Contract-Based Cooperative Spectrum Sharing

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***Abstract*—Providing proper economic incentives is essential for the success of dynamic spectrum sharing. *Cooperative spectrum sharing* is one effective way to achieve this goal. In cooperative spectrum sharing, secondary users (SUs) relay traffics for primary users (PUs), in exchange for dedicated transmission time for the SUs' own communication needs. In this paper, we study the cooperative spectrum sharing under *incomplete information*, where SUs' types (capturing their heterogeneity in relay channel gains and evaluations of power consumptions) are private information and not known by PUs. Inspired by the contract theory, we model the network as a labor market. The single PU is the employer who offers a *contract* to the SUs. The contract consists of a set of contract items representing combinations of spectrum accessing time (i.e., reward) and relaying power (i.e., contribution). The SUs are employees, and each of them selects the best contract item to maximize his payoff. We study the optimal contract design for both weak and strong incomplete information scenarios. First, we provide necessary and sufficient conditions for feasible contracts in both scenarios. In the weak incomplete information scenario, we further derive the optimal contract that achieves the same maximum PU's utility as in the complete information benchmark. In the strong incomplete information scenario, we propose a Decompose-and-Compare algorithm that achieves a close-to-optimal contract. We future show that the PU's average utility loss due to the suboptimal algorithm and the strong incomplete information are both relatively small (less than 2% and 1.3%, respectively, in our numerical results with two SU types).**

I. Introduction

With the explosive development of wireless services and networks, spectrum is becoming more congested and scarce. Dynamic spectrum sharing is a promising approach to increase spectrum efficiency and alleviate spectrum scarcity, as it enables the unlicensed secondary users (SUs) to dynamically access the spectrum licensed to the primary users (PUs) [1]–[4]. The successful implementation of dynamic spectrum sharing requires many innovations in technology, economics, and policy. In particular, it is important to design the sharing mechanism such that PUs have incentive to open their licensed spectrum for sharing, and SUs have incentive to utilize the new spectrum opportunities despite of the potential costs.

Market-driven spectrum trading is a promising paradigm to address the incentive issue in dynamic spectrum sharing. With spectrum trading, PUs temporarily *sell* the spectrum to SUs to obtain either a monetary reward or a performance improvement. A particular interesting trading scheme is *cooperative spectrum sharing*, where SUs relay traffics for PUs in order to get their own share of spectrum. A brief illustration of cooperative spectrum sharing is shown in Figure 1 on the next page. The SUs' transmitters ($\mathrm{ST}_1 \sim \mathrm{ST}_3$) act as cooperative relays for the PU in Phase I and Phase II (Decoding and Forwarding), and transmit their own data in Phase III.

Researchers have only recently started to study cooperative spectrum sharing mechanisms [31]–[34]. The prior results all assumed complete network information, i.e., PUs know SUs' channel conditions, resource constraints, and costs of transmission. This assumption is often too strong for practical networks. In this paper, we study the cooperative spectrum sharing under *incomplete information*. We consider the general case that SUs have different *types* based on their relay channel gains and evaluations of power consumptions. The types are private information, and only an SU knows his own type.

To tackle this problem, we propose a contract-based cooperative spectrum sharing mechanism. Contract theory is an effective tool in designing the incentive compatible mechanism in a monopoly market under incomplete information [39]. The key idea is to offer the right contract items so that all of the agents have the incentive to truthfully reveal their private information. For the spectrum sharing problem, we can imagine the network as a labor market. The PU is the employer and offers a *contract* to the SUs. The contact consists of a set of contract items, which are combinations of spectrum access time (i.e., reward) and relay power (i.e., contribution). The SUs are employees, and each SU selects the best contract item according to his type. *We want to design an optimal contract that maximizes the PU's utility (average data rate) under the incomplete information of SUs' types*.

The main contributions of this paper are as follows:

- *New modeling and solution technique*: As far as we know, this is the first paper that tackles cooperative spectrum sharing under incomplete information based on contract theory.
- *Multiple information scenarios*: We study the optimal contract designing in three different scenarios: complete information (benchmark), weak incomplete information, and strong incomplete information. In the last two incomplete information scenarios, the PU does *not* know the exact type of each SU. The difference between the two scenarios is whether the PU knows the number of SUs in each type (the weak scenario) or only knows the distribution of types (the strong scenario). We will design optimal contracts for all three scenarios.
- *Sufficient and necessary condition for feasible contracts*: Under incomplete information, a contract is feasible if and only if it satisfies the incentive-compatibility (IC) and individually rationality (IR) for each SU. We find necessary and sufficient conditions that satisfy both IC and IR conditions. This discussion helps us understand important properties of a contract and how to design optimal contracts later.
- *Optimal contract design*: In the weak incomplete in-

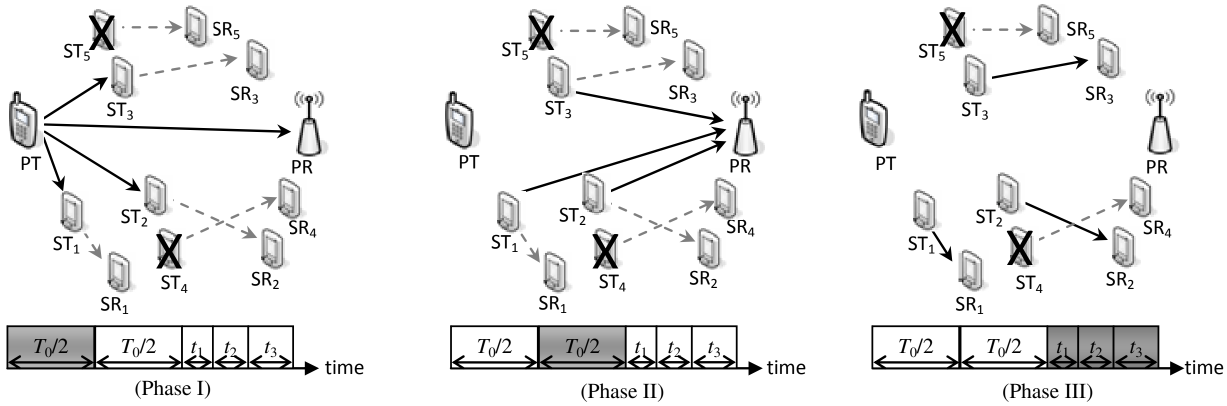

Fig. 1. Cooperative spectrum sharing with three phases in each time slot

TABLE I
FEASIBILITY CONDITIONS AND OPTIMALITY

Network Information	Feasibility	Optimality	Sections in this paper
Complete (benchmark)	IR	Optimal	III
Weak Incomplete	IC & IR	Optimal	IV, V
Strong Incomplete	IC & IR	Close-to-Optimal	IV, VI

formation scenario, we derive the optimal contract that achieves the same maximum PU's utility as in the complete information benchmark. In the strong incomplete information scenario, we propose a Decompose-and-Compare algorithm that obtains a close-to-optimal contract.

- *Performance analysis*: In the strong incomplete information scenario, we quantify the PU's average utility loss due to the suboptimal algorithm (by comparing it with the exhaustive search method) as well as the strong incomplete information (by comparing it with the complete information benchmark). Both kinds of losses are are relatively small, i.e., less than 2% and 1.3% in our numerical results with two SU types, respectively.

The key results and the corresponding section numbers in this paper are are summarized in Table I.

The rest of this paper is organized as follows. In Section II, we provide the system model and problem formulation. In Section III, we propose the optimal contract under complete information (benchmark). In Section IV, we propose the necessary and sufficient conditions for feasible contracts under incomplete information. In Sections V and VI, we derive the optimal and suboptimal contracts in weak and strong incomplete information scenarios, respectively. We present the numerical results in Section VII. We review the related literatures in Section VIII and finally conclude in Section IX.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a cognitive radio network with a primary licensed user (PU) and multiple secondary unlicensed users (SUs) as shown in Fig. 1. Each user is a dedicated transmitter-receiver pair. The PU has the exclusive usage right of the licensed spectrum band, but its transmission suffers from the poor channel condition between its transmitter PT and receiver PR. We represent M SUs by distinct transmitter-receiver pairs $\{ST_k - SR_k\}_{k=1}^{M}$. Each SU wants to have dedicated time to access the licensed band and transmits its own data. The PU can employ a subset of or all SUs to relay its traffic; the *involved* SUs will obtain dedicated transmission time for their own data. The interaction between the PU and the SUs involves three phases as in Fig. 1: Phases I and II for the cooperative communications with a total fixed length of time T_0,[1] and Phase III for the SUs' own transmissions. More specifically, we have

- Phase I: In the first half of the cooperative communications ($T_0/2$), primary transmitter PT broadcasts its data to primary receiver PR and the involved SUs' transmitters (e.g., ST_1, ST_2 and ST_3 in Fig. 1). Note that SU 4 and SU 5 are not involved in this example.
- Phase II: In the remaining half of the cooperative communications ($T_0/2$), the involved SUs' transmitters (STs) decode the data received in Phase I and forward to PU's receiver PR simultaneously using the space-time codes assigned by PU.[2] Through proper choice of space-time codes, SUs' simultaneous relay signals do not interference with each other at the primary receiver PR [36], [37].
- Phase III: PU rewards each involved SU with a dedicated time allocation for that SU's own data (e.g., $\{t_k\}_{k=1}^{3}$ for three involved SUs). SUs access the spectrum using TDMA and do not interfere with each other.

