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Control of MTDC Transmission Systems under Local Information

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***Abstract*—High-voltage direct current (HVDC) is a commonly used technology for long-distance electric power transmission, mainly due to its low resistive losses. In this paper a distributed controller for multi-terminal high-voltage direct current (MTDC) transmission systems is considered. Sufficient conditions for when the proposed controller renders the closed-loop system asymptotically stable are provided. Provided that the closed loop system is asymptotically stable, it is shown that in steady-state a weighted average of the deviations from the nominal voltages is zero. Furthermore, a quadratic cost of the current injections is minimized asymptotically.**

I. Introduction

Transmitting power over long distances while minimizing losses is one of the greatest challenges in today's power transmission systems. Increased distances between power generation and consumption is a driving factor behind long-distance power transmission. One such example are large-scale off-shore wind farms, which often require power to be transmitted in cables over long distances to the mainland power grid. High-voltage direct current (HVDC) power transmission is a commonly used technology for long-distance power transmission. Its higher investment costs compared to AC transmission lines are compensated by its lower resistive losses for sufficiently long distances. The break-even point, i.e., the point where the total costs of overhead HVDC and AC lines are equal, is typically 500-800 km [8]. However, for cables, the break-even point is typically lower than 100 km [3]. Increased use of HVDC for electrical power transmission suggests that future HVDC transmission systems are likely to consist of multiple terminals connected by several HVDC transmission lines. Such systems are referred to as Multi-terminal HVDC (MTDC) systems in the literature.

Maintaining an adequate DC voltage is the single most important practical control problem for HVDC transmission systems. If the DC voltage deviates too far from a nominal operational voltage, equipment could be damaged, resulting in loss of power transmission capability.

Different voltage control methods for HVDC systems have been proposed in the literature. Among them, the voltage margin method (VMM) and the voltage droop method (VDM) are the most well-known methods [6]. These voltage control methods change the active injected power to maintain active power balance in the DC grid and as a consequence, control the DC voltage. A decreasing DC voltage requires increased injected currents through the converters in order to restore the voltage.

The VDM controller is designed so that several converters participate to control the DC voltage through proportional control [7]. All participating terminals change their injected active power to a level proportional to the deviation from the nominal voltage [6]. These decentralized proportional controllers induce static errors in the voltage, which is the main disadvantage of VDM.

The VMM controller on the other hand, is designed so that one terminal is responsible to control the DC voltage, by e.g., a PI controller. The other terminals keep their injected active power constant. The terminal controlling the DC voltage is called a slack terminal. When the slack terminal is no longer able to supply or extract the power necessary to maintain its DC bus voltage within a certain threshold, a new terminal will operate as the slack terminal [5]. The transition between the slack terminals can cause conflicts between the controllers, and requires one or a few terminals to inject all the current needed to maintain an adequate voltage [5].

Distributed control has been successfully applied to both primary and secondary frequency control of AC transmission systems [1], [9]. Recently, distributed controllers have been applied also to secondary frequency control of asynchronous AC transmission systems connected through an MTDC system [4]. In [2], a distributed controller for voltage control of MTDC systems was proposed. It was shown that the controller can regulate the voltages of the terminals, while the injected power is shared fairly among the converters. However, this controller possesses the disadvantage of requiring a terminal dedicated to measuring and controlling the voltage. In this paper, we propose a fully distributed voltage controller for MTDC transmission systems, which possesses the property of fair power sharing, asymptotically minimizing the cost of the power injections.

The remainder of this paper is organized as follows. In Section II, the mathematical notation is defined. In Section III, the system model and the control objectives are defined. In Section IV, a distributed averaging controller is presented, and its stability and steady-state properties are analyzed.

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In Section V, simulations of the distributed controller on a four-terminal MTDC test system are provided, showing the effectiveness of the proposed controller. The paper ends with a discussion and concluding remarks in Section VI.

