

# Dynamic Resource Allocation in Cognitive Radio Networks: A Convex Optimization Perspective

Rui Zhang, Ying-Chang Liang, and Shuguang Cui

**Abstract**—This article provides an overview of the state-of-art results on communication resource allocation over space, time, and frequency for emerging cognitive radio (CR) wireless networks. Focusing on the interference-power/interference-temperature (IT) constraint approach for CRs to protect primary radio transmissions, many new and challenging problems regarding the design of CR systems are formulated, and some of the corresponding solutions are shown to be obtainable by restructuring some classic results known for traditional (non-CR) wireless networks. It is demonstrated that convex optimization plays an essential role in solving these problems, in a both rigorous and efficient way. Promising research directions on interference management for CR and other related multiuser communication systems are discussed.

## I. INTRODUCTION

In recent years, *cognitive radio* (CR) networks, where CRs or the so-called secondary users (SUs) communicate over certain bandwidth originally allocated to a primary network, have drawn great research interests in the academic, industrial, and regulation communities. Accordingly, there is now a rapidly growing awareness that CR technology will play an essential role in enabling *dynamic spectrum access* for the next generation wireless communications, which could hopefully resolve the spectrum scarcity vs. under-utilization dilemma caused by the current static spectrum management policies. Specifically, the users in the primary network, or the so-called primary users (PUs), could be licensed users, who have the absolute right to access their spectrum bands, and yet would be willing to share the spectrum with the unlicensed SUs. Alternatively, both the PUs and SUs could equally coexist in an unlicensed band, where the PUs are regarded as existing active communication links while the SUs are new links to be added. A unique feature of CRs is that they are able to identify and acquire useful environmental information (cognition) across the primary and secondary networks, and thereby adapt their transmit strategies to achieve the best performance while maintaining a required quality of service (QoS) for each coexisting active primary link. Depending on the type of cognitive knowledge collected (e.g., on/off statuses of primary links, PU messages, interference power levels at PU receivers, or primary link performance margins) and the primary/secondary network models of interests (e.g., infrastructure-based vs. ad hoc), many new and challenging problems on the design of CR networks can be formulated, as will be reviewed in this article.

To date, quite a few operation models have been proposed for CRs; however, there is no consensus yet on the terminology used for the associated definitions [1], [2], [3]. Generally speaking, there are two basic operation models for CRs:

Opportunistic Spectrum Access (OSA) vs. Spectrum Sharing (SS). In the OSA model, the SUs are allowed to transmit over the band of interest when all the PUs are not transmitting at this band. One essential enabling technique for OSA-based CRs is *spectrum sensing*, where the CRs individually or collaboratively detect active PU transmissions over the band, and decide to transmit if the sensing results indicate that all the PU transmitters are inactive at this band with a high probability. Spectrum sensing is now a very active area for research; the interested readers may refer to, e.g., [4], [5], [6], [7] for an overview of the state-of-art results in this area. As a counterpart, the SS model allows the SUs to transmit simultaneously with PUs at the same band even if they are active, provided that the SUs know how to control their resultant interference at the PU receivers such that the performance degradation of each active primary link is within a tolerable margin. Thus, OSA and SS can be regarded as the *primary-transmitter-centric* and *primary-receiver-centric* dynamic spectrum access techniques, respectively. Consequently, there will be an inevitable debate on which operation model, OSA or SS, is better to deploy CRs in practical systems; however, a rigorous comparative study for these two models, in terms of spectrum efficiency and implementation complexity tradeoffs, is still open. Generally speaking, SS utilizes the spectrum more efficiently than OSA, since the former supports concurrent PU and SU transmissions over the same band while the latter only allows orthogonal transmissions between them. Moreover, the receiver-centric approach for SS is more effective for CRs to manage the interference to the PU links than the transmitter-centric approach for OSA.

Hence, the SS model for CRs will be focused in this article. It is worth noting that the optimal design approach for SS-based CR networks should treat all coexisting PU and SU links as a giant interference network and jointly optimize their transmissions to maximize the SU network throughput with a prescribed PU network throughput guarantee. From this viewpoint, recent advances in network information theory [8] have provided promising guidelines to approach the fundamental limits of such networks. However, from a practical viewpoint, the centralized design approach for PU and SU networks is not desirable, since PU and SU systems usually belong to different operators and thus it is difficult, if not infeasible, for them to cooperate. Consequently, a *decentralized* design approach is more favorable, where the PU network is designed without the awareness of the existence of the SU network, while the SU network is designed with only partial knowledge (cognition) of the PU network.

Following this (simplified) decentralized approach, there are furthermore two design paradigms proposed for SS-based CRs.

One is based on the ‘‘cognitive relay’’ concept [9], where the SU transmitter allocates only part of its power to deliver the SU messages, and uses the remaining power to relay the PU messages so as to compensate for the additional SU interference experienced at the PU receiver. However, this technique requires non-causal knowledge of the PU messages at the SU transmitter, which may be difficult to realize in practice. In contrast, a more feasible SS design for the SU to protect the PU is to impose a constraint on the maximum SU interference power at the PU receiver, also known as the ‘‘interference temperature (IT)’’ constraint [10], by assuming that the SU-to-PU channels are either perfectly known at the SU transmitters, or can be practically estimated.

In this article, we will focus our study on the IT-based SS model for CRs, namely the IT-SS, as it is a more feasible approach compared with other existing ones. In a wireless communication environment, channels are usually subject to space-time-frequency variation due to multipath propagation, mobility, and location-dependent shadowing. Thus, *dynamic resource allocation* (DRA) becomes an essential technique for CRs to optimally deploy their transmit strategies to maximize the secondary network throughput, where the transmit power, bit-rate, bandwidth, and antenna beam should be dynamically allocated based upon the available channel state information (CSI) of the primary and secondary networks. In particular, this article will focus on DRA problem formulations unique to CR systems under the IT-SS model, and the associated solutions that are non-obvious in comparison with existing results [11] known for the traditional (non-CR) wireless networks. More importantly, we will emphasize the key role of various *convex optimization* techniques in solving these problems.

The remainder of this article is organized as follows. Section II presents different models of the SU network coexisting with the PU network, and various forms of transmit power and interference power (IT) constraints over the SU transmissions. Section III is devoted to the spatial-domain transmit optimization at the SUs for different SU networks subject to transmit and interference power constraints. Section IV extends the results to the more general case of joint space-time-frequency transmit optimization of the SUs, and addresses the important issue on how to optimally set the IT thresholds in CR systems to achieve the best spectrum sharing performance. Finally, conclusions are drawn and future research directions are discussed in Section V.

*Notation:* Lower-case and upper-case bold letters denote vectors and matrices, respectively.  $\text{Rank}(\cdot)$ ,  $\text{Tr}(\cdot)$ ,  $|\cdot|$ ,  $(\cdot)^{-1}$ ,  $(\cdot)^H$ , and  $(\cdot)^{1/2}$  denote the rank of a matrix, trace, determinant, inverse, Hermitian transpose, and square-root, respectively.  $\mathbf{I}$  and  $\mathbf{0}$  denote an identity matrix and an all-zero matrix, respectively.  $\text{Diag}(\mathbf{a})$  denotes a diagonal matrix with diagonal elements given in  $\mathbf{a}$ .  $\mathbb{E}(\cdot)$  denotes the statistical expectation. A circularly symmetric complex Gaussian (CSCG) distributed random vector with zero mean and covariance matrix  $\mathbf{S}$  is denoted by  $\mathcal{CN}(\mathbf{0}, \mathbf{S})$ .  $\mathbb{C}^{m \times n}$  denotes the space of  $m \times n$  complex matrices.  $\|\cdot\|$  denotes the 2-norm of a complex vector.  $\text{Re}(\cdot)$  and  $\text{Im}(\cdot)$  denote the real and imaginary parts of a complex number, respectively. The base of the logarithm function  $\log(\cdot)$  is 2 by default.

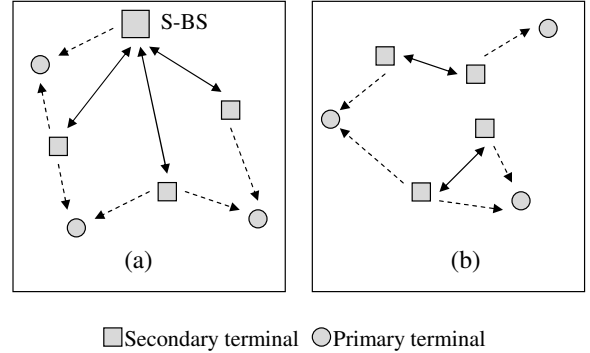


Fig. 1. CR networks: (a) infrastructure-based; (b) ad hoc.

## II. CR NETWORK MODELS

We consider two general types of CR networks, which are of both theoretical and practical interests: One is *infrastructure-based*, as shown in Fig. 1(a), where multiple secondary terminals communicate with a common secondary node referred to as the secondary base station (S-BS); the other is *ad hoc*, as shown in Fig. 1(b), which consists of multiple distributed secondary links. In both types of CR networks, there are coexisting primary terminals operating in the same spectrum band. For the IT-SS model of CRs, the exact operation model of the primary network is not important to our study, provided that all the secondary terminals satisfy the prescribed IT constraints to protect the primary terminals. Without loss of generality, we assume that there are  $K$  secondary links and  $J$  primary terminals in each type of the CR networks.

Consider first the infrastructure-based secondary/CR network with the S-BS coordinating all the CR transmissions, which usually corresponds to one particular cell in a CR cellular network. The uplink transmissions from the SUs to the S-BS are usually modeled by a multiple-access channel (MAC), while the downlink transmissions from the S-BS to different SUs are modeled by a broadcast channel (BC). For the MAC, the equivalent baseband transmission can be represented as

$$\mathbf{y} = \sum_{k=1}^K \mathbf{H}_k \mathbf{x}_k + \mathbf{z} \quad (1)$$

where  $\mathbf{y} \in \mathbb{C}^{M \times 1}$  denotes the received signal at the S-BS, with  $M$  denoting the number of antennas at S-BS;  $\mathbf{H}_k \in \mathbb{C}^{M \times N_k}$  denotes the channel from the  $k$ th SU to S-BS,  $k = 1, \dots, K$ , with  $N_k$  denoting the number of antennas at the  $k$ th SU;  $\mathbf{x}_k \in \mathbb{C}^{N_k \times 1}$  denotes the transmitted signal of the  $k$ th SU; and  $\mathbf{z} \in \mathbb{C}^{M \times 1}$  denotes the noise received at S-BS. We assume that  $\mathbf{x}_k$ 's are independent over  $k$ .

