \in [\mathsf{s}(x)]}{\alpha \in [\mathsf{s}(\psi \mathrel{\wedge\!\!\blacktriangle} x)]}\ \alpha \models \psi$$

$$9\ \frac{\alpha \in [\mathsf{s}(x)]}{\alpha \in [\mathsf{s}(\phi \mathrel{\ulcorner\!\!\blacktriangledown_V} x)]}\ \alpha \models \phi \qquad 10\ \frac{\alpha \in [\mathsf{s}(x)]}{\alpha \in [\mathsf{s}(\chi \mathrel{\ulcorner\!\!\blacktriangledown} x)]} \qquad 11\ \frac{}{\alpha \in [\mathsf{s}(\chi \mathrel{\ulcorner\!\!\blacktriangledown} x)]}\ \alpha \not\models {}^\circ\chi \qquad 12\ \frac{\alpha \in [\mathsf{s}(x)]}{\alpha \in [\mathsf{s}(\nu_{rel}(x))]}$$

Table 10: Additional Rules for Integration ($a \in A, p, q, \geq 0, r > 0$)

$$1\ \frac{\langle F(p),\alpha\rangle \xrightarrow{a} \langle x',\alpha'\rangle, \{\alpha \in [\mathsf{s}(F(q))] \mid q \in U\}}{\langle \int_{u\in U} F(u),\alpha\rangle \xrightarrow{a} \langle x',\alpha'\rangle}\ p \in U$$

$$2\ \frac{\langle F(p),\alpha\rangle \xrightarrow{a} \langle \surd,\alpha'\rangle, \{\alpha \in [\mathsf{s}(F(q))] \mid q \in U\}}{\langle \int_{u\in U} F(u),\alpha\rangle \xrightarrow{a} \langle \surd,\alpha'\rangle}\ p \in U$$

$$3\ \frac{\begin{array}{l}\{\langle F(q),\alpha\rangle \xmapsto{r,\rho} \langle F_1(q),\alpha'\rangle \mid q \in U_1\},\\ \ldots \\ \{\langle F(q),\alpha\rangle \xmapsto{r,\rho} \langle F_n(q),\alpha'\rangle \mid q \in U_n\},\\ \{\langle F(q),\alpha\rangle \not\xmapsto{r}, \alpha \in [\mathsf{s}(F(q))] \mid q \in U_{n+1}\}\end{array}}{\langle \int_{u\in U} F(u),\alpha\rangle \xmapsto{r,\rho} \langle \int_{u\in U_1} F_1(u) + \ldots + \int_{u\in U_n} F_n(u),\alpha'\rangle}\ \begin{array}{l}\{U_1,\ldots U_n\}\\ \text{partition of } U\backslash U_{n+1}, U_{n+1} \subset U\end{array}$$

$$4\ \frac{\{\alpha \in [\mathsf{s}(F(q))] \mid q \in U\}}{\alpha \in [\mathsf{s}(\int_{u\in U} F(u))]}$$

11.3 Bisimulation

A bisimulation is a symmetric binary relation $B \subseteq C_{BPA} \times C_{BPA}$ on configurations with same states, such that if $B(\langle t_1,\alpha\rangle,\langle t_2,\alpha\rangle)$ then,

- for all $a \in A$ and $\langle t_1',\alpha'\rangle \in C_{BPA}$, if $\langle t_1,\alpha\rangle \xrightarrow{a} \langle t_1',\alpha'\rangle$, then $\exists\, \langle t_2',\alpha'\rangle \in C_{BPA}$, such that $\langle t_2,\alpha\rangle \xrightarrow{a} \langle t_2',\alpha'\rangle$ and $B(\langle t_1',\alpha'\rangle,\langle t_2',\alpha'\rangle)$;
- for all $a \in A$ and $\alpha' \in S$, if $\langle t_1,\alpha\rangle \xrightarrow{a} \langle \surd,\alpha'\rangle$, then $\langle t_2,\alpha\rangle \xrightarrow{a} \langle \surd,\alpha'\rangle$;
- for all $(r,\rho) \in D$ and $\langle t_1',\alpha'\rangle \in C_{BPA}$, if $\langle t_1,\alpha\rangle \xmapsto{r,\rho} \langle t_1',\alpha'\rangle$, then $\exists\, \langle t_2',\alpha'\rangle \in C_{BPA}$, such that $\langle t_2,\alpha\rangle \xmapsto{r,\rho} \langle t_2',\alpha'\rangle$ and $B(\langle t_1',\alpha'\rangle,\langle t_2',\alpha'\rangle)$;
- if $\alpha \in [\mathsf{s}(t_1)]$ then $\alpha \in [\mathsf{s}(t_2)]$.

