Antisymmetric tensor $\mathbf{Z}_p$ gauge symmetries in field theory and string theory

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Abstract

We consider discrete gauge symmetries in D dimensions arising as remnants of broken continuous gauge symmetries carried by general antisymmetric tensor fields, rather than by standard 1-forms. The lagrangian for such a general $\mathbf{Z}_p$ gauge theory can be described in terms of a r-form gauge field made massive by a $(r-1)$-form, or other dual realizations, that we also discuss. The theory contains charged topological defects of different dimensionalities, generalizing the familiar charged particles and strings in $D=4$. We describe realizations in string theory compactifications with torsion cycles, or with background field strength fluxes. We also provide examples of non-abelian discrete groups, for which the group elements are associated with charged objects of different dimensionality.

Contents

1   Introduction

Discrete symmetries are ubiquitous in many models of physics beyond the Standard Model. Moreover, the study of their nature is important also at a more fundamental level, since global symmetries, either continuous or discrete, are believed not to exist in consistent quantum theories including gravity, such as string theory (see the early references [1–3], and e.g. [4,5] for recent discussions). Hence, *exact* discrete symmetries should have a gauge nature [6–8] in these theories.

Discrete gauge symmetries in 4d theories have been subject to intense study both in field theory (see references above, also [9,10]) and string theory [11–14][1]. In these papers, the discrete symmetries arise as subgroups of continuous gauge symmetries[2], carried by 1-form fields, broken by their coupling to scalar fields. In a dual formulation

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[1]See also [15,16] and [17–21] for related applications, and [22–24] for discrete symmetries in heterotic orbifolds.

[2]Even in cases when there is no obvious underlying continuous symmetry, the latter can be made manifest in suitable supercritical string extensions decaying through closed tachyon condensation [25].

[5], an underlying $\mathbf{Z}_p$ symmetry is manifest in the existence of a 4d coupling between a 2-form potential B_2 and the gauge 1-form (through its field strength)

$$p \int_{4d} B_2 \wedge F_2. \tag{1.1}$$

Since the question of discrete symmetries in theories of quantum gravity is a fundamental one, it is fair to address it in higher-dimensional theories. The latter can host antisymmetric tensor gauge fields of rank higher than those available in 4d. This paper explores a novel realization of discrete gauge symmetries in string theory[3], based on these higher rank gauge fields. In fact, string theory contains a plethora of higher rank antisymmetric tensor fields, which upon compactification pick up topological couplings generalizing (1.1). This allows to describe sectors in which two massless antisymmetric tensor gauge fields, of ranks appropriate to the spacetime dimension, couple and became massive. The gauge symmetry is 'broken' by a Higgs-like mechanism, but a discrete $\mathbf{Z}_p$ subgroup remains.

The main novelty of these $\mathbf{Z}_p$ gauge symmetries from higher-rank antisymmetric tensors lies in the nature of the charged objects. For discrete symmetries arising from 1-form gauge potentials (coupling to 2-form fields in 4d), the charged objects are $\mathbf{Z}_p$ particles and $\mathbf{Z}_p$ strings. The charged objects under these more general $\mathbf{Z}_p$ symmetries are branes of worldvolume dimension related to the degrees of the form fields involved. Just like particles and strings in 4d, these objects pick up $\mathbf{Z}_p$ phases when surrounding each other. These objects, and the violation of their number conservation mod p, will receive a simple description as suitably wrapped branes in our string theory examples. This description provides an interpretation of the branes characterized by K-theory (or other groups of charges) in terms of topological defects associated to discrete gauge symmetries.

A complementary interpretation of our results is as a refinement of the discussions in [28] (actually predated by [26, 27]), which describe the coupling of two massless antisymmetric tensors of different ranks into one massive antisymmetric tensor. This higher-rank Higgs mechanism motivates a discussion of the phases of the corresponding field theories and their Higgs-confinement dualities. We are thus considering a refinement in which the Higgs-like mechanism leaves an extra $\mathbf{Z}_p$ discrete gauge symmetry.