Similarly, the BC can be represented as

$$\mathbf{y}_k = \mathbf{H}_k^H \mathbf{x} + \mathbf{z}_k, \quad k = 1, \dots, K \quad (2)$$

where  $\mathbf{y}_k \in \mathbb{C}^{N_k \times 1}$  denotes the received signal at the  $k$ th SU; for convenience, we have used the Hermitian transposed uplink channel matrix for the corresponding downlink channel matrix, i.e.,  $\mathbf{H}_k^H$  denotes the channel from the S-BS to the  $k$ th SU;  $\mathbf{x} \in \mathbb{C}^{M \times 1}$  denotes the transmitted signal from S-BS; and  $\mathbf{z}_k \in \mathbb{C}^{N_k \times 1}$  denotes the receiver noise of the  $k$ th SU.

In the case that  $\mathbf{x}$  carries information common to all SUs, the associated downlink transmission is usually called *multicast*, while if  $\mathbf{x}$  carries independent information for different SUs, it is called *unicast*.

Next, consider the ad hoc secondary/CR network, which is usually modeled as an interference channel (IC). For convenience, we assume that for the  $k$ th secondary link,  $k = 1, \dots, K$ , the transmitter is denoted as SU-TX $_k$  and the receiver is denoted as SU-RX $_k$ , although in general a secondary terminal can be both a transmitter and a receiver. The baseband transmission of the IC can be represented as

$$\tilde{\mathbf{y}}_k = \mathbf{H}_{kk}\tilde{\mathbf{x}}_k + \sum_{i=1, i \neq k}^K \mathbf{H}_{ik}\tilde{\mathbf{x}}_i + \tilde{\mathbf{z}}_k, \quad k = 1, \dots, K \quad (3)$$

where  $\tilde{\mathbf{y}}_k \in \mathbb{C}^{B_k \times 1}$  denotes the received signal at SU-RX $_k$ , with  $B_k$  denoting the number of receiving antennas;  $\tilde{\mathbf{x}}_k \in \mathbb{C}^{A_k \times 1}$  denotes the transmitted signal of SU-TX $_k$ , with  $A_k$  denoting the number of transmitting antennas;  $\mathbf{H}_{kk} \in \mathbb{C}^{B_k \times A_k}$  denotes the direct-link channel from SU-TX $_k$  to SU-RX $_k$ , while  $\mathbf{H}_{ik} \in \mathbb{C}^{B_k \times A_i}$  denotes the cross-link channel from SU-TX $_i$  to SU-RX $_k$ ,  $i \neq k$ ; and  $\tilde{\mathbf{z}}_k \in \mathbb{C}^{B_k \times 1}$  denotes the noise at SU-RX $_k$ . It is assumed that  $\tilde{\mathbf{x}}_k$ 's are independent over  $k$ .

Furthermore, we assume that the  $j$ th PU,  $j = 1, \dots, J$ , in each type of the CR networks is equipped with  $D_j$  antennas,  $D_j \geq 1$ . We then use  $\mathbf{G}_{kj} \in \mathbb{C}^{D_j \times N_k}$  to denote the channel from the  $k$ th SU to the  $j$ th PU in the CR MAC,  $\mathbf{F}_j \in \mathbb{C}^{D_j \times M}$  to denote the channel from S-BS to the  $j$ th PU in the CR BC, and  $\mathbf{E}_{kj} \in \mathbb{C}^{D_j \times A_k}$  to denote the channel from SU-TX $_k$  to the  $j$ th PU in the CR IC. Moreover, the receiving terminals in the secondary networks may experience interference from active primary transmitters. For simplicity, we assume that such interference is treated as additional noise at the secondary receivers, and the total noise at each secondary receiving terminal is distributed as a CSCG random vector with zero mean and the identity covariance matrix.

Note that the (spatial) channels defined in the above CR network models are assumed constant for a fixed transmit dimension such as one time-block in a time-division-multiple-access (TDMA) system or one frequency-bin in an orthogonal-frequency-division-multiplexing (OFDM) system. In a wireless environment, these channels usually change over time and/or frequency dimensions as governed by an underlying joint stochastic process. As such, DRA becomes relevant to schedule SUs into different transmit dimensions based on their CSI. In general, the secondary transmitting terminals need to satisfy two types of power constraints for DRA: One is due to their own transmit power budgets; the other is to limit their resulting interference level at each PU to be below a prescribed threshold. These constraints can be applied over each fixed dimension as *peak* power constraints, or over multiple dimensions as *average* power constraints. Without loss of generality, we consider DRA for the secondary network over  $L$  transmit dimensions with different channel realizations, with  $L \geq 1$ . In total, four different types of power constraints can be defined for the secondary network. By taking the CR MAC as an example (similarly as for the CR BC/IC), we have

- **Peak transmit power constraint (PTPC):**

$$\text{Tr}(\mathbf{S}_k[l]) \leq P_k \quad (4)$$

where  $\mathbf{S}_k[l]$  denotes the transmit covariance matrix for the  $l$ th transmit dimension of the  $k$ th SU,  $l \in \{1, \dots, L\}$ ,  $k \in \{1, \dots, K\}$ ;  $P_k$  denotes the  $k$ th SU's peak power constraint that applies to each of the  $L$  transmit dimensions.

- **Average transmit power constraint (ATPC):**

$$\frac{1}{L} \sum_{l=1}^L \text{Tr}(\mathbf{S}_k[l]) \leq \bar{P}_k \quad (5)$$

where  $\bar{P}_k$  denotes the  $k$ th SU's average transmit power constraint over the  $L$  transmit dimensions.

- **Peak interference power constraint (PIPC):**

$$\sum_{k=1}^K \text{Tr}(\mathbf{G}_{kj}[l]\mathbf{S}_k[l]\mathbf{G}_{kj}^H[l]) \leq \Gamma_j \quad (6)$$

where  $\mathbf{G}_{kj}[l]$  denotes the realization of channel  $\mathbf{G}_{kj}$  for a given  $l$ ; and  $\Gamma_j$  denotes the peak interference power constraint for protecting the  $j$ th PU,  $j \in \{1, \dots, J\}$ , which limits the total interference power caused by all the  $K$  SUs across all the receiving antennas of the  $j$ th PU, for each of the  $L$  transmit dimensions.

- **Average interference power constraint (AIPC):**

$$\frac{1}{L} \sum_{l=1}^L \sum_{k=1}^K \text{Tr}(\mathbf{G}_{kj}[l]\mathbf{S}_k[l]\mathbf{G}_{kj}^H[l]) \leq \bar{\Gamma}_j \quad (7)$$

where  $\bar{\Gamma}_j$  denotes the average interference power constraint for the  $j$ th PU to limit the total interference power from the  $K$  SUs, which is averaged over the  $L$  transmit dimensions.

Note that DRA for traditional (non-CR) wireless networks under PTPC and/or ATPC has been thoroughly studied in the literature [12], while the study of DRA subject to PIPC and/or AIPC as well as their various combinations with PTPC/ATPC is unique to CR networks and is relatively new. In order to gain more insights into the optimal DRA designs for CR networks, we will first study the case of a single transmit dimension ( $L = 1$ ) with PTPCs and PIPCs by focusing on the spatial-domain transmit optimization for multi-antenna CRs in Section III, and then study the general case of  $L > 1$  for joint space-time-frequency DRA in CR networks under ATPCs and AIPCs in Section IV.

### III. COGNITIVE BEAMFORMING OPTIMIZATION

In this section, we consider the case of  $L = 1$ , where the DRA for CR networks reduces to the spatial-domain transmit optimization under PTPCs and PIPCs to maximize the CR network throughput. We term this practice as *cognitive beamforming*. In order to investigate the fundamental performance limits of cognitive beamforming, we study the optimal designs with the availability of perfect knowledge on all the channels in the SU networks, and those from all the secondary transmit terminals to PUs. For convenience, we drop the dimension index  $l$  for the rest of this section given  $L = 1$ .



First, it is worth noting that the PIPC given in (6) can be unified with the PTPC given in (4) into a form of *generalized linear transmit covariance constraint* (GLTCC):

$$\sum_{i=1}^K \text{Tr}(\mathbf{W}_i \mathbf{S}_i) \leq w \quad (8)$$

where  $\mathbf{W}_i$ 's and  $w$  are constants. For example, with each PIPC given in (6),  $\mathbf{W}_i = \mathbf{G}_{ij}^H \mathbf{G}_{ij}$ ,  $\forall i$ , and  $w = \Gamma_j$ , while for each PTPC given in (4),  $\mathbf{W}_i = \mathbf{I}$  if  $i = k$  and  $\mathbf{0}$  otherwise, with  $w = P_k$ . Previous studies on transmit optimization for multi-antenna or multiple-input multiple-output (MIMO) systems have mostly adopted some special forms of GLTCC such as the user individual power constraints and sum-power constraint. However, it remains unclear whether such existing solutions are applicable to the general form of GLTCC, which is crucial to the problem of CR MIMO transmit optimization with the newly added PIPCs. In the following, we provide an overview of the state-of-art solutions for this problem under different CR network models, while the developed solutions also apply to the case with the general form of GLTCCs as in (8). From a convex optimization perspective, we next divide our discussions into two parts, which deal with the cases of convex and non-convex problem formulations, respectively.

#### A. Convex Problem Formulation

First, consider the case where the associated optimization problem in a traditional MIMO system without PIPC is *convex*. In such cases, since the extra PIPCs are linear over the SU transmit covariance matrices, the resulting transmit covariance optimization problem for CR systems remains convex; and thus, it can be efficiently solved by standard convex optimization techniques.

**CR Point-to-Point MIMO Channel:** We elaborate this case by first considering the CR point-to-point MIMO channel, which can be treated as the special case with only one active SU link in the MAC, BC, or IC based CR network. Without loss of generality, we will use the notations developed for the CR MAC with  $K = 1$  in the following discussions. Specifically, the optimal transmit covariance to achieve the CR point-to-point MIMO channel capacity under both the PTPC and PIPCs can be obtained from the following problem [13]:

$$\begin{aligned} \text{Max.}_{\mathbf{S}} \quad & \log |\mathbf{I} + \mathbf{H}\mathbf{S}\mathbf{H}^H| & (P1) \\ \text{s. t.} \quad & \text{Tr}(\mathbf{S}) \leq P \\ & \text{Tr}(\mathbf{G}_j \mathbf{S} \mathbf{G}_j^H) \leq \Gamma_j, \quad j = 1, \dots, J \\ & \mathbf{S} \succeq \mathbf{0} \end{aligned}$$

where for conciseness we have removed the SU index  $k$  in the symbol notations since  $K = 1$ , while  $\mathbf{S} \succeq \mathbf{0}$  means that  $\mathbf{S}$  is a positive semi-definite matrix [14].

