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BANDWIDTH REDUCTION IN COGNITIVE RADIO

TARIQ SHAH[1], S. A. HUSSAIN[2] AND ANTONIO APARECIDO DE ANDRADE[3]

Abstract. Due to mushroom development of wireless devices cognitive radio is used to resolve the bandwidth utilization and sacristy problem. The crafty usage of bandwidth in cognitive radio based on error correcting codes is ensured to accomodate un authorized user. This study proposes a transmission model by which a finite sequence of binary cyclic codes constructed by a binary BCH code of length $n = 2^s - 1$, in which all codes have same error correction capability and code rate but sequentially increasing code lengths greater than n. Initially all these codes are carrying data of their corresponding primary users. A transmission pattern is planned in the sprit of interweave model deals the transmission parameters; modulation scheme, bandwidth and code rate. Whenever, any of the primary users having mod of transmission, the binary cyclic code, is not using its allocated bandwidth, the user having its data built by binary BCH code enter and exploit the free path as a secondary user. Eventually whenever the primary user with W bandwidth having binary BCH code for its data transmission, change its status as a secondary user, it just requires the bandwidth less than W.

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Key words: Bandwidth, secondary user, primary user, BCH code, cyclic code, encoding, decoding, Interweave.

1. Introduction

Cognitive radio is a most recent technology in wireless communication by which the spectrum is vigorously utilized when the primary user, the authorized holder of the spectrum, is not in. The idea of cognitive radio is introduced in [9]. Interpreting to this notion the cognitive radio has the capability to evaluate the radio surroundings and boost the decision permitting to the transmission parameters such as modulation scheme, power, carrier frequency, bandwidth and code rate. By [3] power is allocated to entire bandwidth for upsurge in capacity and keep the interference at the primary user at the given start and endure the complete transmission power inside properties.

Alternative inkling of the interference temperature model of [4] is necessary for the primary receiver to dose the interference boundary, henceforth the secondary user can transmit beneath the set level. The fundamental plan in [20] is to matter license spectrum to secondary users and guaranteed the interference perceived through primary users. To protect the primary user commencing the interference triggered by the secondary user through transmission, Srinivasa and Jafar [16] offered an organization of transmission models as: Interweave, overlay and underlay.

By [9], in the interweave model the secondary user utilized the primary under utilized spectrum opportunistically and draw out when the primary wants to in again. Thus the sensing is necessary for the interweave scheme. However, dissimilar type of the sensing procedures are used to identify the primary user and elude the interference shaped by the secondary user. By [6] for primary user detection, three prominent methods known as: energy detection, feature detection and match filter are getting attention. In cognitive radio secondary user, permitted to use a spectrum of primary user devoid of making interference to the primary user. However, the secondary users need to recurrently monitor the management of the spectrum to repel from interfering with the primary user(see [8]). Overlay and underlay stay with spectrally modulated and spectrally encoding (SMSE) procedure all beside with code division multiple access (CDMA) and orthogonal frequency division multiple access (OFDMA). According to [15], in underlay, primary and secondary user transmit the data concurrently

[1]Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan, stariqshah@gmail.com, [2]School of Engineering and Applied Sciences (SEAS), ISRA University, Islamabad Campus, Pakistan, 1009-pd-ee002@isra.edu.pk and [3]Department of Mathematics, São Paulo State University at São José do Rio Preto - SP, Brazil, andrade@ibilce.unesp.br. Acknowledgment to FAPESP by financial support, 2007/56052-8 and 2011/03441-2.

under the situation that the secondary user interferes less than an optimistic beginning with the primary user. Spectrum sensing is not mandatory for underlay. Only the interference limit is compulsory for the secondary user for talented utilization of the spectrum. Henceforth, this interference limitation confines the communication of secondary user to small range. Whereas by [3], some range of the power should be endorsed to the subcarrier of underlay as they also craft extra interference to the primary user. The request of wireless devices is increasing every day. Therefore, efficient utilization of spectrum is a burring subject to decrease the spectrum over crowdedness. By [2], the overlay model permits the concurrent transmission of primary and secondary users. The secondary transmitter is supposed to have awareness of the primary message and use for sinking the interference at its receiver. Precisely, in underlay and overlay models the secondary users can transmit their data at once with the primary users under some limitations: In underlay, secondary users can transmit with enough power due to interference bound fixed at the primary receiver whereas in the case of overlay transmission of secondary users are solitary promising if secondary transmitter recognizes the code-books and channel information. Besides, both of these models do not guarantee that secondary user will not produce interference for primary user during simultaneous transmission. Similarly, the allowing surroundings for these models may damage the transmission of both primary and secondary user.

In [17] wireless mesh network is practiced for terminal to terminal bandwidth allocation which used the routing and scheduling algorithms. The Max - min model is used for allocation of a fair bandwidth. In the [19] TV band is utilized for the cognitive radio over sensing and opportunistically use the vacant frequencies. The diverse protocols are used for centralized and decentralized spectrum distribution.

By using the undisclosed messages for cognitive radio channels first of all fixed the boundaries for channel capacity. Two transmitters having the primary and secondary messages go through the channel which was received at the receivers with distinct primary and secondary messages. By [18], improvement of spectral efficiency in cognitive radio can be achieved by the secondary user through the consent to utilize the untrustworthy part of the spectrum which is assigned to the primary user. Henceforth the optimal bandwidth is required from the numerous bandwidth allocated to the primary and secondary users. Additionally, the secondary users effort to choose the best possible bandwidth out of the different many collections of bandwidths. Furthermore in [18], different spectrum sharing procedure is considered to improve the cognitive radio networks.

In [11] it is established that for a given binary BCH code C_n^0 of length n generated by a polynomial $g(x)$ of $\mathbb{F}_2[x]$ of degree r there exists a sequence of binary cyclic codes $\{C^j_{2^{j-1}(n+1)n}\}_{j\geq 1}$ such that for each $j \geq 1$, the binary cyclic code $C^j_{2^{j-1}(n+1)n}$ has length $2^{j-1}(n+1)n$ and generated by $2^j r$ degree generalized polynomial $g(x^{\frac{1}{2^j}})$ in $\mathbb{F}_2[x, \frac{1}{2^j}\mathbb{Z}_0]$. Furthermore C_n^0 is embedded in $C^j_{2^{j-1}(n+1)n}$ for each $j \geq 1$ and every code of the family $\{C^j_{2^{j-1}(n+1)n}\}_{j\geq 1}$ have same code rate and higher than of the binary BCH code C_n^0. Furthermore, in [11] a decoding algorithm is proposed, by which the binary BCH code C_n^0 can be transmit and decode through any of binary cyclic codes of the family $\{C^j_{2^{j-1}(n+1)n}\}_{j\geq 1}$.