The paper is organized as follows. In Section 2 we provide the field theory description of the phenomenon, starting with the familiar case of $\mathbf{Z}_p$ gauge symmetries from 1-form gauge potentials in section 2.1, and providing the higher-rank general-

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[3]See e.g. [26, 27] for an early appearance of these gauge symmetries in field theory.

ization in section 2.2. In Section 3 we provide explicit string theory realizations, by exploiting the flux catalysis described in [14] (based on the mechanism in [29]). In Section 4, we comment on a realization in compactifications with torsion homology, generalizing [11]. In Section 5, we discuss the realization of non-abelian discrete symmetries from higher-rank form fields. Finally, Section 6 contains some final remarks. Appendix A generalizes the ideas to theories with multiple tensor fields, and clarifies that the discrete symmetries in the original and dual descriptions can be different.

2 Field theory of higher-rank $\mathbf{Z}_p$ gauge symmetries

2.1 $\mathbf{Z}_p$ gauge symmetries from 1-form gauge potentials

We now quickly review the realization of 4d discrete gauge symmetries as subgroups of 'standard' continuous gauge symmetries, i.e. carried by 1-form gauge fields. For simplicity, we stick to the abelian case, which suffices to illustrate the main points.

The description is phrased in terms of (a) a 1-form A_1 and a 0-form ϕ, or (b) in a dual version, the magnetic gauge potential V_1 and the 2-form B_2 [5] (see also [30] for an alternative viewpoint on discrete gauge symmetries). These are subject to gauge invariances

$$
\begin{aligned}
(a) \quad & A_1 \to A_1 + d\lambda \quad , \quad \phi \to \phi + p\,\lambda \\
(b) \quad & B_2 \to B_2 + d\Lambda_1 \quad , \quad V_1 \to V_1 + p\,\Lambda_1.
\end{aligned}
\tag{2.1}
$$

This structures lead to a $\mathbf{Z}_p$ discrete gauge symmetry. The charged objects are $\mathbf{Z}_p$ particles and strings (electrically charged under A_1 and B_2, respectively), whose charge can be violated by suitable instantons and junctions, respectively (coupling to ϕ and V_1, respectively). The processes are associated to the gauge invariant operators [5]:

$$
\exp(-i\phi) \exp\left(i\,p \int_L A_1 \right) \quad , \quad \exp\left(-i \int_C V_1 \right) \exp\left(i\,p \int_\Sigma B_2 \right)
\tag{2.2}
$$

where L is a curve ending at the point P at which $e^{-i\phi}$ is inserted, i.e. $\partial L = P$, and similarly Σ is a surface ending on the curve C, i.e. $\partial\Sigma = C$. The first operator in (2.2) describes p (minimally) charged particles (coupling electrically to A_1) along the worldline L emanating from the point P; the second describes p (minimally) charged strings (coupling to B_2) spanning Σ and emanating from a string junction line C.

The basic structure of the 4d gauge invariant actions in terms of the above fields is

$$
(a) \quad \int_{4d} |d\phi - pA_1|^2 \quad \overset{\text{dual}}{\longleftrightarrow} \quad (b) \quad \int_{4d} |dV_1 - pB_2|^2.
\tag{2.3}
$$

In terms of B_2 and A_1, the $\mathbf{Z}_p$ discrete symmetry is usually identified from the presence of a 4d topological coupling [5]

$$p \int_{4d} B_2 \wedge F_2 \tag{2.4}$$

between the $U(1)$ field strength $F_2 = dA_1$ and the 2-form B_2. We recall that for proper identification of the discrete symmetry, the normalization of B_2 is such that its 4d dual scalar has periodicity 1, and that the minimal $U(1)$ charge is 1.

