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CONFORMALLY EXACT RESULTS FOR $SL(2,\mathbb{R}) \otimes SO(1,1)^{d-2}/SO(1,1)$ COSET MODELS * †

KONSTADINOS SFETSOS

Physics Department

University of Southern California

Los Angeles, CA 90089-0484, USA

ABSTRACT

Using the conformal invariance of the $SL(2,\mathbb{R}) \otimes SO(1,1)^{d-2}/SO(1,1)$ coset models we calculate the conformally exact metric and dilaton, to all orders in the $1/k$ expansion. We consider both vector and axial gauging. We find that these cosets represent two different space–time geometries: ($2d$ black hole)$\otimes \mathbb{R}^{d-2}$ for the vector gauging and ($3d$ black string)$\otimes \mathbb{R}^{d-3}$ for the axial one. In particular for $d = 3$ and for the axial gauging one obtains the exact metric and dilaton of the charged black string model introduced by Horne and Horowitz. If the value of k is finite we find two curvature singularities which degenerate to one in the semi–classical $k \to \infty$ limit. We also calculate the reflection and transmission coefficients for the scattering of a tachyon wave and using the Bogoliubov transformation we find the Hawking temperature.

* e-mail: sfetsos@physics.usc.edu

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1. Introduction

Models of strings propagating in curved backgrounds has been studied extensively by means of Conformal Field Theories (CFT), but most of the effort has been directed to the case of string compactification where the non–compact part of the space–time which contains the time coordinate is flat, i.e described by a trivial CFT, and only the internal part requires a non–trivial CFT. The particular CFT used corresponds to a different classical vacuum of the string theory.

In an attempt to formulate solvable models with a single time coordinate Anti-De-Sitter (ADS) Coset models $G/H = SO(d-1,2)_{-k}/SO(d-1,1)_{-k}$ and their $N=1$ superconformal generalizations were introduced as exact string theories [1]. The important difference with previous treatments is that the time direction can be curved as well, i.e the non–compact part requires a non–trivial CFT. All single time coordinate models based on simple non–compact groups are characterized by G/H cosets and the complete list can be found in [2]. Naturally, by taking non–simple (direct product) groups, one can construct extensions of these models (for a classification see [3]). The spectrum of such theories can in principle be found by using non–compact current algebra techniques [4]. For all of these cosets the action is written in the form of a gauged WZW model [5]. The semi–classical analysis [6] for $k \to \infty$ showed that these are useful models for learning more about string and particle theories in gravitationally singular spaces. Higher dimensional models have been subjected since, to the same semi–classical analysis and various interesting singularities were found [7] [8] [9] [10] [11] [12] [13] [14][3]. Cosmological aspects of coset model are discussed in [15] [16] [17]. Heterotic and type–II superstring actions can be constructed [10] in exactly four space–time dimensions providing useful theories for investigating the physics of the early Universe in the context of string theory. It has been shown that the duality properties of the compactified boson on a circle have their correspondence in this context of string theory [18] [19] [20] [21]. The existence of a *discrete* generalized duality is given in [9] while further dualities based on Killing vectors can be found in [22] [23].

The principal method of investigation of the semi–classical geometries followed ref.[6] that used a Lagrangian method starting from the gauged WZW action. In practice, one can use this method to calculate the lowest fields of the string theory, namely, the metric $G_{\mu\nu}$, the antisymmetric tensor $B_{\mu\nu}$ and, the dilaton field Φ, to lowest order in the $1/k$ expansion. The above fields satisfy [6][7][9][10][11] the perturbative equations for conformal invariance [24]. Another drawback of the gauged WZW method is that one obtains the various fields in only one patch of the group manifold because of the gauge fixing procedure [9][10][11]. A different gauge choice leads to a metric in a different coordinate patch which may bear no resemblance to the previous one (e.g compare [7] to [9] or [11]). In [16] a group theoretical method for the global analysis of any semi–classical geometry, including an explicit solution for the particle geodesics, was formulated and applied explicitly to some cases.

It is well known that a necessary condition for a critical string theory requires that the central charge of the matter part exactly compensates the central charge from the Faddeev–Popov ghosts ($c = -26$ or $c = -15$ when supersymmetric) so that the trace anomaly vanishes. Most of the CFT based on coset models require a value for k which is far from being large. Thus one needs to go beyond the large k limit. Following a Hamiltonian approach to gauged WZW models the authors of [25] formulated a general method for computing the conformally exact metric and dilaton, to all orders in the $1/k$ expansion, for any bosonic, heterotic, or type–II superstring based on a coset G/H and gave explicit results for the $d = 2, 3, 4$ ADS models. In the $k \to \infty$ limit these results tend to those one obtains in the semi–classical approach, to leading order in perturbation theory. In the special case $d = 2$ they were also in agreement with the exact metric and dilaton obtained in a previous computation [21].

In this paper we use the above method to obtain the conformally exact metric and dilaton for a simple class of models involving several abelian factors, i.e $SL(2,\mathbb{R}) \otimes SO(1,1)^{d-2}/SO(1,1)$, which is a d–dimensional model. For $d = 3$ the semi–classical aspects of the model were worked out in [8], for $d = 4$ in [13] and for general d in [3]. The paper is organized as follows. In section 2 we review the general method for computing the conformally exact metric and dilaton with particular emphasis on the $SL(2,\mathbb{R}) \otimes SO(1,1)^{d-2}/SO(1,1)$ coset models. In section 3 we are dealing with the vector gauge and in section 4 with the axial one. In section 5 we consider the scattering of a tachyon wave in the geometry of the coset manifold and we compute the Hawking temperature using the Bogoliubov transformation. Finally we end the paper in section 6 with concluding remarks and discussion.

