Limited Channel Feedback for Coordinated Beamforming under SINR Requirements

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***Abstract*—We consider bit allocation for channel state information (CSI) feedback in coordinated beamforming (CB) systems. The optimization problem under consideration is to minimize the total transmission power subject to the target signal to interference and noise ratio (SINR) constraints. We obtain a suboptimal solution to the problem that provides closed-form expressions for the total feedback rate and the feedback rates of each user and its channels. The advantage of the proposed scheme over the methods with equally distributed feedback rates among the users and/or channels is demonstrated by computer simulation: the CB system employing the proposed scheme causes less outage with less transmission power.**

I. Introduction

Coordinated multi-point transmission (CoMP), also called network multi-input multi-output (MIMO), has been recognized as an effective technique for increasing the sum rate and reducing outages in celluar systems [1]. CoMP is realized by a group of based stations (BSs) that cooperate to form a virtual antenna array distributed over multiple cells. When BSs fully cooperate by sharing both user channel state information (CSI) and data, powerful MIMO techniques such as dirty paper coding and multicell linear preprocessing can be designed for CoMP [2],[3]. This type of CoMP is referred to as *multicell joint processing* (JP). On the other hand, in partial BS cooperation, known as *coordinated beamforming* (CB), the BSs exchange only user CSI and design their respective beamforming vectors [4],[5].

The performance of CoMP depends on the quality of CSI at the BSs. In frequency division duplex (FDD) systems, where users estimate CSI and feedback to BSs via wireless links, limited channel feedback considering the tradeoff between CSI feedback overhead and the system performance is an important design issue, and strategies for CSI feedback in CoMP FDD systems have been proposed recently. For multicell JP, selective CSI feedback methods, which permit only those users with high quality channels to feedback their CSI, have been introduced in [6]-[8]. They include a channel selection algorithm maximizing the average signal to interference noise ratio (SINR) [6], schemes for reducing both CSI feedback overhead and backhaul overhead [7], and algorithms that jointly optimize the channel selection and bit allocation to reduce the mean-loss in achievable rate caused by limited feedback [8]. In addition, for multicell JP, a bit allocation scheme maximizing the quantization accuracy for limited CSI feedback has been developed [9].

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CSI feedback for CB has been investigated in [5], [10]-[12]. The impact of CSI quantization error on the average achievable rate is analyzed [5], and a bit allocation method is developed to reduce the mean-loss in achievable rate caused by delayed limited feedback [10]. While these results have been derived for independent block fading channels, where CSI varies independently with time-slots, [11] assumes channel correlation among time-slots and proposes an optimal feedback-control policy that turns CSI feedback on/off according to channel gain and quantization error. A two stage feedback method for both scheduling and beamforming has been introduced [12].

In this paper, we develop an alternative approach to the feedback bit allocation for CB systems over independent block fading channels. In contrast to the existing schemes in [9]-[12] that allocate bits to each user's channels for a given total CSI feedback rate that is equally distributed among the users, the proposed scheme determines the required total feedback rate and considers bit allocation for the users as well as their channels. In particular, motivated by the bit allocation scheme for single-cell MU-MIMO [13], we formulate an optimization problem minimizing the total transmission power subject to the target SINR constraints and derive a suboptimal solution to the problem. Closed-form expressions for the total feedback rate and the feedback rates of each user and its channels are derived. Simulation results show that the CB system employing the proposed bit allocation scheme can cause less outage and needs less transmission power than the system with equally allocated feedback rates among the users and/or channels.

