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一种应用伪随机二位式信号的极值寻优法

刘 文 江

中国西安交通大学訪問学者

CIP-GEGEVENS KONINKLIJKE BIBLIOTHEEK, DEN HAAG

Liu Wen-Jiang

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ABSTRACT

A new extremum hunting method, using a correlation analysis hunting method, is proposed in this paper. A pseudo-random binary signal has been used as probe signal and the hunting direction is judged by the cross-correlation function between input and output. The input is stirred up on variable mode so that the hunting speed is higher than with other methods, such as common step forward searching method etc. Apart from this, high hunting precision and strong anti-interference ability are other advantages of this method.

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CONTENTS

1. INTRODUCTION

Self-searching optimum control systems, which have block diagram such as in Fig. 1, have been widely used in industrial processes because they do not need a mathematical model. The principle of self-searching optimum control is to make the control system reach its optimum operating state by continuously measuring and controlling. The necessary condition is that the plants must have an extremum characteristic.
The key of self-searching optimum control is how to find the system extremum point automatically.
There are mainly two kinds of self-searching methods, which are the continuous searching and the step forward searching method. [1], The step forward searching method can also be divided into two parts - the static searching method and the dynamic searching method. However, the heavy searching losses and poor anti-interference ability are their common weaknesses. In this paper, we proposed a correlation analysis searching method which use the cross-correlation function of the output and input signals z(t) and u(t), to detect searching direction to the optimum by adding a pseudo random binary signal sequence u(t) in the plant input.

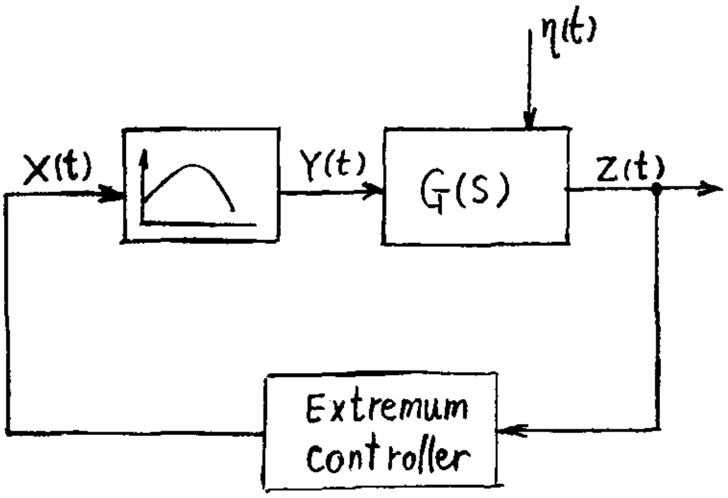

Fig. 1 Block diagram of the extremum control system

Due to the use of correlation analysis, this method has a strong anti-interference ability and the hunting precision has been improved by using an alter-stepping proper input mode.

2. PRINCIPLE OF CORRELATION ANALYSIS HUNTING METHOD

As an example, we study a SISO system as Fig. 1.
Generally, $y(t)$ is unobservable and whether the system operates on its optimum can be determined by measuring the input and output. The automatic hunting ability of a correlation analysis method is acquired by seeking the relationship between I/O cross-correlation function and plant optimum.

Assume the plant $G(s)$ has a transition time T_s pulse response function $h(t)$ and the non-linear characteristic is $y = f(x)$ which has only one optimum, (x_m, y_m). The input and output signals at n steps $x(nT)$, $y(nT)$, $z(nT)$ are represented by X_n, Y_n, Z_n. T is the hunting period; the hunting step is ΔX.