2.2   Higher rank $\mathbf{Z}_p$ discrete gauge symmetries

The structure in the previous section is the only one available in four dimensions. However, in higher dimensions there are gauge symmetries carried by higher-rank antisymmetric tensors, and it is reasonable to exploit them to generate discrete $\mathbf{Z}_p$ gauge symmetries. Conversely, higher dimensions allow the existence of $\mathbf{Z}_p$ charged objects with higher worldvolume dimensionality. Clearly, a straightforward possibility is to consider a 1-form gauge field and a $(D-2)$-form gauge field in D dimensions, coupling through a $B_{D-2} \wedge F_2$; this is a trivial addition of dimensions, in which the 4d $\mathbf{Z}_p$ string is extended to a real codimension-2 $(D-3)$-brane, and has appeared implicit or explicitly in earlier discussions of $\mathbf{Z}_p$ discrete symmetries. In other words, this case can always be dualized into that of a 1-form and a scalar field, i.e. a standard field theory Higgs mechanism.

In this paper we explore $\mathbf{Z}_p$ symmetries whose underlying continuous symmetry involves genuine higher rank antisymmetric tensors, in any dual picture. Due to the difficulties with non-abelian tensor field theories, we stick to the abelian case, although Section 5 contains some discussion on the realization of non-abelian discrete structures.

We consider a theory in D dimensions, with a r-form field A_r and a $(r-1)$-form field ϕ_{r-1}, with the gauge invariance[4]

$$A_r \to A_r + d\lambda_{r-1} \quad , \quad \phi_{r-1} \to \phi_{r-1} + p\,\lambda_{r-1}. \tag{2.5}$$

The notation is obviously chosen to recover the familiar one for $r = 1$, c.f. (2.1a). A gauge invariant action, generalizing (2.3a), is

$$\int_{\mathbf{M}_D} |\,d\phi_{r-1} - pA_r\,|^2. \tag{2.6}$$

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[4]These theories have been considered e.g. in [28] (see also e.g. [31, 32]). As in there, we consider the gauge symmetries to be *compact*, namely there is charge quantization for the extended objects to which they couple. As in the 4d case, normalization is such that the minimal charge is unity.

Notice that in this theory both fields are gauge fields since, on top of (2.5), the lagrangian is invariant under $\phi_{r-1} \to \phi_{r-1} + d\sigma_{r-2}$.

In the above lagrangian, the field A_r eats up the field ϕ_{r-1} and gets massive. Note that this is consistent with the counting of degrees of freedom of antisymmetric tensor gauge fields under the $SO(D-2)$ and $SO(D-1)$ little groups for massless and massive particles:

$$\binom{D-2}{r} + \binom{D-2}{r-1} = \binom{D-1}{r}. \tag{2.7}$$

The gauge symmetry of A_r is broken spontaneously, but a discrete $\mathbf{Z}_p$ symmetry remains. This is a higher-rank analogue of the Higgsing of a $U(1)$ gauge group by eating up the phase of a charge-p scalar. However, the naturally charged objects are not in general codimension-2 $(D-3)$-branes, and point particles, as in the rank-1 case; rather we have $(r-1)$-branes and $(D-r-2)$-branes (electric charges under A_r and magnetic charges under ϕ_{r-1}).

It is straightforward to dualize A_r into its magnetic $(D-r-2)$-form gauge potential V_{D-r-2}, and ϕ_{r-1} into its dual $(D-r-1)$-form gauge potential B_{D-r-1}. They are subject to the gauge invariance c.f. (2.1b)

$$B_{D-r-1} \to B_{D-r-1} + d\Lambda_{D-r-2} \quad , \quad V_{D-r-2} \to V_{D-r-2} + p\,\Lambda_{D-r-2}. \tag{2.8}$$