II. Notation

Let $\mathcal{G}$ be a graph. Denote by $\mathcal{V} = \{1, \ldots, n\}$ the vertex set of $\mathcal{G}$, and by $\mathcal{E} = \{1, \ldots, m\}$ the edge set of $\mathcal{G}$. Let $\mathcal{N}_i$ be the set of neighboring vertices to $i \in \mathcal{V}$. In this paper we will only consider static, undirected and connected graphs. For the application of control of MTDC power transmission systems, this is a reasonable assumption as long as there are no power line failures. Denote by $\mathcal{B}$ the vertex-edge adjacency matrix of a graph, and let $\mathcal{L}_\mathcal{W} = \mathcal{B}W\mathcal{B}^T$ be its weighted Laplacian matrix, with edge-weights given by the elements of the diagonal matrix W. Let $\mathbb{C}^-$ denote the open left half complex plane, and $\bar{\mathbb{C}}^-$ its closure. We denote by $c_{n\times m}$ a vector or matrix of dimension $n \times m$ whose elements are all equal to c. For a symmetric matrix A, $A > 0$ $(A \geq 0)$ is used to denote that A is positive (semi) definite. I_n denotes the identity matrix of dimension n. For simplicity, we will often drop the notion of time dependence of variables, i.e., $x(t)$ will be denoted x for simplicity.

III. Model and problem setup

Consider a MTDC transmission system consisting of n converters, denoted $1, \ldots, n$, see Figure 1 for an example of an MTDC topology. The converters are assumed to be connected by m HVDC transmission lines. The dynamics of converter i is assumed to be given by

$$\begin{aligned} C_i\dot{V}_i &= -\sum_{j\in\mathcal{N}_i} I_{ij} + I_i^{\text{inj}} + u_i \\ &= -\sum_{j\in\mathcal{N}_i} \frac{1}{R_{ij}}(V_i - V_j) + I_i^{\text{inj}} + u_i, \end{aligned} \tag{1}$$

where V_i is the voltage of converter i, C_i is its capacity, I_i^{inj} is the nominal injected current, which is assumed to be unknown but constant over time, and u_i is the controlled injected current. The constant R_{ij} denotes the resistance of the transmission line connecting the converters i and j. Equation (1) may be written in vector-form as

$$\dot{V} = -C\mathcal{L}_R V + CI^{\text{inj}} + Cu, \tag{2}$$

where $V = [V_1, \ldots, V_n]^T$, $C = \text{diag}([C_1^{-1}, \ldots, C_n^{-1}])$, $I^{\text{inj}} = [I_1^{\text{inj}}, \ldots, I_n^{\text{inj}}]^T$, $u = [u_1, \ldots, u_n]^T$ and $\mathcal{L}_R$ is the weighted Laplacian matrix of the graph representing the transmission lines, whose edge-weights are given by the conductances $\frac{1}{R_{ij}}$. The control objectives considered in this paper are twofold.

Objective 1. *The voltages of the converters, V_i, should converge to a value close to the nominal voltage for converter i (V_i^{nom}), after a disturbance has occurred. More precisely, a weighted average of the steady-state errors should be zero:*

$$\lim_{t\to\infty} \sum_{i=1}^n K_i^V \left(V(t) - V_i^{nom}\right) = 0,$$

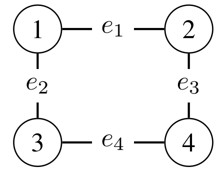

Figure 1. Example of a graph topology of a MTDC system.

for some $K_i^V > 0, i = 1, \ldots, n$.

Remark 1. *It is in general not possible to have $\lim_{t\to\infty} V_i(t) = V_i^{nom}$ for all $i \in \mathcal{V}$, since this would imply that the injected currents are always constant, not allowing for time-varying demand.*

Objective 2. *The cost of the current injections should be minimized asymptotically. More precisely, we require*

$$\lim_{t\to\infty} u(t) = u^*,$$

where u^* is defined by

$$[u^*, V^*] = \underset{[u,V]}{\text{argmin}} \sum_{i\in\mathcal{V}} \frac{1}{2} f_i u_i^2 \quad \textit{s.t.} \quad \mathcal{L}_R V = I^{\textit{inj}} + u, \tag{3}$$

and where $f_i > 0, i = 1, \ldots, n$ are positive constants.

