

Translation in Space by Rotations

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Abstract

A new means of non-photonic electromagnetic propulsion is discussed both theoretically and practically. A history of the problem is carried out pointing out the common pitfalls in the pure experiment line of reasoning adopted by many inventors. We note the problems of all mechanical systems such as rockets and ion engines. All claims of oscillating/pendulating masses claiming free-fall propulsion are hopefully dismissed by a survey of the pitfalls inventors present themselves – we hope that this will form a useful reference at least. Then we look at nascent electromagnetic propulsion ideas - photonic propulsion/light sails. We ask if the large energy requirement for electromagnetic propulsion can be overcome by using other electromagnetic phenomena apart from radiative pressure. The hidden momentum trap is pointed out for ExB devices and a new device is presented and analysed using a well-known phenomena that, seemingly doesn't violate the laws of physics. Finally quantitatively we present an intuitive model using the Electrodynamics formalism to ask how this might be possible and not violate conservation laws that are core to the fabric of space-time we occupy.

Introduction

Flight and space travel isn't easy, high speeds, high aerodynamic forces, immense airframe pressures from the tenuous air or vacuum outside the craft and the pressure within, high temperatures and volatile fuels. Indeed we find much of the craft is given over to present day 'peculiarities' of solution of the flight problem – wings, expensive high specification engines operating at high temperatures and large tanks for fuel taking up much space and weight, increasing drag and reducing range.

Our problem is Newton's Third Law, which on a more profound level¹ (appendix 1) can be understood as due to the isotropy of space. There is no free ride in space; where-ever one performs an experiment – if one shoots a gun into a block of material and find the bullet embeds and the block moves a small distance, it has got to happen the same elsewhere (translational invariance leads to conservation of linear momentum) and similarly if the experiment was rotated (conservation of angular momentum).

Countless observations find we move forward by pushing against something else, the other object recoiling or barely moving if it has large mass. Either we have a short range/time energetic mass ejection system limited in both energy and matter but with much thrust or a long range/time mass ejection system with very little thrust such as the ion engine.

In space flight usually energy is not a consideration as we can contend with putting heavy or unsafe nuclear reactors in the craft but we run out of propellant. Relativity Theory by mass-energy equivalence presents us with possibility of converting our energy into propellant matter but the exchange rate for this transaction is prohibitive. Relativity⁴ gives us this relation between energy and momentum:

$$\frac{\vec{p}}{E} = \frac{\vec{v}}{c^2} \quad \text{eqn. 1}$$

To gain the biggest amount of momentum for our energy input it is best if our propellant leaves its 'nozzle' at light speed. This is the principle of photonic propulsion such as by laser rocket or light sail however approximately $3 \times 10^9 \text{W}$ is needed to give a force of 1N!

Intuitively and that's how ideas start, we need to have something to push against *wherever* we are that could be said to have some kind of mass-energy. If we could couple to some phenomena that would allow this, energy wouldn't really be an issue because our power sources are quite compact and long lasting. We could expect great range in reasonable time and at least interplanetary travel would become commonplace. The only phenomenon that presents itself to current engineering toolsets is electromagnetism, we believe. This force has the necessary long range, strength and commonplace engineering methods to utilise.

To proceed we shall survey devices that are non-starters and give reasons to hopefully guide co-researchers, some are very seductive but flawed by often subtle effects. The sections are as follows:

- Oscillating Masses can fool the wary
- ExB devices and Hidden Momentum
- Dynamic ExB devices are really just Antennas
- Feynman's Disk
- Translation in Space by Rotations
- Early thoughts on Implementation and Engineering of Proposed Propulsion
- Conclusion
- Appendix

Oscillating Masses can fool the wary

There is a hard-core of people who convince themselves that free propulsion in space is possible by some means of rotating masses. Some are gifted amateurs who, as they see it, are unencumbered by schooling and 'personality and thought oppression' of the education system; others are highly trained individuals who have often passed high into the science establishment but observe some effect and maybe suspend some critical facility. We are not knocking these brave people because more often than not progress is made by the Maverick, however we (in the consensus sense) feel sure that the core issue is not being attacked – that of conservation of momentum or more graphically action-reaction. No-one has shown how some system of masses can have one half of the action-reaction pair somehow removed. The NASA website 'Breakthrough Propulsion Physics'¹ has some interesting summaries of the pitfalls and categorisation of devices, a mechanism appealing to conventional ideas has always been found. Most commonly the devices operate and display a differential friction effect whereby a mass in an asymmetric cycle (fast, slow) somehow presses down less on the chassis (and through to the surface as no one ever does these in free-fall) in part of the cycle generating less normal reaction force and less friction and hence net movement.

We observed another type of motion whilst consulting in a project coming from an horizontal component of force from reaction forces. What was happening in this case was a to-and fro motion like waving one's arms fore and aft flapping fashion; the centre of mass of the masses and the chassis was then shifting back and forth. The inventor said it couldn't possibly be friction because he'd done it on ice, an air table and suspended from a pendulum whereupon it stayed to one side. A little thought makes one realise that it operates in principle as a person punting themselves across an ice-rink with a pole: the pole is in compression and one can resolve this force into a normal reaction and *an horizontal component*.

ExB devices and Hidden Momentum

References 3 and 4 give a good account and 'derivation' of the Poynting Vector. It has the most satisfactory form and other attempted derivations give rise to physical effects that just don't exist. Nether-the-less the form that is accepted by everyone leads to some wacky consequences that people didn't quite believe and pushed under the mat as a backwater of classical electrodynamics. The Feynman disk (next section) allows *angular* translation through space, the mechanical part being balanced by the electromagnetic part. Experiment to show this was done in recent times⁵ and we shall discuss this more in the next section where we develop the ideas around our postulated device.

Recently (fifty years ago) people started wondering if linear translation might be possible by such a scheme. Naively people considered a solenoid producing a static magnetic field with a static electric field at right angles to its axis by some other means. According to theory this should have a continual flow of momentum orthogonal to the electric and magnetic fields, though on greater analysis this is not

so. Reference 6 gives an account by Puthoff, Ivison and Little and further references of hidden momentum, which is a Relativistic effect concerning the motion of the charge carriers generating the magnetic field. In a nutshell, those charges at high potential in the superimposed electric field have more potential energy and hence more mass. This 'hidden' effect and the mechanical momentum from it *always* just balance that from the Poynting flow. References 7 and 8 give more detail with reference 8 giving a particularly elegant derivation done a few years ago showing that there is life in 'old' physics, least where people aren't bothered to look or that old theories and concepts need polishing, review and re-presentation.

Dynamic ExB devices are really just Antennas

Following on from the static devices many have been fooled into thinking that if this mechanism of hidden momentum could be defeated then we would have a propulsion device. Somehow in the mind's eye of the inventor it was seen that the constant electric field superimposed on the magnetic field from the solenoid needed to be replaced with one circulating around it. The figure is borrowed from the Puthoff, Ivison and Little account⁶ and shown 'morphed' on the right-hand side as a bulbous shape with movement in the bulbous direction.