2. The general method and the $SL(2,\mathbb{R}) \otimes SO(1,1)^{d-2}/SO(1,1)$ models

In this section we will briefly review the general method for computing the conformally exact metric and dilaton fields for any bosonic σ–model based on a coset G/H as it was developed in [25]. Generalizations of the method for the cases where there is superconformal symmetry can be found in [25]. Let us consider a bosonic string theory for closed strings in d curved space–time dimensions, based on a σ–model CFT with string coordinates X^μ, $\mu = 0, 1 \cdots d-1$. We denote the space–time metric and dilaton fields by $G_{\mu\nu}(X)$ and $\Phi(X)$ respectively. We begin with the most general form of the effective action for the tachyon T in d space–time dimensions

$$\begin{aligned} S[T] &= \int d^d X \sqrt{-G} e^{\Phi} \left(G^{\mu\nu} \partial_\mu T \partial_\nu T - V(T) \right) \\ V(T) &= 2T^2 + \mathcal{O}(T^3) \ , \end{aligned} \tag{2.1}$$

where $V(T)$ is the tachyon potential whose precise form is not necessary for the analysis that follows. From the point of view of the CFT the tachyon is completely defined through the action of the zero modes, L_0 and $\bar{L}_0$, of the stress tensors for the right and left movers. Therefore (2.1) must be equivalent to the following action

$$S_c[T] = \int d^d X \sqrt{-G} e^{\Phi} \left(T(L_0 + \bar{L}_0) T - V(T) \right) \ . \tag{2.2}$$

Comparison of (2.1) with (2.2) determines the form of $L_0 + \bar{L}_0$ as a differential operator in configuration space

$$(L_0 + \bar{L}_0)\ T = - \frac{1}{e^{\Phi} \sqrt{-G}} \partial_\mu G^{\mu\nu} e^{\Phi} \sqrt{-G} \partial_\nu T \ . \tag{2.3}$$

Using the equivalence between gauged WZW models and current algebra coset models G_{-k}/H_{-k} we can write L_0 in terms of the quadratic Casimir operators Δ_G and Δ_H for the group and the subgroup, as follows

$$\begin{aligned} L_0 T &= \left(\frac{\Delta_G}{k-g} - \frac{\Delta_H}{k-h} \right) T \\ \Delta_G &\equiv Tr(J_G)^2, \qquad \Delta_H \equiv Tr(J_H)^2 \ , \end{aligned} \tag{2.4}$$

where J_G, J_H are the group and subgroup operators obeying the appropriate Lie algebras, and g, h are the Coxeter numbers for the group and subgroup respectively. An expression similar to (2.4) can also be written for $\bar{L}_0$. The currents J_G, J_H, $\bar{J}_G$, $\bar{J}_H$ act as first order differential operators on the group parameter space. Consequently the Casimir operators Δ_G, Δ_H, $\bar{\Delta}_G$, $\bar{\Delta}_H$ contain single and double derivatives with respect to all $dimG$ parameters in G. At the tachyon level we require states which are singlets under the gauge group H (acting simultaneously on left and right movers). Thus we can impose the following conditions on the tachyon T

$$\begin{aligned} (J_H + \bar{J}_H)\, T = 0 \ , \qquad & \text{Vector gauging} \\ (J_H - \bar{J}_H)\, T = 0 \ , \qquad & \text{Axial gauging} \ . \end{aligned} \tag{2.5}$$

The second of the above conditions is appropriate only for the currents associated with the abelian part of the subgroup. The number of conditions is $dimH$ and therefore T can only depend on $d = dim(G/H)$ parameters, X^μ (string coordinates), which are H–invariants. Consequently, using the chain rule, we reduce the derivatives in (2.4) to only derivatives with respect to the d string coordinates X^μ. The gauge invariance condition (2.5) implies that $(\Delta_H - \bar{\Delta}_H)\, T = 0$. Using this and the fact that $\Delta_G = \bar{\Delta}_G$ for any group [25], we ensure the physical condition for closed bosonic strings $(L_0 - \bar{L}_0)\, T = 0$. Then using (2.3) and (2.4) one can deduce uniquely the expression for the inverse metric $G^{\mu\nu}$ by comparing the coefficients of the double derivatives $\partial_\mu \partial_\nu T$. Comparison of the single derivative terms $\partial_\mu T$ will give a system of d first order partial differential equations, whose solution determines the dilaton field Φ.

Let us specialize to the $SL(2,\mathbb{R}) \otimes SO(1,1)^{d-2}/SO(1,1)$ coset models. It is convenient to parametrize the group element of $G = SL(2,\mathbb{R}) \otimes SO(1,1)^{d-2}$ as follows [1]

$$g = \begin{pmatrix} g_0 & 0 & \cdots & 0 \\ 0 & g_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_{d-2} \end{pmatrix} , \tag{2.6}$$

where

$$g_0 = \begin{pmatrix} a & u \\ -v & b \end{pmatrix} , \qquad ab + uv = 1 \tag{2.7}$$

and

$$g_i = \begin{pmatrix} \cosh 2r_i & \sinh 2r_i \\ \sinh 2r_i & \cosh 2r_i \end{pmatrix} , \qquad i = 1, 2, \cdots, d-2 \ . \tag{2.8}$$

The infinitesimal generators for $SL(2,\mathbb{R})$ are

$$j_0 = \frac{q_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \quad j_+ = \frac{q_0}{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} , \quad j_- = \frac{q_0}{2} \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} \tag{2.9}$$

[1] We follow closely the notation of [3].

and those for the $SO(1,1)$'s

$$j_i = \frac{q_i}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad i = 1, 2, \cdots, d-2 \ . \tag{2.10}$$