In the interweave model the secondary user exploited the primary under utilized spectrum opportunistically and pull out when the primary wants to in once again. Thus the sensing is necessary for the interweave scheme. Though, dissimilar kind of the sensing processes is used to detect the primary user and escape the interference formed by the secondary user. For primary user uncovering, bulging approaches, energy detection, feature detection and match filter are receiving consideration.

Like interweave scheme, in this study we designed a transmission model built on error correcting codes. Cognitive radio has the ability to gauge the radio environs and boost the decision allowing to the transmission parameters such as modulation scheme, bandwidth, code rate, power, carrier frequency. In this study due to our model formation we address the parameters; modulation scheme, bandwidth and code rate. This model uses a finite embeddings sequence $\{C^j_{2^{j-1}(n+1)n}\}_{j\geq 1}$ of binary cyclic codes introduced in [11] against a binary BCH code C_n^0 in which all codes have same error correction capability and code rate but sequentially increasing code lengths. Initially all these codes including the binary BCH code are carrying data of their corresponding primary users. The procedure advances as, whenever, any of the primary users having mod of transmission, the binary cyclic code, is not using its allocated bandwidth, the user having its data configured by binary BCH code enter and utilize the free path as a secondary user.

2. Basic facts

This section covers some of the basic results associated to monoid ring, cyclic codes and specific on transmission parameters.

To construct a polynomial (n, k)-code C over a finite Galois field $\mathbb{F}_q$, where q is power of some prime, we choose a polynomial $g(x)$ of degree $n-k=r$ from $\mathbb{F}_q[x]$. A message is represented by a polynomial, called the message polynomial, $j(x)$ of degree less than or equal to $k-1$. The code polynomial corresponding to this $j(x)$ is $v(x)$ and is equal to $r(x)+x^{n-k}j(x)$, where $r(x)$ is the remainder of $x^{n-k}j(x)$ after dividing it by $g(x)$. A polynomial-code is an error correcting code whose codewords consists of multiple of a given fixed polynomial $g(x)$ known as the generator polynomial.

Let $(\mathbf{S}, *)$ and $(\mathcal{R}, +, .)$ be a commutative semigroup and an associative unitary commutative ring respectively. The set $\mathcal{SR}$ of all finitely nonzero functions f from $\mathbf{S}$ into $\mathcal{R}$ is a ring with respect to binary operations addition and multiplication defined as $(f+g)(s) = f(s) + g(s)$ and $(fg)(s) = \sum\limits_{t*u=s} f(t)g(u)$, where the symbol $\sum\limits_{t*u=s}$ indicates that the sum is taken over all pairs (t, u) of elements of $\mathbf{S}$ such that $t * u = s$ and it is settled that in the situation where s is not expressible in the form $t * u$ for any $t, u \in \mathbf{S}$, $(fg)(s) = 0$. The set $\mathcal{SR}$ is called a unitary commutative semigroup ring of $\mathbf{S}$ over $\mathcal{R}$. If $\mathbf{S}$ is a monoid, then $\mathcal{SR}$ is called monoid ring. This ring $\mathcal{SR}$ is represented as $\mathcal{R}[\mathbf{S}]$ whenever $\mathbf{S}$ is a multiplicative semigroup and elements of $\mathcal{T}$ are written either as $\sum\limits_{s\in\mathbf{S}} f(s)s$ or as $\sum_{i=1}^{n} f(s_i)s_i$. The representation of $\mathcal{SR}$ will be $\mathcal{R}[x; \mathbf{S}]$ whenever $\mathbf{S}$ is an additive semigroup. A nonzero element f of $\mathcal{R}[x; \mathbf{S}]$ is uniquely represented in the canonical form $\sum_{i=1}^{n} f(s_i)x^{s_i} = \sum_{i=1}^{n} f_i x^{s_i}$, where $f_i \neq 0$ and $s_i \neq s_j$ for $i \neq j$. Of course, the monoid ring $\mathcal{R}[x; \mathbf{S}]$ is a polynomial ring in one indeterminate if $\mathbf{S}$ is the additive monoid $\mathbb{Z}_0$ of non-negative integers.

The concept of degree and order are not generally defined in a semigroup ring. Though if $\mathbf{S}$ is a totally ordered semigroup, then the degree and order of an element of semigroup ring $\mathcal{R}[x; \mathbf{S}]$ is defined as: if $f = \sum_{i=1}^{n} f_i x^{s_i}$ is the canonical form of the nonzero element $f \in \mathcal{R}[x; \mathbf{S}]$, where $s_1 < s_2 < \cdots < s_n$, then s_n is called the degree of f and we write $\deg(f) = s_n$ and similarly the order of f written as $ord(f) = s_1$. Now, if $\mathcal{R}$ is an integral domain, then for $f, g \in \mathcal{R}[x; \mathbf{S}]$, it follows that $\deg(fg) = \deg(f) + \deg(g)$ and $ord(fg) = ord(f) + ord(g)$.

We start by an observation that, for a field $\mathbb{F}$ and an integer $j \geq 0$, the structures of a polynomial ring $\mathbb{F}[x]$ and a monoid ring $\mathbb{F}[x, \frac{1}{j}\mathbb{Z}_0]$ have many interconnections, for instance, for an ordered monoid $\mathbf{S}$, the monoid ring $\mathbb{F}[x, \mathbf{S}]$ is a Euclidean domain if $\mathbb{F}$ is a field and $\mathbf{S} \cong \mathbb{Z}$ or $\mathbf{S} \cong \mathbb{Z}_0$ [5, Theorem 8.4]. Of course here $\frac{1}{j}\mathbb{Z}_0$ is totally ordered and and has an isomorphism with $\mathbb{Z}_0$.

