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ECORE DISCUSSION PAPER

2011/89

Efficiency, Access and the Mixed Delivery of Health Care Services

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CORE DISCUSSION PAPER
2011/46

Efficiency, access and the mixed delivery of health care services

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September 2011

Abstract

The focus of this paper is on the trade-off between cost efficiency and access in the choice of the optimal mix of public and private provision in universal health systems. We model a simple health care market in which the regulator acts as a third payer. Patients need one unit of medical service and differ in the severity of illness. A private and a public hospital are available. The private manager internalizes profits, and has an incentive both to refuse to treat costly patients and to exert effort in cost reduction. The public manager does not internalize profits, and has no incentive to reduce costs or to dump costly patients. We show that, when a relatively efficient effort in cost reduction is available, it is optimal to buy part of the services from the private hospital. This may be the case for procedures that are easier to standardize, such as elective surgery. Since the regulator acts as an insurer for the whole population, a public hospital has to be used as a last resort facility. Imposing a no-dumping constraint on the private hospital is not always optimal since eliciting effort and truthful revelation of costs may become more difficult.

Keywords: health care, regulation, dumping, public and private hospitals.

JEL Classification: I18, L33, L51

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I am grateful to Albert Ma and Fred Schroyen for extremely useful discussions and suggestions. I thank participants to seminars in TSE, CORE and Liège University, and participants to the CESifo Area Conference on Public Sector Economics and to the iHEA conference 2011 for comments and suggestions.

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the author.

1 Introduction

In universal health care systems, the government acts as a third payer for medical services. These services are delivered by a mix of public and private providers. In the last twenty years, procurement arrangements with private hospitals have increased in many industrialized countries, such as Italy, France and the UK. In these systems, there is a range of services that are provided by both public and private hospitals. This trend seems to be motivated more by ideological factors than by strong theoretical and empirical results on the optimal mix of delivery. It is usually claimed that private hospitals are more efficient because they face different costs and are more flexible regarding their personnel. Private managers may also be more sensitive to financial incentives since they internalize profits.[1] If this was indeed true, what is the rationale for keeping some public capacity, as many countries do? The answer is that efficiency is not the only dimension to evaluate health services delivery. In many countries governments have to ensure universal health coverage; this target has been put forward to justify the role of public hospitals.[2] Profit driven private hospitals might reject patients whose cost exceeds the reimbursement from the government.[3] Dumping is the result of a regulation failure: unobservability or contract incompleteness make it impossible to adapt the price to the severity of illness of each patient, within a certain diagnostic group. If such a problem is severe, the regulator might prefer to use public hospitals, where access can be ensured. In many countries, public providers act as last resort, and are considered as safety nets for the poorest and sickest.

The focus of this paper is on the trade-off between cost efficiency and access to care in the choice of the optimal mix of public and private provision. We model a simple health care market where the government acts as a third payer and can contract with a public and a private hospital. A crucial assumption is that all patients are entitled to get the treatment for free. All patients benefit from the treatment in the same way, but differ in the severity of illness and thus in the cost of treatment. Because of contract incompleteness,

[1] The higher efficiency of private hospitals has yet to be shown empirically. For instance, Burgess and Wilson (1996), find contradicting results on the impact of ownership on efficiency and results depend on the measure of technical inefficiency used. Zuckerman *et al.* (1994) use US data and Chang *et al.* (2004) use Taiwan data. They both find that public hospitals are more inefficient than private ones.

[2] In France, the role of the private sector in the provision of publicly funded health services was enhanced by the "Bachelot Law", introduced in 2009. Critics of the law feared that the use of private hospitals would reduce the access to care.

[3] The WHO, discussing the role of the private sector in European health systems, advocates the implementation of regulation restricting"individual entrepreneurial behavior so as to protect core societal objectives in such areas as public health and safety, access, social cohesion and quality of care." (WHO, 2002, p.8.)

ex ante reimbursements cannot depend on cost realizations and the regulator is constrained to use a unitary price, and a fixed remuneration for the public manager. Both the public and the private manager can choose whether to treat patients and can exert effort to reduce variable costs. The incentives faced by the managers, however, depend on the ownership of the hospital. The private manager internalizes profits, and has an incentive both to turn down costly patients and to exert effort in cost reduction. The public manager does not internalize profits, which are appropriated by the the regulator, and has no incentive to reduce costs or to dump costly patients. Since she is the owner, the regulator can also control the waiting time of the public hospital.

We show that it is optimal to use the public hospital as a last resort provider; the regulator buys a share of services from the private hospital if and only if managerial effort leads to high reductions in costs. This result is rather intuitive. Since all patients have to be treated, it is impossible to distort quantities in order to provide incentives to the last resort hospital. The public hospital, which does not dump patients, is the most suited to play this role. Because of the presence of a last resort hospital, it is possible for the regulator to be tough with the private hospital. Eliciting effort from the private manager is less costly for the regulator the higher the efficiency of managerial effort. The model predicts that mixed delivery is optimal for procedures such as elective surgery: in this case an efficient management of the operating rooms might be sufficient to obtain considerable cost reductions. Procedures with smaller margins for cost reductions (for instance, medical ones) should be provided entirely by public hospitals.

The second contribution of the paper is to look at the role of dumping. If the regulator can forbid the private hospital to dump patients, the private manager has to report the cost realization and has to treat all patients as long as profits are non-negative. If the potential reduction in costs due to managerial effort is high, this no-dumping constraint is effective in reducing the cost of provision. If the reduction is small, conversely, the hospital has incentives to misreport costs in order to dump some patients. No reduction in total government expenditures is expected from contracting with the private hospital. Thus, forbidding dumping is not always beneficial from a welfare prospective. This result stems from the trad-off between efficiency and access, and to our knowledge, is novel in the literature.

There is a prolific literature dealing with dumping in the health care sector. As mentioned above, prospective reimbursement schemes may lead to undesired refusal of patients. Ma (1994), Ellis(1997), and Chalkley and Malcomson (2002), among others, show that a mixed cost reimbursement enhances social welfare even though it lowers the power of incentives in cost reduction. We take a different perspective. We do not look at the optimal remuneration scheme, and limit the analysis to the case in which the regulator has only one instrument, the price, because it is impossible to condition the reimbursement on the realization of costs. This assumption is reasonable since the costs of health services may be non verifiable (even *ex post*) and difficult to be contracted upon. In addition, in most industrialized countries, health care systems shifted to prospective remuneration schemes based on diagnostic groups.

The literature comparing public and private ownership considers incomplete contracts as a source of inefficiency in the public sector. Hart et al. (1997), Schmidt (1996) and Laffont and Tirole (1991), for instance, show that public managers invest too little in cost reduction since the government can appropriate the returns of their investment. In fact, because of contract incompleteness, the government/owner cannot commit to leave to the manager any rent related to his performance. This leads to a soft budget constraint: public managers cannot appropriate their profits and are not liable for their losses. In the case of the US hospital industry, Duggan (2000) finds empirical evidence that public managers do not respond to financial incentives as much as their public conterpart since they face a soft budget constraint. The incomplete contracts litterature, however, always leads to dichotomous solution, where either public or private provision is optimal. It does not allow for the presence of mixed delivery systems.

Even if a public and a private hospital coexist in our the model, we do not use the same approach of the mixed oligopolies literature, which assumes different objectives for the public and the private manager and the presence of competition (see Cremer *et al.*, 1989 and De Fraja and Delbono, 1989). In this spirit, Jofre-Bonet (2000) looks at a health care market where a public and a private hospital choose a quality level and compete *à la* Cournot, and the public provider maximizes consumers' surplus. In our model the public hospital is a rent maximizer, and the price is regulated.

Previous works from Ma and Grassi (2008, 2010) are close to our study. They examine the impact of public rationing on the price of private hospitals; the price in the public sector is zero while patients have to pay out of pocket if they visit a private provider. In our model the price of the private hospital is regulated and patients do not pay for the health care they receive. The focus here is on a third payer that may or may not contract with private hospitals.

The paper is organized as follows. In section 2 we present the main features of the model. In section 3, we characterize the optimal mix of public and private provision. In section 4 we study the optimality of a no-dumping constraint for the private hospital. We devote section 5 to concluding remarks. All proofs of lemmas and propositions, when not specified otherwise, are collected in the appendix.

2 The model

The regulator acts as an insurer for the population and must guarantee the treatment to all patients. The size of the sick population is normalized to one and each patient needs one unit of a uniform treatment. Patients face a price equal to zero irrespectively on whether they get the treatment from a public or a private hospital. We focus here exclusively on public patients seeking services in public and private hospitals, while the government acts as a third payer. Patients all get the same benefit v from being treated, but they differ in the severity of their illness, which affects the cost of the treatment they receive. Even if the diagnostic group (the illness) is the same, the cost of treatment is not homogeneous across patients. Elderly patients, for instance, may require a higher level of care, implying higher costs. The severity of illness is denoted by c, with $c \sim U(0,1)$.[4] Patients do not observe the realization of c .

Two hospitals are available on the market; one is publicly owned, the other is privately owned. The public and the private hospitals have the same technology. Treating patient i entails for hospital j a cost:

$$C_{ij} = k_j + c_i,$$

where c_i is the severity of illness and $k_j \in \{\underline{k},\ \bar{k}\}$ is an hospital specific parameter. Each hospital observes both k_j and c_i prior to the treatment; both are private information of the hospital. There are no fixed costs for the treatment.

Managers can influence the hospital specific cost parameter through an efficiency enhancing investment that can be made before production takes place. The investment costs S to the manager, and leads to a low hospital specific cost ($\underline{k}$) with probability $e \in (0,1)$. If no investment is made, the hospital has a high hospital specific ($\bar{k}$) cost with probability one. We assume that S is a private cost of the manager, so that the investment can be interpreted as managerial effort.