II. System Model and problem formulation

We consider a downlink cooperative network consisting of K cells and a central controller that allocate resources based on channel information and SINR requirements from the BSs (Fig.1). The BS in each cell serves one user at a time, and each user receives interference from $K-1$ neighbering cells as well as the desired signal from its BS. The BSs are equipped with M antennas, $M \geq K$, while each user has a single antenna. Beamforming for prenulling the intercell

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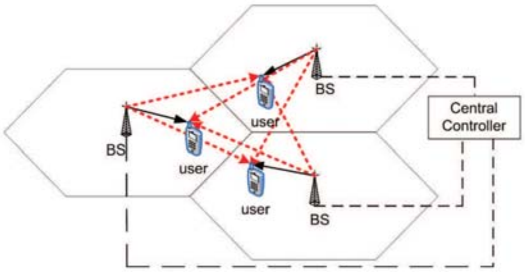

Fig. 1. A cooperative network with a central controller (K=3).

interferences is performed at each BS. The channel between the j^{th} BS and the i^{th} user is denoted by $\mathbf{h}_{ji} \in \mathcal{C}^M$, where $\mathbf{h}_{ii}$ is the channel corresponding to the desired signal and $\mathbf{h}_{ji}, j \neq i$, represents the interference channel. We assume that the m^{th} entry of $\mathbf{h}_{ji}$, say h_{jim} for $1 \leq m \leq M$, is independent identically distributed with $\mathcal{CN}(0, a_{ji}^2)$ where a_{ji}^2 represents the pathloss between the j^{th} BS and the i^{th} user. In addition, the desired and interference channels are assumed to be statistically independent. The received signal of the i^{th} user is given by

$$y_i = \mathbf{h}_{ii}^H \mathbf{v}_i s_i + \sum_{j=1, j\neq i}^{K} \mathbf{h}_{ji}^H \mathbf{v}_j s_j + n_i, \tag{1}$$

where $(\cdot)^H$ denotes the conjugate transpose; $\mathbf{v}_i \in \mathcal{C}^M$ is the beamforming vector; s_i is a transmit symbol from the i^{th} BS; and $n_i \sim \mathcal{CN}(0,1)$ is additive white Gaussian noise. We consider zero-forcing at each BS to nullify intercell interference of the users. Specifically, at the j^{th} BS, the beamforming vector $\mathbf{v}_j$ is designed to satisfy $\overline{\mathbf{h}}_{ji}^H \mathbf{v}_j = 0 \quad for \quad 1 \leq i \leq K, i \neq j$, where $\overline{\mathbf{h}}_{ji}$ is the channel direction given by $\overline{\mathbf{h}}_{ji} = \mathbf{h}_{ji}/g_{ji}$ and $g_{ji} = \|\mathbf{h}_{ji}\|^2$ is the channel gain. The channel directions required for beamforming are estimated by each user and transferred to the BSs through wireless feedback channels with limited rates. We denote the feedback rate for transferring $\overline{\mathbf{h}}_{ji}$ by B_{ji} and develop a procedure for determining $\{B_{ji} | 1 \leq j \leq K, 1 \leq i \leq K, j \neq i\}$, which are the feedback rates for interference channel directions, while assuming sufficiently large feedback rates B_{ii} for the desired channel directions.[2] The total feedback rate for the interference channel directions allocated to the i^{th} user is given by $B_i \triangleq \sum_{j=1, j\neq i}^{K} B_{ji}$.

The limited feedback rate causes intercell interference leakage. When each user employs the random vector quantization (RVQ), an upper bound of the interference leakage from the j^{th} BS to the i^{th} user, denoted as I_{ji}, is given by [14]

$$|\mathbf{h}_{ji}^H \mathbf{v}_j|^2 < g_{ji} 2^{-B_{ji}/(M-1)} \triangleq I_{ji}, \tag{2}$$

and the sum of $\{I_{ji}\}$ at the i^{th} user is $I_i \triangleq \sum_{j=1, j\neq i}^{K} I_{ji}$.

The design problem for determining $\{B_{ji}\}$ is formulated as the minimization of the total transmission power subject to the user's target SINR requirements:

$$\min_{\{B_{ji}\}} \; P_{total} \triangleq \sum_{j=1}^{K} P_j \tag{3a}$$

$$subject \;\; to \;\; (s.t.) \; \mathrm{SINR}_i \geq \gamma_i, \;\; \forall \; i, \tag{3b}$$

