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Discussion

Comment on “Growth uncertainty, generalized disappointment aversion and production-based asset pricing”



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1. Overview

A model that jointly explains aggregate asset price and macroeconomic dynamics has been elusive. The literature in macro-finance has evolved significantly since the early efforts of, e.g., [Jermann \(1998\)](#), [Tallarini \(2000\)](#), and [Boldrin et al. \(2001\)](#), but we still have not arrived at an agreed upon workhorse macro-finance model: fitting one moment better often leads to fitting other moments worse, calibrations are typically quite aggressive in order to quantitatively account for the data,¹ and parsimony is increasingly giving way to parameter proliferation and ‘black-box’ models where it is harder to fully understand all the mechanisms at play.

Relative to this, [Liu and Miao \(2015\)](#) take a more classic, parsimonious approach. They consider a standard real business cycle model with a representative firm and a representative agent, where the technology process is estimated in a straightforward way from the data. While the firm has a standard Cobb–Douglas production function with convex capital adjustment costs, the agent is endowed with generalized disappointment aversion (GDA) preferences and the exogenous technology growth process has time-varying mean and volatility. Introducing GDA preferences into a standard production-based model is the marginal contribution of the paper. Like [Epstein and Zin \(1989\)](#) preferences, GDA preferences allow for a preference for early resolution of uncertainty, and so endogenous long-run consumption risks are priced (see [Kaltenbrunner and Lochstoer 2010](#)). Time-varying mean and volatility in the technology process under Epstein–Zin (EZ) utility has been considered in prior literature (e.g., [Croce 2014](#), [Malkhozov 2014](#)), as has disappointment aversion (DA) preferences ([Campanale et al., 2010](#)). Relative to the latter, which also features first-order risk aversion, GDA preferences can generate endogenously counter-cyclical risk aversion ([Routledge and Zin 2010](#)), which typically generates counter-cyclical risk premiums. Time-varying discount rates can in turn help to increase the volatility of the aggregate dividend claim, which is a sticking point in the literature.

Liu and Miao have three main results. The first mirrors that of [Tallarini \(2000\)](#) and subsequent studies using EZ or DA preferences: increasing agents’ sensitivity to risk (through higher first- and/or second-order risk aversion) has quantitatively

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¹ I will leave it up to the reader to interpret for themselves what constitutes an ‘aggressive’ calibration, but I have in mind situations where preference parameters and/or the technology specification are pushed to the limit relative to what is reasonable given available evidence.

minimal effects on the macro dynamics. The second is that levered returns to the firm payouts (equity) display excess return predictability pattern as in the data. The third is that volatility risk in technology growth also appears the endogenous consumption dynamics and is therefore a priced risk.

2. Comments

In my reading of the paper, the most striking result is that introducing GDA preferences helps with little else, relative to using EZ preferences, than allowing for a reduction in the (second-order) risk aversion coefficient. Of course, a more realistic second-order risk aversion coefficient and introducing first-order risk aversion are important in order to match micro-evidence on risk preferences, but it is not otherwise quantitatively important for the asset price and macro moments Liu and Miao are trying to explain. In fact, the model with high risk aversion and EZ preferences performs almost identically along these dimensions as shown in Table 3 in their paper.

In this comment I will highlight two features of their model. First, as discussed by the authors, introducing GDA preferences does not materially alter the second moments of macro quantities relative to a model with EZ preferences, and differences in asset pricing moments are mainly found for *levered* equity claims. I will expound on why there is little difference in the return dynamics of the claim to the total firm payouts across models and argue that the successes for the levered claim are mainly due to counter-factual properties of the assumed corporate debt in the model. Second, volatility risk, while priced, do not lead to a higher unlevered equity premium and is, in fact, quantitatively not very important for overall risk in the model.

Relative to Epstein–Zin preferences, GDA preferences instead robustly imply substantially higher positive skewness in the pricing kernel. Thus, the likelihood of very high marginal utility states (which happens when outcomes are disappointing) is much higher than with Epstein–Zin preferences, and this has notable implications for assets or policy that are particularly sensitive to these states of the world.