The private manager maximizes profits, since he can appropriate them. In particular, he can influence the average cost of treatment in two ways: by dumping costly patients, which reduces the average c, and by exerting effort in cost reduction, which reduces k in expectation.

The public manager cannot appropriate profits. Whenever profit (or losses) occur in the public hospital, they add to the public sector balance sheet. Unless his compensation depends on the realization of costs,

[4] Assuming a uniform distribution simplifies the exposition and the intuition of the results. In the conclusion we discuss the sensitivity of the results to a change in the distribution of c, and argue that the main qualitative results are robust to such a change.

the public manager has no incentive to reject patients or to exert effort. Similarly, Laffont and Tirole (1991) assume that the government appropriates the returns of the investments made by the public manager. Thus, the incentives of the public manager to invest are weak, if contract incompleteness makes it impossible to compensate him for the incurred cost. [5]

Health treatments can be publicly provided or bought from the private hospital. The only instrument for the regulator is a prospective reimbursement. The underlying assumption for such a remuneration scheme is that, *ex ante*, the regulator cannot write complete contracts, conditional on costs. Even *ex post*, costs are not contractible. Hence, there is no room for renegotiation. The problem of the regulator reduces to choosing the optimal price per treatment p. Contract incompleteness also implies that it is impossible for the regulator to condition the remuneration of managers on the realization of costs. Consequently, the remuneration of the public manager is a fixed transfer (see Schmidt, 1996). The regulator also has the ability to set the extra waiting time at the public hospital, denoted by $w \geq 0$. Without loss of generality, the waiting time at the private hospital is normalized to zero, so w can be interpreted as the total waiting time at the public hospital.

The regulator chooses the price p that maximizes the welfare of the patients net of health expenditures, under the constraint that all patients get the treatment. The profit of the private hospital has zero weight in the welfare function, while the profit of the public hospital is internalized by the regulator.

The timing is as follows:

1- The regulator sets a waiting time w for the public hospital,

2- The regulator sets a price p.

3- The private and public managers choose whether to exert effort at a private cost S.

4- The realization of k occurs. The value of k is private information of the managers.

5- The patients seek care at the hospital they prefer. If dumped by the private hospital, they will visit the public one and get the treatment.

6- The hospitals receive p for each treated patient. The profits of the public hospital are appropriated by the regulator.

As a benchmark, consider the case of a single private hospital that treats all patients (no dumping is possible), receiving a price p per patient. In this scenario the manager has the highest incentives to exert effort, since he cannot reject patients. The manager exerts effort if and only if it permits to reduce total

[5] This situation can apply to countries where the remuneration of public providers has shifted towards a DRG reimbursement. However, the public managers face a soft budget constraint, since they are not liable for incurred losses.

costs, that is to say if and only if :

$$\bar{k} - e\Delta k + \frac{1}{2} - S \leq \bar{k} + \frac{1}{2}.$$

The left hand side represents total costs when effort is exerted, while the right hand side represents total cost when no effort is exerted. Effort takes place if and only if $\bar{e}\Delta k \geq S$: the expected gains in efficiency exceed the private cost to the manager. To rule out cases in which managerial effort never takes place, we will thus make the following assumption:

Assumption 1 $e\Delta k \geq S$.

Another assumption that will hold throughout the paper is:

Assumption 2 $\Delta k \leq 1$.

This assumption ensures that the supports of the random unitary costs $\bar{k}+c$ and $\underline{k}+c$ intersect. This implies that, $\underline{k}+1 \geq \bar{k}$. The economic consequences of this assumption will be discussed in the following paragraph.

In the following, we will solve the problem by backward induction analyzing the behavior of patients and managers first. In the next section we will then solve the problem of the regulator.

2.1 Patients' behavior

As said above, patients face a price equal to zero in each hospital. Consequently, the hospital choice only depends on the waiting time at the public hospital, w (as we said earlier, the waiting time is normalized to zero). We assume that there are no search or switching costs, so that patients can costlessly pass from one hospital to another. Let us denote by $D_{PU}(w)$ and $D_{PR}(w)$ the number of patients seeking care at the public and at the private hospital, with $D_{PU}(w) + D_{PR}(w) = 1$.

If the waiting time w is equal to zero, they are indifferent between the two hospitals. Thus, each hospital has the same demand: $D_{PU}(0) = D_{PR}(0) = 1/2$; the distribution of the patient specific cost is uniform ($c \sim U(0,1)$), and the average patient specific cost is 1/2. If the waiting time w is positive, all patients first visit the private hospital. Thus $D_{PR}(w > 0) = 1$ and $D_{PU}(w > 0) = 0$. In both cases, the private hospital might dump some patients, who would then seek care in the public one.

Summarizing,

$$D_{PR}(w) = \begin{cases} 1/2 & if\ w = 0 \\ 1 & if\ w > 0, \end{cases}$$

and $D_{PU}(w) = 1 - D_{PR}(w)$

In the following, whenever there is no ambiguity, we will use the simplified notations D_{PR} and D_{PU}.

2.2 Private hospital's behavior

The objective of the private manager is to maximize profits. Once the regulated price is set, the manger chooses whether to undertake the investment in cost reduction. Subsequently, $D_{PR}(w)$ patients seek care at the private hospital. Given the realization of the hospital specific cost parameter k_{PR}, patients are either offered a treatment or dumped depending on their patient specific cost. In the following, we proceed by backward induction in order to characterize the minimal regulated price that elicits the investment in cost reduction.

Ex post, $D_{PR}(w)$ patients visit the private hospital. Given the regulated price p, and the realization of k, the hospital treats a patients with cost c if and only if

$$p \geq k_{PR} + c.$$

For any given price p and realization of k_{PR}, the share of patients that receive the treatment, conditional on visiting the private hospital, is

$$d(p,\ k_{PR}) = \begin{cases} 0 & if\ \ p \leq k \\ p - k_{PR} & if\ \ k_{PR} < p \leq k_{PR} + 1 \\ 1 & if\ \ p > k_{PR} + 1 \end{cases} \tag{1}$$

for $k_{PR} \in \{\underline{\mathrm{k}},\ \bar{k}\}$. The total amount of treatments provided by the private hospital is equal to $D_{PR}d(p,\ k_{pr})$. We define $\bar{d}(p) \equiv d(p,\ \bar{k})$ and $\underline{\mathrm{d}}(p) \equiv d(p,\ \underline{\mathrm{k}})$. Assumption 2 implies that $\underline{\mathrm{k}} < \bar{k} \leq \underline{\mathrm{k}} + 1 < \bar{k} + 1$, so that there exists no price p such that $\bar{d}(p) = 0$ and $\underline{\mathrm{d}}(\underline{\mathrm{k}}) = 1$. The private hospital never satisfies the whole demand if the effort in cost reduction has been successful, while it dumps all patients if k_{PR} turns out to be high. In other words, the gain in efficiency due to the effort (Δk) is not too high. [6]

Ex ante, taking p and w as given, the manager decides whether to exert effort or not. The expected profit

[6] This assumption merely facilitates the exposition and does not reduce the generality of the model, as we will discussed later. If $\Delta k > 1$ the private manager would be more prone to exert effort, and contracting with the private hospital more desirable.

when no effort is exerted is

$$\Pi_{PR}(0,\ p,\ w) = D_{PR}\left[\int_0^{\bar{d}(p)}(p-\bar{k}-c)dc\right],$$

where the profit depends on the probability of a low cost realization and on the regulatory parameters. Conversely, the expected profit when the effort is exerted is

$$\Pi_{PR}(e,\ p,\ w) = D_{PR}\left[e\left(\int_0^{\underline{\mathrm{d}}(p)}(p-\underline{\mathrm{k}}-c)dc\right)+(1-e)\left(\int_0^{\bar{d}(p)}(p-\bar{k}+c)dc\right)\right].$$

Let us define $\Delta\Pi(p,\ w)\equiv\Pi_{PR}(e\ p,\ w)-\Pi_{PR}(0\ p,\ w)$ the difference between the expected profits with and without investment in cost reduction. The private manager exerts effort if and only if $\Delta\Pi_{PR}(p,\ w)\geq S$. The following lemma establishes an important property of the function $\Delta\Pi(p,\ w)$.

Lemma 1 *The expected return of managerial effort, $\Delta\Pi(p,\ w)$ is non-decreasing in the regulated price p.*

The incentives to exert effort are increasing in the regulated price. Thus, effort will be made for any price $p\geq\tilde{p}$ such that

$$\Delta\Pi(\tilde{p},w)=S. \tag{2}$$

The following proposition characterizes the minimal price eliciting effort as a function of the cost of effort S.

Proposition 1 *The private hospital exerts effort if and only if the regulated price p is greater than $\tilde{p}$, where*

$$\tilde{p}=\begin{cases}\sqrt{\frac{2S}{eD_{PR}}}+\underline{k}\quad\in\left]\underline{k},\ \bar{k}\right] & if\ \ S/D_{PR}\leq\bar{e}(\Delta k)^2/2\\ \frac{S}{D_{PR}(e\Delta k)}-\frac{\Delta k}{2}+\bar{k}\quad\in\left]\bar{k},\ \underline{k}+1\right] & if\ \ \bar{e}(\Delta k)^2/2<S/D_{PR}\leq e\Delta k\,(1-\Delta k/2)\\ 1+\bar{k}-\sqrt{2\left(\Delta k-\frac{S}{eD_{PR}}\right)}\quad\in\left]\underline{k}+1,\ \bar{k}+1\right] & if\ \ S/D_{PR}>\bar{e}\Delta k\,(1-\Delta k/2)\end{cases} \tag{3}$$

The minimal price eliciting effort is increasing in S and is decreasing in e and in Δk. This is rather intuitive: the higher the efficiency gain from effort (or the lower the cost of effort), the lower the necessary monetary incentives. When the private manager exerts effort, the expected number of treatments increases with the efficiency of the effort. Conversely, the number of patients treated if no effort is exerted is constant. For any price, an increase in the returns to effort leads to an increase in the difference between expected profits with and without effort; this makes it easier for the regulator to elicit effort.