Two configurations $\langle t_1,\alpha\rangle$ and $\langle t_2,\alpha\rangle$ with same states are *bisimulation equivalent* or *bisimilar* written as $\langle t_1,\alpha\rangle \leftrightarrows \langle t_2,\alpha\rangle$, if there exists a bisimulation relation B such that $B(\langle t_1,\alpha\rangle\langle t_2,\alpha\rangle)$.

Two closed terms t_1 and t_2 are called *bisimulation equivalent terms*, written as $t_1 \leftrightarrows t_2$, if $\langle t_1,\alpha\rangle \leftrightarrows \langle t_2,\alpha\rangle$ for all states α.

11.4 Ic-bisimulation

Interference compatible bisimulation or ic-bisimulation relates two BPA_{hs}^{srt} terms when their behaviour is same in all states and this property is maintained by all pairs of subsequent terms obtained through similar action or time steps. Formally, an ic-bisimulation is a symmetric binary relation $B \subseteq \mathcal{C}(P) \times \mathcal{C}(P)$, such that if $B(t_1, t_2)$ then for all states α,

- for all actions $a \in A$ and $\langle t_1', \alpha' \rangle \in C_{BPA}$, if $\langle t_1, \alpha \rangle \xrightarrow{a} \langle t_1', \alpha' \rangle$ then $\exists \langle t_2', \alpha' \rangle \in C_{BPA}$, such that $\langle t_2, \alpha \rangle \xrightarrow{a} \langle t_2', \alpha' \rangle$ and $B(t_1', t_2')$;
- for all actions $a \in A$ and $\alpha' \in S$, if $\langle t_1, \alpha \rangle \xrightarrow{a} \langle \surd, \alpha' \rangle$ then $\langle t_2, \alpha \rangle \xrightarrow{a} \langle \surd, \alpha' \rangle$;
- for all $(r, \rho) \in D$ and $\langle t_1', \alpha' \rangle \in C_{BPA}$, if $\langle t_1, \alpha \rangle \xmapsto{r,\rho} \langle t_1', \alpha' \rangle$ then $\exists \langle t_2', \alpha' \rangle \in C_{BPA}$, such that $\langle t_2, \alpha \rangle \xmapsto{r,\rho} \langle t_2', \alpha' \rangle$ and $B(t_1', t_2')$;
- if $\alpha \in [\mathsf{s}(t_1)]$ then $\alpha \in [\mathsf{s}(t_2)]$.

Two closed terms t_1 and t_2 are called *ic-bisimilar*, written as $t_1 \leftrightarrows t_2$, if there exists a ic-bisimulation

11.5 Proof: $\langle Th, 18 \rangle$ is bisimilar to $\langle Th', 18 \rangle$

$$\begin{array}{ll}
Th & = (T = 18) \mathrel{{}^{\wedge}\!\!\blacktriangle} Th^{on}, \\
Th^{on} & = up \mathrel{\ulcorner\!\!\blacktriangledown}_T Th_0^{on}, \\
Th_t^{on} & = \int_{u \in [t, \infty)} \sigma_{\mathsf{rel}}^{u-t}(Th^{\rightarrow}), \\
Th^{\rightarrow} & = (T = 20) :\rightarrow \\
 & (T^{\bullet} = {}^{\bullet}T) \mathrel{\ulcorner\!\!\blacktriangledown} \widetilde{\widetilde{\widetilde{\mathtt{toff}}}} \cdot Th^{off}, \\
Th^{off} & = down \mathrel{\ulcorner\!\!\blacktriangledown}_T Th_0^{off}, \\
Th_{t'}^{off} & = \int_{u \in [t', \infty)} \sigma_{\mathsf{rel}}^{u-t'}(Th^{\leftarrow}), \\
Th^{\leftarrow} & = (T = 18) :\rightarrow \\
 & (T^{\bullet} = {}^{\bullet}T) \mathrel{\ulcorner\!\!\blacktriangledown} \widetilde{\widetilde{\widetilde{\mathtt{ton}}}} \cdot Th^{on}.
\end{array}$$