We see that (P1) is a convex optimization problem since its objective function is concave over  $\mathbf{S}$  and its constraints define a convex set over  $\mathbf{S}$ . Thus, (P1) can be efficiently solved by, e.g., the interior point method [14]. In the special case of CR multiple-input single-output (MISO) channel, i.e.,  $\mathbf{H}$  degrades to a row-vector denoted by  $\mathbf{h} \in \mathbb{C}^{1 \times N}$ , it can be shown by exploiting the Karush-Kuhn-Tucker (KKT)

optimality conditions of (P1) that transmit beamforming is capacity optimal, i.e.,  $\text{Rank}(\mathbf{S}) = 1$  [13]. Thus, without loss of generality we could write  $\mathbf{S} = \mathbf{v}\mathbf{v}^H$ , where  $\mathbf{v} \in \mathbb{C}^{N \times 1}$  denotes the precoding vector. Accordingly, (P1) for the special case of MISO CR channel is simplified as (P1-S) [13]:

$$\begin{aligned} \text{Max.}_{\mathbf{v}} \quad & \|\mathbf{h}\mathbf{v}\| \\ \text{s. t.} \quad & \|\mathbf{v}\|^2 \leq P \\ & \|\mathbf{G}_j \mathbf{v}\|^2 \leq \Gamma_j, \quad j = 1, \dots, J, \end{aligned}$$

which is non-convex due to the non-concavity of its objective function. However, by observing the fact that if  $\mathbf{v}$  is the solution of (P1-S), so is  $e^{j\theta} \mathbf{v}$  for any arbitrary  $\theta$ , we thus assume without loss of generality that  $\mathbf{h}\mathbf{v}$  is a real number and modify (P1-S) by rewriting its objective function as  $\text{Re}(\mathbf{h}\mathbf{v})$  and adding an additional linear constraint  $\text{Im}(\mathbf{h}\mathbf{v}) = 0$ . Thereby, (P1-S) can be converted into a second-order cone programming (SOCP) [14] problem, which is convex and thus can be efficiently solved by available convex optimization softwares [15]. Alternatively, (P1-S) can be shown equivalent to its Lagrange dual problem [13], which is a convex semi-definite programming (SDP) [14] problem and is thus efficiently solvable [15]. For (P1-S) in the case of one single-antenna PU, a closed-form solution for the optimal precoding vector  $\mathbf{v}$  was derived in [13] via a geometric approach.

In order to reveal the structure of the optimal  $\mathbf{S}$  for (P1), we consider its Lagrange dual problem defined as (P1-D):

$$\text{Min.}_{\boldsymbol{\eta} \succeq \mathbf{0}} \quad d(\boldsymbol{\eta})$$

where  $\boldsymbol{\eta} = [\eta_0, \eta_1, \dots, \eta_J]$  denotes a vector of dual variables for (P1) with  $\eta_0$  associated with the PTPC, and  $\eta_j$  associated with the  $j$ th PIPC,  $j = 1, \dots, J$ , while we have the dual function defined as

$$\begin{aligned} d(\boldsymbol{\eta}) \triangleq \max_{\mathbf{S} \succeq \mathbf{0}} \quad & \log |\mathbf{I} + \mathbf{H}\mathbf{S}\mathbf{H}^H| - \eta_0(\text{Tr}(\mathbf{S}) - P) \\ & - \sum_{j=1}^J \eta_j(\text{Tr}(\mathbf{G}_j \mathbf{S} \mathbf{G}_j^H) - \Gamma_j). \end{aligned} \quad (9)$$

Since (P1) is convex with Slater's condition satisfied [14], the duality gap between the optimal values of (P1) and (P1-D) is zero, i.e., (P1) can be solved equivalently as (P1-D). Accordingly, an iterative algorithm can be developed to solve (P1-D) by alternating between solving  $d(\boldsymbol{\eta})$  for a given  $\boldsymbol{\eta}$  and updating  $\boldsymbol{\eta}$  to minimize  $d(\boldsymbol{\eta})$ . At each iteration,  $\boldsymbol{\eta}$  can be updated by a subgradient-based method such as the ellipsoid method [16], according to the subgradients of  $d(\boldsymbol{\eta})$ , which can be shown equal to  $P - \text{Tr}(\mathbf{S}^*)$  and  $\Gamma_j - \text{Tr}(\mathbf{G}_j \mathbf{S}^* \mathbf{G}_j^H)$  for  $\eta_0$  and  $\eta_j, j \neq 0$ , respectively, where  $\mathbf{S}^*$  denotes the optimal  $\mathbf{S}$  to obtain  $d(\boldsymbol{\eta})$  for a given  $\boldsymbol{\eta}$ . From (9), it follows that  $\mathbf{S}^*$  is the solution of the following equivalent problem (by discarding irrelevant constant terms):

$$\max_{\mathbf{S} \succeq \mathbf{0}} \quad \log |\mathbf{I} + \mathbf{H}\mathbf{S}\mathbf{H}^H| - \text{Tr}(\mathbf{T}\mathbf{S}) \quad (10)$$

where  $\mathbf{T} = \eta_0 \mathbf{I} + \sum_{j=1}^J \eta_j (\mathbf{G}_j^H \mathbf{G}_j)$  is a constant matrix for a given  $\boldsymbol{\eta}$ . In order to solve Problem (10), we introduce an

auxiliary variable:  $\hat{\mathbf{S}} = \mathbf{T}^{1/2} \mathbf{S} \mathbf{T}^{1/2}$ . Problem (10) is then re-expressed in terms of  $\hat{\mathbf{S}}$  as

$$\max_{\hat{\mathbf{S}} \succeq \mathbf{0}} \log \left| \mathbf{I} + \mathbf{H} \mathbf{T}^{-1/2} \hat{\mathbf{S}} \mathbf{T}^{-1/2} \mathbf{H}^H \right| - \text{Tr}(\hat{\mathbf{S}}). \quad (11)$$

The above problem can be shown equivalent to the standard point-to-point MIMO channel capacity optimization problem subject to a single sum-power constraint [17], and its solution can be expressed as  $\hat{\mathbf{S}}^* = \mathbf{V} \mathbf{\Sigma} \mathbf{V}^H$ , where  $\mathbf{V}$  is obtained from the singular-value decomposition (SVD) given as follows:  $\mathbf{H} \mathbf{T}^{-1/2} = \mathbf{U} \mathbf{\Theta} \mathbf{V}^H$ , with  $\mathbf{\Theta} = \text{Diag}([\theta_1, \dots, \theta_T])$  and  $T = \min(M, N)$ , while  $\mathbf{\Sigma} = \text{Diag}([\sigma_1, \dots, \sigma_T])$  follows the standard water-filling solution [17]:  $\sigma_i = (1/\ln 2 - 1/\theta_i^2)^+$ ,  $i = 1, \dots, T$ , with  $(\cdot)^+ \triangleq \max(0, \cdot)$ . Thus, the solution of Problem (10) for a given  $\boldsymbol{\eta}$  can be expressed as  $\mathbf{S}^* = \mathbf{T}^{-1/2} \mathbf{V} \mathbf{\Sigma} \mathbf{V}^H \mathbf{T}^{-1/2}$ .

Next, we present a heuristic method for solving (P1), which leads to a suboptimal solution in general and could serve as a benchmark to evaluate the effectiveness of the above two approaches based on convex optimization. To gain some intuitions for this method, we first take a look at two special cases of (P1). For the first case, supposing that all the PIPCs are inactive (e.g., by setting  $\Gamma_j = \infty, \forall j$ ) and thus can be removed, (P1) reduces to the standard MIMO channel capacity optimization problem under the PTPC only, for which the optimal solution of  $\mathbf{S}$  is known to be derivable from the SVD of  $\mathbf{H}$  [17]. For the second case, assuming that  $\Gamma_j = 0, \forall j$ , the solution for (P1) is then obtained by the ‘‘zero-forcing (ZF)’’ algorithm [18], which first projects  $\mathbf{H}$  into the space orthogonal to all  $\mathbf{G}_j$ 's, and then designs the optimal  $\mathbf{S}$  based on the SVD of the projected channel. Note that the (non-trivial) ZF-based solution exists only when  $N > \sum_{j=1}^J D_j$ . From the above two special cases, we observe that as  $\Gamma_j$ 's decrease, the optimal  $\mathbf{S}$  should evolve along with a sequence of subspaces of  $\mathbf{H}$  with decreasing dimensions as a result of keeping certain orthogonality to  $\mathbf{G}_j$ 's, which motivates a new design method for cognitive beamforming, named as *partial channel projection* [13]. Specifically, let  $\bar{\mathbf{G}}_j = \mathbf{G}_j / \Gamma_j, \forall j$ . Then, define  $\bar{\mathbf{G}} \triangleq [\bar{\mathbf{G}}_1^T, \dots, \bar{\mathbf{G}}_J^T]^T$ . Denote the SVD of  $\bar{\mathbf{G}}$  as  $\bar{\mathbf{G}} = \mathbf{U}_G \mathbf{\Lambda}_G \mathbf{V}_G^H$ . Without loss of generality, assume that the singular values in  $\mathbf{\Lambda}_G$  are arranged in a decreasing order. Then, we propose a generalized channel projection operation:

$$\mathbf{H}_\perp = \mathbf{H} \left( \mathbf{I} - \mathbf{V}_G^{(b)} \left( \mathbf{V}_G^{(b)} \right)^H \right) \quad (12)$$

where  $\mathbf{V}_G^{(b)}$  consists of the first  $b$  columns of  $\mathbf{V}_G$  corresponding to the  $b$  largest singular values in  $\mathbf{\Lambda}_G$ ,  $1 \leq b \leq \min(N-1, \sum_{j=1}^J D_j)$ . Note that  $b$  could also take a zero value for which  $\mathbf{V}_G^{(0)} \triangleq \mathbf{0}$ . Now, we are ready to present the transmit covariance matrix for the partial projection method in the form of its eigenvalue decomposition (EVD) as  $\mathbf{S} = \mathbf{V}_\perp \mathbf{\Sigma}_\perp \mathbf{V}_\perp^H$ , where  $\mathbf{V}_\perp$  is obtained from the SVD of the projected channel  $\mathbf{H}_\perp$ , i.e.,  $\mathbf{H}_\perp = \mathbf{U}_\perp \mathbf{\Lambda}_\perp \mathbf{V}_\perp^H$ . By substituting this new form of  $\mathbf{S}$  into (P1), it can be shown that the problem reduces to maximizing the sum-rate of a set of parallel channels (with channel gains given by  $\mathbf{\Lambda}_\perp$ ) over their power allocation  $\mathbf{\Sigma}_\perp$  subject to  $(J+1)$  linear power constraints, for which the optimal power allocation can be obtained by a generalized

‘‘water-filling’’ algorithm [13]. Note that the partial channel projection works for any values of  $N$  and  $D_j$ 's.