$$\sum_{i=1}^{K} B_i \leq B_T, \;\; and \;\; B_{ji} \in \mathbb{N}_+, \tag{3c}$$

$\forall \; i \; and \; j, \;\; i \neq j$, where P_j is the transmission power of the j^{th} BS; γ_i is the target SINR for the i^{th} user; B_T is the available total feedback rate for the interference channel directions; and $\mathbb{N}_+$ is a set of nonnegative integers. To specify the problem in (3), we consider the worst-case SINR, following the approach in [13]. SINR_i in (3) can be represented as

$$\mathrm{SINR}_i = \frac{P_i |\mathbf{h}_{ii}^H \mathbf{v}_i|^2}{\sum_{j\neq i} P_j |\mathbf{h}_{ji}^H \mathbf{v}_j|^2 + 1} \geq \frac{P_i g_{ii} x_i}{\sum_{j\neq i} P_j I_{ji} + 1}, \tag{4}$$

where $g_{ii} = \|\mathbf{h}_{ii}\|^2$ is the desired channel gain and $x_i = |\overline{\mathbf{h}}_{ii}^H \mathbf{v}_i|^2$ is the beamforming gain. Note in (4) that the transmission power for the i^{th} BS, P_i, can be excessive when x_i is close to zero. To avoid this difficulty, an outage region is defined: we set $P_i = 0$ if $x_i < x_i^o$, where x_i^o is the smallest acceptable directional gain. Under this setting, $x_i \geq x_i^o$ whenever $P_i > 0$, and (4) can be rewritten as

$$\mathrm{SINR}_i \geq \frac{P_i g_{ii} x_i^o}{\sum_{j\neq i} P_j I_{ji} + 1}. \tag{5}$$

The right-hand side (RHS) of (5) is the worst-case SINR, which is a function of $\{I_{ji}\}$ (or $\{B_{ji}\}$).

III. Proposed Bit Allocation

In this section, we first derive an expression for B_{ji} in terms of B_i and $\{g_{ji}\}$. Then, using this expression, the optimization in (3) is converted into an optimization with respect to $\{B_i\}$, and a suboptimal solution for B_i is obtained.

A. Allocating $\{B_{ji}\}$ for a given B_i

When B_i is given, we determine $\{B_{ji} | 1 \leq j \leq K, j \neq i\}$ by minimizing the sum of interference leakage upper bounds, I_i:

$$\min_{\{B_{ji}\}} \;\; I_i \;\; s.t. \; \sum_{j\neq i} B_{ji} \leq B_i \;\; and \;\; B_{ji} \in \mathbb{R}_+,$$

$1 \leq j \leq K, \;\; j \neq i$, where $\mathbb{R}_+$ is the set of non-negative real numbers, and $\{B_{ji}\}$ are relaxed to real numbers to avoid integer programming. This problem is the convex optimization problem, and after some calculation, we can get the following closed-form solution for the optimal B_{ji}, denoted as $\widehat{B}_{ji}$ [16]:

$$\widehat{B}_{ji} = \left(\frac{B_i}{K-1} + (M-1)\left(\log_2(g_{ji}) - \log_2 \overline{g}_i\right) \right)^+, \tag{6}$$

where $(x)^+ = \max(0, x)$ and $\overline{g}_i = \left(\prod_{k\neq i}^{K} g_{ki} \right)^{\frac{1}{K-1}}$ is the geometric mean of the interference channel gains at the i^{th}

[2]In fact, it is assumed that each BS has perfect knowledge on its desired channel. We do not consider bit allocation for B_{ii}.

user. As expected, feedback bits are assigned to the channels in proportion to their channel gains, g_{ji}. If $\{g_{ji}\}$ are the same for all j, each interference channel takes an equal share of $\frac{B_i}{K-1}$.