Let $\mathbb{F}$ be any field and $\frac{1}{j}\mathbb{Z}_0$ is the additive monoid, then $\mathbb{F}[x; \frac{1}{j}\mathbb{Z}_0]$ is a monoid ring. A generalized polynomial $g(x^{\frac{1}{j}})$ of arbitrary degree r in $\mathbb{F}[x; \frac{1}{j}\mathbb{Z}_0]$ is represented as

$$g(x^{\frac{1}{j}}) = g_0 + g_{1,\frac{1}{j}} x^{\frac{1}{j}} + g_{2,\frac{1}{j}} x^{\frac{1}{j}2} + \cdots + g_{r,\frac{1}{j}} x^{\frac{1}{j}r}$$

Andrade and Shah has constructed cyclic codes over a local finite commutative ring $\mathcal{R}$, through the monoid rings $\mathcal{R}[x; \frac{1}{3}Z_0], \mathcal{R}[x; \frac{1}{2}Z_0]$ and $\mathcal{R}[x; \frac{1}{2^2}Z_0]$ in [1], [12] and [13], respectively. However, in [14] the cyclic codes of certain types are discussed corresponding to the ascending chain of monoid rings.

3. Bandwidths of n length BCH code and $2^{j-1}(n+1)n$ lengths cyclic codes

3.1. **Cyclic codes through $\mathbb{F}_q[x]$ and $\mathbb{F}_q[x; \frac{1}{2^j}\mathbb{Z}_0]$.** For any positive integer j, there is a following ascending chain of monoid rings given by

$$\mathbb{F}_q[x] \subset \mathbb{F}_q[x; \frac{1}{2}\mathbb{Z}_0] \subset \mathbb{F}_q[x; \frac{1}{2^2}\mathbb{Z}_0] \subset \cdots \subset \mathbb{F}_q[x; \frac{1}{2^j}\mathbb{Z}_0] \subset \cdots$$

Let $n = 2^s - 1$, where $s \in \mathbb{Z}^+$ and take $(n+1)n = n'$. Thus, $a_0 + a_{\frac{1}{2^j}}\zeta + a_{\frac{2}{2^j}}\zeta^2 + \cdots + a_{\frac{(2^{j-1}n'-1)}{2^j}}\zeta^{2^{j-1}n'-1}$ is a typical element of $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]/((x^{\frac{1}{2^j}})^{2^{j-1}n'} - 1)$, where $a_0, a_{\frac{1}{2^j}}, a_{\frac{2}{2^j}}, \cdots, a_{\frac{(2^{j-1}n'-1)}{2^j}} \in \mathbb{F}_2$ and ζ is the coset $x^{\frac{1}{2^j}} + ((x^{\frac{1}{2^j}})^{2^{j-1}n'} - 1)$. So $f(\zeta) = 0$, where ζ satisfies the relation $\zeta^{2^{j-1}n'} - 1 = 0$.

Now, replace $x^{\frac{1}{2^j}}$ at the place of ζ. Thus, the ring $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]/((x^{\frac{1}{2^j}})^{2^{j-1}n'} - 1)$ becomes $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_{\geq 0}]_{2^{j-1}n'}$ in which the relation $(x^{\frac{1}{2^j}})^{2^{j-1}n'} = 1$ holds. The binary operation multiplication $*$ in the ring $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]_{2^{j-1}n'}$ is modulo the ideal $((x^{\frac{1}{2^j}})^{2^{j-1}n'} - 1)$. So, given $a(x^{\frac{1}{2^j}}), b(x^{\frac{1}{2^j}}) \in \mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]_{2^{j-1}n'}$, we write $a(x^{\frac{1}{2^j}}) * b(x^{\frac{1}{2^j}})$ to denote their product in the ring $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]_{2^{j-1}n'}$ and $a(x^{\frac{1}{2^j}})b(x^{\frac{1}{2^j}})$ to denote their product in the ring $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_{\geq 0}]$. If $\deg(a(x^{\frac{1}{2^j}})) + \deg(b(x^{\frac{1}{2^j}})) < 2^{j-1}n'$, then $a(x^{\frac{1}{2^j}}) * b(x^{\frac{1}{2^j}}) = a(x^{\frac{1}{2^j}})b(x^{\frac{1}{2^j}})$. Otherwise, $a(x^{\frac{1}{2^j}}) * b(x^{\frac{1}{2^j}})$ is the remainder left on dividing $a(x^{\frac{1}{2^j}})b(x^{\frac{1}{2^j}})$ by $(x^{\frac{1}{2^j}})^{2^{j-1}n'} - 1$. In other words, if $a(x^{\frac{1}{2^j}}) * b(x^{\frac{1}{2^j}}) = r(x^{\frac{1}{2^j}})$, then $a(x^{\frac{1}{2^j}})b(x^{\frac{1}{2^j}}) = r(x^{\frac{1}{2^j}}) + ((x^{\frac{1}{2^j}})^{2^{j-1}n'} - 1)q(x^{\frac{1}{2^j}})$ for some generalized polynomial $q(x^{\frac{1}{2^j}})$. Practically, to get $a(x^{\frac{1}{2^j}}) * b(x^{\frac{1}{2^j}})$, we only compute the ordinary product $a(x^{\frac{1}{2^j}})b(x^{\frac{1}{2^j}})$ and then put $(x^{\frac{1}{2^j}})^{2^{j-1}n'} = 1$, $(x^{\frac{1}{2^j}})^{2^{j-1}(n+1)n + \frac{1}{2^j}} = x^{\frac{1}{2^j}}$, and so on. Now, consider $x^{\frac{1}{2^j}} * a(x^{\frac{1}{2^j}})$, and it would be

$$a_{\frac{(2^{j-1}n'-1)}{2^j}} + a_0 x^{\frac{1}{2^j}} + a_{\frac{1}{2^j}}(x^{\frac{1}{2^j}})^2 + \cdots + a_{\frac{(2^{j-1}n'-2)}{2^j}}(x^{\frac{1}{2^j}})^{2^{j-1}n'-1}.$$