Here we assume that each involved SU can successfully decode PU's data in the first phase of cooperative communications. Thus we can focus on the relay links between the STs and PR, which are the performance bottleneck of cooperative communications. This assumption can be relaxed if the PU can perform an initial screening over all SUs as follows. The PU first broadcasts a pilot signal to all SUs, and only those SUs replying correctly can choose to accept the contract and

[1] The time period T_0 is a constant and is determined by PU's MAC layer and Physical layer specifications.

[2] We will discuss the details of the coordination between PU and SUs in Section II-C.

involve in the cooperative communications later. Notice that the involved SUs are often the ones that are close to the primary transmitter PT, and thus are not close to the primary receiver PR. This explains why a $PT-ST$ channel is often better than the corresponding $ST-PR$ channel.

The PU and SUs have conflicting objectives in the above interactions. The PU wants the SUs to relay its traffic with high power levels, which will increase the PU's data rate but reduce the SUs' battery levels. An SU k wants to obtain a large dedicated transmission time t_k, which will increase the SU's own performance but reduce the PU's utility (time average data rate). In Sections II-A and II-B, we will explain in details how the PU and SUs evaluate the trade-off between relay powers and time allocations. In Section II-C, we propose a contract-based framework which brings the PU and SUs together and resolve the conflicts.

A. Primary User Model

In this subsection, we discuss how PU evaluates relay powers and time allocations.

We first derive the PU's achievable data rate during the cooperative communications (i.e., Phases I and II in Fig. 1). Let us denote the set of involved SUs as $\mathcal{N}$ (e.g., $\mathcal{N}=\{1,2,3\}$ in Fig. 1). The received power (at the primary receiver PR) from SU k is p_k, and the time allocation to this SU is t_k. *Without loss of generality, we normalize T_0 to be 1 in the rest of the paper. Then t_k can be viewed as the ratio t_k/T_0.*

- In Phase I, PU's transmitter broadcasts its data, and PU's receiver achieves a data rate (per unit time) of
$$R^{dir}=\log(1+\texttt{SNR}_{PT,PR}). \tag{1}$$
R^{dir} remains as a constant throughout the analysis.
- In Phase II, each involved SU successfully decodes PU's data and forwards to PU's receiver.

Thus the PU's total transmission rate during the cooperative communications (Phases I and II) is ([36])

$$\begin{aligned} r_{PU}^{relay} &= \frac{R^{dir}}{2}+\frac{1}{2}\log\left(1+\sum_{k\in\mathcal{N}}\texttt{SNR}_{ST_k,PR}\right)\\ &= \frac{R^{dir}}{2}+\frac{1}{2}\log\left(1+\frac{\sum_{k\in\mathcal{N}}p_k}{n_0}\right), \end{aligned} \tag{2}$$

where n_0 is the noise power, and constant $1/2$ is due to equal partition of the cooperative communication time into Phase I and Phase II. We can think (2) as the sum of transmission rates of two "parallel" channels, one from the PT to PR and the other from the set of involved SUs' transmitters to PR.

Based on the above discussion, we next compute the PU's average data rate during entire time period (i.e., Phases I, II, and III). The cooperative communications only utilizes $1/(1+\sum_{k\in\mathcal{N}}t_k)$ fraction of the entire time period. The PU's objective is to maximize its **utility** (i.e., average transmission rate during the entire time period) as follows

$$\begin{aligned} u_{PU} &= \frac{1}{1+\sum_{k\in\mathcal{N}}t_k}r_{PU}^{relay} \\ &= \frac{1}{1+\sum_{k\in\mathcal{N}}t_k}\left(\frac{R^{dir}}{2}+\frac{1}{2}\log\left(1+\frac{\sum_{k\in\mathcal{N}}p_k}{n_0}\right)\right), \end{aligned} \tag{3}$$

which is decreasing in total time allocations to SUs ($\sum_{k\in\mathcal{N}}t_k$), and is increasing in the total received power from SUs ($\sum_{k\in\mathcal{N}}p_k$).

We want to emphasize that the utility calculation in (3) assumes that the PU involves at least one SU in the cooperative communication. The PU can also choose to have direct transmissions only in both Phases I and II and does not interact with the SUs (and thus there is no Phase III). The total rate in this direct transmission only approach is R^{dir}. This means that the PU will only choose to use cooperative communications if the utility in (3) is larger than R^{dir}. In the rest of the analysis, we assume that R^{dir} is small such that the PU wants to use cooperative communications. In Section VII, we will further explain what will happen when this is not true.

B. Secondary User Model

Next we discuss how SUs evaluate relay powers $\{p_k\}_{k\in\mathcal{N}}$ and time allocations $\{t_k\}_{k\in\mathcal{N}}$. *We want to emphasize that the relay power is measured at the PU's receiver, not at the SUs' transmitters.* We consider a general model where SUs are heterogeneous in three aspects:

- SUs have different relay channel gains between their transmitters and the PU's receiver ($h_{ST,PR}$). If an SU k wants to reach a received power p_k at the PU's receiver, then it needs to transmit with a power $p_k^t=p_k/h_{ST_k,PR}$.
- SUs can achieve different (fixed) rates to their own receivers (i.e., data rate r_{SU_k} over link ST_k-SR_k) with different (fixed) transmission power (i.e., $p_{SU_k}^t$).
- SUs have different cost C_k per unit transmission power.

Note that the parameters C_k, r_{SU_k}, $p_{SU_k}^t$, and $h_{ST_k,PR}$ are SU k's private information and are only known to himself.

We define an $SU\ k$'s **payoff** as π_{SU_k}, which is the difference between its own transmitted data during time allocation t_k in Phase III and its cost of power consumption during Phase II and t_k in Phase III. That is,

$$\pi_{SU_k}=t_kr_{SU_k}-\left(t_kp_{SU_k}^t+\frac{1}{2}\frac{p_k}{h_{ST_k,PR}}\right)C_k. \tag{4}$$

We assume that every SU is willing to use positive transmission time if it does not need to relay the PU's traffic, i.e., $r_{SU_k}-p_{SU_k}^tC_k\geq 0$ for all $k\in\mathcal{N}$.[3] Notice that (4) is increasing in time allocation t_k, but is decreasing in relay power p_k.

We can further simplify (4) by multiplying both sides by $2h_{ST_k,PR}/C_k$, which leads to the normalized payoff

$$\tilde{\pi}_{SU_k}:=\pi_{SU_k}\frac{2h_{ST_k,PR}}{C_k}=\frac{2h_{ST_k,PR}(r_{SU_k}-C_kp_{SU_k}^t)}{C_k}t_k-p_k. \tag{5}$$

Such normalization does not affect SUs' choice among different relay powers and time allocations. Thus it will not affect the contract design introduced later.

To facilitate later discussions, we define an $SU\ k$'s type as

$$\theta_k:=\frac{2h_{ST_k,PR}(r_{SU_k}-C_kp_{SU_k})}{C_k}>0, \tag{6}$$

which captures all private information of this SU. A large type θ_k means that the SU's own transmission is efficient (a large

[3] If this is not true for an SU, we can simply eliminate it from the network.

r_{SU_k} or a small p_{SU_k}), or it has good channel condition over relay link $ST_k - PR$ (a large channel gain h_{ST_k-PR}), or it has a more efficient battery technology (a small C_k). With (6), we can simplify SU's normalized payoff in (5) as

$$\pi_k(p_k, t_k) := \tilde{\pi}_{SU_k} = \theta_k t_k - p_k. \tag{7}$$

which is decreasing in PU's received power p_k and increasing in PU's time allocation t_k to $SU\ k$.

Since each SU is selfish, a type-θ_k SU wants to choose relay power and time allocation to maximize its payoff in (7). Notice that an SU can always choose not to help the PU and thus receives zero time allocation and zero payoff (i.e., $t_k = p_k = 0$).

C. Contract Formulation under Incomplete Information

After introducing PU's utility in (3) and SUs' (normalized) payoffs in (7), we are ready to introduce the contract mechanism that resolves the conflicting objectives between the PU and SUs.

Contract theory studies how economic decision-makers construct contractual arrangements, generally in the presence of asymmetric (private) information [39]. In our case, the SUs' types are private information, thus their types are only known to themselves. The PU does not know the type of each SU, and needs to design a contract to attract the SUs to participate in cooperative communications.

To better understand the contract design in this paper, we can imagine the PU as the employer and SUs as employees in a labor market. The employer determines the contract, which specifies the relationship between the employee's performance (i.e., received relay powers) and reward (i.e., time allocation). If we denote $\mathcal{P}$ as the set of all possible relay powers and $\mathcal{T}$ as the set of all possible time allocations, then the contract specifies a $t \in \mathcal{T}$ for every $p \in \mathcal{P}$. Each distinct power-time association becomes a contract item. Once a contract is given, each SU will choose the contract item that maximizes its payoff in (7). The PU wants to optimize the contract items to maximize its utility in (3).