The coefficients q_i parametrize the embedding of $H = SO(1,1)$ into the factored $SO(1,1)$'s in G and are normalized to $\sum_{i=0}^{d-2} q_i^2 = 1$. With this normalization the level of $SL(2,\mathbb{R})$ is $q_0^2 k$ and that of the $SO(1,1)$'s in G is $q_i^2 k_i$. Therefore the level of $H = SO(1,1)$ is $q_0^2 k + \sum_{i=1}^{d-2} q_i^2 k_i$. If we consider the infinitesimal transformations $\delta g = g j_a$ (right) and $\delta g = j_a g$ (left), where $a = 0, \pm$ we find the following expressions for the infinitesimal group generators

$$\begin{aligned} J_0 &= \frac{1}{2}(a\partial_a - b\partial_b - u\partial_u) \ , & \bar{J}_0 &= \frac{1}{2}(b\partial_b - a\partial_a - u\partial_u) \\ J_+ &= a\partial_u - v\partial_b \ , & \bar{J}_+ &= b\partial_u - v\partial_a \\ J_- &= -u\partial_a \ , & \bar{J}_- &= -u\partial_b \ . \end{aligned} \tag{2.11}$$

In the previous expressions for the generators, a, b, u were taken as the independent group parameters, while $v = (1-ab)/u$. As we shall see they are more convenient to use in the vector gauge. For the axial gauge a, u, v will be used as independent parameters. In the latter case the generators have the form

$$\begin{aligned} J_0 &= \frac{1}{2}(a\partial_a - u\partial_u + v\partial_v) \ , & \bar{J}_0 &= \frac{1}{2}(v\partial_v - a\partial_a - u\partial_u) \\ J_+ &= a\partial_u \ , & \bar{J}_+ &= b\partial_u - v\partial_a \\ J_- &= b\partial_v - u\partial_a \ , & \bar{J}_- &= a\partial_v \ . \end{aligned} \tag{2.12}$$

It can easily be shown that the $SL(2,\mathbb{R})$ Lie algebra is indeed obeyed for both the left and the right generators separately and that any left commutes with any right generator. For the $SO(1,1)$'s the generators are

$$\begin{aligned} J_i &= \frac{1}{2} q_i \partial_i \ , & \bar{J}_i &= -\frac{1}{2} q_i \partial_i \\ J_H &= q_0 J_0 + \sum_{i=1}^{d-2} J_i \ , & \bar{J}_H &= q_0 \bar{J}_0 + \sum_{i=1}^{d-2} \bar{J}_i \ , \end{aligned} \tag{2.13}$$

where $\partial_i \equiv \frac{\partial}{\partial r_i}$. The central charge for both the right and the left movers is

$$c = \frac{3k}{k-2} + (d-2) - 1 \ . \tag{2.14}$$

Conformal invariance requires that $c = 26$. In what follows we assume that $k > 2$ and therefore $d \leq 26$. If we analytically continue the expressions for the various metrics below, to the range of parameters $k < 2$, $d > 26$ we get unphysical metrics.

3. The vector gauging

In this case using (2.11) and (2.13) the first condition in (2.5) takes the following simple form

$$\partial_u T = 0 \quad \Rightarrow \quad T = T(a, b, r_i) \ . \tag{3.1}$$

Then by using (2.3) and (2.4) we determine the inverse metric [2] $G^{\mu\nu}$, $\mu, \nu = a, b, 1, 2, \cdots, d-2$ (by comparing the coefficients of the double derivatives $\partial_\mu \partial_\nu T$)

$$G^{\mu\nu} = \begin{pmatrix} \sigma^2 a^2 & 2(ab-1) - \sigma^2 ab & -(1-2/k)\frac{\eta_j}{1+\rho^2}\ a \\ 2(ab-1) - \sigma^2 ab & \sigma^2 b^2 & (1-2/k)\frac{\eta_j}{1+\rho^2}\ b \\ -(1-2/k)\frac{\eta_i}{1+\rho^2}\ a & (1-2/k)\frac{\eta_i}{1+\rho^2}\ b & (1-2/k)\big(\frac{\delta_{ij}}{\kappa_i} - \frac{\eta_i \eta_j}{1+\rho^2}\big) \end{pmatrix} \ , \tag{3.2}$$

where $\eta_i \equiv q_i/q_0$, $\kappa_i \equiv k_i/k$, $\rho^2 \equiv \sum_{i=1}^{d-2} \eta_i^2 \kappa_i$, $\sigma^2 \equiv \frac{\rho^2 + 2/k}{1+\rho^2}$, and we obtain a system of two first order partial differential equations (by comparing the single derivative terms) which will determine the dilaton Φ

$$\begin{aligned} \partial_a\big(e^\Phi \sqrt{-G} G^{ab}\big) + \partial_b\big(e^\Phi \sqrt{-G} G^{bb}\big) &= 2 e^\Phi \sqrt{-G}(1 + \frac{1}{2}\sigma^2) b \\ \partial_b\big(e^\Phi \sqrt{-G} G^{ab}\big) + \partial_a\big(e^\Phi \sqrt{-G} G^{aa}\big) &= 2 e^\Phi \sqrt{-G}(1 + \frac{1}{2}\sigma^2) a \ . \end{aligned} \tag{3.3}$$

If we invert the inverse metric we get the following expression for the line element

$$\begin{aligned} ds^2 = & \frac{1/2}{1-(1-2/k)ab}\Big[\frac{1}{k}\frac{1}{ab-1}(bda + adb)^2 - 2\ dadb\Big] \\ & + \frac{1}{1-(1-2/k)ab}\sum_{i=1}^{d-2}(bda - adb)\eta_i \kappa_i\ dr_i \\ & + \frac{1}{1-2/k}\sum_{i,j=1}^{d-2} \kappa_i \big(\delta_{ij} + \frac{\eta_i \eta_j \kappa_j}{1-(1-2/k)ab}\big)\ dr_i dr_j \ . \end{aligned} \tag{3.4}$$

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[2] In what follows we disregard a factor of $\frac{1}{2(k-2)}$ in $G^{\mu\nu}$. We do the same for the axial gauging as well.

