

Black holes production in self-complete quantum gravity

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Abstract

A regular black hole model, which has been proposed by Hayward in [1], is reconsidered in the framework of higher dimensional TeV unification and self-complete quantum gravity scenario [2,3,4]. We point out the “quantum” nature of these objects and compute their cross section production by taking into account the key role played by the existence of a *minimal length* l_0 . We show as the threshold energy is related to l_0 . We recover, in the high energy limit, the standard “black-disk” form of the cross section, while it vanishes, below threshold, faster than any power of the invariant mass-energy $\sqrt{-s}$.

1 Introduction

Astrophysical size black holes are well modeled by classical solutions of the Einstein equations. Theoretical predictions like the existence of accretion disks and plasma jets along rotation axis of spinning black holes have been confirmed by the amazing photos taken by orbiting telescopes.

Much different is the case of “quantum black holes” of microscopic size, whose description calls for a consistent theory of quantum gravity. String theory is presently the only framework providing a finite, anomaly-free, perturbative formulation of gravity at the quantum level. However, even in this framework, black holes are usually treated as classical solutions of certain Super-Gravity

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field theory emerging from a related Super-String model in the “point-like limit”. This approach is powerful enough to shed some light on the microscopic origin of black hole entropy (at least in certain cases) and to introduce higher dimensional objects, i.e. D -branes, as solitonic solutions of the field equations. However, many answers are still to come, e.g. the form of the mass spectrum of a quantum black hole, a satisfactory resolution of the Information Paradox, an explanation of the horizon surface quantization in elementary Planck cells, etc. All these problems are currently under an intensive investigation. Even more compelling, is a satisfactory solution to the “singularity problem”: one expects that in a theory of extended objects, the very concepts of (point-like) curvature singularity should be meaningless. In other words, one would like to show that in a quantum theory of gravity there should not be any curvature singularity neither “naked” nor hidden by an event horizon [5,6]. Unfortunately, this problem cannot be treated in any kind of field theory (including Super-Gravity) where fundamental objects are basically point-like. At a first glance, this kind of physics could appear purely speculative and totally detached from any experimental verification. It may be true. However, TeV quantum gravity is an intriguing spin-off of non-perturbative String Theory, where all four interactions, including gravity, are unified at an energy scale much lower than the Planck energy and, presumably, not too far away from LHC peak energy, i.e. $14 TeV$. The start of LHC runs opens the actual possibility to test “new physics” beyond the Standard Model, hopefully, including signatures of “quantum gravity” phenomena [7,8,9,10,11,12,13].

With this background in mind, we consider an effective approach to the singularity problem, where (semi)classical Einstein equations are used to determine black hole solutions “keeping memory” of their quantum nature.

A common feature of all candidate theory of quantum gravity is the existence of a fundamental length scale where the very concept of space-time as a classical manifold breaks down. Speaking of arbitrarily small distances becomes meaningless and the concept of *minimal length*, l_0 , emerges as a new fundamental constant of Nature on the same footing as the speed of light and Planck quantum of action [14].

String Theory [15,16,17], non-commutative coordinates coherent states [18,19,20,21,22,23], Generalized Uncertainty Principle [24,25], Path Integral Duality [26,27], etc., share this common feature.

In reference [1] an intriguing model of singularity-free black hole was proposed in order to investigate, in a safe environment, back-reaction effects of the Hawking radiation and the late stage of black hole evaporation. There are several ways to change the form of the standard line element in order to include quantum gravity effects [28,29,30,31,32,33,34], [35,36,37] The “effective-model” in [1] has several distinctive features. From our vantage point, we notice that

- it is mathematically simple and allows analytic calculations;
- it encodes the basic features of more “sophisticated” models of quantum

gravity improved black holes, e.g. non-commutative geometry inspired, or Loop-quantum gravity black holes;

- the mass spectrum is bounded from below by an extremal configuration;
- the Hawking temperature vanishes as the extremal configuration is approached.