We consider K types of SUs with types denoted by the set $\Theta = \{\theta_1, \theta_2, ..., \theta_K\}$. Without loss of generality, we assume that $\theta_1 < \theta_2 < ... < \theta_K$. The total number of SUs in type-θ_k is N_k. According to the revelation principle [38], it is enough to consider the class of contract that enables the SUs to truthfully reveal their types. Because of this, it is enough to design a contract that consists K contract items, one for each type. The contract can be written as $\Phi = \{(p_k, t_k), \forall k \in \mathcal{K}\}$ where $\mathcal{K} = \{1, 2, ..., K\}$.

We will derive the optimal contract design for three information scenarios.

- *Complete information in Section III:* This is a benchmark case, where the PU knows each SU's type. We will compute the maximum utility the PU can achieve in this case, which serves as an upper-bound of the PU's achievable utility in the incomplete information scenarios.
- *Weak incomplete information in Section V:* The PU does not know each SU's type, but has knowledge of the number of each type SUs in the market (i.e., N_k for type-θ_k SUs). We will show that the optimal contract in this case achieves the same maximum PU's utility as in the complete information benchmark.
- *Strong incomplete information in Section VI:* The PU only knows the total number of SUs (N) and the distribution of each type, but does not know the number of each type (N_k). The PU needs to design a contract to maximize its *expected* utility.

Once the PU has determined the contract, the interactions between the PU and SUs will follow four steps.

1) The PU broadcasts the contract $\Phi = \{(p_k, t_k), \forall k \in \mathcal{K}\}$ to all SUs.
2) After receiving the contract, each SU chooses one contract item that maximizes its payoff and informs the PU its choice.
3) After receiving all SUs confirmations, the PU informs the involved SUs (i.e., those choosing positive contract items) the space-time codes to use in Phase II and the transmission schedule in Phase III. Note that the length for transmission time for each involved SU is specified by the contract item and the PU can no longer change.
4) The communications start by following three phases in Fig. 1.

III. Optimal Contract Design under complete information: The Benchmark Scenario

In the complete information scenario, the PU knows the type of each SU. We will use the maximum PU's utility achieved in this case as a baseline to evaluate the performance of the proposed contracts under incomplete information in Sections V and VI. Without loss of generality, we assume that $N_k \geq 1$ for all type $k \in \mathcal{K}$.

As the PU knows each SU's type, it can monitor and make sure that each type of SUs accepts only the contract item designed for that type. The PU needs to ensure that the SUs have non-negative payoffs so that they are willing to accept the contract. In other words, the contract needs to satisfy the following individual rationality constraint.

Definition 1: (IR: Individual Rationality): A contract satisfies the individual rationality constraint if each type-θ_k SU receives a non-negative payoff by accepting the contract item for θ_k, i.e.,

$$\theta_k t_k - p_k \geq 0, \forall k \in \mathcal{K}. \tag{8}$$

We say a contract is *optimal* if it yields the maximum utility for the PU under the current information scenario. Different information scenarios may lead to different optimal contracts.

In the complete information scenario, an optimal contract maximizes the PU's utility as follows

$$\max_{\{(p_k,t_k)\geq \mathbf{0}, \forall k\}} \frac{\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\sum_{k\in\mathcal{K}} N_k p_k}{n_0}\right)}{1 + \sum_{k\in\mathcal{K}} N_k t_k}, \tag{9}$$
$$\text{subject to IR Constraints in Eq. } (8).$$

In this paper, the vector operations are component-wise (e.g, $(p_k, t_k) \geq \mathbf{0}$ means that $p_k \geq 0$ and $t_k \geq 0$) unless specified otherwise. Then we have the following result.

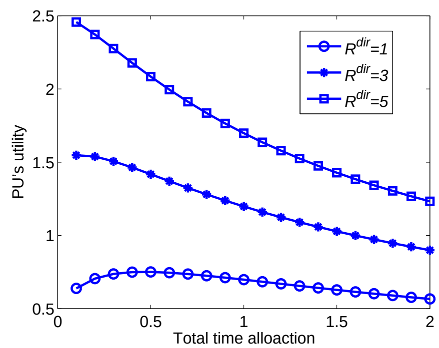

Fig. 2. PU's utility as a function of the total time allocation $\tilde{t}_K^*$ for different direct transmission rates R^{dir}.

Lemma 1: In an optimal contract with complete information, each SU receives zero payoff by accepting the corresponding contract item. In other words, $t_k\theta_k = p_k$ for any $k \in \mathcal{K}$.

Proof. We prove by contradiction. Suppose that there exists an optimal contract item (p_k, t_k) with $\theta_k t_k - p_k > 0$. Since PU's utility in (9) is increasing in p_k and decreasing in t_k, the PU can increase its utility by decreasing t_k until $\theta_k t_k - p_k = 0$. This contradicts with the assumption that (p_k, t_k) with $\theta_k t_k - p_k > 0$ belongs to an optimal contract, and thus completes the proof. ∎

Using Lemma 1, we can replace p_k by $\theta_k t_k$ for each $k \in \mathcal{K}$ and simplify the PU's utility maximization problem in (9) as

$$\max_{\{t_k \geq 0, \forall k\}} \frac{\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\sum_{k\in\mathcal{K}} N_k\theta_k t_k}{n_0}\right)}{1 + \sum_{k\in\mathcal{K}} N_k t_k}. \quad (10)$$

Theorem 1: In an optimal contract with complete information, only the contract item for the highest type is positive and all other contract items are zero. That is $(p_K, t_K) > \mathbf{0}$, and $(p_k, t_k) = \mathbf{0}$ for any $k < K$.

Proof. We prove by contradiction. Suppose that there exists an optimal contract item with $t_k > 0$ for the type-θ_k SUs and $k < K$. The total time allocation is $T' = \sum_{k\in\mathcal{K}} N_k t_k$. Then the PU's utility is

$$u_{PU}^1 = \frac{1}{1+T'}\left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\sum_{k\in\mathcal{K}} \theta_k N_k t_k}{n_0}\right)\right). \quad (11)$$

Next, we show that given a fixed total time allocation T', allocating positive time only to the highest type SUs (i.e., $N_K t_K = T'$) achieves a larger utility for the PU as follows

$$u_{PU}^2 = \frac{1}{1+T'}\left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\theta_K N_K t_K}{n_0}\right)\right). \quad (12)$$

This is because $\theta_K N_K t_K = \theta_K T'$ in (12) and $\sum_{k\in\mathcal{K}} \theta_k N_k t_k < \theta_K T'$ in (11), thus (12) is larger than (11). This contradicts with the optimality of the contract, and thus completes the proof. ∎

Intuitively, the highest type SUs can offer the most help to the PU within the given total time allocation in Phase III.

Using Theorem 1, the optimization problem in (10) can be simplified as

$$\max_{t_K \geq 0} \frac{1}{1 + N_K t_K}\left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\theta_K N_K t_K}{n_0}\right)\right). \quad (13)$$

Since N_K and t_K always appear as a product in (13), we can redefine the optimization variable as $\tilde{t}_K = N_K t_K$ and rewrite (13) as

$$\max_{\tilde{t}_K > 0} \frac{1}{1 + \tilde{t}_K}\left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\theta_K \tilde{t}_K}{n_0}\right)\right), \quad (14)$$

This means the PU's optimal utility does not depend on N_K. When N_K changes, the optimal time allocation per user t_K^* changes inversely proportional to N_K.

At this point, we have successfully simplified the PU's optimization problem from involving $2K$ variables $\{(p_k, t_k), \forall k \in \mathcal{K}\}$ in (9) to a single variable $\tilde{t}_K$ in (14).

The problem (14) is a non-convex optimization problem, as shown in Fig. 2 for various parameters. Although it is difficult to find a closed-form solution, we can use an efficient one-dimensional exhaustive search algorithm to find the global optimal solution $\tilde{t}_K^*$ [40]. We will provide numerical results in Section VII.

IV. Feasible Contracts Under Incomplete Information

In this section, we study the necessary and sufficient conditions for a feasible contract under incomplete information. This will help us derive optimal contracts in Sections V and VI.

A feasible contract includes K power-time items such that any type-θ_k SU prefers the contract item (p_k, t_k) for its type to any other contract item. A feasible contract must satisfy both the individual rationality (IR) constraint in Definition 1 introduced in Section III and the incentive compatibility (IC) constraint defined next.

Definition 2: (IC: Incentive Compatibility): A contract satisfies the incentive compatibility constraint if each type-θ_k SU prefers to choose the contract item for θ_k, i.e.,

$$\theta_k t_k - p_k \geq \theta_k t_j - p_j, \forall k, j \in \mathcal{K}. \quad (15)$$

In summary, the PU's optimization problem is

$$\max_{\{(p_k,t_k),\forall k\}} u_{PU}(\{(p_k, t_k), k \in \mathcal{K}\}), \quad (16)$$
$$\text{subject to } \theta_k t_k - p_k \geq \theta_k t_j - p_j, \ \forall k, j \in \mathcal{K},$$
$$\theta_k t_k - p_k \geq 0, \forall k \in \mathcal{K}$$
$$t_k \geq 0, p_k \geq 0, \ \forall k \in \mathcal{K}.$$

The first two constraints correspond to IC and IR, respectively.

A. Sufficient and Necessary Conditions for Feasibility

Next we provide several necessary and sufficient conditions for the contract feasibility.

Proposition 1: [Necessary condition 1]: For any feasible contract $\Phi = \{(p_k, t_k), \forall k\}$, we have $p_i > p_j$ if and only if $t_i > t_j$.

