

# A Generalization of $F$ -regular Semigroups

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## Abstract

A regular semigroup  $S$  is termed *locally  $F$ -regular*, if each class of the least completely simple congruence  $\xi$  contains a greatest element with respect to the natural partial order. It is shown that each locally  $F$ -regular semigroup  $S$  admits an embedding into a semidirect product of a band by  $S/\xi$ . Further, if  $\xi$  satisfies the additional property that for each  $s \in S$  and each inverse  $(s\xi)'$  of  $s\xi$  in  $S/\xi$  the set  $(s\xi)' \cap V(s)$  is not empty, we represent  $S$  both as a Rees matrix semigroup over an  $F$ -regular semigroup as well as a certain subsemigroup of a restricted semidirect product of a band by  $S/\xi$ .

The main result contains our recent representation theorem for  $F$ -regular semigroups [1], whence McAlister's characterization of  $F$ -inverse semigroups [2].

Finally, we establish that an orthodox semigroup  $S$  is a homomorphic image of an  $F$ -regular semigroup, if and only if it contains an inverse subsemigroup  $S^0$  such that (i) for each  $s \in S$  there is  $s^0 \in S^0$  with  $ss^0s = s$ , (ii) the idempotents of  $S^0$  commute with the idempotents of  $S$ . In particular, we recapture a result due to McFadden, which states that each unit-regular orthodox semigroup is an idempotent separating homomorphic image of a semidirect product of a band with identity by a group [3].

## References

- [1] B. Billhardt, A representation for  $F$ -regular semigroups, *Semigroup Forum* **70** (2005), 243–251.
- [2] D.B. McAlister,  $E$ -unitary inverse semigroups over semilattices, *Glasgow Math. J.* **19** (1978), 1–12.
- [3] R. McFadden, Unit-regular orthodox semigroups, *Glasgow Math. J.* **25** (1984), 229–240.