

1 Introduction

Since the inception of quantum key distribution by Bennett and Brassard [BB84], it has been known that quantum communication permits to achieve protocol tasks that are impossible given only a classical channel. For example, a quantum key distribution scheme [BB84] permits to agree on a secret key that is statistically secret, using only an authenticated but not secret channel. (By statistical security we mean security against computationally unbounded adversaries, also known as information-theoretical security.) In contrast, when using only classical communication, it is easy to see that such a secret key can always be extracted by a computationally sufficiently powerful adversary. Similarly, based on an idea by Wiesner [Wie83], Bennett, Brassard, Crépeau, and Skubiszewska [BBCS91] presented a protocol that was supposed to construct an statistically secure oblivious transfer¹ protocol from a commitment, another feat that is easily seen to be impossible classically.² Oblivious transfer, on the other hand, has been recognized by Kilian [Kil88] to securely evaluate arbitrary functions. Unfortunately, the protocol of Bennett et al. could, at the time, not be proven secure, and the first complete proof of (a variant of) that protocol was given almost two decades later by Damgård, Fehr, Lunemann, Salvail, and Schaffner [DFL⁺09a].

Yet, although the oblivious transfer protocol satisfies the intuitive secrecy requirements of oblivious transfer, in certain cases the protocol might lose its security when used in a larger context. In other words, there are limitations on how the protocol can be composed. For example, no security guarantee is given when several instances of the protocol are executed concurrently (see Section 1.5 for a more detailed explanations of the various restrictions).

The problem of composability has been intensively studied by the classical cryptography community (here and in the following, we use the word classical as opposed to quantum). To deal with this problem in a general way, Canetti [Can01] introduced the notion of Universal Composability, short UC (Pfitzmann and Waidner [PW01] independently introduced the equivalent Reactive Simulatability framework). The UC framework allows to express the security of a multitude of protocol tasks in a unified way, and any UC-secure protocol automatically enjoys strong composability guarantees (so-called universal composability). In particular, such a protocol can be run concurrently with others, and it can be used as a subprotocol of other protocols in a general way. Ben-Or and Mayers [BOM04] and Unruh [Unr04] have shown that the idea of UC-security can be easily adapted to the quantum setting and have independently presented quantum variants of the UC notion. These notions enjoy the same strong compositionality guarantees. Shortly afterwards, Ben-Or, Horodecki, Leung, Mayers, and Oppenheim [BOHL⁺05] showed that many quantum key distribution protocols are quantum-UC-secure.

Our contribution. In this work, we use the UC framework to show the existence of a statistically secure and universally composable oblivious transfer protocol that uses only a commitment scheme. Towards this goal, we first present a new definition of quantum-UC-security. In our opinion, our notion is technically simpler than the notions of Ben-Or and Mayers [BOM04] and Unruh [Unr04].

¹In an oblivious transfer protocol, Alice holds two bitstrings m_0, m_1 , and Bob a bit c . Bob is supposed to get m_c but not m_{1-c} , and Alice should not learn c .

²We remark that, on the other hand, Mayers [May97] shows that also in the quantum case, constructing a statistically secure commitment scheme *without any additional assumption* is impossible. However, under additional assumptions like in the quantum bounded storage model by Damgård, Fehr, Salvail, and Schaffner [DFSS05], statistically secure bit commitment is possible. See Section 1.4 for a discussion of the implications of Mayers' impossibility result for our result.

We believe that this may also help to increase the popularity of this notion in the quantum cryptography community and to show the potential for using UC-security in the design of quantum protocols. Second, we show that a variant of the protocol by Bennett et al. [BBCS91] is indeed a UC-secure oblivious transfer protocol. By composing this protocol with a UC-secure protocol for general multi-party computations by Ishai, Prabhakaran, and Sahai [IPS08], we get UC-secure protocols for general multi-party computations using only commitments and a quantum channel – this is easily seen to be impossible in a purely classical setting.

1.1 Quantum Universal Composability (quantum-UC)

We begin by giving an overview over the UC framework. The basic idea behind the UC framework is to define security by comparison. Given a certain protocol task, say to implement a secure message transfer, we first specify a machine, called the ideal functionality \mathcal{F} that, by definition, fulfills this protocol task securely. E.g., In the case of a secure message transfer, this functionality would take a value x from Alice and give this value to Bob. All communication between parties and the functionality is done over secure channels. Obviously, this functionality \mathcal{F} does exactly what we expect from a secure message transfer. Then, we define what it means for a protocol π to be a secure implementation of \mathcal{F} . Intuitively, we require that π is no less secure than \mathcal{F} . In other words, anything the adversary can do in an execution of π , the adversary could also do in an execution using \mathcal{F} . (And in particular, since \mathcal{F} is secure by definition, the adversary then cannot perform any successful attacks on π either.) This requirement is formally captured by requiring that for any adversary Adv , there is another adversary Sim , the simulator, such that an execution of π with Adv (called the real model) is indistinguishable from an execution of \mathcal{F} with Sim (called the ideal model). And indistinguishability in turn is modeled by requiring that no machine \mathcal{Z} , called the environment, can guess whether it is interacting with the real model or with the ideal model. More precisely:

Definition 1 ((Classical) Universal Composability – informal) *We say π classical-UC-emulates \mathcal{F} if for any adversary Adv there is a simulator Sim such that for all environments \mathcal{Z} we have that the difference between the following probabilities is negligible: The probability that \mathcal{Z} outputs 1 in an execution of \mathcal{Z} , Adv , and π , and the probability that \mathcal{Z} outputs 1 in an execution of \mathcal{Z} , Sim , and \mathcal{F} . (We assume that \mathcal{Z} can freely communicate with the adversary/simulator.)*

In the example of a secure message transfer functionality \mathcal{F} , the functionality would require its inputs x from \mathcal{Z} and then send x back to \mathcal{Z} . In a secure message transfer protocol π , that is, in a protocol π classical-UC-emulating \mathcal{F} , Alice would then have to take the input x from \mathcal{Z} , and Bob would have to output x to \mathcal{Z} (otherwise \mathcal{Z} could trivially distinguish the real and the ideal model). All communication sent between Alice and Bob over insecure channels, however, would be under the control of the adversary. Thus everything the adversary learns from that communication, the simulator would have to be able to produce on its own; in particular, the adversary cannot derive x from that communication since the simulator could not simulate that knowledge (in the ideal model, the simulator cannot get x). This captures the intuitive requirement that a secure message transfer protocol should not reveal the message to the adversary. In a similar way, other properties like the authenticity of the message can be derived from the UC definition.

The UC definition comes in many flavors. For example, in computational classical UC-security we restrict the adversary, simulator, and environment to polynomial-time machines. This variant

is used if we want to show security based on computational assumptions. In statistical classical UC, on the other hand, we quantify over all (possibly unbounded) adversaries, simulators, and environments. This variant is used to model statistical security.

Besides providing a unified way to model the security of various protocol tasks by specifying the ideal functionality, the UC framework allows for very general composition of protocols. Assume a protocol $\sigma^{\mathcal{F}}$ that uses a functionality \mathcal{F} as a building block. That is, in the real model, $\sigma^{\mathcal{F}}$ has access to a functionality \mathcal{F} that performs a certain task in a fully trusted way. (We say, $\sigma^{\mathcal{F}}$ runs in the \mathcal{F} -hybrid model.) Assume that $\sigma^{\mathcal{F}}$ classical-UC-emulates some other functionality \mathcal{G} and that we are given a protocol π that classical-UC-emulates \mathcal{F} . Then the so-called universal composition theorem states that σ^π , the protocol resulting from using the subprotocol π instead of \mathcal{F} , also classical-UC-emulates \mathcal{G} . This even holds if $\sigma^{\mathcal{F}}$ invokes many instances of \mathcal{F} concurrently. Such a composition theorem is very useful for proving the security of larger protocols in a modular way: One first abstracts away a subprotocol (here π) by replacing it by some functionality (here \mathcal{F}), leading to a simpler protocol $\sigma^{\mathcal{F}}$ in the \mathcal{F} -hybrid model that is more amenable to analysis. Then the protocol π is analyzed separately. It should be noted that it was shown by Lindell [Lin03] that no security notion weaker than (a particular variant of) classical UC can have such a composition theorem.

To get a variant of the UC notion suitable for modeling quantum cryptography, we only need to slightly modify the definition: Instead of quantifying over classical adversaries, simulators, and environment, we quantify over quantum adversaries, simulators, and environment. That is, the protocol parties, the adversary, the simulator, and the environment are allowed to store, send, and compute with quantum states. (And in the computational variant of quantum-UC-security, we additionally restrict adversaries, simulators, and environment to be restricted to polynomial-time quantum computations.) Since, in a sense, we only change the machine model, most structural theorems about UC-security, in particular the universal composition theorem, still hold for quantum-UC-security; their proofs are almost identical in the classical and in the quantum setting. We present our model of quantum-UC in Section 2 and give a universal composition theorem for that model.

1.2 UC-secure quantum oblivious transfer

The oblivious transfer (OT) protocol used in this paper is essentially the same as the protocol proposed by Damgård et al. [DFL⁺09a] which in turn is based on a protocol by Bennett et al. [BBCS91]. The basic idea of the protocol is that Alice encodes a random sequence \tilde{x} of bits as a quantum state, each bit randomly either in the computational basis or in the diagonal basis.³ Then Bob is supposed to measure all bits, this time in random bases of his choosing. Then Alice sends the bases she used to Bob. Let $I_=_$ denote the indices of the bits \tilde{x}_i where Alice and Bob chose the same basis, and I_\neq the indices of the bits where Alice and Bob chose different bases. Assume that Bob wants to receive the message m_c out of Alice's messages m_0, m_1 . Then Bob sets $I_c := I_=_$ and $I_{1-c} := I_\neq$ and sends (I_0, I_1) to Alice. Alice will not know which of these two sets is which and hence does not learn c . Bob will know the bits \tilde{x}_i at indices $i \in I_c$. But even a dishonest Bob, assuming that he measured the whole quantum state, will not know the bits at indices I_{1-c} since he used the wrong bases for these bits. Thus Alice uses the bits at I_0 to mask her message m_0 , and the bits at I_1 to mask

³If we were to use photons for transmission, in the computational basis we might encode the bit 0 as a vertically polarized photon and the bits 1 as a horizontally polarized photon. In the diagonal basis we might encode the bit 0 as a 45°-polarized photon, and the bit 1 as a 135°-polarized photon.

her message m_1 . Then Bob can recover m_c but not m_{1-c} . (To deal with the fact that a malicious Bob might have partial knowledge about the bits at I_{1-c} , we use so-called privacy amplification to extract a near uniformly mask from these bits.)

The problem with this analysis is that we have assumed that a malicious Bob measures the whole quantum state upon reception. But instead, Bob could store the quantum state until he learns the bases that Alice used, and then use these bases to measure all bits \tilde{x}_i accurately. Hence, we need to force a dishonest Bob to measure all bits before Alice sends the bases. The idea of Bennett et al. [BBCS91] is to introduce the following test: Bob has to commit to the bases he used and to his measurement outcomes. Then Alice picks a random subset of the bits, and Bob opens the commitments on his bases and outcomes corresponding to this subset of bits. Alice then checks whether Bob's measurement outcomes are consistent with what Alice sent. If Bob does not measure enough bits, then he will commit to the wrong values in many of the commitments, and there will be a high probability that Alice detects this.

It was a long-standing open problem what kind of a commitment needs to be used in order for this protocol to be secure. Damgård et al. [DFL⁺09a] give criteria for the commitment scheme under which the OT protocol can be proven to have so-called stand-alone security; stand-alone security, however, does not give as powerful compositionality guarantees as UC-security (cf. Section 1.5 below). In order to achieve UC-security, we assume that the commitment is given as an ideal functionality. Then we have to show UC-security in the case of a corrupted Alice, and UC-security in the case of a corrupted Bob. The case of a corrupted Alice is simple, as one can easily see that no information flows from Bob to Alice (the commitment functionality does, by definition, not leak any information about the committed values). The case of a corrupted Bob is more complex and requires a careful analysis about the amount of information that Bob can retrieve about Alice's bits. Such an analysis has already been performed by Damgård et al. [DFL⁺09a] in their setting. Fortunately, we do not need to repeat the analysis. We show that that under certain special conditions, stand-alone security already implies UC-security. Since in the case of a corrupted Bob, these conditions are fulfilled, we get the security in the case of a corrupted Bob as a corollary from the work by Damgård et al. [DFL⁺09a].

In Section 5, we show that the OT protocol by Damgård et al. [DFL⁺09a], when using an ideal functionality for the commitment, is statistically quantum-UC-secure. Furthermore, the universal composition theorem guarantees that we can replace the commitment functionality by any quantum-UC-secure commitment protocol.

