Optimal Scheduling and Power Allocation for Two-Hop Energy Harvesting Communication Systems

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Abstract

Energy harvesting (EH) has recently emerged as a promising technique for green communications. To realize its potential, communication protocols need to be redesigned to combat the randomness of the harvested energy. In this paper, we investigate how to apply relaying to improve the short-term performance of EH communication systems. With an EH source and a non-EH half-duplex relay, we consider two different design objectives: 1) short-term throughput maximization; and 2) transmission completion time minimization. Both problems are joint scheduling and power allocation problems, rendered quite challenging by the half-duplex constraint at the relay. A key finding is that directional water-filling (DWF), which is the optimal power allocation algorithm for the single-hop EH system, can serve as guideline for the design of two-hop communication systems, as it not only determines the value of the optimal performance, but also forms the basis to derive optimal solutions for both design problems. Based on a relaxed energy profile along with the DWF algorithm, we derive key properties of the optimal solutions for both problems and thereafter propose efficient algorithms. Simulation results will show that both scheduling and power allocation optimizations are necessary in two-hop EH communication systems.

Index Terms

Energy harvesting, two-hop transmission, directional water-filling, scheduling, power allocation.

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I. INTRODUCTION

The growing concerns on the energy consumption of wireless networks and its associated global warming effects have spurred lots of research activities related to the development of more energy-efficient communication techniques. Energy harvesting (EH) has recently emerged as a promising approach to realize green communications, as it can power the communication devices and networks with renewable energy sources. Communication terminals with EH capability can harvest energy from the environment [1], including solar energy, vibration energy, thermoelectric energy, RF energy, etc. However, as the harvested energy is typically in a small amount and also random, how to guarantee satisfactory short-term performance is a challenging problem.

A. Related Works and Motivations

The potential of EH technology has recently spurred lots of research activities in the wireless communications community. The capacity of a point-to-point link with an EH transmitter was investigated in [2] for the AWGN channel and in [3] for the fading channel, respectively. To achieve the capacity, new transmission policies such as save-and-transmit and best-effort-transmit schemes are required. In addition to these information-theoretic studies, practical transmission policies have also been investigated. An offline packet scheduling problem to minimize the transmission completion time with a discrete EH model and infinite energy storage at the transmitter was first introduced in [4], and this work was later extended in [5] to transmitters with limited energy storage. A directional water-filling algorithm adapted to the instantaneous channel state over a Gaussian fading channel was developed in [6], while channel training optimization was investigated in [7]. Other EH communication systems were also investigated, including the broadcast channel [8], [9], the multiple access channel [10], and the interference channel [11]. These studies uncovered an important and unique property of EH communication systems; namely, even when the channel remains unchanged, the transmit power should adapt to the random energy arrivals. Subsequently, communication protocols need to be revised in EH systems for satisfactory short-term performance.

Multi-hop transmission is often adopted to increase the communication range and utilize the energy more efficiently in wireless communication systems. Hence, it can serve as a potential candidate to enhance the short-term performance of EH communication networks. However, relaying protocols need to be re-designed for the random energy arrival. Therefore, further investigation is required in order to exploit the potential of multi-hop transmissions. Energy harvesting two-hop networks have been studied in [12], [13], [14], but only for some special cases. Specifically, the optimal transmission policy with a non-EH source and an EH relay was developed in [12], while the case of an EH source with two energy arrivals was considered in [13] and [14]. Moreover, these previous works only considered the throughput as the objective.

So far, the optimal transmission policy for a two-hop EH communication system with a general EH source of multiple energy arrivals is still not known. This is in fact a more important case as the EH source has a larger effect on the design and performance of the whole system than the EH relay. Moreover, the scheduling with a general EH source is more challenging. For example, if only the relay is an EH node, the optimal scheduling involves a two-stage transmission [12], but as will be shown in this paper, generally, multi-stage scheduling is needed when the source is an EH node. As power adaptation is also needed at the EH source to combat the random energy arrivals, we are faced with a joint scheduling and power allocation problem.

B. Contributions

In this paper, we will investigate a two-hop communication system with an EH source and a non-EH relay. Two different objectives will be considered. The first is a short-term throughput maximization problem with a given deadline, while the second is a transmission completion time minimization problem with a given amount of data. We find that the optimal power allocation algorithm for the single-hop EH transmission, namely, directional water-filling (DWF), can serve as guideline for the design of the two-hop system. Essentially, the DWF power allocation results for the single-hop transmission will help us decouple the joint scheduling and power allocation problem. Specifically, for any given EH profile at the source, we can first find a related DWF EH profile, which provides a performance upper bound and the optimal transmission policy for which can be derived. Then, we can extend the result to solve the original design problem. Efficient algorithms are then proposed to achieve the performance upper bound, and guarantee optimality. Simulation results shall demonstrate the importance of the adaptive power allocation at the EH source and the optimal scheduling of the source and relay transmission periods. Particularly, both of the fixed power allocation and the fixed scheduling transmission policies always incur a performance loss with a time-varying EH profile, especially when there is a significant difference between the energy levels of the source and relay.

The organization of this paper is as follows. In Section II, we introduce the energy harvesting model and formulate two design problems: Short-term throughput maximization, and transmission completion time minimization. In Sections III and IV, the two problems are solved with the help of the DWF EH profile. Simulation results are given in Section V. Finally, Section VI summarizes our work.

II. System Model and Problem Formulation

We consider a two-hop communication system with a half-duplex decode-and-forward relay, where the direct link between the source and destination is too weak to provide reliable communication. The source is an EH node, all the energy harvested by which is used for communication. The relay is a non-EH node, powered by a battery, to assist the communication between the EH source and its destination. For the data transmission of both the source-relay (S-R) and relay-destination (R-D) channels, we assume that the power-rate relationship follows a non-negative, strictly concave and monotonically increasing function, $R = g(P)$, where P is the instantaneous receive power. These properties are satisfied in many widely used communication models. For the relay transmission, there exists a data causality constraint, which means that at time instant t, the data transmitted by the relay, denoted as $D^{\mathrm{R}}(t)$, should not exceed that transmitted by the source, denoted as $D^{\mathrm{S}}(t)$. For ease of reference, we list the main notations defined in this section in Table I.

A. Energy Harvesting Model

An important factor that determines the performance of an EH system is the *EH profile*, denoted as $E_{\Sigma}^{\mathrm{EH}}(t)$, which models the cumulative harvested energy up to time t. Similar to [4], [6], [12], [13], [14], we consider a discrete-time EH profile, as shown in Fig. 1. Such discrete-time EH profiles can also be used to approximate other EH profiles, and later we will show that our results can deal with more general EH profiles. To demonstrate the impact of EH profiles, we assume that the information of the EH profile in the coming transmission block is available at the beginning of the block, which is also adopted in the previously-mentioned works. We refer to the time instant when the energy arrives as the *energy arrival epoch*, denoted as t_i, $i = 1, 2, ..., N$, with $t_N < T$, where T is the total transmission period, and N is the number of energy arrivals in the whole transmission period. The corresponding amount of harvested energy

Table I

MAIN NOTATIONS

Symbols	Definition
T	Total transmission period
N	Number of energy arrivals in T
$P^{\text{S}}(t)$/$P^{\text{R}}(t)$	Instantaneous transmit power of the source/relay
$D^{\text{S}}(t)$/$D^{\text{R}}(t)$	Amount of data transmitted until t by the source/relay
$E_{\Sigma}^{\text{EH}}(t)$	Cumulative harvested energy at time t
t_k	k-th energy arrival epoch
E_k	Energy harvested in the k-th energy arrival interval
E_{Σ}	Total harvested energy in T
N^{S}	Number of all the source-relay stage pairs
P_M^{R}	Maximum transmit power of the relay
E_{Σ}^{R}	Total amount of energy available at the relay

in the i-th arrival is denoted as E_i, and the respective cumulative energy is $E_{\Sigma}^{\text{EH}}(t_i) \triangleq \sum_{j=1}^{i} E_j$. The time interval between the $(i-1)$-th and the i-th energy arrival epochs is called the i-th *energy arrival interval*. For convenience, the interval between the N-th energy arrival and the ending time T is called the $(N+1)$-th energy arrival interval. Without loss of generality, we assume that $t_1 = 0$, and that the total harvested energy is E_{Σ}.

The utilization of the harvested energy is constrained by the EH profile, which yields the causal energy neutrality constraint [15], i.e., the energy consumed thus far cannot exceed the total harvested energy. Denote the transmit power at time t as $P(t)$, then the energy neutrality constraint can be expressed as

$$\int_0^t P(\tau)\, d\tau \leq E_{\Sigma}^{\text{EH}}(t)\,. \tag{1}$$

Accordingly, a certain EH profile determines a feasible energy consumption domain, and only the transmission policies inside this domain are feasible, as shown in Fig. 1. Due to this energy neutrality constraint, we cannot use the energy arriving in the future, but can save the current energy for future use.

Besides the EH profile, the battery capacity is also an important factor for the EH link performance. In this paper, we assume that the battery capacity is infinite, while the case with a finite capacity will be considered in future work.

