TRIVIAL EXTENSIONS OF GENTLE ALGEBRAS AND BRAUER GRAPH ALGEBRAS

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Abstract. After showing that the class of symmetric special biserial algebras coincides with the class of Brauer graph algebras - a result already generally believed to be true - we show that the trivial extension of a gentle algebra A by its minimal injective co-generator $D(A)$ is isomorphic to a Brauer graph algebra. We do this by constructing a graph with an ordering of its edges around each vertex associated to the gentle algebra A and show that this is the graph of the Brauer graph algebra which is isomorphic to the trivial extension of A. We then show that any admissible cut of a Brauer graph algebra gives a gentle algebra and that the trivial extension of that gentle algebra is the original Brauer graph algebra. As a consequence we show that the trivial extension of a Jacobian algebra of a triangulation of an unpunctured marked Riemann surface is the Brauer graph algebra associated to the internal edges of the triangulation.

The class of special biserial algebras comprises many interesting examples such as gentle algebras, monomial algebras and Brauer graph algebras. It is well-known that they are of tame representation type [30] and their representation theory is well understood. Most recently a renewed interest in gentle algebras has arisen as they appear naturally as Jacobian algebras of triangulations of marked Riemann surfaces without punctures [1].

The class of gentle algebras is special in the sense that it is closed under derived equivalence [29]. Within the class of special biserial algebras, Brauer graph algebras form a class of algebras closed under derived equivalence [14, 26]. The classes of Brauer graph algebras and that of symmetric special biserial algebras coincide (see Theorem 1.1) and they are closely related to gentle algebras: an algebra is gentle if and only if its trivial extension is special biserial [23, 25, 28].

Brauer graph algebras were first considered by Richard Brauer in the context of modular representation theory of finite groups. Since their inception, Brauer graph algebras have been extensively studied by many authors, see for example [3, 24, 14, 15, 26], for a small selection of this literature and more recently [2, 13].

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For a symmetric special biserial algebra Λ given by quiver and relations, Roggenkamp has shown in [26] that if the quiver of Λ has no double arrows then Λ is a Brauer graph algebra. This result seems to be believed to hold in general with no restriction on the number of arrows. Since our subsequent results rely on it, we include a proof. Closely following [26], we associate to every symmetric special biserial algebra Λ a graph G_Λ with a local structure given by a cyclic ordering of the edges around each vertex. Our first result is that the class of symmetric special biserial algebras and that of Brauer graph algebras coincides.

Let k be an algebraically closed field.

Theorem 1.1. *Let $\Lambda = kQ/I$ be a finite dimensional symmetric special biserial algebra and let G_Λ be its graph with local structure. Let B be the Brauer graph algebra associated to G_Λ. Then B is isomorphic to Λ.*

Given a gentle algebra A, the trivial extension $T(A) = A \ltimes D(A)$ of A by its minimal co-generator $D(A)$ is a symmetric special biserial algebra. Thus it is a Brauer graph algebra. In order to identify its Brauer graph we construct, for any gentle algebra A, a graph Γ_A, equipped with a cyclic ordering of the edges around each vertex induced by the maximal paths in A and we show that Γ_A is the Brauer graph of $T(A)$.

Theorem 1.2. *Let $A = kQ/I$ be a finite dimensional gentle algebra and let Γ_A be its graph. Let B be the Brauer graph algebra defined on Γ_A with cyclic ordering induced by the maximal paths in A. Then the trivial extension $T(A)$ of A is isomorphic to B and $G_{T(A)} = \Gamma_A$ where $G_{T(A)}$ is the graph of $T(A)$ as symmetric special biserial algebra.*

We show that every Brauer graph algebra with multiplicity identically one (or alternatively every symmetric special biserial algebra in which all cycles in the relations are no power of a proper sub-cycle) is the trivial extension of a gentle algebra. For this we define the notion of an admissible cut of a Brauer graph algebra. Admissible cuts of this form were first considered in [10, 12], see also [5, 19, 9], and [20].

Theorem 1.3. *Let $\Lambda = kQ_\Lambda/I_\Lambda$ be a Brauer graph algebra with multiplicity one at all vertices in the associated Brauer graph. Let $A = kQ/I$ be an admissible cut of Λ. Then A is gentle and $T(A)$ and Λ are isomorphic.*

This theorem has the following immediate consequence.

Corollary 1.4. *Every Brauer graph algebra with multiplicity function identically one is the trivial extension of a gentle algebra.*

Let (S, M) be a bordered Riemann surface and M a set of marked points in the boundary of S. Given a triangulation T of (S, M), let (Q, I) be the associated bound quiver as defined in [7, 8] and let $A = kQ/I$ be the associated finite dimensional gentle algebra [1]. Denote by T^0 the connected graph given by the edges of T that are not in the boundary of S. The orientation of S induces a cyclic ordering of the edges of T^0 around each vertex and thus T^0 can be viewed as a Brauer graph.

Corollary 1.5. *Let A be a gentle algebra arising from a triangulation T of a marked unpunctured Riemann surface (S, M). Then the trivial extension $T(A)$ of A is isomorphic to the Brauer graph algebra of T^0 where T^0 is the graph consisting of the internal edges of T.*

Note that connections between Jacobian algebras of triangulations of marked Riemann surfaces and Brauer graph algebras have recently been established by several authors. In particular, the connection of mutation, flip of diagonals in triangulations and derived equivalences have been studied in [17, 18], [2] and [21].

Given a triangulation T of a bordered unpunctured Riemann surface (S, M), in [9] surface algebras were defined by cutting T at internal triangles. This gives rise to a partial triangulation of (S, M). A surface algebra A is again a finite dimensional gentle algebra and it can be constructed by associating a bound quiver (Q, I) to the partial triangulation of (S, M) such that $A = kQ/I$, see [9]. We show that the graph T^0 given by the internal edges of the partial triangulation together with the cyclic ordering of edges induced by the orientation of the surface is the Brauer graph associated to the trivial extension of A.