Particularly, take the product $x^{\frac{1}{2^j}} * a(x^{\frac{1}{2^j}})$ in $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]_{2^{j-1}n'}$ as: the $\mathbb{F}_2$-space $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]_{2^{j-1}n'}$ is isomorphic to $\mathbb{F}_2$-space $\mathbb{F}_2^{2^{j-1}n'}$, indeed, $(x^{\frac{1}{2^j}})^{2^{j-1}n'} - 1 = y^{2^{j-1}(n+1)n} - 1$, where $x^{\frac{1}{2^j}} = y$. In fact, we deal the coefficients of generalized polynomials $a(x^{\frac{1}{2^j}}) = a_0 + a_{\frac{1}{2^j}}x^{\frac{1}{2^j}} + \cdots + a_{\frac{(2^{j-1}n'-1)}{2^j}}(x^{\frac{1}{2^j}})^{2^{j-1}n'-1}$ of $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]$, so $c(x^{\frac{1}{2^j}})$ has $2^{j-1}n'$ terms and hence the coefficients in $\mathbb{F}_2$. Corresponding to the polynomial $a(x^{\frac{1}{2^j}})$ of $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]_{2^{j-1}n'}$, there is a $2^{j-1}n'$-tuppled vector $(a_0, a_{\frac{1}{2^j}}, \cdots, a_{\frac{(2^{j-1}n'-1)}{2^j}})$ in $\mathbb{F}_2^{2^{j-1}n'}$. Thus, there is an isomorphism between the vector spaces $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]_{2^{j-1}(n+1)n}$ and $\mathbb{F}_2^{2^{j-1}n'}$, defined by $a \longmapsto a(x^{\frac{1}{2^j}})$.

We observed that, multiplication by $x^{\frac{1}{2^j}}$ in the ring $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_{\geq 0}]_{2^{j-1}n'}$ corresponds to cyclic shift σ in $\mathbb{F}_2^{2^{j-1}n'}$, that is, $x^{\frac{1}{2^j}} * a(x^{\frac{1}{2^j}}) = \sigma(a)(x^{\frac{1}{2^j}})$.

A subspace $\mathcal{C}$ of $\mathbb{F}_2$-space $\mathbb{F}_2^{2^{j-1}n'}$ is a linear code. As already agreed, we recognize every vector $\mathbf{a}$ in $\mathbb{F}^{2^{j-1}n'}$ with the polynomial $a(x^{\frac{1}{2^j}})$ in $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]_{2^{j-1}n'}$, so the ideal $\mathcal{C}^j_{2^{j-1}n'}$ in $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]_{2^{j-1}n'}$ is acyclic code. The elements of the codes $\mathcal{C}^j_{2^{j-1}n'}$ are now referred as codewords or code generalized polynomials.

Note that if $\mathcal{C}^j_{2^{j-1}n'} = (p(x^{\frac{1}{2^j}}))$ is the ideal generated by $p(x^{\frac{1}{2^j}})$, then $p(x^{\frac{1}{2^j}})$ is the generator (generalized) polynomial of $\mathcal{C}^j_{2^{j-1}n'}$ if and only if $p(x^{\frac{1}{2^j}})$ is monic and divides $(x^{\frac{1}{2^j}})^{2^{j-1}n'} - 1$ in $\mathbb{F}_2[x; \frac{1}{2^j}\mathbb{Z}_0]$.

In [11], a link is developed between a binary BCH code $(n, n-r)$ and a sequence $\{2^{j-1}n', 2^{j-1}n' - 2^j r\}_{j\geq 1}$ of binary cyclic codes.

Following [11], if C^0_n is a binary BCH code $(n, n-r)$ based on the positive integers c, δ_1, $q = 2$ and n such that $2 \leq \delta_1 \leq n$ and $n = 2^s - 1$, where $s \in \mathbb{Z}^+$. Thus, the binary BCH code C^0_n has r degree generator polynomial $g(x) = lcm\{j_i(x) : i = c, c+1, \cdots, c+\delta_1 - 2\}$, where $j_i(x)$ are minimal polynomials of ζ^i, for $i = c, c+1, \cdots, c+\delta_1 - 2$. Whereas ζ is the primitive n^{th} root of unity in $\mathbb{F}_{2^l}$. Since $j_i(x)$ divides $x^n - 1$ for each i, it follows that $g(x)$ divides $x^n - 1$. Thus, $\mathcal{C}^0_n$ is a principal ideal in the ring $\mathbb{F}_2[x]_n$ which is generated by $g(x)$.

Theorem 1. [11, Theorem 2] *For a positive integer s, let C^0_n be a binary BCH code of length $n = 2^s - 1$ generated by a polynomial $g(x) = g_0 + g_1 x + \cdots + g_r x^r \in \mathbb{F}_2[x]$ of degree r. Then,*

(1) *there exists a sequence $\{\mathcal{C}^j_{2^{j-1}n'}\}_{j\geq 1}$ of binary cyclic codes such that for each $j \geq 1$, the code $\mathcal{C}^j_{2^{j-1}n'}$ has length $2^{j-1}(n+1)n$, generated by $2^j r$ degree generalized polynomial $g(x^{\frac{1}{2^j}}) = g_0 + g_1(x^{\frac{1}{2^j}})^{2^j} + \cdots + g_{2^j r}(x^{\frac{1}{2^j}})^{2^j r} \in \mathbb{F}_2[x, \frac{1}{2^j}\mathbb{Z}_0]$,*
(2) *the binary BCH code $\mathcal{C}^0_n$ is embedded in each binary cyclic code $\mathcal{C}^j_{2^{j-1}n'}$ for $j \geq 1$,*
(3) *there are embeddings $\mathcal{C}^1_{n'} \hookrightarrow \mathcal{C}^2_{2^1 n'} \hookrightarrow \cdots \hookrightarrow \mathcal{C}^j_{2^{j-1}n'} \hookrightarrow \cdots$ of binary cyclic codes of the sequence $\{C^j_{2^{j-1}n'}\}_{j\geq 1}$.*

In [11] it is established that for a binary BCH code $\mathcal{C}^0_n = (g(x))$ there does not exist any binary BCH code $\mathcal{C}^j_{2^{j-1}n'}$ generated by polynomial $g(x^{\frac{1}{2^j}})$.

The following table for $\mathcal{C}^j_{2^{j-1}n'} = (2^{j-1}n', 2^{j-1}n' - 2^j r)$ can be constructed for varying, n, r, j, where integer $j \geq 0$.