Proof. We divide the proof into two parts. First, we prove that if $p_i > p_j$, then $t_i > t_j$. Due to the IC constraint in (15), we have

$$\theta_i t_i - p_i \geq \theta_i t_j - p_j,$$

i.e.,

$$\theta_i(t_i - t_j) \geq p_i - p_j.$$

Since $p_i > p_j$, we conclude

$$\theta_i(t_i - t_j) \geq p_i - p_j > 0,$$

and thus $t_i > t_j$.

Next we prove that if $t_i > t_j$, then $p_i > p_j$. Due to the IC constraint in (15), we have

$$\theta_j t_j - p_j \geq \theta_j t_i - p_i,$$

which can be transformed to be

$$p_i - p_j \geq \theta_j(t_i - t_j).$$

Since $t_i > t_j$, we conclude

$$p_i - p_j \geq \theta_j(t_i - t_j) > 0,$$

and thus $p_i > p_j$. ■

Proposition 1 shows that an SU contributing more in terms of received power at the PU receiver should receive more time allocation, and vice versa. From Proposition 1, we have the following corollary, saying that the same relay powers must have the same time allocations, and vice versa.

Corollary 1: For any feasible contract $\Phi = \{(p_k, t_k), \forall k\}$, we have $p_i = p_j$ if and only if $t_i = t_j$.

Proposition 2 shows the second necessary condition for contract feasibility.

Proposition 2: [Necessary Condition 2]: For any feasible contract $\Phi = \{(p_k, t_k), \forall k\}$, if $\theta_i > \theta_j$, then $t_i \geq t_j$.

Proof. We prove by contradiction. Suppose that there exists $t_i < t_j$ with $\theta_i > \theta_j$. Then we have

$$\theta_i t_j + \theta_j t_i > \theta_i t_i + \theta_j t_j. \quad (17)$$

On the other hand, the feasible contract satisfies the IC constraints for both type-θ_i and type-θ_j SUs, i.e.,

$$\theta_i t_i - p_i \geq \theta_i t_j - p_j,$$

and

$$\theta_j t_j - p_j \geq \theta_j t_i - p_i.$$

By combining last two inequalities, we have

$$\theta_i t_i + \theta_j t_j \geq \theta_i t_j + \theta_j t_i,$$

which contradicts with (17). This completes the proof. ■

Proposition 2 shows that a higher type SU should be allocated more transmission time. Combined with Proposition 1, we know that a higher type of SU should also contribute more in terms of PU's received power.

From Propositions 1 and 2, we conclude that for a feasible contract, all power-time combination items satisfy

$$0 \leq p_1 \leq p_2 \leq ... \leq p_K, \ 0 \leq t_1 \leq t_2 \leq ... \leq t_K, \quad (18)$$

with $p_k = p_{k+1}$ if and only if $t_k = t_{k+1}$.

The previous propositions help us obtain Theorem 2 as follows.

Theorem 2: [Sufficient and Necessary Conditions]: For a contract $\Phi = \{(p_k, t_k), \forall k\}$, it is feasible if and only if all the following three conditions hold:

- `Contd.a`: $0 \leq p_1 \leq p_2 \leq ... \leq p_K$ and $0 \leq t_1 \leq t_2 \leq ... \leq t_K$;
- `Contd.b`: $\theta_1 t_1 - p_1 \geq 0$;
- `Contd.c`: For any $k = 2, 3, ..., K$,

$$p_{k-1} + \theta_{k-1}(t_k - t_{k-1}) \leq p_k \leq p_{k-1} + \theta_k(t_k - t_{k-1}). \quad (19)$$

We give the proof of Theorem 2 in Appendix A. The conditions in Theorem 2 are essential to the optimal contract design under weak and strong incomplete information in Section V and Section VI.

V. Optimal Contract Design under weak incomplete information

In this section, we will look at the weak incomplete scenario where the PU does not know each SU's type but only knows the number of each type (i.e., N_k for any $k \in \mathcal{K}$). Without loss of generality, we assume that $N_k \geq 1$ for all $k \in \mathcal{K}$. Different from the complete information case, here the PU cannot force an SU to accept certain contract item as the PU does not know the SU's type. Thus we need to consider IC constraint here (while not in the complete information case in Section III).

A conceptually straightforward approach to derive the optimal contract is to solve (16) directly. Going through this route, however, is very challenging as (16) is a non-convex and involves complicated constraints.

Here we adopt a sequential optimization approach instead: we first derive the best relay powers $\{p_k^*(\{t_k, \forall k\}), \forall k\}$ given fixed feasible time allocations $\{t_k, \forall k\}$, then derive the best time allocations $\{t_k^*, \forall k\}$ for the optimal contract, and finally show that there is no gap between the solution $\{(p_k^*, t_k^*), \forall k\}$ obtained from this sequential approach and the one obtained by directly solving (16).

Proposition 3: Let $\Phi = \{(p_k, t_k), \forall k\}$ be a feasible contract with fixed time allocations $\{t_k, \forall k : 0 \leq t_1 \leq ... \leq t_K\}$. The optimal unique relay powers satisfy

$$p_1^*(\{t_k, \forall k\}) = \theta_1 t_1,$$
$$p_k^*(\{t_k, \forall k\}) = \theta_1 t_1 + \sum_{i=2}^{k} \theta_i(t_i - t_{i-1}), \forall k = 2, ..., K. \quad (20)$$

The proof of this proposition is given in Appendix B. Using Proposition 3, we can simplify the PU's optimization problem in (16) as

$$\max_{\{t_k, \forall k\}} \frac{\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\sum_{k\in\mathcal{K}} N_k(\theta_1 t_1 + \sum_{i=1}^{k}\theta_i(t_i - t_{i-1}))}{n_0}\right)}{1 + \sum_{k\in\mathcal{K}} N_k t_k}, \quad (21)$$

$$\text{subject to,} \quad 0 \leq t_1 \leq ... \leq t_K.$$

We can further simplify (21) using Theorem 3 below.

Theorem 3: In an optimal contract with weak incomplete information, only the contract item for the highest SU type is positive and all other contract items are zero. That is $(p_K, t_K) > \mathbf{0}$, and $(p_k, t_k) = \mathbf{0}$ for any $k < K$.

Using Theorem 3, we can simplify the optimization problem in (21) further as

$$\max_{t_K \geq 0} \frac{1}{1 + N_K t_K}\left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\theta_K N_K t_K}{n_0}\right)\right). \quad (22)$$

Notice that (22) under weak incomplete information is the same as (13) under complete information. We thus conclude that our sequential optimization approach (first over $\{p_k, \forall k\}$ and then over $\{t_k, \forall k\}$) results in no loss in optimality, as it achieves the same maximum utility as in the complete information scenario.

To solve problem (22), we can use an efficient one-dimensional exhaustive search algorithm to find the global optimal solution t_K^*. We will provide numerical results in Section VII.

$$\max_{\{(p_k,t_k),\forall k\}} \sum_{n_1=0}^{N} \sum_{n_2=0}^{N-n_1} \cdots \sum_{n_{K-1}=0}^{N-\sum_{i=1}^{K-2} n_i} \frac{Q_{(n_1,\ldots,n_{K-1},N-\sum_{i=1}^{K-1} n_i)}}{1+\sum_{i=1}^{K-1} n_i t_i + (N-\sum_{i=1}^{K-1} n_i)t_K} \left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1+\frac{\sum_{i=1}^{K-1} n_i p_i + (N-\sum_{i=1}^{K-1} n_i)p_K}{n_0}\right)\right). \quad (24)$$

$$\max_{\{t_k,\forall k\}} \sum_{n_1=0}^{N} \sum_{n_2=0}^{N-n_1} \cdots \sum_{n_{K-1}=0}^{N-\sum_{i=1}^{K-2} n_i} \frac{Q_{(n_1,\ldots,n_{K-1},N-\sum_{i=1}^{K-1} n_i)} \left(\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1+\frac{\sum_{i=1}^{K-1} n_i p_i^*(\{t_k,\forall k\}) + (N-\sum_{i=1}^{K-1} n_i)p_K^*(\{t_k,\forall k\})}{n_0}\right)\right)}{1+\sum_{i=1}^{K-1} n_i t_i + (N-\sum_{i=1}^{K-1} n_i)t_K}. \quad (26)$$

subject to, $\quad 0 \le t_1 \le \ldots \le t_K.$

VI. Optimal Contract Design under strong incomplete information

In this section, we study the strong incomplete information scenario, where the PU does not know each SU's type and even the number of each type (i.e., N_k for each $k \in \mathcal{K}$). The PU only knows the total number of SUs N and the probability q_k of each SU belonging to type-θ_k (i.e., $\sum_{k\in\mathcal{K}} q_k = 1$).

Similar to the weak incomplete information scenario, here the PU needs to consider the IC constraint since it cannot force an SU to accept a certain contract item. The difference from Section V is that here the PU does not know whether $N_K = 0$ or $N_K > 0$ for the highest type-θ_K, and thus the simple approach of only providing a positive contract item for type θ_K as in Theorem 3 may not be optimal. If the PU does that and it turns out that $N_K = 0$ in a particular realization, then there will be no SUs participating in the cooperative communications.