The solution to the system of differential equations gives the following result for the dilaton

$$C\ e^{\Phi} = (1-ab)\sqrt{1+\frac{2}{k}\frac{ab}{1-ab}}\ , \tag{3.5}$$

where C is the constant of integration. We have thus found the exact metric and dilaton for the vector gauging of the $SL(2,\mathbb{R}) \otimes SO(1,1)^{d-2}/SO(1,1)$ model. [3] In the $k \to \infty$ limit eq.(3.4) agrees with the semi–classical expression found in [3]. The dilaton in (3.5) is independent of the r_i coordinates and it is the same as the exact dilaton found in [21] for the $2d$ black hole. This fact gives the hint that the two models are very closely related. Indeed as in [3] we can show that there is a coordinate transformation which diagonalizes the metric. In the region where $ab > 1$ we can make the following transformations

$$\begin{aligned} &a = \cosh R\ e^{X_0 + mX_{d-2}}, \qquad b = \cosh R\ e^{-(X_0+mX_{d-2})} \\ &r_i = \sqrt{1-2/k}\ N_{ij}X_j\ , \end{aligned} \tag{3.6}$$

with

$$N_{ij} = \begin{cases} -\dfrac{\rho_j}{\rho_i\sqrt{\kappa_i}} & i = j+1 \\ \dfrac{\sqrt{\kappa_{j+1}}\,\eta_i\,\eta_{j+1}}{\rho_{j+1}\rho_j} & i \le j \ne d-2 \\ \dfrac{\eta_i}{\rho_j(\rho_j^2+1)^{1/2}} & i \le j = d-2 \\ 0 & \text{otherwise} \end{cases}\ , \qquad m = -\sqrt{1-2/k}\ \frac{\rho}{(1+\rho^2)^{\frac{1}{2}}}\ , \tag{3.7}$$

where

$$\rho_i^2 = \sum_{j=1}^{i} \kappa_j \eta_j^2\ , \qquad \text{and} \qquad \rho_{d-2} \equiv \rho\ . \tag{3.8}$$

The matrix elements N_{ij} satisfy the relations

[3] The combination $e^{\Phi}\sqrt{-G}$ is k–independent, as it was conjectured in previous work [9][10][25]. The same is true for the case of the Axial gauging.

$$
\begin{aligned}
&\sum_{l=1}^{d-2} \kappa_l \ N_{li} \ N_{lj} = \delta_{ij} \qquad i, j \neq d-2 \\
&\sum_{l=1}^{d-2} \kappa_l \ N^2_{l,d-2} = \frac{1}{1+\rho^2} \\
&\sum_{l=1}^{d-2} \kappa_l \ \eta_l \ N_{li} = 0 \qquad i \neq d-2 \\
&\sum_{l=1}^{d-2} \kappa_l \ \eta_l \ N_{l,d-2} = \frac{\rho}{(1+\rho^2)^{\frac{1}{2}}} \ .
\end{aligned}
\tag{3.9}
$$

In these new coordinates the metric takes the form

$$
ds^2 = dR^2 - \frac{1}{\tanh^2 R - 2/k} \ dX_0^2 + \sum_{i=1}^{d-2} dX_i^2 \ . \tag{3.10}
$$

The first two terms in (3.10) are the exact metric found in [21] for the $SL(2,\mathbb{R})/SO(1,1)$ $2d$ black hole. Although the embedding of $H = SO(1,1)$ in G was general the resulting geometry coincides with the case $\eta_i = 0$, i.e $H = SO(1,1)$ embedded only in $SL(2,\mathbb{R})$. This is as expected because for the vector gauging $\delta r_i = 0$. Therefore we have proved that the $SL(2,\mathbb{R}) \otimes SO(1,1)^{d-2}/SO(1,1)$ model for the vector gauging, is equivalent to the ($2d$ black hole) $\otimes \mathbb{R}^{d-2}$ model for any k. In the semi–classical limit $k \to \infty$ this was proved in [3]. For the regions $ab < 0$ and $0 < ab < 1$ we can find the conformally exact metric by analytically continue $R \to R + i\pi/2$ and $R \to it$ respectively.

4. The axial gauging

The most interesting case is that of the axial gauging. Then using (2.12) and (2.13) the second condition in (2.5) becomes

$$
(a\partial_a + \sum_{i=1}^{d-2} \eta_i \partial_i) \ T = 0 \quad \Rightarrow \quad T = T(u, v, x_i = r_i - \eta_i \ \ln a) \ . \tag{4.1}
$$

Proceeding as in the previous section for the vector gauging we find for the inverse metric the following expression [4]

[4] See remarks in footnote 2.

$$G^{\mu\nu} = \begin{pmatrix} \sigma^2 u^2 & 2(uv-1)-\sigma^2 uv & -u\eta_j \\ 2(uv-1)-\sigma^2 uv & \sigma^2 v^2 & -v\eta_j \\ -u\eta_i & -v\eta_i & (1-2/k)\frac{\delta_{ij}}{\kappa_i}+\eta_i\eta_j \end{pmatrix} . \tag{4.2}$$

The differential equations which determine the dilaton are similar to (3.3) above with $(a,b)\to(u,v)$. If we invert the inverse metric and solve the system of differential equations we find for the line element

$$\begin{aligned} ds^2 =& \frac{1}{[(1-2/k)(uv-1)-\rho^2-2/k][(1-2/k)(uv-1)-\rho^2]}\Big(-\frac{1-2/k}{2k}(v^2du^2+u^2dv^2) \\ &+\big[(1-2/k)\big((1-1/k+\rho^2)(uv-1)-1/k\big)-\rho^2(1+\rho^2)\big]\ dudv\Big) \\ &+\frac{1}{(1-2/k)(uv-1)-\rho^2}\sum_{i=1}^{d-2}(vdu+udv)\eta_i\kappa_i\ dx_i \\ &+\frac{1}{1-2/k}\sum_{i,j=1}^{d-2}\kappa_i(\delta_{ij}+\frac{\eta_i\eta_j\kappa_j}{(1-2/k)(uv-1)-\rho^2})\ dx_i dx_j \end{aligned} \tag{4.3}$$

and for the dilaton

$$C'e^{\Phi} = (1-uv)\sqrt{[1+\rho^2+(\rho^2+2/k)\frac{uv}{1-uv}][1+\rho^2-2/k+\rho^2\frac{uv}{1-uv}]}\ , \tag{4.4}$$

where C' is the constant of integration. Thus, we have obtained the exact expressions for the metric and the dilaton of the $SL(2,\mathbb{R})\otimes SO(1,1)^{d-2}/SO(1,1)$ model in the axial gauging generalizing the previous semi–classical results of [8] for $d=3$, [13] for $d=4$ and [3] for any d. [5] Our expression for the metric (4.3) is not yet ready to be compared with the corresponding semi–classical expression in [3]. To do so we must specialize to the "gauge"

$$b=\pm a \quad \Rightarrow \quad x_i = r_i - \frac{1}{2}\eta_i\ \ln|1-uv|\ . \tag{4.5}$$

