

Limits on Neutron Lorentz Violation From Pulsar Timing

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Abstract

Pulsars are the most accurate naturally occurring clocks, and data about them can be used to set bounds on neutron-sector Lorentz violations. If $SO(3)$ rotation symmetry is completely broken for neutrons, then pulsars' rotation speeds will vary periodically. Pulsar timing data limits the relevant Lorentz-violating coefficients to be smaller than 1.7×10^{-8} at at least 90% confidence.

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Since the discovery that Lorentz symmetry could be broken spontaneously in string theory [1], there has been a great deal of interest in the study of Lorentz violation. Besides string theory, many other candidate theories of quantum gravity also predict Lorentz violation, at least in certain regimes. If Lorentz violations were observed experimentally, they would be of tremendous importance, potentially telling us a great deal about Planck scale physics. Experimental searches for Lorentz violation have thus far failed to produce any positive results. These searches have included studies of matter-antimatter asymmetries for trapped charged particles [2, 3, 4, 5] and bound state systems [6, 7], determinations of muon properties [8, 9], analyses of the behavior of spin-polarized matter [10, 11], frequency standard comparisons [12, 13, 14, 15], Michelson-Morley experiments with cryogenic resonators [16, 17], Doppler effect measurements [18, 19], measurements of neutral meson oscillations [20, 21, 22, 23], polarization measurements on the light from distant galaxies [24, 25, 26, 27], analyses of the radiation emitted by energetic astrophysical sources [28, 29], and others.

Although the work by Kostelecký and Samuel on string theory [1] has been a major impetus for the study of Lorentz violation in the last ten years, there are also other situations in which Lorentz violation is a possibility. While there is no experimental evidence for Lorentz violation yet, there are numerous theoretical reasons to believe that Lorentz violation may be possible. A Lorentz- and CPT-violating effective field theory, the standard model extension (SME), has been developed in detail [30, 31, 32], to give a general effective field theory parameterization of all possible forms of Lorentz violation. The SME framework is useful because it does not tie us to any particular underlying mechanism for Lorentz symmetry breaking. Many basic issues in the SME, including stability and causality [33] and one-loop renormalization [34, 35], have been examined. The SME contains coefficients parameterizing all possible Lorentz violations. The experiments mentioned above have placed very tight constraints on some of these coefficients; however, the bounds of many of the SME coefficients are still somewhat muddled, and some of the coefficients have not been bounded at all.

Some of the most accurate tests of Lorentz invariance are clock comparison experiments. Today's atomic clocks are extremely accurate, and this makes them wonderful tools for making precision measurements. On the other hand, the most accurate naturally occurring clocks are pulsars, and it is natural to ask whether observations of these rotating neutron stars could also be used to test Lorentz symmetry in a useful fashion. In fact, we shall see that pulsar data can be used to place some interesting bounds on Lorentz violations in the neutron sector.

A certain subset of the SME coefficients will be relevant for our analysis of pulsar data. The Lagrange density for the neutrons in our theory is

$$\mathcal{L} = \bar{\psi}[(\gamma^\mu + c^{\nu\mu}\gamma_\nu)i\partial_\mu - m]\psi, \tag{1}$$

where ψ is the neutron field. The use of covariant notation is possible because the theory is invariant under observer Lorentz transformations. That is, the physics is independent of

the choice of coordinates used by an observer. The theory is not invariant under particle Lorentz transformations, however.

c contains nine parameters that contribute to Lorentz-violating physics at leading order. The trace $c^\mu{}_\mu$ only affects the overall normalization of the electron field, and the antisymmetric part of c has no effects at first order, where it is equivalent to a redefinition of the Dirac matrices. There are many other possible Lorentz-violating terms in the neutron sector. However, the c terms should be the most important in the context of pulsar timing. Although we observe pulsars through the electromagnetic radiation they emit, the details of these radiation processes are unimportant, and we do not need to consider possible Lorentz violations in the photon sector. The neutrons are of course also subject to gravitational interactions, but we shall assume that the gravity sector is conventional. Angular momentum is not conserved according to (1), and this is the effect that we shall seek to exploit; this is similar to what was done in [36], where the precession of the sun's axis in a Lorentz-violating background was considered.