1.3 Quantum lifting and multi-party computation

We are now equipped with a statistically quantum-UC-secure OT protocol π_{QOT} in the commitment-hybrid model. As noted first by Kilian [Kil88], OT can be used for securely evaluating arbitrary functions, short, OT is complete for multi-party computation. Furthermore, Ishai, Prabhakaran, and Sahai [IPS08] showed that for any functionality \mathcal{G} (even interactive functionalities that proceed in several rounds), there is a classical protocol $\rho^{\mathcal{F}_{\text{OT}}}$ in the OT-hybrid model that statistically classical-UC-emulates \mathcal{G} . Thus, to get a protocol for \mathcal{G} in the commitment-hybrid model, we simply replace all invocations to \mathcal{F}_{OT} by invocations of the subprotocol π_{QOT} , resulting in a protocol $\rho^{\pi_{\text{QOT}}}$. We then expect that the security of $\rho^{\pi_{\text{QOT}}}$ follows directly using the universal composition theorem (in its quantum variant). There is, however, one difficulty: To show that $\rho^{\pi_{\text{QOT}}}$ statistically quantum-UC-emulates \mathcal{G} , the universal composition theorem requires that the following premises

are fulfilled: π_{QOT} statistically quantum-UC-emulates \mathcal{F}_{OT} , and $\rho^{\mathcal{F}_{\text{OT}}}$ statistically quantum-UC-emulates \mathcal{G} . But from the result of Ishai et al. [IPS08] we only have that $\rho^{\mathcal{F}_{\text{OT}}}$ statistically *classical*-UC-emulates \mathcal{G} . Hence, we first have to show that the same result also holds with respect to quantum-UC-security. Fortunately, we do not have to revisit the proof of Ishai et al., because we show the following general fact:

Theorem 2 (Quantum lifting theorem – informal) *If the protocols π and ρ are classical protocols, and π statistically classical-UC-emulates ρ , then π statistically quantum-UC-emulates ρ .*

Combining this theorem with the universal composition theorem, we immediately get that $\rho^{\pi_{\text{QOT}}}$ statistically quantum-UC-emulates \mathcal{G} . In other words, any multi-party computation can be performed securely using only a commitment and a quantum-channel. In contrast, we show that in the classical setting a commitment is not even sufficient to compute the AND-function.

We stress that a property like the quantum lifting theorem should not be taken for granted. For example, for the so-called stand-alone model as considered by Fehr and Schaffner [FS09], no corresponding property is known. A special case of security in the stand-alone model is the zero-knowledge property: The question whether protocols that are statistical zero-knowledge with respect to classical adversaries are also zero-knowledge with respect to quantum adversaries has been answered positively by Watrous [Wat06] for particular protocols, but is still open in the general case.

1.4 How to interpret our result

We show that we can perform arbitrary statistically UC-secure multi-party computations, given a quantum channel and a commitment. However, Mayers [May97] has shown that, even in the quantum setting, statistically secure commitment schemes do not exist, not even with respect to security notions much weaker than quantum-UC-security. In the light of this result, the reader may wonder whether our result is not vacuous. To illustrate why our result is useful even in the light of Mayers' impossibility result, we present four possible application scenarios.

Weaker computational assumptions. The first application of our result would be to combine our protocols with a commitment scheme that is only *computationally* quantum-UC-secure. Of course, the resulting multi-party computation protocol would then not be *statistically* secure any more. However, since commitment intuitively seems to be a simpler task than oblivious transfer, constructing a computationally quantum-UC-secure commitment scheme might be possible using simpler computational assumptions, and our result then implies that the same computational assumptions can be used for general multi-party computation.

Physical setup. One might seek a direct physical implementation of a commitment, such as a locked strongbox (or an equivalent but technologically more advanced construct). With our result, such a physical implementation would be sufficient for general multi-party computation. In contrast, in a classical setting one would be forced to try to find physical implementations of OT. It seems that a commitment might be a simpler physical assumption than OT (or at least an incomparable one). So our result reduces the necessary assumptions when implementing general multi-party computation protocols based on physical assumptions. Also, Kent [Ken99] proposes to build commitments based on the fact that the speed of light is bounded. Although it is not clear whether his schemes are UC-secure (and in particular, how to model his physical assumptions in

the UC framework), his ideas might lead to a UC-secure commitment scheme that then, using our result, gives general UC-secure multi-party computation based on the limitation of the speed of light.

Theoretical separation. Our result can also be seen from the purely theoretical point of view. It gives a separation between the quantum and the classical setting by showing that in the quantum setting, commitment is complete for general statistically secure multi-party computation, while in the classical world it is not. Such separations – even without practical applications – may increase our understanding of the relationship between the classical and the quantum setting and are therefore arguably interesting in their own right.

Long-term security. Müller-Quade and Unruh [MQU07b] introduce the concept of long-term UC-security. In a nutshell, long-term UC-security is a strengthening of computational UC-security that guarantees that a protocol stays secure even if the adversary gets unlimited computational power after the protocol execution. This captures the fact that, while we might confidently judge today’s technology, we cannot easily make predictions about which computational problems will be hard in the future. Müller-Quade and Unruh show that (classically) long-term UC-secure commitment protocols exist given certain practical infrastructure assumptions, so-called signature cards. It is, however, likely that their results cannot be extended to achieve general multi-party computation. Our result, on the other hand, might allow to overcome this limitation: Assume that we show that the commitment protocol of Müller-Quade and Unruh is also secure in a quantum variant of long-term UC-security. Then we could compose that commitment protocol with the protocols presented here, leading to long-term UC-secure general multi-party protocols from signature cards.

1.5 Compositionality restrictions in prior work

Above, we claimed that the results of prior work on commitment schemes in the quantum setting have limitations concerning their composability guarantees. We will now briefly explain in which cases composition is possible in prior models, and what the restrictions are. All prior results giving some kind of composability guarantee work in the some variant of the so-called stand-alone model. The basic idea of the stand-alone model is similar to that of the UC model: We specify a protocol π and an ideal functionality \mathcal{F} , and we say that π implements \mathcal{F} in the stand-alone model if for every adversary Adv attacking π (real model), there is a simulator Sim attacking \mathcal{F} (ideal model), so that the real and the ideal model are indistinguishable. But in contrast to the UC model, indistinguishability of the real and the ideal model is not defined with respect to an environment that tries to guess which model it is interacting with. Instead, given fixed inputs for all honest parties, we require that the output of the honest parties and of the adversary (considered as a joint quantum state) is indistinguishable from the output of the functionality and of the simulator (considered as a joint quantum state). The notion of indistinguishability of quantum states is then defined depending of the flavor of the stand-alone model. Security in the stand-alone model is strictly weaker than security in the UC model: the UC environment may introduce additional dependencies between the messages send in the protocol and the protocol inputs/outputs. For example, the environment could give a message that has been sent over an insecure channel by Bob as initial protocol input to Alice. Such dependencies are explicitly excluded in the stand-alone model.

In the classical case, it has been shown by Canetti [Can00] that the stand-alone model allows for sequential composition. Sequential composition means that we are allowed to run several protocols

or several instances of one protocol one after the other without losing security, but we are not allowed to run them concurrently or interleave the protocol steps (as can easily happen if the protocol parties are not careful about their synchronization). Similar results have been obtained in the quantum case by Wehner and Wullschleger [WW08] and by Fehr and Schaffner [FS09] for different variants of the quantum stand-alone model.

There are two main flavors of the quantum stand-alone model: Statistical and computational security. In the first case, adversary and simulator are allowed to be unlimited, and in the second case, adversary and simulator are computationally bounded. Note that when defined like this, statistical stand-alone security does not imply computational stand-alone security because statistical stand-alone security does not guarantee that the simulator corresponding to a computationally bounded adversary is also computationally bounded. The effect of this is slightly paradoxical: one can compose statistically secure protocols with each other, and one can compose computationally secure protocols with each other, but no guarantees are given if one composes a computationally secure protocol with a statistically secure protocol.

We note that the problems arising from an unlimited simulator can be avoided by simply strengthening the statistical stand-alone model and requiring that the simulator is computationally bounded if the adversary is. This is the approach we also take in our modeling of statistical quantum-UC-security.

The protocols analyzed by Wehner and Wullschleger [WW08] and Fehr and Schaffner [FS09] are proven secure in (different variants of) the statistical stand-alone model. Furthermore, the simulator they construct does not run in polynomial time, therefore their results do not imply computational stand-alone security and the difficulties outlined above apply.

The situation concerning the OT protocol analyzed by Damgård, Fehr, Lunemann, Salvail, and Schaffner [DFL⁺09a] is even more subtle. They prove that in the case of a corrupted recipient Bob, their protocol is secure in the computational stand-alone model. Furthermore, for a corrupted sender Alice, the protocol is secure in the statistical stand-alone model with non-polynomial-time simulator. Thus, the protocol can be composed sequentially with other protocols that are computationally secure for corrupted Bob and statistically secure for corrupted Alice; yet it cannot be composed with protocols which are statistically secure for corrupted Bob and computationally secure for corrupted Alice. In particular, the OT protocol cannot be composed with another instance of itself where Bob is the sender. The full version [DFL⁺09b, Section 5] of their paper describes an extension of the underlying commitment scheme which enables the construction of an efficient simulator. With such an extension, sequential composition of their OT protocol with computationally secure protocols is possible.

In all three papers, when composing classical and quantum protocols, it is necessary that even the classical protocols are proven secure with respect to a definition involving quantum adversaries. A result like our quantum lifting theorem (Theorem 2) is an open problem in the stand-alone model.

1.6 Related work

Security models. General quantum security models based on the stand-alone model have first been proposed by van de Graaf [vdG98]. His model comes without a composition theorem. The notion has been refined by Wehner and Wullschleger [WW08] and by Fehr and Schaffner [FS09] who also prove sequential composition theorems. Quantum security models in the style of the UC

model have been proposed by Ben-Or and Mayers [BOHL⁺05] and by Unruh [Unr04]. The original idea behind the UC framework in the classical setting was independently discovered by Canetti [Can01] and by Pfitzmann and Waidner [PW01] (the notion is called Reactive Simulatability in the latter paper).

Quantum protocols. The idea of using quantum communication for cryptographic purposes seems to originate from Wiesner [Wie83]. The idea gained widespread recognition with the BB84 quantum key-exchange protocol by Bennett and Brassard [BB84]. A statistically hiding and binding commitment scheme was proposed by Brassard, Crépeau, Jozsa, and Langlois [BCJL93]. Unfortunately, the scheme was later found to be insecure; in fact, Mayers [May97] showed that statistically hiding and binding quantum commitments are impossible without using additional assumptions. Kent [Ken99] circumvents this impossibility result by proposing a statistically hiding and binding commitment scheme that is based on the limitation of the speed of light. Bennett, Brassard, Crépeau, and Skubiszewska [BBCS91] present a protocol for statistically secure oblivious transfer in the quantum setting. They prove their protocol secure under the assumption that the adversary cannot store qubits and measures each qubit individually. They also sketch an extension that uses a commitment scheme to make their OT protocol secure against adversaries that can store and compute on quantum states. The protocol analyzed in the present paper is, in its basic idea, that extension. Yao [Yao95] gave a partial proof of the extended OT protocol. His proof, however, is incomplete and refers to a future complete paper which, to the best of our knowledge, never appeared. As far as we know, the first complete proof of a variant of that OT protocol has been given by Damgård, Fehr, Lunemann, Salvail, and Schaffner [DFL⁺09a]; their protocol is secure in the stand-alone model. Hofheinz and Müller-Quade [HMQ03] conjectured that the extended OT protocol by Bennett et al. [BBCS91] is indeed UC-secure; in the present paper we prove this claim. Damgård, Fehr, Salvail, and Schaffner [DFSS05] have presented OT and commitment protocols which are statistically secure under the assumption that the adversary has a bounded quantum storage capacity.

Classical vs. quantum security. To the best of our knowledge, van de Graaf [vdG98] was the first to notice that even statistically secure classical protocols are not necessarily secure in a quantum setting. The reason is that the powerful technique of rewinding the adversary is not available in the quantum setting. Watrous [Wat06] showed that in particular cases, a technique similar to classical rewinding can be used. He uses this technique to construct quantum zero-knowledge proofs. No general technique relating classical and quantum security is known; to the best of our knowledge, our quantum lifting theorem is the first such result (although restricted to the statistical UC model).

Miscellaneous. Kilian [Kil88] first noted that OT is complete for general multi-party computation. Ishai, Prabhakaran, and Sahai [IPS08] prove that this also holds in the UC setting. Computationally secure UC commitment schemes have been presented by Canetti and Fischlin [CF01].

1.7 Preliminaries

General. A nonnegative function μ is called negligible if for all $c > 0$ and all sufficiently large k , $\mu(k) < k^{-c}$. A nonnegative function f is called overwhelming if $f \geq 1 - \mu$ for some negligible μ . Keywords in typewriter font (e.g., `environment`) are assumed to be fixed but arbitrary, distinct non-empty words in $\{0, 1\}^*$. $\varepsilon \in \{0, 1\}^*$ denotes the empty word. Given a sequence $x = x_1, \dots, x_n$, and a set $I \subseteq \{1, \dots, n\}$, $x|_I$ denote the sequence x restricted to the indices $i \in I$.

Quantum systems. We can only give a few terse overview over the formalism used in quantum computing. For a thorough introduction, we recommend the textbook by Nielsen and Chuang [NC00, Chap. 1–2]. A (pure) state in a quantum system is described by a vector $|\psi\rangle$ in some Hilbert space \mathcal{H} . In this work, we only use Hilbert spaces of the form $\mathcal{H} = \mathbb{C}^N$ for some countable set N , usually $N = \{0, 1\}$ for qubits or $N = \{0, 1\}^*$ for bitstrings. We always assume a designated orthonormal basis $\{|x\rangle : x \in N\}$ for each Hilbert space, called the computational basis. The basis states $|x\rangle$ represent classical states (i.e., states without superposition). Given several separate subsystems $\mathcal{H}_1 = \mathbb{C}^{N_1}, \dots, \mathcal{H}_n = \mathbb{C}^{N_n}$, we describe the joint system by the tensor product $\mathcal{H}_1 \otimes \dots \otimes \mathcal{H}_n = \mathbb{C}^{N_1 \times \dots \times N_n}$. We write $\langle\Psi|$ for the linear transformation mapping $|\Phi\rangle$ to the scalar product $\langle\Psi|\Phi\rangle$. Consequently, $|\Psi\rangle\langle\Psi|$ denotes the orthogonal projector on $|\Psi\rangle$. We set $|0\rangle_+ := |0\rangle$, $|1\rangle_+ := |1\rangle$, $|0\rangle_\times := \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, and $|1\rangle_\times := \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. For $x \in \{0, 1\}^n$ and $\theta \in \{+, \times\}^n$, we define $|x\rangle_\theta := |x_1\rangle_{\theta_1} \otimes \dots \otimes |x_n\rangle_{\theta_n}$.