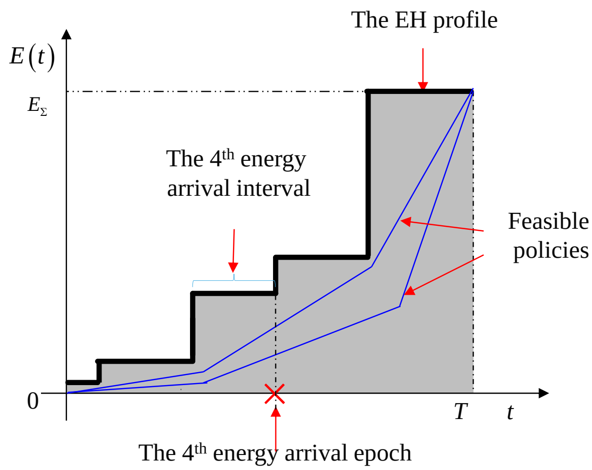

Figure 1. An example of the staircase-shaped EH profile. The power consumption curves under the EH profile denote feasible policies, for which the slope represents the instantaneous transmit power.

B. Transmission Policy

Denote the instantaneous source (or relay) transmit power as $P^{\mathrm{S}}(t)$ (or $P^{\mathrm{R}}(t)$). With a random EH profile, transmitting with a constant power will in general not be optimal and the transmit power needs to be adjusted according to the EH profile. Meanwhile, due to the half-duplex constraint at the relay, i.e., $P^{\mathrm{S}}(t)\, P^{\mathrm{R}}(t) = 0,\ \forall t$, the whole transmission block will be divided into multiple *stages*, in each of which only one side (source or relay) is allowed to transmit. The stage in which the source (relay) transmits is called the *source (relay) stage*, and the union of all the source (relay) stages in the k-th DWF interval is denoted as $\mathfrak{T}_k^{\mathrm{S}}$ ($\mathfrak{T}_k^{\mathrm{R}}$). The length of each source (relay) stage is called a *source (relay) period*[1], and then *scheduling* means the transmission time allocation between the source and relay. Therefore, the design of transmission policies in two-hop EH communication systems involves power allocation at the EH source and scheduling between the source and relay transmissions.

An example of the source/relay scheduling and power allocation is illustrated in Fig. 2, the slope of which represents the instantaneous transmit power. At each time instant, if the

[1] In this paper, we make an ideal assumption that each source or relay stage can take any real value. In practice, there will be a minimum time unit related to the symbol duration, and our results can be round off to serve as approximations. A detailed investigation of this aspect is left to future work.

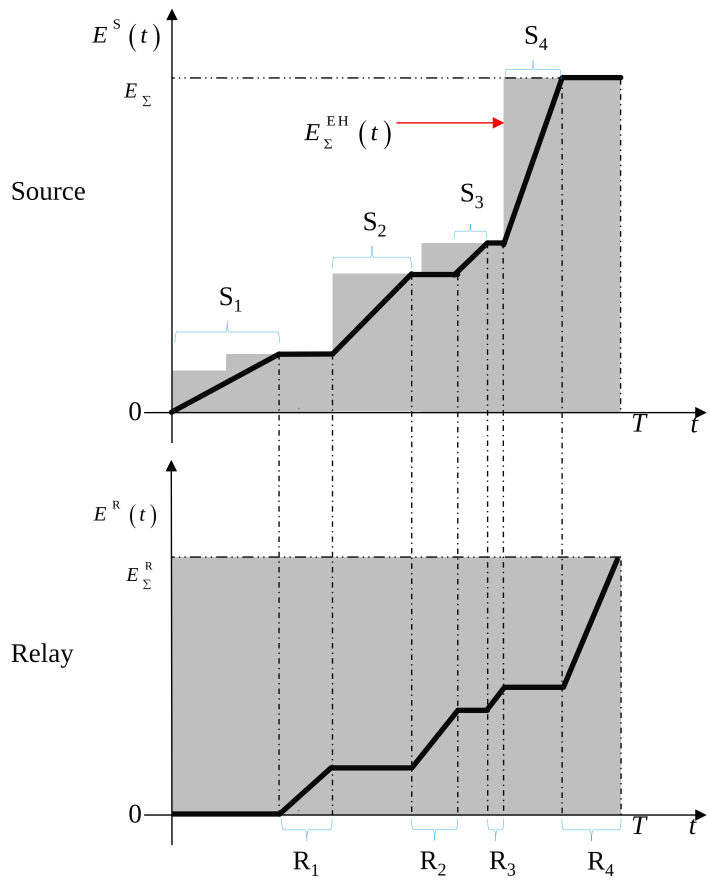

Figure 2. An example of the source/relay scheduling and power allocation, represented by the bold solid curves. On one hand, the slope of the curve represents the instantaneous transmit power. On the other hand, at each time instant, the nonzero source/relay transmit power signifies a source/relay stage. In this example, S_i $(i = 1, 2, 3, 4)$ is a source stage, while R_i $(i = 1, 2, 3, 4)$ is a relay stage.

source/relay transmit power is nonzero, it belongs to a source/relay stage. With the half-duplex constraint, either the source or the relay power consumption curve completely describes a scheduling result. Therefore, we only adopt the source power consumption curve to represent a scheduling result in the rest of this paper. Due to the data causality constraint, the last stage must be a relay stage, while the first stage must be a source stage. The total number of stages should be an even number, and source and relay will take turns to transmit, but the length of each stage will in general be different from each other, as shown in Fig. 2. Each source stage in combination with the relay stage directly after it is called an *S-R stage pair.* The total number of S-R stage pairs is denoted as N^{S}.

For the case of a non-EH source and an EH relay, which was tackled in [12], the relay tends to wait for enough energy, and delaying the relay transmission can always get a better performance without violating either the data or energy feasibility condition. Hence, the optimal scheduling is to divide the whole transmission block into two stages, i.e., a single S-R stage pair. For the system with a non-EH relay and an EH source as in our case, it is the source that wants to wait for enough energy. However, the two-stage transmission will in general be sub-optimal due to the data causality constraint. Therefore, the optimal scheduling is more challenging with an EH source than with an EH relay.

C. Problem Formulation

Since the short-term performance of EH systems is more crucial, in the context of two-hop communications, we consider two different objectives: Short-term throughput maximization and transmission completion time minimization. For the first objective, the total transmission period is fixed, while for the second, the total amount of data that needs to be transmitted is fixed. To make the problem tractable, we assume that both the S-R and R-D channels are static. For the non-EH relay, we consider two kinds of constraints [16]: 1) a peak power constraint, which means that the instantaneous transmit power should not exceed a threshold P_M^{R}, due to the linear operation of power amplifiers or safety regulation; and 2) a total energy constraint, which means that the total consumed energy should not exceed the total available energy E_Σ^{R}, the capacity of the battery.

Given the above, the short-term throughput maximization problem (denoted as RMAX) can be formulated as follows:

$$\textbf{RMAX:} \max_{P^{\mathrm{S}}(t), P^{\mathrm{R}}(t)} \quad D^{\mathrm{R}}(T) \tag{2}$$

$$\text{s.t.} \quad P^{\mathrm{S}}(t) \geq 0, P^{\mathrm{R}}(t) \geq 0, P^{\mathrm{S}}(t) P^{\mathrm{R}}(t) = 0, \tag{3}$$

$$\int_0^t P^{\mathrm{S}}(\tau)\, d\tau \leq E_\Sigma^{\mathrm{EH}}(t), D^{\mathrm{R}}(t) \leq D^{\mathrm{S}}(t), \tag{4}$$

$$\int_0^T P^{\mathrm{R}}(\tau)\, d\tau \leq E_\Sigma^{\mathrm{R}}, P^{\mathrm{R}}(t) \leq P_M^{\mathrm{R}}, t \in [0, T], \tag{5}$$

where (3) is due to the half duplex constraint at the relay, (4) is due to the energy causality constraint and the data causality constraint, (5) is due to the total energy constraint and the peak power constraint at the relay, while the objective function (2) is derived as min $\left\{D^{\mathrm{S}}\left(t\right), D^{\mathrm{R}}\left(T\right)\right\} = D^{\mathrm{R}}\left(T\right)$ due to (4).

Similarly, the transmission completion time minimization problem (denoted as TMIN) is as follows:

$$\begin{aligned}
\textbf{TMIN:} \quad & \min_{P^{\mathrm{S}}(t), P^{\mathrm{R}}(t)} T \\
\text{s.t.} \quad & D^{\mathrm{R}}\left(T\right) = D, \\
& P^{\mathrm{S}}\left(t\right) \geq 0, P^{\mathrm{R}}\left(t\right) \geq 0, P^{\mathrm{S}}\left(t\right) P^{\mathrm{R}}\left(t\right) = 0, \\
& \int_0^t P^{\mathrm{S}}\left(\tau\right) d\tau \leq E_{\Sigma}^{\mathrm{EH}}\left(t\right), D^{\mathrm{R}}\left(t\right) \leq D^{\mathrm{S}}\left(t\right), \\
& \int_0^T P^{\mathrm{R}}\left(\tau\right) d\tau \leq E_{\Sigma}^{\mathrm{R}}, P^{\mathrm{R}}\left(t\right) \leq P_M^{\mathrm{R}}, t \in \left[0, T\right].
\end{aligned}$$

For both RMAX and TMIN problems, due to the half-duplex constraint $P^{\mathrm{S}}\left(t\right) P^{\mathrm{R}}\left(t\right) = 0$, the scheduling and power allocation are coupled. For a given solution, once we modify the scheduling decision, the power allocation result should also be adjusted, and vice versa. Moreover, compared to the two-hop system with a non-EH source in [12], our problem is more challenging, as with an EH source, there will be multiple S-R stage pairs, depending on the EH profile. Therefore, a more complicated scheduling is needed. Since it is difficult to solve the optimization problems directly, we will seek an indirect approach to get the optimal solutions.

