Maximum Correlation of Binary Signals over Fading Channels

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***Abstract*— Multiple access interference (MAI) in CDMA system is related to the correlation of the transmitted signals from multiple users. In this work, we investigate the correlations of binary signal sets over fading channels, particularly the Gold and Kasami signal sets. A lower bound on the maximum correlation with fading is established. Moreover, an asymptotic bound is determined by the length of the sequence and the first and second order statistics of the independent fading distribution, while an approximation to the maximum correlation is obtained for correlative fading channel. To analyze the fading effect on cross correlation, experimental results show that the distribution of cross correlation in slow fading environment is similar to the one without fading.**

I. Introduction and Preliminaries

Code Division Multiple Access (CDMA) allows multiple users to access the same channel simultaneous by assigning a distinct signature sequence to each user. Since the structure of CDMA receivers is based on matched filter design, the output of the detector is the correlation between the desired signal and input signal. *Multiple access interference (MAI)* is described as the sum of correlations of the desired signal and any interfering components in the input signal. Therefore, it is desirable to have small MAI for reliable signal detection when there are many users in a the system.

MAI is upper bounded by the sum of maximum correlations between any pair of transmitted signals in the system. Therefore, deriving lower bounds on the maximum correlation gives an idea of the minimum interference experienced by the users. Welch established lower bounds on maximum cross correlation of complex signals without fading channels [1]. In this work, we study the fading effect on correlation of signals.

A. Binary Signal Sets and Correlation

Let $\underline{\mathbf{s}}^{(j)} = (s_0^{(j)}, s_1^{(j)}, \cdots, s_{L-1}^{j})$, $0 \le j < M$, be M shift-distinct binary sequences of period L where $s_i^{(j)} \in \{0,1\}$. Let $\mathbb{S} = \{\underline{\mathbf{s}}^{(0)}, \underline{\mathbf{s}}^{(1)}, \cdots, \underline{\mathbf{s}}^{(M-1)}\}$ [2],

$$C_{\underline{\mathbf{s}}^{(j)},\underline{\mathbf{s}}^{(k)}}(\tau) = \sum_{i=0}^{L-1} (-1)^{s_{i+\tau}^{(j)} + s_i^{(k)}}, \quad \tau = 0, 1, \cdots$$

$$\delta = \max |C_{\underline{\mathbf{s}}^{(j)},\underline{\mathbf{s}}^{(k)}}(\tau)|, \quad 0 \le \tau < L, \quad 0 \le j,k < M$$

where $\tau \neq 0$ if $j = k$. $C_{\underline{\mathbf{s}}^{(j)},\underline{\mathbf{s}}^{(k)}}(\tau)$ is the correlation between sequence $\underline{\mathbf{s}}^{(j)}$ and $\underline{\mathbf{s}}^{(k)}$ at shift τ. The set $\mathbb{S}$ is said to be a (L, M, δ) signal set and δ is referred to as the maximum correlation of $\mathbb{S}$. A sequence in $\mathbb{S}$ is also called a signal.

B. Gold-pair Sequences

Let $\underline{\mathbf{m}} = \{m_i\}$ be an m-sequence with period $2^n - 1$ for $n = 2m - 1$, where $d = 2^e + 1$ such that $\gcd(e, n) = 1$ and $e < (n+1)/2$. Gold-pair signal set $\mathbb{S}$ is generated by the combination of $\underline{\mathbf{m}}$ and its d-decimation $\underline{\mathbf{m}}_d = \{m_{di}\}$ [3], i.e.,

$$s_i^{(j)} = m_{i+j} + m_{di}, \quad s_i^{(2^n-1)} = m_i, \quad s_i^{(2^n)} = m_{di}$$

for $0 \le i \le 2^n - 2$ and $0 \le j \le 2^n - 2$. $\mathbb{S}$ is a $(2^n - 1, 2^n + 1, 1 + 2^{(n+1)/2})$ signal set and $\underline{\mathbf{s}}^{(j)} = \{s_i^{(j)}\}$ is called a Gold-pair sequence with period $2^n - 1$.