The angular momentum nonconservation effects considered here are quite different than those studied in the torsion pedulum experiment [11], which looks for Lorentz-violating effects in a spin system. The spin pedulum experiment is only sensitive to axial vector effects, because the spin operator is itself an axial vector. However, it should not be surprising that a given experiment will only be sensitive to a subset of the possible Lorentz-violating coefficients. As we shall see, pulsar timing experiments are most sensitive to the c term, which does not have an axial vector form.

The best current bounds on the neutron c coefficients come from reanalysis of older clock comparison experiments [13]. However, most of the bounds combine the neutron c with other coefficients for Lorentz violation, and there could potentially be cancellations between the different coefficients. Only one of the four experiments analyzed in [13] provides bounds on the neutron c coefficients only, and the resulting bounds only cover two of the five possible coefficients that are observable nonrelativistically. None of the clock experiments provide bounds on the combination $c_Q = c_{XX} + c_{YY} - 2c_{ZZ}$, where X , Y , and Z are coordinates in a sun-centered celestial equatorial frame. Moreover, any results gleaned from reanalysis of old experimental data must be considered very cautiously. It is always possible that the experimenters made some unrecorded assumptions when they constructed the experiment, which could throw the results, when analyzed in the context of Lorentz violation, into doubt.

So it would be nice to find alternative bounds on neutron-sector Lorentz violations, and pulsars are natural systems to consider. Our model of a pulsar will be rather crude, but it should allow us to pick out the dominant Lorentz-violating effects. The most important assumptions that we shall make are that a pulsar is indeed composed of predominantly neutrons and that it is supported by degeneracy pressure. For more exotic objects like quark stars, similar bounds ought to be available, but determining them would be substantially more complicated.

The strength of the bounds we may derive will be determined by the empirically ob-

served regularity of the pulses we observe from neutron stars. Real pulsars will wobble slightly even in the absence of Lorentz violation, because they are not perfectly rigid rotating systems. The magnetic and spin axes of a pulsar are not aligned, which automatically implies some asymmetry in the internal angular momentum distribution. If conventional mechanisms of wobbling produce larger effects than any Lorentz violations, then no Lorentz-violating effects would be observable, and this limits the accuracy of our bounds on c . Moreover, the ultimate limit on the accuracy of our bounds is not necessarily the stability of the pulsars themselves. Effects in a pulsar's environment may introduce timing errors as well. Even if the origins of these timing errors are fairly well understood, the quantitative details of how the observed pulses are affected must be determined from empirical fits to the data. A Lorentz violation signal could be hidden amidst the effects of more conventional physics; we must therefore consider the case in which Lorentz violation is the largest source of precession and timing errors in the system. This simplifies the analysis, but it limits the accuracy of our bounds.

All the fermion coefficients in the minimal SME, except c and one unphysical coefficient a , have nontrivial Dirac matrix structures. The sign of any contributions from the other coefficients will depend on the spin states of particles. However, the neutrons in a neutron star are not in a coherent spin state. This results in averaging that will diminish the effects of any spin-dependent Lorentz-violating operators. So only the effects associated with the c coefficients will receive coherent contributions from all the neutrons in a pulsar. We are therefore justified in considering only the c coefficients in the neutron sector.

We shall also consider only those components of c that have observable effects on nonrelativistic physics. Although the neutrons in a pulsar are moving, this motion is not highly relativistic. For a pulsar with a very short rotation period of 2 ms and a radius of 10 km, the surface speed is $v \approx 0.1$. Any effects caused by c_{0j} or $c_{00} = -c_{jj}$ terms will be suppressed by at least one factor of v . In reality, the suppression will be substantially greater, because most of the neutrons in the pulsar are located closer to the core and are moving more slowly than those at the surface; also, there will be some cancellation among the relativistic effects, because neutrons on opposite sides of the rotating pulsar are moving in opposite directions. In light of these facts, and because we shall be considering pulsars with periods significantly larger than 2 ms, we shall neglect all Lorentz-violating effects except those due to c_{jk} .