III. Short-term Throughput Maximization

In this section, we will solve the RMAX problem. We will first show that a kind of DWF EH profile that is based on the original EH profile can provide a throughput upper bound. We will then verify that this upper bound can be achieved by the original EH profile, and efficient algorithms will then be proposed to solve the RMAX problem.

A. The DWF EH profile

In the single-hop EH communication system with random energy arrivals, as shown in Fig. 3, the optimal transmit power allocation yields a directional water-filling interpretation [6], [17], which has the following three properties:

1) The transmit power increases monotonically.

2) The transmit power remains constant between the energy arrivals.
3) Whenever the power level changes, the energy consumed up to that time instant equals the total harvested energy.

According to these properties, the transmit power can only change at some of the energy arrival epochs, and up to these time instants all the energy harvested should be exhausted. We define those points where the power level changes as *DWF points*, denoted as $\left(t_k^{\mathrm{D}}, \sum_{i=1}^{k} E_i^{\mathrm{D}}\right)$, $1 \leq k \leq N^{\mathrm{D}}$, where N^{D} represents the number of DWF points, t_k^{D} is the horizontal coordinate of the DWF point, and E_i^{D} is the amount of the energy harvested between the $(i-1)$-th and i-th DWF points. Particularly, the DWF points exclude the origin but include the ending point. The DWF points can be determined as $t_k^{\mathrm{D}} = t_{i_k}$, $E_k^{\mathrm{D}} = \sum_{j=i_{k-1}+1}^{i_k} E_j$, where all the i_k can be computed iteratively as

$$i_k = \arg \min_{i:t_i \leq T} \left\{ \frac{\sum_{j=i_{k-1}+1}^{i} E_j}{t_i - t_{i_{k-1}}} \right\} \tag{6}$$

with $i_0 = 1$ and $t_0 = 0$. We also define the interval between two adjacent DWF points (or the origin) as a *DWF interval*, and the length of the k-th DWF interval is denoted as T_k^{D}.

Throughout the paper, we will demonstrate that these DWF points obtained in the single-hop case are extremely important for the optimization of the two-hop case. For two-hop communications, according to these DWF points, we can construct a *DWF EH profile*, which only includes the beginning time instants of these DWF intervals as its energy arrival epochs, as illustrated in Fig. 3. Since this DWF EH profile is relaxed from the original EH profile and has a larger feasible domain, it can provide a performance upper bound for the original EH profile. The DWF EH profile can be uniquely determined by the set of all the DWF points, and vice versa. For ease of reference, we list the main notations related to the DWF EH profile in Table II.

B. Throughput Upper Bound from the DWF EH profile

We will first analyze the properties of the optimal transmission policy for an EH source with the DWF EH profile, and then obtain the respective optimal solution based on these properties.

Note that the optimal transmission policy is not unique, i.e., different scheduling decisions may provide the same performance. Unless otherwise mentioned, if the problem has multiple optimal solutions, we select the one with the smallest number of S-R stage pairs. For the DWF EH profile, since all of the energy in each DWF interval is available at the beginning of that

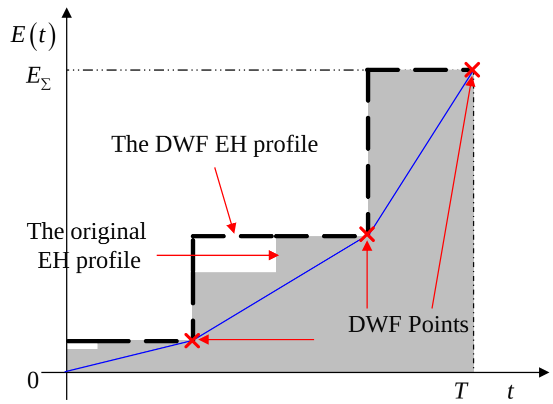

Figure 3. The DWF power allocation for a given EH profile, and its associated DWF EH profile. The edge of the dark area denotes the original EH profile. The DWF points are marked by crosses, while the curve that connects the DWF points is the DWF power allocation curve for the single-hop transmission, the slope of which represents the transmit power.

Table II

MAIN NOTATIONS RELATED TO THE DWF EH PROFILE

Symbols	Definition
i_k	Index of the k-th DWF point in all the energy arrival epochs
t_k^{D}	Time instant of the the k-th DWF point
T_k^{D}	Time duration of the k-th DWF interval
E_k^{D}	Total harvested energy in the k-th DWF interval
N^{D}	Number of all the DWF intervals

DWF interval, the number of S-R stage pairs inside each DWF interval can always attain the minimum value as 1. The properties of the optimal solution to RMAX are listed in Theorem 1.

Theorem 1. *For **Problem RMAX** with a source having the DWF EH profile, the optimal transmission policy has the following properties:*

1) Within the k-th DWF interval, $k = 1, 2, ..., N^{\mathrm{D}}$,

$$P^{\mathrm{S}}(t) = \begin{cases} P_k^{\mathrm{S}} & t \in \mathfrak{T}_k^{\mathrm{S}} \\ 0 & t \in \mathfrak{T}_k^{\mathrm{R}} \end{cases}, \quad P^{\mathrm{R}}(t) = \begin{cases} P_k^{\mathrm{R}} & t \in \mathfrak{T}_k^{\mathrm{R}} \\ 0 & t \in \mathfrak{T}_k^{\mathrm{S}} \end{cases},$$

where both P_k^{S} and P_k^{R} are constants to be determined.

2) At the end of the k-th DWF interval, $k = 1, 2, ..., N^{\mathrm{D}}$,

$$\int_0^{t_k^{\mathrm{D}}} P^{\mathrm{S}}(\tau)\, d\tau = E_{\Sigma}^{\mathrm{EH}}\left(t_k^{\mathrm{D}}\right), \quad D^{\mathrm{R}}\left(t_k^{\mathrm{D}}\right) = D^{\mathrm{S}}\left(t_k^{\mathrm{D}}\right).$$

Proof: The proof is given in Appendix 1. ■

Remark 1. The physical meaning of Property 1 is that the source (relay) should use the same transmit power in all the source (relay) stages within a given DWF interval. Property 2 means that at the end of each DWF interval, the relay empties its data buffer (denoted as the relay data equality); while the source empties its energy buffer (denoted as the source energy equality). Also note that Theorem 1 holds for both the energy and power constraints at the relay.

Next we will derive the optimal solution for Problem RMAX based on the properties in Theorem 1. We first consider an energy constraint at the relay and then extend the result to the general case with both power and energy constraints. Denote the amount of relay energy consumption in the k-th DWF interval as E_k^{R}, and the k-th source period as T_k^{S}. The optimal solution for Problem RMAX with an energy constrained relay is provided in the following Corollary.

Corollary 1. *In the optimal transmission policy for **Problem RMAX** with the DWF EH profile and an energy constrained relay, there is a single S-R stage pair in each DWF interval, i.e., the source and relay stages for the k-th DWF interval ($k = 1, 2, ..., N^{\mathrm{D}}$) are*

$$\mathfrak{T}_k^{\mathrm{S}} = \left[t_k^{\mathrm{D}}, t_k^{\mathrm{D}} + T_k^{\mathrm{S}}\right), \quad \mathfrak{T}_k^{\mathrm{R}} = \left[t_k^{\mathrm{D}} + T_k^{\mathrm{S}}, t_{k+1}^{\mathrm{D}}\right),$$

respectively. The transmit powers in the k-th DWF interval ($k = 1, 2, ..., N^{\mathrm{D}}$) are

$$P_k^{\mathrm{S}} = \frac{E_k^{\mathrm{D}}}{T_k^{\mathrm{S}}}, \quad P_k^{\mathrm{R}} = \frac{E_k^{\mathrm{R}}}{T_k^{\mathrm{D}} - T_k^{\mathrm{S}}},$$

where E_k^{R} and T_k^{S} are obtained by solving Eqns. (13)~(15) in Appendix 2.

Proof: The proof is given in Appendix 2. ■

Next, we consider the case with a power constrained relay, for which another property that is stronger than Property 1 in Theorem 1 can be found, given in the following Lemma.

Lemma 1. *Assume that the source follows a DWF EH profile and the relay is power constrained. Then, when the optimal transmission policy is adopted for* ***Problem RMAX****,* $P^{\mathrm{R}}(t)$ *can only be* 0 *or* P_M^{R}.

Proof: As the relay transmit power is limited by the peak power constraint, at any time transmitting below the peak power will degrade the performance. Hence the relay uses a constant transmit power, which is the peak power. ■

According to Theorem 1 and Lemma 1, the optimal solution with a power constrained relay is provided in the following Corollary.