A pseudo random binary signal (m-sequence) with an amplitude B, length N and clock period ΔT, is added to the input:

$$\left.\begin{aligned} X(t) &= X_n + U(t) \\ Y(t) &= f\left[X_n + U(t)\right] \end{aligned}\right\} \qquad (1)$$

Extending it into a Taylor series at $X = X_n$ gives:

$$y(t) = f(X_n) + f'(X_n)U(t) + \frac{f''(X_n)}{2!}U^2(t) + \dots .$$

When the pseudo random binary signal amplitude B is small, the higher power part of the series can be ignored:

$$y(t) \doteq f(X_n) + f'(X_n)U(t) = Y_n + f'(X_n)U(t) \qquad (2)$$

The relationship between $Z(t)$ and $Y(t)$ is

$$Z(t) = \int_0^t h(v)y(t-v)\,dv$$

If the length value N of $U(t)$ is large enough, for instance $(N-1)\Delta T \gg T_s$, the above formula can be simplified as:

$$Z(t) = \int_0^{(N-1)\Delta T} h(v)y(t-v)\,dv \qquad (3)$$

Assume $k = (t/\Delta T)$, substituting for k, ℓ in equation (2), (3) we obtain:

$$y(k) = y_n + f'(x)u(k) \tag{4}$$

$$z(k) = \sum_{\ell=0}^{N-1} h(\ell)y(k-\ell)\Delta T \tag{5}$$

and the cross-correlation function of the psuedo random signal u(k) and output signal z(k) is

$$\begin{aligned} R_{uz}(\mu) &= E\ \{E[u(k)z(k+\mu)]\} \\ &= \sum_{\ell=0}^{N-1} h(\ell)E[u(k)y(k+\mu-\ell)]\Delta T \\ &= \Delta T\sum_{\ell=0}^{N-1} h(\ell)\left\{ E[u(k)]y_n + E[u(k)u(k+\mu-\ell)]f'(x_n)\right\} \\ &= \Delta T y_n \sum_{\ell=0}^{N-1} h(\ell)E[u(k)] + \Delta T f'(x_n)\sum_{\ell=0}^{N-1} h(\ell)R_{uu}(\mu-\ell) \end{aligned} \tag{6}$$

The mean and autocorrelation function of m-sequence are:

$$E[u(k)] = -\frac{B}{N} \tag{7}$$

$$R_{uu}(\mu-\ell) = \begin{cases} B^2 & \ell=\mu \\ \frac{-B^2}{N} & \mu < \ell < N+\mu \end{cases} \tag{8}$$

Substitute (7), and (8) into equation (6):

$$R_{uz}(\mu) = \Delta T y_n \sum_{\ell=0}^{N-1} h(\ell)\left(-\frac{B}{N}\right) + \Delta T f'(x_n)\left[\frac{N+1}{N}B^2 h(\mu) - \frac{B^2}{N}\sum_{\ell=0}^{N-1} h(\ell)\right]$$

$$= (\frac{N+1}{N}B^2\Delta T) f'(x_n)h(\mu) - \Delta T\frac{B}{N}[y_n+Bf'(x_n)] \sum_{\ell=0}^{N-1} h(\ell)$$

$$= C_1 f'(x_n)h(\mu) + C_2 \tag{9}$$

where

$$C_1 = \frac{N+1}{N} B^2 \Delta T$$

$$C_2 = - \Delta T \frac{B}{N} [y_n + Bf'(x_n)] \sum_{\ell=0}^{N-1} h(\ell) \tag{10}$$

where C_1 is a positive constant. C_2 also is a constant when X_n is fixed, so the sign of $f'(x_n)$ is decided by an equation as follows

$$\text{Sign } [f'(x_n)] = \text{Sign } [R_{uz}(\mu) - C_2] \text{ sign } [h(\mu)] \text{ .} \tag{11}$$

Since y_n, and $f'(x_n)$ are generally unknown, the constant C_2 could not be obtained from equation (10).