The dual gauge-invariant action has the structure

$$\int_{\mathbf{M}_D} |dV_{D-r-2} - p\,B_{D-r-1}|^2. \tag{2.9}$$

This dual description makes manifest an *emergent* $\mathbf{Z}_p$ gauge symmetry[5]. The objects charged under ϕ_{r-1} and V_{D-r-2} are $(r-2)$- and $(D-r-3)$-branes, and play the role of generalized junctions violating the number of $(r-1)$- and $(D-r-2)$-branes in p units, making them $\mathbf{Z}_p$-valued. This follows form the gauge-invariant operators

$$\exp\left(-i\int_{P_{r-1}} \phi_{r-1}\right) \exp\left(i\,p\int_{L_r} A_r\right) \quad , \quad \exp\left(-i\int_{C_{D-r-2}} V_{D-r-2}\right) \exp\left(i\,p\int_{\Sigma_{D-r-1}} B_{D-r-1}\right)$$

where L_r has P_{r-1} as its boundary, $\partial L_r = P_{r-1}$, and similarly $\partial\Sigma_{D-r-1} = C_{D-r-2}$.

By standard arguments, the quantum amplitude of a process involving a (minimally charged) $(r-1)$-branes with worldvolume Σ_r, and a (minimally charged) $(D-r-2)$-brane with worldvolume Δ_{D-r-1} receives a phase

$$\exp\left[\frac{2\pi i}{p} L(\Sigma_r, \Delta_{D-r-1})\right] \tag{2.10}$$

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[5] An important point, not manifest in the examples in [5] is that the emergent discrete symmetry may differ from the original one. This is illustrated explicitly in Appendix A.

where $L(\Sigma_r, \Delta_{D-r-1})$ is the linking number in D-dimensions (the number of times Σ_r surrounds Δ_{D-r-1}, or vice-versa).

For future convenience, it is useful to identify the analogue of the BF coupling (2.4) in 4d. This is the topological coupling

$$p \int_{\mathbf{M}_D} B_{D-r-1} \wedge F_{r+1}, \tag{2.11}$$

where we have introduced the field strength $F_{r+1} = dA_r$.

The construction in this section is basically a refinement of that in [28]. The main novelty is the identification of the unbroken $\mathbf{Z}_p$ symmetry, which reflects in a $\mathbf{Z}_p$-grading of the topological defects in the theories under consideration. In these $\mathbf{Z}_p$ theories, the duality between the Higgs and confinement phases holds as in [28]. Indeed, the Higgs phase of A_r translates into the fact that the dual magnetic $(D-r-3)$-branes cannot exist in isolation but are confined by the p $(D-r-2)$-branes stuck to them.

3 Higher-rank $\mathbf{Z}_p$ symmetries in string theory flux compactifications

A simple way to realize rich sets of $\mathbf{Z}_p$-charged objects, associated to discrete gauge symmetries, is the 'flux catalysis' systematically studied in [14] (based on [29]) for 4d discrete symmetries carried by 1-form gauge fields; see [33–39] for related phenomena. The key idea is that in 4d string compactifications with field-strength flux backgrounds (flux compactifications), the 10d Chern-Simons couplings can produce 4d BF couplings associated to $U(1)$ gauge symmetries broken to $\mathbf{Z}_p$ subgroups.

Clearly, the idea easily generalizes to produce $\mathbf{Z}_p$ gauge symmetries from higher rank antisymmetric tensor fields. In this Section we pursue this suggestion to recover the structures introduces in section 2.2, for compactifications to higher $D > 4$.

For concreteness, we focus on a particular example in $D = 6$. Consider a compactification of type IIA on a real dimension 4 space $\mathbf{X}_4$ (not necessarily $\mathbf{T}^4$ or K3, since we are not particularly interested in supersymmetry). We introduce p units of flux for the RR field strength 4-form $F_4 = dC_3$

$$\int_{\mathbf{X}_4} F_4 = p. \tag{3.1}$$

The 10d Chern-Simons couplings produce the following 6d coupling

$$\int_{10d} C_3 \wedge H_3 \wedge F_4 \rightarrow p \int_{6d} C_3 \wedge H_3. \tag{3.2}$$