Mixed states. If a system is not in a single pure state, but instead is in the pure state $|\Psi_i\rangle \in \mathcal{H}$ with probability p_i (i.e., it is in a mixed state), we describe the system by a density operator $\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$ over \mathcal{H} . This representation contains all physically observable information about the distribution of states, but some distributions are not distinguishable by any measurement and are represented by the same mixed state. The set of all density operators is the set of all positive⁴ operators \mathcal{H} with trace 1, and is denoted $\mathcal{P}(\mathcal{H})$. Composed systems are described by operators in $\mathcal{P}(\mathcal{H}_1 \otimes \dots \otimes \mathcal{H}_n)$. In the following, when speaking about (quantum) states, we always mean mixed states in the density operator representation. A mapping $\mathcal{E} : \mathcal{P}(\mathcal{H}_1) \rightarrow \mathcal{P}(\mathcal{H}_2)$ represents a physically possible operation (realizable by a sequence of unitary transformations, measurements, and initializations and removals of qubits) iff it is a completely positive trace preserving map.⁵ We call such mappings superoperators. The superoperator \mathcal{E}_{init}^m on $\mathcal{P}(\mathcal{H})$ with $\mathcal{H} := \mathbb{C}^{\{0,1\}^*}$ and $m \in \{0, 1\}^*$ is defined by $\mathcal{E}_{init}^m(\rho) := |m\rangle\langle m|$ for all ρ .

Composed systems. Given a superoperator \mathcal{E} on $\mathcal{P}(\mathcal{H}_1)$, the superoperator $\mathcal{E} \otimes id$ operates on $\mathcal{P}(\mathcal{H}_1 \otimes \mathcal{H}_2)$. Instead of saying “we apply $\mathcal{E} \otimes id$ ”, we say “we apply \mathcal{E} to \mathcal{H}_1 ”. If we say “we initialize \mathcal{H} with m ”, we mean “we apply \mathcal{E}_{init}^m to \mathcal{H} ”. Given a state $\rho \in \mathcal{P}(\mathcal{H}_1 \otimes \mathcal{H}_2)$, let $\rho_x := (|x\rangle\langle x| \otimes id)\rho(|x\rangle\langle x| \otimes id)$. Then the outcome of measuring \mathcal{H}_1 in the computational basis is x with probability $\text{tr} \rho_x$, and after measuring x , the quantum state is $\frac{\rho_x}{\text{tr} \rho_x}$. Since we will only performs measurements in the computational basis in this work, we will omit the qualification “in the computational basis”. The terminology in this paragraph generalizes to systems composed of more than two subsystems.

Classical states. Classical probability distributions $P : N \rightarrow [0, 1]$ over a countable set N are represented by density operators $\rho \in \mathcal{P}(\mathbb{C}^N)$ with $\rho = \sum_{x \in N} P(x) |x\rangle\langle x|$ where $\{|x\rangle\}$ is the computational basis. We call a state classical if it is of this form. We thus have a canonical isomorphism between the classical states over \mathbb{C}^N and the probability distributions over N . We call a superoperator $\mathcal{E} : \mathcal{P}(\mathbb{C}^{N_1}) \rightarrow \mathcal{P}(\mathbb{C}^{N_2})$ classical iff there is a randomized function $F : N_1 \rightarrow N_2$ such that $\mathcal{E}(\rho) = \sum_{\substack{x \in N_1 \\ y \in N_2}} \Pr[F(x) = y] \cdot |x\rangle\langle x| \cdot |y\rangle\langle y|$. Classical superoperators describe what can be realized with classical computations. An example of a classical superoperator on $\mathcal{P}(\mathbb{C}^N)$ is $\mathcal{E}_{class} : \rho \mapsto \sum_x \langle x|\rho|x\rangle \cdot |x\rangle\langle x|$. Intuitively, \mathcal{E}_{class} measures ρ in the computational basis and then

⁴We call an operator positive if it is Hermitian and has only nonnegative Eigenvalues.

⁵A map \mathcal{E} is completely positive iff for all Hilbert spaces \mathcal{H}' , and all positive operators ρ over $\mathcal{H}_1 \otimes \mathcal{H}'$, $(\mathcal{E} \otimes id)(\rho)$ is positive.

discards the outcome, thus removing all superpositions from ρ .

2 Quantum Universal Composability

We now present our quantum-UC-framework. For a motivation of the model, we refer to Section 1.1.

Machine model. A machine M is described by an identity id_M in $\{0,1\}^*$ and a sequence of superoperators $\mathcal{E}_M^{(k)}$ ($k \in \mathbb{N}$) on $\mathcal{H}^{state} \otimes \mathcal{H}^{class} \otimes \mathcal{H}^{quant}$ with $\mathcal{H}^{state}, \mathcal{H}^{class}, \mathcal{H}^{quant} := \mathbb{C}^{\{0,1\}^*}$ (the *state transition operators*). The index k in $\mathcal{E}_M^{(k)}$ denotes the security parameter. The Hilbert space \mathcal{H}^{state} represents the state kept by the machine between invocations, and \mathcal{H}^{class} and \mathcal{H}^{quant} are used both for incoming and outgoing messages. Any message consists of a classical part stored in \mathcal{H}^{class} and a quantum part stored in \mathcal{H}^{quant} . If a machine id_{sender} wishes to send a message with classical part m and quantum part $|\Psi\rangle$ to a machine id_{rcpt} , the machine id_{sender} initializes \mathcal{H}^{class} with $(id_{sender}, id_{rcpt}, m)$ and \mathcal{H}^{quant} with $|\Psi\rangle$. (See the definition of the network execution below for details.) The separation of messages into a classical and a quantum part is for clarity only, all information could also be encoded directly in a single register. If a machine does not wish to send a message, it initializes \mathcal{H}^{class} and \mathcal{H}^{quant} with ε .

A network \mathbf{N} is a set of machines with pairwise distinct identities containing a machine \mathcal{Z} with $id_{\mathcal{Z}} = \mathbf{environment}$. We write $ids_{\mathbf{N}}$ for the set of the identities of the machines in \mathbf{N} .

We call a machine M quantum-polynomial-time if there is a uniform⁶ sequence of quantum circuits C_k such that for all k , the circuit C_k implements the superoperator $\mathcal{E}_M^{(k)}$.

Network execution. The state space $\mathcal{H}_{\mathbf{N}}$ for a network \mathbf{N} is defined as $\mathcal{H}_{\mathbf{N}} := \mathcal{H}^{class} \otimes \mathcal{H}^{quant} \otimes \bigotimes_{id \in ids_{\mathbf{N}}} \mathcal{H}_{id}^{state}$ with $\mathcal{H}_{id}^{state}, \mathcal{H}^{class}, \mathcal{H}^{quant} := \mathbb{C}^{\{0,1\}^*}$. Here \mathcal{H}_{id}^{state} represents the local state of the machine with identity id and \mathcal{H}^{class} and \mathcal{H}^{quant} represent the state spaces used for communication. (\mathcal{H}^{class} and \mathcal{H}^{quant} are shared between all machines. Since only one machine is active at a time, no conflicts occur.)

A step in the execution of \mathbf{N} is defined by a superoperator $\mathcal{E} := \mathcal{E}_{\mathbf{N}}^{(k)}$ operating on $\mathcal{H}_{\mathbf{N}}$. This superoperator performs the following steps: First, \mathcal{E} measures \mathcal{H}^{class} in the computational basis, and parses the outcome as $(id_{sender}, id_{rcpt}, m)$. Let M be the machine in \mathbf{N} with identity id_{rcpt} . Then \mathcal{E} applies $\mathcal{E}_M^{(k)}$ to $\mathcal{H}_{id}^{state} \otimes \mathcal{H}^{class} \otimes \mathcal{H}^{quant}$. Then \mathcal{E} measures \mathcal{H}^{class} and parses the outcome as $(id'_{sender}, id'_{rcpt}, m')$. If the outcome could not be parsed, or if $id'_{sender} \neq id_{rcpt}$, initialize \mathcal{H}^{class} with $(\varepsilon, \mathbf{environment}, \varepsilon)$ and \mathcal{H}^{quant} with ε . (This ensures that the environment is activated if a machine sends no or an ill-formed message.)

The output of the network \mathbf{N} on input z and security parameter k is described by the following algorithm: Let $\rho \in \mathcal{P}(\mathcal{H}_{\mathbf{N}})$ be the state that is initialized to $(\varepsilon, \mathbf{environment}, z)$ in \mathcal{H}^{class} , and to the empty word ε in all other registers. Then repeat the following indefinitely: Apply $\mathcal{E}_{\mathbf{N}}^{(k)}$ to ρ . Measure \mathcal{H}^{class} . If the outcome is of the form $(\mathbf{environment}, \varepsilon, out)$, return out and terminate. Otherwise, continue the loop. The probability distribution of the return value out is denoted by $\text{Exec}_{\mathbf{N}}(k, z)$.

⁶A sequence of circuits C_k is uniform if a deterministic Turing machine can output the description of C_k in time polynomial in k .

Corruptions. To model corruptions, we introduce *corruption parties*, special machines that follow the instructions given by the adversary. When invoked, the corruption party P_{id}^C with identity id measures \mathcal{H}^{class} and parses the outcome as $(id_{sender}, id_{rcpt}, m)$. If $id_{sender} = \mathbf{adversary}$, \mathcal{H}^{class} is initialized with m . (In this case, m specifies both the message and the sender/recipient. Thus the adversary can instruct a corruption party to send to arbitrary recipients.) Otherwise, \mathcal{H}^{class} is initialized with $(id, \mathbf{adversary}, (id_{sender}, id_{rcpt}, m))$. (The message is forwarded to the adversary.) Note that, since P_{id}^C does not touch the \mathcal{H}^{quant} , the quantum part of the message is forwarded.

Given a network \mathbf{N} , and a set of identities C , we write \mathbf{N}^C for the set resulting from replacing each machine $M \in \mathbf{N}$ with identity $id \in C$ by P_{id}^C .

Security model. A protocol π is a set of machines with **environment**, **adversary** $\notin ids(\pi)$. We assume a set of identities $parties_\pi \subseteq ids(\pi)$ to be associated with π . $parties_\pi$ denotes which of the machines in the protocol are actually protocol parties (as opposed to incorruptible entities such as ideal functionalities).

An environment is a machine with identity **environment**, an adversary or a simulator is a machine with identity **adversary** (there is no formal distinction between adversaries and simulators, the two terms refer to different intended roles of a machine).

In the following we call two networks indistinguishable if there is a negligible function μ such that for all $z \in \{0, 1\}^*$ and $k \in \mathbb{N}$, $|\Pr[\text{Exec}_N(k, z) = 1] - \Pr[\text{Exec}_M(k, z) = 1]| \leq \mu(k)$. We speak of perfect indistinguishability if $\mu = 0$.

Definition 3 (Statistical quantum-UC-security) *Let protocols π and ρ be given. We say π statistically quantum-UC-emulates ρ iff for every set $C \subseteq parties_\pi$ and for every adversary Adv there is a simulator Sim such that for every environment \mathcal{Z} , the networks $\pi^C \cup \{\text{Adv}, \mathcal{Z}\}$ (called the real model) and $\rho^C \cup \{\text{Sim}, \mathcal{Z}\}$ (called the ideal model) are indistinguishable. We furthermore require that if Adv is quantum-polynomial-time, so is Sim .*

Definition 4 (Computational quantum-UC-security) *Let protocols π and ρ be given. We say π computationally quantum-UC-emulates ρ iff for every set $C \subseteq parties_\pi$ and for every quantum-polynomial-time adversary Adv there is a quantum-polynomial-time simulator Sim such that for every quantum-polynomial-time environment \mathcal{Z} , the networks $\pi^C \cup \{\text{Adv}, \mathcal{Z}\}$ and $\rho^C \cup \{\text{Sim}, \mathcal{Z}\}$ are indistinguishable.*

Note that although $\text{Exec}_{\pi^C \cup \{\text{Adv}, \mathcal{Z}\}}(k, z)$ may return arbitrary bitstrings, we only compare whether the return value of \mathcal{Z} is 1 or not. This effectively restricts \mathcal{Z} to returning a single bit. This can be done without loss of generality (see [Can01] for a discussion this issue; their arguments also apply to the quantum case) and simplifies the definition.

In our framework, any communication between two parties is perfectly secure since the network model guarantees that they are delivered to the right party and not leaked to the adversary. To model a protocol with insecure channels instead, one would explicitly instruct the protocol parties to send all messages through the adversary. Authenticated channels can be realized by introducing an ideal functionality (see the next section) that realizes an authenticated channel. For simplicity, we only consider protocols with secure channels in this work.