Under this change the dilaton is unaffected (still given by (4.4)) whereas the metric takes the following form

[5] See remarks in footnote 3.

$$\begin{aligned} ds^2 =& \frac{1}{(1-2/k)(uv-1)-\rho^2-2/k}\Big[-\frac{1}{4}\frac{\rho^2+2/k}{uv-1}(vdu+udv)^2+(1+\rho^2)\ dudv\Big] \\ &+\frac{1}{1-2/k}\sum_{i,j=1}^{d-2}\kappa_i(\delta_{ij}+\frac{\eta_i\eta_j\kappa_j}{(1-2/k)(uv-1)-\rho^2})\ dr_i dr_j\ , \end{aligned} \tag{4.6}$$

which in the $k \to \infty$ limit is in agreement with the semi–classical expression found in [3]. The metric (4.6) is singular at $uv = 1$ where the "gauge choice" breaks down. This is obviously a coordinate singularity because (4.3) is manifestly non–singular at $uv = 1$. We can easily show that the same change of coordinates (3.6)(3.7)(3.8), made for the vector case, also diagonalizes the metric (4.6) but now with $m = 0$. After rescaling $X_0 \to X_0/(1+\rho^2)^{\frac{1}{2}}$ and $X_{d-2} \to X_{d-2}(\frac{1+\rho^2}{1-2/k})^{\frac{1}{2}}$ the metric in the region where $uv > 1$ takes the following form

$$ds^2 = dR^2 - \frac{dX_0^2}{(1+\rho^2)\tanh^2 R-\rho^2-2/k} + \frac{\tanh^2 R}{(\rho^2+1-2/k)\tanh^2 R-\rho^2}\ dX_{d-2}^2 + \sum_{i=1}^{d-3} dX_i^2\ . \tag{4.7}$$

In the regions $0 < uv < 1$ and $uv < 0$ we obtain the metric by making the same analytic continuations of R as in the vector case. To obtain further insight let us concentrate on the first three terms of the metric. Inspired by the work in ref. [8] we make the following change of variables in the $uv > 1$ region (a similar change can be made in the other two regions).

$$\cosh^2 R = \frac{r_+ - r}{r_+ - r_-}\ , \tag{4.8}$$

where

$$r_+ = M \equiv \sqrt{2/k'}\ (\rho^2+1)\ e^a\ , \qquad r_- = Q^2/M \equiv \sqrt{2/k'}\ (\rho^2+2/k)\ e^a\ . \tag{4.9}$$

The constant a is related to C' in (4.4) and $k' = k-2$ is the renormalized value for the central extension k. After a few rescalings of the variables, the $3d$ non–trivial part of the metric (4.7), and the dilaton (4.4) take the following forms

$$ds_{3d}^2 = -(1-\frac{r_+}{r})\ dt^2 + (1-\frac{r_- - r_q}{r-r_q})\ dx^2 + \frac{k'}{8r^2}(1-\frac{r_+}{r})^{-1}(1-\frac{r_-}{r})^{-1}\ dr^2 \tag{4.10}$$

and

$$\Phi = \frac{1}{2}\ln\big(r(r-r_q)\big) + \frac{1}{2}\ln k' \ , \tag{4.11}$$

where $r_q \equiv 2/k\sqrt{2/k'}\ e^a$. Notice that $r_q \to 0$ when $k \to \infty$. The scalar curvature for the metric (4.10) can also be calculated

$$\begin{aligned} R =& \frac{4}{k'\big(r(r-r_q)\big)^2}\big\{2(r_+ + r_- - r_q)\ r^3 - \big(7r_+r_- - r_q(r_- - r_q)\big)\ r^2 \\ & + r_q r_+(7r_- + r_q)\ r - 3r_q^2 r_+ r_-\big\} \ . \end{aligned} \tag{4.12}$$

The above expressions are the conformally exact metric, dilaton and scalar curvature of the $3d$ model which was analyzed by Horn and Horowitz [8] in the semi–classical limit. These authors showed that, in the $k \to \infty$ limit, the metric describes a black string with mass M and charge Q the same as the quantities defined in (4.9) in the large k limit. It can be seen by inspecting (4.12) that now the exact metric (4.10) has two true curvature singularities at $r = 0$, $r = r_q$ which degenerate to only one singularity, at $r = 0$, in the $k \to \infty$ limit. There are also two coordinate singularities at $r = r_+$ and $r = r_-$ whose interpretation will be given in the next section. It is interesting to take the $\rho^2 \to 0$ limit. In this case one should recover the $2d$ (black hole) $\otimes\ \mathbb{R}$ model since the subgroup $H = SO(1,1)$ is totally embedded in the $SL(2,\mathbb{R})$ factor in G. It can be checked that this is indeed the case. In particular the scalar curvature (4.12) becomes $R = 8/k'(r - 3/k)/r^2$ which has only one singularity at $r = 0$, in agreement with [25].

Therefore we have proven that the $SL(2,\mathbb{R}) \otimes SO(1,1)^{d-2}/SO(1,1)$ coset model for the axial gauging is equivalent to the ($3d$ black string) $\otimes\ \mathbb{R}^{d-3}$ model for any k. In the semi–classical limit this was proved in [3].