This has the structure (2.11) for $r = 2$ (with $B_3 \to C_3$ and $F_3 \to H_3$). There are $\mathbf{Z}_p$-charged 1-branes (arising from fundamental F1-strings) and 2-branes (from D2-branes). Their decay occurs through junctions of worldvolume dimensions 1 and 2, respectively, from the corresponding dual sources; namely, a D4-brane wrapped on $\mathbf{X}_4$ (on which p F1-strings must end [40]), and a NS5-brane wrapped on $\mathbf{X}_4$ (on which p D2-branes must end, by a dual of the Freed-Witten anomaly[6] [29]). See Appendix B of [14] for an overview of these processes.

The M-theory version of the above system is interesting, and arises naturally in the context of the $\mathrm{AdS}_7/\mathrm{CFT}_4$ correspondence. Compactification of M-theory on a 4-manifold down to $D = 7$, with p units of G_4 4-form flux produces a 7d coupling $p \int_{7d} G_4 \wedge C_3$. The corresponding $\mathbf{Z}_p$ discrete symmetry has appeared in [42]. It is amusing to notice that M2-branes correspond to the two kinds of $\mathbf{Z}_p$ topological defects, hence M2-branes pick up $\mathbf{Z}_p$ phases when surrounding each other, in a higher dimensional analogy of anyons in $D = 3$. This interesting behaviour is presumably linked to the elusive system of coincident M5-branes underlying this gauge/gravity duality.

4 Higher-rank $\mathbf{Z}_p$ symmetries in string compactifications with torsion

Discrete gauge symmetries associated to higher-rank forms are briefly mentioned in [43], although related to torsion in homology or K-theory. This very formal discussion can be made very explicit following [11], at least for torsion homology. To show that compactifications with torsion homology can produce higher-rank discrete symmetries, we consider a simple illustrative example. Consider M-theory on a 4-manifold with torsion 1-cycles (and their dual 2-cycles), $H_1(\mathbf{X}_4, \mathbf{Z}) = H_2(\mathbf{X}_4, \mathbf{Z}) = \mathbf{Z}_p$. We focus on the sector of M2-branes on 1-cycles – 7d strings – and M5-branes on 2-cycles – 7d 3-branes – (there is another sector of M2-branes on 2-cycles and M5-branes on 1-cycles, which can be discussed similarly). Following [11], we introduce the Poincaré dual torsion 2- and 3-forms α_2^{tor}, $\tilde{\omega}_3^{\mathrm{tor}}$, satisfying the relations

$$d\omega_1^{\mathrm{tor}} = p \, \alpha_2^{\mathrm{tor}} \quad , \quad d\beta_2^{\mathrm{tor}} = p \, \tilde{\omega}_3^{\mathrm{tor}} \tag{4.1}$$

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[6]Actually [41] considered the case of torsion H_3 flux, and the physical picture for general H_3 appeared in [29]. Still, we stick to the widely used term Freed-Witten anomaly, even for non-torsion fluxes.

where ω_1^{tor} and β_2^{tor} are globally well-defined 1- and 2-forms. The torsion 2- and 3-forms α_2^{tor} and $\tilde{\omega}_3^{\text{tor}}$ are thus trivial in de Rham cohomology, but not in the $\mathbf{Z}$-valued cohomology, i.e. $H_2(\mathbf{X}_4\mathbf{R}) = H_3(\mathbf{X}_4, \mathbf{R}) = \emptyset$, $H_2(\mathbf{X}_4\mathbf{Z}) = H_3(\mathbf{X}_4, \mathbf{Z}) = \mathbf{Z}_p$. The torsion linking number is encoded in the intersection pairing

$$\int_{\mathbf{X}_4} \alpha_2^{\text{tor}} \wedge \beta_2^{\text{tor}} = \int_{\mathbf{X}_4} \omega_1^{\text{tor}} \wedge \tilde{\omega}_2^{\text{tor}} = 1. \tag{4.2}$$

