Beam Profiling with Optical Transition Radiation

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Abstract

One of the simplest applications of optical transition radiation (OTR) to accelerator beam diagnostics is beam profiling. We compare the limits of resolution of beam profiles made using OTR and profiles made using synchrotron radiation. We will discuss the physical basis for the limiting resolution in each case and show that the case of OTR yields essentially the same result as standard diffraction theory.

I. INTRODUCTION

During the last several years, we have developed a number of techniques for measuring the emittance of relativistic electron beams, in collaboration with colleagues at several accelerator facilities [1-4]. Our emittance measurement techniques, based on OTR, have been performed at energies up to about 110 MeV, to date. The emittance measurement involves the simultaneous observation of the OTR radiation pattern in the focal plane of a lens and the image of the beam profile at a beam waist [4]. The OTR radiation pattern from a single foil, or a two foil Wartski OTR interferometer [5], is used to determine the beam divergence. The polarization of the radiation patterns gives information on the horizontal and vertical emittances

Gradually, OTR based beam diagnostics are beginning to be used more and more at accelerator facilities around the world; however we have become aware that some members of the beam diagnostics community mistakenly believe that OTR techniques are limited to relatively moderate energies because of a supposed self-diffraction effect [7]. The purpose of this brief paper is to discuss the physical basis for the limiting resolution of beam profiles using OTR . We will demonstrate that OTR can, in principal, be used for ultra relativistic beam diagnostics. OTR resolution will also be contrasted with the optics of imaging beams with synchrotron radiation (SR), which is well known. Some of the misunderstandings about the optics of OTR imaging comes from inappropriate analogies to SR 's special optical properties.

II. OPTICAL PROPERTIES OF OTR

A. *Angular Distribution*

Two common misunderstandings regarding OTR's optical properties are: 1) that it is inherently "self-diffracting" because it is confined to angles of the order of $1/\gamma$, and 2) that it is "formed" over a length $L \sim \gamma^2\lambda$. We will show why these concepts are wrong in this section.

In Reference [6], we used the model of a charge entering a perfect conductor to illustrate the properties of OTR. This model is excellent for optical wavelengths and the angular distribution derived also applies to x-ray wavelengths.

Figure 1 illustrates the results of deriving the transition radiation properties using the method of image charges.

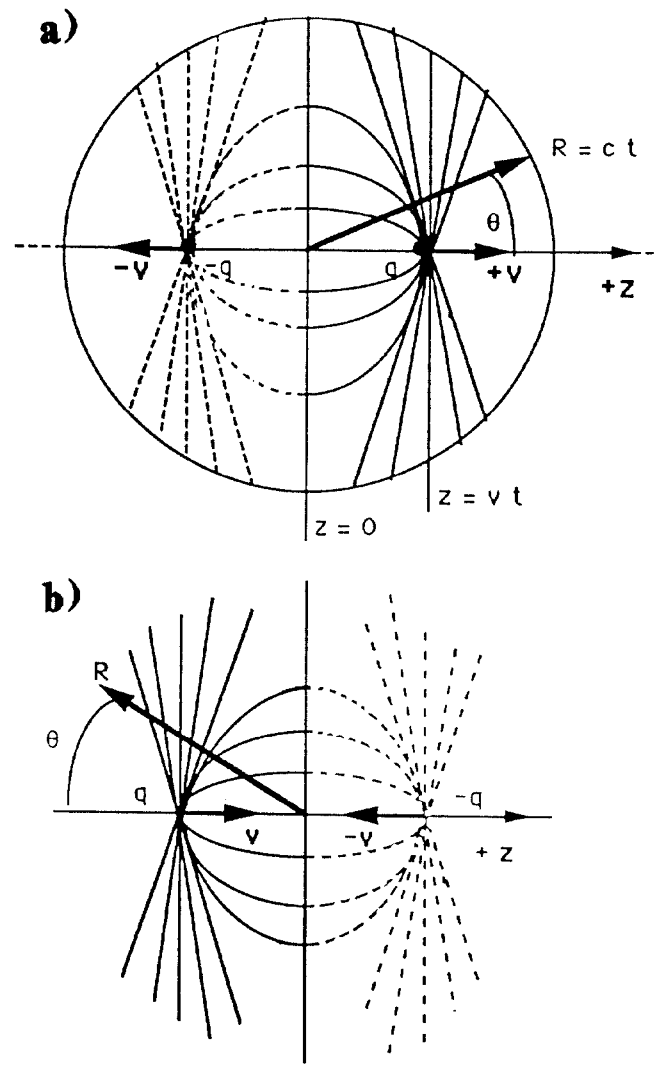

Figure 1. Coulomb and radiation fields generated by a charge q and its image.-q : a) charge emerging from conductor, b) charge entering conductor. Radiation fields exist only on sphere of radius R=ct, when t>0.