The right target for the PU is design a contract to maximize the *expected* utility subject to the IC and IR constraints. As the PU knows the total number of SUs N, then the probability density function of the number of SUs $\{N_k, \forall k\}$ is

$$\begin{aligned} & Q_{(n_1,\ldots,n_{K-1},n_K=N-\sum_{i=1}^{K-1} n_i)} \\ & := \Pr(N_1 = n_1, \ldots, N_{K-1} = n_{K-1}, N_K = N - \sum_{i=1}^{K-1} N_i) \\ & = \frac{N!}{n_1!\ldots n_{K-1}!(N-\sum_{i=1}^{K-1} n_i)!} q_1^{n_1}\ldots q_{K-1}^{n_{K-1}} q_K^{N-\sum_{i=1}^{K-1} n_i}. \end{aligned} \quad (23)$$

The PU's optimization problem can be written in (24) subject to the IC ad IR constraints.

Similar to Section V, here we adopt a sequential optimization approach: we first derive the optimal relay powers $\{p_k^*(\{t_k,\forall k\}),\forall k\}$ with fixed feasible time allocations $\{t_k,\forall k\}$, then derive the optimal time allocations $\{t_k^*,\forall k\}$ for the optimal contract. The difference is that optimality is no longer guaranteed here as explained later.

Proposition 4: Let $\Phi = \{(p_k,t_k),\forall k\}$ be a feasible contract with fixed time allocations $\{t_k, \forall k : 0 \le t_1 \le \ldots \le t_K\}$, then the unique optimal relay powers satisfy

$$\begin{aligned} & p_1^*(\{t_k,\forall k\}) = \theta_1 t_1, \\ & p_k^*(\{t_k,\forall k\}) = \theta_1 t_1 + \sum_{i=2}^{k} \theta_i (t_i - t_{i-1}), \forall k = 2, \ldots, K. \end{aligned} \quad (25)$$

Notice that Proposition 4 under strong incomplete information is actually the same as Proposition 3 under weak incomplete information. Proposition 4 can be proved in a similar manner as Proposition 3, using the fact that PU's expected utility is increasing in $\{p_k,\forall k\}$. The proof is given in Appendix D. Based on Proposition 4, we can simplify the PU's optimization problem in (24) as (26).

Note that (26) is a non-convex optimization problem and all K variables are coupled in the objective function. Furthermore, the number of terms in the objective increases exponentially with the number of types K. Thus it is hard to solve efficiently.

Next, we propose a low computation complexity approximate algorithm, Decompose-and-Compare algorithm, to compute a close-to-optimal solution to (26) efficiently. In this algorithm, we will compare K simple candidate contracts, and pick the one that yields the largest utility for the PU. There are two key ideas behind this heuristic algorithm.

- *One positive contract item per contract:* In each of the K individually optimized candidate contracts, there is only one positive contract item (for one or more types). For the contract item in the kth candidate contract, this positive contract item is offered to SUs with types equal to or larger than type-θ_k. In other words, optimization of each candidate contract only involves a scalar optimization, instead of K variables as in (26).
- *A balance between efficiency and uncertainty:* Among K optimized candidate contracts, we will pick the best one that achieves the best trade-off between efficiency and uncertainty (i.e., maximizing the PU's expected utility). Under strong incomplete information, it is not clear which type is the highest among all SUs existing in a particular network realization. If a candidate contract offers the same positive contract item for types equal to or larger than θ_k, then all SUs in these types will choose to accept that contract item. The corresponding SU's payoff is increasing in type, i.e., a type θ_k SU receives zero payoff and a type-θ_K SU receives the maximum payoff. Thus choosing a candidate contract with a threshold θ_k too low will give too much payoffs to the SUs (and thus reduce the PU's expected utility), but choosing a candidate contract with a threshold too high might lead to the undesirable case that no SUs will be able to participate. This requires us to examine all possibilities (i.e., K candidate contracts) and pick the one with the best performance.

The Decompose-and-Compare algorithm is as follows.

1) *Decomposition:* We construct the K candidate contracts as follows. For the kth candidate contract,
    - Only offers the same contract item $t_k > 0$ to SUs with a type equal to or larger than type θ_k, and zero for the rest of the tyes.
    - Computes the optimal t_k^* that maximizes PU's expected utility in (26).

2) *Comparison:* After determining the above K contract candidates, we choose the one that gives PU the highest expected utility.

In Section VII, we will use numerical results to show that the proposed Decompose-and-Compare algorithm achieves a performance very close to the optimal solution to (26) in most cases.

VII. Numerical Results

Here we use numerical results to show how the PU designs the optimal contract in different information scenarios.

A. Complete and Weak Incomplete Information Scenarios

As shown in Section III and Section V, the optimal contract is the same for complete and weak incomplete information scenarios. By examining the PU's optimization in (22) that applies to both scenarios, we have the following observations.

Observation 1: The PU's optimal utility increases in the highest SU type-θ_K and the PU's direct transmission rate R^{dir}.

Figure 3 shows PU's utility achieved under the optimal contract, which is increasing in both θ_K and direct transmission rate R^{dir}.[4] The dotted baseline denotes the rate R^{dir} achieved by direct transmission only. As R^{dir} increases, the PU has less incentive to share spectrum with the SUs. When R^{dir} is very large, the PU chooses not to use SUs at all, which corresponds to *No Relay Region* in Fig. 3 (where the three curves with different θ_K's are below the baseline).

For the rest of the numerical results, we will only examine the PU's choice of optimal contract, without reiterating the need to compare with R^{dir} and choose direct transmission only if needed.

Observation 2: Figure 4 shows that the PU's optimal total time allocation to the highest type-θ_K SUs, $N_K t_K^*$, decreases in PU's direct transmission rate R^{dir}. When direct transmission rate is zero (i.e., $R^{dir} = 0$), the PU's optimal total time allocation is strictly decreasing in θ_K; when direct transmission rate is positive (i.e., $R^{dir} > 0$), the total time allocation first increases in θ_K and then decreases in θ_K.

When direct transmission rate $R^{dir} = 0$, the PU can only rely on SUs for transmissions and will always allocate positive transmission time to the highest type-θ_K SUs. If we look at the PU's utility in (22) with $R^{dir} = 0$, the logarithmic term $\log(1 + \theta N_K t_K^*)$ plays a more important role than PU's transmission time ratio $\frac{1}{1+N_K t_K^*}$ in this case. When θ_K is small, the PU needs to allocate a large amount time to the SUs to achieve its desirable rate. When θ_K becomes large, the PU can reach a high relay rate by allocating less transmission time to the SUs. This explains why we observe a decrease of $N_K t_K^*$ in θ_K. Appendix E provides a rigorous proof of Observation 2 under $R^{dir} = 0$.

When direct transmission rate $R^{dir} > 0$ (the lower three curves in Fig. 4), the PU has less incentive to allocate transmission time to the SUs especially when the highest SU type-θ_K is small. As θ_K becomes large, PU is willing to allocate more time in exchange of efficient help from SUs. As θ_K becomes very large, the PU only needs to allocate a small amount of time to the SUs in order to obtain enough relay help. The above analysis together explain why the lower three curves in Fig. 4 first increase and then decrease in θ_K.

[4] Without loss of generality, we can normalize n_0 to be 1 in the rest of this paper. Then θ_k can be viewed as the ratio θ_k/n_0.

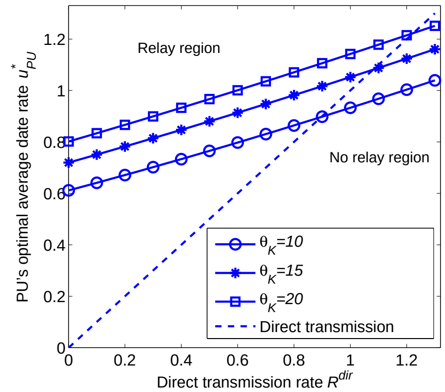

Fig. 3. PU's optimal utility u^*_{PU} as a function of the PU's direct transmission rate R^{dir} and the highest type θ_K. The dotted baseline with 45^o divides the figure into two regions: Relay Region and No Relay Region.

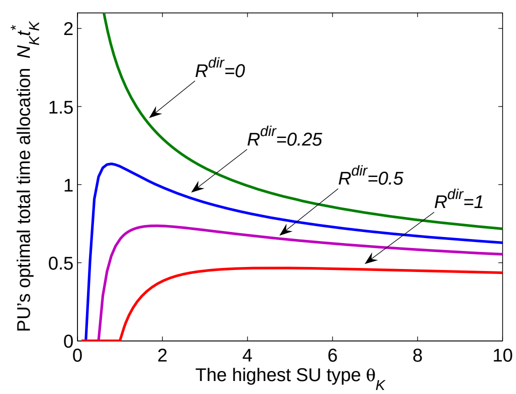

Fig. 4. PU's optimal total time allocation $N_K t_K^*$ as a function of the PU's direct transmission rate R^{dir} and the highest type θ_K.

B. Strong Incomplete Information Scenario

Here we show how PU chooses the optimal contract to maximize its expected utility. As a performance benchmark, we first compute the optimal solution to the PU's expected utility maximization problem in (26) via a K-dimension exhaustive search. We denote the corresponding optimal solution as $E[u_{PU}]^*$. Notice that $E[u_{PU}]^*$ is often smaller than the maximum utility achieved under complete information. The performance gap is due to the strong incomplete information. Next, we will compare the PU's expected utility achieved by the proposed Decompose-and-Compare algorithm with $E[u_{PU}]^*$.