Corollary 1.6. *Let A be a surface algebra of a partial triangulation T of a marked unpunctured Riemann surface (S, M). Then the trivial extension $T(A)$ of A is isomorphic to the Brauer graph algebra of T^0 where T^0 is the graph consisting of the internal edges of T.*

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2. Gentle algebras and trivial extension algebras

Let k be an algebraically closed field and let Q be a finite connected quiver. Let I be an admissible ideal in the path algebra kQ such that kQ/I is a finite dimensional algebra. A path in Q is in the bound quiver (Q, I) if it avoids the relations in I.

Let $D = Hom_k(-, k)$ denote the standard duality of the module category $A{-}mod$ of finitely generated A-modules. A finite dimensional k-algebra A is symmetric if it is isomorphic to $D(A)$ as an A-$A-$bimodule. Let $A^e \simeq A \otimes_k A^{op}$ be the enveloping algebra of A. Unless otherwise stated all modules considered are right modules.

2.1. **Special biserial and gentle algebras.**

We say that a finite dimensional algebra A is special biserial if it is Morita equivalent to an algebra of the form kQ/I where

(S1) Each vertex of Q is the starting point of at most two arrows and is the end point of at most two arrows.

(S2) For each arrow α in Q there is at most one arrow β in Q such that $\alpha\beta$ is not in I and there is at most one arrow γ such that $\gamma\alpha$ is not in I.

We say that an algebra A is gentle if it is Morita equivalent to an algebra kQ/I satisfying (S1), (S2) and

(S3) I is generated by paths of length 2.

(S4) For each arrow α in Q there is at most one arrow δ in Q such that $\alpha\delta$ is in I and there is at most one arrow ε in Q such that $\varepsilon\alpha$ is in I.

An arrow α in Q starts at the vertex $s(\alpha)$ and ends at the vertex $t(\alpha)$. If $p = \alpha_1\alpha_2\ldots\alpha_n$, for arrows α_i, $1 \le i \le n$, is a path in Q then $s(p) = s(\alpha_1)$ and $t(p) = t(\alpha_n)$. In the following we'll collect some well-known facts about gentle algebras, see for example [6] for more details.

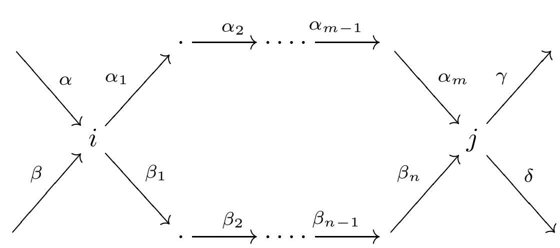
Let $\mathcal{P}$ be the set of paths in (Q, I). Let $\mathcal{M}$ be the set of maximal elements in $\mathcal{P}$, that is all paths $p \in \mathcal{P}$ such that for all arrows α in Q, $\alpha p \notin \mathcal{P}$ and $p\alpha \notin \mathcal{P}$.

In general, for any finite dimensional algebra every path is a subpath of some maximal path and for a gentle algebra that maximal path is unique. Thus in a gentle algebra every arrow is in a unique maximal path. Furthermore, a non-trivial path p in Q is in $\mathcal{P}$ if and only if it is a subpath of a (unique if A gentle) maximal path $m \in \mathcal{M}$ where $m = qpq'$ with q, q' paths in Q.

Suppose kQ/I is a gentle algebra. Every arrow is contained in a unique maximal path, and there are at most two maximal paths starting at any given vertex, and at most two maximal paths ending at any given vertex. It follows that two distinct maximal paths cannot have a common arrow. Hence maximal paths only intersect at a vertex of Q.

Lemma 2.1. *Let $A = kQ/I$ be a gentle algebra. Then the maximal paths in (Q, I) form a basis of* $\mathrm{soc}_{A^e}A$.

Proof: We have $\mathrm{soc}_{A^e}A = \bigoplus_{i,j \text{ vertices in } Q} e_i(\mathrm{soc}_{A^e}A)e_j$ as k-vector spaces. Let i, j be vertices in Q. We will show that the maximal paths from i to j form a basis of $e_i(\mathrm{soc}_{A^e}A)e_j$. Recall that a path p in (Q, I) is maximal if for all arrows α in Q, $p\alpha, \alpha p \in I$ or equivalently if $p\alpha = 0 = \alpha p$ in A. Set $R = \mathrm{soc}_{A^e}A$. Since A is gentle there are at most two non-zero paths from i to j, denote them by $p = \alpha_1\alpha_2\ldots\alpha_m$ and $q = \beta_1\beta_2\ldots\beta_n$. Let $\alpha, \beta, \gamma, \delta$ be arrows in Q such that $t(\alpha) = t(\beta) = i$ and $s(\gamma) = s(\delta) = j$ as in the diagram below.



A case by case analysis will give a basis of e_iRe_j in terms of p and q. Clearly if $\alpha, \beta, \gamma, \delta$ do not occur then p and q are maximal and $\{p, q\}$ is a basis of e_iRe_j.

Now suppose β, γ, δ do not occur but α is an arrow in Q. Then either $\alpha p \notin I$ or $\alpha q \notin I$. Suppose that $\alpha p \notin I$. Then p is not maximal and q is maximal and $\{q\}$ is a basis of e_iRe_j. Similarly, if α, β, γ do not occur but δ is an arrow in Q.

Suppose now that β and δ do not occur but that α and γ are arrows in Q. Then there are two cases: (i) if $\alpha\alpha_1 \in I$ and $\alpha_m\gamma \in I$ then $\{p\}$ is a basis of e_iRe_j, (ii) if $\alpha\alpha_1 \in I$ and $\beta_n\gamma \in I$ then p and q are not maximal and no linear combination x of p and q is such that it is zero by left and right multiplication by all arrows in Q. Hence $e_iRe_j = \{0\}$.

If α and β are arrows in Q and γ and δ do not occur then either (i) $\alpha\alpha_1 \in I$ and $\beta\beta_1 \in I$ or (ii) $\alpha\beta_1 \in I$ and $\beta\alpha_1 \in I$. Assume without loss of generality that (i) holds. Then p and q are not maximal and as above $e_iRe_j = \{0\}$.