C. Kasami-pair Sequences

Let $\underline{\mathbf{m}} = \{m_i\}$ be an m-sequence with a period $2^{n_k} - 1$ for $n_k = 2m$. For $d = 2^m + 1$, a sequence in a Kasami (small) signal set $\mathbb{S}$ is generated by the combination of $\underline{\mathbf{m}}$ and its d-decimation $\underline{\mathbf{m}}_d = \{m_{di}\}$ [4], i.e.,

$$s_i^{(j)} = m_{i+j} + m_{di}, \quad s_i^{(2^m-1)} = m_i$$

for $0 \le i \le 2^n - 2$ and $0 \le j \le 2^m - 2$. $\mathbb{S}$ is a $(2^n - 1, 2^{n/2}, 1 + 2^{n/2})$ signal set and $\underline{\mathbf{s}}^{(j)} = \{s_i^{(j)}\}$ is called a Kasami(small set)sequence with period $2^n - 1$.

D. Multiple Access Interference in Rayleigh Fading Channel

Let $\{x^{(1)}, x^{(2)}, \cdots, x^{(P)}\}$ be the baseband signals that P different users transmit in one symbol duration. Let the user j be assigned the chip sequence $\underline{\mathbf{c}}^{(j)} = \{c_i^{(j)} = (-1)^{s_i^{(j)}}\}$. Assuming the fading channel is frequency-nonselective and the receiver has perfect channel estimation, the i^{th} chip of the input signal at k^{th} receiver is defined by

$$y_i^{(k)} = \sum_{j=0}^{P-1} h_i^{(j)} x_i^{(j)} c_i^{(j)} + n_i^{(k)}$$

After despreading and low-pass filtering for desired user k, the detected signal $\overline{x}^{(k)}$:

$$\begin{aligned}\overline{x}^{(k)} =& x^{(k)} \sum_{i=0}^{N-1} h_i^{(k)} \\ &+ \sum_{j=0, j\neq k}^{P-1} x^{(j)} \sum_{i=0}^{N-1} h_i^{(j)} c_i^{(j)} c_i^{(k)} + \sum_{i=0}^{N-1} n_i^{(k)} c_i^{(k)}\end{aligned} \quad (1)$$

where N is the number of chips in a symbol duration. $h_i^{(j)}$, usually modelled as a random variable, is the wireless channel attenuation experienced by user j experiences, and $n_i^{(k)}$ is real additive white Gaussian noise (AWGN) [5]. In Rayleigh fading channels, the fading amplitudes $h_i^{(j)} = \sqrt{[u_i^{(j)}]^2 + [z_i^{(j)}]^2} \geq 0$, where $u_i^{(j)}$ and $z_i^{(j)}$ are two independent zero-mean Gaussian random variables with variance σ^2 [6].

The second term in the right-hand-side of (1) is the MAI component which is the interference contributed by other simultaneous users in the system. Our work is focused on the periodic correlation of chip sequences over fading channel although partial correlation is actually used in practical signal detection. The purpose of this work is to theoretically investigate the fading effect on binary signals and establish a lower bound on the maximum cross correlation when each chip experiences different fading amplitudes.

II. Correlation of Signals over Fading Channel

A. Lower bound on maximum correlation

Let $\underline{\mathbf{a}} = \{a_i\}$ and $\underline{\mathbf{b}} = \{b_i\}$ be sequences in a signal set $\mathbb{S}$ with a size M. Assume that $a_i = s_i^{(j)}$ is a chip sequence of the j-th user, and $b_i = s_i^{(k)}$ a chip sequence of the k-th user. The periodic correlation of $\underline{\mathbf{a}}$, corrupted by a multiplicative fading $\underline{\mathbf{f}} = \{f_i = h_i^{(j)}\}$, and $\underline{\mathbf{b}}$ in the kth user's receiver is

$$C_{\underline{\mathbf{f}}\cdot\underline{\mathbf{a}},\underline{\mathbf{b}}}(\tau) = \sum_{i=0}^{L-1} f_i \cdot (-1)^{a_i + b_{i+\tau}} \tag{2}$$

where $\underline{\mathbf{f}} \cdot \underline{\mathbf{a}}$ means the sequence whose elements are given by $f_i(-1)^{a_i}$ and L is a period of each sequence $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$. Here, a fading amplitude f_i is a positive random variable with Rayleigh distribution. In this paper, we assume that every user suffers from the same fading amplitude f_i during the same period of time, and the receiver is able to know the accurate value of f_i with the perfect channel estimation. Thus, the maximum correlation of signal set $\mathbb{S}$ is defined by

$$C_1 = \max_{\underline{\mathbf{a}},\underline{\mathbf{b}}\in\mathbb{S}} \max_{0\leq\tau<L} |C_{\underline{\mathbf{f}}\cdot\underline{\mathbf{a}},\underline{\mathbf{b}}}(\tau)|, \; C_2 = \max_{\underline{\mathbf{a}}\in\mathbb{S}} \max_{0<\tau<L} |C_{\underline{\mathbf{f}}\cdot\underline{\mathbf{a}},\underline{\mathbf{a}}}(\tau)|,$$

$$C_{\max} = \max(C_1, C_2). \tag{3}$$

In other words, C_1 and C_2 are cross- and out-of-phase auto-correlation of signals in $\mathbb{S}$ over fading channel, respectively.