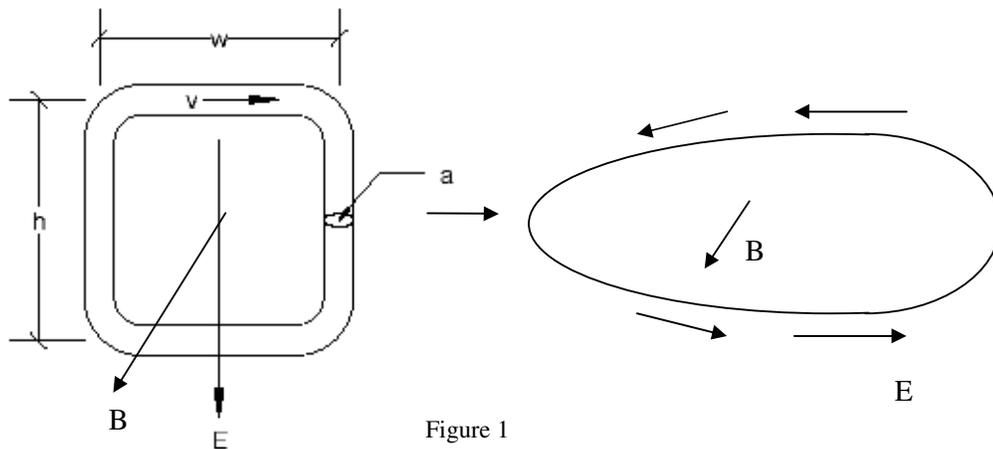


Figure 1

Of course this circulating field would be achieved by changing the magnetic field. The belief system is that field energy of the antenna (to give its correct name!) near-field, sloshing around, is much greater than radiated energy (true) and that when field energy impinges, momentum is imparted in one sense and when it leaves is again imparted in the same sense. Very involved calculations can be done but a moment's thought tells us that once the power supply is included in the set up the momentum from the antenna near-field will cancel and only the puny E/c force from the radiation field will result. Try as we might, placing the power supply at right angles, having the power supply symmetrical, the wires and field will eventually converge on the antenna structure taking up its geometry. We rely on the asymmetrical flow of energy to the antenna to generate net propulsion but at some point the wires feeding it will have the same asymmetric geometry. The energy flowing from one system to the other and hence the momentum will cancel. The radiated momentum dominates; we have just a photon rocket.

Feynman's Disk

Field momentum has long been acknowledged, certainly for radiation. What has been contentious is field momentum in 'static' situations when there is no transmission of wave energy – so called induction fields. Richard Feynman gave a thought experiment to illustrate the point³ in the form of a non-conducting disk (see figure below) with charged electric balls at the periphery. At the centre a solenoid set up an axial field. The apparent paradox is to explain what the disk will do when the solenoid is switched off: a radial electric field will react against the electric field of the balls on the periphery leading to a torque. However according to the design the disk is suspended with nothing to react against and this seems to violate conservation of momentum.

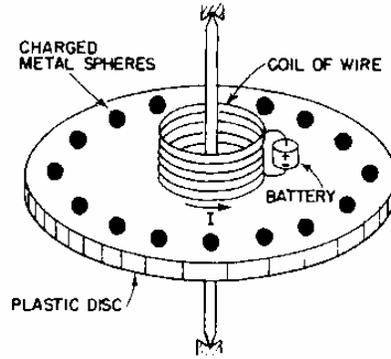


Figure 2 Feynman's Disk

Reference 5 is a short paper describing how this experiment was carried out in comparatively recent times.

Appendix 2 covers in the most fundamental form the derivation of field momentum concepts from the action of the field and particles. Conservation of momentum is expressed (appendices 1 and 2) by:

$$\sum_n \frac{\partial L}{\partial \dot{q}} = 0 \quad \text{eqn. 2}$$

That is the sum of partial differentials of the Lagrangian with respect to velocity of generalised co-ordinates for the system is zero. For the field and a particle density ρ_m and working with the Lagrangian 'density' this reduces to:

$$\rho_m \dot{v} - \rho \mathbf{E} - \mu_0 \mathbf{J} \times \mathbf{H} - \nabla \cdot \mathbf{T} + \frac{\partial}{\partial t} \epsilon_0 \mu_0 \mathbf{E} \times \mathbf{H} = 0 \quad \text{eqn. 3}$$

We understand the second and third terms to be the usual Lorentz force but the terms four and five come from the action of the field S_f which we do not take as a given and so enter the variation (appendix 2). 'T' is the Maxwell Stress Tensor that on taking the divergence ('contraction') leads to a component of the 3-vector of force. Overall the last term (Poynting vector) and the Stress Tensor come from the Stress-Energy Tensor that is derivable from the Electromagnetic Field Tensor (appendix 2). The 3-vector form is less elegant but more expressive to the engineer.

If we integrate the above expression over all space the Maxwell Stress term vanishes so that Lorentz terms is only balanced by the Poynting term – apparently too with static or induction fields. The last term was accepted and understood to be the reaction force on the particles due to radiation. However with the Feynman disk thought experiment the implication was that the field was some kind of fluid body taking up the opposite momentum to the mechanical part⁵. This was seen as some backwater of Classical Electrodynamics that people brushed under the mat and only recently was the experiment performed⁵.

The Feynman disk will rotate *in vacuo* when the field is switched on, translate angularly and stop when the field is switched off. Nature seems to allow this because it happens *at a point* in space and the field momentum and mass momentum cancel. Nature isn't caught with her bloomers down it seems.

In the next section we ask if we can somehow turn this known angular translation into a linear translation.

Translation in Space by Rotations

What are the necessary conditions to achieve a proper translation in space by rotation? Can it be possible? Consider the rod below with two masses at its end:

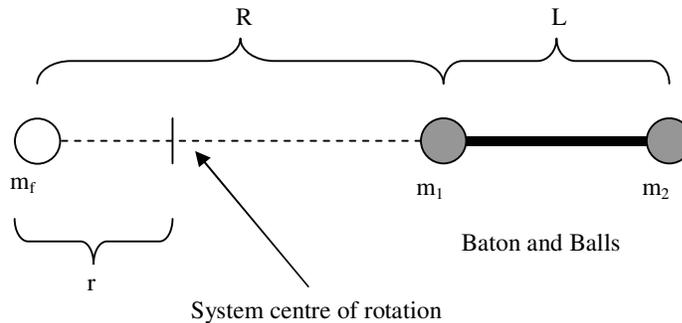


Figure 3

The rod and baton is of length L to the ball centres. Drawn is a dotted line, which isn't a physical device, to the centre of the effect causing the rotation (distance R from mass m_1) as we shall see, this isn't the same as the system centre of rotation at point distance r from mass m_f . The mass m_f is meant to represent the mass of the electromagnetic field we set up.

We shall prove simply that:

- The Centre-of-the-Effect-Causing-Rotation (CECR) is not the same as the System-Centre-of-Rotation (SCR).
- Placing the CECR on the baton only results in rotation about the SCR and hence no translation.
- The CECR when placed outside of the baton and given at least some mass (m_f) allows a rotation about point 'r', that being on a greater diameter can allow translation through space on two such rotations (one on the left hand side then one on the right hand side).