For similar reasons, the minimal price eliciting effort is decreasing in w. The higher the number of patients seeking care at the private facility, the higher the incentive to reduce variable costs.

2.3 Public hospital's behavior

The public manager cannot appropriate profits, and his compensation consists of a fixed wage. Irrespective of the realization of the hospital specific cost parameter, he does not have any incentives to turn down patients. Consequently the public hospital treats all the patients that initially chose it, and the patients that were dumped from the private hospital. The total supply of the public hospital depends on the initial demands and on the behavior of the private hospital, and is equal to $D_{PU} + (1 - d(p,\ k_{PR}))D_{PR} = 1 - D_{PR}d(p,\ k_{PR})$. The average severity of illness of the patients that visit the public hospital first is equal to $1/2$; the severity of illness of the patients that have been dumped from the private hospital is equal to $\int_{d(p,\ k_{PR})}^{1} cdc/(1 - d(p,\ k_{PR})) = (1 + d(p,\ k_{PR}))/2$. Note that the higher the supply of the private hospital, the higher the expected severity of illness of the public patients.

Ex ante, the manager chooses whether to exert effort in cost reduction. Since the cost realization cannot be specified in his contract, it cannot affect the compensation of the manager. Even if a loss occurs *ex post*, it is internalized by the regulator. Faced with such a soft budget constraint, the public manager never exerts any effort. The cost realization at the public hospital is always $\bar{k}$.

Ex post, the profit of the public hospital is equal to

$$\Pi_{PU}(p,\ w,\ k_{PR}) = (1 - D_{PR})\left(p - \bar{k} - \frac{1}{2}\right) + D_{PR}\left(1 - d(p,\ k_{PR})\right)\left[p - \bar{k} - \frac{1 + d(p,\ k_{PR})}{2}\right]. \quad (4)$$

Note that this profit might be either negative or positive. We can derive the profit of the public hospital when the private hospital has high costs $\bar{k}$:

$$\Pi_{PU}(p,\ w,\ \bar{k}) = \left(p - \bar{k} - \frac{1}{2}\right) - D_{PR}\bar{d}(p)\left(p - \bar{k} - \frac{\bar{d}(p)}{2}\right).$$

The profit of the public hospital when the private hospital has low costs $\underline{\mathrm{k}}$ is:

$$\Pi_{PU}(p,\ w,\ \underline{\mathrm{k}}) = \left(p - \bar{k} - \frac{1}{2}\right) - D_{PR}\underline{\mathrm{d}}(p)\left(p - \bar{k} - \frac{\underline{\mathrm{d}}}{2}\right).$$

The first term of each expression represents the profit of the public hospital when it treats all patients. The second term represents the loss in profits due to the fact that $D_{PR}d(p,\ k_{PR})$ patients are treated in the private hospital.

3 Optimal regulation

The regulator acts as an insurer that seeks to provide one unit of treatment to the entire population at minimal cost. Thus the objective function of the regulator is

$$W(p,\ w) = \begin{cases} v - wD_{PU} - p + \Pi_{PU}(p,\ w,\ \underline{k}) & if\ p < \tilde{p} \\ v - wD_{PU} - p + e\Pi_{PU}(p,\ w,\ \underline{k}) + (1-e)\Pi_{PU}(p,\ w,\ \bar{k}) & if\ p \geq \tilde{p} \end{cases}$$

which is the difference between patients' welfare and expected public health expenditures. We continue to proceed by backward induction. First, we characterize the optimal price for any level of the waiting time. Then, we characterize the optimal waiting time for the public hospital.

We consider first two benchmark scenarios. In the first one (called PR), only a private hospital is available. In the second one (called PU), only a public hospital is available. We then turn to the scenario (called MIX) in which the regulator can contract with both a public and a private hospital.

3.1 Government contracting with a single hospital

Consider first the scenario PR, in which the regulator has to provide the service to all patients and is constrained to contract with a private hospital. *Ex post*, the private hospital treats all patients if and only if $p \geq k_{PR} + 1$. Since the only instrument is the price, the regulator has to choose a price *ex ante* that is sufficient to cover costs in any state of the world. Thus, the optimal price, coinciding with the total health expenditures is equal to $p^{PR} = \bar{k} + 1$. As long as $e\Delta k \geq S$, the manager has an incentive to exert effort. However, the returns to effort are completely appropriated by the hospital.

Consider now the scenario PU in which the regulator is constrained to contract with a public hospital. The problem of the regulator reduces to

$$\max_{p,\ w}\ v - w - p + \left[p - \bar{k} - 1/2\right] = v - w - \bar{k} - 1/2$$

Clearly the optimal waiting cost is equal to zero. The welfare function is independent from the price, since ex post the regulator always ends up paying the total production costs of the public hospital. The total public health expenditure is equal to $\bar{k} + 1/2$. Comparing this result with the previous one leads to the following proposition:

Proposition 2 *If only one hospital is used, and the regulator is constrained to treat all patients, then contracting with a public hospital dominates contracting with a private hospital.*

Proof. Straightforward ■

If a single hospital is used as a last resort, it is not possible to distort quantities. If the hospital is private, the regulator has to set a price equal to the highest realization of costs in order to prevent dumping. Thus, the regulator prefers to contract with the public hospital, which treats all patients irrespective of the price. In such a case, total health expenditures equal the average cost. Quite naturally, the possibility to dump is a negative attribute of a hospital when the regulator needs to treat all patients.

3.2 Government contracting with a public and a private hospital

3.2.1 Optimal price

In this section, we consider the MIX scenario, in which the regulator can contract with both a public and a private hospital.

Patients' welfare only depends on the waiting time. Thus, taking the waiting time as given, the problem of the regulator reduces to minimizing the expected health expenditures:

$$\min_{p} \ HE(p,\ w) = \begin{cases} p - \Pi_{PU}(p,\ w,\ \underline{\mathrm{k}}) & if\ p < \tilde{p} \\ p - e\Pi_{PU}(p,\ w,\ \underline{\mathrm{k}}) - (1-e)\Pi_{PU}(p,\ w,\ \bar{k}) & if\ p \geq \tilde{p} \end{cases}.$$

p is the total reimbursement to be given to the private and public hospitals. The second term relates to the production costs of the public hospital. Using the definition of the profit of the public hospital, we have

$$HE(p,\ w) = \begin{cases} \bar{k} + \frac{1}{2} + D_{PR}\bar{d}(p)\left(p - \bar{k} - \frac{\bar{d}(p)}{2}\right) & if\ p < \tilde{p} \\ \bar{k} + \frac{1}{2} + D_{PR}\left[e\underline{\mathrm{d}}(p)\left(p - \bar{k} - \frac{\underline{\mathrm{d}}(p)}{2}\right) + (1-e)\bar{d}(p)\left(p - \bar{k} - \frac{\bar{d}(p)}{2}\right)\right] & if\ p \geq \tilde{p} \end{cases}. \quad (5)$$

This expression has an easy interpretation. $\bar{k} + 1/2$ represent the expected health expenditures if the public hospital exclusively provides the service. The residual term in each line represents the difference between the expected payment to the private hospital for treating $D_{PR}\left(e\underline{\mathrm{d}}(p) + (1-e)\bar{d}(p)\right)$ patients and the total cost of treating the same patients in the public facility. Of course, it is optimal to contract with the private hospital if and only if this difference is negative.

We denote the optimal price by p^{MIX}. We will first establish a property of the objective function of the regulator.

Lemma 2 *Any price $p < \tilde{p}$ is dominated by $p = 0$, where $\tilde{p}$ is the minimal price eliciting high effort defined by equation (3).*

This result is quite intuitive. There is no reason to contract with the private hospital unless it is more efficient than the public one in expectation.

Remark that if the price is equal to zero, $HE(0,\ w) = \bar{k} + 1/2$. This are the total health expenditures when only the public hospital provides the services. A direct consequence of Lemma 2 is that the optimal price is positive if and only if

$$\bar{k} + 1/2 > \min_{p \geq \tilde{p}} \ HE(p,\ w) = \min_{p \geq \tilde{p}} \ \bar{k} + \frac{1}{2} + D_{PR}\left[e\underline{\mathrm{d}}\left(p - \bar{k} - \frac{\underline{\mathrm{d}}(p)}{2}\right) + (1-e)\bar{d}\left(p - \bar{k} - \frac{\bar{d}(p)}{2}\right)\right].$$

In order to characterize the optimal price, we will first find the solution of $\min_{p \geq \tilde{p}} \ HE(p,\ w)$; then, we will compare the result with $\bar{k} + 1/2$.

If $S/D_{PR} \leq \bar{e}(\Delta k)^2/2$ we know from Proposition 1 that $\tilde{p} \in \left]\underline{\mathrm{k}},\ \bar{k}\right]$. Expected health expenditures are piecewise-defined and continuous for any $p \geq \tilde{p}$ and they have the form

$$\begin{cases} \bar{k} + \frac{1}{2} + eD_{PR}(p - \underline{\mathrm{k}})\left(\frac{p - \underline{\mathrm{k}}}{2} - \Delta k\right) & if\ p \in \left]\tilde{p},\ \bar{k}\right] \\ \bar{k} + \frac{1}{2} + D_{PR}\left[\left(\frac{(p-\bar{k})^2}{2}\right) - e\frac{(\Delta k)^2}{2}\right] & if\ p \in \left]\bar{k},\ \underline{\mathrm{k}} + 1\right] \\ \bar{k} + \frac{1}{2} + D_{PR}\left[e\left(p - \bar{k} - 1/2\right) + (1-e)\left(\frac{(p-\bar{k})^2}{2}\right)\right] & if\ p \in \left]\underline{\mathrm{k}} + 1,\ \bar{k} + 1\right] \\ p & if\ p > \bar{k} + 1 \end{cases}$$

It is easy to show that the function is decreasing for any $p \leq \bar{k}$, and increasing for any $p > \bar{k}$. Consequently, if $S/D_{PR} \leq e(\Delta k)^2/2$, the price that minimizes expenditures conditional on eliciting effort is equal to $\bar{k}$. Such a price implies that a positive amount of patients is treated in the private hospital if the hospital-specific cost realization is $\underline{\mathrm{k}}$. Otherwise, all patients are treated by the public hospital and the private hospital makes zero profits. Expected expenditures are equal to $\bar{k} + 1/2 - eD_{PR}(\Delta k)^2/2$, which is smaller than $\bar{k} + 1/2$, the expression of the expected expenditures when $p = 0$. Thus, we can conclude that $p^{MIX} = \bar{k}$ whenever $S/D_{PR} \leq e(\Delta k)^2/2$.