$$\begin{array}{ll}
Th' & = (T = 18) \mathrel{{}^{\wedge}\!\!\blacktriangle} Th'^{on}, \\
Th'^{on} & = up \mathrel{\ulcorner\!\!\blacktriangledown}_T Th_0'^{on}, \\
Th_t'^{on} & = \sigma_{\mathsf{rel}}^{ln2-t}(Th'^{\rightarrow}), \\
Th'^{\rightarrow} & = (T^{\bullet} = {}^{\bullet}T) \mathrel{\ulcorner\!\!\blacktriangledown} \widetilde{\widetilde{\widetilde{\mathtt{toff}}}} \cdot Th'^{off}, \\
Th'^{off} & = down \mathrel{\ulcorner\!\!\blacktriangledown}_T Th_0'^{off}, \\
Th_{t'}'^{off} & = \sigma_{\mathsf{rel}}^{ln3-t'}(Th'^{\leftarrow}), \\
Th'^{\leftarrow} & = (T^{\bullet} = {}^{\bullet}T) \mathrel{\ulcorner\!\!\blacktriangledown} \widetilde{\widetilde{\widetilde{\mathtt{ton}}}} \cdot Th'^{on}
\end{array}$$

where $t \in [0, ln2]$ and $t' \in [0, ln3]$, *up* denotes proposition $(18 \leq T \leq 20 \wedge \dot{T} = -T + 22)$ and *down* denotes proposition $(18 \leq T \leq 20 \wedge \dot{T} = -T + 17)$.

Let $\rho \in \epsilon_{ln4/3}$ be a state evolution that satisfies *up* in interval [0,ln4/3] keeping the variables T and $\dot{T}$ continuously differentiable. Let $\rho(0)(T) = 18$ and $\rho(ln(4/3))(T) = 19$.Then we can write,

$$18 \xmapsto{ln4/3,\rho} 19 \models_T up.$$

The above statement means, "the state with temperature 18 evolves into the state with temperature 19, evolving according to ρ in time $ln(4/3)$, where ρ satisfies *up* and T & $\dot{T}$ remain infinitely often continuously differentiable during the whole delay."

Let ρ' be another state evolution of duration $ln3/2$, such that $\rho' \models up$ and ρ' is smooth for T and $\rho'(0)(T) = 19$ and $\rho'(ln(3/2))(T) = 20$, i.e.,

$$19 \xmapsto{ln3/2,\rho'} 20 \models_T up.$$

Applying ρ and then ρ', i.e., by sequentially composing ρ and ρ' (see section 5.12), we get,

$$18 \xmapsto{ln2, \rho \cdot \rho'} 20 \models_T up.$$

We consider only T in our states (and not $\dot{T}$), for the sake of simplicity. Temperature at any time r, for processes Th^{on} and Th'^{on} is given by T_r^{on} , where $T_r^{on} = (22e^r - 4)/e^r$. Temperature at any time r for processes Th^{off} and Th'^{off} is given by T_r^{off}, where $T_r^{off} = (17e^r + 3)/e^r$.

We give a binary relation $\mathbb{R}$ on configurations of the process Th and process Th'. The configurations are the ones as defined in the operational semantics for BPA_{hs}^{srt} (see table 8).

$$\begin{aligned} \mathbb{R} \quad = \quad & \{(\langle Th, 18\rangle, \langle Th', 18\rangle), \\ & (\langle up \curvearrowright_T Th_r^{on}, T_r^{on}\rangle, \langle up \curvearrowright_T Th_r'^{on}, T_r^{on}\rangle), \\ & (\langle down \curvearrowright_T Th_{r'}^{off}, T_{r'}^{off}\rangle, \langle down \curvearrowright_T Th_{r'}'^{off}, T_{r'}^{off}\rangle) \\ & \mid r \in [0, ln2], r' \in [0, ln3]\}. \end{aligned}$$

We prove that $\mathbb{R}$ is a bisimulation relation. The definition of bisimulation for BPA_{hs}^{srt} terms is given in Section 11.3. Here we give the proof of bisimulation of $(\langle up \curvearrowright_T Th_r^{on}, T_r^{on}\rangle, \langle up \curvearrowright_T Th_r'^{on}, T_r^{on}\rangle)$. The proof that $(\langle down \curvearrowright_T Th_{r'}^{off}, T_{r'}^{off}\rangle, \langle down \curvearrowright_T Th_{r'}'^{off}, T_{r'}^{off}\rangle)$, for all $r' \in [0, ln3]$, fulfills the bisimulation conditions is left to the reader.

