A Unique Inefficient Equilibrium in a Bargaining with Perfect Information

Taiji Furusawa*
Hitotsubashi University

Quan Wen†
Vanderbilt University

June 2003

Abstract

This paper studies a simple bargaining model of perfect information, where players' interim disagreement payoffs decrease over time and the proposer may postpone making an offer without losing the right to propose. The model has a generically unique subgame perfect equilibrium. The equilibrium outcome exhibits inefficient delay if interim disagreement payoffs drop sharply in the near future.

JEL Classification: C72, C73, C78
Keywords: Negotiation, disagreement payoffs, delay.

*Faculty of Economics, Hitotsubashi University, Kunitachi, Tokyo 186-8601 Japan. Email: furusawa@econ.hit-u.ac.jp

†Department of Economics, Vanderbilt University, VU Station B # 351819, 2301 Vanderbilt Place, Nashville, TN 37235-1819, U.S.A. Email: quan.wen@vanderbilt.edu

1 Introduction

Since the seminal work of Rubinstein (1982), it has been well known that many bargaining models, even with complete information, often possess inefficient perfect equilibrium outcomes.[1] Inefficient perfect equilibrium outcomes often result from the multiplicity of equilibrium. They are supported by trigger-type strategies, as in repeated games. Any player who deviates from an inefficient outcome would be punished by his worst perfect equilibrium in the continuation game, which is possible in the existence of multiple subgame perfect equilibrium outcomes.

This paper provides a simple bilateral bargaining model of perfect information, which has a unique and inefficient perfect equilibrium. In the model, players' interim disagreement payoffs are deterministic and decreasing over time, and the proposer in any period may postpone making an offer without losing the right to propose in the following period. To demonstrate our key point, assume for simplicity that disagreement payoffs are positive for some periods and then drop to 0 in the rest of the game. In a contract bargaining between a firm and a union, for example, disagreement payoffs drop dramatically when the firm runs out of inventories and the union runs out of funds to finance the strike. We show that in such a situation there exists a generically unique perfect equilibrium that may feature a delayed agreement until the period where their disagreement payoffs become zero. Unlike many bargaining models of complete information where inefficient equilibria are supported by the existence of multiple perfect equilibria, the model studied here admits a unique and inefficient equilibrium.

The evolution of the first-mover advantage, or equivalently the proposer's advantage, plays a very important role in the enforcing mechanism. The higher the current disagreement payoffs, the higher the responder's reservation value and the higher the proposer's opportunity cost of making an offer. Given that disagreement payoffs decrease over time, the

[1]For example, see Chatterjee and Samuelson (1990), Perry and Reny (1993), Sákovics (1993), Haller and Holden (1990), Fernandez and Glazer (1991), Avery and Zemsky (1994), Busch and Wen (1995), Houba (1997), Busch, Shi and Wen (1998), Kambe (1999), Manzini (1999), and Furusawa and Wen (2003).

proposer may have an incentive to postpone making an offer until the first-mover advantage becomes sufficiently large. Indeed, the proposer will not make any offer until disagreement payoffs drop sharply if he knows it happens in the near future. The proposer's choice of when to realize the first-mover advantage is the main engine of delayed agreement in the unique equilibrium.

2 The Model

Two players bargain about how to share a periodic value of 1 over infinitely many periods. In any period before reaching an agreement, one player, called the proposer, may either make an offer or choose not to make any offer. If the proposer chooses not to make any offer then two players will simply collect their common interim disagreement payoff, denoted by d_t for period t, and the same process will repeat in the following period. In other words, if the proposer decides to postpone making an offer by not making any offer in a period, he will not lose his right to propose (or his first-mover advantage) in the following period. If the proposer makes an offer, on the other hand, the other player, called the responder, may either accept or reject the standing offer. If the responder accepts the offer, the bargaining will be over and two players will receive their agreed-upon shares forever. If the responder rejects the offer, on the other hand, two players will collect their common disagreement payoff and continue to bargain in the following period where they switch their roles in the bargaining so that the responder in the current period becomes the proposer in the following period and vice versa. Two players have a common discount factor $\delta \in (0, 1)$ per bargaining period. By convention, we call the proposer in the first period player 1. For simplicity, we consider the case where players' common disagreement payoffs decrease over time such that

$$d_t = \begin{cases} d & \text{if } t \leq T \\ 0 & \text{if } t \geq T + 1. \end{cases}$$

Let us assume that $d \in (0,\ 1/2)$, so any delayed agreement is inefficient. Notice that this game is of perfect information.