Corollary 2. *In the optimal transmission policy for* ***Problem RMAX*** *with the DWF EH profile and a power constrained relay, there is a single S-R stage pair in each DWF interval, i.e., the source and relay stages for the* k*-th DWF interval (*$k = 1, 2, ..., N^{\mathrm{D}}$*) are*

$$\mathfrak{T}_k^{\mathrm{S}} = \left[t_k^{\mathrm{D}}, t_k^{\mathrm{D}} + T_k^{\mathrm{S}}\right), \quad \mathfrak{T}_k^{\mathrm{R}} = \left[t_k^{\mathrm{D}} + T_k^{\mathrm{S}}, t_{k+1}^{\mathrm{D}}\right),$$

respectively; and the transmit powers in the k*-th DWF interval (*$k = 1, 2, ..., N^{\mathrm{D}}$*) are*

$$P_k^{\mathrm{S}} = \frac{E_k^{\mathrm{D}}}{T_k^{\mathrm{S}}}, \quad P_k^{\mathrm{R}} = P_M^{\mathrm{R}},$$

where T_k^{S} *is the solution of the following equation*

$$T_k^{\mathrm{S}} g\left(\frac{E_k^{\mathrm{D}}}{T_k^{\mathrm{S}}}\right) = \left(T_k^{\mathrm{D}} - T_k^{\mathrm{S}}\right) g\left(P_M^{\mathrm{R}}\right). \tag{7}$$

Proof: We can see that Eqn. (7) satisfies all the conditions in Theorem 1 and Lemma 1, and has a unique solution, so its solution is exactly the optimal source period. The solution of the relay periods and source powers can be easily verified. ■

Corollaries 1 and 2 provide optimal solutions for the RMAX Problem with an energy constrained relay and a power constrained relay, respectively. For the case where the relay has both energy and power constraints[2], the optimal solution can be obtained by Algorithm 1.

Steps 1 and 2 of Algorithm 1 can be proved to be optimal directly via Corollaries 1 and 2. For Step 3, in each **"for"** loop, it can be verified that the relay power equals the peak power

[2]Note that when the peak power constraint is small, e.g., for a sensor node, the power constraint will be the only constraint, while the total energy constraint will not be tight.

Algorithm 1: Optimal solution for the case where the source assumes a DWF EH profile and the relay has both energy and power constraints.

1) $P_M^{\mathrm{R}*} \leftarrow P_M^{\mathrm{R}}$, $P_M^{\mathrm{R}} \leftarrow \infty$.

   Obtain Solution 1 for **Problem RMAX** by Corollary 1.

   If $P_k^{\mathrm{R}} \leq P_M^{\mathrm{R}}$ holds for all $k = 1\colon N^{\mathrm{D}}$ {Only the energy constraint is tight.}

      Solution 1 is the optimal solution.

      Return.

   End if

2) $E_\Sigma^{\mathrm{R}*} \leftarrow E_\Sigma^{\mathrm{R}}$, $E_\Sigma^{\mathrm{R}} \leftarrow \infty$, $P_M^{\mathrm{R}} \leftarrow P_M^{\mathrm{R}*}$.

   Obtain solution 2 for **Problem RMAX** by Corollary 2.

   If $\sum_{k=1}^{N^{\mathrm{D}}} \left(T_k^{\mathrm{D}} - T_k^{\mathrm{S}}\right) P_k^{\mathrm{R}} \leq E_\Sigma^{\mathrm{R}}$ {Only the power constraint is tight.}

      Solution 2 is the optimal solution.

      Return.

   End if

3) $E_\Sigma^{\mathrm{R}} \leftarrow E_\Sigma^{\mathrm{R}*}$. {Both constraints are tight.}

   For $k = 1\colon N^{\mathrm{D}} - 1$

      $P_i^{\mathrm{R}} \leftarrow P_M^{\mathrm{R}}$ for $i = N^{\mathrm{D}} + 1 - k\colon N^{\mathrm{D}}$.

      $E_\Sigma^{\mathrm{R}} \leftarrow E_\Sigma^{\mathrm{R}} - P_M^{\mathrm{R}} \sum_{i=N^{\mathrm{D}}+1-k}^{N^{\mathrm{D}}} \left(T_i^{\mathrm{D}} - T_i^{\mathrm{S}}\right)$.

      Obtain solution for **Problem RMAX** by Corollary 1 with $j = 1\colon N^{\mathrm{D}} - k$.

      If $P_j^{\mathrm{R}} \leq P_M^{\mathrm{R}}$ holds for $j = 1\colon N^{\mathrm{D}} - k$

         The current solution is the optimal solution.

         Return.

      End if

   End for

for all the $i = N^{\mathrm{D}} + 1 - k\colon N^{\mathrm{D}}$, by checking previous steps and the monotonic property (see Lemma 4 in [13]) of the relay transmit power. Furthermore, according to the two Corollaries, we can observe that once the "**for**" loop terminates, the current solution is optimal.

C. Achievability of the Throughput Upper Bound with the Original EH Profile

We have derived the optimal solution for the DWF EH profile, which provides an upper bound of the achievable throughput for the original EH profile. Next we will show how to achieve this upper bound. For the original EH profile, the assumption that each DWF interval includes one S-R stage pair may no longer hold, but by dividing each DWF interval into more stages and carefully designing these stages, we can achieve the throughput upper bound. Define the total source period for the k-th DWF interval as the sum of all source periods in this DWF interval (also the length of $\mathfrak{T}_k^{\mathrm{S}}$), denoted as $T_{k,\Sigma}^{\mathrm{S}}$, then this main result is stated in the following Theorem.

Theorem 2. *The maximum throughput of the considered two-hop communication system with the original EH profile is the same as that under its associated DWF EH profile. In addition, in each DWF interval, the optimal solutions for* ***Problem RMAX*** *with both EH profiles share the same values for the following parameters: 1) the source (relay) transmit power P_k^{S} (P_k^{R}), and 2) the total source period $T_{k,\Sigma}^{\mathrm{S}}$.*

Proof: It can be verified that for any two feasible solutions, as long as they have the same parameters as stated in the Theorem, they achieve the same throughput. Therefore, we need to find a feasible solution for the original EH profile that has the same parameters as those for the DWF EH profile. Without loss of generality, we consider the k-th DWF interval, and treat the solution for the DWF EH profile as a temporary solution. If the temporary power consumption curve is inside the feasible domain of the original EH profile, then the temporary solution is exactly the desired final solution. Otherwise, the source will transmit with power P_k^{S} until all the available energy is used, and then the relay will transmit with power P_k^{R} until it empties its data buffer. We can then restart the source transmission, and iterate as in the previous steps until we reach the end of the DWF interval. This process is illustrated in Fig. 4. It can be easily verified that the new scheduling enjoys the same set of critical parameters as the ones for the DWF EH profile and it does not violate the data or energy constraint. ■

Based on Algorithm 1 and Theorem 2, the optimal solution for the RMAX problem with the original EH profile can be obtained by Algorithm 2. Note that all the i, j, k in the algorithm are integers. If the set of j, i.e., $U_{\{j\}}$, is an empty set, then no j violates the required condition. We omit these two statements in the rest of this paper.

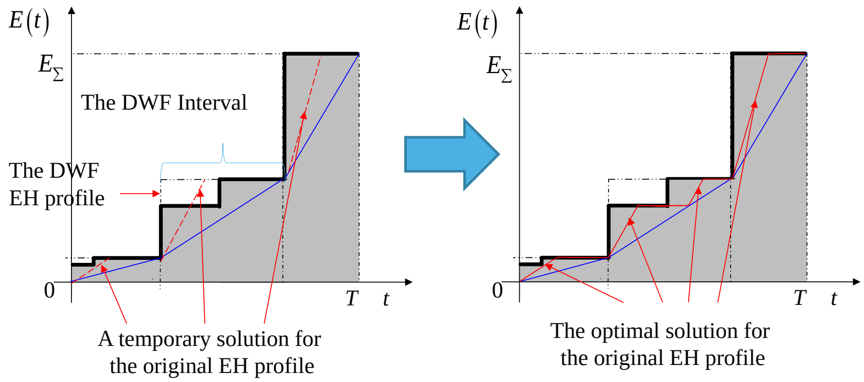

Figure 4. An illustration of how to modify the solution for the DWF EH profile to get the optimal solution for the original EH profile. A temporary solution and the optimal solution are shown by the power allocation curves for the source, the slope of each of which denotes the instantaneous source transmit power. In this example, in the first and third DWF intervals, the temporary solution is already the final solution. However, note that in the second interval, the optimal solution is obtained through the iterative steps described in the proof.

Algorithm 2: The optimal solution for **Problem RMAX**

---

1) Adopt Algorithm 1 to obtain: DWF points $\left(t_k^{\mathrm{D}}, \sum_{i=1}^{k} E_i^{\mathrm{D}}\right)$; solutions of T_k^{S}, P_k^{S}, P_k^{R} ; indices i_k with $k = 1\colon N^{\mathrm{D}}$.

2) **For** $k = 1\colon N^{\mathrm{D}}$

$E_0^{\tau} \leftarrow \sum_{l=1}^{i_{k-1}} E_l$, $\tau_0 \leftarrow t_{i_{k-1}}$, $m_k \leftarrow 1$, $T_{k,1}^{\mathrm{S}} \leftarrow T_k^{\mathrm{S}}$,

$U_{\{j\}} \leftarrow \{j \,|j \le i_k - 1, t_j - \tau_0 > 0\}$.