IV. Distributed MTDC controller

It was shown in [2] that a decentralized proportional droop controller cannot satisfy Objective 1 and 2 simultaneously. Furthermore, a proportional controller can only satisfy Objective 1 or 2 if the proportional gains tend to infinity or 0, respectively. A distributed controller was proposed, which was shown to satisfy Objective 1 and 2 simultaneously. However, this controller requires one specific converter to measure and control the voltage. This controller thus has the disadvantage of being sensitive to failure of this specific terminal. In this section we propose a novel, fully distributed controller for MTDC networks which allows for communication between the converters. This controller does not rely on a single leader, but the voltage regulation is distributed among all converters. The proposed controller takes inspiration from the control algorithms given in [1], [2], and is given by

$$\begin{aligned} u_i &= -K_i^P(V_i - \hat{V}_i - \bar{V}_i) \\ \dot{\hat{V}}_i &= -\gamma \sum_{j\in\mathcal{N}_i} c_{ij} \Big((\hat{V}_i + \bar{V}_i - V_i) - (\hat{V}_j + \bar{V}_j - V_j) \Big) \\ \dot{\bar{V}}_i &= -K_i^V(V_i - V_i^{\text{nom}}) - \delta \sum_{j\in\mathcal{N}_i} c_{ij}(\bar{V}_i - \bar{V}_j). \end{aligned} \tag{4}$$

The first line of the controller (4) can be interpreted as a proportional controller, whose reference value is controlled by the remaining two lines. The second line ensures that the current injections converge to the optimal value through a consensus-filter. The third line is a distributed secondary voltage controller, where each terminal measures the voltage and updates the reference value through a consensus-filter. In vector-form, (4) can be written as

$$\begin{aligned} u &= -K^P(V - \hat{V} - \bar{V}) \\ \dot{\hat{V}} &= -\gamma \mathcal{L}_c(\hat{V} + \bar{V} - V) \\ \dot{\bar{V}} &= -K^V(V - V^{\text{nom}}) - \delta \mathcal{L}_c \bar{V}, \end{aligned} \tag{5}$$

where $K^P = \text{diag}([K_1^P, \dots, K_n^P])$, $K^V = \text{diag}([K_1^V, \dots, K_n^V])$, $V^{\text{nom}} = [V_1^{\text{nom}}, \dots, V_n^{\text{nom}}]^T$ and $\mathcal{L}_C$ is the weighted Laplacian matrix of the graph representing the communication topology, denoted $\mathcal{G}_c$, whose edge-weights are given by c_{ij}, and which is assumed to be connected. Substituting the controller (5) in the system dynamics (2), yields

$$\begin{aligned} \begin{bmatrix} \dot{\bar{V}} \\ \dot{\hat{V}} \\ \dot{V} \end{bmatrix} &= \underbrace{\begin{bmatrix} -\delta \mathcal{L}_C & 0_{n\times n} & -K^V \\ -\gamma \mathcal{L}_C & -\gamma \mathcal{L}_C & \gamma \mathcal{L}_C \\ CK^P & CK^P & -C(\mathcal{L}_R + K^P) \end{bmatrix}}_{\triangleq A} \begin{bmatrix} \bar{V} \\ \hat{V} \\ V \end{bmatrix} \\ &+ \underbrace{\begin{bmatrix} K^V V^{\text{nom}} \\ 0_{n\times 1} \\ CI^{\text{inj}} \end{bmatrix}}_{\triangleq b}. \end{aligned} \tag{6}$$

The following theorem characterizes when the controller (4) stabilizes the system (1), and shows that it has some desirable properties.