Table I

n, r	C^0_n	$C^1_{n'}$	$C^2_{2n'}$	$C^3_{2^2n'}$	$C^4_{2^3n'}$	$\cdots$
$3, 2$	$(3, 1)$	$(12, 8)$	$(24, 16)$	$(48, 32)$	$(96, 64)$	$\cdots$

3.2. **Bandwidth limitations.** Modulation is the process by which information is conveyed by means of an electromagnetic wave. The information is impressed on a sinusoidal carrier wave by varying its amplitude, frequency, or phase. Methods of modulation may be either analog or digital. The power and bandwidth necessary for the transmission of a signal with a given level of quality depends on the method of modulation. There is a classic tradeoff between power and bandwidth that is fundamental to the efficient design of communication systems. There are three types of modulation; Amplitude shift keying (ASK), Frequency shift keying (FSK), and Phase shift keying (PSK). Furthermore, quicker computer processors allow the use of more complex forward error correction coding techniques at high bit rates. Therefore, more spectrum proficient procedures of digital modulation such as 8PSK and 16QAM are more gorgeous, even though the power necessities are higher. Together with powerful coding methods such as concatenated BCH coding, these methods offer the viewpoint of improved spectral efficiency with essentially error-free digital signal transmission.

Let S be the signal set j be the number of signals in the signal set. Suppose $v^{(t)} = (v_0^{(t)}, \cdots, v_{n-1}^{(t)}) \in \mathbb{F}_q^n$ is the codeword of an (n, k)-code corresponding to a message $u^{(t)} = (u_0^{(t)}, \cdots, u_{k-1}^{(t)}) \in \mathbb{F}_q^k$ at time t and we have divided each $v^{(t)}$ into n/m blocks, where $m = log_q j$ and $j = q^m$ (the case $q = 2$). Thus, modulation is a map $M : \mathbb{F}_q^m \to S$ defined as $s^{(t)} = s(v^{(t)})$, where $s^{(t)} \in S$ and S is a subset of N-dimensional real Euclidean space, that is, $S \subset R^N$ [7, Chapter 7].

Following [10], the bandwidth required for an (n, k) code is $W = \frac{R_u}{m}(\frac{1}{R})$, where $m = \log_2 M$, R_u is the source data (transmission) rate and $R = \frac{k}{n}$ the code rate.

The bandwidth may be maximize and minimize, depends upon the minimum and the maximum value of the $n/k = 1/R$ and the value of the, m bits for selection of modulation scheme for different modulation types. These bits may be minimum and maximum for maximum and minimum bandwidth. It can be seen as $W_{\max} = \frac{R_u}{m_{\min}}(\frac{1}{R})_{\max}$ and $W_{\min} = \frac{R_u}{m_{\max}}(\frac{1}{R})_{\min}$. Thus, there are the followings possibilities:

(1) m is fixed but $\frac{1}{R}$ is varying, and
(2) m and $\frac{1}{R}$ both are varying.

For cognitive radio transformation under the interweave model we may get spectrum corresponding to the given sequence $\{\mathcal{C}^j_{2^{j-1}n'}\}_{j=1}^{j_0}$ of binary cyclic codes for data transfer of the primary users. Now, the setup allow the secondary users having the binary BCH code $\mathcal{C}^0_n$ mod for their data transfer. Accordingly the secondary users obtain high speed data transfer as compare to its own scheme of the BCH code $\mathcal{C}^0_n$. Furthermore, since there are embeddings $\mathcal{C}^1_{n'} \hookrightarrow \mathcal{C}^2_{2^1n'} \hookrightarrow \cdots \hookrightarrow \mathcal{C}^{j_0}_{2^{j_0-1}n'}$ of binary cyclic codes of the sequence $\{\mathcal{C}_{2^{j-1}n'^j}\}_{j=1}^{j_0}$ and the binary BCH code $\mathcal{C}^0_n$ is embedded in each of binary cyclic codes $\mathcal{C}^j_{2^{j-1}n'}$ for $1 \leq j \leq j_0$.

It is noticed that corresponding to the code rate $R^0_n = \frac{k}{n}$ of binary BCH code $\mathcal{C}^0_n$, the code rate of binary cyclic code $\mathcal{C}^j_{2^{j-1}n'}$ is $R^j_{2^{j-1}n'} = \frac{2^{j-1}n' - 2^j r}{2^{j-1}n'}$, for each $1 \leq j \leq j_0$. Moreover, $R^0_n \leq R^j_{2^{j-1}n'}$ and $R^j_{2^{j-1}n'}$ is same for each binary cyclic code $\mathcal{C}^j_{2^{j-1}n'}$, that is, $R^1_{n'} = R^2_{2n'} = \cdots = R^{j_0}_{2^{j_0-1}n'}$. This means $1/R^j_{2^{j-1}n'} \leq \frac{1}{R^0_n}$, and therefore, $W^j_{2^{j-1}n'} = \frac{R_u}{m} 1/R^j_{2^{j-1}n'} \leq W^0_n = \frac{R_u}{m}(1/R^0_n)$. Thus, if we transmit data through any of code of the sequence $\{\mathcal{C}^j_{2^{j-1}n'}\}_{j=1}^{j_0}$, the bandwidth $W^j_{2^{j-1}n'}$ for each $j \geq 1$ will be lesser the bandwidth W^0_n required for data transmitted through the binary BCH code $\mathcal{C}^0_n$. Interestingly the same bandwidth $W^j_{2^{j-1}n'} = W$ is required for a user having any type of code from the sequence $\{\mathcal{C}^j_{2^{j-1}n'}\}_{j=1}^{j_0}$ of binary cyclic codes.

For $m = 1, 3$, the relation between bandwidth and code rate is given as $W = w(R_u/m)(1/R) = wR_u/mR$, where w is the bandwidth expansion, R_u is the transmission rate and $R = k/n$ is the code rate. The bandwidth with different code rates is given in the Table III (Chosen codes are from Table 1).

Table II

R_n^0	$R_{n'}^1$	$R_{2n'}^2$	$R_{4n'}^3$	$R_{8n'}^4$
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

For $w = 1.2$ and $R_u = 64\ kbps$.