If $e(\Delta k)^2/2 < S/D_{PR} \leq e\Delta k\,(1 - \Delta k/2)$, we know from Proposition 1 that $\tilde{p} \in \left]\bar{k},\ \underline{\mathrm{k}} + 1\right]$. Expected health expenditures are piecewise-defined and continuous for any $p \geq \tilde{p}$ and they have the form

$$\begin{cases} \bar{k} + \frac{1}{2} + D_{PR}\left[\left(\frac{(p-\bar{k})^2}{2}\right) - e\frac{(\Delta k)^2}{2}\right] & if\ p \in \left]\tilde{p},\ \underline{\mathrm{k}}+1\right] \\ \bar{k} + \frac{1}{2} + D_{PR}\left[e\left(p - \bar{k} - 1/2\right) + (1-e)\left(\frac{(p-\bar{k})^2}{2}\right)\right] & if\ p \in \left]\underline{\mathrm{k}}+1,\ \bar{k}+1\right] \\ p & if\ p > \bar{k}+1 \end{cases}$$

It is easy to show that this function is increasing for any $p \geq \tilde{p}$. Consequently, if $e(\Delta k)^2/2 < S/D_{PR} \leq e\left(\Delta k - (\Delta k)^2/2\right)$, the price that minimize expenditures conditional on eliciting effort is equal to $\tilde{p} = S/D_{PR}(e\Delta k) - \Delta k/2 + \bar{k}$. The private hospital treats a positive amount of patients for any cost realization. Expected health expenditures are equal to $\bar{k} + \frac{1}{2} + D_{PR}\left[(S/D_{PR} - \Delta k/2)^2 - e(\Delta k)^2\right]/2$. They are smaller than $HE(0,\, w)$ if and only if

$$\left(\frac{S}{D_{PR}\,(e\Delta k)} - \frac{\Delta k}{2}\right)^2 - e(\Delta k)^2 \leq 0 \iff S/D_{PR} \leq e(\Delta k)^2\left(\sqrt{e} + \frac{1}{2}\right).$$

In conclusion, if $e(\Delta k)^2/2 < S/D_{PR} \leq min\left\{e\Delta k\,(1 - \Delta k/2)\,,\, e(\Delta k)^2\left(\sqrt{e} + \frac{1}{2}\right)\right\}$, then the optimal price is the minimal price eliciting managerial effort, i.e. $p^{MIX} = \tilde{p}$.

If $S/D_{PR} > e\Delta k\,(1 - \Delta k/2)$, we know from Proposition 1 that $\tilde{p} \in \left]\underline{\mathrm{k}}+1,\ \bar{k}+1\right]$. Expected health expenditures are piecewise-defined and continuous for any $p \geq \tilde{p}$ and they have the form

$$\begin{cases} \bar{k} + \frac{1}{2} + D_{PR}\left[e\left(p - \bar{k} - 1/2\right) + (1-e)\left(\frac{(p-\bar{k})^2}{2}\right)\right] & if\ p \in \left]\tilde{p},\ \bar{k}+1\right] \\ p & if\ p > \bar{k}+1 \end{cases}$$

It is easy to show that this function is increasing for any $p \geq \tilde{p}$. Consequently, if $S/D_{PR} > e\Delta k\,(1 - \Delta k/2)$, the price that minimize expenditures conditional on eliciting effort is equal to $\tilde{p} = 1 + \bar{k} - \sqrt{2\,(\Delta k - S/eD_{PR})}$. The private hospital treats all the patients if the cost realization is low, and a positive share of patients if the cost realization is high. Expected health expenditures are equal to

$$\bar{k} + \frac{1}{2} + D_{PR}\left[e\left(1/2 - \sqrt{2\left(\Delta k - \frac{S}{eD_{PR}}\right)}\right) + (1-e)\frac{1}{2}\left(1 - \sqrt{2\left(\Delta k - \frac{S}{eD_{PR}}\right)}\right)^2\right].$$

They are smaller than $HE(0,\, w)$ if and only if

$$e\left(\frac{1}{2}-\sqrt{2\left(\Delta k-\frac{S}{eD_{PR}}\right)}\right)+(1-e)\frac{1}{2}\left(1-\sqrt{2\left(\Delta k-\frac{S}{eD_{PR}}\right)}\right)^2\leq 0$$
$$\Longleftrightarrow\; S/D_{PR}\leq e\left(\Delta k-(1+\sqrt{e})^{-2}/2\right).$$

In conclusion, if $e\Delta k\,(1-\Delta k/2)<S/D_{PR}\leq e\left(\Delta k-(1+\sqrt{e})^{-2}/2\right)$, then the optimal price is the minimal price eliciting managerial effort, i.e. $p^{MIX}=\tilde{p}$.

The following proposition formally characterizes the optimal price.

Proposition 3 *If the regulator is constrained to treat all patients and can use both a public and a private hospital, the cost minimizing regulated price price depends on the efficiency of the managerial effort. Define* $\bar{S}\equiv min\left\{e(\Delta k)^2\,(\sqrt{e}+1/2)\,,\; e\left(\Delta k-(1+\sqrt{e})^{-2}/2\right)\right\}$. *Three cases might arise:*

(i) If $S/D_{PR}\leq e(\Delta k)^2/2$, *then* $p^{MIX}=\bar{k}$.

(ii) If $e(\Delta k)^2/2<S/D_{PR}\leq\bar{S}$, *then* $p^{MIX}=\tilde{p}$, *where* $\tilde{p}$ *is the minimal price eliciting high effort defined by equation (3).*

(iii) If $S/D_{PR}>\bar{S}$, *then* $p^{MIX}=0$, *and no treatment is bought from the private hospital.*

Point (i) of Proposition 3 has been proved above. The proof of points (ii) and (iii) is provided in the appendix.

The intuition for this result is very simple. It is not possible to get effort and truthful revelation from the last resort hospital since quantities cannot be distorted. Consequently, it is always optimal to use the public hospital as a last resort since it does not dump patients and the price is equal to the average cost. However, if the effort in cost reduction is efficient, the price permitting to elicit effort in the private hospital is relatively low, and contracting with the private hospital is expenditure minimizing due to gains in efficiency.

If the cost of effort is very low with respect to its benefit, then the price offered to the private hospital is positive and elicits high effort. The private hospital treats some patients only if the hospital-specific cost realization is $\underline{k}$. Otherwise, all patients are treated by the public hospital. If costs turn out to be high for the private hospital, it is optimal to get the patients treated at average cost by the public one. The private hospital profit is equal to zero if costs turn out to be high.

For intermediate efficiency of the managerial effort, the regulator still finds it optimal to contract with the private hospital, but the minimal price that elicits effort is higher than in the previous scenario. In this case, the private hospital earns a positive profit even if its cost turn out to be high. This is suboptimal *ex* *post*, since it would be less costly for the regulator to get all the treatment in the private hospital. However, *ex ante*, the expected gains in efficiency justify the extra rent guaranteed to the private manager.

Finally, if the effort in cost reduction is very costly with respect to its benefit, no treatment is bought from the private hospital. In this case, eliciting effort is too costly for the regulator because it implies a high agency rent to the private manager.

Summarizing, our results suggests that the regulator should contract with private hospitals for services involving high margins of cost reduction. This might be the case of elective surgery, where the hospital managers can engage in relatively cheap and reliable efficiency-enhancing investments, such as rational use of operating rooms. For treatments involving high investment for low efficiency gain, the production should be concentrated in public facilities. For instance, emergency services are good candidates for exclusive public provision, as their demand is stochastic and it is impossible to schedule treatments in advance.

3.2.2 Optimal waiting time

In the first stage of the game, the regulator sets the waiting time at the public hospital. If the waiting time was equal to zero, patients would be indifferent between the public and private hospital. Thus, 1/2 of the patients would visit the public hospital, while 1/2 would visit the private hospital first, and the public one only if they happened to be dumped. Conversely, if the waiting time is greater than zero, all patients will visit the private hospital first. We can prove the following result:

Proposition 4 *If the regulator is constrained to treat all patients and can use both a public and a private hospital, the optimal waiting time depends on the efficiency of the managerial effort. Two cases might arise:*

(i) If $S \leq \bar{S}$ then $w^{MIX} = \epsilon$, with $\epsilon > 0$, $\epsilon < \bar{\epsilon}$ and $\bar{\epsilon}$ very small.

(ii) If $S > \bar{S}$, then $w^{MIX} = 0$.

Note that it is optimal to contract with the private hospital (by setting a price greater than zero) if and only if $e\underline{d}\left(p - \bar{k} - \underline{d}/2\right) + (1-e)\bar{d}\left(p - \bar{k} - \bar{d}/2\right) < 0$, irrespective of the waiting time. This inequality can be rewritten as $\left(e\underline{d}(p) + (1-e)\bar{d}(p)\right)p < \left(e\underline{d}(p) + (1-e)\bar{d}(p)\right)\bar{k} + e\left(\underline{d}(p)\right)^2/2 + (1-e)\left(\bar{d}(p)\right)^2/2$, meaning that the expected reimbursement to the private hospital is smaller than the cost of treating the same patients in the public hospital. In such a case, it is clear that the regulator wants all patients to visit the private hospital first. If this were not the case, the public hospital would treat some low cost patients at a higher cost than its private counterpart. A positive waiting time ensures that patients are allocated optimally across hospitals.