The line element is given by

$$ds^2 = - \left(1 - \frac{r_s r^2}{r^3 + r_s l_0^2} \right) dt^2 + \left(1 - \frac{r_s r^2}{r^3 + r_s l_0^2} \right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) , \quad (1)$$

where, $r_s = 2MG_N$ and l_0 is a new fundamental constant on the same ground as c and \hbar . In order to keep some degree of generality, we do not choose any specific value for l_0 and consider it as a free, model-dependent, parameter ranging from the TeV up to the Planck scale.

This metric is a solution of the Einstein equations with the following energy-momentum tensor:

$$T_0^0 = -\rho = -\frac{1}{8\pi} \frac{3l_0^2 r_s^2}{(r^3 + r_s l_0^2)^2} , \quad (2)$$

$$T_r^r = p_r = -\rho , \quad (3)$$

$$T_\theta^\theta = p_\theta = \frac{1}{8\pi} \frac{6l_0^2 r_s^2 (r^3 - r_s l_0^2)}{(r^3 + r_s l_0^2)^3} , \quad (4)$$

$$T_\phi^\phi = p_\phi = p_\theta \quad (5)$$

The “weak-point” of the model is that the form of T_ν^μ is not recovered by an underlying theory, it is assumed in order to source the (1) field.

It is immediate to check that at short distance the metric is a regular deSitter geometry characterized by an *effective* cosmological constant of “Planckian” size $\Lambda_{eff} \equiv \frac{3}{l_0^2}$. Thus, the curvature singularity is replaced by a core of ultra-dense deSitter vacuum. *This is the key mechanism to stop matter to collapse into a singular, infinite density configuration.* The deSitter vacuum plays the role of “Planckian foam”, a chaotic state of violent quantum gravitational fluctuations disrupting the very fabric of the space-time continuum³. In this paper we are going to provide a generalization of the line element 1 in the framework of TeV quantum gravity and compute the production cross section

³ From a formal point of view, one can say that the “regularization” of the curvature singularity can be encoded into the substitution rule

$$\frac{1}{r} \longrightarrow \frac{r^2}{r^3 + r_s l_0^2} \quad (6)$$

for this type of black holes, by taking into account the existence of a minimal length. In this case, the “hoop-conjecture” [38] must be properly modified to comply with the fact that an impact parameter smaller than l_0 is physically meaningless.

2 *TeV* “quantum” black hole

In extending the metric (1) in $d + 1$ dimensions we choose a slightly different definition of l_0 for reasons which will become clear later on

$$ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2 d\Omega_{d-1} , \quad (7)$$

$$f(r) = 1 - \frac{2MG_*r^2}{r^d + 2\frac{d-2}{d}MG_*l_0^2} \quad (8)$$

where, $d\Omega_{d-1}$ is the infinitesimal solid angle in $d - 1$ dimensions and G_* is the *TeV*-scale gravitational coupling with dimensions $[G_*] = L^{d-1} = E^{1-d}$. In our scenario the total mass energy of the system equals the invariant mass of the two colliding partons, i.e.

$$M = \sqrt{-s} \quad (9)$$

where $-s$ is the Mandelstam variable representing the center of mass energy squared.

The position of the horizon(s) is determined by the equation $f(r_H) = 0$. This is an algebraic equation of degree d . For arbitrary $d > 3$ one can plot M as a function of r_H :

$$M = \frac{1}{2G_*} \frac{r_H^d}{r_H^2 - \frac{d-2}{d} l_0^2} \quad (10)$$

For an assigned value of M equation (10) has two solutions, i.e. $r_H = r_{\pm}$, provided $M > M_0$, where M_0 is the absolute minimum of the function. The minimum corresponds to the degenerate horizon of an extremal black hole and is determined by the two conditions