All pairs in $\mathbb{R}$ have same states which is the first condition for bisimulation. (We refer to the operational semantic rules given in tables 8,9 and 10.)

$\langle Th, 18\rangle$ cannot perform action $\widetilde{\widetilde{toff}}$ because of conditional $T = 20$, and $\langle Th', 18\rangle$ cannot perform action $\widetilde{\widetilde{toff}}$ because of a delay of $ln2$ time units.

$$\langle Th, 18\rangle \not\xrightarrow{toff}$$
$$\langle Th', 18\rangle \not\xrightarrow{toff}$$

They can only delay as follows:

$19 \in [\mathsf{s}(Th^{\rightarrow})]$ From Rule 7 Table 9

$\langle \sigma_{\mathsf{rel}}^{ln4/3}(Th^{\rightarrow}), 18\rangle \xmapsto{ln4/3,\rho} \langle Th^{\rightarrow}, 19\rangle$ From Rule 6 Table 8

$\langle \int_{u\in[0,\infty)} \sigma_{\mathsf{rel}}^{u-0} Th^{\rightarrow}, 18\rangle \xmapsto{ln4/3,\rho}$

$\langle \int_{u\in[ln4/3,\infty)} \sigma_{\mathsf{rel}}^{u-ln4/3} Th^{\rightarrow}, 19\rangle$ From Rule 3 Table 10

$\langle Th_0^{on}, 18\rangle \xmapsto{ln4/3,\rho} \langle Th_{ln4/3}^{on}, 19\rangle$

$\langle up \curvearrowright_T Th_0^{on}, 18\rangle \xmapsto{ln4/3,\rho} \langle up \curvearrowright_T Th_{ln4/3}^{on}, 19\rangle$ From Rule 26 Table 8

$^1\langle Th, 18\rangle \xmapsto{ln4/3,\rho} \langle up \curvearrowright_T Th_{ln4/3}^{on}, 19\rangle$ From Rule 23 Table 8

Similarly for $< Th', 18 >$,

$\langle \sigma_{\mathsf{rel}}^{ln2}(Th'^{\rightarrow}), 18\rangle \xmapsto{ln4/3,\rho} \langle \sigma_{\mathsf{rel}}^{ln2-ln4/3} Th'^{\rightarrow}, 19\rangle$ From Rule 5 Table 8

$\langle Th_0'^{on}, 18\rangle \xmapsto{ln4/3,\rho} \langle Th_{ln4/3}'^{on}, 19\rangle$

$\langle up \curvearrowright_T (Th_0'^{on}), 18\rangle \xmapsto{ln4/3,\rho} \langle up \curvearrowright_T Th_{ln4/3}'^{on}, 19\rangle$ From Rule 26 Table 8

$^2\langle Th', 18\rangle \xmapsto{ln4/3,\rho} \langle up \curvearrowright_T Th_{ln4/3}'^{on}, 19\rangle$ From Rule 23 Table 8

Similarly following two transitions can be derived:

$$^3\langle up \curvearrowright_T Th_{ln4/3}^{on}, 19\rangle \xmapsto{ln3/2,\rho'} \langle up \curvearrowright_T Th_{ln2}^{on}, 20\rangle$$
$$^4\langle up \curvearrowright_T Th_{ln4/3}'^{on}, 19\rangle \xmapsto{ln3/2,\rho'} \langle up \curvearrowright_T Th_{ln2}'^{on}, 20\rangle$$

$\langle up \curvearrowright_T Th_{ln4/3}^{on}, 19\rangle$ cannot perform action $\widetilde{\widetilde{\widetilde{toff}}}$ because of conditional $T = 20$, and $\langle up \curvearrowright_T Th_{ln4/3}'^{on}, 19\rangle$ cannot perform action $\widetilde{\widetilde{\widetilde{toff}}}$ because a delay of $ln(3/2)$ time units is still left in the process term. All configurations $\langle up \curvearrowright_T Th_r^{on}, T_r^{on}\rangle$ and $\langle up \curvearrowright_T Th_r'^{on}, T_r^{on}\rangle$, with r less than $ln2$ cannot perform action $\widetilde{\widetilde{\widetilde{toff}}}$ because of the same reasons.