B. Allocation of $\{B_i\}$

To convert the optimization in (3) into an optimization with respect to $\{B_i\}$, we represent SINR$_i$ and P_{total} in terms of $\{B_i\}$ by exploiting (6). Let us define $\widehat{B}'_{ji}$ by dropping the $(\cdot)^+$ from $\widehat{B}_{ji}$ in (6)[3], and substitute $\widehat{B}'_{ji}$ for B_{ji} in (2). The resulting I_{ji}, denoted as $\widehat{I}_{ji}$ is given by

$$\widehat{I}_{ji} = \overline{g}_i 2^{-\eta_i}, \quad \forall j \neq i, \tag{7}$$

where $\eta_i = B_i/((M-1)(K-1))$. Note that $\{\widehat{I}_{ji}\}$ are the same for all j. Using $\{\widehat{I}_{ji}\}$ in (5), the SINR can be rewritten as

$$\text{SINR}_i \geq \frac{P_i g_{ii} x_i^o}{\overline{g}_i 2^{-\eta_i} \sum_{j\neq i} P_j + 1} = \frac{P_i g_{ii} x_i^o}{\overline{g}_i 2^{-\eta_i}(P_{total} - P_i) + 1}. \tag{8}$$

The last term in (8) does not depend on $\{B_{ji}\}$ but depends on B_i, and an upper bound of P_{total} in (3) can be obtained by setting this term to be equal to γ_i, i.e.,

$$\frac{P_i g_{ii} x_i^o}{\overline{g}_i 2^{-\eta_i}(P_{total} - P_i) + 1} = \gamma_i. \tag{9}$$

By solving this equality and computing $\sum_{i=1}^K P_i$, we get

$$P_{total} = \sum_{i=1}^K P_i = \frac{\sum_{i=1}^K \alpha_i}{1 - \sum_{i=1}^K \alpha_i \overline{g}_i 2^{-\eta_i}}, \tag{10}$$

where $\alpha_i = \frac{\gamma_i}{g_{ii}x_i^o + \gamma_i \overline{g}_i 2^{-\eta_i}}$. If we use this expression for P_{total} in (3), the optimization problem is simplified as follows

$$\min_{\{B_i\}} \; P_{total} \text{ in } (10) \tag{11a}$$

$$s.t. \sum_{i=1}^K B_i \leq B_T, \; and \; B_i \in \mathbb{R}_+, \; \forall\, i. \tag{11b}$$

Comparing the problems in (3) and (11a), the SINR constraint in (3) is dropped in (11a) because P_{total} in (10) is derived from the equality in (9), and $\{B_i\}$ are relaxed to non-negative real numbers to avoid integer programming. The problem (11a) is not yet a convex optimization problem and we need to simplify further the objective function to derive a closed-form expression for B_i. The simplification process is as follows. We ignore the numerator in (10) and consider the maximization of the denominator, which is equivalent to

$$\begin{aligned}\min_{\{B_i\}} \sum_{i=1}^K \alpha_i \overline{g}_i 2^{-\eta_i} &= \min_{\{B_i\}} \sum_{i=1}^K \frac{1}{g_{ii}x_i^o/(\gamma_i \overline{g}_i 2^{-\eta_i}) + 1} \\ &\simeq \min_{\{B_i\}} \sum_{i=1}^K \frac{1}{g_{ii}x_i^o/(\gamma_i \overline{g}_i 2^{-\eta_i})}.\end{aligned} \tag{12}$$

[3] $\widehat{B}'_{ji} = \widehat{B}_{ji}$ if $\overline{g}_i 2^{-\frac{B_i}{(M-1)(K-1)}} \leq \min_j g_{ji}$. This inequality usually holds because, in general, $B_i > (M-1)(K-1)$ and $\overline{g}_i$ is not exceedingly greater than $\min_j g_{ji}$ when BSs cooperate to suppress intercell interference.

The approximation for the last term in (12) can be justified because $g_{ii}x_i^o/(\gamma_i \overline{g}_i 2^{-\eta_i}) > 1$ under the equality in (9). Using (12), the optimization problem in (11a) is rewritten as

$$\min_{\{B_i\}} \; \sum_{i=1}^K \frac{1}{g_{ii}x_i^o/(\gamma_i \overline{g}_i 2^{-\eta_i})} \quad s.t. \;\; (11b). \tag{13}$$

This is a convex optimization problem, and after some calculation, we can obtain the following closed-form expression for the optimal B_i:

$$\widehat{B}_i = \left(\frac{B_T}{K} + (M-1)(K-1)\log_2 \beta_i - (M-1)(K-1)\log_2 \overline{\beta}\right)^+, \tag{14}$$

where $\beta_i = \frac{\gamma_i \overline{g}_i}{g_{ii}x_i^o}$ and $\overline{\beta} = \left(\prod_j^K \frac{\gamma_j \overline{g}_i}{g_{jj}x_i^o}\right)$ is the geometric mean of β_i. If $\{\beta_i\}$ are the same for all i, each user takes the same share of $\lfloor \frac{B_T}{K} \rfloor$; otherwise, a user with a larger β_i (or equivalently, a larger target SINR γ_i and a smaller desired channel gain g_{ii}) uses a higher feedback rate.