2.1. The macro-finance separation result revisited

The preferences are given by

$$V_t = \{ (1-\beta)C_t^\rho + \beta\mu_t^\rho \}^{1/\rho}, \quad (1)$$

where μ_t is a certainty equivalent whose form determines whether the preferences are standard constant relative risk aversion (CRRA), Epstein–Zin (EZ), or GDA:

$$\mu_t = \begin{cases} E_t[V_{t+1}^\rho]^{1/\rho} & \text{CRRA} \\ E_t[V_{t+1}^\alpha]^{1/\alpha} & \text{EZ} \\ E_t[V_{t+1}^\alpha]^{1/\alpha} - \eta E_t[\mathbf{d}_{t+1} \{ (\kappa\mu_t)^\alpha - V_{t+1}^\alpha \}] & \text{GDA} \end{cases} \quad (2)$$

Here $\alpha = 1 - \gamma \neq 0$, $\rho = 1 - 1/\psi \neq 0$, where γ is the risk aversion coefficient and ψ is the intertemporal elasticity of substitution, as in Liu and Miao (2015). As is well known, EZ preferences allow for a separation between risk aversion and elasticity of substitution ($\alpha \neq \rho$), while GDA also introduces disappointment aversion through the indicator variable \mathbf{d}_{t+1} which takes the value 1 if a disappointing outcomes occurs ($V_{t+1}/\mu_t < \kappa$), and 0 otherwise. Thus, the differences in risk preferences across these three models only affects the certainty equivalent, while the elasticity of intertemporal substitution governs the trade-off between current consumption (and thus investment) today and the certainty equivalent of future consumption.

Since the production function is standard Cobb–Douglas, agents cannot hedge particular (e.g., disappointing) technology outcomes using the capital stock. All the agent can do is to engage in more or less precautionary saving. Given the capital accumulation equation

$$K_{t+1} = (1-\delta)K_t + \phi(I_t/K_t)K_t, \quad (3)$$

investment I_t today is transformed into a pre-determined amount of capital next period, K_{t+1} , through the capital transformation function $\phi(\cdot)$. This feature of technology and capital accumulation combined with the recursive, isoelastic preferences in Eq. (1) are what gives rise to Tallarini's (2000) separation result: increasing risk aversion does not materially change the dynamics of investment/consumption decisions in these models. It will, of course, change the risk-free rate and the risk premium. Before we get to those, however, it is useful to consider the dynamics of the unlevered equity claim.

Given that the production function and capital accumulation equation are homogeneous of degree one, the value of the firm (FV_t) and the firm payout, D_t , are

$$FV_t = \frac{K_{t+1}}{\phi'(I_t/K_t)}, \quad (4)$$

$$D_t = \alpha Y_t - I_t, \quad (5)$$

Table 1

Macro moments and the unlevered firm return across models. The table shows annualized moments from three economies where preferences are either (i) constant relative risk aversion (CRRA), (ii) Epstein–Zin (EZ), or (iii) generalized disappointment aversion (GDA). R_{firm} is the net return to the (unlevered) firm, R_f is the net risk-free rate, Δc is the log consumption growth, Δi is the log investment growth, Δy is the log output growth, Δz is the log technology growth, and M_t is the pricing kernel. $E[x]$ and $\sigma[x]$ denote the mean, standard deviation of x , while $SR[x]$ denotes the Sharpe ratio (the ratio of the mean and standard deviation) of x .

Annualized moments	CRRA	EZ	GDA
	$\gamma = 1/2$ $\eta = 0$	$\gamma = 10$ $\eta = 0$	$\gamma = 10$ $\eta = 2.45$ $\kappa = 0.989$
$\sigma[\Delta c](\%)$	0.80	0.81	0.78
$\sigma[\Delta i](\%)$	3.25	3.22	3.10
$\sigma[\Delta y](\%)$	1.58	1.58	1.58
$E[I/Y]$	0.33	0.34	0.36
$E[R_{firm}](\%)$	2.63	2.36	1.73
$\sigma[R_{firm}](\%)$	0.44	0.43	0.41
$\text{Corr}[R_{firm}, \Delta z]$	0.95	0.94	0.94
$E[R_{firm} - R_f](\%)$	< 0.01	0.09	0.32
$\sigma[R_{firm} - R_f](\%)$	0.39	0.38	0.37
$SR[R_{firm} - R_f]$	0.00	0.24	0.86
$E[R_{firm}](\%)$	2.63	2.27	1.41
$\sigma(M_t)/E[M_t]$	0.00	0.30	1.65

where Y_t is the output and α is the capital share in the production function. Note that firm value and dividends are only a function of real quantities. Thus, if the investment/consumption decision is roughly the same across models, so must the firm and dividend dynamics be. Accordingly if the return volatility of the unlevered equity claim is too low in the Power Utility (CRRA) case, it will also be too low in the EZ and GDA cases.