Figure 1a shows the Lorentz contracted Coulomb fields of a relativistic charge emerging from a conductor as a bundle of field lines centered on q. The radiation field is on the sphere of radius R=ct and the Coulomb fields are nonzero only inside this sphere. Figure 1b shows the situation for a charge q entering a conductor. In this case backward OTR appears on

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the sphere of radius R=ct and the Coulomb fields disappear at time t=0. The radiation is in phase everywhere on this sphere, however the field strengths are a function of θ, given by:

$$E_r = 0$$

$$E_\theta = B_\phi = \frac{2\beta q \sin\theta}{R\,(1-\beta^2\cos^2\theta)}\ , \qquad (1)$$

where β=v/c. The peak fields occur at $\theta=\sin^{-1}(\beta\gamma)$, however, since the radiation is a spherical wave centered at R=0, there is no uncertainty in its position of origin. Note that it takes a finite time for the Coulomb fields to propagate along the surface of the conductor inside the radius R.

It has been suggested that, since the Fourier component of wavelength λ of the Coulomb fields of a relativistic particle in vacuum extend out a distance ~γ λ perpendicular to the velocity vector, there would be an uncertainty of this amount in the position of origin of the photon produced at a boundary. Further, it is incorrectly suggested that this will limit the resolution of OTR images to γ λ. The above discussion of Figure 1 shows that the presence of a boundary modifies the Coulomb fields in such away as to confine them inside the sphere whose boundary contains the radiation fields.

Another misunderstanding arises in connection with the concept of "formation length". The terminology is misleading because it refers to the distance along the z-direction in Figure 1 over which the radiation fields on the hemisphere at R=ct remain in phase with the Coulomb fields centered on the charge q. When the charge has traveled a distance $Z \sim \gamma^2\lambda$, the radiation fields begin to get ahead of the particle fields. Note that the relative phase of the Coulomb fields, where they intersect the hemisphere, varies as a function of θ, thus the "formation length" is a function of θ, with maximum value at θ=0. The formation length does not apply to backward OTR.

This characteristic length Z, associated with the phase between a particle field and co-moving radiation, is essentially the same as is found in the case of undulator and free electron laser radiation. It *does not* refer to a distance over which a photon is supposedly "formed" and therefore *cannot* be a basis for suggesting that OTR is subject to a depth of field limit to resolution similar to the synchrotron radiation depth of field problem discussed below.

Figure 2 shows the ratio of radiated OTR intensity per unit frequency contained inside a cone of half-angle θ_{max} to the intensity integrated over the entire hemisphere. This ratio is plotted versus θ_{max} in units of 1/γ for two energies of electrons, 1 and 10 GeV. Figure 2 demonstrates that, even for ultra relativistic electrons, a substantial portion of the OTR occurs at angles θ >> 1/γ; therefore the concept of a sharply limited angular distribution of order 1/γ and an associated "self-diffraction" effect is *not valid.*

B. Diffraction Limit of OTR Images

We have just discussed how the OTR production and its characteristic properties don't create any strange resolution limitations. Now we will summarize the result of a calculation of diffraction of OTR by a lens which focuses a spherical OTR wave front to a Gaussian image point on the optical axis of an optical system at a distance f from the aperture of radius a of the lens.The distance from the axis in the image plane is r, as shown in Figure 3. A point in the image plane at r subtends an angle $\theta = \sin^{-1}(r/f)$. As described in Reference [6], we have used standard scalar diffraction theory, assuming f >>a, and a >>λ; however, we replaced the usual constant pupil function with one with the angular behavior of E_θ of Eq (1), where now θ of Eq. (1) becomes α in Figure 3.

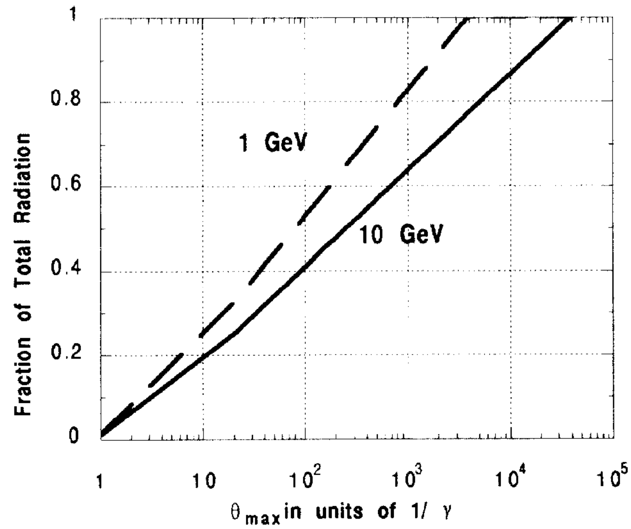

Figure 2. Fraction of total OTR radiation per unit frequency contained in cone of half-angle θ_{max} as a function of θ_{max} , in units of 1/γ , for 1 and 10 GeV electrons.