Using Rules {7,26} of Tab 8 and derivations $1, 2, 3, 4$ above, we can derive,

$$\langle Th, 18\rangle \xmapsto{ln2,\rho\cdot\rho'} \langle up \curvearrowright_T Th_{ln2}^{on}, 20\rangle$$
$$\langle Th', 18\rangle \xmapsto{ln2,\rho\cdot\rho'} \langle up \curvearrowright_T Th_{ln2}'^{on}, 20\rangle$$

Now we compare the behaviour of $\langle up \curvearrowright_T Th_{ln2}^{on}, 20\rangle$ and $\langle up \curvearrowright_T Th_{ln2}'^{on}, 20\rangle$.

$\langle up \curvearrowright_T Th_{ln2}^{on}, 20\rangle$ cannot wait further as proposition up will not be satisfied in a delay (keeping the temperature continuous). The delay duration of $\langle up \curvearrowright_T Th_{ln2}'^{on}, 20\rangle$ is zero, so it also cannot wait further. They can both perform action $\widetilde{\widetilde{\widetilde{toff}}}$ as described below:

$\langle Th^{\rightharpoonup}, 20\rangle \xrightarrow{toff} \langle Th^{off}, 20\rangle$ From Rules {1,16,27,18} of Table 8

$\langle Th^{\rightharpoonup} + \int_{u\in(ln2,\infty)} \sigma_{\mathsf{rel}}^{u-ln2} Th^{\rightharpoonup}, 20\rangle$
$\xrightarrow{toff} \langle Th^{off}, 20\rangle$ From Rule 1 of Tab 10, Rule 8 of Tab 8

$\langle \int_{u\in[ln2,\infty)} \sigma_{\mathsf{rel}}^{u-ln2} Th^{\rightharpoonup}, 20\rangle \xrightarrow{toff} \langle Th^{off}, 20\rangle$ Rule 3 of Tab 8

$\langle Th^{on}_{ln2}, 20\rangle \xrightarrow{toff} \langle Th^{off}, 20\rangle$

$\langle up \curvearrowright_T Th^{on}_{ln2}, 20\rangle \xrightarrow{toff} \langle Th^{off}, 20\rangle$ From Rule 24 Table 8

Similarly

$\langle Th'^{\rightharpoonup}, 20\rangle \xrightarrow{toff} \langle Th'^{off}, 20\rangle$ From Rule {1,27} Table 8

$\langle \sigma_{\mathsf{rel}}^{ln2-ln2} Th'^{\rightharpoonup}, 20\rangle \xrightarrow{toff} \langle Th'^{off}, 20\rangle$ From Rule 2 Table 8

$\langle Th'^{on}_{ln2}, 20\rangle \xrightarrow{toff} \langle Th'^{off}, 20\rangle$

$\langle up \curvearrowright_T Th'^{on}_{ln2}, 20\rangle \xrightarrow{toff} \langle Th'^{off}, 20\rangle$ From Rule 24 of Tab 8

And $(\langle Th^{off}, 20\rangle, \langle Th'^{off}, 20\rangle)$ are in $\mathbb{R}$.

Thus relation $\mathbb{R}$ is a bisimulation relation.

11.6 Proof: $\langle T \triangle Th, *\rangle$ is not bisimilar to $\langle T \triangle Th', *\rangle$

where, $*$ indicates a state with temperature hidden, i.e.,

$$* =_T 18 =_T 19 =_T 20 \text{ etc}$$

We prove that $\langle T \triangle Th, *\rangle$ behaves as $\langle Th''', *\rangle$, where,

$$Th''' = \int_{u\in(0,\infty)} \sigma_{\mathsf{rel}}^{u}(\widetilde{\widetilde{\widetilde{toff}}}) \cdot \sigma_{\mathsf{rel}}^{*}(\widetilde{\widetilde{\widetilde{ton}}}) \cdot Th'''';$$
$$Th'''' = \sigma_{\mathsf{rel}}^{*}(\widetilde{\widetilde{\widetilde{toff}}}) \cdot \sigma_{\mathsf{rel}}^{*}(\widetilde{\widetilde{\widetilde{ton}}}) \cdot Th''''.$$

i.e, initially $\langle T \triangle Th, *\rangle$ can actually perform action $\widetilde{\widetilde{\widetilde{toff}}}$ at any time $t > 0$. Once it has performed a $\widetilde{\widetilde{\widetilde{toff}}}$ and a $\widetilde{\widetilde{\widetilde{ton}}}$ action then it behaves as Th'''.