The suppression of the observable effects of c_{0j} and c_{00} by powers of v deserves some explanation. If we look at the energy-momentum relation, the absolute magnitude of the contributions proportional c_{0j} or c_{00} are not suppressed. Nonrelativistically, in fact, a contribution involving, say, c_{01} will be larger than one involving c_{12} by a factor of $\mathcal{O}(m/|p_2|)$. However, no contribution can be identified as a signature of Lorentz violation unless its effects on particles with different rest coordinates are compared. c_{0j} and c_{00} break boost invariance, and so have different effects for particles in different rest frames. These different rest frames are connected by Lorentz boosts. The observable differences in quantities come from the $-\vec{v} \cdot \vec{p}$ part of the boost transformation expression $\gamma(E - \vec{v} \cdot \vec{p})$

and so are suppressed by one factor of $\mathcal{O}(\vec{v} \cdot \vec{p}/m)$ for each time index on $c_{\mu\nu}$. The net result is a suppression of observable boost invariance violation effects relative to rotation invariance invariance ones by powers of v . This general argument has been verified by the calculation of specific quantities in, for example, [13].

The Lorentz-violating c_{jk} coefficients will affect the moments of inertia of a neutron star. These modifications are easy to determine, because to a very good approximation, a neutron star is spherical. We are justified in neglecting any deviations from sphericity not related to Lorentz violation by our assumption that the largest wobble effects that we observe are due to the Lorentz violation. This holds even with the Lorentz violation, because the determination of the star's shape is basically a static problem, and c does not affect the isotropy of the neutrons' degeneracy pressure [37].

To leading order, the effect of the c_{jk} is simply to replace the standard kinetic energy, $\vec{p}^2/2m$, with $(\vec{p}^2 - 2c_{jk}p_j p_k)/2m$, and we see that only the symmetric part $c_{(jk)} = c_{jk} + c_{kj}$ contributes. In some coordinate frame, $c_{(jk)}$ will be diagonal, and this is the frame of the principal axes. To calculate the effective moment of inertia around the z -axis in this frame, we may imagine that the star is rotating around this axis. The kinetic energy (and hence I_{33}) is then proportional by an integral over the mass distribution, with the integrand proportional to $(1 - 2c_{11})p_1^2 + (1 - 2c_{22})p_2^2$ (instead of simply $p_1^2 + p_2^2$). Since the mass distribution is symmetric around the axis of rotation, the p_1^2 and p_2^2 terms contribute equally, and the net effect is to multiply the conventional expression for the moment of inertia I_{33} by $1 - (c_{11} + c_{22})$. Since the overall normalization of the moment of inertia is an empirically determined quantity, any Lorentz-violating effect that contributes only to the trace I_{jj} will not be detectable, and an overall rescaling of I allows us to replace the factor $1 - c_{11} - c_{22}$ with $1 + c_{33}$. Therefore the observable effect of Lorentz violation will simply be to modify the moment of inertia to $I_0(1 + c_{33})$, where I_0 is its value in the absence of rotation invariance violation. It immediately follows that the complete tensor of inertia is $I_{jk} = I_0 \left[\delta_{jk} + \frac{1}{2}c_{(jk)} \right]$.

The preceding arguments suppose that the distribution of the neutrons in momentum space is not significantly modified by the Lorentz violation; however, this is a well justified supposition. A neutron star can be roughly seen as a sphere of degenerate neutron matter confined by a gravitational field. The structure of the Fermi sea is largely unchanged by the Lorentz violation. Most occupied neutron states in a star lie deep in the Fermi sea, and these states will be occupied regardless of whether or not Lorentz violation exists; this is true whether the Lorentz violation in question is in the neutron sector or the strong interaction sector. The shape of the Fermi surface will be distorted, but this only affects the occupations numbers of states near the top of the sea. This is a small fraction of the total number of neutrons present, so any resulting effects will be suppressed; unlike the changes to the moments of inertia, these are not coherent effects in which all the particles in the Fermi sea contribute equally.

So the central question is how the modified I affects the observed pattern of pulses. In fact, a nonzero c will generally lead to slow periodic modulations in the pulse frequency

as the axis of rotation wobbles. Working out this effect can be tricky; however, we are aided by our empirical knowledge that the rotation is extremely stable. Therefore, we are justified in assuming that any wobbles are small oscillations around a stable rotation axis, which according to the “tennis racket theorem” must be the principal axis with either the largest or smallest moment of inertia. If the dominant effect causing the spin precession is Lorentz violation, we may separate $\vec{\omega}$ into $\vec{\omega}_0 + \vec{\delta\omega}$, where $\vec{\omega}_0$ lies along one of the eigenvectors of $c_{(jk)}$, and $\vec{\delta\omega}$ is the fluctuating part.