While $I(j) = P_k^{\mathrm{S}}(t_j - \tau_0) - \left(\sum_{l=1}^{j} E_l - E_0^{\tau}\right) \le 0$ does not hold for all the $j \in U_{\{j\}}$

{The solution for the DWF EH profile is not feasible for the original EH profile.}

$j' \leftarrow \min\{j\,|j \in U_{\{j\}}, I(j) > 0\}$, $E_0^{\tau} \leftarrow \sum_{l=1}^{j'} E_l$, $T_{k,m_k}^{\mathrm{S}} \leftarrow \frac{E_0^{\tau}}{P_k^{\mathrm{S}}}$.

{Obtain actual solution by dividing each DWF interval into multiple S-R stage pairs.}

Update $U_{\{j\}}$ with the new $\tau_0 \leftarrow T_{k,m_k}^{\mathrm{S}} \frac{T_k^{\mathrm{D}}}{T_k^{\mathrm{S}}}$.

$m_k \leftarrow m_k + 1$.

End while

End for

For the k-th DWF interval, the solutions of the source (or relay) periods are $\left(T_{k,1}^{\mathrm{S}}, T_{k,2}^{\mathrm{S}}, ..., T_{k,m_k}^{\mathrm{S}}\right)$ (or $\frac{T_k^{\mathrm{D}} - T_k^{\mathrm{S}}}{T_k^{\mathrm{S}}}\left(T_{k,1}^{\mathrm{S}}, T_{k,2}^{\mathrm{S}}, ..., T_{k,m_k}^{\mathrm{S}}\right)$), while the source (or relay) power is P_k^{S} (or P_k^{R}).

Return.

The procedures in Algorithm 2 can be divided into two parts. The initial step is to obtain the solution for the DWF EH profile, which can serve as a temporary solution for the original EH profile. The objective of Step 2 is to obtain the actual solution based on the temporary solution. Also note that we actually only need to determine the scheduling, as the power allocation has been completed in Step 1.

Remark 2. We see that the DWF power allocation result for the single-hop transmission, which determines the DWF EH profile, can serve as a design guideline for the two-hop communication system. First, it determines the value of the optimal throughput. Second, it allows us to decouple the joint scheduling and power allocation problem, and provides the optimal power allocation solution eventually. This indicates that the DWF power allocation plays a central role in EH communication systems.

Remark 3. Though we only consider the staircase-shaped EH profiles in this paper, this algorithm can be directly applied for more general EH profiles. In fact, for an EH source node, whether the EH profile is continuous or discrete, as long as its cumulative EH curve is between the DWF EH profile and the DWF power allocation curve, its optimal transmission policy is provided by Algorithm 2 and its throughput is the same with that of the DWF EH profile.

IV. Transmission Completion Time Minimization

Problem TMIN is the dual problem of RMAX, where the total transmission time is unknown and needs to be minimized. For the single-hop case, an algorithm[3] similar to DWF was proposed in [4] to minimize the transmission time. Unfortunately, in contrast to RMAX, the result of the single-hop DWF power allocation for TMIN cannot provide the respective DWF EH profile that we need in the two-hop case. Hence, the main challenge is to obtain the required DWF points, and then the algorithm for RMAX can be applied for the scheduling and power allocation.

A. Obtaining All the DWF Points with a Power Constrained Relay

We will first consider a power constrained relay. From Theorem 2, we can see that with the optimal scheduling and power allocation, the same amount of data can be transmitted with either the DWF EH profile or the original EH profile, i.e., only the DWF points will affect the

[3] In this section, the DWF algorithm refers to this particular algorithm (i.e., not the one we used in Section III).

transmission completion time. According to Theorems 1 and 2, along with the properties of the DWF power allocation [4], we are now ready to characterize the DWF points for the TMIN Problem.

Lemma 2. *For a given amount of data D and its associated minimum transmission completion time T, the following equation set is the necessary and sufficient condition for the set of points $\left(t_{i_k}, \sum_{j=1}^{i_k} E_j\right)$ (k=1,2,...,N^{D}) to be the DWF points of the given EH profile:*

$$\sum_{k=1}^{N^{\mathrm{D}}} T_k^{\mathrm{S}} g\left(P_k^{\mathrm{S}}\right) = D, \tag{8}$$

and for k=1,2,...,$N^{\mathrm{D}}-1$,

$$i_k = \arg\min_{i:s_i \le T} \left\{ \frac{\sum_{j=i_{k-1}+1}^{i} E_j}{t_i - t_{i_{k-1}}} \right\}, \tag{9}$$

$$T_k^{\mathrm{D}} = t_{i_k} - t_{i_{k-1}}, P_k^{\mathrm{S}} = \frac{\sum_{j=i_{k-1}+1}^{i_k} E_j}{t_{i_k} - t_{i_{k-1}}}, \tag{10}$$

$$T_k^{\mathrm{S}} g\left(P_k^{\mathrm{S}}\right) = \left(T_k^{\mathrm{D}} - T_k^{\mathrm{S}}\right) g\left(P_M^{\mathrm{R}}\right). \tag{11}$$

Proof: (9) is due to the DWF algorithm (See (6)), while (8) is due to the transmission data constraint. Finally, (10) and (11) can be obtained directly from Corollary 2. ■

Lemma 2 provides the necessary and sufficient condition for a set of points to be the DWF points. However, the optimization in condition (9) requires the value of T, which can not be determined before the optimization is finished, i.e., the condition (9) needs a non-causal knowledge, which makes it difficult to obtain the DWF points through Lemma 2.

Next we will show that this non-causal requirement can be removed. According to Properties 1 and 3 of DWF, the first DWF point is the point where the source transmit power changes for the first time. To obtain the first DWF point, we only need to replace T in (9) with an earlier deadline before which the source transmit power must have changed at least once. All the other DWF points can be obtained iteratively using a similar approach by updating the remaining energy and data. Based on this, we propose Algorithm 3 to find all the DWF points and also the transmission completion time.

Theorem 3. *All the DWF points found in Algorithm 3 satisfy the conditions in Lemma 2.*

Proof: The proof for Theorem 3 is given in Appendix 3. ■

Algorithm 3: Finding all the DWF points and the transmission completion time for the power constrained relay case.

$i_0 \leftarrow 0$, $k \leftarrow 1$, $T^{\mathrm{R}} \leftarrow \frac{D}{g\left(P_M^{\mathrm{R}}\right)}$, $j_0 \leftarrow \min\left\{j \,|s_j > T^{\mathrm{R}}\right\}$.

While $D > 0$

$E_{k,j}^{\mathrm{req}} \leftarrow (t_j - T^{\mathrm{R}})\, g^{-1}\left(\frac{D}{t_j - T^{\mathrm{R}}}\right)$, with $j = j_0$: N

Obtain $\tilde{i_k}$ and t'_k by $\tilde{i_k} = \min\left\{j \left| E_{k,j}^{\mathrm{req}} \le \sum_{l=1}^{j} E_l \right.\right\}$ and $g\left(\frac{\sum_{i=1}^{\tilde{i_k}} E_i}{t'_k - T^{\mathrm{R}}}\right) = \frac{D}{t'_k - T^{\mathrm{R}}}$.

{$\tilde{i_k}$ is the previously-mentioned deadline for the k-th DWF point.}

$T_k^{\mathrm{D}} \leftarrow t'_k$, $E_k^{\mathrm{D}} \leftarrow \sum_{i=1}^{\tilde{i_k}} E_i$, $T_k^{\mathrm{R}} \leftarrow T^{\mathrm{R}}$.

If $\frac{\sum_{i=1}^{\tilde{i_k}} E_i}{t'_k} t_j \le \sum_{l=1}^{j} E_l$, $j \in U_{\{j\}} = \left\{j \left| 0 < j < \tilde{i_k} \right.\right\}$

{Constant power transmission is permitted by the DWF EH profile before $\tilde{i_k}$.}

Break.

End if

$i_k \leftarrow \arg\min\limits_{i<\tilde{i_k}} \left\{\frac{\sum_{j=i_{k-1}+1}^{i} E_j}{t_i - t_{i_{k-1}}}\right\}$, $T_k^{\mathrm{D}} \leftarrow t_{i_k}$, $E_k^{\mathrm{D}} \leftarrow \sum_{j=i_{k-1}+1}^{i_k} E_j$.

{i_k is the k-th DWF point.}

$T_k^{\mathrm{R}} \leftarrow (T_k^{\mathrm{D}} - T_k^{\mathrm{R}})\, g\left(\frac{E_k^{\mathrm{D}}}{T_k^{\mathrm{D}} - T_k^{\mathrm{R}}}\right) = T_k^{\mathrm{R}} g\left(P^{\mathrm{R}}\right)$.

$t_{j-i_k} \leftarrow t_j - t_{i_k}$, $E_{j-i_k} \leftarrow E_j$, for all the $i_k \le j \le N$.

$D \leftarrow D\frac{T^{\mathrm{R}} - T_k^{\mathrm{R}}}{T^{\mathrm{R}}}$, $T^{\mathrm{R}} \leftarrow T^{\mathrm{R}} - T_k^{\mathrm{R}}$, $k \leftarrow k+1$.

{All parameters are updated for obtaining the remaining DWF points.}

End while

$N^{\mathrm{D}} = k$, the transmission completion time is $T = t_{N^{\mathrm{D}}}^{\mathrm{D}}$, coordinates of DWF points are $\left(\sum_{j=1}^{k} T_j^{\mathrm{D}}, \sum_{j=1}^{k} E_j^{\mathrm{D}}\right)$.