Taking moments about SCR (point 'r'):

$$\begin{aligned}
 m_f r &= m_1 (R - r) + m_2 (R - r + L) \\
 &\Rightarrow \\
 r &= \frac{(m_1 + m_2)R + m_2 L}{m_f + m_1 + m_2} \qquad \text{eqn. 4}
 \end{aligned}$$

- We can prove the first bullet point by noting that if all the masses and distance are finite and positive 'r' is non-zero and doesn't lie at the CECR.
- If we set R to zero, that is place the CECR on the baton, the SCR must lie somewhere on the baton dependent on the masses.
- If R is positive and m_f finite then the CECR must lie outside of the baton and the SCR is outside of the baton too.

To this last point we can calculate the force 'field' around the baton:

At point m_1 :

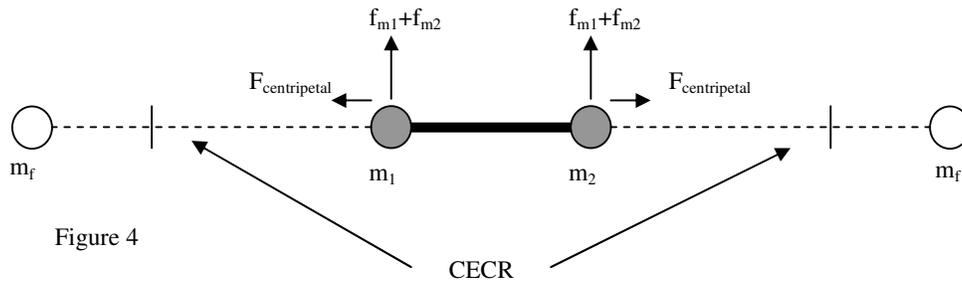
$$f_{m_1} = \frac{\tau(m_f + m_1 + m_2)}{Rm_f - Lm_2} \quad \text{eqn. 5}$$

At point m_2 :

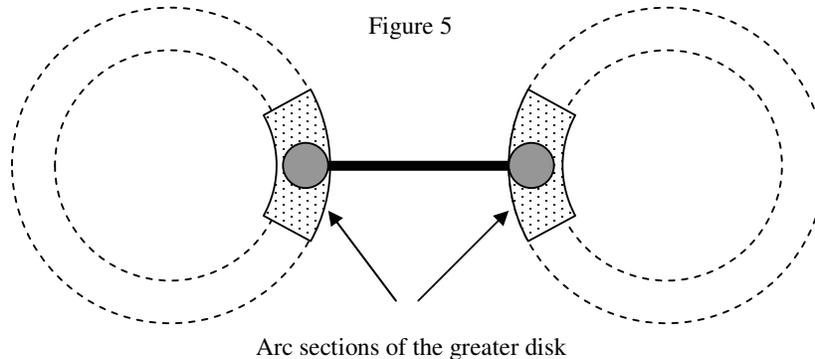
$$f_{m_2} = \frac{\tau(m_f + m_1 + m_2)}{(R + L)m_f + Lm_1} \quad \text{eqn. 6}$$

Taking the case when baton masses are equal, and the CECR is outside of the baton ($R > 0$) clearly the two expressions are not equal (in magnitude) if we attribute some mass to the field. Otherwise the forces are equal and opposite and just form a couple with the SCR within the baton.

If we use two such rotations on either side of the baton (one anti-clockwise the other clockwise) the illusion of distant rotation is complete. The necessary centripetal forces causing the rotation place the baton in tension but we can assume it is rigid. The torque forces sum and act in the same direction forming a *translational* force.



We envision forming two 'virtual' Feynman Disks by having arc sections on the craft that would be part of greater disk whose centre is physically outside of the craft.



The forces all resolve to give the illusion of the greater disks and rotation about a distant point, as we can see by the earlier diagram. A real rotation caused by say two fixed electric motors (bolted to the ground) outside of the baton would compete and place the baton in tension. Equilibrium would be reached whereby the tension in the baton would resolve equal and opposite the torque forces – it would rotate a short way. In the propulsion case our 'stator' is the electromagnetic field itself and not being fixed to anything would be 'dragged' and rotated. We calculate these forces (appendix 3), which amount to radiation resistance but note that their order of magnitude is $1/c^3$ down on acceleration effects produced by rate of change of the Poynting Vector and so are negligible.

As we shall see shortly, most of the field's mass is in the magnetic part and this can be made to appear outside of the craft by the following arrangement (fig. 6), so the Feynman disks are not 'virtual' but really do exist in space outside the craft. The force that the electrets experience is not merely a body force reacting against the solenoids because axial symmetry does not permit this⁵; the force is really in reactance to the field momentum. Also it matters not that the fields are generated by solenoids on the craft, electrodynamics is *field formulated* and the sources do not exert influence by spooky action at distance – Nature cuts her apron strings from the fields once they are propagated, they exist in their own right. All that matters is the arrangement of the fields.

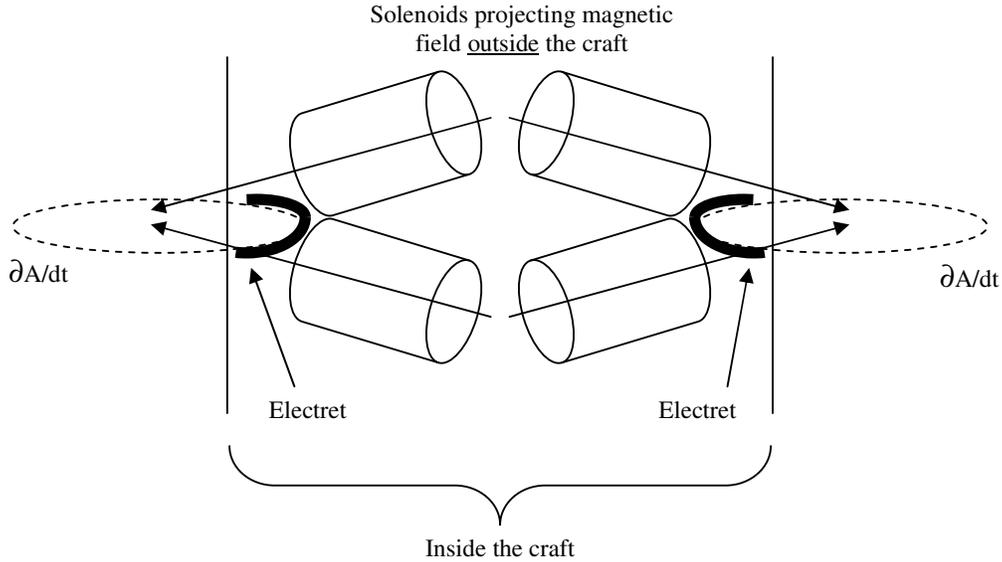


Figure 6

We note that the Feynman disk received an angular impulse when the field was switched on: it rotated and then received the opposite impulse when the field was switched off. Overall it had angularly translated a distance in space. The effect is very feeble and one-shot operation on our linear translation scheme would result in a little kick at the journey's start and a little brake at its end.