Table III

m	$W_n^0\ kHz$	$W_{n'}^1\ kHz$	$W_{2n'}^2\ kHz$	$W_{4n'}^3\ kHz$	$W_{8n'}^4\ kHz$
1	$W_3^0 : 236.4$	$W_{12}^1 : 118.2$	$W_{24}^2 : 118.2$	$W_{48}^3 : 118.2$	$W_{96}^4 : 118.2$
3	$W_3^0 : 78.8$	$W_{12}^1 : 39.4$	$W_{24}^2 : 39.4$	$W_{48}^3 : 39.4$	$W_{96}^4 : 39.4$

4. A transformation model for cognitive radio

In communication Transmission Control Protocol (TCP) systematically interlaced through RF front end, Physical layer, Data link layer, Network layer, Transport Layer, and Upper Lyer. However, in this study we deal Data link layer, which addresses the error correcting codes.

Secondary user has an opportunistic accesses to the spectrum slum in the interweave model whenever the primary user is not in and pull out when the primary user desires to in once more. Therefore the codes constructed in this study may provide an excellent scheme for wireless communication in which interference issue is handled amicably. We propose a transmission model for Cognitive radio based on error correcting codes which ensures the non interference across the users.

Here in the following a sketch of the model is presented.

It is assumed that the primary users family $\{\mathcal{P}_{2^{j-1}n'}^j\}_{j=1}^{j_0}$ correspondingly use the family $\{\mathcal{C}_{2^{j-1}n'}^j\}_{j=1}^{j_0}$ of binary cyclic codes for its data transmission. Since there are embeddings

$$\begin{array}{ccccccc} \mathcal{C}_{n'}^1 & \hookrightarrow & \mathcal{C}_{2^1n'}^2 & \cdots & \hookrightarrow & \mathcal{C}_{2^{j_0-1}n'}^{j_0} \\ \circlearrowleft & & \circlearrowleft & & & \circlearrowleft \\ \mathcal{C}_n^0 & & \mathcal{C}_n^0 & & & \mathcal{C}_n^0 \end{array}$$

of binary cyclic codes of the sequence $\{\mathcal{C}_{2^{j-1}n'}^j\}_{j=1}^{j_0}$ and the binary BCH code $\mathcal{C}_n^0$ is embedded in each of binary cyclic code $\mathcal{C}_{2^{j-1}n'}^j$ for $1 \leq j \leq j_0$.

Binary cyclic codes of the sequence $\{\mathcal{C}_{2^{j-1}n'}^j\}_{j=1}^{j_0}$ are used for data transmission of the sequence $\{\mathcal{P}_{2^{j-1}n'}^j\}_{j=1}^{j_0}$ of primary users with corresponding bandwidths $\{W_{2^{j-1}n'}^j\}_{j=1}^{j_0}$ such that $W_{n'}^1 = W_{2n'}^1 = \cdots = W_{2^{j_0-1}n'}^{j_0} = W$ and the total bandwidth j_0W is required for simultaneous transmission. As binary BCH code $\mathcal{C}_n^0$ requires bandwidth $W_n^0 \geq W$, so the data transmission of the sequence $\{\mathcal{P}_{2^{j-1}n'}^j\}_{j=1}^{j_0}$ of primary users and the user $\mathcal{P}^0$ is $j_0W + W_n^0$.

Whenever all users $\{\mathcal{P}_{2^{j-1}n'}^j\}_{j=1}^{j_0}$ and $\mathcal{P}^0$ transmitting their data at a glance considered to be the primary users. However, the user $\mathcal{P}^0$ enter as a secondary user and opportunistically can use any of path of the sequence $\{\mathcal{P}_{2^{j-1}n'}^j\}_{j=1}^{j_0}$ of primary users whenever any of them is not using its allotted bandwidth. Here it is noticed that the data of the secondary user $\mathcal{P}^0$ is configurated with the binary BCH code $\mathcal{C}_n^0$ and it requires bandwidth higher than any of the bandwidth required for the data of any primary user of the sequence $\{\mathcal{P}_{2^{j-1}n'}^j\}_{j=1}^{j_0}$. Consequently, with high rate and with less bandwidth secondary user $\mathcal{P}^0$ can transmit its data.

4.1. **How the model work.** Notions

$j = 0$, $1 \leq j \leq 2^{j_0-1}n'$.

$\mathcal{P}_n^0$: primary user corresponding to the binary BCH code $\mathcal{C}_n^0$.

$\mathcal{P}_{2^{j-1}n'}^j$: j-th primary user corresponding to the binary cyclic code $\mathcal{C}_{2^{j-1}n'}^j$.

m^j: information symbols for jth user.

E^j: j-th encoder for m^j.

$j_{\mathcal{P}^j}$: modulation for E^j.

$\blacksquare$: bandwidth required for user $\mathcal{P}^j_{2^{j-1}n'}$ for each j.
$\blacklozenge\blacksquare$: bandwidth required for user $\mathcal{P}^0_n$.
$\blacklozenge\blacksquare\blacksquare\blacksquare\blacksquare\cdots\blacksquare$: $W^0_n + j_0W$, total bandwidth.
$Dj_{\mathcal{P}^j}$: j-th demodulation
D^j: j-th decoder.

The data of $\mathcal{P}^j$, for each j, is modulated through $\mathbf{M}_{\mathcal{P}^j}$, where $\mathbf{M}_{\mathcal{P}^j}$ is a modulation map, i.e., $\mathbf{M}_{\mathcal{P}^j} : \mathbb{F}^m_q \to S_{\mathcal{P}^j}$, where $S_{\mathcal{P}^j}$ is the signal set, $j_{\mathcal{P}^j}$ is the number of signals in the signal sets $S_{\mathcal{P}^j}$. However, for $q = 2$ it follows that $j_{\mathcal{P}^j} = 2^m$, where m is a positive integer.