Table 1 shows relevant moments for the return to the unlevered equity claim ($R_{firm,t}$) for the three different specifications (CRRA, EZ, and GDA), where I have added the CRRA case relative to the analysis in Liu and Miao as well as total investment return moments. The parameters and the technology dynamics are the same as those given in Table 1 and Table 2 of Liu and Miao (2015), where variation in the risk attitudes across models is explicitly given in Table 1 in this comment.²

The table shows that there are only minor differences in the volatility of output, consumption, and investment growth between the three models, as well as in the average investment to output ratio. It is therefore unsurprising, given the above argument, that the return volatility of the unlevered claim to firm payouts, R_{firm} , is also very similar across models (in all cases the annualized return volatility is about 0.4% per year).

Thus, changing the certainty equivalent from a CRRA form to either an EZ form or a GDA form does not materially change the return volatility of the unlevered equity claim. Due to higher risk in the EZ and GDA models, the agent engages in more precautionary savings and the average investment to output ratio is slightly higher. This is reflected in a lower average return to capital, and so the unconditional return to the unlevered equity claim decreases from 2.63% in the CRRA case to 1.73% in the GDA case. Thus, higher risk aversion in these cases lead to a lower expected return to capital, though the annualized risk premium is increasing (from < 0.01% in the CRRA case to 0.32% in the GDA case) as the risk-free rate also drops due to an increased precautionary savings demand. The fact that the macro dynamics are very similar across models also holds conditionally, as the authors show in their Table 4.

The low return volatility of the unlevered equity claim makes it hard to match the high equity risk premium we see in the data. Both the EZ and the GDA models match the Sharpe ratio of the equity claim, so matching the equity premium is at this point about increasing the quantity of risk. In the data, equity return volatility is high, about 15% per year as reported by the authors, which means that financial leverage must increase the return volatility by a factor of more than 30 for the model to match the data along this dimension.

Before I turn to financial leverage as modeled by the authors, I would like to point out that we do have measures of the historical unlevered aggregate equity return volatility. In particular, Larrain and Yogo (2008) use data from the Flow of Funds accounts, as well as Compustat to construct aggregate historical asset returns (i.e., the return to the claim on all firm payouts). In the 1926–2004 Flow of Funds sample, they compute this return volatility to be 12.2%. In the 1971–2004

² The numbers I present in this comment are slightly different from those presented in Liu and Miao (2015), presumably due to a different numerical solution methodology. I use 16 quadrature nodes for the Normal technology shock and 96 grid points for capital, where capital normalized by technology is on a grid from 30 to 110. I iterate on the log value function normalized by technology and log-linearly extrapolate between the 96 grid points when numerically integrating over future value function outcomes.

Table 2

The term spread and levered equity returns. The table shows annualized moments from three economies where preferences are either (i) constant relative risk aversion (CRRA), (ii) Epstein–Zin (EZ), or (iii) generalized disappointment aversion (GDA). R_{Equity} is the net return to the levered equity claim, R_f is the net risk-free rate, y^{10} is the log yield on a 10-year zero coupon, default-free real bond, r_f is the log of the gross risk-free rate. $E[x]$ and $\sigma[x]$ denote the mean, standard deviation of x .

Annualized moments	CRRA	EZ	GDA
	$\gamma = 1/2$ $\eta = 0$	$\gamma = 10$ $\eta = 0$	$\gamma = 10$ $\eta = 2.45$ $\kappa = 0.989$
$E[y^{10} - r_f](\%)$	-0.01	-0.20	-0.88
$\sigma[y^{10} - r_f](\%)$	0.12	0.13	0.15
$E[\text{debt}/\text{equity}]$	2.70	2.23	1.50
$E[R_{Equity} - R_f](\%)$	0.05	0.87	2.17
$\sigma[R_{Equity} - R_f](\%)$	4.22	3.58	2.35
$SR[R_{Equity} - R_f]$	0.01	0.24	0.92

Compustat sample, they compute it to be 10.8%. Relative to these numbers, the return volatility of the unlevered equity claim in all three models is still too low by a factor of more than 20.

