Concerning Logics of Abduction

-On Integrating Abduction and Induction-

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1 Introduction

The larger context in which the discussion here should be placed is our effort to develop a logic of abduction that models Peirce's abduction ([Fann, 1970], [Kruijff, 1995]) as far as logically justifiable, and computationally feasible. The reason for trying to model Peirce's abduction is that it is in Peirce's theory that abduction is an explicitly *distinguished, logical kind of inference*, which infers a *novel* explanation for a *surprising* observation. We will argue that it is the combination novelty/surprise that form the core of abduction's ubiquity, constituting its peculiar relation to reality via perception. What is more, these distinctive qualities can be exploited in a cooperative interaction with other forms of reasoning, notably induction, as we will discuss here in the context of machine learning. (Our own application of abduction is primarily in the field of natural language interpretation by so-called communicative agents would provide an interesting example as for "perceptive interaction" but appears to be less illustrative for the relation abduction-induction.)

1.1 Overview

We will commence with discussing novelty in (abductive) reasoning, thereby elucidating the position we are taking in the debate on the nature of abduction and induction, and focusing on the issue of modeling logical properties of abduction. Subsequently, we will present a view on integrating abduction and induction, and particularly, how such could be done. The aim we have with this paper is to provide a discussion of some issues appertaining to the potential of abduction in Artificial Intelligence, set against a background which is on the one hand philosphical, but justified by means of mathematical formalization of its primary arguments concerning abduction, and on the other hand driven by practical application of the theory to for example machine learning or the development of robust, communicative agents.

2 Novelty

2.1 Peirce's Mature Theory

In Peirce's opinion, abduction is the only kind of reasoning that can come up with a conclusion that -eventually- could lead to new knowledge. The reason for this claim lies in the intricate relation between perception, perceptual judgment, and the formation of an explanatory hypothesis. The importance of this relation is reflected in the "two faces" of the *surprise*. One side of the coin is that the observation was surprising because we had erroneous or incomplete knowledge concerning it. We expected to observe something different, and the surprise is genuine - for otherwise we would not have been surprised in the first place. The other side is that the surprise is genuine (as Peirce put it, "A man cannot startle himself, by jumping up with an exclamation of **Boo!**"; cf. [Kruijff, 1995], p.20) and that we *acknowledge that*. Both aspects of surprise lead Peirce to argue that part of the hypothetical explanation is formed from perceptual judgments that acknowledge the fact that observation is as it is rather than that the judgments arise from our background knowledge. Abductively inferred hypothetical explanations therefore *possibly* present novel knowledge, in that they contain aspects of reality previously unknown, or known differently. Note that the explanations are *hypothetical*, presenting only a *possibility*: The need to be verified in further inquiry before they should be incorporated into our knowledge.

2.2 Counterarguments

There are several counterarguments to this view. A straightforward one is that the view seems to open the way for sheer voluntarism in what should be accepted as explanatory. Should we just accept any observed aspect as explanatory? We recall in [Kruijff, 1994] that this question between *is/ought-to* is not new, and argue that opposing voluntarism should not result in reverting to a (modernist/Kantian) ought-to perspective but rather in perceive of the two principles as *interacting*. Or, as Cain puts the issue in [Cain, 1991] when considering abduction in integrated learning, abduction is apparently ubiquituous but should be constrained in order to be really usefull.

Others have put forward that besides abductively inferred conclusions one can also argue that *inductively inferred conclusions* can express novelty. For example, Josephson argues in [Josephson, 1996] that inductive generalization is an instance of abduction, because a generalization explains the premisses from which it has been derived by linking the particular to the general case (metalevel). As put in [Flach and Kakas, 1996], "all crows are black" does not explain why crows are black, but it does explain why "we observe a black thing whenever we observe a crow." Furthermore, just like in abduction, it is the 'best' generalization that is of interest. Or, in the words of Michalski, if we would consider inductive inference as "a process of generating descriptions that imply original facts in the context of background knowledge", then we could consider inductive generalization and abduction as special cases of that definition (op.cit. [Flach, 1996]).