Theorem 1. *Consider an MTDC network described by* (1)*, where the control input u_i is given by* (4) *and the injected currents I^{inj} are constant. Let $K^P = F^{-1}$, where $F = \text{diag}([f_1, \dots, f_n])$. It is easily shown that A as defined in* (6)*, has one eigenvalue equal to* 0*. If all other eigenvalues lie in the open complex left half plane, then:*

1) $\lim_{t\to\infty} \sum_{i=1}^n K_i^V \left(V(t) - V_i^{nom}\right) = 0$
2) $\lim_{t\to\infty} u(t) = u^*$*, where u^* is defined as in Objective 2.*

The relative voltage differences are also bounded and satisfy $\lim_{t\to\infty} |V_i(t) - V_i(t)| \leq 2I^{max} \sum_{i=2}^n \frac{1}{\lambda_i}$, where $I^{max} = \max_i |I^{tot}|$ and $I^{tot} = \lim_{t\to\infty} I^{inj} + u(t)$, and λ_i denotes the i'th eigenvalue of $\mathcal{L}_R$.

Proof: It is easily verified that the right-eigenvector of A corresponding to the eigenvalue is $v_1 = 1/\sqrt{2n}[1_{1\times n}, -1_{1\times n}, 0_{1\times n}]^T$. Since b as defined in (6), is orthogonal to v_1, $\lim_{t\to\infty}[\bar{V}(t), \hat{V}(t), V(t)]$ exists and is finite, by the assumption that all other eigenvalues lie in the open complex left half plane. Hence, we consider any stationary solution of (6)

$$\begin{aligned} \begin{bmatrix} 0_{n\times 1} \\ 0_{n\times 1} \\ 0_{n\times 1} \end{bmatrix} &= \begin{bmatrix} -\delta \mathcal{L}_C & 0_{n\times n} & -K^V \\ -\gamma \mathcal{L}_C & -\gamma \mathcal{L}_C & \gamma \mathcal{L}_C \\ CK^P & CK^P & -C(\mathcal{L}_R + K^P) \end{bmatrix} \begin{bmatrix} \bar{V} \\ \hat{V} \\ V \end{bmatrix} \\ &+ \begin{bmatrix} K^V V^{\text{nom}} \\ 0_{n\times 1} \\ CI^{\text{inj}} \end{bmatrix}. \end{aligned} \tag{7}$$

Premultiplying (7) with $[1_{1\times n}, 0_{1\times n}, 0_{1\times n}]$ yields

$$-1_{1\times n} K^V(V + V^{\text{nom}}) = -\sum_{i=1}^n K_i^V V(t) + \sum_{i=1}^n K_i^V V_i^{\text{nom}}.$$

The $n+1$:th to $2n$:th lines of (7) imply

$$\begin{aligned} \mathcal{L}_C(\bar{V} + \hat{V} - V) = 0_{n\times 1} &\Rightarrow \\ (\bar{V} + \hat{V} - V) = k_1 1_{n\times 1} &\Rightarrow \\ u = K^P(\bar{V} + \hat{V} - V) &= k_1 K^P 1_{n\times 1} \end{aligned}$$

Now finally, premultiplying (7) with $[0_{1\times n}, 0_{1\times n}, 1_{1\times n}C^{-1}]$ yields

$$\begin{aligned} &1_{1\times n}\left(K^P(\bar{V} + \hat{V} - V) + I^{\text{inj}}\right) = 1_{1\times n}\left(k_1 K^P 1_{n\times 1} + I^{\text{inj}}\right) \\ &= k_1 \sum_{i=1}^n K_i^P + \sum_{i=1}^n I_i^{\text{inj}}, \end{aligned}$$

which implies $k_1 = -\left(\sum_{i=1}^n I_i^{\text{inj}}\right) / \left(\sum_{i=1}^n K_i^P\right)$. The bound on $\lim_{t\to\infty} |V_i(t) - V_i(t)|$ follows from the proof of Theorem 3 in [2]. Since $K^P = F^{-1}$, any stationary solution of (6) satisfies $u = k_1 F^{-1} 1_{n\times 1}$. On the other hand, the KKT condition for the optimization problem (3) is $Fu = \lambda 1_{n\times 1}$. Since (3) is convex, the KKT condition is necessary and sufficient. This implies that any stationary solution of (6) solves (3). ■

While Theorem 1 establishes an exact condition when the distributed controller (4) stabilizes the MTDC system (1), it does not give any insight in how to choose the controller parameters to achieve a stable closed loop system. The following theorem gives a sufficient stability condition for a special case.