$$f(r_0) = 0 , \quad (11)$$

$$f'(r_0) = 0 , \quad (12)$$

By solving the system (11),(12) we get

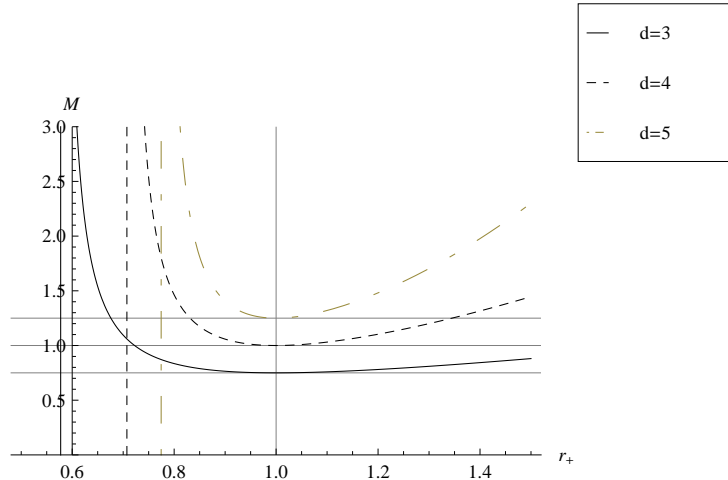


Fig. 1. Plot of $M(r_H)$ vs r_+ for $d = 3, 4, 5$. The position of the minimum, which corresponds to the extremal black hole, is independent from d . l_0 is our unit of length.

$$r_0 = l_0, \quad (13)$$

$$M_0 = \frac{1}{4G_*} \frac{d}{l_0^{d-2}}, \quad (14)$$

Equation (13) is a new and interesting result showing that *there cannot exist black holes with radius smaller than the minimal length*. This is not only a self-consistency check of our model, but also the proof that extremal and near-extremal black holes are “quantum” object as their size falls in a quantum gravity fluctuations dominated range. At the same time, the existence of this kind of objects makes impossible to probe shorter distances. It is important to recall that any further increase in energy makes black holes bigger reducing the resolution power, and not viceversa as in the case of ordinary particles. This is the crux of the quantum gravity self-completeness scenario which has been recently discussed in [2,3,4]. Furthermore, it is worth noticing that r_0 is the same for any d being determined by the “universal” constant l_0 alone.

More in detail, we have three possible cases:

- $M > M_0$ the metric has an (outer) Killing horizon, $r_H = r_+$, and an (inner) Cauchy horizon $r_H = r_-$;
- $M = M_0$ the two horizons coincide and the metric describe an *extremal* black hole;
- $M < M_0$ there are no horizon and the line element is sourced by a particle-like object. If M is not too small with respect to M_0 , let us call these objects “*quasi-black holes*” to remark that they fall in the intermediate range of masses between particles and black holes.

Thus, we meet a further nice feature of the model: it smoothly interpolates between “particles” and black holes by increasing the total energy of the system. The transition between point-like objects and black holes is defined by the mass M_0 of the extremal configuration. A detailed investigation of the thermodynamics properties of the black hole (8) is postponed to a future paper, while we proceed in the next section to the calculation of the production cross section.

3 Cross section

It is generally assumed that the production cross section for a black hole of radius r_H is simply its transverse area $\sigma(s) = \pi r_H^2(s)$. Such a “black disk” cross section encodes the *hoop-conjecture* [38]: if two partons collides with energy $\sqrt{-s}$ and *impact parameter* b , black holes can be produced if

$$b \leq r_H(s) \tag{15}$$

Thus, the scattering cross section for partons of impact parameter b reads

$$\frac{d\sigma}{db} = 2\pi b \Theta_H[r_H(s) - b] \tag{16}$$

where, Θ_H is the Heaviside step-function. By integration over the un-observable b parameter, one gets the cross section

$$\sigma(s) = 2\pi \int_0^\infty db b \Theta_H[r_H(s) - b] = \pi r_H^2(s) \tag{17}$$

As simple as that, this result is a little too naive. If equation (17) is literally taken, we conclude that the probability to produce a black hole is non-zero even at arbitrary low energy. This is a result conflicting with all the known particle phenomenology. When a classical argument like the hoop-conjectured is mismatched with a quantum cross section, the result can often be unsatisfactory.