Let us assume μ is equal to μ_1 and μ_2 respectively.
Substitute it into equation (9).

$$R_{uz}(\mu_1) = C_1 f'(x_n)h(\mu_1) + C_2$$

$$R_{uz}(\mu_2) = C_1 f'(x_n)h(\mu_2) + C_2$$

so we obtain:

$$R_{uz}(\mu_1) - R_{uz}(\mu_2) = C_1 f'(x_n) [h(\mu_1) - h(\mu_2)]$$

$$\text{Sign } [f'(x_n)] = \text{Sign } [R_{uz}(\mu_1) - R_{uz}(\mu_2)] \text{ Sign } [h(\mu_1) - h(\mu_2)]$$

Once the sign of $f'(x_n)$ has been decided, the direction of the next hunting step is determined, Fig. 2. The input at the (n+1)th step is x_{n+1}:

$$x_{n+1} = x_n + \text{sign } [f'(x_n)]\Delta x \tag{13}$$

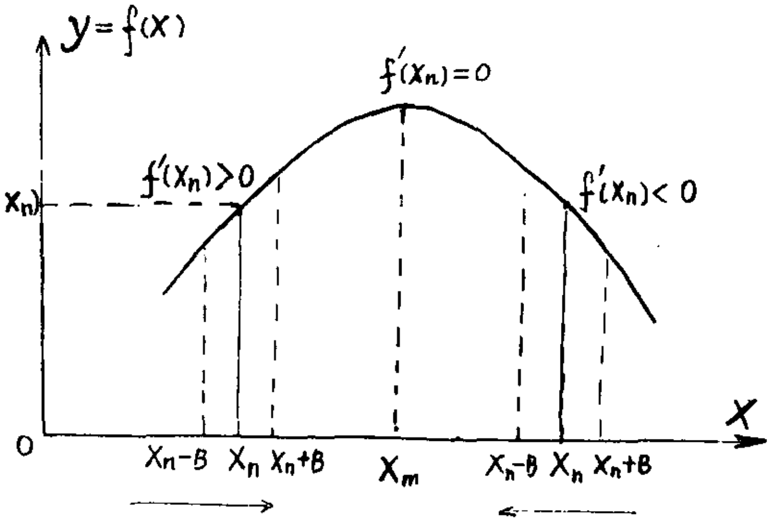

Fig. 2 The searching direction of the next step

We also can obtain the cross-correlation function $R_{uz}(\mu)$ by adding (m+1) periods of a pseudo random signal at input end,

$$R_{uz}(\mu) = \frac{1}{mn}\sum_{k=0}^{MN-1} u(k)z(k+\mu) \qquad (m = 1,\ 2,\ 3,\ 4) \qquad (14)$$

From the equation above, we obtain the optimum hunting formulae as follows:

$$\begin{aligned}
&x(k) = x_n + u(k)\ ;\\
&R_{uz}(\mu) = \frac{1}{mN}\sum_{k=0}^{mN-1} u(k)Z(k+\mu)\ ;\\
&\text{Sign}\ [f'(x_n)] = \text{Sign}\ [R_{uz}(\mu_2) - R_{uz}(\mu_1)]\ ;\\
&x_{n+1} = x_n + \Delta x\ \text{sign}\ [f'(x_n)]\ .
\end{aligned} \qquad (15)$$

So, the self-hunting extremum control can be realized by the formulae above.

3. SIMULATION RESULTS

To examine the correctness of the method, simulation is necessary. The optimum characteristics and delay of the control plant are very easy to simulate. The second order inertia link can be obtained from 4th-

order Runge-Kuta method.
The older step hunting method formulae were shown in chapter 16 of reference [1]. Here we show the simulation result on interference, in cases where the laboratory plant transfer function is $G(s) = \frac{Ke^{-\tau s}}{(T_1 s+1)\ (T_2+1)}$; in the equation, $T_1 = T_2 = 10s$, $\tau = 20s$, $K=1$.
The pseudo random signal is an m-sequence with $N=31$, $\Delta T=5s$. The plant extremum characteristics is $y=0.4(10-x)x$; the optimum is at $x_m=5$, $y_m=10$. Some simulation results are shown in Fig. 3 and Fig. 4. Fig. 3 shows the simulation results of the cross-correlation analysis hunting method.
Fig. 4 shows the result of general step hunting method.