Theorem 2. *The matrix A as defined in* (6)*, always has one eigenvalue equal to* 0*. Assume that $\mathcal{L}_C = \mathcal{L}_R$, i.e. that the topology of the communication network is identical to the topology of the MTDC system. Assume furthermore that $K^P = k^P I_n$, i.e. the controller gains are equal. Then the remaining eigenvalues lie in the open complex left half plane if*

$$\frac{\gamma + \delta}{2k^P} \lambda_{\min}\left(\mathcal{L}_R C^{-1} + C^{-1}\mathcal{L}_R\right) + 1 > 0 \tag{8}$$

$$\frac{\gamma\delta}{2k^P} \lambda_{\min}\left(\mathcal{L}_R^2 C^{-1} + C^{-1}\mathcal{L}_R^2\right) + \min_i K_i^V > 0 \tag{9}$$

$$\begin{aligned} &\lambda_{\max}\left(\mathcal{L}_R^3\right) \frac{\gamma\delta}{k^{P^2}} \\ &\leq \left(\frac{\gamma + \delta}{2k^P} \lambda_{\min}\left(\mathcal{L}_R C^{-1} + C^{-1}\mathcal{L}_R\right) + 1\right) \\ &\left(\frac{\gamma\delta}{2k^P} \lambda_{\min}\left(\mathcal{L}_R^2 C^{-1} + C^{-1}\mathcal{L}_R^2\right) + \min_i K_i^V\right) \end{aligned} \tag{10}$$

Remark 2. *By choosing γ and δ sufficiently small, and choosing k^P and $\min_i K_i^V$ sufficiently large, the inequalities* (8)–(10) *can always be satisfied. Intuitively, this implies that the consensus dynamics in the network should be sufficiently slow compared to the voltage dynamics.*

Proof of Theorem 2: The characteristic equation of A is given by equation (11). Clearly, this equation has a solution