Let us proceed on the basis that the time average of the Poynting force (cross product term eqn. 3) can be made non-zero (more on this later) and calculate the magnitude of these forces and see the effect of variation of parameters such as radial distance to CECR, field strength, the length of the arc-section in the craft etc. We aim for simplicity and a ballpark figure so we make approximations:

First let us sum equations 5 and 6 ($m_1 = m_2 = m$) to get the total force:

$$f = -\frac{\pi m_f (2m + m_f)(2R + L)}{(Lm - Rm_f)(Lm + Lm_f + Rm_f)} \quad \text{eqn. 7}$$

The mass of the field is:

$$\begin{aligned} & \frac{1}{c^2} \frac{\epsilon_0}{2} \int (E^2 + c^2 B^2) dV \\ & \approx \frac{\epsilon_0}{2} B^2 V \end{aligned}$$

This is scaled to take account that only a sector of the disk exists that we represent by the radius R and arc sector length S, thus:

$$m_f = \frac{S}{4\pi R} \epsilon_0 B^2 V \quad \text{eqn. 8}$$

From equation 3 we note that the structure is in internal mechanical equilibrium so the electric field and stress tensor do not come into play. Also the magnetic field travels with the structure so the relative velocity is zero, thus the Lorentz term is not important. What we are left with is the Poynting term that gives the torque. If we cycle the electric and magnetic fields at frequency F: $E(t)=E_{\max}Ft$ and $B(t)=B_{\max}Ft$, the expression for the torques becomes:

$$\tau = \frac{1}{2} \epsilon_0 BEVF \times R$$

We take an average for τ over the cycle and we scale the expression by the average value. The disks are just part sectors so we must scale the by the sector length S, i.e. by $S/2\pi R$ ($E \times B$ is an expression for energy/power which is proportional to volume) obtaining finally:

$$\tau = \frac{\epsilon_0}{4\pi} BEVFS \quad \text{eqn. 9}$$

Now we can substitute expressions 8 and 9 in 7[†]:

$$f = -\frac{1}{4} \frac{\epsilon_0^2 EB^3 FV^2 S^2 (8\pi mR + \epsilon_0 B^2 VS)(2R + L)}{R(4\pi mL - \epsilon_0 B^2 VS)(\epsilon_0 B^2 VLS + \epsilon_0 B^2 VRS + 4\pi mL R)}$$

Which can approximate down to:

$$f = -\frac{\epsilon_0^2 EB^3 V^2 FS^2 (2R + L)}{8\pi mL^2 R} \quad \text{eqn. 10}$$

Let us substitute some 'achievable' figures into the expression to estimate the force:

$$B = 100T \quad E = 10^7V/m \quad F = 1GHz \quad L=R=S = 1m \quad m = 10kg \quad V = 10m^3$$

We obtain a magnitude of around 1N. Obviously some very high specification materials and methods are required but there is no reason why we can't re-capture the field energy on each cycle (something like an LCR circuit or cavity oscillator) to leave a very energy efficient process.

An issue already raised is the radiation resistance of Lorentz frictional force from moving the electric field around dielectric. Reference 4 and appendix 3 gives this as:

$$f_{res} = \frac{2e}{3c^3} \ddot{d} \quad \text{eqn. 11}$$

And so is totally negligible.

The proceeding analysis was on the basis that the momentum time average was not zero over a cycle. We shall delve into this more:

$$\langle p \rangle = \frac{1}{T} \int_T \left[\epsilon_0 \int_V (B \times E) dV \right] dt \quad \text{eqn. 12}$$

Consider this for a plane electromagnetic wave: $B=B_m \sin(\omega t)$ and $E=E_m \sin(\omega t)$. Thus $\langle p \rangle$ will be $B_m E_m / 2$ integrated over the volume and so an electromagnetic wave transfers momentum and this figure is independent of frequency. In our propulsion case the electric field is fixed to that of the electret's field, thus the integral is:

[†] We note that $f \rightarrow 0$ as $R \rightarrow \infty$ and $\theta \rightarrow 0$ by writing $S=R\theta$, $V \propto R$ (since it is a cylinder). We get f in terms $O(R\theta^2) \rightarrow 0$.

$$\langle p \rangle = \frac{1}{T} \int_T \left[\epsilon_0 E \int_V B_m \sin(\omega t) dV \right] dt$$

If we can cancel the second half of the cycle we will achieve a force proportional to the cycling frequency, otherwise the net force is zero. Put another way, since the force is the time derivative of the momentum this follows:

$$\langle f \rangle = \frac{1}{T} \int_T \left[\frac{d}{dt} \int_V \epsilon_0 (B \times E) dV \right] dt$$

$$\langle f \rangle = \frac{1}{T} \left[\int_V \epsilon_0 (B \times E) dV \right]_T - \frac{1}{T} \left[\int_V \epsilon_0 (B \times E) dV \right]_0 \quad \text{eqn. 13}$$

Thus for a cyclical process the average force is zero. We shall try to understand if there is a way around this but one thing is sure, for the cyclical process it is impossible. There is an impulse in the forward sense on switch-on and one in the reverse sense on switch-off. One might concoct some scheme of using two such devices and rotating them at right angles to the intended direction of movement so that their switch-off transients are harmlessly set in opposition (either compression or tension of the member connecting them). The killer argument against any such scheme is that as the field sweeps out its path in space that, momentum is conserved at each stage: when the field increases as it enters the region and then decreases as it leaves the region.

A more promising approach is to somehow cancel the E-field around the collapsing magnetic flux on the second half of the cycle. This attacks the problem head on; the force on the electret is due to the tangential electric field of the magnetic flux collapse. We cannot just cancel the E-field from the electret (discharging it, etc.) because the force from the Poynting vector will be the differential of the product of the B and E fields – returning to our earlier point, any form of cycling of these fields will lead to net zero force. We have to cancel the imposed electrical field by switching solenoid(s) whose field centres are internal to the craft at the same instant as the externally projected magnetic field changes. Being internal, the solenoids will only cause rotation about the system centre of mass; there is no effect on the translational forces and this rotation can be cancelled anyhow by a counter arrangement. Figure 7 is an elaboration of figure 6 and shows the cancellation scheme. The external field projecting solenoids are not shown for clarity.