For illustration purposes, we consider only two types of SUs: $\theta_1 < \theta_2$. The PU only knows the total number of SUs N and the probabilities q_1 and q_2 of two types with $q_1 + q_2 = 1$.

In the Decompose-and-Compare algorithm, we first consider two candidate contracts. The first candidate contract optimizes

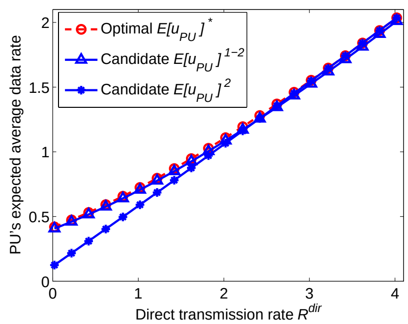

Fig. 5. Comparison among PU's optimal expected utility using optimal exhaustive search method ($E[u_{PU}]^*$) and the two candidate contracts of the Decompose-and-Compare algorithm as a function of the PU's direct transmission rate R^{dir}. Other parameters are $q_1 = 0.9$, $N = 2$, $\theta_1 = 4$, and $\theta_2 = 10$.

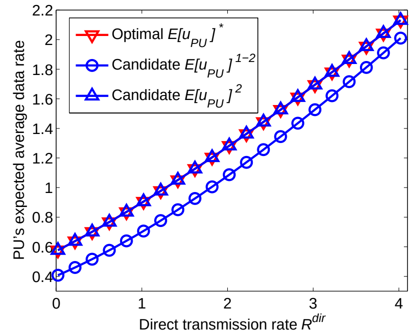

Fig. 6. Comparison among PU's optimal expected utility using optimal exhaustive search method ($E[u_{PU}]^*$) and the two candidate contracts of the Decompose-and-Compare algorithm as a function of the PU's direct transmission rate R^{dir}. Other parameters are $q_1 = 0.5$, $N = 5$, $\theta_1 = 4$, and $\theta_2 = 10$.

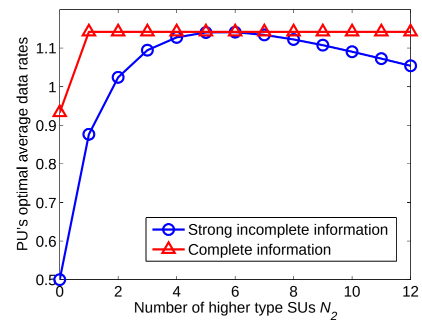

Fig. 7. Comparison among PU's optimal utilities for any SU number realization under different information scenarios. Other parameters are $q_1 = 0.5$, $N = 12$, $R^{dir} = 1$, $\theta_1 = 10$, and $\theta_2 = 20$.

the same positive contract item $t_1 = t_2 > 0$ for both types. The PU's corresponding maximum expected utility is $E[u_{PU}]^{1-2}$. The second candidate contract optimizes the positive contract item $t_2 > 0$ and chooses $t_1 = 0$. The PU's corresponding maximum expected utility is $E[u_{PU}]^2$. Then we pick the candidate contract that leads to a larger PU's expected utility as the solution of the Decompose-and-Compare algorithm.

To evaluate the performance of this approximate algorithm, we consider two different parameter regimes.

1) Large q_1^N: This means that the probability that all SUs belong to the low type-θ_1 is large. This happens when the total number of SUs N is small and probability q_1 is large. Figure 5 shows the PU's expected utility obtained from the Decompose-and-Compare algorithm ($\max(E[u_{PU}]^{1-2}, E[u_{PU}]^2)$) and the optimal exhaustive search method ($E[u_{PU}]^*$) as functions of PU's direct transmission rate R^{dir}. We can see that the candidate contract that offers the same positive contract items to both types (i.e., $E[u_{PU}]^{1-2}$) achieves a close-to-optimal performance with all values of R^{dir} simulated here. This is because very often the PU needs to rely on the low type-θ_1 SUs to relay its traffic.

We also notice that the candidate contract that offers a positive contract item to the high type SU (i.e., $E[u_{PU}]^2$) also achieves a close-to-optimal performance (even larger than $E[u_{PU}]^{1-2}$) when R^{dir} is large. This is because the PU with a large R^{dir} relies less on the SUs, and will have more incentive to employ only the high type SUs when they are available.

2) Small q_1^N: This means that the probability that at least one SU belongs to the high type-θ_2 (i.e., $1 - q_1^N$) is large. Figure 6 shows the PU's expected utility obtained from Decompose-and-Compare algorithm and the optimal exhaustive search method as a function of R^{dir}. We can see that $E[u_{PU}]^2$ is always better than $E[u_{PU}]^{1-2}$ and achieves a close-to-optimal performance under all choices of R^{dir}. This is because very often the PU can rely on the high type SUs only to relay its traffic.

Observation 3: The performance of the proposed Decompose-and-Compare algorithm achieves a close-to-optimal performance (i.e., less than 2% according to Fig. 5 and Fig. 6) under the strong incomplete information.[5]

Next we study how the strong incomplete information reduces PU's utility comparing with the complete information benchmark. First, we note that PU's contract in the strong incomplete information scenario does not depend on N_1 and N_2, as the PU targets at optimizing the expected rate and does not know the realization information. However, the actual PU's utility (not the expected value) does depend on N_1 and N_2. Figure 7 shows the PU's utility under different information scenarios and different user number realizations (i.e., any realization of N_1 and N_2 for a fixed $N_1 + N_2 = N$). Here, q_1^N is small and the curve with strong incomplete information corresponds to offering $t_2 > 0$ and $t_1 = 0$ (as in Fig. 6). The (very close to) optimal contract under strong incomplete information can be obtained by using the Decompose-and-Compare algorithm. The optimal contract under complete information changes as N_1 and N_2 change.

Observation 4: Figure 7 shows that the PU's optimal utility under strong incomplete information achieves the maximum value (close to the one under complete information) when the realized SU numbers is close to the expected value (i.e., $N_1 = N_2 = 6$ in this example as $q_1 = q_2 = 0.5$).

In Fig. 7, the largest performance gap between the two curves happens when $N_2 = 0$. In this case, the optimal contract under complete information satisfies $t_1 > 0$ and $t_2 = 0$, as there are no type-θ_2 users. However, the PU under strong incomplete information chooses $t_1 = 0$ and $t_2 > 0$ to maximize the PU's expected utility. Such mismatch means that the PU under strong incomplete information has no SUs serving as relays, and can only achieve a utility equal to half of the direct transmission rate (only in Phase I). However, this parameter setting only happens with a very small probability $q_1^N = 0.5^{12} \approx 2.4 \times 10^{-4}$.

A more meaning comparison is the PU's *average* utility loss due to strong incomplete information. We can first compute the PU's average utility under strong incomplete information, which is the optimal objective of the PU's expected utility

[5] Due to the page limit, we only discuss two-type case here. We have similar numerical results for cases of more than two types.

maximization problem in (26) (via an K-dimension exhaustive search). Then we can compute the PU's average utility under complete information by calculating the weighted sum (weighted by the probability of each parameter (N_1, N_2)) of the 13 values on the upper curve in Fig. 7. In this example, the ratio is 0.9874. This means that the PU's average utility loss due to strong incomplete information is very small (i.e., less than 1.3%).

TABLE II
A SUMMARY OF SPECTRUM TRADING LITERATURES

Network Information	Money-Exchange	Resource-Exchange
Complete	Pricing: [19]–[21]	Stackelberg: [31]–[34]
Incomplete	Contract: [22], [23] Auction: [24]–[30]	Contract: This paper

VIII. Related Work

A. Cognitive Radio and Dynamic Spectrum Sharing

There are several comprehensive surveys on cognitive radio and dynamic spectrum sharing [1]–[4]. Here we will only highlight some recently proposed models. In [5], Huang *et al.* proposed a distributed spectrum access scheme that enables multiple SUs to collectively protect the PU while adapting to changes in PU activity patterns. In [6], Xing *et al.* considered dynamic spectrum sharing with quality of service guarantee for each secondary link under interference temperature constraint. In [7], Cao *et al.* introduced a distributed spectrum management architecture, where nodes share spectrum resource fairly by making independent actions following spectrum rules. In [8], Mehonen *et al.* discussed how to precisely model spectrum maps with spatial statistics and random fields, which is crucial for dynamic spectrum sharing. In [9], Mehonen and Petrova *et al.* further proposed a topology engine to perform the collecting and processing of spatial information, and discussed both technical and architectural issues in enabling such an approach. In [10], Buddhikot discussed the potential benefits of applying cognitive radio technologies in cellular networks. The above results focused on the technical aspect of dynamic spectrum sharing without considering the issue of economic incentives.

B. Spectrum Trading for Dynamic Spectrum Sharing

Recent years have witnessed a growing body of literatures on the economic aspect of dynamic spectrum sharing. In [11], Pierre *et al.* studied the problem of how to determine the right balance between exclusive use (typically market mechanisms approach) and licence-exempt use of spectrum. In [12], Lehr and Jesuale examined the economic, policy, and market challenges of enabling spectrum pooling, which is considered as the first step toward dynamic spectrum accessing/sharing. In [13], Chapin *et al.* investigated time-limited leases for innovative radios such as cognitive radio and dynamic spectrum access from both technical and policy perspectives. In [14], Delaere and Ballon discussed the multi-level standardization and business models for cognitive radio. In [15], Hwang *et al.* discussed the feasibility of cognitive radio based network and application scenarios from the aspects of regulation, policy and market structure. In [16], Peha discussed policies that can enable or facilitate the use of many spectrum-sharing arrangements.