In addition, a very low waiting time is sufficient for D_{PR} to be equal to one. Thus, the optimal sorting of patients across hospitals can be achieved at a very low cost in terms of patients' welfare.

3.2.3 Market shares, profits and expected health expenditures

If the cost of effort is small with respect to the efficiency gain of effort ($S \leq e(\Delta k)^2/2$), the regulator always wants to elicit effort in the private sector. In this case, the reimbursement per patient is equal to $\bar{k}$, the private hospital treats $\Delta k \leq 1$ patients only when the realization of k is $\underline{k}$ and the expected number of patients treated in the private sector is $\bar{e}\Delta k$. The market share of the public hospital is positive whatever the realization of costs in the private hospital. The presence of the public hospital permits to leave no rents to the private one if the latter does manage to reduce costs. The expected profit of the private hospital is equal to $e(\Delta k)^2/2$ and is thus increasing in the return to effort and decreasing in S. Expected health expenditures are

$$HE(p^{MIX}) = \bar{k} + \frac{1}{2} - e\frac{(\Delta k)^2}{2}, \tag{6}$$

which are decreasing with e and Δk and is independent of S.

If S is bigger with respect to the efficiency gain, the social planner has to leave some rent to the private manager even if the realization of the costs is $\bar{k}$. In such a case, the optimal price is equal to the minimal price that permits to elicit high effort. In this range of parameters, two cases have to be considered, depending on the level of S.

If $S \leq \bar{S} \leq e\left(\Delta k - (\Delta k)^2/2\right)$, the optimal price $\tilde{p} = S/e\Delta k - \Delta k/2 + \bar{k}$ belongs to the interval $\left]\bar{k},\ \underline{k}+1\right]$. The private hospital treats $S/e\Delta k + \Delta k/2 < 1$ patients if the realization of the cost parameter is $\underline{k}$ and $S/e\Delta k - \Delta k/2 < 1$ patients if the realization of the cost parameter is $\bar{k}$. Note that both quantities are increasing in S and decreasing in Δk and e in the relevant range of parameters. The expected profit of the private hospital is equal to $e(\tilde{p} - \underline{k})^2/2 + (1-e)(\tilde{p} - \bar{k})^2/2 - S$ which is increasing in S and decreasing in e and Δk . In fact, the higher the cost of effort or the lower the efficiency gain, the higher the regulated price. Expected health expenditures are

$$HE(p^{MIX}) = \bar{k} + \frac{1}{2} - \frac{e(\Delta k)^2}{2} + \frac{1}{2}\left(\frac{S}{e\Delta k} - \frac{\Delta k}{2}\right)^2, \tag{7}$$

which are decreasing in e and in Δk, while it is increasing in S. Of course, if higher profits have to be given to the private hospital, the cost of provision is bigger.

If $\bar{S} > S > e\left(\Delta k - (\Delta k)^2/2\right)$, the optimal price $\tilde{p} = 1 + \bar{k} - \sqrt{2\left(e\Delta k - S\right)/\bar{e}}$ belongs to the interval

$\left]\underline{k}+1,\ \bar{k}+1\right]$. The private hospital treats all patients if the hospital-specific cost parameter is low, and a share $1-\sqrt{2\left(e\Delta k-S\right)/e}$ in case of high cost $\bar{k}$. This quantity is decreasing in Δk and e, and increasing in S. In fact, the minimal price eliciting effort decreases in the efficiency gain of the effort in cost reduction and increases in S. The expected profit of the hospital is equal to $\tilde{p}\left[e+(1-e)\left(\tilde{p}-\bar{k}\right)\right]$, which is increasing in S and decreasing in Δk . Expected health expenditures are

$$HE(p^{MIX}) \quad = \quad 1+\bar{k}-\sqrt{2\left(\frac{e\Delta k-S}{e}\right)}+(1-e)\left(\frac{e\Delta k-S}{e}\right). \qquad (8)$$

Again, the cost is decreasing in Δk, and e while it is increasing in S. The interpretation is similar to the one of the previous case.

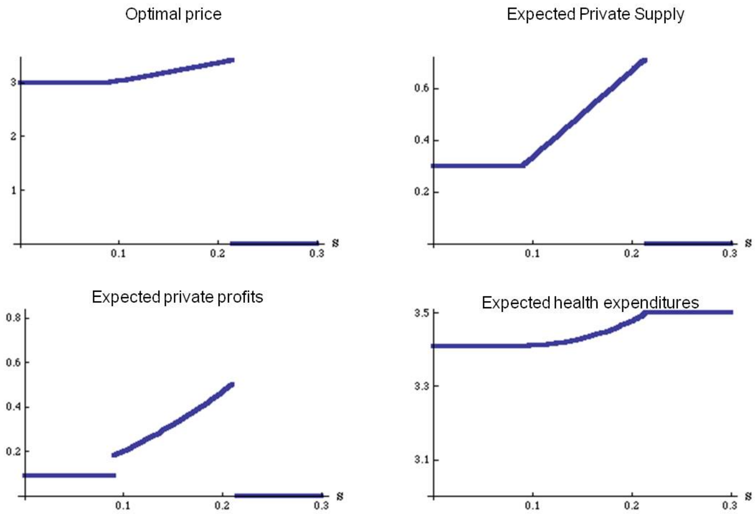
Finally, when S is very high with respect to the efficiency gain of effort, it becomes too expensive for the regulator to contract with the private hospital. In this Eliciting effort from the private manager is too costly with respect to the potential gains.

An illustration of the optimal price, private expected production and profits, and expected health expenditures as a function of S presented in Figure 1. This figure shows that the model predicts a non monotonic relationship between some of these quantities and the cost of managerial effort (its gains being fixed).

Expected health expenditures decrease in the efficiency of managerial effort. Conversely, both the regulated price and the expected amount of services provided by the private hospital exhibit a non monotonic relationship with the efficiency of managerial effort. If this efficiency is low, the price equals zero and the private hospital is excluded from production. As the efficiency of effort exceeds a critical value, it becomes optimal for the regulator to elicit effort, but this is only possible for a high regulated price. However, the latter decreases in the efficiency of effort until a certain threshold, after which it is constant and equal to $\bar{k}$. The expected amount of services provided follows a similar path. The profit of the private hospital is also non monotonic in the efficiency of effort. If efficiency is small, the profit is equal to zero. As the efficiency of effort exceeds a critical value, profits are very high since both the price and the expected quantities are high. They subsequently decrease. When the effort is very efficient in reducing costs, the price is constant and low. However, profits might rise again due to higher expected quantities. [7]

[7] Note that Assumption 2 plays a role in the results. If Δk was bigger than one, this would enhance the role of the private hospital. In this case, setting a price in the interval $\left]\underline{k}+1,\ \bar{k}\right]$, the regulator could elicit effort without the private hospital's producing in the bad state of the world.

Figure 1: Optimal price, expected private supply, private profit and health expenditures as a function of S ($e = 0.5$, $\Delta k = 0.6$).



4 No-dumping constraint on the private hospital

Our previous results depend on the hypothesis that the private hospital can freely dump the patients whose cost exceeds the regulated price. We now turn to the case when the private hospital is forbidden to dump. We will assume a waiting time $w > 0$ in the public hospital.[8] The timing is the same as in the previous sections. However, in period one, the regulator can also decide whether to enforce or not a no-dumping constraint on the private hospital. Under this constraint, the private manager has to report the hospital-specific cost realization. Depending on the reported costs, the regulator obliges the private hospital to treat the maximal possible amount of patients, under the constraint of non negative profits.

We show that it is not always in the interest of the regulator to enforce the constraint. The no-dumping constraint reduces the rent left to the private hospital. Consequently, eliciting effort and truthful revelation of the cost type may be impossible under a no-dumping constraint. In the following section we show that, if the efficiency of the effort is not too high, the low cost hospital has an incentive to mimic high cost type in order to get a positive profit.

[8] We can show that a positive waiting time is indeed optimal, but we will omit the proof here, since it is very similar to the proof of Proposition 4.

4.1 Private hospital's behavior when dumping is banned

In this section, we assume that the regulator can use both hospitals and can oblige the private hospital not to dump. We will call this regime MND. In many health care systems, this might be a relevant setting. The price (the DRG reimbursement) is often set *ex ante* by law. *Ex post*, the government may contract on quantities with private hospitals, and enforce a no-dumping regulation.[9]

All patients visit the private hospital first. The hospital-specific cost parameter k_{PR} is still private information of the hospital. Once the realization of k_{PR} occurs, the private hospital reports k', and has to treat the maximum amount of patients under the constraint of non-negative profits.

For any price p offered to the private hospital and for any reported k', the private hospital has to treat $\hat{d}$ patients, where $\hat{d}$ satisfies:

$$\hat{d}(p,\ k') = \arg\max_{d}\ d$$
$$s.t.\ \int_0^d (p - k' - c)dc \geq 0.$$

Thus,

$$\hat{d}(p,\ k') = \begin{cases} 0 & if\ p \leq k' \\ 2(p-k') & if\ k' < p \leq k' + \frac{1}{2} \\ 1 & if\ p > k' + \frac{1}{2} \end{cases}$$

Ex post, the profit of the private manager is thus equal to:

$$\int_0^{\hat{d}(p,k')} (p - k_{PR} - c)\, dc,$$

where k' and k_{PR} are the reported cost parameter and the real cost parameter. The manager of a low cost private hospital might have an incentive to mimic the high cost type. On the one hand, truthfully reporting $\underline{k}$ permits to treat more patients, and get a higher revenue $\hat{d}(p,\ k')p$. On the other hand, mimicking the high cost type permits to get rid of patients with a high severity of illness and reduces total costs $\hat{d}(p,\ k')\left(\underline{k} + \hat{d}(p,\ k')/2\right)$. The behavior of the manager depends on which of these effects dominates. The following proposition characterizes the behavior of the manager depending on the regulated price.