2.1 Ideal functionalities

In most cases, the behavior of the ideal model is described by a single machine \mathcal{F} , the so-called ideal functionality. We can think of this functionality as a trusted third party that perfectly implements

the desired protocol behavior. For example, the functionality \mathcal{F}_{OT} for oblivious transfer would take as input from Alice two bitstrings m_0, m_1 , and from Bob a bit c , and send to Bob the bitstring m_c . Obviously, such a functionality constitutes a secure oblivious transfer. We can thus define a protocol π to be a secure OT protocol if π quantum-UC-emulates \mathcal{F}_{OT} where \mathcal{F}_{OT} denotes the protocol consisting only of one machine, the functionality \mathcal{F}_{OT} itself. There is, however, one technical difficulty here. In the real protocol π , the bitstring m_c is sent to the environment \mathcal{Z} by Bob, while in the ideal model, m_c is sent by the functionality. Since every message is tagged with the sender of that message, \mathcal{Z} can distinguish between the real and the ideal model merely by looking at the sender of m_c . To solve this issue, we need to ensure that \mathcal{F} sends the message m_c in the name of Bob (and for analogous reasons, that \mathcal{F} receives messages sent by \mathcal{Z} to Alice or Bob). To achieve this, we use so-called dummy-parties [Can01] in the ideal model. These are parties with the identities of Alice and Bob that just forward messages between the functionality and the environment.

Definition 5 (Dummy-party) *Let a machine P and a functionality \mathcal{F} be given. The dummy-party \tilde{P} for P and \mathcal{F} is a machine that has the same identity as P and has the following state transition operator: Let $\text{id}_{\mathcal{F}}$ be the identity of \mathcal{F} . When activated, measure $\mathcal{H}^{\text{class}}$. If the outcome of the measurement is of the form $(\text{environment}, \text{id}_P, m)$, initialize $\mathcal{H}^{\text{class}}$ with $(\text{id}_P, \text{id}_{\mathcal{F}}, m)$. If the outcome is of the form $(\text{id}_{\mathcal{F}}, \text{id}_P, m)$, initialize $\mathcal{H}^{\text{class}}$ with $(\text{id}_P, \text{environment}, m)$. In all cases, the quantum communication register is not modified (i.e., the message in that register is forwarded).*

Note the strong analogy to the corruption parties (page 12).

Thus, if we write π quantum-UC-emulates \mathcal{F} , we mean that π quantum-UC-emulates $\rho_{\mathcal{F}}$ where $\rho_{\mathcal{F}}$ consists of the functionality \mathcal{F} and the dummy-parties corresponding to the parties in π . More precisely:

Definition 6 *Let π be a protocol and \mathcal{F} be a functionality. We say that π statistically/computationally quantum-UC-emulates \mathcal{F} if π statistically/computationally quantum-UC-emulates $\rho_{\mathcal{F}}$ where $\rho_{\mathcal{F}} := \{\tilde{P} : P \in \text{parties}_{\pi}\} \cup \{\mathcal{F}\}$.*

For more discussion of dummy-parties and functionalities, see [Can01].

Using the concept of an ideal functionality, we can specify a range of protocol tasks by simply defining the corresponding functionality. Below, we give the definitions of various functionalities. All these functionalities are classical, we therefore do not explicitly describe when the registers $\mathcal{H}^{\text{class}}$ and $\mathcal{H}^{\text{quant}}$ are measured/initialized but instead describe the functionality in terms of the messages sent and received.

Definition 7 (Commitment) *Let A and B be two parties. The functionality $\mathcal{F}_{\text{COM}}^{B \rightarrow A, \ell}$ behaves as follows: Upon (the first) input (commit, x) with $x \in \{0, 1\}^{\ell(k)}$ from B , send committed to A . Upon input open from B send (open, x) to A . All communication/input/output is classical.*

We call B the sender and A the recipient.

Definition 8 (Oblivious transfer (OT)) *Let A and B be two parties. The functionality $\mathcal{F}_{\text{OT}}^{A \rightarrow B, \ell}$ behaves as follows: When receiving input (s_0, s_1) from A with $s_0, s_1 \in \{0, 1\}^{\ell(k)}$ and $c \in \{0, 1\}$ from B , send $s := s_c$ to B . All communication/input/output is classical.*

We call A the sender and B the recipient.⁷

⁷We used A as the sender in the description of the OT functionality, and as the recipient in the description of the commitment functionality. We do so to simplify notation later; our protocol for OT from A to B will use a commitment from B to A .

Definition 9 (Randomized oblivious transfer (ROT)) *Let A and B be two parties. The functionality $\mathcal{F}_{\text{ROT}}^{A \rightarrow B, \ell}$ behaves as follows: If A is uncorrupted, when receiving input $c \in \{0, 1\}$ from B , choose $s_0, s_1 \in \{0, 1\}^{\ell(k)}$ uniformly and send (s_0, s_1) to A and $s := s_c$ to B . If A is corrupted, when receiving input (s_0, s_1) from A with $s_0, s_1 \in \{0, 1\}^{\ell(k)}$ and $c \in \{0, 1\}$ from B , send $s := s_c$ to B . All communication/input/output is classical.*

2.2 Elementary properties of UC-security

Lemma 10 (Reflexivity, transitivity) *Let π , ρ , and σ be protocols. Then π quantum-UC-emulates π . If π quantum-UC-emulates ρ and ρ quantum-UC-emulates σ , then π quantum-UC-emulates σ .*

This holds both for statistical and computational quantum-UC-security.

Proof. We first consider the case of statistical quantum-UC-security.

For any adversary Adv and any set C , with $\text{Sim} := \text{Adv}$, we have that $\pi^C \cup \{\text{Adv}, \mathcal{Z}\}$ and $\pi^C \cup \{\text{Sim}, \mathcal{Z}\}$ are equal and hence perfectly indistinguishable for all \mathcal{Z} . If Adv is quantum-polynomial-time, so is $\text{Sim} = \text{Adv}$. Thus π quantum-UC-emulates ρ .

Assume that π quantum-UC-emulates ρ and ρ quantum-UC-emulates σ . Fix an adversary Adv and a set C . Then there is a simulator Sim such that for all \mathcal{Z} , $\pi^C \cup \{\text{Adv}, \mathcal{Z}\}$ and $\rho^C \cup \{\text{Sim}, \mathcal{Z}\}$ are indistinguishable. Furthermore, for the adversary $\text{Adv}' := \text{Sim}$, there is a simulator Sim' such that $\rho^C \cup \{\text{Sim}, \mathcal{Z}\} = \rho^C \cup \{\text{Adv}', \mathcal{Z}\}$ and $\sigma^C \cup \{\text{Sim}', \mathcal{Z}\}$ are indistinguishable for all \mathcal{Z} . Since indistinguishability is transitive, $\pi^C \cup \{\text{Adv}, \mathcal{Z}\}$ and $\sigma^C \cup \{\text{Sim}', \mathcal{Z}\}$ are indistinguishable for all \mathcal{Z} . Finally, if Adv is quantum-polynomial-time, so is $\text{Adv}' = \text{Sim}$, and thus also Sim' . Thus π quantum-UC-emulates σ .

In the case of computational quantum-UC-security, the proof is identical, except that we quantify over quantum-polynomial-time Adv and \mathcal{Z} . \square

Dummy-adversary. In the definition of UC-security, we have three entities interacting with the protocol: the adversary, the simulator, and the environment. Both the adversary and the environment are all-quantified, hence we would expect that they do, in some sense, work together. This intuition is backed by the following fact which was first noted by Canetti [Can01]: Without loss of generality, we can assume an adversary that is completely controlled by the environment. This so-called dummy-adversary only forwards messages between the environment and the protocol. The actual attack is then executed by the environment.

Definition 11 (Dummy-adversary $\text{Adv}_{\text{dummy}}$) *When activated, the dummy-adversary $\text{Adv}_{\text{dummy}}$ measures $\mathcal{H}^{\text{class}}$; call the outcome m . If m is of the form $(\text{environment}, \text{adversary}, m')$, initialize $\mathcal{H}^{\text{class}}$ with m' . Otherwise initialize $\mathcal{H}^{\text{class}}$ with $(\text{adversary}, \text{environment}, m)$. In all cases, the quantum communication register is not modified (i.e., the message in that register is forwarded).*

Note the strong analogy to the dummy-parties (Definition 5) and the corruption parties (page 12).

Lemma 12 (Completeness of the dummy-adversary) *Assume that π quantum-UC-emulates ρ with respect to the dummy-adversary (i.e., instead of quantifying over all adversaries Adv , we fix $\text{Adv} := \text{Adv}_{\text{dummy}}$). Then π quantum-UC-emulates ρ .*

This holds both for statistical and computational quantum-UC-security.

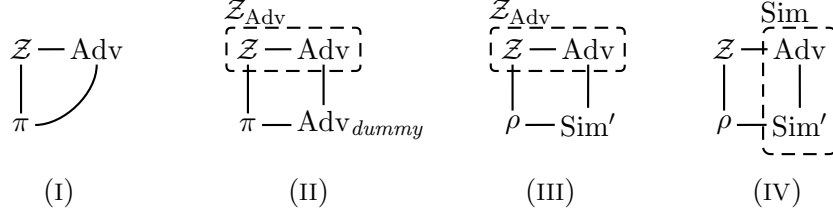


Figure 1: Completeness of the dummy-adversary: proof steps

Proof. We first consider the case of statistical quantum-UC-security.

Assume that π statistically quantum-UC-emulates ρ with respect to the dummy-adversary. Fix an adversary Adv . We have to show that there exists a simulator Sim such that for all environments \mathcal{Z} we have that $\pi \cup \{\text{Adv}, \mathcal{Z}\}$ and $\rho \cup \{\text{Sim}, \mathcal{Z}\}$ are indistinguishable. Furthermore, if Adv is quantum-polynomial-time, Sim has to be quantum-polynomial-time, too.

For a given environment \mathcal{Z} , we construct an environment \mathcal{Z}_{Adv} that is supposed to interact with $\text{Adv}_{\text{dummy}}$ and internally simulates \mathcal{Z} and Adv , and that routes all messages sent by the simulated Adv to π through $\text{Adv}_{\text{dummy}}$ and vice versa. Then $\pi \cup \{\text{Adv}, \mathcal{Z}\}$ and $\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}_{\text{Adv}}\}$ are perfectly indistinguishable. (Cf. networks (I) and (II) in Figure 1.) Since π statistically quantum-UC-emulates ρ with respect to the dummy-adversary, we have that $\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}_{\text{Adv}}\}$ and $\rho \cup \{\text{Sim}', \mathcal{Z}_{\text{Adv}}\}$ are indistinguishable for some Sim' and all \mathcal{Z} . (Cf. networks (II) and (III).) Since $\text{Adv}_{\text{dummy}}$ is quantum-polynomial-time, so is Sim' . We construct a machine Sim that internally simulates Sim' and Adv (network (IV)). Then $\rho \cup \{\text{Sim}', \mathcal{Z}_{\text{Adv}}\}$ and $\rho \cup \{\text{Sim}, \mathcal{Z}\}$ are perfectly indistinguishable. Summarizing, $\pi \cup \{\text{Adv}, \mathcal{Z}\}$ and $\rho \cup \{\text{Sim}, \mathcal{Z}\}$ are indistinguishable for all environments \mathcal{Z} . Furthermore, since Sim' is quantum-polynomial-time, we have that Sim is quantum-polynomial-time if Adv is. This concludes the proof in the case of statistical quantum-UC-security.

The proof in the case of computational quantum-UC-security is identical, except that we consider only quantum-polynomial-time Adv and \mathcal{Z} , and thus have that \mathcal{Z}_{Adv} , Sim' , and Sim are quantum-polynomial-time. \square

2.3 Universal composition

For some protocol σ , and some protocol π , by σ^π we denote the protocol where σ invokes (up to polynomially many) instances of π . That is, in σ^π the machines from σ and from π run together in one network, and the machines from σ access the inputs and outputs of π . (That is, σ plays the role of the environment from the point of view of π . In particular, \mathcal{Z} then talks only to σ and not to the subprotocol π directly.) A typical situation would be that $\sigma^\mathcal{F}$ is some protocol that makes use of some ideal functionality \mathcal{F} , say a commitment functionality, and then σ^π would be the protocol resulting from implementing that functionality with some protocol π , say a commitment protocol. (We say that $\sigma^\mathcal{F}$ is a protocol in the \mathcal{F} -hybrid model.) One would hope that such an implementation results in a secure protocol σ^π . That is, we hope that if π quantum-UC-emulates \mathcal{F} and $\sigma^\mathcal{F}$ quantum-UC-emulates \mathcal{G} , then σ^π quantum-UC-emulates \mathcal{G} . Fortunately, this is the case:

Theorem 13 (Universal Composition Theorem) *Let π , ρ , and σ be quantum-polynomial-time protocols. Assume that π quantum-UC-emulates ρ . Then σ^π quantum-UC-emulates σ^ρ .*

This holds both for statistical and computational quantum-UC-security.

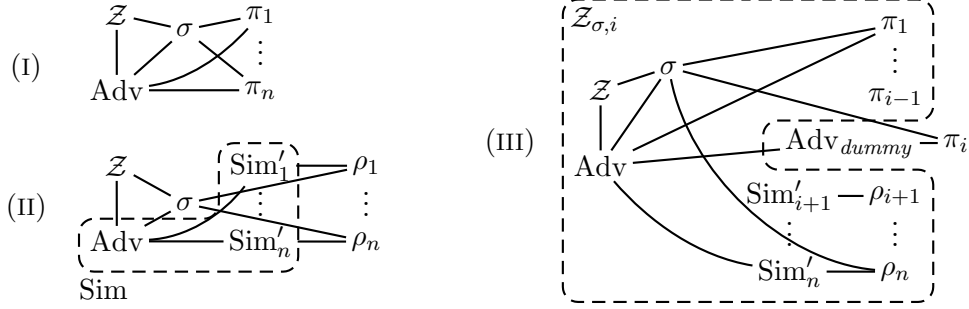


Figure 2: Networks occurring in the proof sketch of Theorem 13. Network (I) represents the real model, (II) the ideal model, and (III) the hybrid case. To avoid cluttering, in (III), the connections to π_{i-1} , Sim'_{i+1} , and ρ_{i+1} have been omitted.