Return.

Remark 4. Algorithm 3 not only obtains all the DWF points and the transmission completion time $T = t_{N^{\mathrm{D}}}^{\mathrm{D}}$, but also determines the optimal power allocation ($P_k^{\mathrm{S}} = \frac{E_k^{\mathrm{D}}}{T_k^{\mathrm{D}} - T_k^{\mathrm{R}}}$) and scheduling ($T_k^{\mathrm{S}}$ and T_k^{R}) for the DWF EH profile, i.e., the temporary solution for the original EH profile.

B. Obtaining All the DWF Points with an Energy Constrained Relay

For the energy constrained relay case, the relay cannot always transmit with a constant power, which adds to the difficulty of optimization. Therefore, Algorithm 3 cannot be directly

Algorithm 4: Finding all the DWF points and the transmission completion time for the energy constrained relay case.

Obtain P^{R} by $\frac{E_{\Sigma}^{\mathrm{R}}}{P^{\mathrm{R}}} g\left(P^{\mathrm{R}}\right) = D$.

Obtain all the $N^{\mathrm{D}'}$ temporary DWF points by Algorithm 3.

$k_l \leftarrow 0$, $k_u \leftarrow N^{\mathrm{D}'}$.

Find $\hat{k}$ that satisfies $R\left(\hat{k}\right) \leq D$ and $R\left(\hat{k}+1\right) > D$ using bisection.

{The $\hat{k}$-th DWF interval contains the ending time.}

If $R\left(\hat{k}\right) = D$

  $T \leftarrow \sum_{i=1}^{\hat{k}} T_i^{\mathrm{D}}$.

Else

  $t_l \leftarrow \sum_{i=1}^{\hat{k}} T_i^{\mathrm{D}}$, $t_u \leftarrow \sum_{i=1}^{\hat{k}+1} T_i^{\mathrm{D}}$.

  For $t \in [t_l, t_u)$, use bisection to find $\hat{t}$ that makes $R'\left(\hat{t}\right) = D$.

  $T \leftarrow \hat{t}$. {This step is to decide the exact value of the ending time.}

End

T is the transmission completion time, the DWF points and solutions are obtained by Corollary 1 for this T.

Return.

applied. However, we can first force the relay to transmit with a constant power in the whole transmission period to obtain an upper bound for the transmission time. We can then find the optimal transmission time between this upper bound and the beginning time. The algorithm of obtaining all the DWF points for the energy constrained relay case is given in Algorithm 4, in which $R(k)$ denotes the throughput by adopting Corollary 1 with $T \leftarrow \sum_{i=1}^{k} T_i^{\mathrm{D}}$, while $R'(t)$ denotes the throughput by adopting Corollary 1 and the single-hop DWF in (9) with $T \leftarrow t$.

Remark 5. A similar procedure as Algorithm 4 can be derived to obtain the DWF points for the case where the relay has both power and energy constraints.

C. *Optimal Transmission Policy*

With Algorithms 3 and 4, the optimal solution to TMIN with the original EH profile can be found as follows:

1) Obtain all the DWF points $\left(t_k^{\mathrm{D}}, \sum_{i=1}^{k} E_i^{\mathrm{D}}\right)$, as well as the respective parameters of P_k^{S} (P_k^{R}), T_k^{S} (T_k^{R}) for $1 \leq k \leq N^{\mathrm{D}}$, by Algorithm 4.

2) Obtain the optimal transmission policy by Step 2 of Algorithm 2.

Similar to the RMAX problem, the DWF EH profile serves as the building block for solving the TMIN problem. This novel approach to study the two-hop EH communication system, by first investigating the DWF EH profile and then extending the result to the original EH profile, has the potential to solve other design problems.

V. Simulation Results

In this section, we provide simulation results to demonstrate the performance of our proposed algorithms and to show the importance of both scheduling and power allocation optimizations in two-hop EH communication systems.

A. Short-term Throughput Maximization

We first investigate the throughput maximization problem. To verify the importance of both the scheduling and power allocation, we consider two baseline policies: Fixed scheduling and fixed power allocation. For the first policy, we fix a two-equal-stage scheduling but adopt the optimal power allocation in each stage. In the second one, we adopt the optimal scheduling locally inside each energy arrival interval so that the source and the relay transmit the same amount of data, but a fixed transmit power allocation is applied at the source. All the policies are also compared with a throughput upper bound, assuming a non-EH source with the same amount of total energy.

We consider a band-limited additive white Gaussian noise channel, with bandwidth W = 1MHz and noise power spectral density $N_0 = 10^{-19}$W/Hz. We assume that the path loss H is 100dB, and the transmission block length is 100ms. The rate-power function is $r = W \log_2\left(1 + \frac{PH}{N_0 W}\right) = \log_2\left(1 + \frac{P}{10^{-3}}\right)$ Mbps. The number of energy arrivals in a time interval of length t follows a Poisson distribution with mean $\lambda_e t$ and is in the unit of E_0^{EH}. The *average EH rate* is then defined as $P^{\mathrm{EH}} = E_0^{\mathrm{EH}} \lambda_e$, e.g., if $E_0^{\mathrm{EH}} = 1\mathrm{mJ}$ and $\lambda_e = 1\mathrm{sec}^{-1}$, then $P^{\mathrm{EH}} = 1\mathrm{mJ/sec} = 1\mathrm{mw}$. The simulation is run for 1000 random EH realizations with $\lambda_e = 1\mathrm{sec}^{-1}$. Both the throughput versus the source average EH rate with the relay peak power $P_M^{\mathrm{R}} = 10\mathrm{mw}$ and the throughput versus the relay peak power with the average EH rate $P^{\mathrm{EH}} = 3\mathrm{mw}$ are shown in Fig. 5. The

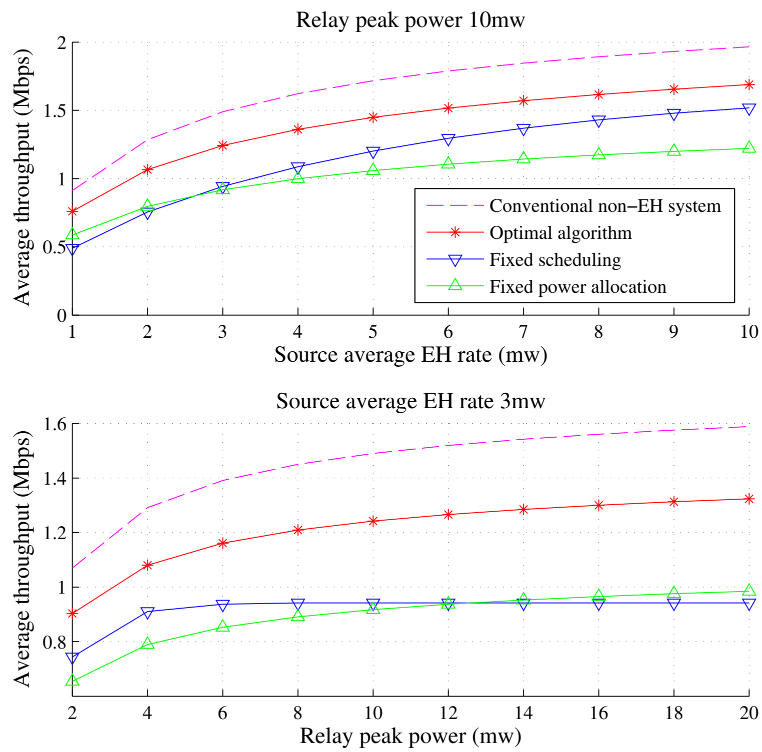

Figure 5. Simulation results of the throughput versus the source average EH rate and the relay peak power respectively.

numerical values of the different simulation parameters are selected based on the EH wireless sensor STM 11x [18], which can harvest energy at the rate of 3mw with a peak transmit power of 10mw.

We see that there is a large throughput gap between our optimal solution and the two baseline policies, and there is a performance loss compared to the upper bound due to the random EH profile. The first baseline case is constrained by the fixed scheduling, as the equal-length stages cannot adapt to different EH levels of the source. Therefore, even when the source has enough energy and can spend less time to transmit, the source stage will still occupy a fixed period. The second baseline policy also performs poorly, and this is because the power allocation is not adapted to the source EH profile. These two observations demonstrate the importance of both the optimal scheduling and power allocation for two-hop EH systems. We shall also mention that the performance of both fixed policies is always not good as long as the source EH profile varies with time. Meanwhile, the fixed scheduling policy performs especially poorly when the source and relay power levels differ significantly.