[9] In Italy, for instance, the DRG reimbursement is set by low at the national level. Regions contract with private hospitals on the maximum amount of services to be provided, given this reimbursement. See Francese and Romanelli (2010).

Proposition 5 *Consider the case when the private hospital is forbidden to dump.*

If $\Delta k \leq 1/2$*, the private manager exerts effort in cost reduction and truthfully reports the cost realization if and only if* $p \geq \bar{k} + 1/2$*.*

If $\Delta k > 1/2$*, any price induces the truthful revelation of the hospital specific cost parameter. For any price* $p \geq \hat{p} = \underline{k} + 1/2 + S/e$*, the manager exerts effort. The price* $\hat{p}$ *always belongs to the interval* $\left]\underline{k} + 1/2,\ \bar{k} + 1/2\right]$*.*

The intuition for this result is very simple. If a private hospital cannot dump patients freely, it might report a high hospital-specific cost parameter in order to be allowed to reject patients with a high severity of illness. If the efficiency differential is low ($\Delta k \leq 1/2$), $\hat{d}(p,\ \underline{k}) - \hat{d}(p,\ \bar{k})$ is quite low. Thus, if a manager mimics the high cost hospital, the loss in revenues is not too high. At the same time, the hospital can get rid of some high cost patients and reduce its total costs. For any $p < \bar{k} + 1/2$, it can be shown that the reduction in costs exceeds the loss in revenues. Thus, mimicking is a dominant strategy. If $p \geq \bar{k} + 1/2$, there is no gain from mimicking, since $\hat{d}(p,\ \underline{k}) = \hat{d}(p,\ \bar{k})$.

If the efficiency differential is high ($\Delta k > 1/2$), mimicking the high cost type is never profitable for the private manager, since it entails a loss in revenues larger than the reduction in total costs. The minimal price eliciting effort is always greater than $\underline{k} + 1/2$. If the hospital-specific cost parameter turns out to be low, the private hospital treats all patients.

4.2 Government contracting with a public and a private hospital when dumping is banned

Consider first the case when $\Delta k \leq 1/2$. If $p < \bar{k} + 1/2$, it has been shown in Proposition 4 that the private hospital always reports high costs. Expected health expenditures are equal to:

$$HE(p) = \bar{k} + \frac{1}{2} + \hat{d}(p,\ \bar{k}) \left(p - \bar{k} - \frac{\hat{d}(p,\ \bar{k})}{2} \right).$$

Since $p < \bar{k} + 1/2$, either $\hat{d}(p,\ \bar{k}) = 0$ or $\hat{d}(p,\ \bar{k}) = 2(p - \bar{k})$. In both cases, expected health expenditures are the same as $HE(0) = \bar{k} + 1/2$ and the regulator is indifferent between any price in the interval $\left[0,\ \bar{k} + 1/2\right[$. If $p \geq \bar{k} + 1/2$, then $\hat{d}(p,\ \bar{k}) = \hat{d}(p,\ \underline{k}) = 1$. In this case $HE(p) = p \geq \bar{k} + 1/2$. Such a price is clearly suboptimal for the regulator.

When $\Delta k > 1/2$, expected health expenditures are equal to:

$$HE(p) = \begin{cases} \bar{k} + \frac{1}{2} + \hat{d}(p,\ \bar{k})\left(p - \bar{k} - \frac{\hat{d}(p,\ \bar{k})}{2}\right) & if\ p < \hat{p} \\ \bar{k} + \frac{1}{2} + e\left[\hat{d}(p,\ \underline{\text{k}})\left(p - \bar{k} - \frac{\hat{d}(p,\underline{\text{k}})}{2}\right)\right] + (1-e)\left[\hat{d}(p,\ \bar{k})\left(p - \bar{k} - \frac{\hat{d}(p,\ \bar{k})}{2}\right)\right] & if\ p \geq \hat{p} \end{cases}$$

If $p < \hat{p}$, then no effort is exerted by the private manager, $k_{PR} = 0$ and health expenditures are equal to $\bar{k} + 1/2$. The regulator is indifferent between any price in the interval $[0,\ \hat{p}[$.

If $p \geq \hat{p}$, then the private hospital exerts effort and treats all the patients when the hospital specific cost parameter is equal to $\underline{\text{k}}$. Consequently, $HE(p) = \bar{e}p + (1-\bar{e})\left(\bar{k} + 1/2\right)$.This function is increasing in p. Thus, in this range of prices, costs are minimized if $p = \hat{p}$. To check that $HE(\hat{p})$ is a global minimum it is sufficient to compare it with the cost of provision when $p = 0$. This is always the case, since $\hat{p} \leq \bar{k} + 1/2$. Thus, the cost minimizing price is $p^{MND} = \hat{p} = \underline{\text{k}} + 1/2 + S/e$.

In such a case, it is possible for the regulator to reduce costs by using a hospital that is constrained not to dump. Expected health expenditures are equal to

$$HE(p^{MND}) \quad = \quad \bar{k} + \frac{1}{2} + S - \bar{e}\Delta k$$

Note that this expression is smaller than $\bar{k} + 1/2$ under Assumption 1. We have proved the following result:

Proposition 6 *Assume that the regulator is able to impose a no-dumping constraint on the private hospital.*

If $\Delta k \leq 1/2$*, the regulator cannot reduce expected health expenditures by contracting with the private hospital.*

If $\Delta k > 1/2$*, the regulator can reduce expected health expenditures by offering to the private hospital a price* $p^{MND} = \underline{k} + 1/2 + S/e$.

The no-dumping constraint reduces the rent attainable with truthful revelation, pushing managers to mimic the high type. Only a price high enough can elicit truthful revelation. Furthermore, contracting with the private hospital is optimal if and only if the price elicits both effort and truthful revelation. When $\Delta k \leq 1/2$, in order to elicit effort and truthful revelation, the regulator should set such a high price that she prefers to use exclusively the public hospital. Conversely, when $\Delta k > 1/2$, the price that elicits effort and truthful is not too high. Thus, contracting with the private hospital reduces expected expenditures.

Comparing the cost of provision when dumping is allowed and when a no-dumping constraint is enforced, it is possible to establish the following proposition.

Proposition 7 *Assume that the regulator can costlessly impose a no-dumping constraint on the private hospital.*

If $\Delta k \leq 1/2$, and $S \leq \bar{S}$ then it is optimal for the regulator not to enforce the constraint.

If $\Delta k > 1/2$, it is optimal for the regulator to enforce the constraint.

In all other cases, the regulator does not contract with the private hospital.

The regulator faces a trade-off between efficiency and access to care. If the hospital is free to dump, it can choose the profit maximizing number of patients. Thus, allowing dumping ensures a greater profit for the private hospital. This permits to obtain effort for lower prices than in the no-dumping scenario.

Consequently, imposing a no-dumping constraint is not always optimal. For low values of Δk, it is indeed optimal to let the private hospital dump patients. The no-dumping constraint implies too little profits when the cost parameter is low and truthfully reported, and the private manager lies in order to reject patients with a high severity of illness. In such a case, the regulator is not able to appropriate the benefit of managerial effort. Allowing the private hospital to dump reduces expected health expenditures, while the public hospital insures that all patients are treated.

When Δk is larger, however, it is possible to elicit effort and truthful revelation by fixing a price greater than $\underline{k} + 1/2$. Eliciting the investment in cost reduction is not too costly for the regulator since the benefit of effort is high and internalized by the hospital through a positive profit in case of low cost realizations. The model predicts that the production should be concentrated in the private hospital if $k^{pr} = \underline{k}$. If the cost parameter is high, the private hospital may still serve a positive share of patients.

5 Conclusion

This paper characterizes conditions under which a regulator optimally contracts with both a public and a private hospital for the production of health services. The private manager can appropriate profits, while the public one cannot. We show that it is optimal to contract with the private hospital whenever a very efficient managerial effort in cost reduction is available. If not, only the public hospital should produce, despite its lower cost efficiency.

Another finding of the paper is that imposing a no-dumping constraint on the private hospital is not always beneficial for the regulator, since it makes it more difficult to elicit effort and truthful revelation of the hospital-specific cost parameter. This result is somehow counterintuitive, but is economically justified by the presence of asymmetric information on costs leading to a the trade-off between efficiency and access to care.

An interesting extension of the paper would be to consider the role of physicians in hospitals' dumping decision. This analysis is not included in the paper and is left for further investigation. Wright (2007) argues that the regulator might use a salary instead than a fee for service to remunerate physicians in public hospitals in order to attract in the public sector more fairness prone physicians not engaging in dumping.

Another line of research concerns how different distributions of the patient specific cost parameter c affect the optimal public-private mix. In fact, health costs are not likely to be distributed uniformly. Usually most patients have similar severities of illness, while some outliers are very costly to treat. A change in the variance of the severities of illness affects the contract offered to the private hospital and the optimality of a no-dumping constraint. It is easy to show that, if high severities of illness are relatively more rare (the distribution is first order stochastically dominated by the uniform distribution), the regulator finds it more profitable to contract with the private hospital: even if the price is low, only few patients get dumped, and it is easier to provide incentives to the private manager.

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Appendix. Proofs of Lemmas and Propositions

Proof of Lemma 1

Since the supply of the private hospital is discontinuous in p, to prove the lemma we will analyze in turn all the possible cases that might arise.

- If $p < \underline{k}$, no patient are treated by the private hospital and profit is equal to zero both in case of effort and in case of non effort. $\Delta\Pi(p,\ w)$ is equal to zero and constant in p.