A slightly different approach is taken by Flach in [Flach, 1996]. There, he discusses Peirce's different views on abductive reasoning, being Peirce's early, syllogistic view, and Peirce's late, inferential view (cf. [Fann, 1970]). By arguing that Peirce's late theory of abduction combines the earlier syllogisms for Hypothesis and Induction into one logical inference, Flach concludes that two different stances towards the relation between abduction and induction follow from adhering to either the syllogistic perspective, or to the inferential perspective. Favoring the syntactically oriented syllogistic view means stressing the differences between abduction and induction, whereas those who adhere to the inferential perspective, one "therefore view[s] induction as a special case of abduction." ([Flach and Kakas, 1996])[1].

Notwithstanding that I do believe that it is defendable that inductively inferred generalizations *can* present novelty, I disagree with above mentioned argumentations. It appears from [Flach and Kakas, 1996] that Josephson's argumentation can be captured as "extending [abduction] to include hypothesis selection as well." In the light of Josephson's [Josephson and Josephson, 1994], this does not seem to be about whether to consider abduction purely as generating hypotheses, or as *putting forward some likely hypotheses* (i.e. as generation+selection, as in Peirce's mature view). Rather, Josephson appears to immediately go from hypothesis to rule, and by arguing that inductive generalization is an instance of abduction, one could thus conclude that abduction outputs rules. The explanation can be accepted because it has "a strong abductive justification"[2]. In my opinion, there is a confusion here of the *modality* of the conclusions of the various kinds of inferences.

This becomes particularly problematic in Flach's argumentation. In the syllogisms, the copula does *not* have one and the same modality. In case of Barbara, the "are" expresses a "must be": If all the beans from this bag are white, and these beans are from this bag, then these beans *must be* white. But in the case of Hypothesis, inferring a case from a rule and a result, the "are" is a "possibly", whereas in case of the inductive syllogism, the "are" expresses a tendentiality ("most likely"-hood)[3]

Even though abduction and induction are both ampliative kinds of reasoning, they are so in different ways. Their conclusions' modalities are different, showing the separate ways in which these kinds of reasoning try to deal with (different kinds of) incompleteness of a theory. Simply put, abduction tries to overcome the kind of incompleteness of a theory that regards describing, explaining, observations, whereas induction tries to overcome incompleteness in the sense of *applicability* of available descriptions to more phenomena (for example, this appears to be the kind novelty arising from Thagard's system described in [Thagard, 1988]). It is in this sense that I do agree with saying that inductive generalizations may express novelty: They express novelty in the sense of extending the coverage of a theory in terms of *facts*. The novelty that abductive explanations express is the *possible* way in which the theory's description of reality can be extended. We can examplify this by the role observation plays in abduction and induction, respectively. For abduction, observation is a source of inspiration, whereas for induction, observation is a touchstone. Following up on Aliseda's [Aliseda, 1996], we would therefore like to propose that different forms of explanatory reasoning can be obtained by instantiating *four* parameters: Not only the kind of inference involved in explaining, the kind of observation that needs to be explained, and the kind of explanations, but also the role perception plays in the inference process.

2.3 A Mathematical Model of Novelty

Given then that novelty is -arguably- so intricably related to the concept of abduction, can we say that AI

[1]In [Kruijff, 1995] we present more counterarguments; for example, that for the formation of the (crucial) perceptual judgments one need not rely on a "guessing instinct", but that a critical *and* interactive, acknowledging process of interpretation is a good alternative.

[2]True, [Josephson, 1996] *is* about justification, not about reasoning processes. But couldn't the view be rephrased as "It is an explanation because we have found it to be a general explanation"?

[3]Furthermore, one might put the philosophical interpretation of Peirce's mature theory of abduction in [Flach, 1996] to discussion, but this does not seem the time nor the place to do so.

abduction frameworks explicitly address, or silently assume, novelty? Hardly; nor should they, in many cases. Novelty need not be desirable, or needed at all. There is a whole spectrum of abductive inferences, ranging from forming hypotheses based on a complete domain-model, to the kind of abduction that generates and selects hypothetical explanations that can *possibly* lead to new knowledge but only so after ample testing. Some treatises on abduction do address the issue of novelty, though. In [Kruijff, 1995], we reflect on the possibility of generating genuine novelty in the context of abduction in AI, whereas Pirri, in [Pirri, 1995] develops a logical framework for abduction that is capable of dealing with novelty. However, neither of them considers how such novelty could be generated; nor, as far as the author is aware of, do others. Conceiving of a logical system as a *pair* of an inference relation and its operational characteristics (cf. Gabbay's [Gabbay, 1996]), any logic of abduction thus remains in a sense incomplete if we *do* want to generate genuine novelty, but do not describe how such could be done.