We shall consider $\vec{\delta\omega}$ as a small correction, but we do not really expect it to be all that much smaller than $\vec{\omega}_0$. The spin orientation of pulsar is determined by the orientation of its stellar progenitor, which is essentially random. The quantity $\langle |\vec{\delta\omega}| \rangle / |\vec{\omega}_0|$, with the time average of the fluctuating part, characterizes just how closely aligned $\vec{\omega}$ is with an eigenvector of $c_{(jk)}$; for a particular neutron star, the probability that this quantity is smaller than, for example, $\frac{1}{3}$ is less than $4 \left[\pi \left(\frac{1}{3} \right)^2 \right] / 4\pi \sim 10^{-1}$. (The factor of four comes from the fact that there are four stable directions along which $\vec{\omega}_0$ may lie.) However, while we do not expect $\vec{\delta\omega}$ to be strongly suppressed, the leading order analysis still captures the essential behavior; including possible higher-order corrections does not weaken the calculated bounds on c in any way.

In the principal axis frame, the Euler equations are, to leading order in $\vec{\delta\omega}$,

$$\begin{bmatrix} \dot{\delta\omega}_1 \\ \dot{\delta\omega}_2 \\ \dot{\delta\omega}_3 \end{bmatrix} = \begin{bmatrix} 0 & (c_{22} - c_{33})\omega_{03} & (c_{22} - c_{33})\omega_{02} \\ (c_{33} - c_{11})\omega_{03} & 0 & (c_{33} - c_{11})\omega_{01} \\ (c_{11} - c_{22})\omega_{02} & (c_{11} - c_{22})\omega_{01} & 0 \end{bmatrix} \begin{bmatrix} \delta\omega_1 \\ \delta\omega_2 \\ \delta\omega_3 \end{bmatrix}. \quad (2)$$

It is obvious from the form of these equations that the physical effects can depend only on the differences in the eigenvalues of $c_{(jk)}$. Since $\vec{\omega}_0$ is along one of the principal axes, two of its components vanish. Taking the nonzero component to be ω_{03} , the eigenvalues of the matrix in (2) are approximately 0 and $\pm i\omega_{03}\sqrt{(c_{33} - c_{11})(c_{33} - c_{22})}$. So the instantaneous angular velocity $\vec{\omega}$ will undergo oscillations with frequency

$$\Omega = \omega_0 \sqrt{(c_{33} - c_{11})(c_{33} - c_{22})}. \quad (3)$$

Ω^{-1} sets the scale of the time variation in the rotation frequency (unless c_{11} and c_{22} are equal); our bounds come from the fact that the Ω arising from Lorentz violation must be smaller than the frequency of the fastest observed modulation in $\omega = |\vec{\omega}|$. The instantaneous rotation frequency is a quantity that we can observe directly, because the pulsar beam sweeps over us every rotation. The magnitude of the angular velocity varies with time as

$$|\vec{\omega}| \approx |\vec{\omega}_0| + \frac{\langle |\vec{\delta\omega}| \rangle^2}{4 |\vec{\omega}_0|} (c_{11} - c_{22}) \cos(2\Omega t + \psi), \quad (4)$$

where we have evaluated each of the terms to lowest nonvanishing order in c . If the angular momentum is oriented precisely along an eigenvector of $c_{(jk)}$, there is no variation

in $|\vec{\omega}|$ and hence no observable effect. However, we have already argued that a very precise alignment of $\vec{\omega}$ and an eigenvector is unlikely, and that it is almost 90% likely that $\alpha \equiv \langle |\delta\vec{\omega}| \rangle / |\vec{\omega}_0| > \frac{1}{3}$.

There will be no time variation in ω if any two of the eigenvalues of $c_{(jk)}$ are equal. This complicates our task of setting bounds on the coefficients. If $SO(3)$ rotation invariance is broken, yet there is still an unbroken $SO(2)$ subgroup of the Lorentz group (so that the universe has cylindrical symmetry), we would expect to see no signal. The c_Q part of c has cylindrical symmetry, so it cannot be bounded by these measurements.