We begin with our main theorem about the maximum correlation of a signal set. All detailed proofs about Theorems will be given in a full paper.

Theorem 1: Let $\mathbb{S}$ be a signal set of size M. Each sequence in $\mathbb{S}$ is a binary periodic sequence with a period L. Then, maximum correlation defined by (3) is lower bounded by

$$C_{\max} \geq \sqrt{\frac{ML \cdot \sum_{i=0}^{L-1} f_i^2 - (\sum_{i=0}^{L-1} f_i)^2}{ML-1}}.$$

Proof: We expand a signal set $\mathbb{S}$ to a signal set $\mathbb{T}$ with a size ML including every shift of each distinct sequences in $\mathbb{S}$. Consider

$$\begin{aligned} B &= \sum_{\underline{\mathbf{a}},\underline{\mathbf{b}}\in\mathbb{T}} |C_{\underline{\mathbf{f}}\cdot\underline{\mathbf{a}},\underline{\mathbf{b}}}(0)|^2 \\ &\leq ML \cdot (ML-1) \cdot C_{\max}^2 + ML \cdot C_{\underline{\mathbf{f}}\cdot\underline{\mathbf{a}},\underline{\mathbf{a}}}(0)^2. \end{aligned} \tag{4}$$

Also, B is expanded to

$$\begin{aligned} B &= \sum_{\underline{\mathbf{a}},\underline{\mathbf{b}}\in\mathbb{T}} \left(\sum_{i=0}^{L-1} f_i(-1)^{a_i+b_i}\right) \cdot \left(\sum_{j=0}^{L-1} f_j(-1)^{a_j+b_j}\right) \\ &= \sum_{i,j} f_i f_j \cdot \left(\sum_{\underline{\mathbf{a}}\in\mathbb{T}} (-1)^{a_i+a_j}\right)^2 \geq \sum_{i=0}^{L-1} f_i^2 (ML)^2. \end{aligned} \tag{5}$$

The inequality is from the fact that f_i and f_j are positive. From (4) and (5),

$$ML(ML-1) \cdot C_{\max}^2 + ML \cdot (\sum_{i=0}^{L-1} f_i)^2 \geq \sum_{i=0}^{L-1} f_i^2 (ML)^2.$$

Hence, the result is true. ∎

Corollary 1: Let f_i, $0 \leq i \leq L-1$, be a fading amplitude with a mean μ and a variance σ^2. If f_i is statistically independent and identically distributed (i.i.d) for sufficiently large L, then $C_{\max}$ is lower bounded by

$$C_{\max} \geq \sqrt{\frac{ML^2(\mu^2+\sigma^2) - L^2\mu^2}{ML-1}}.$$

Proof: In the lower bound in Theorem 1, if f_i's are i.i.d for sufficiently large L,

$$\sum_{i=0}^{L-1} f_i = L \cdot \mu, \quad \sum_{i=0}^{L-1} f_i^2 = L \cdot (\mu^2 + \sigma^2) \tag{6}$$

from the weak law of large numbers [7]. Hence, the corollary is immediate from this representation. ∎

Remark 1: In Corollary 1, if no fading environment is assumed, $\mu = 1, \sigma = 0$. Then, $C_{\max}$ normalized by a period is given by

$$C_{\max}^{(norm)} = \frac{C_{\max}}{L} \geq \sqrt{\frac{M-1}{ML-1}}.$$

This is exactly the same result as Welch's lower bound in [1]. Basically, this work is a specific example of Welch's work introducing a random variable f_i to correlation.