The root of the problem is that we let the impact parameter to range over arbitrary small values while quantum gravity introduces sever limitations to the very concept of arbitrarily small lengths. In our case, it is meaningless to think about an impact parameter smaller than l_0 . We can translate this feature by requiring that our effective model of “quantum” black holes breaks for $b < l_0$, and the cross section vanishes faster than any power of s . This kind

of asymptotic behavior can be achieved by introducing an exponential cut-off in the integration measure in (17)

$$db \longrightarrow db e^{-l_0^2/b^2} \quad (18)$$

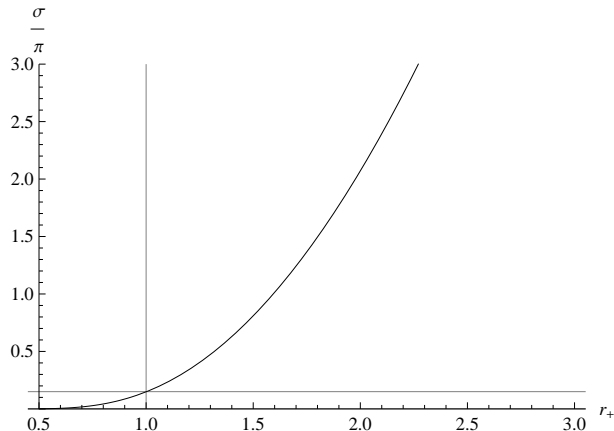


Fig. 2. Plot of $\sigma(s)/\pi$ vs r_+ . We set $l_0 = 1$. The black disk limit is reached for $r_+ \gg 1$.

The resulting cross section reads

$$\sigma(s) = \pi l_0^2 \Gamma[-1; l_0^2/r_H^2(s)] \quad (19)$$

where, $\Gamma[-1; l_0^2/r_H^2(s)]$ is the upper incomplete Gamma function which is defined as

$$\Gamma(\alpha; x) \equiv \int_x^\infty dt t^{\alpha-1} e^{-t} \quad (20)$$

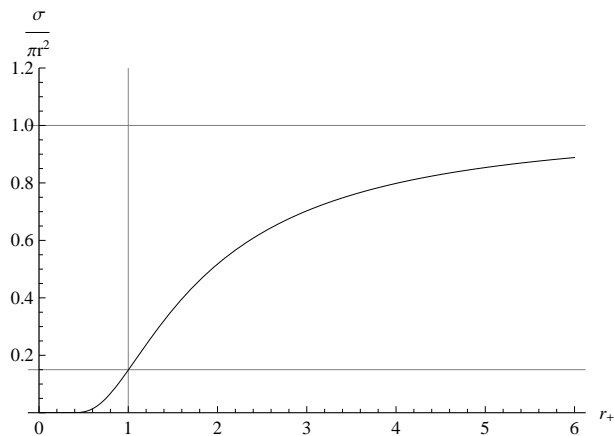


Fig. 3. Plot of $\sigma(s)/\pi r_+^2$. The horizontal asymptote represents the black disk limit.

At low energy, which is $\sqrt{-s} \ll M_0$, the cross section vanishes as

$$\sigma(s) \approx \pi l_0^2 \left(\frac{r_H^4}{l_0^4} \right) e^{-l_0^2/r_H^2} \quad (21)$$

in agreement with our requirement.

In the opposite limit we need the following relation

$$x^{-\alpha} \Gamma(\alpha; x) \longrightarrow -\frac{1}{\alpha}, \quad x \longrightarrow 0 \quad (22)$$

Thus, for $\sqrt{-s} \gg M_0$, we recover the black disk cross section

$$\sigma(s) \approx \pi l_0^2 \left(\frac{l_0^2}{r_H^2} \right)^{-1} = \pi r_H^2(s) \quad (23)$$

The cross section (19) smoothly interpolates between the (high energy) black disk limit and an exponentially decreasing behavior below threshold. Immediately above threshold, it is energetically preferred to produce (near-) extremal black holes. The corresponding cross section can be estimated to be

$$\sigma(\sqrt{-s} = M_0) = \pi l_0^2 \Gamma[-1; 1] \approx 0.15 \pi l_0^2 \quad (24)$$