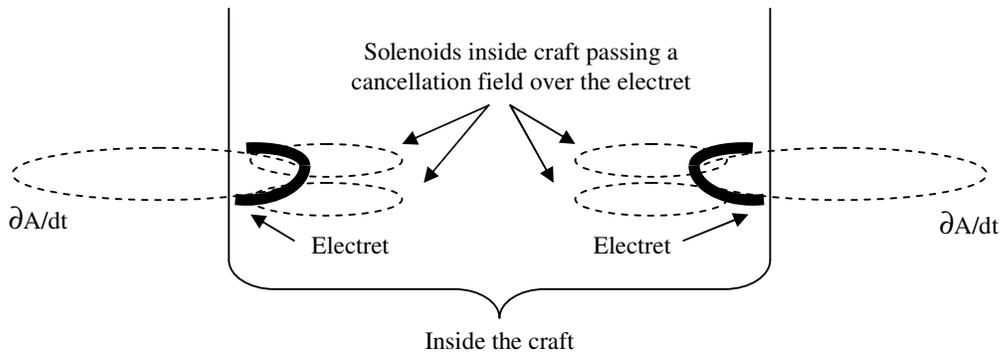


Figure 7

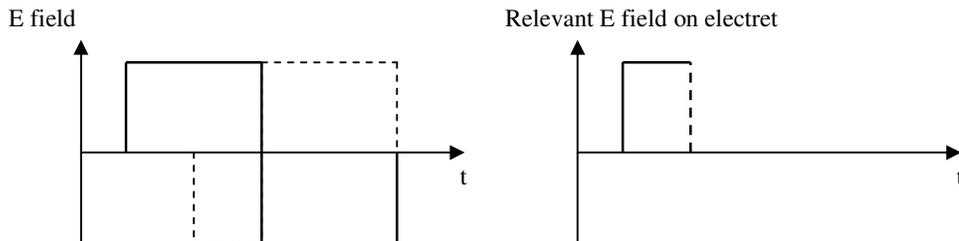


Figure 8

Figure 8 shows the E field from the collapse of the magnetic fields. The left-hand figure shows the field from the externally projecting solenoids and the dotted line from the inner cancellation solenoids. A staggered cancellation pulse will resolve into an E field at the electret in one sense only.

As to the momentum balance, that requires further work. It may go away with the burst of electromagnetic radiation as the external magnetic field collapses – though that is hard to argue, as momentum flux is independent of frequency whereas for this device it is linear with frequency (Appendix 4). As mentioned in the Graham and Lahoz paper⁵ it seems that the electromagnetic momentum in the static induction case ‘behaves like a superfluid’ circulating in opposition to the mechanical part. If the cancellation scheme does work (more experiment or theory) it is only vague to suggest that it is somehow dissipated on the zero state of the field and pushes ultimately against this huge mass-energy.

Work along the lines of a sub-ensemble of quantized harmonic oscillators, representing the macroscopic fields of the device, being randomised by zero-point interaction on field switch-off by the greater ensemble of the ground-state of the field and losing their momentum by a many bodied interaction, seems a promising candidate mechanism. It is similar to a high-speed jet of water colliding with the ocean.

Next we shall tentatively suggest some practical schemes that really should be expanded in another paper.

Early thoughts on Implementation and Engineering of Proposed Propulsion

At this stage the author has given only cursory thought to making a device, being more concerned with the thought experiment and the theory base. However a twin pronged attack by theory and experiment could be warranted if theory proves contentious. The magnitude of the forces can be small, especially more so with rough, early engineering technique. Suggestions are to use some kind of ‘field re-cycler’ or regenerator to efficiently cycle the electric and magnetic fields to avoid having to waste power on each cycle; something like an LCR (or cavity oscillator) circuit should prove possible. As regards materials, special high permittivity and permeability materials abound such as barium titanate and metallic glasses respectively and these will act to boost fields for those not with the budget for superconducting materials.

As always, early prototypes are rough and if the approach proves correct, considerable manpower donated to the project will no doubt lead to high performance units. The approach has the potential to generate large forces efficiently, certainly when compared to other electromagnetic propulsion schemes.

Conclusion

We have seen a method of propulsion using known phenomena and standard methods that are apparently permissible within the conservation laws barring something subtle along the lines of another hidden momentum trap (see appendix 4). One needs to be clear in one’s mind to the three momentum terms which are: radiative, static field energy related and induction fields (the forces on the electret). Respectively their momentum contributions are miniscule, cyclical (hence null) and linear in frequency and potentially large (appendix 4). Further enquiry may produce something beautiful, fundamental and profound or ‘just’ prosaic on the engineering plane.

Appendix 1 – Conservation of Momentum

This presentation is largely a précis of relevant sections in reference 2. It is placed here in conjunction with the rest of the text of this paper to form good immediate reference.

The greatest, most unifying law in physics is the Principle of Least Action. Rather than specify differential equations that describe the time evolution of the system per instant, the Least Action principle, for one form, comes up with a mathematical function based on position and velocity that describes the system *globally*. This is called the Lagrangian and is a function of kinetic and potential energy of the system. By mathematical transformations the two approaches (local and global) can be shown to be the same.

All this extra mathematics is not for pedantry though as the approach is extremely economic and brings all of the laws of physics into the same method. In this system certain laws or truths of Nature become readily apparent, regardless of the physical phenomena and a new depth of understanding becomes clear. The author considers this important in such a field like propulsion where people have trouble seeing the “wood from the trees”. Many claimed devices beggar belief and intuitively one thinks Nature just can’t be like that! The Least Action approach provides flesh for the bones of that intuition through conservation laws.

The action (units Js) is the time integral of the Lagrangian:

$$S = \int_{t_1}^{t_2} L(q, \dot{q}, t) dt \quad \text{eqn. A1.1}$$

The Principle of Least Action is invoked thus:

$$\delta S = \delta \int_{t_1}^{t_2} L(q, \dot{q}, t) dt = 0 \quad \text{eqn. A1.2}$$

The q and dq/dt can be expressed as so-called ‘generalised co-ordinates’ where they don’t have to literally be Cartesian co-ordinates but any convenient abstraction such as angle and radius for instance. Indeed the power of the approach can lead to further abstraction in describing anything that is subject to the variation (the field potential in the action for the field⁴).

Taking the first variation² we arrive at Lagrange’s equations:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0 \quad \text{eqn. A1.3}$$

Which is just really Newton’s 2nd Law ($ma-F=0$) but derived in a more fundamental and general way.

Homogeneity/Isotropy of Space-time and relativity principle leads to Inertia

Space being the same in all directions and times (we perform an experiment in one place at one time in one direction and obtain the same result another time, another place in another direction and find the same result) must mean that the Lagrangian (L) cannot explicitly contain a radius vector, t . Also we know that if we are travelling at speed in a steady fashion we cannot detect differences (this argument is non-Relativistic⁴) in our experiment thus L is a function of v^2 . Applying this to Lagrange’s equation A.3 and noting that q is \mathbf{r} and dq/dt is \mathbf{v} and $\partial L/\partial \mathbf{r} = 0$ since L doesn’t contain \mathbf{r} explicitly we find:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial v} \right) = 0$$

$$\Rightarrow v = \text{const} \quad \text{eqn. A1.4}$$

That is, in an inertial frame, motion goes with uniform velocity. This is the law of inertia and this follows from homogeneity/isotropy of space-time and relativity, which follow from experiment.