Fundamentally, the problem in the CRRA, EZ, and GDA models is too low capital adjustment cost, which makes the aggregate prices too smooth. One cannot simply increase capital adjustment cost as this would lead to too smooth investment, and so what the model needs is a strong and time-varying incentive to invest that would allow for high capital adjustment costs while still matching the high investment volatility in the data. But, that is absent given the relatively smooth, stochastic trend technology process and the isoelastic nature of the preferences.³

2.2. The effect of financial leverage

Liu and Miao (2015) introduce financial leverage as in Jermann (1998), where the firm issues 15 year risk-free debt so as to maintain an average debt to equity ratio of 1 (see Equation (9) in their paper for the resulting equity dividend dynamics). Note that if the firm maintained this leverage ratio by borrowing each period at the short-term risk-free rate, return volatility and the risk premium would simply double relative to the unlevered case. Instead, Table 3 in their paper shows that the return volatility increases by about a factor of 5 (from 0.38 to 2.06) and the risk premium increases by almost a factor of 10 (from 0.25 to 2.36). These increases are remarkable and deserve a closer look.

Table 2 shows the average spread between the 10-year real default-free rate and the 1-quarter real risk-free rate, which is particularly negative for the GDA and EZ models. In other words, the debt of the firm on average earns a negative risk-premium, which is inconsistent with the historical average return to corporate debt. Note that in Jermann (1998), due to the habit formation preferences, long-term debt is risky and so this issue does not arise in that model. Since levered equity effectively is long the firm and short a default-free bond portfolio (which has a negative risk premium in the Liu and Miao model), the levered equity risk premium is more strongly increasing in leverage than if debt has a zero risk premium. Related, the negative risk premium on debt implies that the return on debt is negatively correlated with the return to the firm, and so leverage increases return volatility by more than just the leverage ratio. This explains why the chosen leverage increases the risk premium so drastically in the model.⁴

Further, the negative correlation between debt and firm value leads to highly counter-cyclical leverage ratios which increase return predictability as leverage is (much) higher in bad times. Note that the reason the levered equity risk premium is counter-cyclical in the model is that leverage induces counter-cyclical return volatility. In the data, however, aggregate financial leverage is only weakly related to market return volatility (see, e.g., Schwert 1989).

Since corporate debt historically has yielded an average positive risk premium, the leverage Liu and Miao introduce should perhaps not be regarded simply as financial leverage but instead more broadly as coming from some source of operating leverage, due to, e.g., labor market frictions. Regardless of the interpretation, in future research, it would be beneficial to ensure that the leverage chosen conforms broadly to available empirical evidence, particularly when key model results rely on the leverage channel.

³ Kaltenbrunner and Lochstoer (2010) show that high return volatility due to high capital adjustment costs can be obtained if technology is persistent but stationary in levels and if the elasticity of intertemporal substitution is low, but that this model generates a much too high and volatile risk-free rate.

⁴ I also note that I get a somewhat higher leverage ratio than the authors in the GDA model, and that the unlevered risk premium given in Table 1 for this case is also a little higher than they report. I find that the risk premium and return volatility in the GDA model both increase by a little more than a factor of 6 when introducing financial leverage.