We have seen that below threshold black hole cannot exist. They are both off-mass shell and too small to be considered as physical object. Thus, we need to understand the physical meaning of the exponentially suppressed, but non-zero, tail of the below threshold cross section. As our model smoothly interpolates among different kind of physical objects

$$\text{particles} \longleftrightarrow \text{quasi-black holes} \longleftrightarrow \text{black holes}$$

we propose to continue the cross section (21) below threshold in the following way. A quasi-black hole is a particle-like object characterized by a Compton wave-length $\lambda_C = 1/\sqrt{-s}$. From the vantage point of self-complete quantum gravity the transition between particle and black holes implies to replace $\lambda_C(s)$ with $r_+(s)$ (or, viceversa) as a characteristic length of the object itself. In order to fit the asymptotic behavior (21), we replace (19) below threshold with

$$\sigma(s) = \pi l_0^2 \Gamma[-1; \lambda_C^2(s)/l_0^2], \quad (25)$$

Cross section (25) smoothly joins to (19) at the critical point $\sqrt{-s} = M_0$ and exponentially vanishes at low energy as

$$\sigma(s) \approx \pi l_0^2 \frac{l_0^4}{\lambda_C(s)^4} e^{-\lambda_C(s)^2/l_0^2}, \quad \lambda_C(s) \gg l_0 \quad (26)$$

Thus, we can say that below threshold cross section (25) describe the production of quasi-black holes which are very massive objects with respect to ordinary particles, but not yet heavy enough to collapse into black holes.

It is worth to remark, that even after crossing the threshold and approaching the geometric limit, “Trans-Planckian” scale is never probed as $r_+ > l_0$. The fate of near-extremal, non-thermal black holes will be discussed in the next section.

4 Conclusions and speculations

In this last section we would like to offer some speculations about one of the most intriguing features of black thermodynamics, i.e. horizon surface quantization in elementary Planck cells.

In spite of its simplicity, our model can offer a simple recipe for the mass spectrum and the late stages of Hawking evaporation. The temperature

$$T_H = \frac{d-2}{4\pi r_+} \left(1 - \frac{l_0^2}{r_+^2} \right), \quad r_+ \geq l_0 \quad (27)$$

vanishes as the genuine quantum regime is approached, i.e. $r_+ \rightarrow l_0$. In this phase, thermal behavior is negligible and we expect different decay modes to provide the dominant contribution.

The crux of our argument is once again relation (13). As l_0 is the minimal length, the area of the event horizon of the extremal configuration is entitled to be seen as the fundamental “*quantum of area*”

$$A_0 = 4\pi l_0^{d-1} \quad (28)$$

From this vantage point, we can see any non-extremal black hole as an “*areal excitation*” of the *ground state* given by the extremal configuration, in the sense that the area of the event horizon is an integer multiple of A_0 . This is our quantization condition:

$$A_H = 4\pi n l_0^{d-1}, \quad n = 1, 2, 3, \dots \quad (29)$$

The relation (29) takes into account the cellular structure of the horizon where the quantum of area $4\pi l_0^{d-1}$ plays the role which is generally assigned to the Planck area l_{Pl}^2 .

From equation (29) it is immediate to derive the quantization rule for the event horizon radius

$$r_+ = n^{1/(d-1)} l_0 \quad (30)$$

Finally, by inserting (30) in (10) one gets the mass spectrum

$$M_n = \frac{l_0^{d-2}}{2G_*} \frac{n^{d/(d-1)}}{n^{2/(d-1)} - (d-2)/d} \quad (31)$$

The mass spectrum is bounded from below by the mass of the extremal black hole, i.e. $M_{n=1} = M_0$, and approaches a continuum in the semi-classical limit $n \gg 1$.

The emerging scenario suggests that large black holes decay thermally, while small objects decay quantum mechanically by emitting quanta of energy $\delta M = M_{n+1} - M_n$. Thus, the late stage of the black hole evolution turns out to be quite different from the semi-classical thermal emission and much more similar to the decay of an ordinary unstable particle. On a qualitative ground, this conclusion is in agreement with the results obtained in [39], where it is shown that quantum black holes decay into a limited number of particles estimated to be between six and twenty.

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