Homogeneity of Space leads to Conservation of Momentum

Let us consider the change to a system's (hence the sum) Lagrangian function by changing the radius vector \mathbf{r} to $\mathbf{r}+\boldsymbol{\epsilon}$, that is a translation in space:

$$\delta L = \sum_a \frac{\partial L}{\partial r_a} \delta r_a = \boldsymbol{\epsilon} \sum_a \frac{\partial L}{\partial r_a} \quad \text{eqn. A1.5}$$

As we know from experience there is no change in the experiment so $\delta L=0$ but $\boldsymbol{\epsilon}$ is non-zero so:

$$\begin{aligned} \Rightarrow \sum_a \frac{\partial L}{\partial r_a} &= 0 \\ \Rightarrow \frac{d}{dt} \sum_a \frac{\partial L}{\partial v_a} &= 0 \end{aligned}$$

The last step is obtained from eqn. A1.3. Thus we have a conserved quantity, called the momentum:

$$\bar{\mathbf{P}} = \sum_a \frac{\partial L}{\partial v_a} = \sum_a m_a v_a \quad \text{eqn. A1.6}$$

Incidentally conservation of angular momentum and energy proceed in much a similar manner². This is expressed in Relativistic Mechanics elegantly as the conservation of ‘‘momenergy’’^{4, 9}. The unity of space and time is exposed, its homogeneity and isotropy. Indeed just as we can translate an experiment in space and obtain the same results, we can transfer an experiment in time; as far as we know, the physical constants of the Universe don't change and we get the same result.

My point to this appendix: The Universe seems to be a bland canvass to work upon. We must tread carefully when we purport propulsion or inertia reducing devices (free energy devices too) and fundamentally understand what that means – we have given a preferred direction to space. This begs the question, by what mechanism or just *how* do you do that?

Appendix 2 – The Lagrangian of the Electromagnetic Field and Charged Particles

We shall derive equation 3 in the main text of this paper. Please consult relevant sections in reference 4 for development of the action when electromagnetic forces are at play.

Equation 27.7⁴ shows the action for field and particles to be:

$$S = -\sum \int mcds - \sum \int \frac{e}{c} A_k dx^k - \frac{1}{16\pi c} \int F_{ik} F^{ik} d\Omega \quad \text{eqn. A2.1}$$

We write this in 3-space form more amenable to engineers as:

$$S = -\sum \int mc^2 \sqrt{1 - \frac{v^2}{c^2}} dt + \sum \int \frac{e}{c} \vec{A} \cdot \vec{v} dt - \sum \int e\phi dt - \frac{1}{8\pi} \int (E^2 - H^2) dV dt \quad \text{eqn. A2.2}$$

Where

The four potential is $A^i=(\phi, \mathbf{A})$ in contravariant form

Where

ϕ is the scalar potential

\mathbf{A} is the vector potential

And

The four potential in co-variant form is $A_i=(\phi, -\mathbf{A})$

And

The electromagnetic field tensor is $F_{ik}=(\mathbf{E},\mathbf{H})$ or $F^{ik}=(-\mathbf{E},\mathbf{H})$

$$F_{ik} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -H_z & H_y \\ -E_y & H_z & 0 & -H_x \\ -E_z & -H_y & H_x & 0 \end{pmatrix} \quad F^{ik} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -H_z & H_y \\ E_y & H_z & 0 & -H_x \\ E_z & -H_y & H_x & 0 \end{pmatrix}$$

Equation A2.1 is made up of three parts: $S = S_m + S_{mf} + S_f$ that is a mechanical part, a field and mechanical part and a field part. If we are not concerned with electromagnetic interactions we ignore S_{mf} and S_f and obtain the familiar action and Lagrangian that solely describes classical mechanics:

$$L = -\sum mc^2 \sqrt{1 - \frac{v^2}{c^2}} \quad \text{eqn. A2.3}$$

When the field is *taken as a given* and we include the action S_{mf} which only includes the field potentials and charged entities of the system. The following Lagrangian is obtained:

$$L = -\sum mc^2 \sqrt{1 - \frac{v^2}{c^2}} + \sum \frac{e}{c} \vec{A} \cdot \vec{v} - \sum e\phi \quad \text{eqn. A2.4}$$

From the above we can derive the Lorentz force by use of Lagrange's equation⁴ A1.3.

$$\frac{d\vec{p}}{dt} = e\vec{E} + \frac{e}{c} \vec{v} \times \vec{H} \quad \text{eqn. A2.5}$$

In doing so the electric and magnetic fields have been derived from ultimately the four potential and thus two of Maxwell's equations (curl \mathbf{E} and div \mathbf{H}) are defined. Addition of the field action S_f allows the other two Maxwell equations to be derived⁴ and we see that A2.1 is an incredibly compact statement of Classical Electrodynamics.

When we don't take the field as given we must also include S_f . This allows the interaction of the particles to generate new fields and more profoundly, lets the field 'live' as an interacting system whose changes propagate at speed c . The field becomes very real and as we shall see has energy density and momentum.

In 3-space we have seen how we can relate the Lagrangian to the quantity called momentum and derive its conservation law. A similar procedure can be carried out[†] in 4-space Relativistically where the Energy-Momentum Tensor, T^{ik} , links the concepts of energy and momentum. Conservation of 'momenergy' is expressed as:

$$\frac{\partial T_i^k}{\partial x^k} = 0 \quad \text{eqn. A2.6}$$

The vanishing of a four divergence expresses a conservation law. Let us give an example with the conservation of charge:

$$\text{div } \vec{j} = -\frac{\partial \rho}{\partial t} \quad \text{eqn. A2.7}$$

[†] Though more involved and with the Lagrangian density for greater generality especially with regards to fields.

In 3-space form the rate of loss of charge in an infinitesimal cube is related to the flow of current (a vector) from the infinitesimal cube (so current density). It's just like emptying a tank. In Relativity time and space are unified and so the time part and spatial part ($\text{div} = (\partial/\partial x)\mathbf{i} + (\partial/\partial y)\mathbf{j} + (\partial/\partial z)\mathbf{k}$) of our charge/current concept^{2,3} become a "four-vector" $j_i = (\rho, \mathbf{j})$. The 4-form of the gradient operator is $(\partial/\partial t, -\nabla)$ and so when we take the dot product of this operator with the current four-vector we see that the conservation law is expressed readily in Relativity as:

$$\nabla_i j_i = 0$$

We draw further analogy: we can apply Gauss' Law³ to say a region of space with a charge expressed in differential form (div) and derive the total charge in a volume; or we could form a volume around a distribution of current expressed differentially and obtain the current into and out of the volume. Similarly we can apply a 4-dimensional version of Gauss' Law to our energy-momentum tensor integrating over a hyper-surface and find the flow of momentum to and from a volume in space. We obtain the momentum from the energy-momentum tensor thus:

$$P^i = \frac{1}{c} \int T^{ik} dS_k \quad \text{eqn. A2.8}$$

Thus we relate the Lagrangian to a Lagrangian density (i.e. dealing with infinitesimal volumes not macroscopic objects) to the energy-momentum tensor and to the momentum to and from a volume. Now we have the means to work out momentum concepts on something as nebulous as a field.