If we additionally have that σ quantum-UC-emulates \mathcal{G} , from the transitivity of quantum-UC-emulation (Lemma 10), it immediately follows that σ^π quantum-UC-emulates \mathcal{G} .

The compositionality guarantee given by Theorem 13 is often called *universal composability*. One should not confuse universal composability with UC-security. Although UC security implies universal composability, it has been shown by Hofheinz and Unruh [HU05, HU06, Unr06] that – in the classical setting at least – universal composability is a strictly weaker notion than UC security.

Proof of Theorem 13. We first show Theorem 13 for the case of computational quantum-UC-security. Thus, our goal is to prove that under the assumptions of Theorem 13, σ^π computationally quantum-UC-emulates σ^ρ . Since σ is quantum-polynomial-time, σ invokes at most a polynomial number n of instances of its subprotocol π or ρ . Since π quantum-UC-emulates ρ , there is a quantum-polynomial-time simulator Sim' such that for all environments \mathcal{Z} we have that $\pi \cup \{\text{Adv}_{dummy}, \mathcal{Z}\}$ and $\rho \cup \{\text{Sim}', \mathcal{Z}\}$ are indistinguishable. In the following, we call Sim' the dummy-simulator.

Let a quantum-polynomial-time adversary Adv be given (that is supposed to attack σ^π). We construct a simulator Sim that internally simulates the adversary Adv and n instances $\text{Sim}'_1, \dots, \text{Sim}'_n$ of the dummy-simulator Sim' . The simulated adversary Adv is connected to the environment and to the protocol σ , but all messages between Adv and the i -th instance π_i of π are routed through the dummy-simulator-instance Sim'_i (which is then supposed to transform these messages into a form suitable for instances of ρ). The simulator Sim is depicted by the dashed box in network (II) in Figure 2.

We have to show that for any environment \mathcal{Z} we have that $\sigma^\pi \cup \{\text{Adv}, \mathcal{Z}\}$ and $\sigma^\rho \cup \{\text{Sim}, \mathcal{Z}\}$ are indistinguishable, i.e., that the output of \mathcal{Z} in the networks (I) and (II) in Figure 2 is statistically indistinguishable.

For this, we construct a hybrid environment $\mathcal{Z}_{\sigma,i}$. (It is depicted as the dashed box in network (III) in Figure 2.) This environment simulates the machines \mathcal{Z} , Adv , the protocol σ , instances π_1, \dots, π_{i-1} of the real protocol π , and instances $\text{Sim}'_{i+1}, \dots, \text{Sim}'_n$ and $\rho_{i+1}, \dots, \rho_n$ of the dummy-simulator Sim' and the ideal protocol ρ , respectively. The communication between \mathcal{Z} , Adv , and σ is directly forwarded by $\mathcal{Z}_{\sigma,i}$. Communication between Adv and the j -th protocol instance is forwarded as follows: If $j < i$, the communication is simply forwarded to π_j . If $j > i$, the communication is routed through the corresponding dummy-simulator Sim'_j (which is then supposed to transform these messages into a form suitable for ρ_j). And finally, if $j = i$, the communication is passed to

the adversary/simulator outside of $\mathcal{Z}_{\sigma,i}$. Communication between σ and the instances of π or ρ is directly forwarded.

We will now show that there is a negligible function μ such that $|\Pr[\text{Exec}_{\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}_{\sigma,i}\}}(k, z) = 1] - \Pr[\text{Exec}_{\rho \cup \{\text{Sim}', \mathcal{Z}_{\sigma,i}\}}(k, z) = 1]| \leq \mu(k)$ for any security parameter k and any $i = 1, \dots, n$. For this, construct an environment \mathcal{Z}_σ which expects as its initial input a pair (i, z) , and then runs $\mathcal{Z}_{\sigma,i}$ with input z . Since $\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}\}$ and $\rho \cup \{\text{Sim}', \mathcal{Z}\}$ are indistinguishable for all quantum-polynomial-time environments \mathcal{Z} , there exists a negligible function μ such that the difference of $\Pr[\text{Exec}_{\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}_{\sigma,i}\}}(k, z) = 1] = \Pr[\text{Exec}_{\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}_\sigma\}}(k, (i, z)) = 1]$ and $\Pr[\text{Exec}_{\rho \cup \{\text{Sim}', \mathcal{Z}_{\sigma,i}\}}(k, z) = 1] = \Pr[\text{Exec}_{\rho \cup \{\text{Sim}', \mathcal{Z}_\sigma\}}(k, (i, z)) = 1]$ is bounded by $\mu(k)$ for all i, k, z .

The game $\text{Exec}_{\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}_{\sigma,i}\}}(k, z)$ is depicted as network (III) in Figure 2 (except that we denoted the external copy of π with π_i). Observe that $\text{Exec}_{\rho \cup \{\text{Sim}', \mathcal{Z}_{\sigma,i+1}\}}(k, z)$ (note the changed index $i+1$) contains the same machines as $\text{Exec}_{\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}_{\sigma,i}\}}(k, z)$ (when unfolding the simulation performed by $\mathcal{Z}_{\sigma,i}$ into individual machines) up to the fact that the communication with the i -th instance of π is routed through the dummy-adversary $\text{Adv}_{\text{dummy}}$. However, the latter just forwards messages, so $\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}_{\sigma,i}\}$ and $\rho \cup \{\text{Sim}', \mathcal{Z}_{\sigma,i+1}\}$ are perfectly indistinguishable.

Using the triangle inequality, it follows that $|\Pr[\text{Exec}_{\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}_{\sigma,n}\}}(k, z) = 1] - \Pr[\text{Exec}_{\rho \cup \{\text{Sim}', \mathcal{Z}_{\sigma,1}\}}(k, z) = 1]|$ is bounded by $n \cdot \mu(k)$ which is negligible. Moreover, $\text{Exec}_{\pi \cup \{\text{Adv}_{\text{dummy}}, \mathcal{Z}_{\sigma,n}\}}(k, z)$ and $\text{Exec}_{\sigma \cup \{\text{Adv}, \mathcal{Z}\}}(k, z)$ describe the same game (up to unfolding of simulated submachines and up to one instance of the dummy-adversary). Similarly, $\text{Exec}_{\rho \cup \{\text{Sim}', \mathcal{Z}_{\sigma,1}\}}(k, z)$ and $\text{Exec}_{\sigma \cup \{\text{Sim}, \mathcal{Z}\}}(k, z)$ describe the same game (up to unfolding of simulated submachines). Thus $|\Pr[\text{Exec}_{\sigma \cup \{\text{Adv}, \mathcal{Z}\}}(k, z) = 1] - \Pr[\text{Exec}_{\sigma \cup \{\text{Sim}, \mathcal{Z}\}}(k, z) = 1]|$ is negligible and thus $\sigma \cup \{\text{Adv}, \mathcal{Z}\}$ and $\sigma \cup \{\text{Sim}, \mathcal{Z}\}$ are indistinguishable. Furthermore, since Adv and Sim' are quantum-polynomial-time, so is Sim .

Since this holds for all \mathcal{Z} , and the construction of Sim does not depend on \mathcal{Z} , we have that σ^π computationally quantum-UC-emulates σ^ρ .

The case of statistical quantum-UC-security is shown analogously, except that Adv and \mathcal{Z} may be unbounded, and Sim is only quantum-polynomial-time if Adv is. \square

3 Relating classical and quantum-UC

We call a machine classical if its state transition operator is classical. A protocol is classical if all its machines are classical.

Using this definition we can reformulate the definition of statistical classical UC in our framework.

Definition 14 (Statistical classical-UC-security) *Let protocols π and ρ be given. We say π statistically classical-UC-emulates ρ iff for every set $C \subseteq \text{parties}_\pi$ and for every classical adversary Adv there is a classical simulator Sim such that for every classical environment \mathcal{Z} , $\pi^C \cup \{\text{Adv}, \mathcal{Z}\}$ and $\rho^C \cup \{\text{Sim}, \mathcal{Z}\}$ are indistinguishable. We furthermore require that if Adv is probabilistic-polynomial-time, so is Sim .*

Note that classical statistical UC is essentially the same as the notion of statistical UC-security defined by Canetti [Can01].⁸ Thus, known results for statistical UC-security carry over to the setting of Definition 14.

⁸Details such as the machine model and message scheduling are defined differently, of course. But since these

The next theorem guarantees that if a classical protocol is statistically classical UC-secure, then it is also statistically quantum-UC-secure. This allows, e.g., to first prove the security of a protocol in the (usually much simpler) classical setting, and then to compose it with quantum protocols using the universal composition theorem (Theorem 13).

Theorem 15 (Quantum lifting theorem) *Let π and ρ be classical protocols. Assume that π statistically classical-UC-emulates ρ . Then π statistically quantum-UC-emulates ρ .*

Proof. Given a machine M , let $\mathcal{C}(M)$ denote the machine which behaves like M , but measures incoming messages in the computational basis before processing them, and measures outgoing messages in the computational basis. More precisely, the superoperator $\mathcal{E}_{\mathcal{C}(M)}^{(k)}$ first invokes \mathcal{E}_{class} on $\mathcal{H}^{class} \otimes \mathcal{H}^{quant}$, then invokes $\mathcal{E}_M^{(k)}$ on $\mathcal{H}^{state} \otimes \mathcal{H}^{class} \otimes \mathcal{H}^{quant}$, and then again invokes \mathcal{E}_{class} on $\mathcal{H}^{class} \otimes \mathcal{H}^{quant}$. Since it is possible to simulate quantum Turing machines on classical Turing machines (with an exponential overhead), for every machine M , there exists a classical machine M' such that $\mathcal{C}(M)$ and M' are perfectly indistinguishable.⁹

We define the classical dummy-adversary $\text{Adv}_{dummy}^{class}$ to be the classical machine that is defined like Adv_{dummy} (Definition 11), except that in each invocation, it first measures \mathcal{H}^{class} , \mathcal{H}^{quant} , and \mathcal{H}^{state} in the computational basis (i.e., it applies \mathcal{E}_{class} to $\mathcal{H}^{state} \otimes \mathcal{H}^{class} \otimes \mathcal{H}^{quant}$) and then proceeds as does Adv_{dummy} . Note that $\text{Adv}_{dummy}^{class}$ is probabilistic-polynomial-time.

By Lemma 12, we only need to show that for any set C of corrupted parties, there exists a quantum-polynomial-time machine Sim such that for every machine \mathcal{Z} the real model $\pi^C \cup \{\mathcal{Z}, \text{Adv}_{dummy}\}$ and the ideal model $\rho^C \cup \{\mathcal{Z}, \text{Sim}\}$ are indistinguishable.

The protocol π is classical, thus π^C is classical, too, and thus all messages forwarded by Adv_{dummy} from π^C to \mathcal{Z} have been measured in the computational basis by π^C , and all messages forwarded by Adv_{dummy} from \mathcal{Z} to π^C will be measured by π^C before being used. Thus, if Adv would additionally measure all messages it forwards in the computational basis, the view of \mathcal{Z} would not be modified. More formally, $\pi^C \cup \{\mathcal{Z}, \text{Adv}_{dummy}\}$ and $\pi^C \cup \{\mathcal{Z}, \text{Adv}_{dummy}^{class}\}$ are perfectly indistinguishable. Furthermore, since both π^C and $\text{Adv}_{dummy}^{class}$ measure all messages upon sending and receiving, $\pi^C \cup \{\mathcal{Z}, \text{Adv}_{dummy}^{class}\}$ and $\pi^C \cup \{\mathcal{C}(\mathcal{Z}), \text{Adv}_{dummy}^{class}\}$ are perfectly indistinguishable. Since it is possible to simulate quantum machines on classical machines (with an exponential overhead), there exists a classical machine \mathcal{Z}' that is perfectly indistinguishable from $\mathcal{C}(\mathcal{Z})$. Then $\pi^C \cup \{\mathcal{C}(\mathcal{Z}), \text{Adv}_{dummy}^{class}\}$ and $\pi^C \cup \{\mathcal{Z}', \text{Adv}_{dummy}^{class}\}$ are perfectly indistinguishable. Since $\text{Adv}_{dummy}^{class}$ and \mathcal{Z}' are classical and $\text{Adv}_{dummy}^{class}$ is polynomial-time, there exists a classical probabilistic-polynomial-time simulator Sim (whose construction is independent of \mathcal{Z}') such that $\pi^C \cup \{\mathcal{Z}', \text{Adv}_{dummy}^{class}\}$ and $\rho^C \cup \{\mathcal{Z}', \text{Sim}\}$ are indistinguishable.

Then $\rho^C \cup \{\mathcal{Z}', \text{Sim}\}$ and $\rho^C \cup \{\mathcal{C}(\mathcal{Z}), \text{Sim}\}$ are perfectly indistinguishable by construction of \mathcal{Z}' . And since both ρ^C and Sim measure all messages they send and receive, $\rho^C \cup \{\mathcal{C}(\mathcal{Z}), \text{Sim}\}$ and $\rho^C \cup \{\mathcal{Z}, \text{Sim}\}$ are perfectly indistinguishable.

Summarizing, we have that $\pi^C \cup \{\mathcal{Z}, \text{Adv}_{dummy}\}$ and $\rho^C \cup \{\mathcal{Z}, \text{Sim}\}$ are indistinguishable for all quantum-polynomial-time environments \mathcal{Z} . Furthermore, Sim is classical probabilistic-polynomial-

details also considerably change between different versions of the full version [Can05], we feel justified in saying that the notion of statistical classical UC is essentially the same as that formulated by Canetti.