- If $p \in \left]\underline{k},\ \bar{k}\right]$, some patients are treated if the hospital specific cost parameter is low. Otherwise, all patients are dumped. Then
$$\Delta\Pi(p,\ w) = D_{PR}\left(e\frac{(p-\underline{k})^2}{2}\right),$$
which is increasing in p. Moreover, $\Delta\Pi(\bar{k},\ w) = D_{PR}\left(e(\Delta k)^2/2\right)$.

- If $p \in \left]\bar{k},\ \underline{k}+1\right]$, the private hospital treats a positive share of $D_{PR}(w)$ whichever the realization of k. Then
$$\Delta\Pi(p,\ w) = D_{PR}\left[e\left(\frac{(p-\underline{k})^2}{2} - \frac{(p-\bar{k})^2}{2}\right)\right],$$
which is increasing in p . Moreover, $\lim_{p\to\bar{k}}\Delta\Pi(p,\ w) = D_{PR}\left(e(\Delta k)^2/2\right)$, and $\Delta\Pi(\underline{k}+1,\ w) = D_{PR}\left[e\left(\Delta k - (\Delta k)^2/2\right)\right]$.

- If $p \in \left]\underline{k}+1,\ \bar{k}+1\right]$, the private hospital treats all patients if the hospital specific cost parameter is low, and a positive share of patients if it is high. Then,
$$\Delta\Pi(p,\ w) = D_{PR}\left[e\left(p-\underline{k}-\frac{1}{2}-\frac{(p-\bar{k})^2}{2}\right)\right],$$
which is increasing in p . Moreover, $\lim_{p\to\underline{k}+1}\Delta\Pi(p,\ w) = D_{PR}\left[e\left(\Delta k - (\Delta k)^2/2\right)\right]$ and $\Delta\Pi(\bar{k}+1,\ w) = D_{PR}\left(e\Delta k\right)$.

- Finally, if $p > \bar{k}+1$, the private hospital treats all the patients whichever the realization of k. Then
$$\Delta\Pi(p,\ w) = D_{PR}\left(e\Delta k\right),$$
which is constant in p. Moreover $\lim_{p\to\bar{k}+1}\Delta\Pi(\bar{k}+1,\ w) = D_{PR}\left(e\Delta k\right)$.

We have shown that the function $\Delta\Pi(p,\ w)$ is piecewise-defined and continuous in p. Furthermore, it is non-decreasing in any interval in which it is piecewise-defined. Thus, the function is monotonically non decreasing. **Similarly, we can prove that the function is increasing in w.** $\blacksquare$

Proof of Proposition 1

- Consider first the case in which $S/D_{PR} < \left(e(\Delta k)^2/2\right)$. From the preceding proof we know that $\Delta\Pi(p,\ w) > S$ for any $p > \underline{\mathrm{k}}$. Using the monotonicity of $\Delta\Pi(p,\ w)$, and $\Delta\Pi(p,\ w) = 0\ \ \forall p < \underline{\mathrm{k}}$, we can conclude that $\tilde{p} \in \left]\underline{\mathrm{k}},\ \bar{k}\right]$. The value of $\tilde{p}$ is given by

$$D_{PR}\left(e\frac{(\tilde{p}-\underline{\mathrm{k}})^2}{2}\right) = S \iff \tilde{p} = \sqrt{\frac{2S}{eD_{PR}}} + \underline{\mathrm{k}}.$$

- Consider now the case in which $e(\Delta k)^2 \leq S/D_{PR} < e\Delta k\,(1-\Delta k/2)$. From the preceding proof, we know that $\Delta\Pi(p,\ w) < S$ for any $p < \bar{k}$ and that $\Delta\Pi(p,\ w) > S$ for any $p > \underline{\mathrm{k}} + 1$. Using the monotonicity of $\Delta\Pi(p,\ w)$, we can conclude that $\tilde{p} \in \left]\bar{k},\ \underline{\mathrm{k}}+1\right]$. The value of $\tilde{p}$ is given by

$$D_{PR}\left[e\left(\frac{(\tilde{p}-\underline{\mathrm{k}})^2}{2} - \frac{(\tilde{p}-\bar{k})^2}{2}\right)\right] = S \iff \tilde{p} = \frac{S}{D_{PR}\,(e\Delta k)} - \frac{\Delta k}{2} + \bar{k}.$$

- Consider finally the case in which $S/D_{PR} > \bar{e}\Delta k\,(1-\Delta k/2)$. From the preceding proof, we know that $\Delta\Pi(p,\ w) < S$ for any $p < \underline{\mathrm{k}} + 1$ and $\Delta\Pi(p,\ w) > S$ for any $p > \bar{k} + 1$. Using the monotonicity of $\Delta\Pi(p,\ w)$, we can conclude that $\tilde{p} \in \left]\underline{\mathrm{k}}+1,\ \bar{k}+1\right]$. The value of $\tilde{p}$ is given by

$$D_{PR}\left[e\left(\tilde{p} - \underline{\mathrm{k}} - \frac{1}{2} - \frac{(\tilde{p}-\bar{k})^2}{2}\right)\right] = S \iff \tilde{p} = 1 + \bar{k} - \sqrt{2\left(\Delta k - \frac{S}{eD_{PR}}\right)}\ . \blacksquare$$

Proof of Lemma 2

First of all, note that if $p = 0$ then only the public hospital produce. Then, $HE(0) = \bar{k} + 1/2$. Consider now a price $p^{'} < \tilde{p}$. The private hospital does not exert any effort at this price. Expected health expenditures

$HE(p')$ are then equal to

$$p' D_{PR}\bar{d}(p',\ w) + \left[1 - D_{PR}\bar{d}(p',\ w)\right]\left[\bar{k} + \left(1 + D_{PR}\bar{d}(p',\ w)\right)/2\right].$$

It is easy to prove that $HE(p') = \bar{k} + 1/2 + (p - \bar{k})D_{PR}\bar{d}(p',\ w) + (\bar{d}(p',\ w))^2/2$. Since $\bar{d}(p',\ w)$ is equal to zero whenever $p' - \bar{k} \leq 0$, and positive whenever $p' - \bar{k} > 0$, we can conclude that $HE(p') \geq HE(0)$ for any $p^{'} < \tilde{p}$.■

Proof of Proposition 3, points (ii) and (iii)

We have shown in section 3 that

$$e(\Delta k)^2/2 \quad < \quad S/D_{PR} \leq min\left\{e\left(\Delta k - \Delta k^2/2\right),\ e(\Delta k)^2\left(\sqrt{e} + \frac{1}{2}\right)\right\} \Longrightarrow p = \tilde{p} \tag{9}$$

and

$$e\left(\Delta k - \Delta k^2/2\right) < S/D_{PR} \leq e\left(\Delta k - (1 + \sqrt{e})^{-2}/2\right) \Longrightarrow p = \tilde{p} \tag{10}$$

Note that $e(\Delta k)^2\left(\sqrt{\bar{e}} + 1/2\right) \leq e\left(\Delta k - (\Delta k)^2/2\right) \iff \Delta k \leq 1/(\sqrt{\bar{e}}+1)$. In this case, $e\left(\Delta k - (1 + \sqrt{e})^{-2}/2\right) \leq e\left(\Delta k - \Delta k^2/2\right)$, and condition (10) cannot be satisfied. Thus, $p^{MIX} = \tilde{p}$ if and only if $e(\Delta k)^2/2 < S \leq e(\Delta k)^2\left(\sqrt{\bar{e}} + 1/2\right)$.

Conversely, $e(\Delta k)^2\left(\sqrt{\bar{e}} + 1/2\right) < e\left(\Delta k - (\Delta k)^2/2\right) \iff \Delta k > 1/(\sqrt{e}+1)$. In this case, $e\left(\Delta k - (1 + \sqrt{e})^{-2}/2\right) > e\left(\Delta k - \Delta k^2/2\right)$ and condition (10) is satisfied. Then, $p^{MIX} = \tilde{p}$ if and only if $e(\Delta k)^2/2 < S \leq e\left(\Delta k - (1 + \sqrt{e})^{-2}/2\right)$.

Summarizing $p^{MIX} = \tilde{p}$ if and only if $e(\Delta k)^2/2 < S/D_{PR} \leq min\left\{e(\Delta k)^2\left(\sqrt{e} + \frac{1}{2}\right),\ e\left(\Delta k - (1 + \sqrt{e})^{-2}/2\right)\right\}$.■

Proof of Proposition 4

Assume that $w^{MIX} = 0$.

If $2S \leq \bar{S}$, the optimal price is $p^{MIX}(0) \geq \tilde{p}(0) \geq 0$. Since $\tilde{p}$ is decreasing in w, this price also elicits effort if the waiting time is positive. The difference between the total cost of provision in the two cases, $HE(p^{MIX}(0),\ 0) - HE(p^{MIX}(0),\ w > 0)$ is equal to

$$-\frac{1}{2}\left[e\underline{\mathrm{d}}(p^{MIX}(0))\left(p^{MIX}(0) - \bar{k} - \frac{\underline{\mathrm{d}}(p^{MIX}(0))}{2}\right) + (1 - e)\bar{d}(p^{MIX}(0))\left(p^{MIX}(0) - \bar{k} - \frac{\bar{d}(p^{MIX}(0))}{2}\right)\right].$$

This difference is negative if and only if

$$p^{MIX}(0)\left[e\underline{\mathrm{d}}(p^{MIX}(0))+(1-e)\bar{d}(p^{MIX}(0))\right]>$$
$$\bar{k}\left[e\underline{\mathrm{d}}(p^{MIX}(0))+(1-e)\bar{d}(p^{MIX}(0))\right]+e\frac{\left(\underline{\mathrm{d}}(p^{MIX}(0))\right)^2}{2}+(1-\bar{e})\frac{\left(d(p^{MIX}(0),\ \underline{\mathrm{k}})\right)^2}{2},$$

which is a contradiction with $p^{MIX}(0)>0$ being cost minimizing. In fact, if this inequality holds, it is optimal to delegate all the production to the public hospital (it is optimal to contract with the private hospital if and only if, in expectation, the latter gets a smaller price than the average costs at the public hospital). Thus, the inequality does not hold and we can write $HE(p^{MIX}(0),\ 0)\geq HE(p^{MIX}(0),w>0)\geq HE(p^{MIX}(w),w>0)$. A positive waiting time is expenditure minimizing. Since any positive waiting time w permits to reduce expenditures, by pushing all patients to visit the private hospital first, $w^{MIX}=\epsilon$, with $\epsilon>0$, $\epsilon<\bar{\epsilon}$ and $\bar{\epsilon}$ very small.