Suppose that α, β, and γ are arrows in Q and that δ does not occur. Then as above we can assume without loss of generality that $\alpha_m\gamma \in I$ and hence that p is maximal but q is not and $\{p\}$ is a basis of e_iRe_j.

If all arrows α, β, γ and δ occur in Q then both p and q cannot be maximal and $e_iRe_j = \{0\}$.

All remaining cases are covered by similar arguments. □

2.2. **Brauer graph algebras.** In this section we define symmetric Brauer graph algebras.

We call a finite connected graph Γ a Brauer graph if Γ is equipped with a cyclic ordering of the edges around each vertex and if for every vertex ν in Γ there is an associated strictly positive integer $e(\nu)$ called the *multiplicity of* ν.

To a Brauer graph Γ we associate a quiver Q_Γ where the vertices of Q_Γ correspond to the edges in Γ. Let x and y be two distinct vertices in Q_Γ corresponding to edges E_x and E_y in Γ. Then there is an arrow $x \xrightarrow{\alpha} y$ in Q_Γ if the edge E_y is a direct successor of the edge E_x in the cyclic ordering around a vertex in Γ. If the edge E_x is a leaf with leaf vertex ν with $e(\nu) \geq 2$ then E_x is its own successor in the cyclic ordering and there is an arrow $x \xrightarrow{\alpha} x$. If $e(\nu) = 1$ then no such arrow exists.

It follows from the construction of Q_Γ that every vertex ν of Γ gives rise to an oriented cycle in Q_Γ unless ν is a leaf with leaf vertex ν where $e(\nu) = 1$. Furthermore, no two cycles corresponding to distinct vertices have a common arrow.

Define on $\mathcal{Q}_\Gamma$ a set of relations ρ_Γ as follows. Let E_x be an edge in Γ with vertices ν and ν' where if ν (respectively ν') is a leaf vertex then $e(\nu) \neq 1$ (respectively $e(\nu') \neq 1$). Denote by $C_{\nu,x}$ and $C_{\nu',x}$ the corresponding cycles in Q_Γ starting at vertex x in Q_Γ. Let $C_{\nu,x}^{e(\nu)}$ be the $e(\nu)$-th power of $C_{\nu,x}$. Then $C_{\nu,x}^{e(\nu)} - C_{\nu',x}^{e(\nu')} \in \rho_\Gamma$.

Now suppose the edge E_x is such that ν is a leaf with $e(\nu) = 1$. Then $C_{\nu',x}\alpha \in \rho_\Gamma$ where α is the arrow in $C_{\nu'}$ starting at x. Finally if α, β are two arrows in Q_Γ such that $t(\alpha) = s(\beta) = x$ where α is in $C_{\nu,x'}$ for $s(\alpha) = x'$ and β is in $C_{\nu',x}$ with $\nu \neq \nu'$ then $\alpha\beta \in \rho_\Gamma$.

The algebra $B_\Gamma = kQ_\Gamma/I_\Gamma$ where I_Γ is the ideal generated by ρ_Γ is called the Brauer graph algebra associated to the Brauer graph Γ. Note that B_Γ is a finite dimensional symmetric special biserial algebra, that is B_Γ satisfies (S1) and (S2). Furthermore, it immediately follows from the definition of Brauer graph algebras that they satisfy condition (S4).

2.3. **Symmetric special biserial algebras.** Let $\Lambda = kQ/I$ be a finite dimensional symmetric special biserial algebra.

If $\Lambda = kQ/I$ is a special biserial algebra then without loss of generality we can assume that a set of relations ρ generating I contains only zero relations and commutativity relations of the form $p - q$ for p, q paths in Q such that $p, q \notin \rho$.

Since Λ is symmetric special biserial the projective indecomposable modules are uniserial or biserial. We adopt the following notation. Let s be a vertex in Q. Then if the projective indecomposable P_s at s is uniserial there exists a unique non-zero path p in (Q, I) with $s(p) = t(p) = s$ and p is a k-basis of $\mathrm{soc}P_s$. Let e_s be the trivial path at s. Then we write $P_s = P_s(p, e_s) = P_s(p)$ or $P(p)$ for short. If P_s is biserial then there exist two distinct non-trivial paths p, q in (Q, I) with $s(p) = s(q) = t(p) = t(q) = s$ such that $p - q \in I$ and $p = q$ is a k-basis for $\mathrm{soc}P_s$. We write $P_s = P_s(p, q)$ or $P(p, q)$ for short. Since Λ is symmetric the projective indecomposable at s is also the injective indecomposable at s and hence the paths p and q are maximal. It is a direct consequence of (S2) that p and q do not start or end with a common arrow. Furthermore, if there is a relation $p - q \in I$ where $p, q \notin I$ then $P_s = P(p, q)$ where $s = s(p)$. This follows directly from (S2) and the fact that $\mathrm{rad}\, P_s / \mathrm{soc}\, P_s$ is a direct sum of two uniserial modules.

Remark. Suppose that B is a Brauer graph algebra with Brauer graph Γ. With the notation above let $C_{\nu,x}^{e(\nu)}, C_{\nu',x}^{e(\nu')}$ be two the maximal paths defined by two vertices ν, ν' of Γ connected by an edge E_x corresponding to a vertex $x \in Q_\Gamma$. Then the projective indecomposable B-module at x is given by $P_x(C_{\nu,x}^{e(\nu)}, C_{\nu',x}^{e(\nu')})$ where $C_{\nu,x}^{e(\nu)}$ or $C_{\nu',x}^{e(\nu')}$ might be the trivial path at x if ν (respectively ν') is a leaf with $e(\nu) = 1$ (respectively $e(\nu') = 1$). In the latter case the projective indecomposable is uniserial.