⁹More precisely, for any set of machines N , the networks $N \cup \{M\}$ and $N \cup \{\mathcal{C}(M)\}$ are perfectly indistinguishable.

time and hence quantum-polynomial-time and its construction does not depend on the choice of \mathcal{Z} . Thus π statistically quantum-UC-emulates ρ . \square

3.1 The computational case

We now formulate a computational analogue to the quantum lifting theorem (Theorem 15) from the previous section. We cannot, however, expect a theorem of the following form: If π computationally classical-UC-emulates ρ , then π computationally quantum-UC-emulates ρ . For example, if the security of π is based on the hardness of the discrete logarithm, then π may computationally classical-UC-emulate ρ , but certainly π does not computationally quantum-UC-emulate ρ – a quantum-polynomial-time adversary can easily compute discrete logarithms using Shor’s algorithm [Sho94]. Thus, in order to get a computational quantum lifting theorem, we need to give the adversary in the classical setting the same computational power as in the quantum setting. Classical machines that are as powerful as quantum-polynomial-time machines, we call QPPT machines.

Definition 16 (Quantum-strong PPT) *A classical machine M is said to be QPPT (quantum-strong probabilistic polynomial-time) if there is a quantum-polynomial-time machine \tilde{M} such that for any network N , $N \cup \{M\}$ and $N \cup \{\tilde{M}\}$ are perfectly indistinguishable (short: M and \tilde{M} are perfectly indistinguishable).*

Definition 17 (QPPT classical UC security) *Let protocols π and ρ be given. We say π QPPT classical-UC-emulates ρ iff for every set $C \subseteq \text{parties}_\pi$ and for every QPPT adversary Adv there is a QPPT simulator Sim such that for every QPPT environment \mathcal{Z} , the networks $\pi^C \cup \{\text{Adv}, \mathcal{Z}\}$ and $\rho^C \cup \{\text{Sim}, \mathcal{Z}\}$ are indistinguishable.*

Theorem 18 (Quantum lifting theorem – computational) *Let π and ρ be classical protocols. Assume that π QPPT classical-UC-emulates ρ . Then π computationally quantum-UC-emulates ρ .*

Proof. We define $\mathcal{C}(M)$ and $\text{Adv}_{dummy}^{class}$ as in the proof of Theorem 15.

By Lemma 12, we only need to show that for any set C of corrupted parties, there exists a quantum polynomial-time machine Sim such that for every quantum-polynomial-time machine \mathcal{Z} the real model $\pi^C \cup \{\mathcal{Z}, \text{Adv}_{dummy}\}$ and the ideal model $\rho^C \cup \{\mathcal{Z}, \text{Sim}\}$ are indistinguishable.

The protocol π is classical, so is π^C is classical, and thus all messages forwarded by Adv_{dummy} from π^C to \mathcal{Z} have been measured in the computational basis by π^C , and all messages forwarded by Adv_{dummy} from \mathcal{Z} to π^C will be measured by π^C before being used. Thus, if Adv would additionally measure all messages it forwards in the computational basis, the view of \mathcal{Z} would not be modified. More formally, $\pi^C \cup \{\mathcal{Z}, \text{Adv}_{dummy}\}$ and $\pi^C \cup \{\mathcal{Z}, \text{Adv}_{dummy}^{class}\}$ are perfectly indistinguishable. Furthermore, since both π^C and $\text{Adv}_{dummy}^{class}$ measure all messages upon sending and receiving, $\pi^C \cup \{\mathcal{Z}, \text{Adv}_{dummy}^{class}\}$ and $\pi^C \cup \{\mathcal{C}(\mathcal{Z}), \text{Adv}_{dummy}^{class}\}$ are indistinguishable. By definition of QPPT machines, and since $\mathcal{C}(\mathcal{Z})$ is quantum-polynomial-time, there is a QPPT machine \mathcal{Z}' that is perfectly indistinguishable from $\mathcal{C}(\mathcal{Z})$. Then $\pi^C \cup \{\mathcal{C}(\mathcal{Z}), \text{Adv}_{dummy}^{class}\}$ and $\pi^C \cup \{\mathcal{Z}', \text{Adv}_{dummy}^{class}\}$ are perfectly indistinguishable. Since $\text{Adv}_{dummy}^{class}$ and \mathcal{Z}' are QPPT machines, there exists a QPPT simulator Sim' (whose construction is independent of \mathcal{Z}') such that $\pi^C \cup \{\mathcal{Z}', \text{Adv}_{dummy}^{class}\}$ and $\rho^C \cup \{\mathcal{Z}', \text{Sim}'\}$ are indistinguishable.

Then $\rho^C \cup \{\mathcal{Z}', \text{Sim}'\}$ and $\rho^C \cup \{\mathcal{C}(\mathcal{Z}), \text{Sim}'\}$ are perfectly indistinguishable by construction of \mathcal{Z}' . And since both ρ^C and Sim' measure all message they send and receive, $\rho^C \cup \{\mathcal{C}(\mathcal{Z}), \text{Sim}'\}$ and $\rho^C \cup$

$\{\mathcal{Z}, \text{Sim}'\}$ are perfectly indistinguishable. Since Sim' is a QPPT machine, by definition there exists a quantum-polynomial-time machine Sim such that Sim and Sim' are perfectly indistinguishable. Then $\rho^C \cup \{\mathcal{Z}, \text{Sim}'\}$ and $\rho^C \cup \{\mathcal{Z}, \text{Sim}\}$ are perfectly indistinguishable.

Summarizing, we have that $\pi^C \cup \{\mathcal{Z}, \text{Adv}_{\text{dummy}}\}$ and $\rho^C \cup \{\mathcal{Z}, \text{Sim}\}$ are perfectly indistinguishable for all quantum-polynomial-time environments \mathcal{Z} . Furthermore, Sim is quantum-polynomial-time and its construction does not depend on the choice of \mathcal{Z} . Thus π computationally quantum-UC-emulates ρ . \square

A word of caution: While the statistical quantum lifting theorem (Theorem 15) can be directly applied to existing statistically UC-secure protocols, the computational variant of this theorem cannot be directly applied to existing proofs. Although proving that a classical protocol is QPPT classical UC-secure is probably simpler than directly performing the proof in the quantum setting, at various places in a proof of QPPT classical UC-security one has to prove that the machines one constructed from the adversary/environment are QPPT. (This needs to be done whenever a proof step is done by reduction, and when showing that the final simulator is QPPT). As long as the constructed machines simulate the original adversary as a black-box without rewinding, this will be straightforward. However, when the constructed machine internally rewinds a QPPT machine, showing that the constructed machine is also QPPT will be non-trivial. Thus, to apply Theorem 18 to an existing protocol, we need to carefully revisit the original proof, and we need to be aware of the fact that the closure properties of the class of QPPT machines are not the same as those of the class of PPT machines.

In this context, we formulate the following open problem: Can we formulate the class of all QPPT machines as the class of all probabilistic-polynomial-time machines relative to a suitable oracle? More precisely, is the following conjecture true?

Conjecture 19 *There exists an oracle \mathcal{O} (e.g., the decision oracle of a BQP-complete problem) such that a classical machine M is QPPT if and only if there exists an oracle machine $\hat{M}^{\mathcal{O}}$ which runs in probabilistic-polynomial-time and which is perfectly indistinguishable from M .*

A positive answer to this question would allow rewinding of QPPT machines (since an oracle machine $\hat{M}^{\mathcal{O}}$ can be rewound). However, the impact of such a positive answer would not be limited to our setting; we expect that it would also allow a simple analysis of classical protocols in the quantum stand-alone model, and of classical zero-knowledge proofs in the quantum setting.

4 Relation to the stand-alone model

In this section, we show that security in the quantum stand-alone model does, in some cases, already imply quantum-UC-security. We will need this result as a tool for reusing parts of the proof given by Damgård et al. [DFL⁺09a] for their OT protocol. We first review the necessary parts of the stand-alone model as defined by Fehr and Schaffner [FS09]. For details, see their paper.

The basic idea behind the stand-alone model is similar to that of the UC model. We are given a protocol π and a functionality \mathcal{F} , and we call the protocol π secure if any attack on π can be simulated in an ideal model where the simulator only has access to the functionality \mathcal{F} .¹⁰ We will only need the special case of a two-party protocol in which Alice does not take any input. In

¹⁰In the stand-alone model, one usually call this functionality a function because it is required to be non-interactive, first taking inputs from all parties, and then sending the computed outputs to all parties.

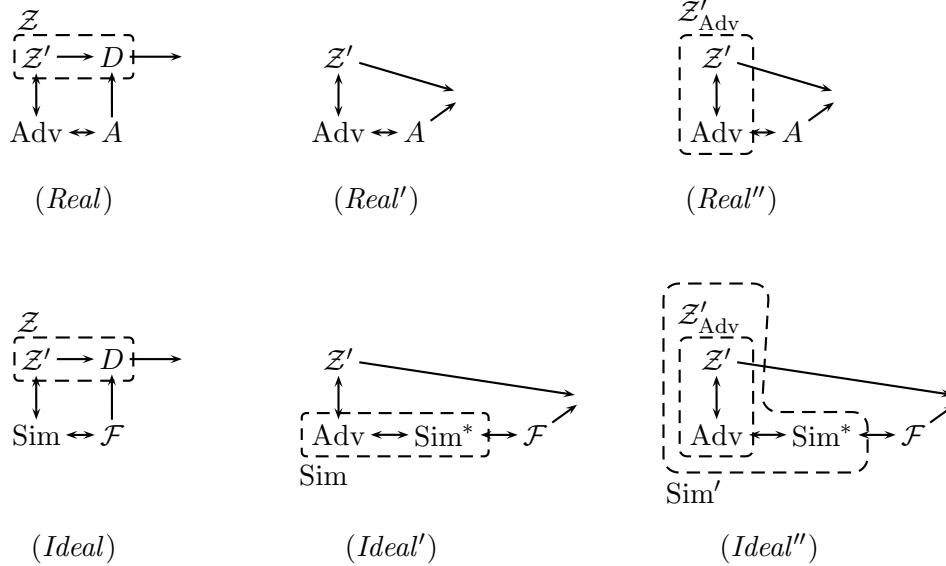


Figure 3: Networks occurring in the proof of Theorem 20. Dashed boxes represent machines that internally simulate other machines. Arrows between machines represent communication, and arrows leaving the network represent the overall output of the network (indistinguishability is defined in terms of that output). Dummy-parties and corruption parties are omitted for simplicity.

this case, we say that the protocol π implements \mathcal{F} in the statistical quantum stand-alone model for corrupted Bob if the following holds: For any adversary Adv , there is a simulator Sim such that for any quantum state ρ_{adv} , the trace distance between the states ρ_{real} and ρ_{ideal} is negligible. Here, the state ρ_{real} is defined to be the joint state consisting of the output of Alice and of the adversary after a protocol execution in which the adversary gets ρ_{adv} as his initial input. The state ρ_{ideal} is defined to be the joint state consisting of the output of Alice and of the simulator after an execution in which the simulator first gets ρ_{adv} as his initial input, then may give arbitrary inputs to \mathcal{F} in the name of Bob, then gets the outputs for Bob from \mathcal{F} , and then produces his output.

Theorem 20 *Fix a protocol π with parties Alice and Bob (not using any ideal functionality). Assume that in this protocol Alice takes no input, and that Alice does not accept messages after sending her output.*

Assume that the protocol π implements a two-party functionality \mathcal{F} in the statistical quantum stand-alone model for corrupted Bob.

Assume that the corresponding simulator is quantum-polynomial-time, that the simulator internally simulates the adversary as a black-box (and in particular, the description of the simulator does not otherwise depend on the adversary), that the simulator does not rewind the adversary, and that the simulator outputs the state output by the internally simulated adversary.

Then π statistically quantum-UC-emulates \mathcal{F} in the case of corrupted Bob.

Proof. Fix an environment \mathcal{Z} . By Lemma 12, we have to construct a simulator Sim such that the probability that \mathcal{Z} outputs 1 in the real and ideal model is negligibly close. (This simulator needs

to be independent of the choice of \mathcal{Z} .) Here, the real model *Real* consists of the environment \mathcal{Z} , the dummy-adversary $\text{Adv} := \text{Adv}_{\text{dummy}}$, the honest party A (Alice), and the corruption party B^C . The ideal model *Ideal* consists of the environment \mathcal{Z} , the simulator Sim , the functionality \mathcal{F} , the dummy-party \hat{A} , and the corruption party B^C .

Alice does not accept any messages after sending her output, so we can assume without loss of generality that \mathcal{Z} does not send any messages to Alice after receiving her output. Since Adv is the dummy-adversary, we can assume that \mathcal{Z} also does not send any messages to Adv after receiving Alice's output (since these messages would be routed through Adv through B^C and then to Alice and ignored). Thus, we can assume without loss of generality that after receiving Alice's output, \mathcal{Z} does not send any messages, but performs a measurement D on its state and Alice's output with some outcome $d \in \{0, 1\}$. Then \mathcal{Z} terminates with output d . Thus we can represent \mathcal{Z} as consisting internally of two machines \mathcal{Z}' and D . The machine D gets the outputs of \mathcal{Z}' and Alice and outputs d . This situation is depicted in Figure 3, network *Real*.