Finally, in this section we derive B_T. Equations (9) and (10) indicate the existence of $\{P_i\}$ and $\{B_i\}$ satisfying the SINR requirement in (9) if the denominator of (10) is positive. This condition is met if $g_{ii}x_i^o/(\gamma_i \overline{g}_i 2^{-\eta_i}) + 1 > K, \;\; \forall i$, or equivalently, $\eta_i > \log_2(K-1) + \log_2 \beta_i$. Assuming that the RHS of this inequality is positive,[4] we compute $\sum_{i=1}^K \eta_i$ to get

$$\sum_{i=1}^K \eta_i = \frac{B_{total}}{(M-1)(K-1)} > K\log_2(K-1) + \sum_{i=1}^K \log_2 \beta_i. \tag{15}$$

From this inequality, B_T is determined as

$$B_T = \left\lceil (M-1)(K-1)\left(K\log_2(K-1) + \log_2 \overline{\beta}\right)\right\rceil, \tag{16}$$

which is the smallest B_{total} satisfying the inequality in (15). B_T increases logarithmically with $\overline{\beta}$.

C. Comments on Operation

The proposed algorithm evaluates, first, B_T in (16), second, $\widehat{B}_i$ in (14), and then, $\widehat{B}_{ji}$ in (6). The central controller collects $\{g_{ji}\}$ and $\{\gamma_i\}$ from the users and determines B_T, $\widehat{B}_i$, and $\widehat{B}_{ji}$. The solutions in (6) and (14) do not take integer values, and thus, it is necessary to perform simple rounding or the steepest descent search to find integer solutions. The simulation results in the next section are obtained by rounding.

Since $\{g_{ji} | 1 \leq j \leq K, j \neq i\}$ are available at the i^{th} user, $\{\widehat{B}_{ji} | 1 \leq j \leq K, j \neq i\}$ can also be evaluated by the i^{th} user once $\widehat{B}_i$ is transferred from the central controller. The quantized channel directions of each user are transferred to all BSs. Then, BSs recover the channel directions $\overline{\mathbf{h}}_{ji}$ and obtain their beamforming vectors $\{\mathbf{v}_i\}$ and the corresponding $\{x_i\}$ in (4). If $x_i < x_i^o$, then an outage, referred to as the *directional* outage, is declared; otherwise, $\{x_i\}$ are reported to the central controller.

Given $\{x_i\}$, the central controller evaluates the required power $\{P_i\}$ by solving $\frac{P_i g_{ii}}{\sum_{j\neq i} P_j I_{ji} + 1} = \gamma_i, \;\; \forall i$, (see (4)).

[4] $\beta_i \geq 1$ if $\gamma_i > g_{ii}x_i^o/\overline{g}_i$, which usuall holds when $\gamma_i > 1$ and $x_i^o \ll 1$.

TABLE I

FOUR CASES WITH DIFFERENT USER LOCATIONS AND TARGET SINR VALUES. FOR EACH CASE B_T, AND B_i ARE OBTAINED USING (16) AND (14), RESPECTIVELY, AND SHOWN IN THE UNIT OF BITS.