Remark 2: In (4), the equality is satisfied if $|C_{\underline{\mathbf{f}}\cdot\underline{\mathbf{a}},\underline{\mathbf{b}}}(0)|$ is identical over all sequences $\underline{\mathbf{a}}, \underline{\mathbf{b}} \in \mathbb{T}$. It implies that as the correlation function $C_{\underline{\mathbf{f}}\cdot\underline{\mathbf{a}},\underline{\mathbf{b}}}(\tau)$ of any pair of sequences in $\mathbb{S}$ over fading channel is flatter, the actual maximum correlation approaches to the lower bound.

B. Lower bound for a Gold signal set

The lower bound in Theorem 1 can be further developed for a Gold signal set.

Theorem 2: In a Gold signal set with $L = 2^n - 1$ and $M = 2^n + 1$ for odd n, the maximum correlation of sequences over fading channel satisfies

$$C_{\max,Gold} \geq \sqrt{\frac{2^{2n} \sum_{i=0}^{2^n-2} f_i^2 - (\sum_{i=0}^{2^n-2} f_i)^2}{2^{2n}-1}}.$$

Proof: In (5), we can write

$$\sum_{\underline{\mathbf{a}}\in\mathbb{T},i\neq j}(-1)^{a_i+a_j}=\sum_{t=0}^{L-1}(-1)^{m_{d(i+t)}+m_{d(j+t)}}\cdot\left(\sum_{r=0}^{L-1}(-1)^{m_{i+r+t}+m_{j+r+t}}+1\right)+\sum_{t=0}^{L-1}(-1)^{m_{i+t}+m_{j+t}}$$

where $\{m_t\}$ is an m-sequence which constitutes a Gold sequence $\{a_i\}$. From the shift-and-add property, and the balance property of m-sequences [8],

$$\sum_{r=0}^{L-1}(-1)^{m_{i+r+t}+m_{j+r+t}}=\sum_{t=0}^{L-1}(-1)^{m_{i+t}+m_{j+t}}=-1$$

for $i\neq j$. Thus, $\sum_{\underline{\mathbf{a}}\in\mathbb{T},i\neq j}(-1)^{a_i+a_j}=-1$, and (5) becomes

$$\begin{aligned}B&=\sum_i f_i^2\left(\sum_{\underline{\mathbf{a}}\in\mathbb{T}}(-1)^{a_i+a_i}\right)^2+\sum_{i,j,i\neq j}f_if_j\left(\sum_{\underline{\mathbf{a}}\in\mathbb{T}}(-1)^{a_i+a_j}\right)^2\\&=\sum_i f_i^2\cdot(ML)^2+\left(\sum_i f_i\right)^2-\sum_i f_i^2.\end{aligned}\tag{7}$$

From (4) and (7), the lower bound in Theorem 1 is slightly changed for a Gold signal set, and the result follows. ■

For an independtly and identically distributed f_i for a sufficiently large n, we have asymptotic bound, or

$$\begin{aligned}C_{\max,Gold}^{(asymp)}&\geq\sqrt{\frac{2^{2n}(2^n-1)(\mu^2+\sigma^2)-(2^n-1)^2\mu^2}{2^{2n}-1}}\\&\approx\sqrt{2^n(\mu^2+\sigma^2)}\end{aligned}\tag{8}$$

from (6). It is noted that the asymptotic lower bound in a Gold signal set is determined by the length of the sequence and the first and second order statistics of the distribution of fading.

If we assume a slow fading environment, the contribution of $C_{\max}^2$ to B will be approximately a half. It is because when each fading amplitude is almost constant over a period, then $|C_{\underline{\mathbf{f}}\cdot\underline{\mathbf{a}},\underline{\mathbf{b}}}(\tau)|^2$ in (4) is approximately two-valued with equal distribution similar to the one of a Gold signal set without fading [2], and one of them is negligible compared to the other. Hence, (4) can be changed into

$$B\approx\frac{ML(ML-1)}{2}C_{\max}^2+ML\left(\sum_{i=0}^{L-1}f_i\right)^2.\tag{9}$$

From (9) and (7), approximated maximum correlation of sequences in a Gold signal set is

$$C_{\max,Gold}^{(slow)}\approx\sqrt{2}\cdot\sqrt{\frac{2^{2n}\sum_{i=0}^{2^n-2}f_i^2-(\sum_{i=0}^{2^n-2}f_i)^2}{2^{2n}-1}}\tag{10}$$

which is a $\sqrt{2}$ times of lower bound in a Gold signal set. From this approximation, we note that as fading gets slower, the actual maximum correlation approaches to $C_{\max,Gold}^{(slow)}$.