In [Kruijff, 1997a], we describe "genuine novelty" as follows. If we would represent abduction as the inference of an explanation ϵ for an observed phenomenon ϕ, from ϕ and $\epsilon \rightsquigarrow \phi$ (or, more general, $\mathcal{R}(\epsilon, \phi)$), then the fact that ϕ is surprising means that both ϵ *and* $\mathcal{R}(\epsilon, \phi)$ are *novel*. Given some background theory Θ, and considering that as our only source for forming explanations, neither $\Theta \models \epsilon$ nor $\Theta \models \mathcal{R}(\epsilon, \phi)$ holds. For otherwise we would not have been surprised in the first place. Rephrased, the novelty in abduction could be given abstractly as a pair $\langle \epsilon, \mathcal{R}(\epsilon, \phi) \rangle$.

Subsequently, the paper describes a mathematical formalization of the operational characteristics of the generation of explanations that *do* present possible novelty as described above. Thereby use is made of a technique recently developed by Feferman (in [Feferman, 1996]), called the *unfolding* of a theory. The idea behind unfolding is Gödel's call for 'axioms of infinity', axioms which should be "abdundant in their verifiable consequences" ([Gödel, 1990],p.183) and should decide previously undecided set-theoretical propositions (see my [Kruijff, 1997b] for a mathematic-philosophical discussion). In other words, these axioms should provide *explanations*, verifiable ones, which, when added to a theory, would extend that theory in a non-trivial way and expanding its explanatory power. As such, unfolding provides an interesting formal tool. The paper introduces a formal notion of abductive unfolding, $\mathfrak{U}_{\mathcal{ABD}}$, which extends a theory in a non-trivial way by obtaining novel explanations $\langle \epsilon, \mathcal{R}(\epsilon, \phi) \rangle$ for an observation ϕ, given a theory Θ. If we would want to make distinction between causality and entailment, as Josephson proposes in [Josephson and Josephson, 1994], then what our extension could be taken to stand for as explanations the observations as entailment-consistent with the theory, but in need of further verification when their entailments would be taken causally. The computational aspects of the approach have not be investigated yet, but it appears that infinite-state automata, used for example to model concurrent processes, are a suitable way to do so.

2.4 Logical Properties of Abduction

We are currently inquiring after the logical properties of abduction that (partly) arise from the operational characterization of generation, $\mathfrak{U}_{\mathcal{ABD}}$. Similar studies have been conducted by for example Pirri [Pirri, 1995], Aliseda [Aliseda, 1996], Konolige [Konolige, 1996], Console et al [Luca Console and Torasso, 1991], and from a more algorithmic perspective, by Josephson et al [Josephson and Josephson, 1994], Bylander et al [Tom Bylander and Josephson, 1991], and in [Kruijff, 1995].

There is, obviously, the question of how could we model abduction *logically*. The perspective taken in [Kruijff, 1997a] focuses on the process of reasoning, like the tableaux methods of Pirri in [Pirri, 1995], rather than taking a model-theoretic approach which essentially characterizes the outcome of the inference. But perhaps the question of how to model *abduction* logically is more intriguing ... Several issues have risen concerning how to model the process of abductive inference.

First, should abduction be modeled as a process of "first generation, then selection" (the 'classical' view), or should generation and selection be *interleaved* (i.e. should the economical criteria of selection guide the generation)? Examples of systems based on the latter view can be found in [Josephson and Josephson, 1994]. A philosophical argument for it can be found in [Kruijff, 1995], trying to construe a model of interpretation that provides the same functionality as Peirce's guessing instinct, and in various discussions concerning the role of abduction and induction in learning (e.g. Dimopoulos and Kakas' [Dimopoulos and Kakas, 1996], [Nguifo, 1996]), abduction and induction are interleaved in a process of gradual refinement.