These forms are assumed to be eigenstates of the Laplacian [11], corresponding to massive modes; they can be usefully exploited to describe dimensional reduction of the antisymmetric tensor fields, in particular, the M-theory 3- and 6-forms

$$C_3 = \phi_1 \wedge \alpha_2^{\text{tor}} + A_2 \wedge \omega_1^{\text{tor}} \quad , \quad C_6 = B_4 \wedge \beta_2^{\text{tor}} + V_3 \wedge \tilde{\omega}_3^{\text{tor}}. \tag{4.3}$$

The corresponding field strengths contain the structures

$$dC_3 = (d\phi_1 + pA_2) \wedge \alpha_2^{\text{tor}} + \dots \quad , \quad dC_6 = (dV3 + pB_4) \wedge \tilde{\omega}_3^{\text{tor}} + \dots \tag{4.4}$$

which (modulo a trivial sign redefinition) imply the gauge invariances (2.5), (2.8). Accordingly, the 11d kinetic term for $G_4 = dC_3$ (and its dual) lead to 7d actions with the structure (2.6), (2.9). The dimensional reduction we have just sketched thus relates the underlying torsion homology with the $\mathbf{Z}_p$ gauge theory lagrangians of section 2.2.

5 The non-abelian case

Non-abelian discrete gauge symmetries are interesting[7]. In 4d, the non-abelian character can be detected by letting two strings (with charges given by non-commuting group elements a, b) cross, and watching the appearance of an stretched string (with charge given by the commutator $c = aba^{-1}b^{-1}$). In string theory realizations, this follows from brane creation processes when the underlying branes are crossed [50].

In general dimension D, we can look for similar effects, the only difference being that the objects have richer dimensionality. Consider the following table, which describes the geometry of two branes (denoted 1 and 2) which cross and lead to the creation of brane 3

	$\overbrace{}^{d_1}$	$\overbrace{}^{d_2}$	$\overbrace{}^{d_3}$	
Brane 1	$- \cdots -$	$- \cdots -$	$\times \cdots \times$	$\times$
Brane 2	$- \cdots -$	$\times \cdots \times$	$- \cdots -$	$\times$
Brane 3	$- \cdots -$	$\times \cdots \times$	$\times \cdots \times$	$-$

The symbols $-$ and $\times$ denote that the brane spans or does not span the corresponding dimension, and obviously $d_1 + d_2 + d_3 + 1 = D$. The last entry corresponds to the single overall transverse dimensions to branes 1 and 2, on which the crossing proceeds, and along which the created brane 3 stretches.

As a concrete example, involving discrete gauge symmetries arising from torsion homology c.f. section 4, consider type IIB compactified on a 5-manifold with a $\mathbf{Z}_p$ torsion 3-cycle, self-intersecting over the dual $\mathbf{Z}_p$ torsion 1-cycle (the AdS$_5 \times \mathbf{S}^5/\mathbf{Z}_3$ geometry in [15] is a realization for $p = 3$). The theory contains 5d 2-branes arising from NS5-branes on the torsion 3-cycle, a further set of 5d 2-branes from D5-branes on the torsion 3-cycle, and a set of 5d 2-branes from D3-branes on the torsion 1-cycle. The crossing of NS5- and D5-branes produces D3-branes [50], leading to the above 2-brane crossing effect (with $d_1 = 2$, $d_2 = d_3 = 1$ in the above table); the resulting discrete group is non-abelian, and is given by a Δ_{27} (for general $\mathbf{Z}_p$ torsion, a discrete Heisenberg group [13], see also [16]).