C. Lower bound for a Kasami (small) signal set

Theorem 3: In a Kasami signal set with $L=2^n-1$ and $M=2^m$ for $n=2m$, the maximum correlation of sequences over fading channel satisfies

$$C_{\max,Kasami}\geq\sqrt{\frac{2^m(2^n-1)\sum_{i=0}^{2^n-2}f_i^2-(\sum_{i=0}^{2^n-2}f_i)^2}{2^m(2^n-1)-1}}.$$

Proof: From sequences in a Kasami (small) signal set,

$$\sum_{\underline{\mathbf{a}}\in\mathbb{T},i\neq j}(-1)^{a_i+a_j}=\sum_{t=0}^{L-1}(-1)^{m_{i+t}+m_{j+t}}\cdot\left(\sum_{r=0}^{M-2}(-1)^{m_{d(i+t)+r}+m_{d(j+t)+r}}+1\right)=0$$

in (5) for $M=2^m$ from the shift-and-add property, and the balance property of m-sequences. Hence, B in (7) is given by

$$B=\sum_i f_i^2\cdot\left(\sum_{\underline{\mathbf{a}}\in\mathbb{T}}(-1)^{a_i+a_i}\right)^2=\sum_i f_i^2(ML)^2.\tag{11}$$

From (4) and (11), the result follows immediately. ■

For an independtly and identically distributed f_i for a sufficiently large n,

$$\begin{aligned}C_{\max,Kasami}^{(asymp)}&\geq\sqrt{\frac{2^m(2^n-1)^2(\mu^2+\sigma^2)-(2^n-1)^2\mu^2}{2^m(2^n-1)-1}}\\&\approx\sqrt{2^n(\mu^2+\sigma^2)}\end{aligned}\tag{12}$$

from (6). Similar to a Gold signal set, the asymptotic lower bound for a Kasami signal set depends on the length of the sequence, and the first and second order statistics of the distribution of fading.

With a slow fading assumption,

$$B\approx ML\cdot(ML-1)\cdot C_{\max}^2+ML\cdot C_{\underline{\mathbf{f}}\cdot\underline{\mathbf{a}},\underline{\mathbf{a}}}(0)^2$$

because $|C_{\underline{\mathbf{f}}\cdot\underline{\mathbf{a}},\underline{\mathbf{b}}}(\tau)|^2$ is assumed to be almost constant in a Kasami signal set when each fading amplitude is almost constant [2]. In a similar way to a Gold signal set,

$$C_{\max,Kasami}^{(slow)}\approx\sqrt{\frac{2^m(2^n-1)\sum_{i=0}^{2^n-2}f_i^2-(\sum_{i=0}^{2^n-2}f_i)^2}{2^m(2^n-1)-1}},\tag{13}$$

which means the Kasami (small) signal set achieves its lower bound over a fading channel as fading gets slower.

III. Simulation Results

To analyze the statistical behavior of correlation of signals over Rayleigh fading, channels with independent and correlated fading amplitudes are simulated respectively. Independent channels have i.i.d. fading for each chip in the signal. On the other hand, the correlation of fading amplitudes in correlative channel is characterized by f_DT_c where f_D is the maximum doppler spread duration [6].

Recall that all users in the system are assumed to suffer from the same fading amplitudes and the receiver achieves perfect

TABLE I

AVERAGE $\kappa_{\max}^{(norm)}$ OF GOLD AND KASAMI SIGNAL SETS

Fading Type	Gold $n=5$ $\kappa_{\max,Gold}^{(norm)}$	Kasami $n=6$ $\kappa_{\max,Kasami}^{(norm)}$
independent	0.62	0.43
$f_D T_c$ = 1	0.46	0.25
$f_D T_c$ = 0.01	0.31	0.22
$f_D T_c$ = 0.0001	0.21	0.16
Fading Type	Gold $n=7$ $\kappa_{\max,Gold}^{(norm)}$	Kasami $n=8$ $\kappa_{\max,Kasami}^{(norm)}$
independent	0.36	0.25
$f_D T_c$ = 1	0.26	0.16
$f_D T_c$ = 0.01	0.23	0.12
$f_D T_c$ = 0.0001	0.13	0.08
Fading Type	Gold $n=9$ $\kappa_{\max,Gold}^{(norm)}$	Kasami $n=10$ $\kappa_{\max,Kasami}^{(norm)}$
independent	0.21	0.12
$f_D T_c$ = 1	0.13	0.09
$f_D T_c$ = 0.01	0.13	0.08
$f_D T_c$ = 0.0001	0.08	0.03