Second, should we take *epistemological assumptions* into account when modeling abduction? It is known from the literature that, for example, assuming independence between parts making up an explanation enables one to formalize abduction using closure semantics or probabilistic techniques. Furthermore, Poole has for example argued that dependency is a characteristic of a model, and not of reality. Ergo, assuming independence is actually a realistic assumption. Others have argued that it is essential for abduction to take into account interaction between these parts making up an explanation, because such can have an influence on what can actually be explained (e.g. see [Josephson and Josephson, 1994]). Does the assumption affect what kind of rules we should formulate for abduction? Logically seen, it would. Interaction, or dependency, could be more naturally modelled with substructural logics, underlying or controlling the structural logic describing the domain.

3 On Integrating Abduction and Induction

3.1 Views on Integration

It appears that there are quite different views on "integration" in the literature on abduction. Some consider "integration" to be "*unifiable* into one formal framework"; witness for example discussions at the ECAI'96 workshop on abduction and induction. Induction is then usually seen as a kind of abduction. Others rather perceive of "integration" as *cooperation*. Denecker, in his response to [Flach and Kakas, 1996], dubs the first view as belonging to the "philosophical school", the second to the "empirical school," whereas Flach couples the perspectives to his syllogism/inference distinction (see above).

Flach and Kakas, in [Flach and Kakas, 1996], observe that "if two such [conceptually different forms of ampliative reasoning from incomplete information] should exist, we need to clarify the differences in their operation and also the difference in the tasks that they can achieve." Whether or not the above made distinctions are appropriate, it should be obvious from the discussions above that I entirely agree with Flach and Kakas' "question". The opinion put forward here is that what abduction does, is generate an explanatory *hypothesis*, which purports to explain a surprising observation - but which, nevertheless, is hypothetical by nature. What induction does is extending the factual coverage. Regarding integration, I believe that the *cooperative view* on integration brings out these differences, and thereby exploits the "best of worlds" of each kind of reasoning.

The cooperative view on abduction and induction has been proposed by for example Aliseda [Aliseda, 1996], Cain, [Cain, 1991], Mephu Nguifo [Nguifo, 1996], and Dimopoulos and Kakas [Dimopoulos and Kakas, 1996]. Except for Aliseda, all of the authors are concerned with *learning*, aiming to provide mechanisms with which various kinds of incompleteness usually occurring in domain models can be overcome. The general pattern is that abduction is used to prepare explanations that later can be generalized by induction, possibly including theory revision (i.e. modification of the background theory itself).

Below we will put forward some ideas concerning extending that view.

3.2 On Including Deduction

It has already been argued for that abduction is capable of bringing about exiting results. What we would like to argue for here is that, actualy, the more exiting these results are, the more abduction needs to be integrated with other forms of reasoning. Such is not only needed for establishing the validity of the results immediately following from abduction; particularly, other forms of reasoning should assist in constraining, and guiding further abductive inference. (Whereas, on the other hand, if we are sure of the validity of the explanations that are abductively inferred, because they are derived from a complete domain model, then abduction is relatively self-sufficient.)

As for constraining abduction, Cain argues in [Cain, 1991] that deduction and induction should be used to constrain abduction in cases of incomplete explanations. Two possible solutions to incomplete explanations would be to either abandon the explanation, or generalize particular rules such that the incomplete assumptions are no longer needed. However, such can possibly give rise to the voluntarism noted earlier. Hence, constraining methods are needed. Deduction can provide one way, namely by requiring that each assumption (signifying incompleteness) has to have a supporting explanation that is completely deductive except for the assumption itself. What this requirement results in is a constraint on the number of assumptions that should be made, making the system rather conservative. Induction can provide another way, by creating new rules that explain the assumption(s) in the explanation. Briefly put, the deductive closure of the model is expanded by induction so as to include the assumptions.

Are there more arguments possible/available why deduction should be included in a cooperative integration between abduction and induction? Clearly, deduction could be used to infer predictions that can be based upon an abductively inferred explanation, predictions that could be generalized by means of induction. More concrete insight could perhaps be obtained via the following questions below.

In [Dimopoulos and Kakas, 1996], abduction is proposed to assist in identifying "useful learning problems 'hidden' in the new information (observations) given to the system" (§4.3), although the problem of finding out which observations are particularly important for a specific learning problem is not further addressed (§4.2). If we recall the discussion of abduction in the previous section, especially the focus on the relation between perception and abduction, then it appears that abduction might have even more to say about these problems.