Lemma 2.2. *Let $\Lambda = kQ_\Lambda/I_\Lambda$ and $\Delta = kQ_\Delta/I_\Delta$ be symmetric special biseral algebras. Suppose $Q = Q_\Lambda = Q_\Delta$ and suppose that at every vertex the projective indecomposable modules of Λ and Δ have the same k-basis given by paths in Q. Then the algebras Λ and Δ are isomorphic.*

Proof: We will show that $I_\Lambda = I_\Delta$. Denote the projective indecomposable module of Λ (respectively Δ) at vertex s by P_s^Λ (respectively P_s^Δ). It is enough to show that every generating relation for I_Λ must also be a generating relation for I_Δ. Suppose $\alpha_1 \dots \alpha_n$ is a path in Q with $\alpha_1 \dots \alpha_n \in I_\Lambda$. Now assume that

$\alpha_1 \dots \alpha_n \notin I_\Delta$. Then there is a path p_1 in Q such that $p = \alpha_1 \dots \alpha_n p_1$ and $P_s^\Delta = P(p,q)$ for some possibly trivial path q and where $s = s(\alpha_1)$. But $p \in I_\Lambda$ since $\alpha_1 \dots \alpha_n \in I_\Lambda$. Therefore the projective $P_s^\Lambda \neq P(p,q)$, a contradiction and thus $\alpha_1 \dots \alpha_n \in I_\Delta$. By symmetry of the argument this implies that $\alpha_1 \dots \alpha_n \in I_\Lambda$ if and only if $\alpha_1 \dots \alpha_n \in I_\Delta$. Now suppose that for p, q paths in Q with $p, q \notin I_\Lambda$ and $0 \neq p - q \in I_\Lambda$. By (S2) p and q do not start with the same arrow (since otherwise $p = q$). But then the projective P_s^Λ with $s = s(p) = s(q)$ is biserial. Hence $P_s^\Lambda = P(p,q)$. But then $P(p,q) = P_s^\Delta$ and thus $p - q \in I_\Delta$. □

2.3.1. *Graph of a symmetric special biserial algebra.* In [26] Roggenkamp showed that if the quiver of a symmetric biserial algebra Λ has no double arrows then Λ is a Brauer graph algebra. We will adapt Roggenkamp's construction of the Brauer graph associated to a symmetric special biserial algebra to show that this result holds in general. This more general result is generally believed to be true, see for example, [2, 4]. For completeness we will give a proof.

Let $\Lambda = kQ/I$ be a symmetric special biserial algebra. We now use the projective indecomposable Λ-modules to define a graph G_Λ with a cyclic ordering of the edges around each vertex. As mentioned above we follow Roggenkamp's construction for this. However, instead of using the Loewy structure of projective indecomposables as in [26] we consider the arrows and paths defining the projective indecomposables. This eliminates any ambiguity arising from the existence of double arrows. Recall that the projective indecomposable Λ-modules are either uniserial or biserial and that they are denoted by $P_s(p)$ and $P_s(p,q)$ respectively where p, q are maximal paths from a vertex s in Q to itself. For the purpose of this construction we consider the trivial path e_s at vertex s to be a maximal cyclic path at s if the projective indecomposable at s is uniserial and we set $P_s(p) = P_s(p, e_s)$. This implies that there are two maximal cyclic paths at every vertex (one of them possibly being the trivial path). For non-trivial maximal cyclic paths p and q at vertex s there are strictly positive integers m and n such that $p = p_0^m$ and $q = q_0^n$ where p_0 and q_0 are cycles starting and ending at s which are no proper power of cycles of shorter length. We call p_0 and q_0 irreducible cycles. Set $e(p) = m$ and $e(q) = n$ and call it the *multiplicity* of p and respectively of q. Trivial maximal paths always have multiplicity one.

Suppose $p = p_0^m$ is a maximal path as above where $p_0 = s \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \cdots \xrightarrow{\alpha_{k-1}} s_k \xrightarrow{\alpha_k} s$. Define the *p-cycle of* p to be the sequence $\mu(p) = (s, s_1, s_2, \dots, s_k)$ of vertices in Q. If the trivial path e_s at a vertex s is maximal, we set $\mu(e_s) = s$.

Remark. (1) If $\tilde{p}$ is a cyclic rotation of p then $e(\tilde{p}) = e(p)$.

(2) The vertices of Q occurring in $\mu(p)$ need not all be different. However, since A is special biserial, each one can occur no more than twice.

(3) If $\tilde{p}$ is a cyclic rotation of p then $\mu(\tilde{p})$ is a cyclic rotation of $\mu(p)$.

Set $p \sim \tilde{p}$ if $\tilde{p}$ is a cyclic rotation of p. This defines an equivalence relation on the set of cyclic rotations of p. Denote the equivalence class of p by $\nu(p)$ and call it a *vertex*. If $p = p_0^{e(p)}$ with p_0 irreducible then we call the rotation class of p_0 a *vertex cycle*.

The vertices V of G_Λ consist of the equivalence classes $V = \{\nu(p) \mid p \text{ maximal path in } (Q,I)\}$. To each vertex $\nu(p)$ we associate its multiplicity $e(p)$. Note that this is well-defined since $e(p) = e(\tilde{p})$ for $p \sim \tilde{p}$. To the vertex $\nu(p)$ we now attach *germs of edges* labelled by the vertices of Q contained in $\mu(p)$. Note that the arrows in p_0 induce a linear order on the vertices in $\mu(p)$ by setting $s_i < s_{i+1}$ for $0 \leq i \leq k-1$ and we can complete this to a cyclic order by setting $s_k < s$. In turn this defines a cyclic ordering of the germs around $\nu(p)$. Call this the *local structure of* G_Λ.

Remark. (1) The local structure of G_Λ is not changed if we replace $\mu(p)$ by $\mu(\tilde{p})$ where $p \sim \tilde{p}$. Hence denote the p-cycle $\mu(p)$ by μ_ν where $\nu = \nu(p)$.

(2) In the set $\{\mu_\nu\}$ each vertex of Q appears exactly twice. Thus for every vertex in Q there are exactly two germs labelled by that vertex. Note that a vertex of Q can appear twice in one μ_ν, labelling two distinct germs.

Definition 2.3. *The graph G_Λ of a symmetric special biserial algebra Λ is given by the vertices $\nu \in V$ with cyclic ordering given by $\mu(p)$ for some maximal path p such that $\mu(p) = \mu_\nu$. There is an edge from ν to ν' if the cycles μ_ν and $\mu_{\nu'}$ have a common vertex of Q. In this case we join the two corresponding germs to a genuine edge.*

Note that if a vertex of Q occurs twice in μ_ν then there is a loop in G_Λ.