The bounds we can derive will therefore be somewhat curious in form. If the lowest frequency of any observed modulations of ω is ϖ , and the instantaneous angular frequency is known to an accuracy of $\Delta\omega$, then *either* $2\Omega < \varpi$ *or* $|c_{11} - c_{22}| < 4\alpha^{-2}\Delta\omega/\omega_0$. The second possibility occurs if the pulsar’s wobbles result in a change in ω too small to be observed. Because of the dual “either/or” nature of these bounds, it is impossible with our methods to bound any the $c_{(jk)}$ coefficients individually. Yet we can learn something about the characteristic order of magnitude of the coefficients, assuming there are no special cancellations. However, there may well be special cancellations; we know that if $c_{(jk)}$ is cylindrically symmetric—certainly not an unreasonable possibility—there will be no signal. So we must be careful making generalizations.

So it is important to understand which components of c these kinds of measurements are actually sensitive to. The measurable quantities are really the eigenvalue differences, but all three differences must be nonzero in order for the Lorentz violation to give a nonzero signal. The eigenvalue differences are bounded in two distinct ways, but in general, the relevant sensitivities will be tied together experimentally, and we expect ϖ and $\Delta\omega$ not to be too different. At 90% confidence (this confidence level reflecting the probability that the wobbles are not suppressed by a too small $\alpha < \frac{1}{3}$), we may place the following bound on the c_{jk} coefficients:

$$\min(|c_{11} - c_{22}|, |c_{11} - c_{33}|, |c_{22} - c_{33}|) < \left[\frac{\max(\varpi/2, 36\Delta\omega)}{\omega_0} \right]. \quad (5)$$

This is still in the frame where $c_{(jk)}$ is diagonal, so c_{11} , c_{22} , and c_{33} are eigenvalues. The bound (5) is therefore effectively independent of the pulsar orientation. This is actually advantageous, because many pulsars have largely unknown spin orientations.

To get actual bounds on the c coefficients, we must now look at data from real pulsars. We shall rely primarily on data from the pulsar PSR B1509–58. This neutron star has been under observation for decades, with no glitches (small but abrupt changes in the period) observed. The timing data have been analyzed very carefully, in a fully phase-coherent fashion [38]. The pulsar has a frequency ν of about 6.6 Hz, known to a fractional accuracy of about 5×10^{-10} . The highest-frequency residuals seen in the timing plots in [38] have periods on the order of 2 years. (The dominant residuals have periods in the 5–10 year range.) The precision with which the frequency is known controls the strength of our

limit, and we have that

$$\min(|c_{11} - c_{22}|, |c_{11} - c_{33}|, |c_{22} - c_{33}|) < 1.7 \times 10^{-8} \quad (6)$$

at 90% confidence. The limit that could be derived from ϖ would be an order of magnitude better, if the resolution of the instantaneous frequency could be improved.

As stated, this bound initially appears to be at 90% confidence. What limits this is the possibility that the pulsar's spin direction could be very closely aligned with an eigenvector of $c_{(jk)}$, which would suppress the variation in ω . However, it seems even more unlikely that there is a quirk in the orientation of PSR B1509–58 when the data from other pulsars are included. For example, the data from PSR B0540–69 have been subjected to the same kind of analysis [39]. The analysis is trickier, since PSR B0540–69 has been observed to glitch, and a much more complicated fitting procedure is required. Naively, the results seem to imply an even stronger bound than (6), although the difficulties of the data analysis leave some lingering questions. We therefore treat this data as merely further confirmation that the result (6) is not merely the result of statistical fluke.

It also is not completely impossible, although it seems very unlikely, that some other kind of effect could be masking a signal for Lorentz violation. For example, orbital motions at a frequency close to 2Ω would produce Doppler shifts affecting the observed pulsation frequency, and there could conceivably be cancellation between the two effects. However, this possibility requires a significant conspiracy of circumstances. In fact, the bound (6) is probably still relatively conservative.

The bound (6) is complementary to the bounds found in [13]. Clean bounds on just c have previously only been available for $c_{XX} - c_{XY}$ and c_{XY} . These are different degrees of freedom than are constrained here. The limits in [13] are very tight, at the 10^{-27} level, much better than pulsar bounds. However, there are lingering questions about their validity, because they rely on reanalyses of older experimental data sets. So the present astrophysical bounds are interesting because they are more secure and because of their complementary structure.

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