One can similarly construct more exotic examples, in which the non-abelian symmetry group elements are associated to objects of *different* dimensionality. For instance, consider type IIA compactified on the same geometry as above, i.e. a 5-manifold with torsion 3- and 1-cycles. The theory contains 5d 2-branes from NS5-branes on the torsion 3-cycle, a set of 5d 1-branes from D4-branes on the torsion 3-cycle, and a further set of 5d 1-branes from D2-branes on the torsion 1-cycle. The crossing of NS5- and D4-branes produces D2-branes; in 5d the process corresponds to crossing a 2-brane with a 1-brane, with the creation of another kind of 1-brane (hence we have $d_1 = 1$, $d_2 = 2$, $d_3 = 1$). The resulting discrete Heisenberg symmetry group is exotic, since its elements are associated to objects of different dimensionality. A similar phenomenon already occurs in the (abelian) context of D-brane charge classification by K-theory, where in certain examples the charges in a K-theory group correspond to branes in cohomology classes of different degree (e.g. [43] quotes the example of $\mathbf{RP_7}$, where the torsion cohomology is $\mathbf{Z}_2 \oplus \mathbf{Z}_2 \oplus \mathbf{Z}_2$, with the torsion K-theory is $\mathbf{Z}_8$).

Similar examples could be worked out involving branes whose charges are $\mathbf{Z}$-valued in (co)homology, but which are actually torsion due to the presence of background fluxes. We refrain from a systematic discussion, hoping that the above examples suffice to illustrate the main idea.

6 Final remarks

In this paper we have considered discrete gauge symmetries remaining from broken continuous gauge symmetries carried by general antisymmetric tensor fields. We have described the field theory for these general $\mathbf{Z}_p$ gauge theories, in several dual realizations[8]. We have described abelian and non-abelian realizations in string theory, in compactifications with torsion cycles, or generating torsion by flux catalysis. We would like to conclude with a few remarks:

• The case of 1-form gauge symmetries broken by scalars can be elegantly described in the language of gaugings in supergravity. It would be interesting to develop such a description for the higher-rank case.

• The non-abelian structure of Section 5 is intriguing, as it points to some underlying non-abelian (broken) symmetry involving higher-rank antisymmetric tensors (possibly of different degree). It would be interesting to explore the existence of this underlying structure more directly, possibly in terms of non-abelian gaugings.

• Recent holographic discussions of the gravitational dual to certain superconductors (e.g. helical or striped phase p-wave superconductors [51]) involve 2-form fields with topological couplings to 1-form gauge fields in $D = 5$. It would be interesting to explore possible holographic applications of our higher-rank antisymmetric tensor field theories, and their discrete symmetries.

We hope this paper triggers further progress into understanding discrete gauge symmetries, and the role of higher-rank gauge potentials, in field theory and string theory.

Acknowledgments

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[8]In fact, we have described in appendix A that the discrete gauge symmetry can in general change in the dual description, in the presence of additional continuous gauge symmetries.

A   Multiple antisymmetric tensors

In this section we comment on subtle points arising when the topological couplings between antisymmetric tensor gauge fields involve several fields of each kind. The analysis is similar to section 2.2, with additional subtleties in identifying the emergent discrete gauge symmetry in the dual description (using ingredients partially noticed in [14]).

A.1   Field theory description

For instance, consider a single r-form gauge field A_r made massive by coupling to several $(r-1)$-form fields ϕ_{r-1}^k in D dimensions, with the lagrangian

$$\int_D \sum_k |d\phi_{r-1}^k - p_k A_r|^2. \tag{A.1}$$

This is gauge invariant under

$$
\begin{aligned}
A_r &\rightarrow A_r + d\lambda_{r-1} \\
\phi_{r-1}^k &\rightarrow \phi_{r-1}^k + p_k \lambda_{r-1}.
\end{aligned}
\tag{A.2}
$$

The potential A_r actually eats up only one linear combination of the fields ϕ_{r-1}^k, while the orthogonal linear combinations remain as massless $(r-1)$-form fields. Denoting $p = \mathrm{g.c.d}(p_k)$, the massive gauge symmetry leaves a remnant $\mathbf{Z}_p$ gauge symmetry. This follows from the structure of $\mathbf{Z}_p$ charged $(r-1)$-brane states, whose number can be violated by operators

$$\exp\left(-i\int_{P_{r-1}} \phi_{r-1}^k\right) \exp\left(i\int_{L_r} p_k A_r\right). \tag{A.3}$$

Each such vertex creates p_k $(r-1)$-branes, so by Bezout's lemma, there exists a set of vertices which (minimally) violates their number in p units, making the $(r-1)$-branes $\mathbf{Z}_p$-valued. In addition, the theory enjoys the continuous gauge invariance associated to the orthogonal combinations of the ϕ_{r-1}^k's.