In this proof we frequently refer to the derivations obtained in the previous section and the operational semantic rules given in section 2. The definitions of ρ and ρ' are the same as in the previous proof. We define ρ^* and ρ'^* to be two state evolutions that differ from ρ and ρ' only in their evolutions of T and $\dot{T}$ respectively. Let ρ_t be a state evolution in ϵ_t that satisfies up. Let $\rho^*_t =_T \rho_t$.

The behaviour of $\langle T \triangle Th, *\rangle$ depends on the behaviour of Th in any state. Th can only exist in state 18 (as its signal is only true at temperature 18). After delaying for some $t > 0$ time units, it becomes Th^{on}_t whose signal is true in the range $[18, 20]$ of temperature. Thus $< T\triangle Th_t^{on}, * >$ can behave as $< Th_t^{on}, \alpha >$, where α is a state with $T \in [18, 20]$.

$$\langle Th, 18\rangle \xmapsto{t,\rho_t} \langle up \curvearrowright_T Th_t^{on}, \alpha_t\rangle$$
$$\rightarrow \langle T\triangle Th, *\rangle \xmapsto{t,\rho_t^*} \langle T\triangle(up \curvearrowright_T Th_t^{on}), *\rangle \quad \text{Rule 3 of table 1}$$
where $t \in (0, ln2]$ and $\alpha_t = (22e^t - 4)/t$.

Now the behaviour of $\langle T\triangle(up \curvearrowright_T Th_t^{on}), *\rangle$, depends on the behaviour of $up \curvearrowright_T Th_t^{on}$ in any state.

$$\langle up \curvearrowright_T Th_t^{on}, 20\rangle \xrightarrow{\widetilde{\widetilde{toff}}} \langle Th^{off}, 20\rangle \quad \text{Refer to previous proof}$$
$$\langle T\triangle(up \curvearrowright_T Th_t^{on}), *\rangle \xrightarrow{\widetilde{\widetilde{toff}}} \langle T\triangle(Th^{off}), *\rangle \quad \text{Rule 1 table 1}$$

$T\triangle(Th_t^{on})$ can also delay indefinitely for time greater than $ln2$ as is shown in the following derivations. Consider the process Th_{ln2}^{on}(that has already delayed for $ln2$ time units) in state 18.

$$\langle up \curvearrowright_T Th_{ln2}^{on}, 18\rangle \xmapsto{ln4/3,\rho} \langle up \curvearrowright_T Th_{ln8/3}^{on}, 19\rangle \quad \text{From Rule \{27,6\} table 8}$$
$$\langle T\triangle(up \curvearrowright_T Th_{ln2}^{on}), *\rangle \xmapsto{ln4/3,\rho^*} \langle T\triangle(up \curvearrowright_T Th_{ln8/3}^{on}), *\rangle \quad \text{From Rule 3 table 1}$$

Waiting of $\langle T\triangle(up \curvearrowright_T Th_{ln8/3}^{on}), *\rangle$ can be repeated by considering the behaviour of $up \curvearrowright_T Th_{ln8/3}^{on}$ again in state 18. Once Th has performed action $\widetilde{\widetilde{\widetilde{toff}}}$ and $\widetilde{\widetilde{ton}}$, then in the recursive call Th^{on} is invoked instead of Th. And $T\triangle(Th^{on})$ can perform action $\widetilde{\widetilde{\widetilde{toff}}}$ at any time.

Whereas $\langle T\triangle(Th'), 18\rangle$ behaves as $\langle Th'', 18\rangle$, where

$$Th'' = \sigma_{\mathsf{rel}}^{ln2}(\widetilde{\widetilde{\widetilde{toff}}}) \cdot \sigma_{\mathsf{rel}}^{ln3}(\widetilde{\widetilde{ton}}).$$

(The proof of this is left to the reader.)

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