Histories and strategies are defined as usual. Let a_i denote player i's share. Then a finite outcome consists of the agreement (a_1, a_2) and the period t' in which the agreement is reached. Player i's average discounted payoff in such a finite outcome is a_i if $t' = 1$, and

$$(1-\delta)\sum_{t=1}^{t'-1} \delta^{t-1} d_t + \delta^{t'-1} a_i \quad \text{if } t' \geq 2.$$

In the case of infinite disagreement, each player receives d_t in any period t so that the common average discounted payoff equals

$$(1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} d_t = (1-\delta^T)d.$$

3 The Subgame Perfect Equilibrium

First we show that despite of the proposer's ability to postpone making an offer, the Rubinstein solution is a unique perfect equilibrium in any subgame since period $T+1$ in which the disagreement payoff becomes 0. Owing to the symmetry, we state and prove most of our results in terms of the proposer and responder's strategies.

Proposition 1 *In any period $t \geq T+1$, there is a unique perfect equilibrium in which the proposer demands $1/(1+\delta)$ and the responder accepts the offer if and only if the proposer demands no more than $1/(1+\delta)$.*

Proof: It is straightforward that the prescribed strategy profile is subgame perfect. Here, we prove the uniqueness. Let M and m be the supremum and infimum of the proposer's equilibrium payoffs, respectively. If the proposer does not make an offer, his equilibrium payoff should not be higher than δM nor lower than δm. If the proposer makes an offer then the responder will be the proposer in the following period if the offer is rejected. Therefore, the responder will not accept any offer that is less than δm, and will certainly accept any offer that is greater than δM. The proposer chooses a better alternative so that

$$\max\{1-\delta M,\ \delta m\} \ \leq\ m\ \leq\ M\ \leq\ \max\{1-\delta m,\ \delta M\}. \tag{1}$$

Suppose $1-\delta m \leq \delta M$. Then the last inequality in (1) becomes $M \leq \delta M$, which implies $m = M = 0$. This contradicts to our supposition of $1 - \delta m \leq \delta M$. Therefore, we must have $1 - \delta m > \delta M$, or equivalently $1 - \delta M > \delta m$. It then follows from (1) that $1 - \delta M \leq m$ and $M \leq 1 - \delta m$, which in turn yields $m = M = 1/(1+\delta)$. Lastly, notice that the prescribed strategy profile is the unique perfect equilibrium that yields payoff $1/(1+\delta)$ to the proposer, which concludes this proof. **Q.E.D.**

Given the unique equilibrium outcome in period $T+1$ by Proposition 1, we now derive a unique perfect equilibrium in the model by backward induction. There are totally $T+1-t$ number of periods where the disagreement payoff remains to be $d \in (0, 1/2)$ between period t and period T. The next proposition demonstrates that under certain conditions, the proposer will not make any offer until period $T+1$. If a player receives d for $T+1-t$ periods and $1/(1+\delta)$ since period $T+1$ onward, the present value of his average payoff will be

$$v(t) \equiv (1 - \delta^{T+1-t})d + \frac{\delta^{T+1-t}}{1+\delta}. \tag{2}$$

Since $d < 1/2 < 1/(1+\delta)$, the function v is strictly increasing. Now, we define t^* by $t^* = 0$ if $v(t) > 1/2$ for all $1 \leq t \leq T$ and otherwise

$$t^* = \max\left\{t \leq T \mid v(t) \leq \frac{1}{2}\right\}. \tag{3}$$

Then, $v(t) > 1/2$ for any $t^*+1 \leq t \leq T$ if $t^* \leq T-1$. Now, we are ready to state Proposition 2.