We then define a network *Real'* which contains \mathcal{Z}' instead of \mathcal{Z} . See Figure 3. Let $\rho(\text{Real}')$ denote the joint output of \mathcal{Z}' and Alice. Note that this output is not a single bit (as in Definition 3) but a quantum state. Note that when applying the measurement D to $\rho(\text{Real}')$, the distribution of the measurement outcome is the distribution of the output of \mathcal{Z}' in *Real*.

We then define a network *Real''* which results from *Real'* by replacing \mathcal{Z}' and Adv by a single machine $\mathcal{Z}'_{\text{Adv}}$ which internally simulates \mathcal{Z}' and Adv . See Figure 3. Then $\rho(\text{Real}') = \rho(\text{Real}'')$.

Now, since π implements \mathcal{F} in the statistical quantum stand-alone model, and since $\mathcal{Z}'_{\text{Adv}}$ is a valid adversary in the quantum stand-alone model (it only interacts with the honest parties, but does not provide inputs or get the outputs), we have that there is a simulator Sim' such that the trace distance between $\rho(\text{Real}'')$ and $\rho(\text{Ideal}'')$ is negligible. Here *Ideal''* is the network consisting of Sim' and \mathcal{F} .

By assumption, the simulator Sim' internally simulates $\mathcal{Z}'_{\text{Adv}}$ as a black box and outputs what the simulated $\mathcal{Z}'_{\text{Adv}}$ outputs. Hence we can represent Sim' as internally consisting of some two machines: the adversary $\mathcal{Z}'_{\text{Adv}}$, and some machine Sim^* that interacts with $\mathcal{Z}'_{\text{Adv}}$. The construction of Sim^* does not depend on $\mathcal{Z}'_{\text{Adv}}$, and Sim^* is quantum-polynomial-time since Sim' is quantum-polynomial-time by assumption. The output of Sim' is that of $\mathcal{Z}'_{\text{Adv}}$. Note further that $\mathcal{Z}'_{\text{Adv}}$ by construction also consists of two internally simulated machines \mathcal{Z}' and Adv and outputs what \mathcal{Z}' outputs. So the output of Sim' is that of the internal \mathcal{Z}' . See Figure 3, network *Ideal''*.

The simulator Sim' internally simulates \mathcal{Z}' , Adv , and Sim^* . We define *Ideal'* by replacing Sim' in *Ideal''* by \mathcal{Z}' and Sim , where Sim is defined to internally simulate Adv and Sim^* . See Figure 3. Then $\rho(\text{Ideal}'') = \rho(\text{Ideal}')$.

Thus the trace distance between $\rho(\text{Real}')$ and $\rho(\text{Ideal}')$ is negligible. Furthermore, when applying the measurement D to $\rho(\text{Real}')$, the distribution of the measurement outcome is the distribution of the output of \mathcal{Z} in *Real*. Similarly, when applying the measurement D to $\rho(\text{Ideal}')$ is the distribution of the output of \mathcal{Z} in *Ideal*. Thus the statistical distance between the output of \mathcal{Z}' in *Real* and in *Ideal* is negligible. Thus *Real* and *Ideal* are indistinguishable.

Furthermore, since Sim consists of Adv and Sim^* , it is independent of \mathcal{Z} . And since Sim^* is quantum-polynomial-time, Sim is quantum-polynomial-time if Adv is. Thus π statistically quantum-UC-emulates \mathcal{F} in the case of corrupted Bob. \square

Parameters: Integers $n, m > n, \ell$, a family \mathbf{F} of universal hash functions.

Parties: The sender Alice and the recipient Bob.

Inputs: Alice gets no input, Bob gets a bit c .

1. Alice chooses $\tilde{x}^A \in \{0, 1\}^m$ and $\tilde{\theta}^A \in \{+, \times\}^m$ and sends $|\tilde{x}^A\rangle_{\tilde{\theta}^A}$ to Bob.
2. Bob receives the state $|\Psi\rangle$ sent by the sender. Then Bob chooses $\tilde{\theta}^B \in \{+, \times\}^m$ and measures the qubits of $|\Psi\rangle$ in the bases $\tilde{\theta}^B$. Call the result \tilde{x}^B .
3. For each i , Bob commits to $\tilde{\theta}_i^B$ and \tilde{x}_i^B using one instance of $\mathcal{F}_{\text{COM}}^{B \rightarrow A, 1}$ each.
4. Alice chooses a set $T \subseteq \{1, \dots, m\}$ of size $m - n$ and sends T to Bob.
5. Bob opens the commitments of $\tilde{\theta}_i^B$ and \tilde{x}_i^B for all $i \in T$.
6. Alice checks $\tilde{x}_i^A = \tilde{x}_i^B$ for all i with $i \in T$ and $\tilde{\theta}_i^A = \tilde{\theta}_i^B$. If this test fails, Alice aborts.
7. Let x^A be the n -bit string resulting from removing the bits at positions $i \in T$ from \tilde{x}^A . Define θ^A, x^B , and θ^B analogously.
8. Alice sends θ^A to Bob.
9. Bob sets $I_c := \{i : \theta_i^A = \theta_i^B\}$ and $I_{1-c} := \{i : \theta_i^A \neq \theta_i^B\}$. Then Bob sends (I_0, I_1) to Alice.
10. Alice chooses $s_0, s_1 \in \{0, 1\}^{\ell(k)}$ and $f_0, f_1 \in \mathbf{F}$, output (s_0, s_1) , and computes $m_i := s_i \oplus f_i(x^A|_{I_i})$ for $i = 1, 2$. Then Alice sends f_0, f_1, m_0, m_1 to Bob.
11. Bob outputs $s := m_c \oplus f_c(x^B|_{I_c})$.

Figure 4: Protocol π_{QROT} for randomized oblivious transfer.

5 Oblivious transfer

Definition 21 (OT protocols) *The protocol π_{QROT} is defined in Figure 4. Fix a commitment scheme com . The protocol $\pi_{\text{QROT}}^{\text{com}}$ is defined like π_{QROT} , but instead of using the functionality \mathcal{F}_{COM} , the commitment scheme com is used. The protocol π_{QOT} is defined like π_{QROT} , with the following modifications: Alice takes as input two $\ell(k)$ -bit strings v_0, v_1 . In Step 10, Alice additionally sends t_0, t_1 with $t_i := s_i \oplus v_i$. Bob outputs $s \oplus t_c$ instead of s in Step 11.*

We first analyze π_{QROT} and will then deduce the security of π_{QOT} from that of π_{QROT} .

We first state the trivial cases (note for the uncorrupted case that we assume secure channels):

Lemma 22 *The protocol π_{QROT} statistically quantum-UC-emulates $\mathcal{F}_{\text{ROT}}^{A \rightarrow B, \ell}$ in the case of no corrupted parties and in the case of both Alice and Bob being corrupted.*

5.1 Corrupted Alice

Lemma 23 *The protocol π_{QROT} statistically quantum-UC-emulates $\mathcal{F}_{\text{ROT}}^{A \rightarrow B, \ell}$ in the case of corrupted Alice.*

Proof. First, we describe the structure of the real and ideal model in the case that the party A (Alice) is corrupted:

In the real model, we have the environment \mathcal{Z} , the adversary Adv , the corruption party A^C , the honest party B (Bob), and the $2m$ instances of the commitment functionality \mathcal{F}_{COM} . The adversary controls the corruption party A^C , so effectively he controls the communication with Bob and the inputs of \mathcal{F}_{COM} . Bob's input (a choice bit c) is chosen by the environment, and the environment also gets Bob's output (a bitstring $s \in \{0, 1\}^\ell$). See Figure 5(a).

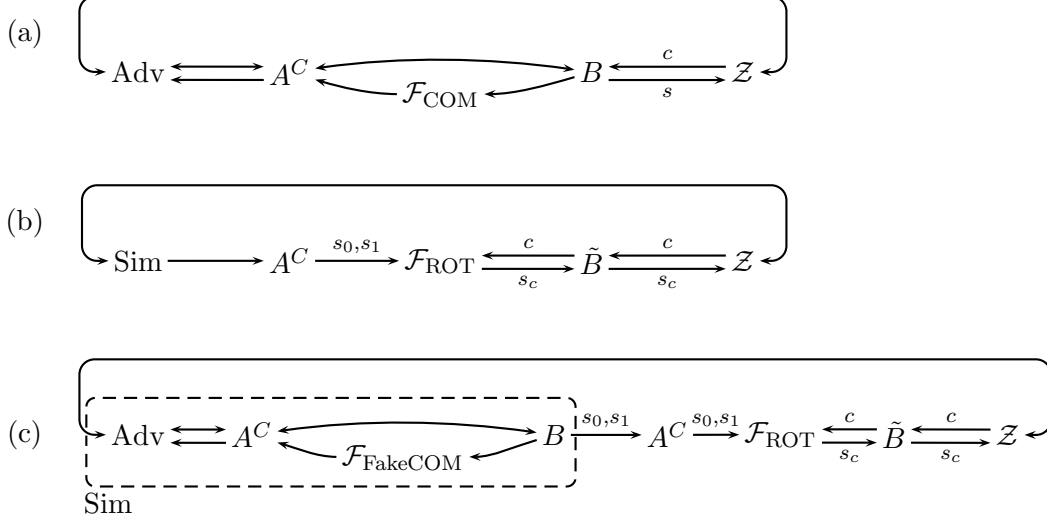


Figure 5: Networks occurring in the proof of Lemma 23. The dashed box represents the machine Sim that internally simulates Adv, A^C , $\mathcal{F}_{\text{FakeCOM}}$ and B .

In the ideal model, we have the environment \mathcal{Z} , the simulator Sim (to be defined below), the corruption party A^C , the dummy-party \tilde{B} , and the randomized OT functionality \mathcal{F}_{ROT} . The simulator Sim controls the corruption party A^C and hence effectively chooses the inputs s_0, s_1 of \mathcal{F}_{ROT} .¹¹ The input c of \mathcal{F}_{ROT} is chosen by the dummy-party \tilde{B} and thus effectively by the environment \mathcal{Z} . The output $s := s_c$ of \mathcal{F}_{ROT} is given to the dummy-party \tilde{B} and thus effectively to the environment \mathcal{Z} . See Figure 5(b).

To show Lemma 23, we need to find a simulator Sim such that, for any environment \mathcal{Z} , the real model and the ideal model are indistinguishable. To do so, we start with the real model, and change the machines in the real model step-by-step until we end up with the ideal model containing a suitable simulator Sim (which we define below in the description of Game 6). In each step, we show that network before and after the step are perfectly indistinguishable.

Game 1. We replace \mathcal{F}_{COM} by a commitment functionality $\mathcal{F}_{\text{FakeCOM}}$ in which Bob (the sender) can cheat. That is, in the commit phase, $\mathcal{F}_{\text{FakeCOM}}$ expects a message `commit` from B (instead of `(commit, x)`), and in the open phase, $\mathcal{F}_{\text{FakeCOM}}$ expects a message `(open, x)` (instead of `open`) and then sends `(open, x)` to Alice. We also change Bob's implementation accordingly, i.e., when Bob should commit to a bit b , he stores that bit b and gives it to $\mathcal{F}_{\text{FakeCOM}}$ when opening the commitment. Obviously, this change leads to a perfectly indistinguishable network (since Bob still opens the commitment in the same way).

Game 2. Since Bob uses $\mathcal{F}_{\text{FakeCOM}}$ instead of \mathcal{F}_{COM} , he does not use the outcomes \tilde{x}_i^B of his measurements before Step 5 (for $i \in T$) or Step 11 (for $i \notin T$) of the protocol. Thus, we modify Bob so that he performs the measurements with outcomes \tilde{x}_i^B ($i \in T$) in Step 5 (in particular, after learning T), and the measurements with outcomes x_i^B in Step 11. Delaying the measurements leads to a perfectly indistinguishable network.

Game 3. The bits x_i^B with $i \in I_{1-c}$ are never used by Bob. Thus we can modify Bob to use

¹¹Remember that, if Alice is corrupted, \mathcal{F}_{ROT} behaves like \mathcal{F}_{OT} and takes inputs s_0, s_1 from Alice.

the bases θ_i^A instead of θ_i^B for these bits without changing the output of \mathcal{Z} . Furthermore, since $\theta_i^A = \theta_i^B$ for $i \in I_c$, we can modify Bob to also use the bases θ_i^A instead of θ_i^B when measuring x_i^B with $i \in I_c$. Summarizing, we modify Bob to use θ^A instead of θ^B , and we get a perfectly indistinguishable network.

Game 4. The bases θ^B are chosen randomly by Bob, and they are only used to compute the sets I_0 and I_1 . We change Bob to instead pick (I_0, I_1) as a random partition of $\{1, \dots, n\}$. Since this leads to the same distribution of (I_0, I_1) and since θ^B is not used elsewhere, this leads to a perfectly indistinguishable network.

Game 5. In Step 11, we change Bob to compute $s_i := m_i \oplus f_i(x^B|_{I_i})$ for $i = 0, 1$ and to output $s := s_c$. This leads to the same value of s as the original computation $s := m_c \oplus f_c(x^B|_{I_c})$, hence the resulting network is perfectly indistinguishable from the previous one. Note that now, Bob only uses the choice bit c to pick which of the two values s_0, s_1 to output.

Game 6. We now construct a machine Sim that internally simulates the machines Adv, A^C , $\mathcal{F}_{\text{FakeCOM}}$, and Bob. We let Sim run with an (external) corruption party A^C , and when (the simulated) Bob computes s_0, s_1 in Step 11, Sim instructs the (external) corruption party A^C to input s_0, s_1 into \mathcal{F}_{ROT} (instead of letting Bob output $s = s_c$). Then \mathcal{F}_{ROT} will, given input c from the dummy-party \tilde{B} , output s_c to the dummy-party \tilde{B} . The dummy-party \tilde{B} then forwards s_c to the environment \mathcal{Z} . See Figure 5(c). The only difference with respect to the previous network (besides a regrouping of machines) is that now s_c is computed by \mathcal{F}_{ROT} from s_0, s_1 . However, \mathcal{F}_{ROT} computes s_c in the same way as Bob would have done. Thus, the resulting network is perfectly indistinguishable from the previous one.