		User 1	User 2	User 3
Case 1	(D_{ii}, γ_i)	(450m, 5dB)	(450m, 5dB)	(450m, 5dB)
B_T = 87.6	B_i	29.2	29.2	29.2
Case 2	(D_{ii}, γ_i)	(450m, 5dB)	(450m, 10dB)	(450m, 15dB)
B_T = 117.5	B_i	29.2	39.2	49.1
Case 3	(D_{ii}, γ_i)	(350m, 5dB)	(400m, 5dB)	(450m, 5dB)
B_T = 74.2	B_i	22.5	24.7	27.0
Case 4	(D_{ii}, γ_i)	(350m, 5dB)	(400m, 10dB)	(450m, 15dB)
B_T = 104.2	B_i	22.5	34.7	47.0

TABLE II

COMPARISON AMONG THE PROPOSED, LOCAL, AND EQUAL ALLOCATION SCHEMES WHEN THE B_T FOR EACH CASE IS GIVEN BY TABLE I. HERE P_{total} IS IN dB SCALE WITH REFERENCE TO THE NOISE VARIANCE.

	Proposed allocation		Local allocation		Equal allocation	
	power outage [%]	P_{total}	*power* outage [%]	P_{total}	*power* outage [%]	P_{total}
Case 1	0.07	23.53	0.53	23.82	1.35	25.31
Case 2	0.05	30.76	1.66	32.66	4.00	34.09
Case 3	0.05	21.85	0.53	22.36	2.02	24.94
Case 4	0.06	30.71	3.63	31.79	9.93	33.54

If the resulting $\{P_i\}$ are nonnegative for all i, then $\{P_i\}$ are transferred to the BSs; otherwise, an outage is declared. This type of outage is referred to as a *power* outage. BSs transmit their symbols with power $\{P_i\}$ and the beamforming vector $\{\mathbf{v}_i\}$.

IV. SIMULATION RESULTS

We examined the performance of the proposed bit allocation scheme through computer simulation. For comparison, two additional schemes were also considered: one is called the *local* allocation scheme that assigns $\{B_{ji}\}$ according to (6) but has equal $\{B_i\}$ for all i, and the other is the naive *equal* allocation scheme with equal $\{B_{ji}\}$ for all $i,\ j,\ i \neq j$ and equal $\{B_i\}$ for all i. The simulation parameters are as follows: $K = 3$, $M = 4$, and the pathloss between each BS and the users are given by $10\log_{10} a_{ji}^2 = 128.1 + 37.6\log_{10}(D_{ji}/1,000)$, where D_{ji} is the distance between the j^{th} BS and the i^{th} user [17]. The cell radius is $500m$, and four cases with different user locations and target SINR values are considered (Table I). For each case, D_{ii} is fixed but the angle between the i^{th} user and its BS is uniformly distributed in between 0^o and 120^o. We set $x_i^0 = 0.1$, which causes the directional outage of the i^{th} user with a probability of 0.03, and $B_{ii} = 10$ bits. The results in this simulation are obtained through $10,000$ channel realizations.

Table I shows B_T and B_i obtained from (16) and (14), respectively, for each case/user. As expected, the case (user) associated with larger (D_{ii}, γ_i) values needs a larger B_T (B_i). For example, Case 2 is associated with the largest (D_{ii}, γ_i) for each i and needs the largest B_T; $\{B_i\}$ for this case is greater than or equal to those for the other cases. Note that $B_1 = B_2 = B_3$ for Case 1. This happened because (D_{ii}, γ_i) are the same for all i.

Table II compares the probability of power outage and the P_{total} of the proposed, local, and equal allocation schemes when B_T for each case is given by Table I (for each case the three schemes are assigned identical B_T). The proposed scheme outperformed the others: it needed the least power and yet caused the least outage probability, as expected, the equal allocation performed the worst. Comparing the P_{total} of the proposed and that of the local allocation, they are close to each other in Case 1, where (D_{ii}, γ_i) are the same for all i, but their difference reaches 1.9dB in Case 2.

V. CONCLUSION

A bit allocation scheme for CSI feedback in CB systems has been developed by considering the minimization of transmission power under target SINR constraints. A suboptimal solution to the optimization was obtained to provide closed-form expressions for the total feedback rate and the feedback rates of each user and its channels. The simulation results demonstrated the advantage of the proposed scheme over those with equally allocated feedback rates among the users and/or channels.

Further work in this area includes extension of the proposed optimization to add target outage probability constraints.

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