In Fig. 2, we plot the achievable rate of a CR MIMO channel under the PTPC and PIPCs with the optimal transmit covariance solution for (P1) via the convex optimization approach, against those with suboptimal covariance solutions via the partial channel projection method with different values of  $b$ . The system parameters are given as follows:  $M = N = 4$ ,  $J = 2$ ,  $D_1 = D_2 = 1$ , and  $\Gamma_1 = \Gamma_2 = 0.1$ . The SU achievable rate is plotted vs. the SU PTPC,  $P$ . It is observed that the optimal covariance solution obtained via the convex optimization approach yields notable rate gains over suboptimal solutions via the heuristic method, for which the optimal value of  $b$  (the number of SU-to-PU channel dimensions to be nulled) to maximize the SU achievable rate increases with the SU PTPC.

**CR MIMO-MAC:** The solutions proposed for the CR point-to-point MIMO channel shed insights to transmit optimization for the CR MIMO-MAC defined in (1) with  $K > 1$ . Assume that in the CR MIMO-MAC, the optimal multiuser detection is deployed at the S-BS to successively decode different SU messages from the received sum-signal. We then consider the problem for jointly optimizing SU transmit covariance matrices to maximize their weighted sum-rate subject to individual PTPCs and joint PIPCs. This problem is referred to as weighted sum-rate maximization (WSRMax). Without loss of generality, we assume that the given user rate weights satisfy that  $\mu_1 \geq \mu_2 \geq \dots \geq \mu_K \geq 0$ ; thus, the optimal decoding order of users at the S-BS to maximize the weighted sum-rate is in accordance with the reverse user index [19]. Accordingly, the WSRMax for the CR MIMO-MAC can be expressed as

$$\begin{aligned} \text{Max.}_{\mathbf{S}_1, \dots, \mathbf{S}_K} \quad & \sum_{k=1}^K \mu_k \log \frac{\left| \mathbf{I} + \sum_{i=1}^k \mathbf{H}_i \mathbf{S}_i \mathbf{H}_i^H \right|}{\left| \mathbf{I} + \sum_{i=1}^{k-1} \mathbf{H}_i \mathbf{S}_i \mathbf{H}_i^H \right|} \quad (P2) \\ \text{s. t.} \quad & \text{Tr}(\mathbf{S}_k) \leq P_k, \quad k = 1, \dots, K \\ & \sum_{k=1}^K \text{Tr}(\mathbf{G}_{kj} \mathbf{S}_k \mathbf{G}_{kj}^H) \leq \Gamma_j, \quad j = 1, \dots, J \\ & \mathbf{S}_k \succeq \mathbf{0}, \quad k = 1, \dots, K. \end{aligned}$$

Reordering terms in the objective function of (P2) yields

$$\begin{aligned} & \sum_{k=1}^{K-1} (\mu_k - \mu_{k+1}) \log \left| \mathbf{I} + \sum_{i=1}^k \mathbf{H}_i \mathbf{S}_i \mathbf{H}_i^H \right| \\ & + \mu_K \log \left| \mathbf{I} + \sum_{i=1}^K \mathbf{H}_i \mathbf{S}_i \mathbf{H}_i^H \right| \quad (13) \end{aligned}$$

From the above new form of the objective function, it can be verified that (P2) is a convex optimization problem over  $\mathbf{S}_k$ 's. Thus, similarly as for (P1), (P2) can be solved by an interior-point-method based algorithm or an iterative algorithm via solving the equivalent Lagrange dual problem, for which the details are omitted here for brevity.

It is noted that (P2) is for the case with the optimal non-linear multiuser decoder at the S-BS, while in practice the low-complexity linear decoder is usually more preferable. The

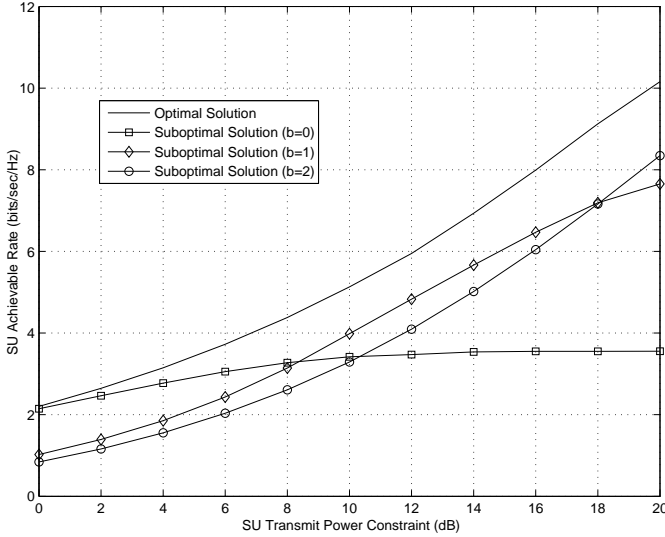


Fig. 2. Comparison of the achievable rates for the CR MIMO channel under the PTPC and PIPCs with the optimal transmit covariance solution for (P1) via the convex optimization approach vs. suboptimal covariance solutions via the partial channel projection method with different values of  $b$ .

use of linear instead of non-linear decoder at the receiver will change the user achievable rates for the CR MIMO-MAC, thus resulting in new problem formulations for transmit optimization. For example, in [20], the authors have considered the CR SIMO-MAC (single-antenna for each SU transmitter) with a linear decoder at the receiver, where the power allocation across the SUs is optimized to maximize their signal-to-interference-plus-noise ratios (SINRs) at the receiver subject to both transmit and interference power constraints.

### B. Non-Convex Problem Formulation

Next, we consider the case where the optimization problems in the associated traditional (non-CR) MIMO systems are *non-convex*. It thus becomes more challenging whether these non-convex problems with the addition of convex PIPCs in the corresponding CR MIMO systems can be efficiently solvable. In the following, we present some promising approaches to solve these problems for the CR MIMO-BC and MIMO-IC.

**CR MIMO-BC:** First, consider the CR MIMO-BC defined in (2) under both the PTPC at the S-BS and  $J$  PIPCs each for one of the  $J$  PUs, which can be similarly defined as for the MAC case in (4) and (6), respectively. We focus on the unicast downlink transmission for the CR BC, while for the case of multicast, the interested readers may refer to [21]. For the purpose of exposition, we consider two commonly adopted design criteria for the traditional multi-antenna Gaussian BC in the literature: One is for the MIMO-BC deploying the non-linear “dirty-paper-coding (DPC)” at the transmitter [22], which maximizes the weighted sum-rate of all the users (i.e., the WSRMax problem); the other is for the MISO-BC (single-antenna for each SU receiver) deploying only linear encoding at the transmitter, which maximizes the minimum SINR among all the users, referred to as “SINR balancing”.

Specifically, the WSRMax problem for the CR MIMO-BC

can be formulated as:

$$\begin{aligned} \text{Max.}_{\mathbf{Q}_1, \dots, \mathbf{Q}_K} \quad & \sum_{k=1}^K \mu_k \log \frac{|\mathbf{I} + \mathbf{H}_k^H \left( \sum_{i=k}^K \mathbf{Q}_i \right) \mathbf{H}_k|}{|\mathbf{I} + \mathbf{H}_k^H \left( \sum_{i=k+1}^K \mathbf{Q}_i \right) \mathbf{H}_k|} \quad (\text{P3}) \\ \text{s. t.} \quad & \sum_{k=1}^K \text{Tr}(\mathbf{Q}_k) \leq P \\ & \text{Tr} \left( \mathbf{F}_j \left( \sum_{k=1}^K \mathbf{Q}_k \right) \mathbf{F}_j^H \right) \leq \Gamma_j, \quad j = 1, \dots, J \\ & \mathbf{Q}_k \succeq \mathbf{0}, \quad k = 1, \dots, K \end{aligned}$$

where  $\mathbf{Q}_k \in \mathbb{C}^{M \times M}$  denotes the covariance matrix for the transmitted signal of S-BS intended for the  $k$ th SU,  $k = 1, \dots, K$ ;  $\mu_k$ 's are the given user rate weights; and  $P$  denotes the transmit power constraint for the S-BS. Without loss of generality, we assume that  $\mu_1 \geq \mu_2 \geq \dots \geq \mu_K \geq 0$ ; thus, in (P3) the optimal encoding order of users for DPC to maximize the weighted sum-rate is in accordance with the user index [22]. Note that (P3) is non-convex with or without the PIPCs due to the fact that the objective function is non-concave over  $\mathbf{Q}_k$ 's for  $K \geq 2$ . As a result, unlike (P1) for the point-to-point CR channel, the standard Lagrange duality method cannot be applied for this problem. For (P3) in the case without the PIPCs, a so-called “BC-MAC duality” relationship was proposed in [23] to transform the non-convex MIMO-BC problem into an equivalent convex MIMO-MAC problem, which is solvable by efficient convex optimization techniques such as the interior point method. In [24], another form of BC-MAC duality, the so-called “mini-max duality” was explored to solve the MIMO-BC problem under a special case of GLTCC: the per-antenna transmit power constraint. However, these existing forms of BC-MAC duality are yet unable to handle the case with arbitrary numbers of GLTCCs, which is the case for (P3) with both the PTPC and PIPCs.