Reversing the sign of M is equivalent to reversing the signs of r, r_- and r_q. Therefore we restrict ourselves to $M > 0$. As in [8] we can distinguish three different cases

(i) The black string with $0 < Q < M$ $(0 < r_- < r_+)$

This is the generic case. As in the semi–classical $k \to \infty$ limit, the coordinate singularities at $r = r_+$ and $r = r_-$ can be interpreted as an event and an inner horizon respectively.

To see the effect the finite value of k has on the structure of the manifold we will consider the geodesic equations. They have the following form

$$\frac{k'}{8}\dot{r}^2 = E^2 r(r-r_-) - P^2(r-r_q)(r-r_+) + \alpha(r-r_+)(r-r_-) \ , \tag{4.13}$$

where E, P are two conserved quantities, determined by the initial conditions, associated with the two killing vectors in the t and x directions and $\alpha = 0$ (-1) for null (time–like) trajectories. For large r, the right hand side is non–negative if $E^2 - P^2 + \alpha \geq 0$. Let us first consider the time–like trajectories. We can check that the right hand side of (4.13) becomes negative at $r = r_q > 0$. Therefore trajectories reach a minimum value for r and they never reach either singularity. The same is true in the semi–classical limit $k \to \infty$, where no trajectory can hit the $r = 0$ singularity. Now we turn to the case of null trajectories. In the $k \to \infty$ limit we can prove using (4.13) that for $E^2 - P^2 > 0$

$$r_{\min} = \begin{cases} 0 & \text{if} \quad E^2 r_- - P^2 r_+ \leq 0 \\ \frac{E^2 r_- - P^2 r_+}{E^2 - P^2} & \text{otherwise} \end{cases} \tag{4.14}$$

and for $E^2 - P^2 = 0$

$$r_{\min} = 0 \ , \qquad \text{if} \quad E^2 - P^2 = 0 \ . \tag{4.15}$$

Therefore under certain initial conditions null trajectories can arrive to the singularity at $r = 0$. Qualitatively the behavior is similar to the case of the Reissner–Nördstrom metric of Einstein's general relativity. However when k is finite the situation changes drastically. In contrast with the $k \to \infty$ case null trajectories can never hit the $r = r_q$ singularity. Instead they reach a minimum value which can be found using (4.13)

$$r_{\min} = \begin{cases} r_0/2 + \sqrt{r_0^2/4 + P^2/(E^2 - P^2) r_q r_+} & E^2 - P^2 > 0 \\ 2/k \ r_+ & E^2 - P^2 = 0 \ . \end{cases} \tag{4.16}$$

with

$$r_0 = \frac{E^2 r_- - P^2(r_+ + r_q)}{E^2 - P^2} \ . \tag{4.17}$$

In all cases the turning point lies inside the inner horizon. In the region inside the two singularities no time–like or null trajectory is allowed because in such case the right hand side of (4.13) is manifestly negative. This is related to the fact that in this region all variables (t, x and r) become space–like as one can see by inspecting (4.10). Finally, we consider trajectories in the region where r takes negative values. It can easily be seen that, null trajectories reach the singularity at $r = 0$ only if $P^2 = 0$. In contrast if $k \to \infty$

this is possible for $E^2 r_- - P^2 r_+ \geq 0$. In either case, time–like trajectories never hit the singularity.

Now we consider some thermodynamic properties of the black string. In general, one can deduce the Hawking temperature associated with the event horizon by considering the metric in the Euclidean regime $t \to i\theta$, in the neighborhood of the event horizon. Then if we introduce the parametrization

$$r = r_+(1 + \beta^2 z^2) \ , \qquad \beta^2 = \frac{2}{k'}\big(1 - \frac{r_-}{r+}\big) \ . \tag{4.18}$$

the metric (4.10) close to the horizon $r = r_+$ ($z = 0$) can be written as

$$ds_E^2 \sim dz^2 + \beta^2 z^2 \ d\theta^2 + \frac{r_+ - r_-}{r_+ - r_q} \ dx^2 \ . \tag{4.19}$$

The horizon represents a conical singularity of the solutions of the Euclideanized equations which can be removed if the imaginary time θ is taken to be periodic with (period) $= 2\pi/\beta$. The temperature is identified with the inverse period [26]. Therefore the temperature of the black string is

$$T = \frac{1}{\pi}\sqrt{\frac{1}{2k'}\big(1 - \frac{r_-}{r_+}\big)} \ . \tag{4.20}$$

This is of the same form (except for the replacement $k \to k'$) as the expression for the temperature found in [8]. The statistical description of the Hawking radiation is inappropriate when the back reaction of the emitted radiation starts to become important [27]. For the black string this happens in the extremal limit (see below). [6] We will reevaluate the Hawking temperature in the next section using the Bogoliubov transformation.

(ii) The extremal limit $Q = M$ ($r_- = r_+$)

In the limit where $q_0 \to 0$ ($\rho^2 \to \infty$) or equivalently $Q \to M$ ($r_- \to r_+$) the embedding of $H = SO(1,1)$ inside $SL(2, \mathbb{R})$ is zero and therefore we expect that the metric (4.10) reduces to the metric appropriate for the Anti-de-Sitter space manifold of $SL(2, \mathbb{R}) \sim SO(2,1)$. Indeed if we change variables to

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[6] One condition which must be satisfied for the statistical description to be valid is $\frac{\partial T}{\partial M}\big|_Q << 1$ [27]. This condition is catastrophically violated in the extremal limit.

$$y = \frac{8(\frac{r}{r_+} - 1)}{k'(1 - \frac{r_-}{r_+})^{1/2}} \ , \quad \hat{t} = \Big(1 - \frac{r_-}{r_+}\Big)^{1/4} t \ , \quad \hat{x} = \frac{(1 - \frac{r_-}{r_+})^{1/4}}{(1 - \frac{r_-}{r_+})^{1/2}} x \tag{4.21}$$

and take the double scaling limit $r \to r_+$ and $r_- \to r_+$ the metric (4.10) can be written as

$$ds^2_{ads} = \frac{k'}{8}\Big(y(-d\hat{t}^2 + d\hat{x}^2) + \frac{1}{y^2}\ dy^2\Big) \ . \tag{4.22}$$

The fact that it describes an Anti-de-Sitter space can be seen by noticing the boost invariance along the string and by calculating the Ricci tensor. The latter reads $R_{\mu\nu} = -\frac{4}{k'} g_{\mu\nu}$. One notices that the geometry is non–singular, but there is still a horizon at $y = 0$. The metric (4.22) is exactly the same as the one found in [8] in the semi–classical limit. [7] This was expected by the authors of [28] on the basis that after some appropriate transformations the metric (4.22) describes a plane–front wave of the same type several authors [29] proved that it solves the σ–model perturbative equations to all orders in the string coupling $(1/k)$. As we have seen this follows trivially in the Hamiltonian formalism.