First, let us go back to surprise. An observation is surprising if we were expecting something else, or nothing at all - the latter arguably also being an expectation.

Naturally, we base these expectations on our theory, and it seems reasonable to assume that we know on *what assumptions* we base a particular expectation. Assuming a finite set of assumptions, let us say that we *doubt* that set which let us to the wrong expectation. Could that set be characterized in terms of abducibles (predicates signifying descriptive incompleteness of the theory), and inducibles (a class of abducibles used in different explanations for the same observation, signifying where theory revision can be performed; [Dimopoulos and Kakas, 1996])? If we would do so, what we would do is trying to find an explanation for why we were mistaken. Note that to explain what went wrong is not about explaining the surprising fact(s), but about how an explanation for the surprising facts might be fitted into our model.

In other words, an explanation of the doubted set could include abducibles signifying what descriptive incompleteness led to the surprise, or "complete" predicates that (therefore) might have to be replaced by new rules (in the sense of Cain's DUCTOR). Similarly, we could perform the usual abductive inference that would try to form an explanation for the observations, based on the background theory. The question that immediately comes up now is of course, *Are these two explanations different?*. Because we are not describing a concrete system, it is hard to do more than reflect on this question, but let us assume that they *are* different (one reason being that the explanations are dealing with different observations: the expected respectively the actual observation). What could be envisionable consequences?

One consequence could be that we could *deductively* infer from the difference between the doubt-explanation and the observation-explanation which abducibles are particularly relevant. Transforming them into inducibles, this could (at least partially, possibly) provide an answer to the question after which abducibles would pose "useful learning problems." In [Kruijff, 1997a], we attempt to use doubt in a similar manner, namely to provide us with the basis in the theory that should be unfolded in order to provide possible explanations for the surprising observation. Doubt is thus used as a "beacon" in learning as a process of improving our knowledge.

Another envisionable consequence, or use of deduction, is that we can infer from the relevant abducibles what we should be trying to observe (predictions). As usual, what we aren't looking for we don't see, but deductively inferred predictions (in the form of self-construed negative/positive examples?) could possibly open eyes.

Obviously, the above conceptual sketches should be elaborated on, and as such state suggestions for further research rather than concrete results. But the intuition is that deduction can be advantegously put to use by *bridging the gap between a hypothesis and a rule*: Whatever we infer from a hypothesis need not be, and in some cases *should not be* (e.g. "when in doubt..."), immediately taken as 'the rule'.

4 Conclusions

The paper presents a view on abduction which takes its inspiration in Peirce's mature theory of abduction. As such, the aspects of perception, surprise, and novelty have been stressed, and reference has been made to (formal) elaborations of the actual functioning of these aspects - pointing out how we go beyond Peirce's theory. The position that, besides abduction, also induction can give rise to novelty has been considered from various viewpoints. Here, we take an approach different from [Josephson, 1996] and [Flach, 1996], arguing that the approaches advocated there do not pay due attention to the different modalities involved in abduction and induction. The importance of these modalities is revealed by the different kind of novelty each form of reasoning leads to, arising from the different function observation/perception has in each form. We propose therefore that, following up on Aliseda's [Aliseda, 1996], different forms of explanatory reasoning can be obtained by instantiating *four* parameters: Not only the kind of inference involved in explaining, the kind of observation that needs to be explained, and the kind of explanations, but also the role perception plays in the inference process.

Regarding the integration of abduction and induction, we take the position of the "cooperative" view, conceiving of it as the view that brings out the differences between the two kinds of reasoning the best, arguably being the most advantegeous way. Arguments for the further integration of deduction into the cooperative view have been given by referring to [Cain, 1991] where deduction is employed to constrain deduction, and by theoretizing about how deduction could be used to guide further abduction/learning processes by infering relevant abducibles from explanations regarding doubt, and regarding the observations.

Finally, we would like to conclude with some questions for further (further) research:

- Considering doubt as guiding learning should we formulate a logic of doubt, or a logic of doubt and belief?
- How could we formulate a *spectrum of logical systems of abduction*, based on explanatory power?
- Are there different degrees of novelty, and could all these different degrees of novelty be obtained using computational methods (cf. [Anderson, 1987], [Kruijff, 1995])?

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