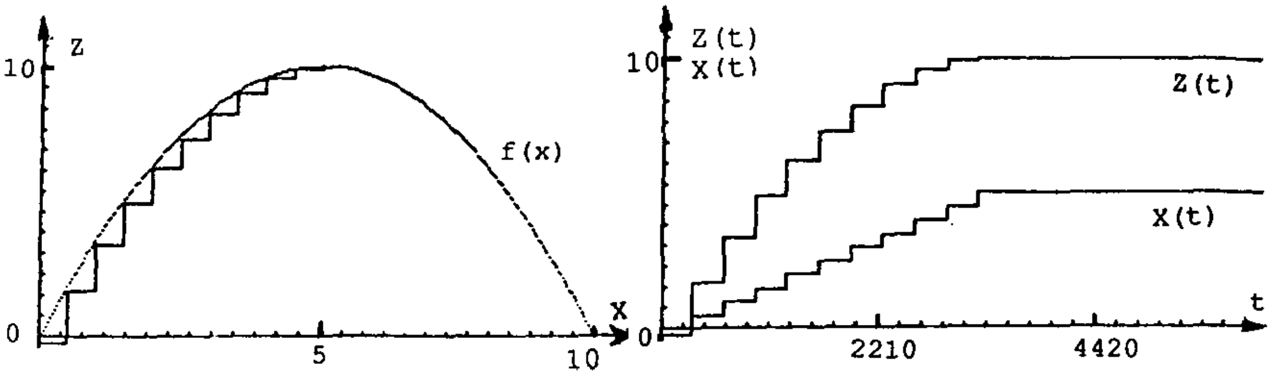

Fig. 3 Simulation result using correlation analysis hunting method

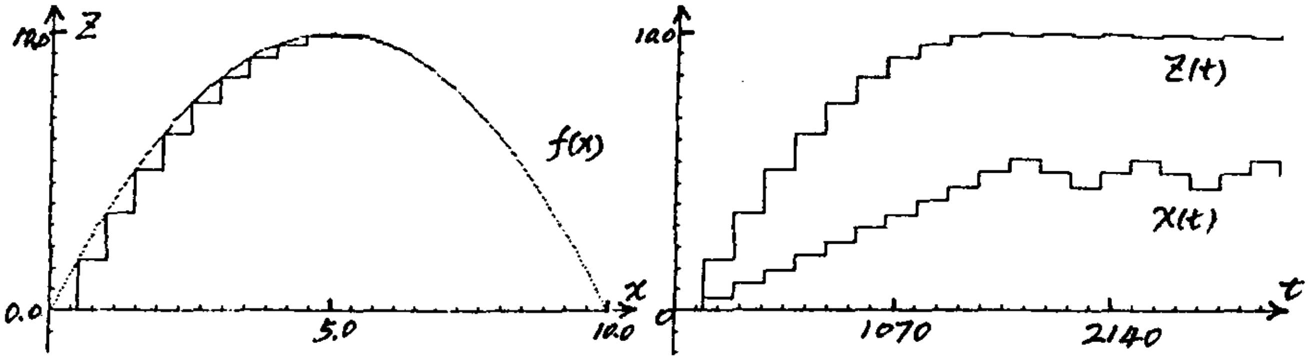

Fig. 4 Simulation result of general step hunting method

With the simulation results shown above, we can see that the correlation analysis hunting method has a higher hunting precision and the hunting curves can be settled on the optimal point, whereas the curves of the general hunting method are oscillating around the optimum.