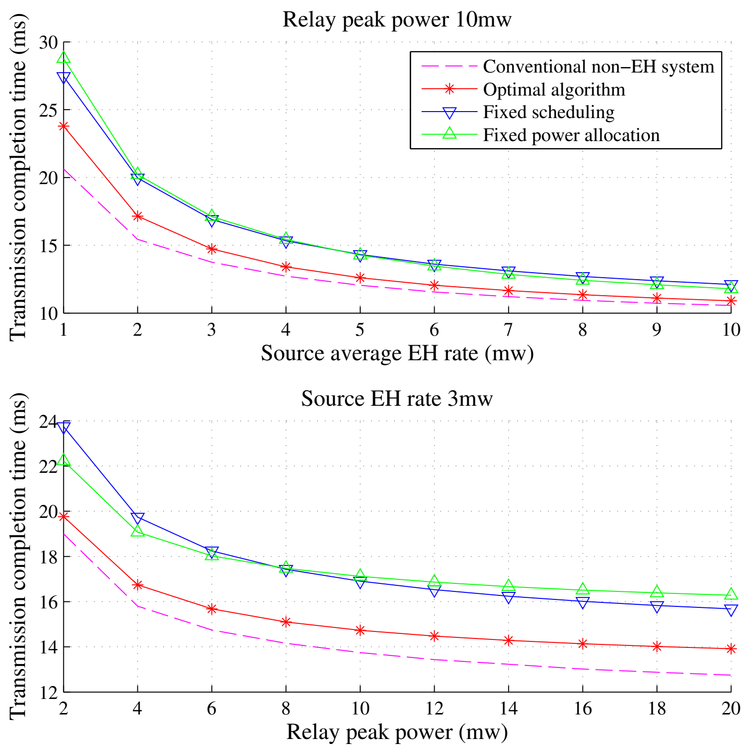

Figure 6. Simulation results of the transmission completion time versus the source average EH rate and the relay peak power respectively.

B. Transmission Completion Time Minimization

For the transmission completion time minimization, we will also compare the optimal policy with two baseline policies along with a performance upper bound. All the other settings are the same as those of the RMAX Problem except that in the first baseline policy, the only two stages are not of the same length, but in each of them the same amount of data is transmitted.

The simulation is run for 1000 random EH realizations, with the same parameter setting as that of Fig. 5. The total amount of data that needs to be transmitted is 20kbits. The transmission completion time versus the source average EH rate with the relay peak power $P_M^{\mathrm{R}} = 10$mw and the transmission completion time versus the relay peak power with the average EH rate $P^{\mathrm{EH}} = 3$mw are shown in Fig. 6. Compared with the two baseline policies, we can notice a great performance improvement when employing the optimal scheduling and power allocation. Again, this result demonstrates the importance of both of the optimal scheduling and power allocation.

VI. Conclusions

In this paper, we investigated the optimal transmission policy for two-hop communication systems with an EH source assisted by a non-EH relay, by either maximizing the throughput with a fixed transmission period or minimizing the transmission completion time with a fixed amount of data. Compared to the case with an EH relay, the system design, especially the S-R transmission scheduling, becomes more challenging with an EH source. Our results have shown that the DWF power allocation algorithm for single-hop EH communications provides useful design insights in solving the problem. In particular, the proposed DWF EH profile is a promising design tool for two-hop EH communication systems.

Appendix 1 Proof of Theorem 1

We only provide the proof for the case where the relay is with a total energy constraint, while the case where the relay has both power and energy constraints can be similarly obtained.

Property 1. This property can be proved according to Lemma 1 in [13]. Furthermore, we can conclude that a transmission policy cannot be optimal if either the source or the relay uses different transmit powers in any given DWF interval.

Property 2. For brevity and without loss of generality, we only provide the proof for the first DWF interval with $N^{\mathrm{D}} > 1$, while the proof for the other intervals can be similarly obtained. The whole proof consists of the following three cases:

Case 1: The source transmit power changes in the second DWF interval.

We define an *equivalent scheduling* as the one that achieves the same performance with the given scheduling decision but without violating the feasibility condition. We will prove the relay data equality by contradiction. Denote the last source period in the first DWF interval as ε_1, and the amount of data transmitted in the first relay stage after the first DWF interval as ξ_1. Assume that the data equality does not hold and the amount of data transmitted by the source is larger than the relay by an amount of ξ_2 at the end of the first DWF interval. Then, we can always find an equivalent scheduling, as Scheduling 1 in Fig. 7, where the beginning part of the second DWF interval is for the relay stage, and the amount of data transmitted in it is $\xi = \min(\xi_1, \xi_2)$. We denote this time period as ε_2. Next, from Scheduling 1, we can obtain another equivalent scheduling as Scheduling 2 in Fig. 7, by exchanging the last time period of $\varepsilon = \min(\varepsilon_1, \varepsilon_2)$ of the last source stage in the first DWF interval, with the relay stage at the beginning of the second DWF interval. Note that previous steps preserve the transmit powers for both the source and the relay. After such re-scheduling, we can see that in the second DWF interval of Scheduling 2, the source adopts a non-constant power, so we can always find a new scheduling with a constant transmit power that achieves a higher throughput, based on Property 1. Hence, the assumption does not hold, and the data equality is proved.

We will now prove the source energy equality with the help of the relay data equality property. This part of the proof can be extended to the other two cases. We assume that the source energy equality does not hold. We can then always move the beginning part of the source stage in the second DWF interval to the end of the source stage of the first DWF interval. Meanwhile we can delay the relay stage so that the source exhausts all the energy at the end of the first DWF interval, as shown in Fig. 8. This can be achieved without violating either the energy or the data causality constraint, while preserving the same throughput. It is obvious that the new scheduling result violates the data equality property, and can always be replaced by another policy with a better throughput, according to the previous proof. The source energy equality is therefore also proved.

Case 2: In the second DWF interval, the relay transmit power changes.

According to Lemma 7 in [13], when the relay transmit power changes, at least one of the following situations happens: The relay exhausts all the energy at that time instant, or the relay has already transmitted all the data from the source. Otherwise, it can always be replaced by another scheduling decision with a better throughput. But for a non-EH relay it is obvious that the energy buffer can only be empty at the end of the whole transmission period. Hence, the relay must empty its data buffer. As such, the relay data equality is proved.

Case 3: In the second DWF interval, both the source and relay transmit powers do not change.

We will prove that this case in fact cannot exist. We denote the index of the DWF interval at the end of which either the source or relay transmit power changes for the first time as k. Denote the time average of the source transmit power in $[0,t]$ as $P_{ave}\left(t\right)=\frac{\int_0^t P^{\mathrm{S}}(\tau)d\tau}{t}$ for $t\in(0,t_k^{\mathrm{D}}]$, then we can verify that $P_{ave}\left(t_1^{\mathrm{D}}\right)\geq\frac{E_k^{\mathrm{S}}}{t_k^{\mathrm{D}}}$. However, due to Properties 1 and 3 of DWF, the total harvested energy $E_{\Sigma}^{\mathrm{EH}}\left(t_1^{\mathrm{D}}\right)$ should satisfy $\frac{E_{\Sigma}^{\mathrm{EH}}\left(t_1^{\mathrm{D}}\right)}{t_1^{\mathrm{D}}}<\frac{E_k^{\mathrm{S}}}{t_k^{\mathrm{D}}}$ at the first DWF point. By combining these two aspects, we can obtain $P_{ave}\left(t_1^{\mathrm{D}}\right)>\frac{E_{\Sigma}^{\mathrm{EH}}\left(t_1^{\mathrm{D}}\right)}{t_1^{D}}$, which contradicts the energy causality

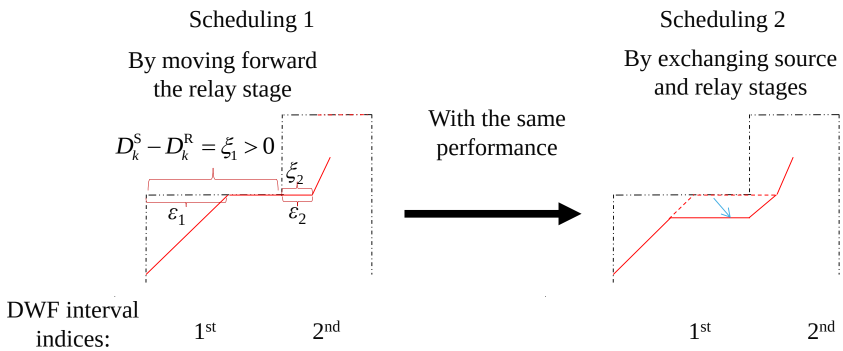

Figure 7. Illustration of the proof of Case 1 for property 2 in Theorem 1. The solid curve represents the power consumption at the source, i.e., the scheduling of the source, and the dashed curve represents the power consumption curve of the previous step. ξ_1/ξ_2 represent the amount of data, while $\varepsilon_1/\varepsilon_2$ represent the time duration.

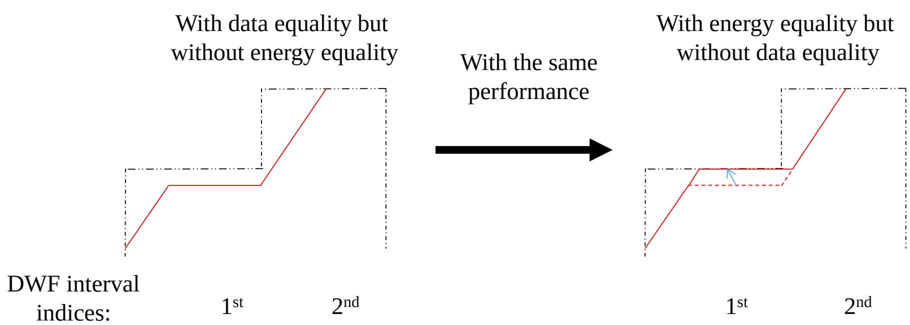

Figure 8. The method of obtaining the scheduling result with the same performance but violating the data equality property, for the proof of the source energy equality in Property 2 of Theorem 1.

constraint that $E_{\Sigma}^{\mathrm{EH}}\left(t_1^{\mathrm{D}}\right) \geq E_1^{\mathrm{S}} = P_{ave}\left(t_1^{\mathrm{D}}\right) t_1^{\mathrm{D}}$. Hence, Case 3 is not feasible.