Market-driven spectrum trading (e.g., [17] [18]) is a promising paradigm to address the incentive issue in dynamic spectrum sharing. We can classify spectrum trading models into two types: money-exchange and resource-exchange. In the former type, SUs pay PUs in the form of (virtual) money for the usage of spectrum (e.g., [19]–[29]); in the latter type, SUs provide communication resources (e.g., the transmission power in our model) for PUs' transmissions in exchange for the usage of spectrum (e.g., [31]–[34]).

There has been extensive research on the money-exchange spectrum trading model, often in the form of pricing, auction, and contract. *Pricing* is often used when the seller knows precisely the value of the resource being sold. In [19], Kloeck *et al.* proposed an integrated pricing, allocating, and billing system for cognitive radio networks. In [20], Wang *et al.* proposed a joint power and spectrum allocation scheme using a distributed pricing strategy to improve the network's performance. In [21], Niyato *et al.* proposed three different pricing models for cognitive radio networks with different objectives.

Contract is more effective in the case where the seller only knows limited information (e.g., distribution) of the buyers' valuation of the resource. By motivating the buyers truthfully reveal their private valuations, the seller can allocate the resource to maximize its own benefit or the social efficiency. In [22], Gao *et al.* proposed a quality-price contract for the spectrum trading in a monopoly spectrum market. In [23], Kalathil *et al.* proposed a contract-based spectrum sharing mechanism to avoid possible manipulating in auction.

When the seller has no knowledge about the value of the resource being sold, *Auction* becomes an effective approach. By letting the bidders bid for the resource in a truthful manner, the seller can efficiently allocate the resource without knowing the value before hand. In [24], Huang *et al.* proposed two divisible auction mechanisms for power allocation in spectrum sharing to achieve efficiency and fairness, respectively. In [25], Li *et al.* proposed several truthful (strategy-proof) spectrum auction mechanisms to achieve the efficiency closed to social optimal. In [26], Gandhi *et al.* proposed a real-time spectrum auction framework to distribute spectrum among users under interference constraints. In [27], Gao *et al.* proposed a multi-shot spectrum auction mechanism to achieve social optimal efficiency in dynamic spectrum sharing. In [28] and [29], Zheng *et al.* proposed truthful single-side spectrum auction and truthful double spectrum auction, respectively, both considering spectrum reuse among users. Wang *et al.* in [30] proposed a general framework for truthful online double auction for spectrum sharing.

Money-exchange spectrum trading is most effective when PUs have some temporarily unused spectrum. However, when PUs' own demands are high or the primary channels' capacities are low (e.g., due to shadowing and deep fading), there will be hardly any resource left for sale. In this case, resource-exchange spectrum trading can be a better choice.

Cooperative spectrum sharing is an effective form of resource-exchange spectrum trading [31]–[34], wherein PUs utilize SUs as cooperative relays. Such cooperation can significantly improve PUs' data rate and thus can free up spectrum resources for SUs. Existing cooperative spectrum sharing mechanisms are usually based on Stackelberg game formulations with complete information [31]–[34]. In this paper, we consider the cooperative spectrum sharing under *incomplete information*, and propose a contract-based cooperative spectrum sharing mechanism. As far as we know, this is the first work considering the cooperative spectrum sharing under incomplete information.

We summarize the key literatures of spectrum trading in Table II.

IX. Conclusion

We study the cooperative spectrum sharing between one PU and multiple SUs, where the SUs' types are private information. We model the network as a monopoly market, in which the PU offers a contract and the each SU selects the best contract item according to his type. We study the optimal contract designing for multiple information scenarios. We first provide the necessary and sufficient conditions for feasible contracts under any incomplete information. For the weak incomplete information scenario, we derive the optimal contract that achieves the same PU's utility as in the complete information benchmark. For the strong incomplete information scenario, we propose a Decompose-and-Compare algorithm that achieves a close-to-optimal PU's expected utility. Both the PU's average utility loss due to the suboptimal algorithm and the strong incomplete information are small in our numerical example (less than 5% and 1.3% in our numerical results with two SU types).

This work represents a small step towards establishing a general framework of understanding incomplete information in dynamic spectrum sharing. As the next step plan, we will consider more incomplete information structures: (1) the PU does not know the distribution of SUs' types, and (2) each SU knows other SUs' types but PU does not know (a similar setting with a different application has been studied in [35]). We also want to understand how to design contracts in a market with multiple PUs and multiple SUs.

Appendix

A. Proof of Theorem 2

1) Proof of sufficient conditions: We use mathematical induction to prove this proposition. Let us denote $\Phi(n)$ as a subset which contains the first n power-time combinations in the complete contract Φ (i.e., $\Phi(n) = \{(p_k, t_k), |k = 1, ..., n\}$).

We first show that $\Phi(1)$ is feasible. Since there is only one SU type, the contract is feasible if it satisfies IR constraint in (8). This is true due to `Contd.b` in Proposition 2.

We next show that if contract $\Phi(k)$ is feasible, then we can construct the new contract $\Phi(k+1)$ by adding new item (p_{k+1}, t_{k+1}) and show that the new contract is also feasible. To achieve this, we need to show two results:

- *Result I:* the IC and IR constraints for type-θ_{k+1} SUs:

$$\begin{cases} \theta_{k+1}t_{k+1} - p_{k+1} \geq \theta_{k+1}t_i - p_i, \ \forall i = 1, ..., k \\ \theta_{k+1}t_{k+1} - p_{k+1} \geq 0, \end{cases} \quad (27)$$

- *Result II:* for types $\theta_1, ..., \theta_k$ already contained in the contract $\Phi(k)$, the IC constraints are still satisfied after adding the new type θ_{k+1}:

$$\theta_i t_i - p_i \geq \theta_i t_{k+1} - p_{k+1}, \forall i = 1, ..., k. \quad (28)$$

  Note that the new contract $\Phi(k+1)$ will satisfy the IR constraints of all type θ_1 to θ_k SUs as the original contract $\Phi(k)$ is feasible.

Proof of Result I in (27): First, we prove the IC constraint for type θ_{k+1}. Since contract $\Phi(k)$ is feasible, the IC constraint for a type-θ_i SU must hold, i.e.,

$$\theta_k t_i - p_i \leq \theta_k t_k - p_k, \forall i = 1, ..., k.$$

Also, the right inequality of (19) in `Contd.c` can be transformed to

$$p_{k+1} \leq p_k + \theta_{k+1}(t_{k+1} - t_k).$$

By combining the above two inequalities, we have

$$\theta_k t_i - p_i + p_{k+1} \leq \theta_k t_k + \theta_{k+1}(t_{k+1} - t_k), \forall i = 1, ..., k. \quad (29)$$

Notice that $\theta_{k+1} > \theta_k$ and $t_k \geq t_i$ in `Contd.a`. We also have

$$\theta_{k+1} t_k - \theta_{k+1} t_i \geq \theta_k t_k - \theta_k t_i.$$

By substituting this inequality into (29), we have

$$\theta_{k+1} t_{k+1} - p_{k+1} \geq \theta_{k+1} t_i - p_i, \quad (30)$$

which is actually the IC constraint for type θ_{k+1}.

Next, we show the IR constraint for type θ_{k+1}. Since $\theta_{k+1} > \theta_i$ for any $i \leq k$, then

$$\theta_{k+1} t_i - p_i > \theta_i t_i - p_i.$$

Using (30), we also have

$$\theta_{k+1} t_{k+1} - p_{k+1} \geq \theta_{k+1} t_i - p_i.$$

By combining the last two inequalities, we have

$$\theta_{k+1} t_{k+1} - p_{k+1} \geq 0,$$

which proves the IR constraint.

Proof of Result II in (28): Since contract $\Phi(k)$ is feasible, the IC constraint for type θ_i holds, i.e.,

$$\theta_i t_k - p_k \leq \theta_i t_i - p_i, \forall i = 1, ..., k.$$

Also, we can transform the left inequality of (19) in `Contd.c` to

$$p_k + \theta_k(t_{k+1} - t_k) \leq p_{k+1}.$$

By combining the above two inequalities, we conclude

$$\theta_i t_k + \theta_k(t_{k+1} - t_k) \leq \theta_i t_i - p_i + p_{k+1}. \quad (31)$$

Note that $\theta_k \geq \theta_i$ for any $i \leq k$ and $t_{k+1} \geq t_k$ in `Contd.a`. We also have

$$\theta_k t_{k+1} - \theta_k t_k \geq \theta_i t_{k+1} - \theta_i t_k.$$

By combining the above two inequalities, we conclude

$$\theta_i t_{k+1} - p_{k+1} \leq \theta_i t_i - p_i, \forall i = 1, \ldots, k$$

which is actually the IC constraint for type θ_i.