If $2S>\bar{S}$ no production is optimally delegated to the private hospital when $w^{MIX}=0$. The optimal price is $p^{MIX}(0)=0$. If $S\leq\bar{S}$, from Proposition 3 we know that, whenever $w>0$, a positive price reduces expenditures with respect to a pure public provision. Thus a positive waiting time is welfare enhancing. Conversely, if $S>\bar{S}$ there are no benefits from a positive waiting time, and $w^{MIX}=0$.

Proof of Proposition 5

Consider an hospital whose cost realization is $\underline{\mathrm{k}}$. If $p<\underline{\mathrm{k}}+1/2$, the profit of the hospital is equal to zero whenever the manager truthfully reports the low cost parameter. If $p\geq\bar{k}$, mimicking the high cost type leads to a positive production and a positive profit. Thus, it is optimal for the manager to misreport the cost parameter. If $p<\bar{k}$, mimicking the high cost type leads to a profit equal to zero. The manager has no incentive to misreport. In this case, however, the expected profit is equal to zero, and the manager has no incentive to exert effort in the first place. If $p>\underline{\mathrm{k}}+1/2$, different cases might arise depending on the level of Δk. We consider separately the case when $\Delta k\leq 1/2$ and the case when $\Delta k>1/2$

- If $\Delta k\leq 1/2$, then $\underline{\mathrm{k}}<\bar{k}\leq\underline{\mathrm{k}}+1/2<\bar{k}+1/2$. A price $p\geq\underline{\mathrm{k}}+1/2$ induces the private hospital to treat all patients if it $\bar{k}$ and a positive amount of patients if it reports high costs.

  If $p\in\left]\underline{\mathrm{k}}+1/2,\ \bar{k}+1/2\right]$, if the manager truthfully reports $k'=\underline{\mathrm{k}}$, he gets a profit equal to $p-\underline{\mathrm{k}}-1/2$; by mimicking the high cost type, the manager gets a profit equal to$2\Delta k(p-\bar{k})$. Truthfully reporting the cost parameter is thus incentive compatible if and only if

$$p - \underline{k} - \frac{1}{2} - 2\Delta k(p - \bar{k}) \geq 0,$$

which is impossible whenever $p \in \left]\underline{k} + 1/2,\ \bar{k} + 1/2\right]$ and $\Delta k \leq 1/2$. Thus, mimicking the high cost type is the best strategy for a low cost hospital.

If, if $p > \bar{k} + 1/2$, the hospital treats all patients irrespectively on the reported cost parameter. There is no incentive to misreport. *Ex ante*, the manger exerts high effort if and only if

$$p - e\underline{k} - (1-e)\bar{k} - \frac{1}{2} - S \geq p - \bar{k} - \frac{1}{2} \iff S \leq e\Delta k,$$

which is always true under Assumption 1.

Thus, the private hospital always mimics the high cost type, in order to appropriate the rent associated to the investment in cost reduction, unless it receives a price $p > \bar{k} + 1/2$.

- If $\Delta k > 1/2$, then $\underline{k} < \underline{k} + 1/2 < \bar{k} < \bar{k} + 1/2$. There exist some price that induces the private hospital to treat all the patients in case of low costs and not to treat any patients in case of high costs.

  If the $p \in \left]\underline{k} + 1/2,\ \bar{k}\right]$ the private hospital does not treat any patient and the profit is equal to zero whenever the manager reports a cost parameter is $\bar{k}$. If the realization of the cost parameter is $\underline{k}$ and the manager truthfully reports costs, all the patients have to be treated and profits are equal to $p - \underline{k} - 1/2 \geq 0$. The manager has no incentives to misreports costs. *Ex ante*, the investment in cost reduction is made if and only if

$$\bar{e}\left(p - \underline{k} - \frac{1}{2}\right) \geq S \iff p \geq \hat{p} = \underline{k} + \frac{1}{2} + \frac{S}{e}.$$

  Remark that the minimal price necessary to elicit effort is indeed lower than $\bar{k}$ if and only if $S \leq e\Delta k - e/2$.

  If $p \in \left]\bar{k},\ \bar{k} + 1/2\right]$, profits are equal to zero if the cost parameter turns out to be $\bar{k}$. In case of low costs, if the manager truthfully reports the cost realization, profits are equal to $(p - \underline{k} - 1/2)$. If the manager mimics the high cost type, the profit is equal to $2\Delta k(p - \bar{k})$. Truthfully reporting the cost parameter is thus incentive compatible if and only if the difference between these two profits is positive:

$$p - \underline{\mathrm{k}} - \frac{1}{2} - 2\Delta k(p - \bar{k}) \geq 0.$$

which is always true if $\Delta k > 1/2$. *Ex ante*, the hospital exerts effort if and only if

$$e\left(p - \frac{1}{2} - \underline{\mathrm{k}}\right) - S \geq 0 \iff p \geq \hat{p} = \underline{\mathrm{k}} + \frac{1}{2} + \frac{S}{e}.$$

The minimal price necessary to exert effort is lower than $\bar{k} + 1/2$ for any $S \leq e\Delta k$.

Finally, if $p > \bar{k} + 1/2$ the private hospital will treat all patients in case of high and low costs. Thus, the private manager does not have any incentive to misreport the realization of costs. *Ex ante*, the hospital exerts effort if and only if

$$p - e\underline{\mathrm{k}} - (1-e)\bar{k} - \frac{1}{2} - S \geq p - \bar{k} - \frac{1}{2} \iff S \leq e\Delta k,$$

which is always the case under assumption 1. Thus, the minimal price eliciting effort is always smaller than $\bar{k} + 1/2$.

Summarizing, the private manager always reports truthfully. He exerts effort for any price greater or equal to $\hat{p} = \underline{\mathrm{k}} + 1/2 + S/e$.

Proof of Proposition 7

To prove the proposition, we have to compare , $HE(p^{MIX})$ and $HE(p^{MND})$.

- If $\Delta k \leq 1/2$, we know from Proposition 5 that $p^{MND} = 0$ and expected health expenditures if the no-dumping constraint is enforced are equal to $\bar{k}+1/2$. Thus, setting the hospital free to refuse patients is optimal whenever $p^{MIX} > 0$ This is true for any $e(\Delta k)^2/2 < S \leq \bar{S}$.
- If $\Delta k > 1/2$ and the no-dumping constraint is enforced, expected health expenditure are equal to $HE(p^{MND}) = \bar{k} + 1/2 + S - \bar{e}\Delta k$. $HE(p^{MIX})$ depend on the efficiency of managerial effort.

  If $S > \bar{S}$, then $p^{MIX} = 0$ and $HE(p^{MIX}) = \bar{k} + 1/2 \geq HE(p^{MND})$.

If $S \leq \bar{e}(\Delta k)^2/2$, then $p^{MIX} = \bar{k}$ and from equation (6) we have:

$$HE(p^{MIX}) = \bar{k} + \frac{1}{2} - e\frac{(\Delta k)^2}{2},$$

which is greater than $HE(p^{MND})$ for any $S \leq \bar{e}(\Delta k)^2/2$.

If $e(\Delta k)^2/2 < S \leq e(\Delta k)^2 \left(\sqrt{\bar{e}} + 1/2\right) \leq e(\Delta k - (\Delta k)^2/2)$, then $p^{MIX} = \tilde{p}$ and from equation (7) we have:

$$HE(p^{MIX}) = \left(\frac{S}{e\Delta k} - \frac{\Delta k}{2}\right)^2 - \frac{e(\Delta k)^2}{2} + \bar{k} + \frac{1}{2}.$$

which is greater than $HE(p^{MND})$ for any $S \leq e(\Delta k - (\Delta k)^2/2)$.

Finally, if $e\left(\Delta k - (\Delta k)^2/2\right) < S \leq e\left(\Delta k - (1+\sqrt{e})^{-2}/2\right)$, then $p^{MIX} = \tilde{p}$ and from equation (8) we have

$$HE(p^{MIX}) = 1 + \bar{k} - \sqrt{2\left(\frac{e\Delta k - S}{e}\right)} + (1 - \bar{e})\left(\frac{e\Delta k - S}{e}\right).$$

This expression is greater than $HE(p^{MND})$ if and only if

$$\frac{1}{2} - \sqrt{A} + \frac{1}{2}(1 - e)A \geq S - e\Delta k,$$

where $A = 2(e\Delta k - S)/e$. The inequality can then be rewritten as $1 - 2\sqrt{A} + A \geq 0$. This is true whenever $A \leq 1$. Thus, a necessary and sufficient condition for $HE(p^{MIX}) \geq HE(p^{MND})$ is:

$$2(e\Delta k - S)/e \leq 1 \iff S \geq e\Delta k - \frac{e}{2}.$$

This condition is always satisfied when $S > e\left(\Delta k - (\Delta k)^2/2\right)$ since $\Delta k < 1$.

Summarizing, if the efficiency gain related to the effort in cost reduction is relatively high ($\Delta k > 1/2$), then it is optimal for the regulator to impose a no-dumping constraint on the private firm.