$$
\begin{aligned}
0 &= \det(sI_{3n}-A) = \begin{vmatrix} sI_n+\delta\mathcal{L}_C & 0_{n\times n} & K^V \\ \gamma\mathcal{L}_C & sI_n+\gamma\mathcal{L}_C & -\gamma\mathcal{L}_C \\ -CK^P & -CK^P & SI_n+C(\mathcal{L}_R+K^P) \end{vmatrix} = \begin{vmatrix} sI_n+\delta\mathcal{L}_C & 0_{n\times n} & K^V \\ -sI_n & sI_n+\gamma\mathcal{L}_C & -\gamma\mathcal{L}_C \\ 0_{n\times n} & -CK^P & SI_n+C(\mathcal{L}_R+K^P) \end{vmatrix} \\
&= s^n \begin{vmatrix} sI_n+\delta\mathcal{L}_C & 0_{n\times n} & K^V \\ -I_n & I_n+\frac{\gamma}{s}\mathcal{L}_C & -\frac{\gamma}{s}\mathcal{L}_C \\ 0_{n\times n} & -CK^P & SI_n+C(\mathcal{L}_R+K^P) \end{vmatrix} \\
&= s^n \left|sI+\delta\mathcal{L}_C\right|^{-1} \begin{vmatrix} sI_n+\delta\mathcal{L}_C & 0_{n\times n} & K^V \\ -sI-\delta\mathcal{L}_C & sI_n+\gamma\mathcal{L}_C+\delta\mathcal{L}_C+\frac{\gamma\delta}{s}\mathcal{L}_C^2 & -\gamma\mathcal{L}_C+\frac{\gamma\delta}{s}\mathcal{L}_C^2 \\ 0_{n\times n} & -CK^P & SI_n+C(\mathcal{L}_R+K^P) \end{vmatrix} \\
&= s^n \left|sI+\delta\mathcal{L}_C\right|^{-1} \begin{vmatrix} sI_n+\delta\mathcal{L}_C & 0_{n\times n} & K^V \\ 0_{n\times n} & sI_n+\gamma\mathcal{L}_C+\delta\mathcal{L}_C+\frac{\gamma\delta}{s}\mathcal{L}_C^2 & -\gamma\mathcal{L}_C-\frac{\gamma\delta}{s}\mathcal{L}_C^2+K^V \\ 0_{n\times n} & -CK^P & SI_n+C(\mathcal{L}_R+K^P) \end{vmatrix} \\
&= s^n \begin{vmatrix} sI_n+\gamma\mathcal{L}_C+\delta\mathcal{L}_C+\frac{\gamma\delta}{s}\mathcal{L}_C^2 & -\gamma\mathcal{L}_C-\frac{\gamma\delta}{s}\mathcal{L}_C^2+K^V \\ -CK^P & SI_n+C(\mathcal{L}_R+K^P) \end{vmatrix} \\
&= s^n \left|SI_n+C(\mathcal{L}_R+K^P\right| \left|CK^P\right| \begin{vmatrix} \left(sI_n+\gamma\mathcal{L}_C+\delta\mathcal{L}_C+\frac{\gamma\delta}{s}\mathcal{L}_C^2\right)\cdot & \\ K^{P^{-1}}C^{-1}\left(sI_n+C(\mathcal{L}_R+K^P)\right) & -\gamma\mathcal{L}_C-\frac{\gamma\delta}{s}\mathcal{L}_C^2+K^V \\ -SI_n-C(\mathcal{L}_R+K^P) & SI_n+C(\mathcal{L}_R+K^P) \end{vmatrix} \\
&= \left|CK^P\right| \Big| \left[\gamma\delta\mathcal{L}_C^2K^{P^{-1}}\mathcal{L}_R\right] + s\left[(\delta+\gamma)\mathcal{L}_CK^{P^{-1}}\mathcal{L}_R+\delta\mathcal{L}_C+\gamma\delta\mathcal{L}_C^2K^{P^{-1}}C^{-1}+K^V\right] \\
&\quad +s^2\left[K^{P^{-1}}\mathcal{L}_R+I_n+(\gamma+\delta)\mathcal{L}_CK^{P^{-1}}C^{-1}\right]+s^3\left[K^{P^{-1}}C^{-1}\right]\Big| \triangleq \left|CK^P\right| \det\left(Q(s)\right)
\end{aligned} \tag{11}
$$

only if $x^TQ(s)x = 0$ has a solution for some $x : \|x\| = 1$. Substituting $K^P = k^PI_n$ and $\mathcal{L}_C = \mathcal{L}_R$, this equation becomes

$$
\begin{aligned}
0 = x^TQ(s)x &= \underbrace{\frac{\gamma\delta}{k^P}x^T\mathcal{L}_R^3x}_{a_0} \\
&+ s\,\underbrace{x^T\left[\frac{\delta+\gamma}{k^P}\mathcal{L}_R^2+\delta\mathcal{L}_R+\frac{\gamma\delta}{k^P}\mathcal{L}_R^2C^{-1}+K^V\right]x}_{a_1} \\
&+ s^2\,\underbrace{x^T\left[\frac{1}{k^P}\mathcal{L}_R+I_n+\frac{\gamma+\delta}{k^P}\mathcal{L}_RC^{-1}\right]x}_{a_2} \\
&+ s^3\,\underbrace{\frac{1}{k^P}x^TC^{-1}x}_{a_3}.
\end{aligned} \tag{12}
$$