As an aside, explaining the concept and need of a tensor is beyond the scope of this appendix^{3,4} but a tensor, which is just like a matrix, allows us to define an object independent of a co-ordinate system. We find 'we need more numbers' to describe each point of our object or field but by a process akin to matrix multiplication, we can 'contract'⁴ the description down to a normal vector when we impose a co-ordinate. Expressing laws independent of a co-ordinate system is called making laws co-variant and ultimately is the goal of Relativity i.e. Physics.

We list the components of the energy-momentum tensor, which describes the flow of energy and momentum to a volume:

$$T^{ik} = \begin{bmatrix} W & S_x/c & S_y/c & S_z/c \\ S_x/c & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ S_y/c & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz} \\ S_z/c & -\sigma_{zx} & -\sigma_{zy} & -\sigma_{zz} \end{bmatrix} \quad \text{eqn. A2.9}$$

Where

- $T^{00} = W$ is the energy density in the volume
- $(1/c)T^{0\alpha} = g$ is the momentum density (via A2.8, the spatial components of P^i is momentum)
- $cT^{0\alpha} = S$ is the energy flow per unit volume per unit time across the volume
(via A2.8 the vector formed contracting along T^{0k} is energy related,
if $T^{00} = W$ is energy density then spatial components are flow of energy)
- $\sigma_{\alpha\beta}$ is the Maxwell Stress Tensor

If T is symmetrical (and can always be made so⁴) $T^{0\alpha} = T^{\alpha 0}$ then

$$\vec{S} = \vec{g}c^2 \quad \text{eqn. A2.10}$$

Equation A2.10 is a very important equation of Relativity but can just be thought of as $E=mc^2$ with a flow of energy and a flow of mass hence momentum.

Without derivation⁴ (from the action of the field S_f) we quote the energy-momentum tensor for the electromagnetic field relating it to the electromagnetic field tensor:

$$T^{ik} = \frac{1}{4\pi} \left(-F^{il} F_l^k + \frac{1}{4} g^{ik} F_{lm} F^{lm} \right) \quad \text{eqn. A2.11}$$

We are then able to relate components of the energy momentum tensor to the electric and magnetic fields. Thus:

$$T_{00} = W = \frac{E^2 + H^2}{8\pi}$$

$$cT^{0\alpha} = \epsilon_0 \mu_0 \vec{E} \times \vec{H} \quad \text{Poynting Vector}$$

$$\sigma_{\alpha\beta} = \text{Maxwell Stress Tensor}$$

Thus when we compute the momentum via A2.8 and differentiate with respect to time we obtain:

$$\rho_m \dot{v} - \nabla \cdot T + \frac{\partial}{\partial t} \epsilon_0 \mu_0 \mathbf{E} \times \mathbf{H} = 0$$

When we include S_{mf} and the force from action of the field and mechanical part we obtain the Lorentz force. In total then for $S_m + S_{mf} + S_f$ we obtain equation 3 in the main text.

Appendix 3 – Lagrangian of the Field to Third Order and Radiation Resistance

In the previous appendix we showed how the action for the field must be included in the total action to get the correct electrodynamics of charges subject to the field. The underlying reason for this is that fields transmit their information at the speed of light – it is not instantaneous, there is no mystical action at a distance as in Newtonian mechanics.

Rather than use the full might of the field action term S_f we can expand the field terms in S_{mf} by use of retarded potentials and approximate:

$$\phi = \int \frac{\rho_{t-R/c}}{R} dV \quad \vec{A} = \frac{1}{c} \int \frac{\vec{j}_{t-R/c}}{R} dV \quad \text{eqn. A3.1}$$

That is, we use the fields not at ‘this’ instant from the charges generating the field to the charge being influenced but earlier to allow travel time. We do not go into full details here⁴ but essentially what happens is that we expand the potentials in a Taylor series of powers of R/c with the proviso that the charge distribution doesn’t change significantly in the time R/c to make the approximation work.

When we expand to the third order important interesting effects of the electromagnetic field interaction with particles become apparent. Reference 4 gives these potentials as:

$$\phi^{(3)} = -\frac{1}{6c^3} \frac{\partial^3}{\partial t^3} \int R^3 \rho dV \quad \vec{A}^{(2)} = -\frac{1}{c^2} \int \vec{j} dV \quad \text{eqn. A3.2}$$

We note that we only go to terms of second order in the vector potential because in expression A2.2 that it is multiplied by $1/c$ anyway. We summarise the derivation in reference 4: a gauge transformation is made to eliminate the third order scalar term; this doesn’t matter, as the physics is the same leaving only the second order vector potential term.

$$\vec{A}^{(2)} = -\frac{2}{3c^2} \sum e\dot{v} \quad \text{eqn. A3.3}$$

The integration over a charge distribution has been reduced to a sum over individual charges. This second order term generates no magnetic field ($\mathbf{H}=\text{curl } \mathbf{A}$) as it does not contain spatial co-ordinates explicitly but it does generate an electric field ($\mathbf{E}=-\frac{1}{c}\partial\mathbf{A}/\partial t$) and hence force on a charge:

$$f = -\frac{2e}{3c^3}\ddot{d} \quad \text{eqn. A3.4}$$

Appendix 4 – Answers to an internal critique of an earlier draft of the paper

Introduction

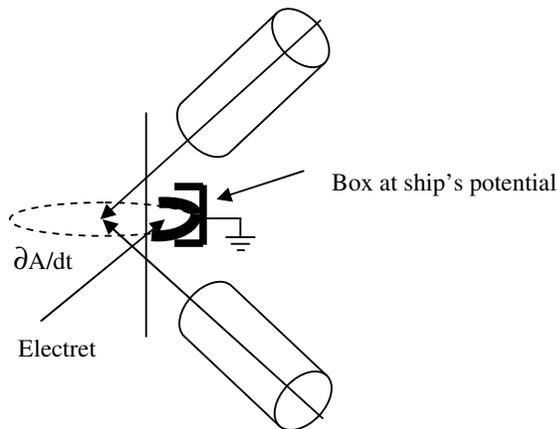
In the said paper a device was discussed whose purpose was to allow a craft to accelerate by a means of propellant-less electromagnetic propulsion. Simple linear translation is forbidden by “hidden momentum” however Nature does allow angular translation as exemplified in the Feynman disk. It was the task of the paper to convert these angular motions into linear motion. Now there is no doubt that the arrangement of disks will lead to linear motion however the effect is symmetric with the craft returning to its starting point after the cycle “field on, field off”. Thus a scheme was contrived to eliminate the second half of the cycle to allow the craft to accelerate by cancelling the effect of the changing magnetic field on the electret (‘decoupling’ the electromagnetic aspects of the system from mechanical part). If such a scheme is possible at all one surely must ask, what happens to the compensating momentum, how is it carried away? Just by theoretical reasoning it is important to ask this question as fundamental arguments kill all engineering contrivances.

Three ideas become apparent if net motion is at all possible or not:

- The belief that some strange hidden momentum effect prevents the device from operating.
- The belief that a ‘chunk’ of electromagnetic mass in the fields set up somehow heads rearwards and is discarded.
- The belief that it is a photon rocket with electromagnetic wave energy heading rearwards.
- The belief that the field cancellation scheme can’t work, as there is no distinction between rotations on different radii. The cancelling solenoid will just return the craft to its initial position.