The graph G_Λ is a graph with a cyclic ordering of the edges at each vertex. Thus together with the multiplicity of the maximal paths (defined above) associated to the corresponding vertices of G_Λ, the graph G_Λ is a Brauer graph. Let B be the corresponding Brauer graph algebra. It follows immediately from the definition of the cyclic ordering in G_Λ which is induced by the arrows in Q that the projective indecomposable B-modules have the same k-basis of paths in Q as the projective indecomposable Λ-modules. Combining this with Lemma 2.2 proves Theorem 1.1 which for the convenience of the reader we restate here.

Theorem 1.1 *Let $\Lambda = kQ/I$ be a symmetric special biserial algebra and let G_Λ be its graph with local structure as defined above. Let B be the symmetric Brauer graph algebra associated to G_Λ, then B is isomorphic to Λ.*

3. Trivial extensions of gentle algebras

Let A be a finite dimensional associative k-algebra. The trivial extension $T(A)$ of A by its minimal injective co-generator $D(A)$ is the algebra $T(A) = A \ltimes D(A)$. As a k-vector space $T(A)$ is given by $A \oplus D(A)$ with the multiplication defined by $(a,f)(b,g) = (ab, ag + fb)$, for $a, b \in A$ and $f, g \in D(A)$. It is well-known that the trivial extension algebra $T(A)$ is a symmetric algebra.

As recalled in the introduction it is well-known that A is gentle if and only if $T(A)$ is special biserial. Therefore the trivial extension of a gentle algebra is a symmetric special biserial algebra. Hence it follows from Theorem 1.1 that $T(A)$ is a Brauer graph algebra.

It is proven in [11, 2.2] that the vertices of the quiver $Q_{T(A)}$ of $T(A)$ correspond to the vertices of the quiver Q_A of A and that the number of arrows from a vertex i to a vertex j in $T(A)$ is equal to the number of arrows from i to j in Q_A plus the dimension of the k-vector space $e_j(\mathrm{soc}_{A^e} A)e_i$.

The injective $T(A)$-bimodule homomorphism $D(A) \longrightarrow T(A)$ gives a short exact sequence of $T(A)$-bimodules

$$0 \longrightarrow D(A) \longrightarrow T(A) \longrightarrow A \longrightarrow 0.$$

For every vertex i in $Q_{T(A)}$, let $e_i \in T(A)$ be the corresponding primitive idempotent. Then by left multiplication with e_i we obtain a short exact sequence of right $T(A)$-modules

$$0 \longrightarrow e_i D(A) \longrightarrow e_i T(A) \longrightarrow e_i A \longrightarrow 0$$

where $e_i T(A)$ is the projective-injective indecomposable $T(A)$-module at vertex i where $e_i A$ is the projective A-module at i and $e_i D(A)$ is the injective A-module at i (see for example [27]).

3.1. **Construction of the graph of a gentle algebra.**

Given an indecomposable gentle algebra $A = kQ/I$, we will define a graph Γ_A associated to A.

As in the construction of the graph associated to a symmetric special biserial algebra, we will now extend the set $\mathcal{M}$ of maximal paths in (Q, I). Let $\overline{\mathcal{M}}$ be the set containing $\mathcal{M}$ and all those trivial paths associated to vertices i of Q that are a sink with a single arrow ending at i, a source with a single arrow starting from i or such that there is a single arrow α ending at i and a single arrow β starting at i and $\alpha\beta \notin I$. We will still call the elements of $\overline{\mathcal{M}}$ maximal paths. As a consequence of the above definition every vertex of Q lies in exactly two distinct maximal paths in $\overline{\mathcal{M}}$.

Let $\Gamma_A = (V_0, V_1, f : V_1 \to V_0 \times V_0)$ be the graph where V_0 is the (extended) set of maximal paths $\overline{\mathcal{M}}$, V_1 is the set of vertices of Q and f is the map sending an edge a_i corresponding to a vertex i of Q to the two maximal paths passing through i (note that f is well-defined since we have enlarged the set $\mathcal{M}$ by some trivial paths). Denote by $\nu(m)$ the vertex of Γ_A corresponding to the maximal path m in $\overline{\mathcal{M}}$.

Lemma 3.1. *Each maximal path m in $\mathcal{M}$ gives rise to a linear order of the edges connected to the corresponding vertex $\nu(m)$ in Γ_A.*

Proof: Let $m = a_1 \xrightarrow{\alpha_1} a_2 \xrightarrow{\alpha_2} \cdots \xrightarrow{\alpha_n} a_{n+1}$ be a maximal path in $\mathcal{M}$ and let $\nu(m)$ be the corresponding vertex in Γ_A. Then each a_i corresponds to an edge in Γ_A determined by $f(a_i) = (m, n_i)$ where n_i is the second maximal path in $\overline{\mathcal{M}}$ going through the vertex a_i of Q. Thus as edges in Γ_A, all a_i, for $1 \le i \le n+1$, are connected to $\nu(m)$. Then setting $a_i < a_j$ if there exists a subpath p of m with $s(p) = a_i$ and $t(p) = a_j$ defines a linear order on the set $\{a_1, a_2, \ldots, a_{n+1}\}$. □

Note that maximal paths in A correspond to maximal fans in Γ_A. A fan is a subgraph of Γ_A such that all edges in the subgraph are connected to a common vertex.

An example of a graph associated to a gentle algebra is given by the internal edges of a triangulation of an unpunctured Riemann surface: Let T be a triangulation of an unpunctured Riemann surface with marked points and let A be the associated gentle algebra [1]. Then $\Gamma_A = T^0$ where T^0 is the graph corresponding to the internal edges of T (see Section 4 for more details).