(iii) The solution for M < Q ($r_+ < r_-$)

It can be seen from the definitions (4.9) that the conformal field theory construction we have followed so far allows only solutions with $Q < M$. However, as in [8], if we gauge a different subgroup of $SL(2, \mathbb{R})$, namely that generated by $(j_+ + j_-)$ in (2.9) we get solutions with $M < Q$. As in [8] we can obtain those solutions by setting $\tilde{r}^2 = r - Q^2/M$ in (4.10). Then the metric reads (using a notation with M and Q this time)

$$\tilde{ds}^2_{3d} = -\frac{Q^2 - M^2 + M\tilde{r}^2}{Q^2 + M\tilde{r}^2}\ dt^2 + \frac{M\tilde{r}^2}{Q^2 - Mr_q + M\tilde{r}^2}\ dx^2 + \frac{k'}{2}\frac{M}{Q^2 - M^2 + M\tilde{r}^2}\ d\tilde{r}^2 \ . \tag{4.23}$$

This metric, for $0 < \tilde{r} < \infty$ has no horizons and no curvature singularities. However, it does have a conical singularity which can be removed by identifying x with period

$$\pi \sqrt{2k' \frac{Q^2 - Mr_q}{Q^2 - M^2}} \ .$$

As in the semi–classical case [8], this changes the structure of the space-time at infinity from $\mathbb{R}^3 \to \mathbb{R}^2 \times S^1$. In fact, if we take the limit $M \to 0$, $Q \to 0$ keeping $Q^2/M = fixed$

[7] One can check that in the extremal limit the scalar curvature (4.12) reduces to $R = -12/k'$ which is compatible with the result for the Ricci tensor above.

the metric (4.23) reduces to the sum of $-dt^2$ and the conformally exact metric for the Euclidean $2d$ black hole [6][21].

5. Scattering off the black string.

In this section will describe the scattering of a tachyon wave off the black string, in the generic case $0 < Q < M$, by solving (2.3) for the $3d$ metric (4.10) and the dilaton (4.11). Because of the independence of this equation on the variables t and x we look for solutions with the following form

$$\begin{aligned} &T(t,x,r) = e^{-iEt}e^{-iNx}T(r) \\ &E = 4\big(2k(\rho^2+1)\big)^{-1/2}\mu \ , \quad N = 4(2k\rho^2)^{-1/2}\nu \ , \end{aligned} \tag{5.1}$$

where $\mu, \nu \in R$ and the various factors in the expressions for E, N were introduced for later convenience. We change variables to

$$\begin{aligned} &z = \frac{r_+ - r}{r_+ - r_-} \\ &T = z^{\frac{c-1}{2}}(1-z)^{\frac{a+b-c}{2}}\Psi \ , \end{aligned} \tag{5.2}$$

where the constants a, b, c are defined as

$$\begin{aligned} &a = j + 1 + i(\epsilon|\mu| - \epsilon'|\nu|) \\ &b = -j + i(\epsilon|\mu| - \epsilon'|\nu|) \\ &c = 1 + 2i\epsilon|\mu| \\ &\epsilon, \epsilon' = \pm \ , \end{aligned} \tag{5.3}$$

provided the eigenvalue of L_0, takes the appropriate form for a coset model

$$\begin{aligned} L_0 &= -\frac{j(j+1)}{k-2} - \frac{1}{k}\Big(\frac{\mu^2}{\rho^2+1} - \frac{\nu^2}{\rho^2}\Big) \\ &= -\frac{1}{k'}\Big(j(j+1) + (r_+ - r_-)\Big(\frac{\mu^2}{r_+} - \frac{\nu^2}{r_- - r_q}\Big)\Big) \end{aligned} \tag{5.4}$$

and j takes values in a representation of $SL(2,\mathbb{R})$ [30]. Then Ψ satisfies the standard hypergeometric equation

$$\Big(z(1-z)\frac{d^2}{dz^2} + (c - (1+a+b)z)\frac{d}{dz} - ab\Big)\Psi = 0 \ , \tag{5.5}$$

with solution for $|z| \leq 1$

$$\Psi(z) = c_1 F(a,b,c;z) + c_2 z^{1-c} F(a+1-c, b+1-c, 2-c; z) \ , \tag{5.6}$$

where c_1, c_2 are two arbitrary constants, which can be determined by imposing the appropriate boundary conditions. We want to describe the scattering of the tachyon off the black string geometry. As we shall see, in this case, j should belong to the principal series representation of $SL(2,\mathbb{R})$ i.e $j = i\sigma - \frac{1}{2}, \sigma \in \mathbb{R}$. Let us first consider a solution which, in the asymptotically flat region $r \to \infty$, reduces to the sum of two waves, one ingoing and the other outgoing, and represents a wave which disappears into the event horizon for $r \to r_+$. We call this type of solutions T_{out} for reasons which will become apparent. The appropriate choice for the various constants is $c_2 = 0$, $\epsilon = -1$ and $\epsilon' = 1$. Then one can check that indeed the solution has the right asymptotic behavior

$$T_{out} \sim (-1/z)^{i|\mu|} \ , \qquad z \to 0^- \ (r \to (r_+)^+) \tag{5.7}$$