Proposition 2 *If $t^* \leq T-1$, the proposer will not make any offer in any period t such that $t^*+1 \leq t \leq T$,.*

Proof: It follows from Proposition 1 that if the proposer does not make any offer in period T, he will be the proposer in period $T+1$ with a payoff of $1/(1+\delta)$, which gives the proposer the average payoff $v(T)$ in period T. If the proposer makes an offer, on the other hand, the responder will reject any offer that gives him less than $v(T)$ since he would receive d in period T and become the proposer in period $T+1$ by rejecting the standing offer. Thus the proposer obtains no more than $1-v(T)$ by making an offer in period T. Since $t^*+1 \leq T$ and v is increasing,

$$v(T) \geq v(t^*+1) > \frac{1}{2},$$

which implies

$$1-v(T) < v(T).$$

Therefore, the proposer will not make any offer in period T.

Next, we consider period t such that $t^* < t < T$, and suppose the proposer will not make any offer in every period between $t+1$ and T. If the proposer does not make any offer in period t then he will not make any offer until period $T+1$, which yields $v(t)$ to the proposer. If the proposer makes an offer in period t then the responder will reject any offer that gives him less than $v(t)$, which in turn leaves no more than $1-v(t)$ to the proposer. Again since v is increasing, we have

$$v(t) \geq v(t^*+1) > \frac{1}{2},$$

which implies

$$1-v(t) < v(t).$$

Therefore, the proposer will not make any offer in period t. The uniqueness of the perfect equilibrium in period $T+1$ guarantees the uniqueness of perfect equilibrium actions in period t for $t^*+1 \leq t \leq T$. Backward induction then concludes the proof. **Q.E.D.**

Proposition 2 implies that if $t^*=0$, player 1 will not make any offer until period $T+1$ in which two players will agree on the Rubinstein outcome. What happens if $t^* \geq 1$? The proof of Proposition 2 also implies that the proposer makes an acceptable offer in period t^*. In a more general framework, it is possible that the proposer prefer not to make any offer in some periods before period t^*. In the current setup of the model, however, the proposer always prefers making an acceptable offer in any period before t^*, as Proposition 3 shows. Nevertheless, the agreement in the subgame perfect equilibrium derived from backward induction is generally different from the Rubinstein solution corresponding to the same sequence of disagreement payoffs $\{d_t\}_{t=1}^{\infty}$.

Proposition 3 *In period $t \leq t^*$, the proposer makes an offer in which he demands*

$$\frac{(1+\delta^{t^*-t})[1-(1-\delta)d]}{1+\delta} - \delta^{t^*-t}\left[1-v\left(t^*\right)\right] \quad \text{if } t^*-t \text{ is odd,} \tag{4}$$

$$\frac{(1-\delta^{t^*-t})[1-(1-\delta)d]}{1+\delta} + \delta^{t^*-t}\left[1-v\left(t^*\right)\right] \quad \text{if } t^*-t \text{ is even.} \tag{5}$$

The responder accepts the offer if and only if the proposer demands less than whichever of the relevant values in (4) and (5).

Proof: We prove Proposition 3 by mathematical induction. The proof of Proposition 2 shows that the proposer makes an acceptable offer in period t^*. We first show that the proposer in period t^*-1 also makes an acceptable offer. Then we show that given the proposers in periods $t-1$ and t for $t \leq t^*$ make acceptable offers, the proposer in period $t-2$ also makes an acceptable offer, which will complete mathematical induction.