In [25], a general method was proposed to solve various MIMO-BC optimization problems under multiple GLTCCs, thus including the CR MIMO-BC WSRMax problem given in (P3). For this method, the first step is to combine all  $(J+1)$  power constraints in (P3) into a single GLTCC as shown in the following optimization problem:

$$\begin{aligned} \text{Max.}_{\mathbf{Q}_1, \dots, \mathbf{Q}_K} \quad & \sum_{k=1}^K \mu_k \log \frac{|\mathbf{I} + \mathbf{H}_k^H \left( \sum_{i=k}^K \mathbf{Q}_i \right) \mathbf{H}_k|}{|\mathbf{I} + \mathbf{H}_k^H \left( \sum_{i=k+1}^K \mathbf{Q}_i \right) \mathbf{H}_k|} \\ \text{s. t.} \quad & \text{Tr} \left( \mathbf{A} \sum_{k=1}^K \mathbf{Q}_k \right) \leq Q \\ & \mathbf{Q}_k \succeq \mathbf{0}, \quad k = 1, \dots, K \quad (14) \end{aligned}$$

where  $\mathbf{A} = \lambda_0 \mathbf{I} + \sum_{j=1}^J \lambda_j \mathbf{F}_j^H \mathbf{F}_j$ , and  $Q = \lambda_0 P + \sum_{j=1}^J \lambda_j \Gamma_j$  with  $\lambda_0, \lambda_1, \dots, \lambda_J$  being non-negative constants. For a given set of  $\lambda_i$ 's,  $i = 0, \dots, J$ , let the optimal value of the above problem be denoted by  $F(\lambda_0, \lambda_1, \dots, \lambda_J)$ . Clearly,  $F(\lambda_0, \lambda_1, \dots, \lambda_J)$  is an upper bound on the optimal value of (P3) since any feasible solutions for (P3) must satisfy the constraints of Problem (14) for a given set of  $\lambda_i$ 's. Interestingly, it can be shown that the optimal value of (P3)



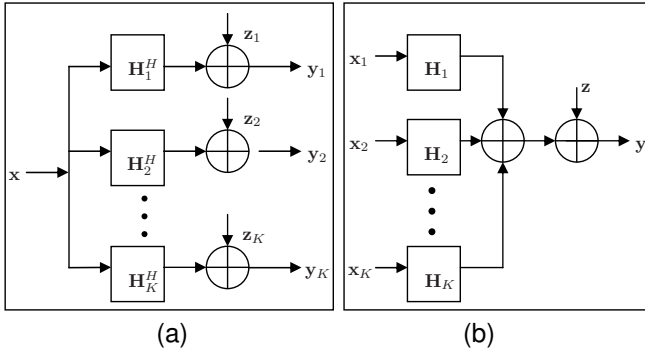


Fig. 3. Generalized MIMO MAC-BC Duality: (a) Primal MIMO-BC channel with downlink channels  $\mathbf{H}_k^H$  and receiver noise vectors  $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ ,  $k = 1, \dots, K$ , and a GLTCC:  $\text{Tr}(\mathbf{A} \sum_{k=1}^K \mathbf{Q}_k) \leq Q$ ; (b) Dual MIMO-MAC with uplink channels  $\mathbf{H}_k$ ,  $k = 1, \dots, K$  and receiver noise vector  $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{A})$ , and a sum-power constraint:  $\sum_{k=1}^K \text{Tr}(\mathbf{S}_k) \leq Q$ . The MIMO-BC and dual MIMO-MAC have the same achievable rate region [25].

is equal to the minimum value of function  $F(\lambda_0, \lambda_1, \dots, \lambda_J)$  over all non-negative  $\lambda_i$ 's [25]. Therefore, (P3) can be resolved by iteratively solving Problem (14) for a given set of  $\lambda_i$ 's and updating  $\lambda_i$ 's towards their optimal values to minimize function  $F(\lambda_0, \lambda_1, \dots, \lambda_J)$ . Specifically,  $\lambda_i$ 's can be updated via the ellipsoid method according to the subgradients of  $F(\lambda_0, \lambda_1, \dots, \lambda_J)$ , which can be shown [25] equal to  $P - \sum_{k=1}^K \text{Tr}(\mathbf{Q}_k^*)$  and  $\Gamma_j - \text{Tr}(\mathbf{F}_j (\sum_{k=1}^K \mathbf{Q}_k^*) \mathbf{F}_j^H)$  for  $\lambda_0$  and  $\lambda_j$  ( $j \neq 0$ ), respectively, where  $\mathbf{Q}_k^*$ 's are the solution of Problem (14) for the given  $\lambda_k$ 's.

Furthermore, Problem (14) with a given set of  $\lambda_k$ 's can be solved by applying the *generalized BC-MAC duality* proposed in [25], which extends the existing forms of BC-MAC duality [23], [24] to transform the MIMO-BC problem subject to a single GLTCC as in Problem (14) to an auxiliary (dual) MIMO-MAC problem subject to a corresponding sum-power constraint. Specifically, it is shown in [25] that the MIMO-BC as in Problem (14) and the dual MIMO-MAC, as depicted in Fig. 3, have the same achievable rate region. Accordingly, the optimal objective value (weighted sum-rate) of Problem (14) for the primal MIMO-BC can be obtained as that of the following equivalent problem for the dual MIMO-MAC:

$$\begin{aligned}
 & \text{Max.}_{\mathbf{S}_1, \dots, \mathbf{S}_K} \sum_{k=1}^{K-1} (\mu_k - \mu_{k+1}) \log \left| \mathbf{A} + \sum_{i=1}^k \mathbf{H}_i \mathbf{S}_i \mathbf{H}_i^H \right| \\
 & \quad + \mu_K \log \left| \mathbf{A} + \sum_{i=1}^K \mathbf{H}_i \mathbf{S}_i \mathbf{H}_i^H \right| \\
 & \text{s. t.} \quad \sum_{k=1}^K \text{Tr}(\mathbf{S}_k) \leq Q \\
 & \quad \mathbf{S}_k \succeq \mathbf{0}, \quad k = 1, \dots, K.
 \end{aligned} \tag{15}$$

Similar to (P2), the above problem is a WSRMax problem for the MIMO-MAC subject to a single sum-power constraint, which is convex and thus can be efficiently solvable by, e.g., the interior point method. After solving Problem (15), the optimal user transmit covariance solutions for the MIMO-MAC,  $\mathbf{S}_k^*$ 's, can be transformed to the corresponding ones for the original MIMO-BC,  $\mathbf{Q}_k^*$ 's, via a MAC-BC covariance transformation algorithm given in [25]. Furthermore, it is

worth noting that with  $K = 1$ , the above method can be shown equivalent to that developed for (P1) in the CR point-to-point MIMO channel case based on the Lagrange duality.

Consider next the SINR balancing problem for the CR MISO-BC, which can be expressed as:

$$\begin{aligned}
 & \text{Max.}_{\alpha, \mathbf{v}_1, \dots, \mathbf{v}_K} \quad \alpha & (\text{P4}) \\
 & \text{s. t.} \quad \frac{\|\mathbf{h}_k^H \mathbf{v}_k\|^2}{1 + \sum_{i \neq k} \|\mathbf{h}_k^H \mathbf{v}_i\|^2} \geq \alpha, \quad k = 1, \dots, K \\
 & \quad \sum_{k=1}^K \|\mathbf{v}_k\|^2 \leq P \\
 & \quad \sum_{k=1}^K \|\mathbf{F}_j \mathbf{v}_k\|^2 \leq \Gamma_j, \quad j = 1, \dots, J
 \end{aligned}$$

where  $\alpha$  denotes an achievable SINR for all the SUs;  $\mathbf{v}_k \in \mathbb{C}^{M \times 1}$  denotes the precoding vector for the transmitted signal of S-BS intended for the  $k$ th SU; and  $\mathbf{h}_k$  represents  $\mathbf{H}_k$  for the MISO-BC case. Similarly as for (P1-S), by treating  $\mathbf{h}_k^H \mathbf{v}_k$  on the left-hand side (LHS) of each SINR constraint in (P4) as a positive real number [26], it can be shown that (P4) for a given  $\alpha$  is equivalent to a SOCP feasibility problem and thus efficiently solvable [15]. For a given  $\alpha$ , if the associated SOCP problem is feasible, we know that the optimal solution of (P4) for  $\alpha$ , denoted by  $\alpha^*$ , must satisfy  $\alpha^* \geq \alpha$ ; otherwise,  $\alpha^* < \alpha$ . Based on this fact,  $\alpha^*$  can be found by a simple bisection search [14]; with  $\alpha^*$ , the corresponding optimal solution for  $\mathbf{v}_k$ 's in (P4) can also be obtained. The above technique has also been applied in [27] for (P4) without the PIPCs.

The SINR balancing problem for the conventional MISO-BC without the PIPCs has also been studied in [28], where an algorithm was proposed using the virtual uplink formulation and a fixed-point iteration. However, this algorithm cannot be extended directly to deal with multiple PIPCs for the case of CR MISO-BC. Similarly as for the previous discussions on the WSRMax problem for the CR MIMO-BC where a generalized MIMO MAC-BC duality holds, a counterpart beamforming duality also holds for the MISO-BC and SIMO-MAC [25]. With this duality result, the SINR balancing problem (P4) for the CR MISO-BC can be converted into an equivalent problem for the dual SIMO-MAC, where the efficient iterative algorithm in [28] can be directly applied. The interested readers may refer to [25] for the details of this method.

**CR MIMO-IC:** Second, consider the CR MIMO-IC given in (3), subject to both the PTPCs for the  $K$  SU-TXs and the PIPCs for the  $J$  PUs, which can be similarly defined as for the MAC case in (4) and (6), respectively. From an information-theoretic perspective, the capacity region for the Gaussian IC under PTPCs, which consists of all the simultaneously achievable rates of all the users, still remains unknown in general even for the case of  $K = 2$  and  $A_k = B_k = 1$ ,  $k = 1, 2$  [29]. A pragmatic approach that leads to suboptimal achievable rates in the Gaussian IC is to restrict the system to operate in a decentralized manner, i.e., allowing only single-user encoding and decoding by treating the co-channel interferences from the other users as additional Gaussian noises. For this approach, transmit optimization for the CR MIMO-IC reduces to finding

a set of optimal transmit covariance matrices for the  $K$  SU links, denoted by  $\mathbf{R}_k \in \mathbb{C}^{A_k \times A_k}$ ,  $k = 1, \dots, K$ , to maximize the secondary network throughput under both the PTPCs and PIPCs. More specifically, the WSRMax problem for the CR MIMO-IC can be expressed as:

$$\begin{aligned} \text{Max.}_{\mathbf{R}_1, \dots, \mathbf{R}_K} \quad & \sum_{k=1}^K \mu_k \log \left| \mathbf{I} + \left( \mathbf{I} + \sum_{i \neq k} \mathbf{H}_{ik} \mathbf{R}_i \mathbf{H}_{ik}^H \right)^{-1} \mathbf{H}_{kk} \mathbf{R}_k \mathbf{H}_{kk}^H \right| \\ \text{s. t.} \quad & \text{Tr}(\mathbf{R}_k) \leq P_k, \quad k = 1, \dots, K \\ & \sum_{k=1}^K \text{Tr}(\mathbf{E}_{kj} \mathbf{R}_k \mathbf{E}_{kj}^H) \leq \Gamma_j, \quad j = 1, \dots, J \\ & \mathbf{R}_k \succeq \mathbf{0}, \quad k = 1, \dots, K \end{aligned} \quad (\text{P5})$$

where  $\mu_k$ 's are the given non-negative user rate weights. We see that (P5) is non-convex with or without the PIPCs due to the fact that the objective function is non-concave over  $\mathbf{R}_k$ 's for  $K > 1$ . As a result, there are no efficient algorithms yet to obtain the globally optimal solution for this problem. For the same problem setup, there have been recent progresses on characterizing the maximum achievable "degrees of freedom (DoF)" for the user sum-rate (i.e.,  $\mu_k = 1, \forall k$ ) [30].