Clearly (12) has one solution $s = 0$ for $x = \frac{a}{\sqrt{n}}[1,\dots,1]^T$, since this implies that $a_0 = 0$. The remaining solutions are stable if and only if the polynomial $a_1 + sa_2 + s^2a_3 = 0$ is Hurwitz, which is equivalent to $a_i > 0$ for $i = 1,2,3$ by the Routh-Hurwitz stability criterion. For $x \neq \frac{a}{\sqrt{n}}[1,\dots,1]^T$, we have that $a_0 > 0$, and thus $s = 0$ cannot be a solution of (12). By the Routh-Hurwitz stability criterion, (12) has only stable solutions if and only if $a_i > 0$ for $i = 0,1,2,3$ and $a_0a_3 < a_1a_2$. Since this condition implies that $a_i > 0$ for $i = 1,2,3$, there is no need to check this second condition explicitly. Clearly $a_3 > 0$ since $K^{P^{-1}}$ and C^{-1} are diagonal with positive elements. It is easily verified that $a_2 >$ if (8) holds, since $\mathcal{L}_R \geq 0$. Similarly, $a_1 > 0$ if (9) holds, since also $\mathcal{L}_R^2 \geq 0$ and $x^TK^Vx \geq \min_i K_i^V$. In order to assure that $a_0a_3 < a_1a_2$, we need furthermore to upper bound a_0a_3. The following bound is easily verified

$$
a_0a_3 < \lambda_{\max}\left(\mathcal{L}_R^3\right)\frac{\gamma\delta}{k^{P^2}}\max_i C_i.
$$

Using this, together with the obtained lower bounds on a_1 and a_2, we obtain that (10) is a sufficient condition for $a_0a_3 < a_1a_2$. ■

V. Simulations

Simulations of an MTDC system were conducted using MATLAB. The MTDC was modelled by (1), with u_i given by the distributed controller (4). The topology of the MTDC system is given by Figure 1. The capacities are assumed to be $C_i = 123.79$ µF for $i = 1,2,3,4$, while the resistances are assumed to be $R_{12} = \Omega$, $R_{13} = \Omega$, $R_{24}\Omega$, $R_{34}0.0065$ Ω. The controller parameters were set to $K_i^P = 1\ \Omega^{-1}$ for $i = 1,2,3,4$, $\gamma = 0.005$ and $c_{ij} = R_{ij}^{-1}\ \Omega^{-1}$ for all $(i,j) \in \mathcal{E}$. Due to the long geographical distances between the DC converters, communication between neighboring nodes is assumed to be delayed with delay τ. While the nominal system without time-delays is verified to be stable according to Theorem 1, time-delays might destabilize the system. It is thus of importance to study the effects of time-delays further. The dynamics of the controller (4) with time delays thus become

$$\begin{aligned} u_i &= K^P(\hat{V}_i(t) - V_i(t)) \\ \dot{\hat{V}}_i &= K_i^V(V^{\text{nom}} - V_i(t)) \\ &-\gamma \sum_{j\in\mathcal{N}_i} c_{ij} \left((\hat{V}_i(t') - V_i(t')) - (\hat{V}_j(t') - V_j(t')) \right), \end{aligned} \tag{13}$$

where $t' = t - \tau$. The injected currents are assumed to be initially given by $I^{\text{inj}} = [300, 200, -100, -400]^T$ A, and the system is allowed to converge to the stationary solution. Since the injected currents satisfy $I_i^{\text{inj}} = 0$, $u_i = 0$ for $i = 1, 2, 3, 4$ by Theorem 1. Then, at time $t = 0$, the injected currents are changed due to changed power loads. The new injected currents are given by $I^{\text{inj}} = [300, 200, -300, -400]^T$ A. The step response of the voltages V_i and the controlled injected currents u_i are shown in Figure 2. For the delay-free case, i.e., $\tau = 0$ s, the voltages V_i are restored to their new stationary values within 2 seconds. The controlled injected currents u_i converge to their stationary values within 8 seconds. The simulation with time delays $\tau = 0.4$ s, show that the controller is robust to moderate time-delays. For a time delay of $\tau = 0.5$ s, the system becomes unstable.

VI. Discussion and Conclusions

In this paper we have proposed a fully distributed controller for voltage and current control in MTDC networks. We show that under certain conditions, there exist controller parameters such that the closed-loop system is stabilized. We have shown that the proposed controller is able to maintain the voltage levels of the converters close to the nominal voltages, while at the same time, the injected current is shared amongst fairly the converters. This control procedure asymptotically minimizes the cost of injecting power.