The simulation of the anti-interference ability is carried out by adding a random disturbance $\eta(t)$, corresponding to the measurement noise at the plant output (see Fig. 1). The results are shown in Fig. 5 and 6, where the p-p value of $\eta(t)$ is 20% of the output maximum value. It is obvious that the correlation analysis hunting method has a stronger anti-interference ability and is more effective compared with the general hunting method when the disturbance becomes stronger.

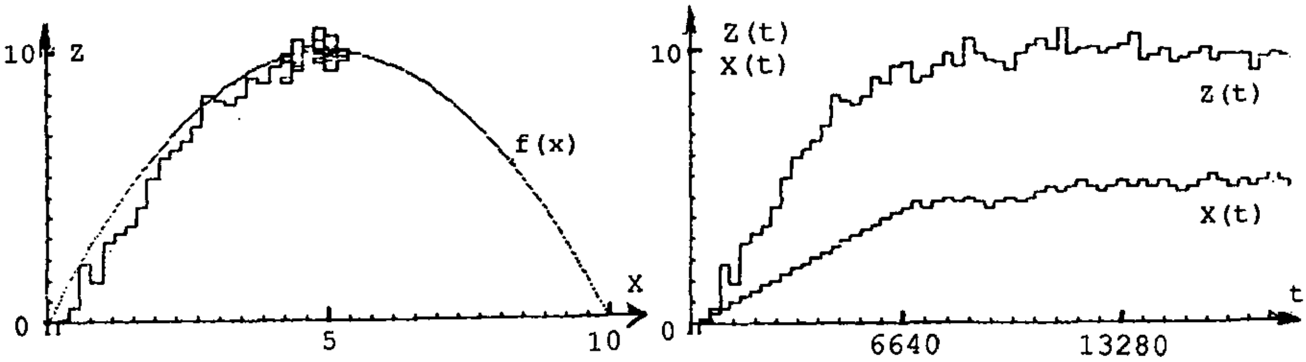

Fig. 5 Simulation result of the correlation analysis method, with a random disturbance present

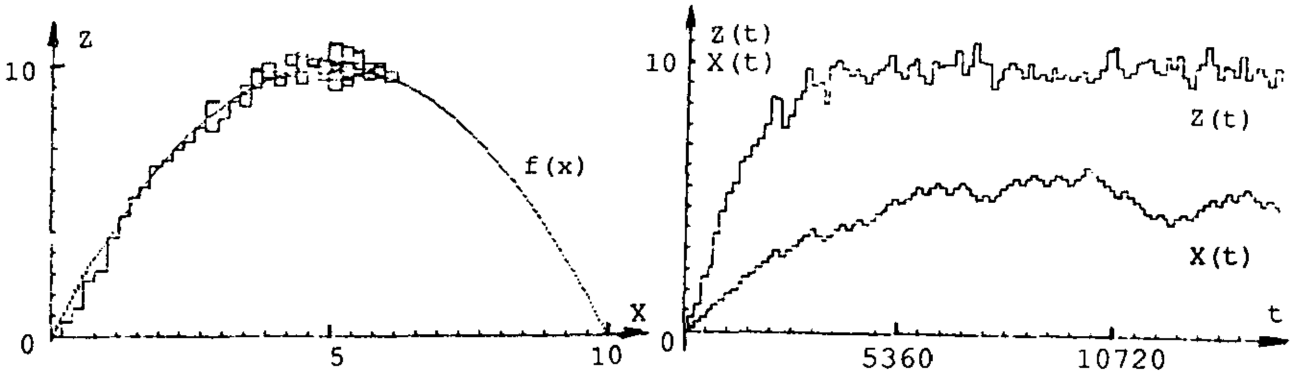

Fig. 6 Simulation result of general step hunting method, with a random disturbance present

4. CONCLUSIONS

The theoretical analysis and computer simulation have shown that the correlation analysis hunting method can catch the system optimum point and make the hunting loss zero in the ideal situation. The correlation analysis hunting method overcomes the disadvantage of the poor anti-interference ability which the general hunting method has and it will be easier to be apply in industrial processes.

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