We will now construct an extended quiver Q_E. It follows from the proof of Lemma 3.1 that if $a_1 < a_2 < \cdots a_{n+1}$ is the linear order defined by a maximal path $m \in \mathcal{M}$ then the vertex $\nu(m)$ together with the edges $a_1, a_2, \ldots, a_{n+1}$ can be embedded locally into the (clockwise oriented) plane such that the linear order corresponds to the clockwise order of the edges a_i around $\nu(m)$. Furthermore, completing the linear order $a_1 < a_2 < \cdots a_{n+1}$ to a cyclic order exactly corresponds to adding a single arrow $a_{n+1} \xrightarrow{\beta_m} a_1$.

Define Q_E to be the quiver with

- set of vertices given by the vertices in Q and
- set of arrows given by the arrows of Q together with a set of new arrows $\{\beta_m |$ for every $m \in \mathcal{M}$ where $s(\beta_m) = t(m)$ and $t(\beta_m) = s(m)\}$.

Lemma 3.2. *With the notations above the quiver Q_E is the quiver of the trivial extension algebra $T(A)$ of A.*

Proof: By [11] and Lemma 2.1 the arrows of $Q_{T(A)}$ are given by the arrows of Q plus for every maximal path $m \in \mathcal{M}$ a new arrow β_m such that $s(\beta_m) = t(m)$ and $t(\beta_m) = s(m)$. □

The following Lemma follows directly from the definitions of Q_E and the definition of the quiver of a Brauer graph algebra.

Lemma 3.3. *With the notations above the quiver Q_E is the quiver of the Brauer graph algebra with associated Brauer graph Γ_A and with cyclic ordering of the edges of Γ_A around each vertex induced by the arrows in Q_E.*

We can now prove Theorem 1.2 which we restate here for convenience.

Theorem 1.2 *Let $A = kQ/I$ be a gentle algebra and let Γ_A be its graph. Let B be the Brauer graph algebra defined on Γ_A with cyclic ordering induced by the maximal paths in A and with multiplicity equal to one at every vertex. Then the trivial extension $T(A)$ of A is isomorphic to B and $G_{T(A)} = \Gamma_A$ where $G_{T(A)}$ is the graph of $T(A)$ as symmetric special biserial algebra.*

Proof: The Brauer graph algebra associated to Γ_A and also $T(A)$ are both symmetric special biserial algebras. The projective indecomposable modules of the Brauer graph algebra can be read of the graph Γ_A as described above. The projective indecomposable $T(A)$-modules are given by short exact sequences as described in section 3 above. That is

$$0 \longrightarrow e_i D(A) \longrightarrow e_i T(A) \longrightarrow e_i A \longrightarrow 0$$

for every vertex i in Q. Let m_i and n_i be the two non-trivial maximal paths of A going through the vertex i. Suppose

$$m_i = i_0 \xrightarrow{\alpha_0} i_1 \xrightarrow{\alpha_1} \cdots \xrightarrow{\alpha_{k-1}} i_k = i \xrightarrow{\alpha_k} \cdots \xrightarrow{\alpha_{m-1}} i_m$$

and

$$n_i = j_0 \xrightarrow{\gamma_0} j_1 \xrightarrow{\gamma_1} \cdots \xrightarrow{\gamma_{l-1}} j_l = i \xrightarrow{\gamma_l} \cdots \xrightarrow{\gamma_{l-1}} j_n.$$

Then e_iA is the string module $M(w)$ with string $w = i_m \xleftarrow{\alpha_{m-1}} \cdots i_{k+1} \xleftarrow{\alpha_k} i \xrightarrow{\gamma_l} j_{l+1} \cdots \xrightarrow{\gamma_{n-1}} j_n$ and $e_iD(A)$ is the string module $M(v)$ with string $v = i_0 \xrightarrow{\alpha_0} i_1 \cdots i_{k-1} \xrightarrow{\alpha_{k-1}} i \xleftarrow{\gamma_{l-1}} j_{l-1} \cdots \xleftarrow{\gamma_0} j_0$. Then in $Q_{T(A)}$ there are arrows $i_m \xrightarrow{\beta_{m_i}} i_0$ and $j_n \xrightarrow{\beta_{n_i}} j_0$ such that the projective-injective indecomposable $T(A)$-module $e_iT(A)$ is given by the biserial module that has the simple at i as top and socle and whose heart $e_iT(A)/\operatorname{rad} e_iT(A)$ is given by a direct sum of uniserial modules given by the direct strings $i_{k+1} \xrightarrow{\alpha_{k+1}} \cdots \xrightarrow{\alpha_{m-1}} i_m \xrightarrow{\beta_{m_i}} i_0 \xrightarrow{\alpha_0} i_1 \xrightarrow{\alpha_1} \cdots \xrightarrow{\alpha_{k-2}} i_{k-1}$ and $j_{l+1} \xrightarrow{\gamma_{l+1}} \cdots \xrightarrow{\gamma_{n-1}} j_n \xrightarrow{\beta_{n_i}} j_0 \xrightarrow{\gamma_0} j_1 \xrightarrow{\beta_1} \cdots \xrightarrow{\beta_{l-2}} j_{l-1}$. Denote by a_i the edge of Γ_A corresponding to the vertex i of Q and let $\nu(m_i)$ and $\nu(n_i)$ be the vertices at either end of a_i corresponding to the maximal paths m_i and n_i. Then we have seen that there is a local embedding of Γ_A into the oriented plane such that the orientation of the plane gives rise to a cyclic ordering of the edges i_s around $\nu(m_i)$ and j_t around $\nu(n_i)$. Then by definition, the projective-injective indecomposable B-module P_i^B at vertex i has the same k-basis of paths in Q_E as $e_iT(A)$.