Point One

Hidden momentum arises from considerations regarding the relativistic fluid of electrons constituting the current in the solenoid^{7,8}. Subject to the field from the electret, the charge carriers experience a change in their mass from the potential energy they achieve in the field. This argument is only relevant in the static case which this isn’t but to further dispel it consider the diagram below, which is a variant of figure 6 in the paper:



The electret is shielded by a box open at one end to the outside of the craft and the external field. The box is held at ship’s potential and thus the hidden momentum argument is not valid.

Point Two

Consulting figure 6 of the device paper we see that electromagnetic energy is projected outwards from the craft but this is not the net mechanism by which the craft is propelled forwards. The energy is recalled to the craft at the end of the cycle. The net momentum from this setting up and removal of the external field is zero. Sure enough the flow of energy can be represented by a Poynting vector of the propagation of the changing E and B fields but it is distinct from the propulsive effect of the changing E field on the electret. One displays momentum transfer by acting on the solenoid (the radiation field, see point three), the other on the electret (the induction field). Thus we have two momentum density terms on a half cycle:

$$g_{Solenoid} = \epsilon_0 \frac{\partial}{\partial t} (B^2)$$

$$g_{Electret} = \epsilon_0 (B \times E)$$

For the first expression it is easier to divide the known final expression for the energy density by c^2 than compute the time varying Poynting expression. So clearly the argument that the propulsion comes from ejection of field energy of the solenoid is not valid.

Point Three

A photon rocket is a puny thing; the majority of the energy developed goes into the rearward beam and not the kinetic energy of the craft. *In no way is it implied by the Feynman disk that radiation emanates and provides all the momentum balance.* In appendix 3 of the paper we see that radiation effects are put off to the second order in the vector potential and the force is of the order:

$$\vec{A}^{(2)} = -\frac{2}{3c^2} \sum e \dot{v}$$

The E field from the above expression is then $\mathbf{E} = -(1/c) \partial \mathbf{A} / \partial t$. The force on charged entities constituting the solenoid current from the radiation field is thus of the order of $1/c^3$ down on the forces generated on the electret by the induction fields:

$$\epsilon_0 \int_V \frac{\partial}{\partial t} (B \times E) dV$$

In fact the radiation is so miniscule that we didn't even calculate it and can say with very high accuracy that all the energy developed goes into the kinetic energy of the craft, much as though we were pushing against a very large mass.

We draw attention to equation 12 and the paragraphs following it on page 8 of the paper rather than reproduce it here.

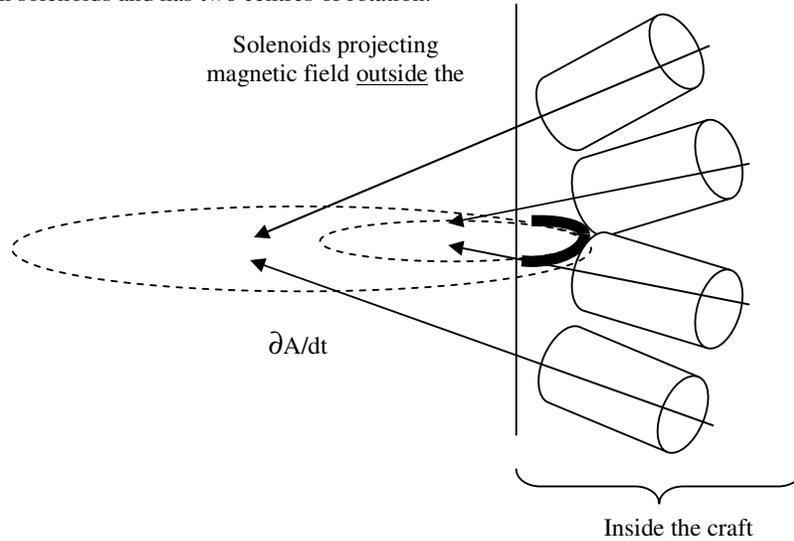
To stress the point again that we are dealing with two different effects, the momentum exchange from the radiation field and the force on the electret from the induction field (point two), note that for a photon rocket momentum flux is independent of frequency, however for the device it is linear in frequency.

It seems fanciful to say that the device is pushing against the ground state but we have shown that something is seriously amiss.

Point Four

This point is related to the fact that one can translate through space by rotation because rotations do not commute. Thus we shall show that point four is not valid.

Consider a simplified device. It is not designed to accelerate but comes to rest in the original frame and translated in a one-shot operation. There is no field cancellation scheme. It consists of two sets of projection solenoids and has two centres of rotation:



Non-commutation of Rotations

We can represent the process of rotation in a Cartesian system by matrices. Sequences of rotations then become a string of matrix multiplication operations. Each operation can be considered to transform to a new co-ordinate system the result of the multiplication is the co-ordinates as viewed from the old system.

A rotation by angle θ is given by:

$$R_{\theta} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

It is easy to verify that $R_{\theta} R_{\theta}^{-1} = I$, that is, a rotation and its reverse rotation give back the original position vector.

Let us model the simplified device by a sequence of operations taken non-commutatively as the rotation of a position vector $(D \ 0)$ by the 1st angle about a radius D , a shift in the co-ordinates and rotation to affect a rotation by the 2nd angle about a radius d , then rotate by minus the 1st angle and then rotate by minus the 2nd angle. Thus:

$$R_{\beta}^{-1} \left[R_{\alpha}^{-1} \left[R_{\beta} \left[R_{\alpha} \begin{pmatrix} D \\ 0 \end{pmatrix} - \begin{pmatrix} D-d \\ 0 \end{pmatrix} \right] + \begin{pmatrix} D-d \\ 0 \end{pmatrix} \right] - \begin{pmatrix} D-d \\ 0 \end{pmatrix} \right] + \begin{pmatrix} D-d \\ 0 \end{pmatrix}$$

After these operations the position vector is:

$$\begin{pmatrix} 2D - d - (D - d)(\cos \alpha + \cos \beta - \cos(\alpha + \beta)) \\ - (D - d)(\sin \alpha + \sin \beta - \sin(\alpha + \beta)) \end{pmatrix}$$

We can see that if the radii are different and the angles non-zero the final position vector is not equal to the start.

Conclusion to answer of the critique

It has been shown that there is two distinct momentum transfer terms, one for the external field and the other for the electret. However the net momentum exchange from setting up of the external field is zero. As regards the mechanism for the propulsion by action of the electret it has been made clear that there are two mechanisms of momentum transfer; one mechanism is by radiation fields, the other is by induction fields. The radiation fields are miniscule, of order $1/c^3$ down on the induction fields. Added to this too, radiation can only ever produce a force independent of frequency but the device is linear in frequency. The device is thus not a photon rocket.

Objections to the field cancellation scheme have, we hope, been agreeably dispelled. It is clear that one can translate through space by two rotations on different radii. The implication of this being that the cancellation solenoid doesn't merely return the craft to its original position.

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