and

$$T_{out} \sim \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)} \ (-z)^{-i\sigma-1/2} + \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)} (-z)^{i\sigma-1/2} \ , \quad z \to -\infty \ (r \to +\infty) \ . \tag{5.8}$$

where the first term in (5.8) represents an ingoing wave and the second an outgoing one. The expressions for the reflection and transmission amplitudes are

$$\begin{aligned} R_+ &= \frac{\cosh \pi(\sigma - |\mu| - |\nu|) \ \cosh \pi(\sigma - |\mu| + |\nu|)}{\cosh \pi(\sigma + |\mu| + |\nu|) \ \cosh \pi(\sigma + |\mu| - |\nu|)} \\ T_+ &= \frac{\sinh 2\pi\sigma \ \sinh 2\pi|\mu|}{\cosh \pi(\sigma + |\mu| + |\nu|) \ \cosh \pi(\sigma + |\mu| - |\nu|)} \ . \end{aligned} \tag{5.9}$$

We see that part of the wave gets reflected in the event horizon. The other part will enter the event horizon and will be absorbed by the black string. The same is true in the case of the $2d$ black hole [21] for which the results follow from our formulas if we set $\nu = 0$. A similar analysis for scattering in the naked singularity region, where $r < 0$, gives for the reflection and transmission amplitudes the following results

$$\begin{aligned} R_- &= \frac{\cosh \pi(\sigma - |\mu| - |\nu|) \ \cosh \pi(\sigma + |\mu| - |\nu|)}{\cosh \pi(\sigma + |\mu| + |\nu|) \ \cosh \pi(\sigma - |\mu| + |\nu|)} \\ T_- &= \frac{\sinh 2\pi\sigma \ \sinh 2\pi|\nu|}{\cosh \pi(\sigma + |\mu| + |\nu|) \ \cosh \pi(\sigma - |\mu| + |\nu|)} \ . \end{aligned} \tag{5.10}$$

We see that the naked singularity is not a perfect reflector as in the $2d$ case [21].

Next we construct a solution, which we call T_{in}, by imposing different boundary conditions. Namely, we demand a solution which, close to the event horizon $r \to r_+$ behaves as the sum of an ingoing with an outgoing wave, and reduces to an ingoing wave for $r \to \infty$. Then the appropriate choice for the constants is $\epsilon = -1$, $\epsilon' = 1$ and

$$\frac{c_1}{c_2} = -\frac{\Gamma(2-c)\Gamma(a)\Gamma(c-b)}{\Gamma(c)\Gamma(a+1-c)\Gamma(1-b)} \ . \tag{5.11}$$

The asymptotic behavior now is

$$T_{in} \sim c_1(-1/z)^{i|\mu|} + c_2(-1/z)^{-i|\mu|} \ , \qquad z \to 0^- \ (r \to (r_+)^+) \tag{5.12}$$

and

$$T_{in} \sim \big(c_1 \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)} + c_2 \frac{\Gamma(2-c)\Gamma(b-a)}{\Gamma(b+1-c)\Gamma(1-a)}\big)(-z)^{-i\sigma-1/2} \ , \quad z \to -\infty \ (r \to +\infty) \ . \tag{5.13}$$

One can easily see that

$$T_{in}(z;\sigma,|\mu|,|\nu|) = c_1 T_{out}(z;\sigma,|\mu|,|\nu|) + c_2 T^*_{out}(x;-\sigma,|\mu|,-|\nu|) \ . \tag{5.14}$$

The states T_{in} and T_{out} form two different bases in terms of which any state can be expanded. Consequently there are two distinct Fock spaces corresponding to two different vacua. One can follow a standard procedure (see for instance [31]) to show that the expectation value of the occupation number operator N_{out} for the $\{T_{out}\}$ basis in the vacuum of the $\{T_{in}\}$ basis is

$$\begin{aligned} {}_{in}\langle 0|N_{out}|0\rangle_{in} &= \frac{1}{\frac{|c_1|^2}{|c_2|^2} - 1} \\ &= \left(\frac{\cosh\pi(\sigma+|\mu|-|\nu|)\ \cosh\pi(\sigma+|\mu|+|\nu|)}{\cosh\pi(\sigma-|\mu|-|\nu|)\ \cosh\pi(\sigma-|\mu|+|\nu|)} - 1 \right)^{-1} \ . \end{aligned} \tag{5.15}$$

This is not zero indicating the fact that the two bases are inequivalent. We can define the Hawking temperature T by rewriting (5.15) in the following form

$${}_{in}\langle 0|N_{out}|0\rangle_{in} \equiv \frac{1}{e^{\frac{M}{T}} - 1} \ , \tag{5.16}$$

where E, defined in (5.1), is the eigenvalue of the time–like vector $i\partial_t$. One can easily show that, when $\sigma \to \infty$, T tends to the same temperature defined in (4.20) above. For σ small the corresponding expression is different and depends explicitly on the value of σ. We see that for the "out" observers the "in" vacuum is full of particles in a heat bath at temperature T.

6. Discussion and concluding remarks

By making use of the conformal properties of the $SL(2,\mathrm{I\!R}) \otimes SO(1,1)^{d-2}/SO(1,1)$ coset model we proved, for any k, that it describes geometries equivalent to the ($2d$ black hole) $\otimes\, \mathrm{I\!R}^{d-2}$ model for the vector gauging, and to the ($3d$ black string) $\otimes\, \mathrm{I\!R}^{d-3}$ model for the axial one. We gave the conformally exact expressions for the metric the dilaton and the scalar curvature. We have seen for the $3d$ case that the finite value of k has some important consequences. One of them is the appearance of a second curvature singularity not present in the semi–classical $k \to \infty$ limit. Finally we calculated the transmission and reflection coefficients for the scattering of the tachyon off the black string and using the Bogoliubov transformation we found the Hawking temperature. According to (4.7) same conclusions follow for $d \geq 4$.

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