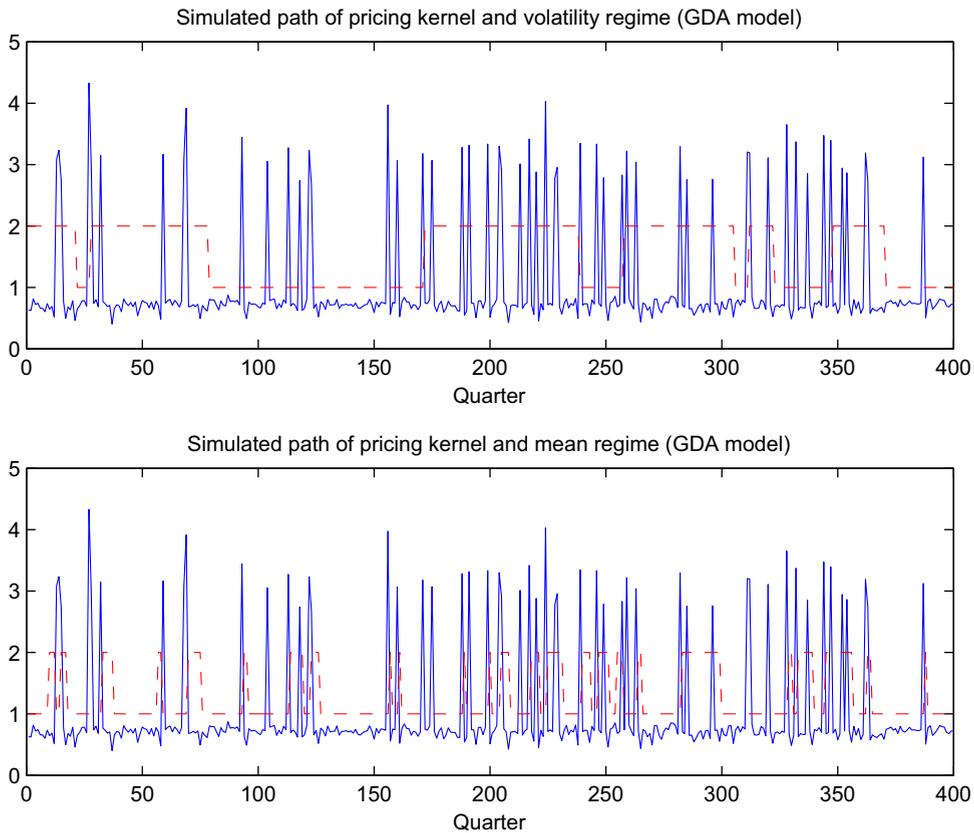


Fig. 1. The figure shows a 400 quarter simulated path of the GDA economy. The solid line shows realizations of the pricing kernel, whereas the dashed line either gives the volatility (top panel) or mean (bottom panel) state. A value of 1 denotes the 'good' state (high growth or low volatility), while a value of 2 denotes the 'bad' state (low growth or high volatility).

2.3. Is volatility risk important in the model?

The authors state that volatility risk is an important part of the model, but many of the reported results point to the opposite. Consider for instance Table 3 in their paper, which shows the unconditional moments. Comparing the 'Benchmark' GDA case with the 'GDAC' case, where the latter has time-varying means but constant volatility, it is clear that adding volatility risk slightly *decreases* the risk premium for the unlevered claim (from 0.27% to 0.25%), while it slightly increases the risk premium for the levered claim (from 2% to 2.36%). The price of risk increases from 1.59 to 1.62. In other words, while there is an effect, it is quite small. Volatility shocks are priced, but they are not a major source of the total risk in the economy.

In fact, it is clear that the regular technology shocks (the ε 's) and the shocks to the mean regime are what gives rise to most of the overall risk. To illustrate this, Fig. 1 shows a simulated path of 400 quarters of the pricing kernel for the GDA model, as well as the mean (bottom panel) and volatility regimes (top panel). Every time a disappointing outcome occurs, the pricing kernel spikes, and these spikes are the main source of the volatility of the pricing kernel. Note, however, that these spikes are typically not associated with changes in the volatility regime. In fact, as the bottom panel shows, the spikes are most associated with changes in the mean regime, or simply occur when a sufficiently negative technology shock hits. Fig. 2 shows the same set of simulations but for the EZ model. Again, most of the variation in the pricing kernel comes from the two other shocks and not from volatility shocks.

2.4. A robust implication of GDA

From Figs. 1 and 2 it is clear that a robust difference between the EZ model and the GDA model is the strong positive skewness in the pricing kernel introduced by allowing for first-order risk aversion. Alvarez and Jermann (2005) and Backus et al. (2014) suggest the use of entropy to measure the dispersion in the pricing kernel. Entropy has the advantage of accounting for non-normalities in a convenient way and provides the upper bound for the log risk premium on any asset in the economy. The average, annualized entropy in the GDA model is 72% versus 4.5% in the EZ model, which indicates the GDA model has the potential to account for, say, the high log risk premiums observed for certain option market trading strategies, such as selling out of the money puts on the S&P500.