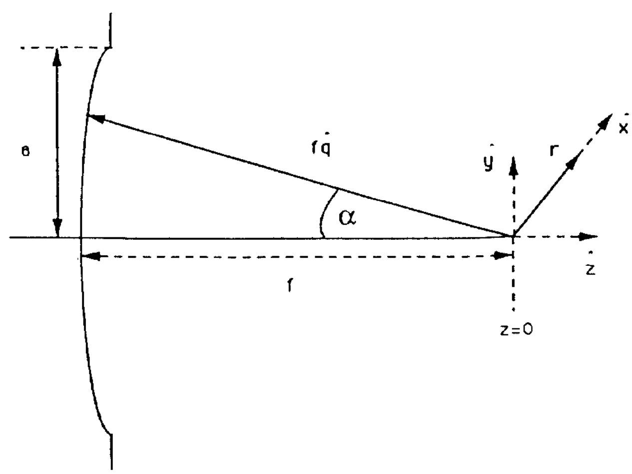

Figure 3. Diffraction of converging spherical waves at a circular aperture. Image plane is at z=0.

Figure 4 shows the results of the diffraction calculation for OTR, i.e.he diffracted amplitude of a point source imaged at z=0 in Figure 3 as a function of x=ka sin θ. For comparison, we show the standard plane wave diffraction result for a constant pupil function. Figure 4a is for the case of an aperture angle $\alpha_{max}= 1/\gamma = 0.01$, while Figure 4b is for the case of $\alpha_{max} >> 1/\gamma$ and is valid for *any* value of γ. We see that when $\alpha_{max}= 1/\gamma$, the diffraction is very close to the standard diffraction pattern, while it is only slightly broader when $\alpha_{max} >> 1/\gamma$, and resembles an apodized diffraction pattern. Therefore OTR diffraction limits are almost the same as the standard plane wave diffraction, i. e. this calculation shows that self-diffraction *does not* significantly alter the resolution of OTR images.

III. Comparison of OTR and synchrotron radiation

The properties of SR are summarized in Reference [8]. For the purpose of comparison to OTR, we will summarize those aspects of SR which affect the resolution of beam profiles imaged in SR. First, SR is formed at every point on the orbit in a bending magnet, therefore a horizontal limiting aperture is required to limit the length of the orbit which is imaged. Secondly, SR is more narrowly directed than OTR for frequencies $\omega < \omega_c$, the critical frequency. In contrast, the OTR radiation pattern is essentially independent of frequency. For an orbit of radius ρ and observed arc length s=ρθ, there is an apparent increase of the source width in the horizontal plane, which is

$$\Delta = \rho\theta^2/2 = s^2 / 2\rho. \qquad (2)$$

The diffraction limit to the resolution is

$$\delta x \sim \lambda/\theta. \qquad (3)$$

Equating Eqs. (2) and (3) gives an optimum angle $\theta \sim (\lambda/\rho)^{1/3}$ which minimizes diffraction and depth of field distortion. Even the apparent beam size in the direction perpendicular to the orbit is increased by the depth of field. In practice, the vertical aperture dimension is chosen to match the natural opening angle of SR and x-ray wavelengths are used to increase the resolution. Since OTR is formed at the planar interface of the radiating foil, it does not suffer from the depth of focus problem and the aperture of the imaging system can be made large enough to obtain the required diffraction limited resolution in accordance with Figure 4.

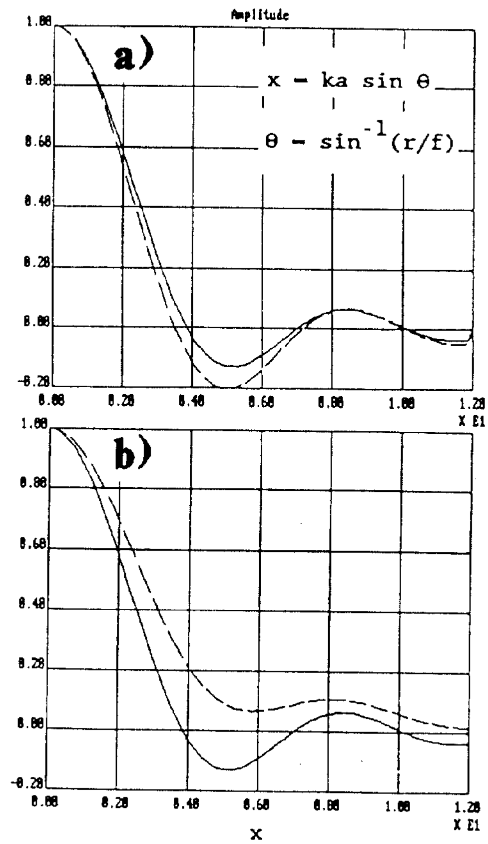

Figure 4. Comparison of standard (solid curve) and OTR (dashed curve) diffraction amplitudes for: a) aperture angle $\alpha_{max}= 1/\gamma$ and b) $\alpha_{max} >> 1/\gamma$.

IV. REFERENCES

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