<u>A cognitive radio transmission model</u>

$$\begin{array}{cccccccccc}
 & \mathcal{P}^1_{n'} & & \mathcal{P}^2_{2n'} & & \mathcal{P}^3_{2^2n'} & \cdots & \mathcal{P}^j_{2^{j-1}n'} & \cdots & \mathcal{P}^{j_0}_{2^{j_0-1}n'} \\
\mathcal{P}^0_n \Rsh & \downarrow & \rightarrow & \downarrow & \rightarrow & \downarrow & \rightarrow & \mathcal{P}^0_n \hookrightarrow\downarrow & & \downarrow \\
\downarrow & & & & & & & & & \\
m^0 & m^1 & & m^2 & & m^3 & & m^0 \hookrightarrow m^j & & m^{j_0} \\
\downarrow & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
E^0 & E^1 & & E^2 & & E^3 & & E^0 \hookrightarrow E^j & & E^{j_0} \\
\downarrow & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
\mathbf{M}_{\mathcal{P}^0} & \mathbf{M}_{\mathcal{P}^1} & & \mathbf{M}_{\mathcal{P}^2} & & \mathbf{M}_{\mathcal{P}^3} & & \mathbf{M}_{\mathcal{P}^0} \leftrightarrow \mathbf{M}_{\mathcal{P}^j} & & \mathbf{M}_{\mathcal{P}^{j_0}} \\
\downarrow & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
\blacklozenge\blacksquare & \blacksquare & & \blacksquare & & \blacksquare & \cdots & \blacksquare & \cdots & \blacksquare \\
\downarrow & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
D\mathbf{M}_{\mathcal{P}^0} & D\mathbf{M}_{\mathcal{P}^1} & & D\mathbf{M}_{\mathcal{P}^2} & & D\mathbf{M}_{\mathcal{P}^3} & & D\mathbf{M}_{\mathcal{P}^0} \leftrightarrow D\mathbf{M}_{\mathcal{P}^j} & & D\mathbf{M}_{\mathcal{P}^{j_0}} \\
\downarrow & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
D^0 & D^1 & & D^2 & & D^3 & & D^0 \hookrightarrow D^j & & D^{j_0} \\
\downarrow & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
\mathcal{P}^0_n & \mathcal{P}^1_{n'} & & \mathcal{P}^2_{2n'} & & \mathcal{P}^3_{2^2n'} & \cdots & \mathcal{P}^0_n \leftrightarrow \mathcal{P}^j_{2^{j-1}n'} & \cdots & \mathcal{P}^{j_0}_{2^{j_0-1}n'}
\end{array}$$

Transmission steps: Let $j = 0$ and $1 \leq j \leq j_0$.

(1) **All users are primary users**:
   (a) Data of the $\mathcal{P}^j$, for each j, users transform into the set m^j of message bits.
   (b) For each j, the set m^j of message bits encoded through encoder E^j.
   (c) For each j, the set E^j of encoded messages modulated through $\mathbf{M}_{\mathcal{P}^j}$.
   (d) For each j, the set $\mathbf{M}_{\mathcal{P}^j}$ of modulated codewords passing through the channel having bandwidth $W^j = W$.
   (e) For each j, the corresponding transmitted signals of $\mathbf{M}_{\mathcal{P}^j}$ are demodulated.
   (f) For each j, the received signals corresponding to $\mathbf{M}_{\mathcal{P}^j}$ are decoded through decoder D^j.
   (g) The end of whole process is the destination of data of all users.

(2) **All users are not primary users**: almost all steps of data transmission are same as I. Though, the user $\mathcal{P}^0$ enter as a secondary user and opportunistically can use any of the path of the sequence $\{\mathcal{P}^j_{2^{j-1}n'}\}^{j_0}_{j=1}$ of primary users whenever any of them is not using its allotted bandwidth. For instance, if the primary user $\mathcal{P}^j_{2^{j-1}n'}$ is not in, then the user $\mathcal{P}^0$ transmitted its data configurated by the binary BCH code $\mathcal{C}^0_n$ through the binary cyclic code $\mathcal{C}^j_{2^{j-1}n'}$ used for data of primary user $\mathcal{P}^j_{2^{j-1}n'}$.

4.2. **Illustration.** Let $n = 2^2 - 1 = 3$, $\delta = 3$, $c = 1$, $p(x) = x^2 + x + 1$ a primitive polynomial of degree 2 and $\mathbb{F}_{2^2} = \frac{\mathbb{F}_2[x]}{(p(x))} = \{a_0 + a_1\zeta : a_0, a_1 \in \mathbb{F}_2\}$, where ζ satisfies the relation $\zeta^2 + \zeta + 1 = 0$. Using this relation, we obtain $\{0, \zeta, 1+\zeta\}$. Let $m_i(x)$ be the minimal polynomial of ζ^i, for $i = c, c+1, \cdots, c+\delta-2$. Thus, $m_1(x) = x^2+x+1$ and hence $g(x) = lcm\{m_i(x) : i = c, c+1, \cdots, c+\delta-2\} = x^2+x+1$. The code $C^0_3 = (g(x)) \subset \mathbb{F}_2[x]_3$ is a binary BCH code based on the positive integers $c = 1$, $\delta = 3$, $q = 2$ and $n = 3$ such that $2 \leq \delta \leq n$ with $gcd(n, 2) = 1$. Consequently, $\{\mathcal{C}^j_{2^{j-1}n'}\}_{j\geq1} = \{\mathcal{C}^j_{2^{j-1}(3+1)3}\}_{j\geq1} = \{\mathcal{C}^j_{3\times2^{j+1}}\}_{j\geq1}$ is the sequence $\{(3 \times 2^{j+1}, 3 \times 2^{j+1} - 2^{j+1})\}_{j\geq1}$

of binary cyclic codes corresponding to the BCH code $\mathcal{C}_3^0$. For instance $\mathcal{C}^1_{2^{1-1}(3+1)3} = \mathcal{C}^1_{12}$, a $(12, 8)$ code which is generated by $g(x^{\frac{1}{2}}) = (x^{\frac{1}{2}})^{2(2)} + (x^{\frac{1}{2}})^2 + 1 = (x^{\frac{1}{2}})^4 + (x^{\frac{1}{2}})^2 + 1$ in $\mathbb{F}_2[x, \frac{1}{2^1}\mathbb{Z}_0]$ and $C^2_{2^{2-1}(3+1)3} = C^2_{24}$ is a $(24, 16)$ code which is generated by $g(x^{\frac{1}{2^2}}) = (x^{\frac{1}{2^2}})^{2^2(2)} + (x^{\frac{1}{2^2}})^{2^2} + 1 = (x^{\frac{1}{2^2}})^8 + (x^{\frac{1}{2^2}})^4 + 1$ in $\mathbb{F}_2[x, \frac{1}{2^2}\mathbb{Z}_0]$, and so on.