TABLE II

κ_{min} OF GOLD AND KASAMI SIGNAL SETS IN INDEPENDENT FADING

Gold Signal Sets			
n_g	$\kappa_{min,Gold}$	lower bound in (2)	asymptotic bound in (8)
5	9.95	7.19	8.00
7	27.32	15.85	16.00
9	65.71	30.85	32.00
Kasami Signal Sets			
n_k	$\kappa_{min,Kasami}$	lower bound in (3)	asymptotic bound in (12)
6	19.27	11.17	11.31
8	45.23	23.28	22.63
10	89.69	43.26	45.25

TABLE III

κ_{min} AND $C_{\max,Gold}^{(slow)}$ OF GOLD SIGNAL SETS IN CORRELATED FADING

n_g	$f_D T_c$	$\kappa_{min,Gold}$	lower bound in (2)	$C_{\max,Gold}^{(slow)}$
5	1	7.15	5.35	7.56
	0.01	7.37	5.09	7.20
	0.0001	6.39	3.89	5.51
7	1	18.07	10.16	14.37
	0.01	19.76	11.35	16.05
	0.0001	14.87	9.79	13.85
9	1	45.29	21.89	30.95
	0.01	39.12	18.77	26.54
	0.0001	36.62	24.45	34.58

channel estimation, experiment is run ten times for a signal set in a particular channel environment but with different random fading amplitudes. In each trial, $\kappa_{\max}$ and $\kappa_{\min}$ are computed for Gold signal sets with $n = \{5, 7, 9\}$ and Kasami signal sets with $n = \{6, 8, 10\}$ as defined by

$$\kappa_{\max} = \max_{\{k,i\}} C_{\max}, \quad \kappa_{\min} = \min_{\{k,i\}} C_{\min} \tag{14}$$

where $k = 0, ..., M-1$ and $i = k+1, ..., M-1$. For each signal set in a given simulated fading channel, the computational space is equal to $\binom{M}{2} \times L \approx M^2 \times L$. It is infeasible to compare all possible pairs of signals in Gold signal set with $n = 9$ Thus, experiments are performed only for signal pairs $\{k, i\}$ where $k = 0, ..., 10$ and $i = k + 1, ..., M - 1$.

A. Normalized Maximum Cross Correlation

$\kappa_{\max}$ represents the worst cross correlation of the signal sets. We can define the normalization $\kappa_{\max}^{(norm)} = \kappa_{\max}/L$. Table I shows the average $\kappa_{\max}^{(norm)}$ over all trials for Gold and Kasami signals in various fading. We can see that $\kappa_{\max}^{(norm)}$ decreases when the sequence period increases or fading becomes slower.

B. Lower Bound on Maximum Cross Correlation

It is found that κ_{min}, therefore all cross correlations in the signal set, satisfy the derived lower bounds in all trials. In the following tables, results from one trial of each experiment are shown. As we can see from Table II, the lower bounds in (2) and (3) approach the asymptotic bounds in independent fading for both Gold and Kasami signal sets.

Table III and IV compare κ_{min} and the lower bounds for Gold and Kasami signal sets respectively. We do not list $C_{\max,Kasami}^{(slow)}$ in the table because it achieves the lower bound as shown in (13). The results demonstrate that the lower bounds become tighter and $C_{\max,Gold}^{(slow)}$ is more accurate when the fading gets slower.

C. Cross Correlation Distributions

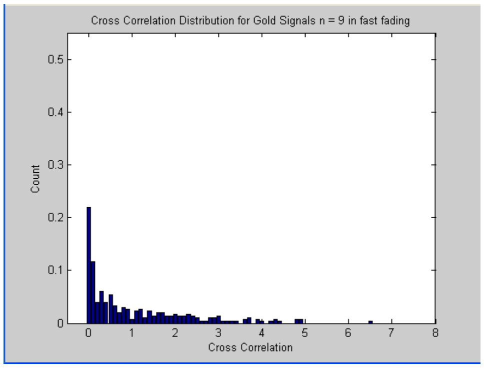
The normalization of $|C_{\underline{\mathbf{f}} \cdot \underline{\mathbf{a}}, \underline{\mathbf{b}}}(\tau)|$ in (2) gives the distributions of correlations in a signal set,

$$C_{\underline{\mathbf{f}} \cdot \underline{\mathbf{a}}, \underline{\mathbf{b}}}^{(norm)}(\tau) = \frac{|C_{\underline{\mathbf{f}} \cdot \underline{\mathbf{a}}, \underline{\mathbf{b}}}(\tau)|^2}{E[|C_{\underline{\mathbf{f}} \cdot \underline{\mathbf{a}}, \underline{\mathbf{b}}}(\tau)|^2]} \tag{15}$$