APPENDIX 2 PROOF OF COROLLARY 1

Based on Theorem 1, **Problem RMAX** can be simplified with only equality constraints:

$$\max_{\substack{T_k^{\mathrm{S}}, E_k^{\mathrm{R}} \\ k=1,2,\ldots,N^{\mathrm{D}}}} D^{\mathrm{R}}(T) = \sum_{k=1}^{N^{\mathrm{D}}} \left(T_k^{\mathrm{D}} - T_k^{\mathrm{S}}\right) g\left(\frac{E_k^{\mathrm{R}}}{T_k^{\mathrm{D}} - T_k^{\mathrm{S}}}\right)$$

$$\text{s.t.} \quad T_k^{\mathrm{S}} g\left(\frac{E_k^{\mathrm{D}}}{T_k^{\mathrm{S}}}\right) = \left(T_k^{\mathrm{D}} - T_k^{\mathrm{S}}\right) g\left(\frac{E_k^{\mathrm{R}}}{T_k^{\mathrm{D}} - T_k^{\mathrm{S}}}\right),$$

$$\sum_{k=1}^{N^{\mathrm{D}}} E_k^{\mathrm{R}} = E_{\Sigma}^{\mathrm{R}},$$

In this new expression, there are N^{D} variables T_k^{S}, and N^{D} variables E_k^{R}, $k = 1, ..., N^{\mathrm{D}}$. The equation of the first constraint for each k in fact only includes two variables: T_k^{S} and E_k^{R}. We can obtain a unique T_k^{S} as a function of E_k^{R} due to the monotonicity of the function $g(\cdot)$, denoted as $T_k^{\mathrm{S}} = f_k\left(E_k^{\mathrm{R}}\right)$. Although an explicit expression of the function f_k is difficult to obtain, we can check that it is a monotonically increasing function by obtaining its inverse function $E_k^{\mathrm{R}} = f_k^{-1}\left(T_k^{\mathrm{S}}\right) = \left(T_k^{\mathrm{D}} - T_k^{\mathrm{S}}\right) g^{-1}\left(\frac{T_k^{\mathrm{S}}}{T_k^{\mathrm{D}} - T_k^{\mathrm{S}}} g\left(\frac{E_k^{\mathrm{D}}}{T_k^{\mathrm{S}}}\right)\right)$. Problem RMAX can then be simplified as an unconstrained problem with the new objective function as

$$
\begin{aligned}
D^{\mathrm{R}}(T) = & \sum_{k=1}^{N^{\mathrm{D}}-1} \left(T_k^{\mathrm{D}} - f_k\left(E_k^{\mathrm{R}}\right)\right) g\left(\frac{E_k^{\mathrm{R}}}{T_k^{\mathrm{D}} - f_k\left(E_k^{\mathrm{R}}\right)}\right) \\
& + \left(T_{N^{\mathrm{D}}}^{\mathrm{D}} - f_{N^{\mathrm{D}}}\left(E_{\Sigma}^{\mathrm{R}} - \sum_{k=1}^{N^{\mathrm{D}}-2} E_k^{\mathrm{R}}\right)\right) g\left(\frac{E_{\Sigma}^{\mathrm{R}} - \sum_{k=1}^{N^{\mathrm{D}}-2} E_k^{\mathrm{R}}}{T_k^{\mathrm{D}} - f_{N^{\mathrm{D}}}\left(E_{\Sigma}^{\mathrm{R}} - \sum_{k=1}^{N^{\mathrm{D}}-2} E_k^{\mathrm{R}}\right)}\right)
\end{aligned}
\tag{12}
$$

over only $\left(N^{\mathrm{D}} - 1\right)$ variables E_k^{R}. It can be verified that $\frac{d^2 D^{\mathrm{R}}(T)}{d^2 E_k^{\mathrm{R}}} < 0$, for each given index k and a given set of $\{E_i^{\mathrm{R}}, i = 1, 2, ..., k-1, k+1, ..., N^{\mathrm{D}} - 1\}$. Also $\frac{dD^{\mathrm{R}}(T)}{dE_k^{\mathrm{R}}} > 0$ for $E_k^{\mathrm{R}} = 0$, and $\frac{dD^{\mathrm{R}}(T)}{dE_k^{\mathrm{R}}} < 0$ for $E_k^{\mathrm{R}} = E_{\Sigma}^{\mathrm{R}} - \sum_{i=1, i\neq k}^{N^{\mathrm{D}}-1} E_i^{\mathrm{R}}$. So the value of E_k^{R} that maximizes the objective function is always located inside the interval $\left(0, E_{\Sigma}^{\mathrm{R}} - \sum_{i=1, i\neq k}^{N^{\mathrm{D}}-1} E_i^{\mathrm{R}}\right)$, and the partial derivative of the objective function over each E_k^{R} should equal zero, i.e., $\frac{dD^{\mathrm{R}}(T)}{dE_k^{\mathrm{R}}} = 0$ for $k = 1, 2, ..., N^{\mathrm{D}} - 1$. As a result, for the optimal solution, the following equations must be satisfied

$$
T_k^{\mathrm{S}} g\left(\frac{E_k^{\mathrm{D}}}{T_k^{\mathrm{S}}}\right) = \left(T_k^{\mathrm{D}} - T_k^{\mathrm{S}}\right) g\left(\frac{E_k^{\mathrm{R}}}{T_k^{\mathrm{D}} - T_k^{\mathrm{S}}}\right), \tag{13}
$$

$$
\frac{dD^{\mathrm{R}}(T)}{dE_k^{\mathrm{R}}} = 0, k = 1, 2, ..., N^{\mathrm{D}} - 1, \tag{14}
$$

$$
\sum_{k=1}^{N^{\mathrm{D}}} E_k^{\mathrm{R}} = E_{\Sigma}^{\mathrm{R}}. \tag{15}
$$

Consequently, the necessity of the equation set is proved.

On the other hand, both Eqns. (13) and (14) have a unique solution for each k, and we can further verify that the whole equation set also has a unique solution. Therefore, the sufficiency of this equation set is also proved.

APPENDIX 3 PROOF OF THEOREM 3

It is obvious that the DWF points obtained through Algorithm 3 satisfy all the other parts of Lemma 2 except for (9). In Algorithm 3, when determining i_k, i.e., the index of the k-th DWF

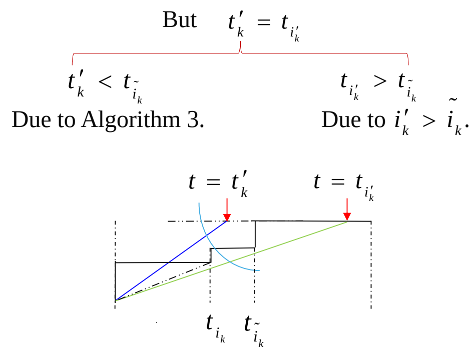

Figure 9. The contradiction of time, for the proof of Theorem 3.

interval in all the energy arrival epochs, we have only checked

$$i_k = \arg\min_{i<\tilde{i}_k} \left\{ \frac{\sum_{j=i_{k-1}+1}^{i} E_j}{t_i - t_{i_{k-1}}} \right\}, \tag{16}$$

which is a local operation, but Lemma 2 requires a global condition

$$i_k = \arg\min_{i:t_i \le T} \left\{ \frac{\sum_{j=i_{k-1}+1}^{i} E_j}{t_i - t_{i_{k-1}}} \right\}. \tag{17}$$

Hence, we need to prove that the former expression infers the latter one, which will be shown by contradiction. For an arbitrary k, we first get i_k from (16), and i'_k from (17), and we assume that $i'_k > \tilde{i}_k > i_k$. Based on the original EH profile, we can construct a new EH profile, of which the part $t \in [0, t_{i_k}]$ is the same as the original EH profile, while in the interval $\left(t_{i_k}, t_{i'_k}\right]$ there is no energy arrival. Due to the third property of DWF, a constant source power transmission for both EH profiles should be adopted in $\left[0, t_{i'_k}\right]$. Furthermore, we can verify that the feasibility of i'_k for the new EH profile is equivalent to that of the original EH profile, the proof of which is omitted. We can then focus on the proof for the new EH profile.

For the new EH profile, since $i'_k > \tilde{i}_k$, we have $t_{i'_k} > t_{\tilde{i}_k}$. On the other hand, due to the calculation procedure in the algorithm, the obtained t'_k should satisfy $t'_k < t_{\tilde{i}_k}$. However, based on the fact that the equation $g\left(\frac{E_0}{t}\right) t = D_0$ over t with arbitrary positive D_0 and E_0 has a unique solution (as long as it has a solution), we can obtain $t_{i'_k} = t'_k$. This contradiction of $t_{\tilde{i}_k} < t_{i'_k} = t'_k < t_{\tilde{i}_k}$ is illustrated in Fig. 9. Therefore, the assumed case does not hold for the new EH profile, nor for the original EH profile.

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