Next, we discuss some feasible solutions for (P5). First, it is worth noting that for (P5) in the case without the PIPCs, a commonly adopted suboptimal approach is to iteratively optimize each user's transmit covariance subject to its individual PTPC with the transmit covariances of all the other users fixed. This decentralized approach has been first proposed in [31], [32] to obtain some local optimal points for (P5) with the PTPCs only, where they differ in that the one in [31] maximizes the user individual rate at each iteration, while the one in [32] maximizes the user weighted sum-rate. It is also noted that a parallel line of works with similar iterative user optimizations has been pursued in the single-antenna but multi-carrier based interference channels such as the wired discrete-multi-tone (DMT) based digital subscriber line (DSL) network [33], and the wireless OFDM based ad hoc network [34]. One important question to answer for such iterative algorithms is under what conditions the algorithm will guarantee to converge to a local optimal point. This problem has been addressed in the contexts of both multi-carrier and multi-antenna based interference channels in, e.g., [35], [36], via game-theoretic approaches.

However, the above iterative approach cannot be applied directly to solve (P5) with both the PIPCs and PTPCs, since each PIPC involves all the user transmit covariances and is thus not separable over the SUs. Thus, a feasible approach for (P5) is to decompose each of the  $J$  PIPCs into a set of interference-power constraints over the  $K$  SU-TXs, i.e., for the  $j$ th PIPC,  $j \in \{1, \dots, J\}$ ,

$$\text{Tr}(\mathbf{E}_{kj} \mathbf{R}_k \mathbf{E}_{kj}^H) \leq \Gamma_j^{(k)}, \quad k = 1, \dots, K \quad (16)$$

where  $\Gamma_j^{(k)}$  is a constant, and all  $\Gamma_j^{(k)}$ 's,  $k = 1, \dots, K$ , satisfy  $\sum_k \Gamma_j^{(k)} \leq \Gamma_j$  such that the  $j$ th PIPC is guaranteed. Then, the iterative algorithm works here, where each SU link independently optimizes  $\mathbf{R}_k$  to maximize its achievable rate

under its PTPC and  $J$  interference-power constraints given by (16), with all other  $\mathbf{R}_i$ 's,  $i \neq k$ , fixed. It is observed that the resulting problem is in the same form of our previously studied (P1) for the CR point-point MIMO channel; thus, similar techniques developed for (P1) can be applied. Note that a suboptimal method for this problem in the same spirit of the partial channel projection method to reduce the design complexity for each SU transmit covariance matrix has also been proposed in [37]. Moreover, it is noted that  $\Gamma_j^{(k)}$ 's,  $j = 1, \dots, J$ ,  $k = 1, \dots, K$ , can be searched over the SUs to further improve their weighted sum-rate.

Alternatively, assuming that a centralized optimization is feasible with the global knowledge of all the channels in the SU network, as well as those from different SU-TXs to all PUs, another heuristic algorithm for (P5) was proposed in [38]. By rewriting the SU transmit covariance matrices into their equivalent precoding vectors and power allocation vectors, this algorithm iteratively updates the SU transmit precoding vectors (based on the "network duality" [39]) or the power allocation vectors (by solving geometric programming (GP) problems [40]), with the others being fixed.

It is worth pointing out that there are other problem formulations different from (P5) to address the transmit optimization for the CR MIMO-IC. In [41], a new criterion was proposed to design the SU link transmission in a CR MISO-IC via an alternative decentralized approach, where each SU-TX independently designs its transmit precoding vector to maximize the ratio between the received signal power at the desired SU-RX and the resulted total interference power at all the PUs, in order to regulate the interference powers at PUs. Moreover, the above discussions are all based on the assumption that each SU-RX treats the interferences from all the other SU links as additional noises, which is of practical interest since it simplifies the receiver design for each SU link. However, due to independent cross-link channels between SU terminals, it may be possible that a SU-RX could occasionally observe "strong" interference signals from some co-existing SU-TXs and thus be able to decode their messages via multiuser detection techniques and then cancel the associated interferences. With such "opportunistic" multiuser detection at each SU-RX, the achievable rate of each SU link becomes a function of not only its own transmit covariance, but also those of the other SUs as well as their instantaneous transmit rates. Thus, the corresponding transmit optimization for the CR MIMO-IC leads to new and more challenging problem formulations than (P5); the interested readers may refer to [42], [43].

#### IV. JOINT SPACE-TIME-FREQUENCY DRA OPTIMIZATION

In the previous section, we have studied DRA for different CR networks at a single transmit dimension in time/frequency, by focusing on spatial-domain transmit optimization under the *peak* transmit and interference power constraints (PTPC and PIPC). In this section, we bring the additional time and/or frequency dimensions into the DRA problem formulations, by applying the *average* transmit and interference power constraints (ATPC and AIPC) in CR networks. Consider the DRA over  $L$  time/frequency dimensions, for which all the required



channel knowledge is assumed to be known. Taking the CR MAC as an example (similar arguments can be developed for the CR BC/IC), under both the ATPCs and AIPCs given in (5) and (7), respectively, a generic problem formulation for DRA optimization can be formulated as:

$$\begin{aligned} \text{Max.} \quad & C(\{\mathbf{S}_k[l]\}) \\ \text{s. t.} \quad & (5), (7) \end{aligned} \quad (\text{P6})$$

where  $\{\mathbf{S}_k[l]\}$  denotes the set of  $\mathbf{S}_k[l]$ 's,  $k = 1, \dots, K$ , and  $l = 1, \dots, L$ , while  $C(\cdot)$  is an arbitrary utility function to measure the CR network performance. We assume that  $C(\cdot)$  is separable over  $l$ 's, i.e.,  $C(\{\mathbf{S}_k[l]\}) = \frac{1}{L} \sum_{l=1}^L U_l(\mathbf{S}_1[l], \dots, \mathbf{S}_K[l])$  with  $U_l(\cdot)$ 's denoting individual utility functions. Since both the ATPC and AIPC involve  $L$  transmit covariance matrices, the *Lagrange dual decomposition* (see, e.g., a tutorial paper [44]) is a general method to deal with this type of average constraints for optimization over a number of parallel dimensions, which is explained as follows. By introducing a set of dual variables,  $\nu_k$ 's, each for one of the  $K$  ATPCs, and  $\delta_k$ 's, each for one of the  $J$  AIPCs, the Lagrange dual problem of (P6) can be written as (P6-D):

$$\text{Min.} \quad d(\boldsymbol{\nu}, \boldsymbol{\delta})$$

$$\boldsymbol{\nu} \succeq \mathbf{0}, \boldsymbol{\delta} \succeq \mathbf{0}$$

with  $\boldsymbol{\nu} = [\nu_1, \dots, \nu_K]$ ,  $\boldsymbol{\delta} = [\delta_1, \dots, \delta_J]$ , and the dual function

$$\begin{aligned} d(\boldsymbol{\nu}, \boldsymbol{\delta}) \triangleq & \max_{\mathbf{S}_k[l] \succeq \mathbf{0}, \forall k, l} C(\{\mathbf{S}_k[l]\}) - \sum_{k=1}^K \nu_k \left( \frac{1}{L} \sum_{l=1}^L \text{Tr}(\mathbf{S}_k[l]) \right. \\ & \left. - \bar{P}_k \right) - \sum_{j=1}^J \delta_j \left( \frac{1}{L} \sum_{l=1}^L \sum_{k=1}^K \text{Tr}(\mathbf{G}_{kj}[l] \mathbf{S}_k[l] \mathbf{G}_{kj}^H[l]) - \bar{\Gamma}_j \right). \end{aligned} \quad (17)$$

Since the dual problem (P6-D) is convex regardless of the convexity of the primal problem (P6) [14], (P6-D) can be efficiently solved by the ellipsoid method according to the subgradients of the dual function  $d(\boldsymbol{\nu}, \boldsymbol{\delta})$ , similarly as in our previous discussions, provided that the maximization problem in (17) is solvable for any given set of  $\boldsymbol{\nu}$  and  $\boldsymbol{\delta}$ . It is interesting to observe that this maximization problem can be decomposed into  $L$  parallel subproblems each for one of the  $L$  dimensions, and all of these subproblems have the same structure and are thus solvable by the same algorithm, a practice known as ‘‘dual decomposition’’. Without loss of generality, we drop the dimension index  $l$  and express each subproblem as

$$\max_{\mathbf{S}_k \succeq \mathbf{0}, \forall k} U(\mathbf{S}_1, \dots, \mathbf{S}_K) - \sum_{k=1}^K \text{Tr}(\mathbf{B}_k(\nu_k, \boldsymbol{\delta}) \mathbf{S}_k) \quad (18)$$

where  $\mathbf{B}_k(\nu_k, \boldsymbol{\delta}) = \nu_k \mathbf{I} + \sum_{j=1}^J (\delta_j \mathbf{G}_{kj}^H \mathbf{G}_{kj})$  is a constant matrix for the given  $\nu_k$  and  $\boldsymbol{\delta}$ ,  $k = 1 \dots, K$ .

We then discuss the following two cases. For the first case, consider that  $U_l(\cdot)$  is a concave function over  $\mathbf{S}_k[l]$ 's,  $\forall l$  (e.g., the point-to-point CR channel capacity in (P1), or the weighted sum-rate for the CR MIMO-MAC in (P2)). Then, (P6) is convex and thus the duality gap between the optimal values of (P6) and (P6-D) is zero, i.e., (P6) and (P6-D) are

equivalent problems. Furthermore, each subproblem in (18) is also convex. Thus, the dual decomposition method solves (P6) via its dual problem (P6-D), which is decomposable into  $L$  convex subproblems. For the second case, as a counterpart, consider that  $U_l(\cdot)$  is non-concave over  $\mathbf{S}_k[l]$ 's (e.g., the weighted sum-rate for the CR MIMO-BC/MIMO-IC in (P3)/(P5)). As a result, (P6) is non-convex and the duality gap between (P6) and (P6-D) may not be zero. Furthermore, the subproblem (18) is also non-convex. For this case, even when the optimal solutions of the  $L$  subproblems are obtainable, the optimal value of (P6-D) in general only serves as an upper bound on that of (P6). However, in [45] it is pointed out that if a set of so-called ‘‘time-sharing’’ conditions are satisfied by a non-convex optimization problem, the duality gap for this problem and its dual problem is indeed zero. Furthermore, for the class of DRA problems in the form of (P6), the associated time-sharing conditions are usually satisfied asymptotically as  $L \rightarrow \infty$  under some cautious considerations on the continuity of channel distributions [46]. Therefore, the dual decomposition method could still be applied to solve (P6) in the non-convex case for sufficiently large values of  $L$ , provided that the optimal solutions for the subproblems in (18) are obtainable (e.g., a variation of (P3) for the CR MIMO-BC). However, with finite values of  $L$ , how to efficiently solve (P6) in the case of non-concave objective functions is still open.

With the above discussions on the general approaches to design joint space-time-frequency DRA for CR networks, we next present some examples of unique interests to CR systems.