Follow the Table I and label the corresponding bandwidths as:

$$\begin{array}{rcl} \text{For} m & = & 1 \\ W_3^0 & : & 236.4 : \blacksquare\blacksquare \\ W_{12}^1 & = & W_{24}^2 : 118.2 : \blacksquare \end{array}$$

$$\begin{array}{rcl} \text{For} m & = & 3 \\ W_3^0 & : & 69.78 : \blacksquare\blacksquare \\ W_{12}^1 & = & W_{24}^2 : 34.89 : \blacksquare \end{array}$$

$$\begin{array}{ccc} & \mathcal{P}_{12}^1 & \mathcal{P}_{24}^2 \\ \mathcal{P}_3^0 \Rsh & \downarrow & \downarrow \\ \downarrow & & \\ \mathbb{F}_2 \hookrightarrow & \mathbb{F}_2^8 & \mathbb{F}_2^{16} \\ \downarrow & & \\ \downarrow & \downarrow & \downarrow \\ \mathcal{C}_3^0 \subseteq \mathbb{F}_2^3 & \mathcal{C}_3^0 \hookrightarrow \mathcal{C}_{12}^1 \subseteq \mathbb{F}_2^{12} & \mathcal{C}_{24}^2 \subseteq \mathbb{F}_2^{24} \\ \downarrow & \downarrow & \downarrow \\ \mathbf{M}_{\mathcal{P}^0} & \mathbf{M}_{\mathcal{P}^0} \leftrightarrow \mathbf{M}_{\mathcal{P}^1} & \mathbf{M}_{\mathcal{P}^2} \\ \downarrow & \downarrow & \downarrow \\ \blacksquare\blacksquare & \blacksquare & \blacksquare \\ \downarrow & \downarrow & \downarrow \\ D\mathbf{M}_{\mathcal{P}^0} & D\mathbf{M}_{\mathcal{P}^0} \leftrightarrow D\mathbf{M}_{\mathcal{P}^1} & D\mathbf{M}_{\mathcal{P}^2} \\ \downarrow & \downarrow & \downarrow \\ \mathcal{C}_3^0 & \mathcal{C}_3^0 \hookrightarrow \mathcal{C}_{12}^1 & \mathcal{C}_{24}^2 \\ \downarrow & \downarrow & \downarrow \\ \mathcal{P}_n^0 & \mathcal{P}_n^0 \leftrightarrow \mathcal{P}_{12}^1 & \mathcal{P}_{24}^2 \end{array}$$

If the data of user $\mathcal{P}_3^0$ is transmitted through the binary BCH code C_3^0 for any modulation scheme, it requires double bandwidth than the bandwidth required for its transmission through the path of any of the binary cyclic codes C_{12}^0 and C_{24}^0.

In the following we illustrate the decoding steps. Consider the binary BCH code C_3^0.

The canonical generator matrix of binary cyclic $(12, 8)$ code C_{12}^1 is given by

$$G^1 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix},$$

which is obtained by the generator polynomial $g(x^{\frac{1}{2}}) = 1 + (x^{\frac{1}{2}})^2 + (x^{\frac{1}{2}})^4$, whereas the parity check-matrix with check polynomial $h(x^{\frac{1}{2}}) = 1 + (x^{\frac{1}{2}})^2 + (x^{\frac{1}{2}})^6 + (x^{\frac{1}{2}})^8$ is given

$$H^1 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

Syndrome table is given by

	coset leader	syndrome
e_0	000000000000	0000
e_1	100000000000	1000
e_2	010000000000	0100
e_3	001000000000	1010
e_4	000100000000	0101
e_5	000010000000	0010
e_6	000001000000	0001
e_7	110000000000	1100
e_8	100100000000	1101
e_9	100001000000	1001
e_{10}	011000000000	1110
e_{11}	010010000000	0110
e_{12}	001100000000	1111
e_{13}	001001000000	1011
e_{14}	000110000000	0111
e_{15}	000011000000	0011

Let $b = 101 \in \mathbb{F}_2^3$ be the received vector of binary BCH code C_3^0. Thus, its polynomial representation is $b(x) = 1+x^2$ in $\mathbb{F}_2[x]_3$ and corresponding received polynomial in the cyclic code C_{12}^1 is given by $b'(x^{\frac{1}{2}}) = 1+(x^{\frac{1}{2}})^4$ in $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0]_{12}$ by using [11, Theorem 1], and its vector representation will be $b' = 100010000000$ in $\mathbb{F}_2^{12}$ and $(b') = b'(H^1)^T = 1010 = S(e_3)$. Hence, the corrected codeword in C_{12}^1 is $a' = b' + e_3 = 101010000000$ and its polynomial representation is $a'(x^{\frac{1}{2}}) = 1 + (x^{\frac{1}{2}})^2 + (x^{\frac{1}{2}})^{2(2)}$ in $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0]_{12}$. Thus, the corresponding corrected codeword in binary BCH code C_3^0 is $a(x) = 1 + x + x^2$ in $\mathbb{F}_2[x]_3$, i.e., $a = 111$.

In the similar fashion we can decode received vector of C_3^0 through the decoding of any member of the family $\{C^j_{2^{j-1}(n+1)n}\}_{j\geq 1}$ instead of C_{12}^1.

5. Conclusion

This study proposed a novel interweave inclined transmission model for cognitive radio. The data of primary user $\mathcal{P}^0$ is configurated and transmitted through the binary BCH code $\mathcal{C}_n^0$. However, the data of the family $\{\mathcal{P}^j_{2^{j-1}n'}\}_{j=1}^{j_0}$ of primary users is configurated by the family $\{\mathcal{C}^j_{2^{j-1}n'}\}_{j=1}^{j_0}$ having sequentially increasing code lengths but with same code rate. Due to the modulation scheme every member of $\{\mathcal{C}^j_{2^{j-1}n'}\}_{j=1}^{j_0}$ requires same bandwidth but lesser than required for the binary BCH code $\mathcal{C}_n^0$.

Initially all these codes are carrying data of their corresponding primary users. A transmission pattern is planned in the spirit of interweave model in such a way that the user $\mathcal{P}^0$ observe and opportunistically avail the channel path of any of the Primary users of the family $\{\mathcal{P}^j_{2^{j-1}n'}\}_{j=1}^{j_0}$ not utilizing its allotted bandwidth.

This study can also be extended for a set of different n, the length of the binary BCH code $\mathcal{C}_n^0$. Consequently, a multiple transformation model can be designed.

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