We will now consider the situation where either n_i or m_i correspond to the trivial path at i. Assume $n_i = e_i$. Then $e_iT(A)$ is uniserial corresponding to the string $i_k = i \xrightarrow{\alpha_k} i_{k+1} \xrightarrow{\alpha_{k+1}} \cdots \xrightarrow{\alpha_{m-1}} i_m \xrightarrow{\beta_{m_i}} i_0 \xrightarrow{\alpha_0} i_1 \xrightarrow{\alpha_1} \cdots \xrightarrow{\alpha_{k-1}} i_k = i$. Then by an argument similar to the one above we see that the edge a_i in Γ_A corresponding to i is a leaf and that P_i^B, the projective indecomposable B-module at vertex i has the same k-basis as $e_iT(A)$. Then by Lemma 2.2, the algebras $T(A)$ and B are isomorphic. From the structure of the projective indecomposable modules for B and $T(A)$ it immediately follows that the graphs Γ_A and $G_{T(A)}$ are isomorphic. That the cyclic orderings of the edges around each vertex in Γ_A and $G_{T(A)}$ coincide, follows from the fact that both cyclic orderings are induced by the arrows in Q_E. □

4. Admissible Cuts in Symmetric Special Biserial Algebras

One way of deleting arrows in quivers and constructing new algebras in this way is through the notion of admissible cuts of finite dimensional algebras which has been studied for example in [5, 10, 19], [9], and see also [20].

Given a Brauer graph algebra $\Lambda = kQ_\Lambda/I_\Lambda$ with multiplicity one at all vertices of the corresponding Brauer graph, we define an *admissible cut D of Q_Λ* to be a subset of arrows in Q_Λ formed of exactly one arrow in every vertex cycle in Q_Λ (see section 2.3.1 for the definition of a vertex cycle). Note that this corresponds to the cutting set defined in [12]. An *admissible cut of Λ* is the algebra kQ_Λ/J_Λ where J_Λ is the ideal generated by $I_\Lambda \cup D$.

Admissible cuts defined as above are a way of constructing gentle algebras from symmetric special biserial algebras. This is very closely related to the results in [10] and [12].

We now prove Theorem 1.3 which states:

Theorem 1.3 *Let $\Lambda = kQ_\Lambda/I_\Lambda$ be a Brauer graph algebra with multiplicity one at all vertices in the associated Brauer graph. Let A be an admissible cut of Λ. Then A is gentle and $T(A)$ and Λ are isomorphic.*

Proof: Let $Q = Q_\Lambda \setminus D$ where D is an admissible cut of Q_Λ. Then define I to be the ideal $I_\Lambda \cap kQ$ and let J_Λ be the ideal of Λ generated by $I_\Lambda \cup D$. Then it follows from the second isomorphism theorem that $A = kQ_\Lambda/J_\Lambda \simeq kQ/I$.

We first show that A is gentle. Since $Q \subset Q_\Lambda$, it clearly satisfies (S1). All zero relations in I_Λ are of the form $p - q$ and $p\alpha$ for p, q irreducible cycles and α an arrow in Q_Λ or are monomial relations $\alpha\beta$ of lengths two where α and β belong to two distinct vertex cycles. By definition of D, any maximal cyclic path p appearing in a relation generating I_Λ, contains exactly one arrow in D. Thus it is not in $I = I_\Lambda \cap kQ$ and therefore I is generated by paths of length two only. In order to show that (S4) holds, suppose that α, β, β' are arrows in $Q \subset Q_\Lambda$ such that $\alpha\beta \in I$ and $\alpha\beta' \in I$. Then $\alpha\beta \in I_\Lambda \supset I$ and also $\alpha\beta' \in I_\Lambda \supset I$ and $\beta = \beta'$, since (S4) holds for Λ. A similar argument holds for arrows preceding α and thus (S4) holds for kQ/I. To show (S2), suppose α, β, β' are arrows in Q such that $\alpha\beta$ and $\alpha\beta'$ are non-zero in kQ. Suppose further that $\alpha\beta \notin I$ and $\alpha\beta' \notin I$. Since $I = I_\Lambda \cap kQ$ this implies $\alpha\beta \notin I_\Lambda$ and $\alpha\beta' \notin I_\Lambda$. Because (S2) holds for Λ this implies $\beta = \beta'$. By a symmetric argument for arrows preceding α, it follows that (S2) holds for kQ/I.

There is a bijection between vertex cycles in Λ and maximal paths in A. Namely let $s \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} \dots \xrightarrow{\alpha_{k-1}} s_k \xrightarrow{\alpha_k} s$ be a vertex cycle ν in Λ. Suppose $\alpha_i \in D$ for some $0 \leq i \leq k$. Then no other arrow in ν is in D and $p = s_{i+1} \xrightarrow{\alpha_{i+1}} s_{i+2} \dots s_k \xrightarrow{\alpha_k} s \xrightarrow{\alpha_0} s_1 \dots s_{i-1} \xrightarrow{\alpha_{i-1}} s_i$ is a path in (Q, I). As subpaths of p, we have $\alpha_{i-1}\alpha_i \notin I$ and $\alpha_i\alpha_{i+1} \notin I$. If there exists an arrow β in Q with $s(\beta) = t(\alpha_{i-1})$ then by (S2) $\alpha_{i-1}\beta \in I$. And hence $p\beta \in I$. Similarly, if there exists an arrow $\gamma \neq \alpha_i$ in Q such that $t(\gamma) = s(\alpha_{i+1})$ then $\gamma p \in I$. Hence p is a maximal path in (Q, I). Conversely, every non-zero path in Λ is a subpath of a unique vertex cycle. Thus if p is a maximal path in A then p is a subpath of a vertex cycle ν. But since no two distinct maximal paths in A can have a common arrow and since we have cut exactly one arrow in each vertex cycle of Λ, p is the path that starts at the end of the cut arrow in ν and ends at the start of the cut arrow in ν. Thus the vertices of the graph G_Λ of the symmetric special biserial algebra Λ and the vertices of the graph Γ_A of the gentle algebra A coincide.

Furthermore, the vertices of Q_Λ in a p-cycle associated to a vertex cycle ν correspond up to rotation to the vertices of the corresponding maximal path p in (Q, I). Recall further that the edges in G_Λ are given by connecting the two p-cycles containing the same vertex and that the edges in Γ_A are given by connecting the two maximal paths containing the same vertex. The cyclic order of the edges around a vertex in G_Λ is induced by the arrows in the corresponding vertex cycle and the maximal paths in (Q, I) induce the linear order of the edges in Γ_A. As described above the latter can be extended to a cyclic ordering. With this induced cyclic ordering on Γ_A, the graphs G_Λ and Γ_A are isomorphic and have the same cyclic ordering of the edges around each vertex. Thus by Theorem 1.2 there is an isomorphism $T(A) \simeq \Lambda$. □

Note that a Brauer graph algebra with multiplicity function identically equal to one corresponds to a symmetric special biserial algebra kQ/I where all relations in I that are not monomial of length 2, are given by irreducible cycles, that is cycles that are no proper power of a cycle.