#### A. TDMA/FDMA Constrained DRA: When Is It Optimal?

Time-/frequency-division multiple-access (TDMA/FDMA), which schedules only one user for transmission at each time/frequency dimension, is usually preferable in practice due to their implementation ease. For the TDMA/FDMA based CR MAC (similar arguments hold for the CR BC/IC), the optimal DRA over  $L$  transmit dimensions to maximize the sum-capacity of the SUs can be formulated as (P6) with properly chosen functions for  $U_l(\cdot)$ 's, where for any given  $l$ ,  $U_l(\cdot)$  is expressed as ( $l$  is dropped for conciseness)

$$U(\mathbf{S}_1, \dots, \mathbf{S}_K) = \begin{cases} \log |\mathbf{I} + \mathbf{H}_k \mathbf{S}_k \mathbf{H}_k^H| & \mathbf{S}_i = \mathbf{0}, \forall i \neq k \\ 0 & \text{otherwise.} \end{cases} \quad (19)$$

Note that  $U(\cdot)$  defined above implies the TDMA/FDMA constraint, i.e., only scheduling one user for transmission at a given dimension with a positive contribution to the sum-capacity. However, it can be shown that  $U(\cdot)$  is non-concave over  $\mathbf{S}_k$ 's in this case and as a result, the corresponding (P6) is non-convex. Nevertheless, according to our previous discussions, since the time-sharing conditions hold approximately when  $L \rightarrow \infty$ , the dual decomposition method can be applied to solve (P6) for this case with very large values of  $L$ , where the optimal solution of the associated subproblem at each dimension given in (18) can be obtained by finding the SU (selected for transmission) with the largest objective value of the following problem (which is of the same form as Problem

(10) and thus solvable in a similar way):

$$\max_{\mathbf{S}_k \succeq \mathbf{0}} \log |\mathbf{I} + \mathbf{H}_k \mathbf{S}_k \mathbf{H}_k^H| - \text{Tr}(\mathbf{B}_k(\nu_k, \delta) \mathbf{S}_k). \quad (20)$$

An important question to investigate for TDMA/FDMA based DRA is how much the performance is degraded as compared with the optimal DRA that allows more than one users to transmit at a given dimension. From an information-theoretic viewpoint, it is thus pertinent to investigate the conditions for the optimality of TDMA/FDMA, i.e., when they are optimal to achieve the system sum-capacity. For the traditional single-antenna fading MAC under the user ATPCs over time, it has been shown in [47] that TDMA is optimal for achieving the ergodic/long-term sum-capacity. This result has been shown to hold for the fading CR MAC and CR BC under both the ATPCs and AIPCs in [48], where by exploiting the KKT optimality conditions of the associated optimization problems, the optimality conditions for TDMA in other cases of combined peak/average transmit/interference power constraints have been characterized. For the traditional single-antenna IC with interference treated as noise, the optimality of TDMA/FDMA for the sum capacity has been investigated under the ATPCs in [49], [50]. It would be interesting to extend these results to the case of CR IC under the additional PIPCs and/or AIPCs.

### B. Peak vs. Average Interference Power Constraints: A New Interference Diversity

From a SU's perspective, it is obvious that the ATPC/AIPC is more flexible than the PTPC/PIPC for DRA under the same power threshold and thus results in a larger SU link capacity. However, from a PU's perspective, it remains unknown whether the AIPC or PIPC causes more PU link performance degradation. Intuitively speaking, the PIPC should be more favorable than the AIPC since the former limits the interference power at the PU to be below certain threshold at each time/frequency dimension, while the latter results in variations of interference power levels over different dimensions although their average level is kept below the same threshold as that for the PIPC.

Somehow surprisingly, in [51] it is shown that for the single-antenna PU fading channel subject to the interference from a SU transmitter, the AIPC is in fact better than its PIPC counterpart under the same average power threshold in terms of minimizing the PU capacity losses, which holds for the cases of both ergodic and outage capacities of the PU channel, with/without power control. To illustrate this result, we consider for simplicity the case without the PU link power control, i.e., the PU transmits with a constant power,  $Q$ , over all the fading states. Suppose that the PU link channel power gain is denoted by  $h_p$ , and that from the SU transmitter to the PU receiver denoted by  $h_{sp}$ . Next, consider the following two cases, where the interference power from the SU transmitter at the PU receiver, denoted by  $I_{sp} = h_{sp} p_s$ , with  $p_s$  denoting the SU transmit power, is fixed over all the fading states in Case I (corresponding to the case of PIPC), and is allowed to be variable in Case II (corresponding to the case of AIPC). For both cases, a constant interference power threshold  $\Gamma$  is set

and is assumed to hold with equality, i.e., for Case I,  $I_{sp}^{(I)} = \Gamma$ , for all the fading states, while for Case II,  $E(I_{sp}^{(II)}) = \Gamma$ . Taking the PU ergodic capacity as an example, which can be expressed as (assuming unit-power receiver Gaussian noise):

$$C_p = E \left( \log \left( 1 + \frac{h_p Q}{1 + I_{sp}} \right) \right). \quad (21)$$

Let  $C_p^{(I)}$  and  $C_p^{(II)}$  denote the values of  $C_p$  in Cases I and II, respectively. The following equalities/inequalities then hold

$$\begin{aligned} C_p^{(I)} &= E_{h_p} \left( \log \left( 1 + \frac{h_p Q}{1 + \Gamma} \right) \right) \\ &= E_{h_p} \left( \log \left( 1 + \frac{h_p Q}{1 + E(I_{sp}^{(II)})} \right) \right) \\ &\leq E_{h_p} \left( E_{I_{sp}} \left( \log \left( 1 + \frac{h_p Q}{1 + I_{sp}^{(II)}} \right) \right) \right) \\ &= C_p^{(II)} \end{aligned} \quad (22)$$

where (22) is due to the Jensen's inequality (see, e.g., [17]) and the convexity of the function  $f(x) = \log \left( 1 + \frac{\kappa}{1+x} \right)$  where  $\kappa$  is any positive constant and  $x \geq 0$ . Thus, it follows that given the same average power of the interference,  $\Gamma$ , it is desirable for the PU to have the instantaneous interference power  $I_{sp}$  fluctuate over fading states (Case II) rather than stay constant (Case I), to achieve a larger ergodic capacity.

In general, the results in [51] reveal a new *interference diversity* phenomenon for SS-based CR networks, i.e., the randomized interference powers from the secondary network can be more advantageous over deterministic ones across different transmit dimensions over space, time, or frequency for minimizing the resulted primary network capacity losses. Further investigations are required on interference diversity driven DRA for CR or other spectrum sharing systems.

### C. Beyond Interference Temperature: Exploiting Primary Link Performance Margins

So far, we have studied DRA for CR networks based on the IT constraints for protecting the PU transmissions. Given that the IT constraints in general conservatively lead to an upper bound on the PU capacity loss due to the interference from the SUs [13], [52], it would be possible to improve the spectrum sharing capacities for both the SUs and PUs over the IT-based methods if additional cognition on the PU transmissions is available at the CR transmitters. For example, by exploiting CSI of the PU links, the CRs could allocate transmit/interference powers more flexibly over the dimensions where the PU channels exhibit poor conditions, without degrading too much the PU link performances. These PU "null" dimensions could come up in time, frequency, or space. Thus, the IT constraints could be replaced by the more relevant *primary link performance margin constraints* [52], [53] for the design of DRA in CR networks, in order to optimally exploit the available primary link performance margins to accommodate the interference from the SUs. Following this new paradigm, many new and challenging DRA problems can be formulated for CR networks. As an example, consider the

same setup with a pair of single-antenna PU and SU links over fading channels as in the previous subsection. Instead of applying the conventional AIPC:  $E(h_{sp}p_s) \leq \Gamma$ , over the SU power allocation, we may apply the following PU ergodic capacity constraint [52]

$$E\left(1 + \frac{h_p Q}{1 + h_{sp} p_s}\right) \geq \bar{C}_p \quad (23)$$

where  $\bar{C}_p$  is a given threshold for the minimum PU ergodic capacity. Note that the new constraint in (23) is more directly related to the PU transmission than the conventional AIPC. However, it can be verified that the constraint in (23) is non-convex over  $p_s$  in general, thus resulting in more challenging SU power allocation problems than that with the convex AIPC. The optimal power allocation rules for the SU link subject to the AIPC vs. the newly introduced PU ergodic capacity constraint given by (23) are compared in [52], where it is shown that the new constraint achieves notable rate improvements for both the PU and SU links over the conventional AIPC.

## V. CONCLUSIONS AND DIRECTIONS FOR FUTURE WORK

Dynamic resource allocation (DRA) has become an essential building block in CR networks to exploit various cognitions over both the primary and secondary networks for CR transmit optimization subject to certain required primary protection. In this article, we have presented an extensive list of new, challenging, and unique problems for designing the optimal DRA in CR networks, and demonstrated the key role of various convex optimization techniques in solving the associated design problems. In addition to those open issues as highlighted in our previous discussions, other promising areas of practical and theoretical interests are discussed as follows, which open an avenue for future work.

**Robust Cognitive Beamforming:** In our previous discussions on cognitive beamforming, we have observed that the knowledge of channels from each secondary transmitting terminal to all PUs is essential to the design optimization. However, since the primary and secondary networks usually belong to different operators, it is difficult for the PUs to feed back the required CSI to the CRs. As a result, the SU usually needs to rely on its own observations over the received signals from the primary terminals to extract the required CSI [54]. Nevertheless, the estimated CSI on the SU-to-PU channels may contain errors, which should be taken into account for the design of practical CR systems. This motivates a new and challenging research direction on robust designs for cognitive beamforming to cope with imperfect CSI [55], [56]. More investigations on the robust cognitive beamforming designs for more general CR networks and CSI uncertainty models are appealing.

**Active Interference-Temperature Control:** In this article, we have focused on the design of CR networks subject to the given interference-power constraints for protecting the PUs. We have also discussed some promising rules on how to optimally set the IT constraints in the CR network to achieve the best spectrum sharing throughput. These results lead to a new and universal design paradigm for interference management in CR or other related multiuser communication systems [57], [58],

via appropriately setting the IT levels across the coexisting links. The active IT control approach to interference management for multiuser communication systems is relatively new, and more research endeavors are required along this direction.