In the dual realization, we have a single potential V_{D-r-2} and several potentials B_{D-r-1}^k, with lagrangian

$$\int_D |dV_{D-r-2} - \textstyle\sum_k p_k B_{D-r-1}^k|^2. \tag{A.4}$$

There are gauge invariances under

$$
\begin{aligned}
B_{D-r-1}^k &\rightarrow B_{D-r-1}^k + d\Lambda_{D-r-2}^k \\
V_{D-r-2} &\rightarrow V_{D-r-2} + \textstyle\sum_k p_k \Lambda_{D-r-2}^k
\end{aligned}
\tag{A.5}
$$

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(on top of the dual gauge transformation $V_{D-r-2} \to V_{D-r-2} + d\sigma_{D-r-3}$). One combination of the continuous gauge symmetries, given by $\sum_k (p_k/p)T_k$ (where T_k is the generator of the kth gauge transformation), is actually broken to a discrete subgroup $\mathbf{Z}_q$, with $q = \sum_k (p_k)^2/p$ [14]. Hence, the discrete part of the emergent gauge group in the dual description is different from the original one; this is a novel feature as compared with the system in [5] and in Section 2. The $\mathbf{Z}_q$ structure follows from the structure of charged $(D - r - 2)$-brane states, which are created by operators

$$\exp\left(-i\int_{C_{D-r-2}} V_{D-r-2}\right)\exp\left(i\int_{\Sigma_{D-r-1}}\sum_k p_k\, B^k_{D-r-1}\right). \tag{A.6}$$

This violates T_k charge conservation in p_k units, and hence $\sum_k (p_k/p)T_k$ in $q = \sum_k (p_k)^2/p$ units.

The fact that the original $\mathbf{Z}_p$ and the emergent $\mathbf{Z}_q$ gauge symmetries are different is not in contradiction with charge quantization of the dual charged objects, i.e. the $\mathbf{Z}_p$ $(r-2)$-branes and the $\mathbf{Z}_q$ $(D-r-2)$-branes, because of the presence of additional charges under the additional continuous gauge symmetries in the system.

Clearly, a similar (but more involved) analysis can be carried out when there are several fields of each kind. We leave this for the interested reader.

A.2   A string theory example

It is easy to use e.g. the flux catalysis of Section 3 to obtain concrete examples of the above structure. For instance, we consider the example of type IIA compactified on K3 to $D = 6$, with background F_2 flux. Specifically, we introduce two basis of 2-cycles $\{\alpha_k\}$, $\{\beta_k\}$, with $\alpha_k \cdot \beta_l = \delta_{kl}$, and define

$$\int_{\alpha_k} F_2 = p_k. \tag{A.7}$$

There a 6d topological coupling arising as follows

$$\int_{10d} B_2 \wedge F_2 \wedge F_6 \quad \longrightarrow \quad \sum_k \int_{6d} p_k\, B_2 \wedge \hat{F}^k_4 \tag{A.8}$$

where

$$\hat{F}^k_4 = \int_{\beta_k} F_6. \tag{A.9}$$

This mixed term has the structure to complete into the square

$$\int_{6d} |d\phi^k_1 - p_k B_2|^2 \tag{A.10}$$

where we have introduced the 6d duals of of $\hat{C}^k_3$, given by $\phi^k_1 = \int_{\alpha_k} C_3$. This has the structure (A.1) with a trivial notation change.

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