The following Corollary immediately follows from Theorem 1.3.

Corollary 1.4 *Every Brauer graph algebra with multiplicity function identically one is the trivial extension of a gentle algebra.*

Finally we end the paper with two applications to gentle algebras associated to marked Riemann surfaces.

Let S be a connected oriented Riemann surface with boundary ∂S and let M be a non-empty finite set of points in the boundary ∂S. Call the pair (S, M) an unpunctured surface. Let T be an ideal triangulation of (S, M) and denote by T^0 the internal arcs of T, that is arcs not connecting two adjacent marked points on the same boundary component. In [1] and in [16], a finite dimensional algebra is associated to the triple (S, M, T), called in [16] the Jacobian algebra of (S, M, T). In [1] this algebra was shown to be gentle.

Corollary 1.5 *Let A be a gentle algebra arising from a triangulation T of a marked unpunctured Riemann surface (S, M). Then the trivial extension $T(A)$ of A is isomorphic to the Brauer graph algebra of T^0 where T^0 is the graph consisting of the internal edges of T with cyclic ordering induced by the orientation of S.*

Proof: Let $\Lambda = KQ_\Lambda/I_\Lambda$ be the Brauer graph algebra associated to T^0 and let $A = kQ_A/I_A$ be the (gentle) Jacobian algebra associated to the triangulation T of (S, M). We start by constructing an admissible cut $J = kQ_J/I_J$ of Λ. For this embed the quiver Q_Λ into (S, M, T) via T^0. That is every edge of T^0 corresponds to a vertex of Q_Λ and the arrows of Q_Λ correspond to the ordering of the edges of T^0 in the cyclic ordering induced by the orientation of S. Then each vertex of T^0 that is not a leaf vertex, corresponds to a cycle in Q_Λ and there is exactly one arrow in every such cycle crossing either one or two boundary segments of T (crossing one boundary segment precisely when the vertex under consideration is the only marked point in its boundary component of (S, M)). The collection of these arrows is an admissible cut D of Q_Λ and $Q_A = Q_\Lambda \setminus D = Q_J$. For an example, see figure 1 below.

Let ρ_Λ be the set of relations generating I_Λ as described in Section 2.2. Then $\rho_\Lambda \cap kQ_J$ is a generating set of relations for I_J. Let ρ_A be a set of relations generating I_A. Then ρ_A consists of paths $\alpha\beta$ of lengths two where α and β are two consecutive arrows in an internal triangle of T. As arrows of Q_Λ, α and β are in two distinct vertex cycles of Λ and $\alpha\beta \in \rho_\Lambda$. Hence, since $Q_A = Q_J$,

$\alpha\beta \in \rho_\Lambda \cap kQ_J$. Conversely, any non-zero path $\alpha\beta$ in ρ_J is given by two arrows α and β belonging to two distinct vertex cycles of Λ. Thus $\alpha\beta$ corresponds to two consecutive arrows in an internal triangle of T. From this we conclude that the generating sets of I_J and I_A coincide and hence A is isomorphic to J. The fact that $T(A)$ is isomorphic to Λ then follows from Theorem 1.3. □

Example. We consider the gentle Jacobian algebra given by the triangulation of the marked annulus in figure 1.

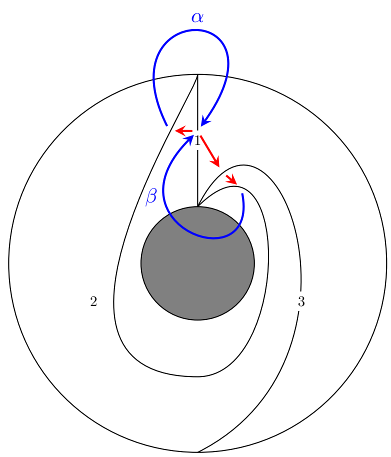


Figure 1: Triangulation of an annulus and associated arrows as described in Corollary 1.5 and its proof.

The quiver given by the red arrows is the quiver of the gentle Jacobian algebra associated to the triangulation T of the annulus in figure 1. Note that the vertices of this quiver correspond to the three internal edges of T, marked 1,2, and 3. These edges constitute precisely the graph T^0. The quiver given by the red arrows together with the blue arrows is the quiver of the trivial extension of A or equivalently the quiver of the Brauer graph algebra associated to T^0. The blue arrows form the admissible cut D described in the proof of Corollary 1.5 above. Note that α corresponds to an arrow crossing two distinct boundary segments of (S, M) and β is an arrow crossing a single boundary segment of (S, M).

Given a triangulation T of (S, M), in [9] surface algebras were defined by cutting T at internal triangles. This gives rise to a partial triangulation of (S, M). A surface algebra A is again a finite dimensional gentle algebra and it can be constructed by associating a bound quiver (Q, I) to the partial triangulation of (S, M) such that $A = kQ/I$ (see [9]).

Corollary 1.6 *Let A be a surface algebra of a partial triangulation T of an unpunctured Riemann surface (S, M) with set of marked points M. Then the trivial extension $T(A)$ of A is isomorphic to the Brauer graph algebra of T^0 where T^0 is the graph consisting of the internal edges of T and with cyclic orientation induced by the orientation of S.*

Proof: Again, as in the proof of Corollary 1.5, we construct an admissible cut J of the Brauer graph algebra associated to T^0 and we show that J is isomorphic to A. However, in A not all relations result from paths of lengths two lying in an internal triangle. There are also relations generated by paths of lengths two lying in a quadrilateral resulting from the 'cut' of an internal triangle (see [9]). However, these relations also appear in the corresponding Brauer graph algebra and its admissible cut. Taking this into account, the proof then is similar to the proof of Corollary 1.5. □

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