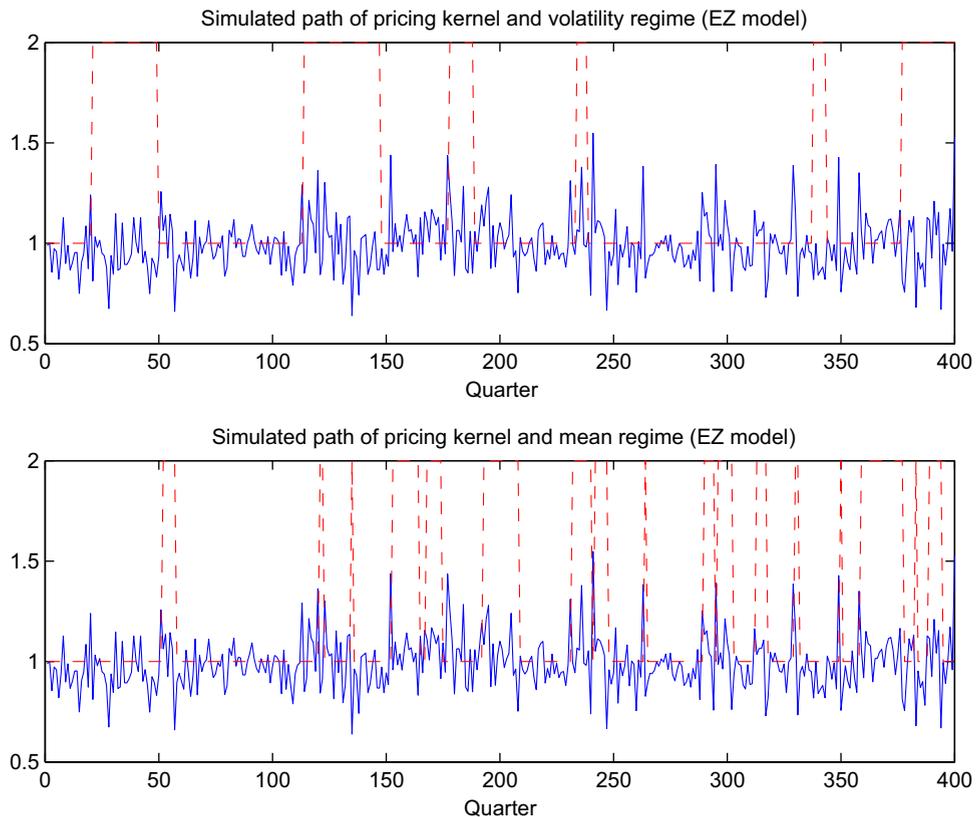


Fig. 2. The figure shows a 400 quarter simulated path of the EZ economy. The solid line shows realizations of the pricing kernel, whereas the dashed line either gives the volatility (top panel) or mean (bottom panel) state. A value of 1 denotes the 'good' state (high growth or low volatility), while a value of 2 denotes the 'bad' state (low growth or high volatility).

In general, GDA preferences have strong implications for assets that pay off when outcomes are disappointing versus assets that do not, but this is an aspect of the model the authors do not focus on given their analysis of a standard, one-firm production-based model. It is here, though, GDA preferences will have the most different implications relative to EZ preferences with high risk aversion. For instance, one could imagine that a model focused on the variance risk premium or on policy implications for climate change—both of which are sensitive to whether an outcome is disappointing or not—may lead to quite different conclusions depending on whether one chooses GDA or EZ preferences.

In sum, adding GDA preferences in a standard real business cycle model has overall only quite small effects on both macro quantities and the standard asset pricing moments relative to EZ preferences. In particular, obtaining high equity return volatility, while matching investment and risk-free rate dynamics, is still a major issue in this class of models. There are substantial differences in pricing between EZ and GDA models, which, as I have highlighted here, shows up in the skewness of the pricing kernel and therefore in the value of assets that are correlated with disappointing events. It would be interesting in future research to consider settings where these aspects of the model are more prominent.

Of course, an important benefit of adding first-order risk aversion is that one can now potentially match micro evidence, or introspective arguments, on the magnitude of both first- and second-order risk aversion and still obtain a high price of risk. This is not a dimension the authors have emphasized in their calibrations, which still feature quite high second-order risk aversion, but it is a dimension along which the model is likely to be quite successful.

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