2) Proof of necessary conditions: It is easy to check that the sufficient conditions in Proposition 2 are also necessary for a feasible contract. Specifically, `Contd.a` is the same as necessary conditions summarized in (18). `Contd.b` is same as the necessary IR constraint for the lowest type θ_1 in a feasible contract. The left inequality of `Contd.c` can be derived from the necessary IC constraint for type θ_{k-1} in a feasible contract, and the right inequality of `Contd.c` can also be derived from the necessary IC constraint for type θ_k. ■

B. Proof of Proposition 3

First, it is not difficult to check that the relay powers in (20) satisfy the sufficient conditions of contract feasibility in Theorem 2. We skip the details here due to the page limit.

Next we prove the optimality and uniqueness of the solutions in (20).

1) Proof of optimality: We first show that the relay powers in (20) maximize the PU's utility given fixed time allocations, i.e., $\{p_k^*, \forall k\}$ maximize

$$\frac{1}{1+\sum_{k\in\mathcal{K}} N_k t_k}\left(\frac{R^{dir}}{2}+\frac{1}{2}\log(1+\frac{\sum_{k\in\mathcal{K}} N_k p_k}{n_0})\right). \quad (32)$$

We prove by contradiction. Suppose that these exists another feasible relay powers $\{\tilde{p}_k, \forall k\}$ which achieves a better solution than $\{p_k^*, \forall k\}$ in (20). Since (32) is increasing in total relay power, we must have $\sum_{k\in\mathcal{K}} N_k\tilde{p}_k > \sum_{k\in\mathcal{K}} N_k p_k^*$. Thus we have at least one relay power $\tilde{p}_k > p_k^*$ for one type θ_k.

If $k=1$, then $\tilde{p}_1 > p_1^*$. Since $p_1^* = \theta_1 t_1$, then $\tilde{p}_1 > \theta_1 t_1$. But this violates the IR constraint for type θ_1. Then we must have $k>1$.

Since $\{\tilde{p}_k, \forall k\}$ is feasible, then $\{\tilde{p}_k, \forall k\}$ must satisfy the right inequality of `Contd.c` in Theorem 2. Thus we have

$$\tilde{p}_k \le \tilde{p}_{k-1} + \theta_k(t_k - t_{k-1}).$$

By substituting $\theta_k(t_k - t_{k-1}) = p_k^* - p_{k-1}^*$ as in (20) into the this inequality, we have

$$\tilde{p}_{k-1} > p_{k-1}^*.$$

Using the above argument repeatedly, we finally obtain that

$$\tilde{p}_1 > p_1^* = \theta_1 t_1,$$

which violates the IR constraint for type-θ_1 again.

2) Proof of uniqueness: We next prove that the relay powers in (20) is the unique solution that maximizes (32). We also prove by contradiction. Suppose that there exists another $\{\bar{p}_k, \forall k\} \neq \{p_k^*, \forall k\}$ such that $\sum_{k\in\mathcal{K}} N_k\bar{p}_k = \sum_{k\in\mathcal{K}} N_k p_k^*$ in (32). Then there is at least one relay power $\bar{p}_i < p_i^*$ and one relay power $\bar{p}_j > p_j^*$. We can focus on type-θ_j and $\bar{p}_j > p_j^*$. By using the same argument before, we have $\bar{p}_1 > p_1^* = \theta_1 t_1$. But this violates the IR constraint for type θ_1. ■

C. Proof of Theorem 3

Proof. We prove by contradiction. Suppose that there exists an optimal contract item with $t_k > 0$ for the type-θ_k SUs with $k < K$. The total time allocation is $T' = \sum_{k\in\mathcal{K}} N_k t_k$ in this case. Then PU's utility is

$$u_{PU}^1 = \frac{\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\sum_{k\in\mathcal{K}} N_k(\theta_1 t_1 + \sum_{i=2}^{k}\theta_i(t_i - t_{i-1}))}{n_0}\right)}{1+T'}. \quad (33)$$

Next we show that given a fixed total time allocation T', allocating positive time only to the highest type SUs (i.e., $N_K t_K = T'$) achieves a larger utility for the PU as follows

$$u_{PU}^2 = \frac{\frac{R^{dir}}{2} + \frac{1}{2}\log\left(1 + \frac{\theta_K N_K t_K}{n_0}\right)}{1+T'}. \quad (34)$$

This is because $\theta_K N_K t_K = \theta_K T'$ in (34) and

$$\sum_{k\in\mathcal{K}} N_k(\theta_1 t_1 + \sum_{i=2}^{k}\theta_i(t_i - t_{i-1})) < \theta_K T'$$

in (33), thus (34) is larger than (33). This contradicts with the optimality of the contract, and thus we completes the proof. ■

D. Proof of Proposition 4

First, it is not difficult to check that the relay powers in (25) satisfy the sufficient conditions of contract feasibility in Theorem 2.

Next we prove the optimality of the solutions in (25).

1) Proof of optimality: We first show that the relay powers in (25) maximize the PU's expected utility given fixed time allocations, i.e., $\{p_k^*, \forall k\}$ is the solution to (24). We prove by contradiction. Suppose that these exists another feasible relay powers $\{\tilde{p}_k, \forall k\}$ which achieves a better solution than $\{p_k^*, \forall k\}$ in (25). Since PU's expected utility in (24) is increasing in total relay power, we must have $\sum_{k\in\mathcal{K}} N_k\tilde{p}_k > \sum_{k\in\mathcal{K}} N_k p_k^*$. Thus we have at least one relay power $\tilde{p}_k > p_k^*$ for one type θ_k.

If $k=1$, then $\tilde{p}_1 > p_1^*$. Since $p_1^* = \theta_1 t_1$, then $\tilde{p}_1 > \theta_1 t_1$. But this violates the IR constraint for type θ_1. Then we must have $k>1$.

Since $\{\tilde{p}_k, \forall k\}$ is feasible, then $\{\tilde{p}_k, \forall k\}$ must satisfy the right inequality of `Contd.c` in Theorem 2. Thus we have

$$\tilde{p}_k \le \tilde{p}_{k-1} + \theta_k(t_k - t_{k-1}).$$

By substituting $\theta_k(t_k - t_{k-1}) = p_k^* - p_{k-1}^*$ as in (25) into the this inequality, we have

$$\tilde{p}_{k-1} > p_{k-1}^*.$$

Using the above argument repeatedly, we finally obtain that

$$\tilde{p}_1 > p_1^* = \theta_1 t_1,$$

which violates the IR constraint for type-θ_1 again.

2) Proof of uniqueness: We next prove that the relay powers in (25) is the unique solution that maximizes (24). We also prove by contradiction. Suppose that there exists another $\{\bar{p}_k, \forall k\} \neq \{p_k^*, \forall k\}$ such that $\sum_{k\in\mathcal{K}} N_k\bar{p}_k = \sum_{k\in\mathcal{K}} N_k p_k^*$ in (24). Then there is at least one relay power $\bar{p}_i < p_i^*$ and one relay power $\bar{p}_j > p_j^*$. We can focus on type-θ_j and $\bar{p}_j > p_j^*$. By using the same argument before, we have $\bar{p}_1 > p_1^* = \theta_1 t_1$. But this violates the IR constraint for type θ_1. ■

E. Proof of Observation 2 for $R^{dir} = 0$

Proof. When $R^{dir} = 0$, the PU will always allocate positive time to the highest type SUs (i.e., $t_K^* > 0$). Its total time allocation is decreasing in the highest type θ_K. PU's utility in (22) can be written as

$$u_{PU}(t_K) = \frac{1}{2(1+N_K t_K)}\log(1+\theta_K N_K t_K). \quad (35)$$

Denote the total time allocation to SUs as T'. We can rewrite (35) as a function of T':

$$u_{PU}(T') = \frac{1}{2(1+T')}\log(1+\theta_K T'),$$

which can be shown as a concave function of T'. Since $t_K^* > 0$, we conclude that the optimal $T'^* = N_K t_K * > 0$ and satisfies

$$\frac{du_{PU}(T')}{dT'}\mid_{T'=T'^*} = \frac{\frac{\theta_K(1+T'^*)}{1+\theta_K T'^*} - \log(1+\theta_K T'^*)}{2(1+T'^*)^2} = 0,$$

i.e.,

$$F(\theta_K, T'^*) := \theta_K(1+T'^*) - (1+\theta_K T'^*)\log(1+\theta_K T'^*) = 0. \quad (36)$$

Since

$$\frac{\partial F(\theta_K, T'^*)}{\partial \theta_K} = 1 - T'^* \log(1+\theta_K T'^*),$$

$$\frac{\partial F(\theta_K, T'^*)}{\partial T'^*} = -\theta_K \log(1+\theta_K T'^*),$$

thus

$$\frac{dT'^*}{d\theta_K} = -\frac{\partial F/\partial \theta_K}{\partial F/\partial T'^*} = \frac{1 - T'^*\log(1+\theta_K T'^*)}{\theta_K \log(1+\theta_K T'^*)}. \quad (37)$$

Since $\theta_K T'^* > 0$, we have

$$\log(1+\theta_K T'^*) - \theta_K T'^* < 0.$$

By substituting this inequality into (36), we conclude

$$1 - T'^* \log(1+\theta_K T'^*) < 0,$$

and thus $\frac{dT'^*}{d\theta_K}$ in (37) is negative. Hence, the optimal total time allocation T'^* is decreasing in the highest type θ_K. ∎

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