## REFERENCES

- [1] J. Mitola and G. Q. Maguire, "Cognitive radio: making software radios more personal," *IEEE Pers. Commun.*, vol. 6, no. 4, pp. 13-18, Aug. 1999.
- [2] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Sig. Proces. Mag.*, vol. 24, no. 3, pp. 79-89, May 2007.
- [3] A. Goldsmith, S. A. Jafar, I. Marić, and S. Srinivasay, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894-914, May, 2009.
- [4] Z. Quan, S. Cui, and A. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE J. Sel. Topics Sig. Proces.*, vol. 2, no. 1, pp. 28-40, Feb. 2008.
- [5] Y.-C. Liang, Y. Zeng, E. C. Y. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1326-1337, Apr. 2008.
- [6] B. H. Juang, Y. Li, and J. Ma, "Signal processing in cognitive radio," *Proc. IEEE*, vol. 97, no. 5, pp. 805-823, May 2009.
- [7] Y. H. Zeng, Y.-C. Liang, A. T. Hoang, and R. Zhang, "A review on spectrum sensing for cognitive radio: challenges and solutions," *EURASIP J. Advances in Sig. Proces.*, Article ID 381465, 2010.
- [8] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 3, pp. 388-404, Mar. 2000.
- [9] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 1813-1827, May 2006.
- [10] M. Gastpar, "On capacity under receive and spatial spectrum-sharing constraints," *IEEE Trans. Inf. Theory*, vol. 53, no. 2, pp. 471-487, Feb. 2007.
- [11] Z.-Q. Luo and W. Yu, "An introduction to convex optimization for communications and signal processing," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1426-1438, Aug. 2006.
- [12] E. Biglieri, J. Proakis, and S. Shamai (Shitz), "Fading channels: information-theoretic and communications aspects," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2619-2692, Oct. 1998.
- [13] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. S. Topics Sig. Proces.*, vol. 2, no. 1, pp. 88-102, Feb. 2008.
- [14] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.
- [15] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming," available [online] at <http://stanford.edu/~boyd/cvx>.
- [16] R. G. Bland, D. Goldfarb, and M. J. Todd, "The ellipsoid method: a survey," *Operations Research*, vol. 29, no. 6, pp. 1039-1091, 1981.
- [17] T. Cover and J. Thomas, *Elements of information theory*, New York: Wiley, 1991.
- [18] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. Sig. Proces.*, vol. 52, no. 2, pp. 461-471, Feb. 2004.
- [19] D. Tse and S. Hanly, "Multi-access fading channels-Part I: polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Trans. Inf. Theory*, vol. 44, no. 7, pp. 2796-2815, Nov. 1998.
- [20] L. Zhang, Y.-C. Liang, and Y. Xin, "Joint beamforming and power control for multiple access channels in cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol.26, no.1, pp.38-51, Jan. 2008.
- [21] K. Phan, S. Vorobyov, N. Sidiropoulos, and C. Tellambura, "Spectrum sharing in wireless networks via QoS-aware secondary multicast beamforming," *IEEE Trans. Sig. Proces.*, vol. 57, no. 6, pp. 2323-2335, June 2009.
- [22] H. Weingarten, Y. Steinberg, and S. Shamai (Shitz), "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," *IEEE Trans. Inf. Theory*, vol. 52, no. 9, pp. 3936-3964, Sep. 2006.
- [23] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2658-2668, Oct. 2003.
- [24] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Sig. Proces.*, vol. 55, no. 6, pp. 2646-2660, June 2007.
- [25] L. Zhang, R. Zhang, Y.-C. Liang, Y. Xin, and H. V. Poor, "On the Gaussian MIMO BC-MAC duality with multiple transmit covariance constraints," submitted to *IEEE Trans. Inf. Theory*. Available [online] at arXiv:0809.4101.



- [26] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*, CRC Press, 2001.
- [27] A. Wiesel, Y. C. Eldar, and S. Shamai (Shitz), "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Sig. Process.*, vol. 54, no. 1, pp. 161-176, Jan. 2006.
- [28] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 18-28, Jan. 2004.
- [29] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49-60, Jan. 1981.
- [30] V. R. Cadambe and S. A. Jafar, "Interference alignment and the degrees of freedom for the K user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425-3441, Aug. 2008.
- [31] M. F. Demirkol and M. A. Ingram, "Power-controlled capacity for interfering MIMO links," in *Proc. IEEE Vehicular Tech. Conf. (VTC)*, vol. 1 pp. 187-191, 2001.
- [32] S. Ye and R. S. Blum, "Optimized signaling for MIMO interference systems with feedback," *IEEE Trans. Sig. Process.*, vol. 51, pp. 2839-2848, Nov. 2003.
- [33] W. Yu, G. Ginis, and J. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE J. Sel. Areas Commun.*, vol. 20, no.5, pp. 1105-1115, June 2002.
- [34] J. Huang, R. Berry, and M. L. Honig, "Distributed interference compensation in wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 5, pp. 1074-1084, May 2006.
- [35] Z.-Q. Luo and J.-S. Pang, "Analysis of iterative waterfilling algorithm for multiuser power control in digital subscriber line," *EURASIP J. Appl. Sig. Process.*, Article ID 24012, 2006.
- [36] G. Scutari, D. P. Palomar, and S. Barbarossa, "Competitive design of multiuser MIMO systems based on game theory: a unified view," *IEEE J. Sel. Areas Commun.* vol. 25, no. 7, pp. 1089-1103, Sep. 2008.
- [37] G. Scutari, D. P. Palomar, and S. Barbarossa, "Cognitive MIMO radio," *IEEE Sig. Proces. Mag.*, vol. 25, no. 6, Nov. 2008.
- [38] S. J. Kim and G. B. Giannakis, "Optimal resource allocation for MIMO ad hoc cognitive radio networks," in *Proc. Annual Allerton Conf. Commun. Control Comput.*, pp. 39-45, Sep. 2008.
- [39] B. Song, R. Cruz, and B. Rao, "Network duality for multiuser mimo beamforming networks and applications," *IEEE Trans. Commun.*, vol. 55, no. 3, pp. 618-630, Mar. 2007.
- [40] M. Chiang, C. W. Tan, D. P. Palomar, D. Neill, and D. Julian, "Power control by geometric programming," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2640-2651, July 2007.
- [41] S. Yiu, M. Vu, and V. Tarokh, "Interference reduction by beamforming in cognitive networks," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, pp. 1-6, Dec. 2008.
- [42] A. Tajer, N. Prasad, and X. Wang, "Beamforming and rate allocation in MISO cognitive radio networks," *IEEE Trans. Sig. Proces.*, vol. 58, no. 1, pp. 362-377, Jan. 2010.
- [43] R. Zhang and J. Cioffi, "Iterative spectrum shaping with opportunistic multiuser detection," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, June 2009.
- [44] D. P. Palomar and M. Chiang, "A tutorial on decomposition methods for network utility maximization," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1439-1451, Aug. 2006.
- [45] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310-1322, July 2006.
- [46] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: complexity and duality," *IEEE J. S. Topics Sig. Proces.*, vol. 2, no. 1, pp. 57-73, Feb. 2008.
- [47] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multi-user communications," in *Proc. IEEE Int. Conf. Comm. (ICC)*, pp. 331-335, 1995.
- [48] R. Zhang, S. Cui, and Y.-C. Liang, "On ergodic sum capacity of fading cognitive multiple-access and broadcast channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 11, pp. 5161-5178, Nov. 2009.
- [49] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE J. Sel. Areas in Commun.*, vol. 25, no. 3, pp. 517- 528, Apr. 2007.
- [50] S. Hayashi and Z.-Q. Luo, "Spectrum management for interference-limited multiuser communication systems," *IEEE Trans. Inf. Theory*, vol. 55, no. 3, pp. 1153-1175, Mar. 2009.
- [51] R. Zhang, "On peak versus average interference power constraints for protecting primary users in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2112-2120, Apr. 2009.
- [52] R. Zhang, "Optimal power control over fading cognitive radio channels by exploiting primary user CSI," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Nov. 2008.
- [53] X. Kang, R. Zhang, Y.-C. Liang, and H. K. Garg, "Optimal power allocation for cognitive radio under primary user outage capacity constraint," in *Proc. IEEE Int. Conf. Commun. (ICC)*, June 2009.
- [54] R. Zhang, F. Gao, and Y.-C. Liang, "Cognitive beamforming made practical: effective interference channel and learning-throughput tradeoff," to appear in *IEEE Trans. Commun.*
- [55] L. Zhang, Y.-C. Liang, Y. Xin, and H. V. Poor, "Robust cognitive beamforming with partial channel state information," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4143-4153, Aug. 2009.
- [56] G. Zheng, K.-K. Wong, and B. Ottersten, "Robust cognitive beamforming with bounded channel uncertainties," to appear in *IEEE Trans. Sig. Proces.*
- [57] L. Zhang, R. Zhang, Y.-C. Liang, Y. Xin, and S. Cui, "On the relationship between the multi-antenna secrecy communications and cognitive radio communications," in *Proc. Annual Allerton Conf. Commun. Control and Comput.*, 2009.
- [58] R. Zhang and S. Cui, "Cooperative interference management in multi-cell downlink beamforming," submitted to *IEEE Trans. Sig. Proces.*. Available [online] at arXiv: 0910.2771.

## AUTHORS:

*Rui Zhang* (rzhang@i2r.a-star.edu.sg) received the B.Eng. and M.Eng. degrees in electrical and computer engineering from National University of Singapore in 2000 and 2001, respectively, and the Ph.D. degree in electrical engineering from Stanford University, California, USA, in 2007. He is now a Senior Research Fellow with the Institute for Infocomm Research (I2R), Singapore. He also holds an Assistant Professorship position in electrical and computer engineering with National University of Singapore. He has authored/co-authored more than 100 refereed international journal and conference papers. His current research interests include cognitive radios, cooperative communications, and multiuser MIMO systems.

*Ying-Chang Liang* (ycliang@i2r.a-star.edu.sg) is presently a Senior Scientist in the Institute for Infocomm Research (I2R), Singapore. He also holds adjunct associate professorship positions in Nanyang Technological University (NTU) and National University of Singapore (NUS). He has been teaching graduate courses in NUS since 2004 and was a visiting scholar with the Department of Electrical Engineering, Stanford University, CA, USA, from Dec 2002 to Dec 2003. His research interest includes cognitive radio, dynamic spectrum access, reconfigurable signal processing for broadband communications, space-time wireless communications, wireless networking, information theory and statistical signal processing.

*Shuguang Cui* (cui@tamu.edu) received Ph.D in Electrical Engineering from Stanford University in 2005, M.Eng in Electrical Engineering from McMaster University, Canada, in 2000, and B.Eng. in Radio Engineering with the highest distinction from Beijing University of Posts and Telecommunications, China, in 1997. He is now working as an assistant professor in Electrical and Computer Engineering at the Texas A&M University, College Station, TX. His current research interests include cross-layer optimization for resource-constrained networks and network information theory. He has been serving as the associate editors for the IEEE Communication Letters and IEEE Transactions on Vehicular Technology, and the elected member for IEEE Signal Processing Society SPCOM Technical Committee.