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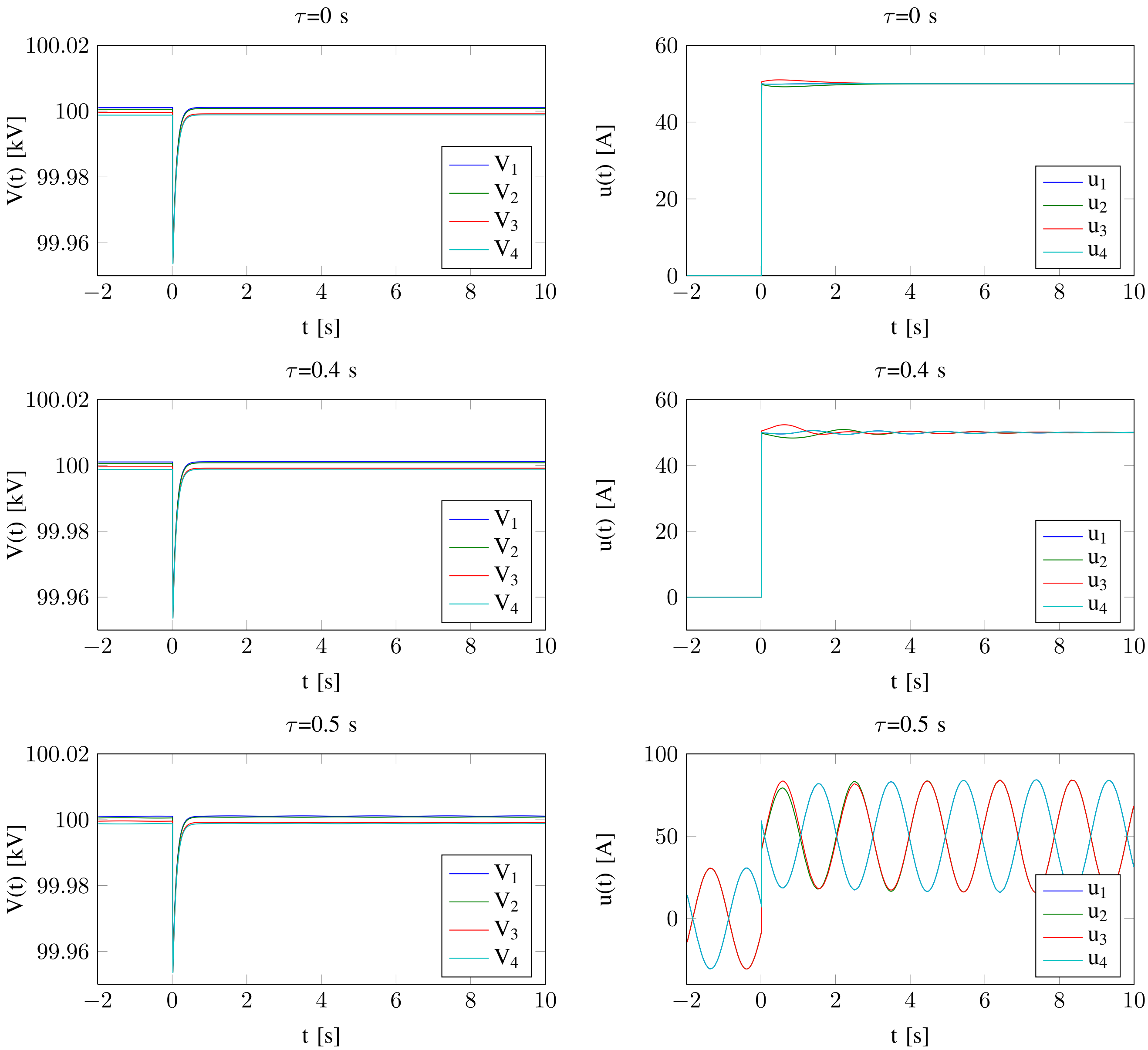

Figure 2. The figure shows the voltages V_i and the controlled injected currents u_i of the converters for different time-delays τ on the communication links. The system model is given by (1), and u_i is given by the distributed controller (13).