Since the network from Game 6 (Figure 5(c)) is identical to the ideal model (Figure 5(b)), and since the real model is perfectly indistinguishable from the network from Game 6, we have that the real and the ideal network are perfectly indistinguishable.

Furthermore, Sim is quantum-polynomial-time if Adv is, and the construction of Sim does not depend on the choice of the environment \mathcal{Z} . Thus the protocol π_{QROT} statistically quantum-UC-emulates $\mathcal{F}_{\text{ROT}}^{A \rightarrow B, \ell}$ in the case of corrupted Alice. \square

5.2 Corrupted Bob

We call a commitment scheme trivially extractable if, given the messages exchanged during the commit phase, it is efficiently possible to determine the value to which the commitment will be opened. Obviously, this directly contradicts the hiding property of the commitment, so trivially extractable commitments are not overly useful. However, we need such commitments as an intermediate construction in the following proofs. An example of a trivially extractable commitment is one which sends the committed message in clear during the commit phase.

Corollary 24 (Stand-alone quantum OT [DFL⁺09a]) *Let $0 < \alpha < 1$ and $0 < \lambda < \frac{1}{4}$ be constants. Assume $m = \lceil n/(1 - \alpha) \rceil$ and $\ell = \lfloor \lambda n \rfloor$ and that n grows at least linearly in the security parameter k .*

Assume that com is a statistically binding, trivially extractable commitment scheme. Then $\pi_{\text{QROT}}^{\text{com}}$ implements $\mathcal{F}_{\text{ROT}}^{A \rightarrow B, \ell}$ in the statistical quantum stand-alone model.

The corresponding simulator is quantum-polynomial-time, internally simulates the adversary as a black-box, does not rewind the adversary, and outputs the state output by the internally simulated adversary.

Note that Damgård et al. [DFL⁺09a] prove a slightly different result. First, it only assumes that the commitment scheme com is extractable in the so-called common reference string (CRS) model. That is, a globally known and trusted string, the CRS, is available to all parties, and it is possible to extract the committed value when one is allowed to choose the CRS oneself. A trivially extractable commitment can be seen as a special case with a zero-length CRS. Second, it only assumes that the scheme is computationally binding, and thus only proves security in the computational quantum stand-alone model. If we assume that the commitment is statistically binding instead, the same proof shows security in the statistical quantum stand-alone model. Third, they analyze the protocol $\pi_{\text{QOT}}^{\text{com}}$, but the proof trivially adapts to $\pi_{\text{QROT}}^{\text{com}}$.

Lemma 25 *Under the same assumptions on n, m, ℓ as in Corollary 24, the protocol π_{QROT} statistically quantum-UC-emulates $\mathcal{F}_{\text{ROT}}^{A \rightarrow B, \ell}$ in the case of corrupted Bob.*

Proof. Let com be the following encryption scheme: To commit to a message m , the sender sends (commit, m) , and the recipient always accepts the commitment. To open the commitment, the sender sends open , and the recipients accepts and output m . Obviously, this commitment is not hiding. However, it is easily seen to be statistically binding and trivially extractable.

Consider the protocol π_{QROT} . Here Bob sends the messages (commit, m) and open to the commitment functionality, while in the protocol $\pi_{\text{QROT}}^{\text{com}}$, Bob sends these messages directly to Alice. In other words, the machine Alice in $\pi_{\text{QROT}}^{\text{com}}$ can be represented as a machine that internally simulates the machine Alice from π_{QROT} and the ideal functionality \mathcal{F}_{COM} . Thus, as long as Alice is honest, $\pi_{\text{QROT}}^{\text{com}}$ statistically quantum-UC-emulates \mathcal{F}_{ROT} in the case of corrupted Bob if and only if π_{QROT} statistically quantum-UC-emulates \mathcal{F}_{ROT} in the case of corrupted Bob.

By Corollary 24, $\pi_{\text{QROT}}^{\text{com}}$ implements \mathcal{F}_{COM} in the statistical quantum stand-alone model in the case of corrupted Bob with a simulator having the special properties listed in Corollary 24. Thus, by Theorem 20, $\pi_{\text{QROT}}^{\text{com}}$ statistically quantum-UC-emulates \mathcal{F}_{ROT} in the case of corrupted Bob. Thus π_{QROT} statistically quantum-UC-emulates \mathcal{F}_{ROT} in the case of corrupted Bob. \square

Theorem 26 *Let $0 < \alpha < 1$ and $0 < \lambda < \frac{1}{4}$ be constants. Assume $m = \lceil n/(1 - \alpha) \rceil$ and $\ell = \lfloor \lambda n \rfloor$ and that n grows at least linearly in the security parameter.*

Then the protocol π_{QROT} statistically quantum-UC-emulates $\mathcal{F}_{\text{ROT}}^{A \rightarrow B, \ell}$.

Proof. Immediate from Lemmas 22, 23, and 25.

Theorem 27 *Let $0 < \alpha < 1$ and $0 < \lambda < \frac{1}{4}$ be constants. Assume $m = \lceil n/(1 - \alpha) \rceil$ and $\ell = \lfloor \lambda n \rfloor$ and that n grows at least linearly in the security parameter.*

Then the protocol π_{QOT} (Definition 21) statistically quantum-UC-emulates $\mathcal{F}_{\text{OT}}^{A \rightarrow B, \ell}$.

Proof. Consider the following protocol π'_{QOT} in the \mathcal{F}_{ROT} -hybrid model. Given inputs $v_0, v_1 \in \{0, 1\}^{\ell(k)}$ for Alice and a bit c for Bob, Bob invokes \mathcal{F}_{ROT} with input c . Then Alice gets random $s_0, s_1 \in \{0, 1\}^{\ell(k)}$, and Bob gets $s = s_c$. Then Alice sends t_0, t_1 with $t_i := v_i \oplus s_i$ to Bob. And Bob outputs $s \oplus t_c$. It is easy to see that π'_{QOT} statistically classical-UC-emulates \mathcal{F}_{OT} . Hence, by the quantum lifting theorem (Theorem 15), π'_{QOT} statistically quantum-UC-emulates \mathcal{F}_{OT} . Note that the protocol π_{QOT} is the protocol resulting from replacing, in π'_{QOT} , calls to \mathcal{F}_{ROT} by calls to the subprotocol π_{QROT} . Furthermore, π_{QROT} statistically quantum-UC-emulates \mathcal{F}_{ROT} by Theorem 26. Hence, by the composition theorem (Theorem 13), π_{QOT} statistically quantum-UC-emulates \mathcal{F}_{OT} . \square

6 Multi-party computation

Theorem 28 *Let \mathcal{F} be a classical probabilistic-polynomial-time functionality.¹² Then there exists a protocol π in the \mathcal{F}_{COM} -hybrid model that statistically quantum-UC-emulates \mathcal{F} . (Assuming the number of protocol parties does not depend on the security parameter.)*

Proof. Ishai, Prabhakaran, and Sahai [IPS08] prove the existence of a protocol $\rho^{\mathcal{F}_{\text{OT}}}$ in the \mathcal{F}_{OT} -hybrid model that statistically classical-UC-emulates \mathcal{F} (assuming a constant number of parties). By the quantum lifting theorem (Theorem 15), $\rho^{\mathcal{F}_{\text{OT}}}$ statistically quantum-UC-emulates \mathcal{F} . By Theorem 27, π_{QOT} statistically quantum-UC-emulates \mathcal{F}_{OT} . Let $\pi := \rho^{\pi_{\text{QOT}}}$ be the result of replacing invocations to \mathcal{F}_{OT} in $\rho^{\mathcal{F}_{\text{OT}}}$ by invocations of the subprotocol π_{QOT} (as described in Section 2.3). Then by the universal composition theorem (Theorem 13), π statistically quantum-UC-emulates $\rho^{\mathcal{F}_{\text{OT}}}$. Using the fact that quantum-UC-emulation is transitive (Lemma 10), it follows that π statistically quantum-UC-emulates \mathcal{F} . \square

We proceed to show that the result from Theorem 28 is possible only in the quantum setting. That is, we show that there is a natural functionality that cannot be statistically classical-UC-emulated in the commitment-hybrid model. To show this impossibility result, we first need the following lemma.

Lemma 29 *There is no classical two-party protocol (that runs in a polynomial number of rounds) in the commitment-hybrid model that has the following properties:*

- *Let $a \in \{0, 1\}$ denote Alice's input, and $b \in \{0, 1\}$ Bob's input. Then Alice's and Bob's output is $a \cdot b$ with overwhelming probability.*
- *The view of Alice in the case $(a, b) = (0, 0)$ is statistically indistinguishable from the view of Alice in the case $(a, b) = (0, 1)$.*
- *The view of Bob in the case $(a, b) = (0, 0)$ is statistically indistinguishable from the view of Bob in the case $(a, b) = (1, 0)$.*

In all three cases we assume that Alice and Bob honestly follow the protocol (i.e., Alice and Bob are honest-but-curious). The view of a party consists of all messages sent and received by that party together with its input and random choices.

Proof. Assume a protocol π satisfying the properties from Lemma 29. We assume without loss of generality that the last message sent in an execution of π contains the output of Alice. We transform π into a protocol π' that does not use commitments. Namely, when Alice would commit to a value m , she instead sends `committed` to Bob, and when she would open that commitment, she sends m to Bob. Analogously, we remove Bob's commitments. The resulting protocol π' still satisfies the properties from Lemma 29 since we only consider honest-but-curious parties.

We use Lemma 33 from [MQU07a]: Let $U, \tilde{U}, L, \tilde{L}$ be interactive machines that send only a polynomially-bounded number of messages. Let $\langle U, L \rangle$ denote the transcript of the communication

¹²Subject to certain technical restrictions stemming from the proof by Ishai et al. [IPS08]: Whenever the functionality gets an input, the adversary is informed about the length of that input. Whenever the functionality makes an output, the adversary is informed about the length of that output and may decide when this output is to be scheduled.

in an interaction of U and L . Assume that $\langle U, L \rangle \approx \langle \tilde{U}, L \rangle \approx \langle U, \tilde{L} \rangle$ where \approx denotes statistical indistinguishability. Then $\langle U, L \rangle \approx \langle \tilde{U}, \tilde{L} \rangle$.

Let U be a machine executing Alice's program in π' on input 0, and let \tilde{U} execute Alice's program on input 1. Let L and \tilde{L} execute Bob's program on inputs 0 and 1, respectively. Then the properties in Lemma 29 guarantee that $\langle U, L \rangle \approx \langle \tilde{U}, L \rangle \approx \langle U, \tilde{L} \rangle$. Hence $\langle U, L \rangle \approx \langle \tilde{U}, \tilde{L} \rangle$. This implies that the communication between Alice and Bob in π' is indistinguishable in the cases $a = b = 0$ and $a = b = 1$. This is a contradiction to the fact that in the first case, the last message contains the output $ab = 0$, and in the second case, the last message contains the output $ab = 1$. \square

Definition 30 (AND) *The functionality \mathcal{F}_{AND} expects an input $a \in \{0, 1\}$ from Alice and $b \in \{0, 1\}$ from Bob. Then it sends $a \cdot b$ to Alice and Bob.*

Theorem 31 (Impossibility of classical multi-party computation) *There is no classical probabilistic-polynomial-time protocol π in the \mathcal{F}_{COM} -hybrid model such that π statistically classical-UC-emulates \mathcal{F}_{AND} .*

Proof. The statistical UC-security of π would imply the properties listed in Lemma 29. Hence by Lemma 29 such a protocol π does not exist. \square

7 Conclusions

We have given a definition of quantum-UC-security that provides strong composability guarantees for quantum protocols. We have shown that in this model, it is possible to construct statistically secure oblivious transfer protocols given only commitments. Furthermore, we showed that a protocol which is secure in the statistical *classical* UC model is also secure in the statistical *quantum* UC model. This simplifies the modular design of quantum protocols and allows us to construct UC-secure general multi-party computation protocols given only commitments.

Directions for future work include:

- Combine the UC framework and the bounded quantum storage model. In this model, Danggård, Fehr, Salvail, and Schaffner [DFSS05] have constructed statistically hiding and binding commitment schemes and statistically secure OT protocols. If variants of these protocols can be shown secure in the UC framework, this would allow to construct general UC-secure multi-party computation protocols, only assuming that the adversary has a certain upper bound on his quantum storage.
- Combine our result with the protocols for long-term classical UC-secure commitments by Müller-Quade and Unruh [MQU07b] (see Section 1.4). If their protocols can be shown to be secure in the quantum setting, this would enable general long-term secure multi-party computation based on practical setup-assumptions (the availability of signature cards).
- Find efficient constructions. Our protocol invokes a commitment for each qubit sent by Alice. In some settings, a commitment can be quite expensive. For example, commitment protocols in the bounded quantum storage model have a large quantum communication complexity. In this setting, the efficiency of our protocol could be improved considerably if we were able to use few string commitments instead of committing to each bit individually.

- Find analogues to the quantum lifting theorem in other security models. In the stand-alone model, it is an open question whether classically secure protocols are secure in the quantum setting, too. Similarly, we do not know whether classically secure zero-knowledge proofs are in general secure against quantum adversaries.

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