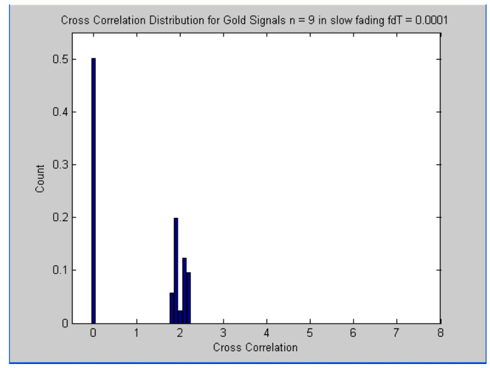
where $E[.]$ denotes the expectation of random variable. Figure 1 and 2 present the distribution of cross correlation for Gold and Kasami signals respectively. From Figure 1(b) and 2(b), we can see that the distribution of a signal set in very slow fading is similar to the one without fading. This verifies the results of Section II about the slow fading approximation to the maximum correlation, and implies that the fading effect on a signal set is negligible when the fading becomes very slow.

IV. CONCLUSIONS

In conclusions, a lower bound on the maximum correlation over fading channel has been established for binary signal sets, particularly the Gold and Kasami signal sets. Asymptotic bound and slow fading approximation of the maximum correlation are obtained for independent and correlative channels

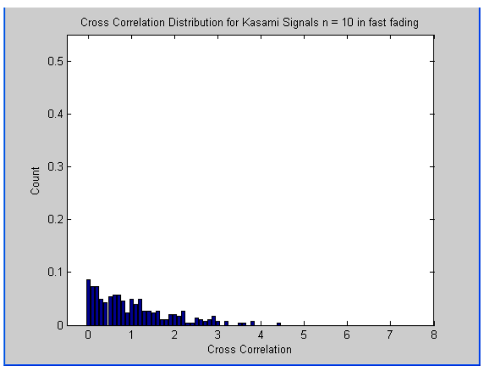


(a) Independent Fading

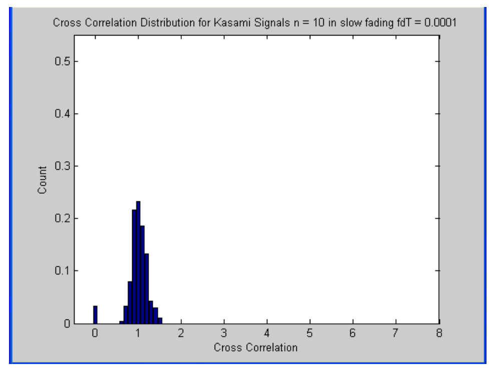


(b) Correlative Fading with $f_D Tc = 0.0001$

Fig. 1. Cross Correlation Distributions of Gold Signals $n = 9$



(a) Independent Fading



(b) Correlative Fading with $f_D Tc = 0.0001$

Fig. 2. Cross Correlation Distributions of Kasami Signals $n = 10$

TABLE IV

κ_{min} OF KASAMI SIGNAL SETS IN CORRELATED FADING

n_k	$f_D T_c$	κ_{min}	lower bound in (3)
6	1	14.53	8.83
	0.01	11.85	7.60
	0.0001	10.79	8.85
8	1	29.75	16.33
	0.01	24.94	15.28
	0.0001	21.08	19.05
10	1	66.46	33.50
	0.01	67.76	34.14
	0.0001	25.73	21.38

respectively. The distribution of cross correlation in a signal set with fading is also analyzed.

Given a frequency-nonselective Rayleigh fading channel, experimental results show that the lower bound is tighter in slow fading channel than in independent channel. Specifically for Gold signal sets, the slow fading approximation of the maximum cross correlation also becomes more accurate when the fading gets slower. The distribution of cross correlation in correlative fading environment is similar to the one without fading, which implies that the fading effect on the signal sets is negligible over slow fading channels.

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