Let w_t denote the proposer's demand in period t. Proposition 1 shows that $w_{T+1} = 1/(1+\delta)$. Since the definition of t^* implies $v\left(t^*+1\right) > 1/2$, we obtain

$$\begin{aligned} v(t^*) &> \frac{1}{2} - \left[v(t^*+1) - v(t^*)\right] \\ &= \frac{1}{2} - \delta^{T-t^*}(1-\delta)(w_{T+1}-d). \end{aligned}$$

Since the proposer in period t^* makes an acceptable offer, we then have

$$w_{t^*} = 1 - v(t^*) < \frac{1}{2} + \delta^{T-t^*}(1-\delta)(w_{T+1}-d).$$

Together with the fact that the definition of t^* implies $v(t^*) \leq 1/2$, we obtain

$$\begin{aligned} (1-\delta)d + \delta w_{t^*} &< (1-\delta)d + \delta\left[\frac{1}{2} + \delta^{T-t^*}(1-\delta)(w_{T+1}-d)\right] \\ &= \frac{\delta}{2} + (1-\delta)v(t^*) \leq \frac{1}{2}. \end{aligned} \tag{6}$$

Therefore, the proposer in period t^*-1 makes an acceptable offer.

Now, we suppose that the proposers in periods $t-1$ and t make acceptable offers, and show that the proposer in period $t-2$ also makes an acceptable offer. Since the proposers in periods t and $t-1$ make acceptable offers, it must be the case that $w_t > 1/2$ and

$$w_{t-1} = 1 - (1-\delta)d - \delta w_t \leq 1 - (1-\delta)d - \frac{\delta}{2}.$$

Consequently, we obtain

$$\begin{aligned}(1-\delta)d + \delta w_{t-1} &\leq (1-\delta)d + \delta\left[1 - (1-\delta)d - \frac{\delta}{2}\right] \\ &= \frac{\delta}{2} + (1-\delta)\left[(1-\delta)d + \frac{\delta}{2}\right] \\ &< \frac{\delta}{2} + \frac{1-\delta}{2} = \frac{1}{2}.\end{aligned}$$

Therefore, the proposer makes an acceptable offer in period $t-2$. By induction, the proposer makes an acceptable offer in every period before and in period t^*. The acceptable demands $\{w_t\}_{t=1}^{t^*}$ follow the law of motion that the responder in period t is indifferent between accepting and rejecting the demand w_t. Since the responder would receive d in period t and w_{t+1} from the period $t+1$ if he rejects the standing offer, we have

$$1 - w_t = (1-\delta)d + \delta w_{t+1}, \quad \text{with} \quad w_{t^*} = 1 - v(t^*).$$

The above dynamic system yields the values of w_t as given by (4) and (5). **Q.E.D.**

Propositions 1, 2, and 3 assert that the equilibrium may not be stationary, having three possible phases. Proposition 1 predicts a unique perfect equilibrium outcome as in the Rubinstein model, after the disagreement payoffs drop to 0 in period $T+1$. Proposition 2 states that during periods between t^*+1 and T, the proposer would choose not to make any offer. If $t^* = 0$, therefore, the equilibrium exhibits delay in agreement for T periods.

If $t^* \geq 1$, on the other hand, the proposer would make the acceptable offer in t^* and in every period before then. In the measure zero event that $v(t^*) = 1/2$, the proposer in period t^* is indifferent between making the acceptable offer and making no offer at all. Then there will be multiple perfect equilibria such that the proposer adopts a mixed strategy on these two actions in period t^*. Equilibrium actions in other periods are the same as those described above. In particular, the equilibrium actions before period t^* are not affected by this multiplicity since the proposer's continuation payoff at the beginning of t^* is $1/2$ in any such mixed strategy equilibrium. Proposition 4 summarizes the subgame perfect equilibrium in the model.

Proposition 4 *There generically exists a unique subgame perfect equilibrium:*

(i) If $t^* = 0$, player 1 does not make any offer until period $T + 1$ in which he demands $1/(1 + \delta)$;

(ii) If $t^* > 0$, player 1 makes the acceptable offer (either (4) or (5)) in the first period.

Figure 1 depicts how the equilibrium outcomes vary with the parameters (δ, d). Since v is an increasing function of t and $v(t)$ increases as d or δ rises, t^* is weakly decreasing with respect to d and δ. Therefore, the higher d and δ, the more likely is player 1 to keep a silence until he makes the offer in period $T + 1$. Figure 1 depicts three representative regions of (δ, d) where $t^* = T$, $T - 1$, and $T - 2$, respectively.

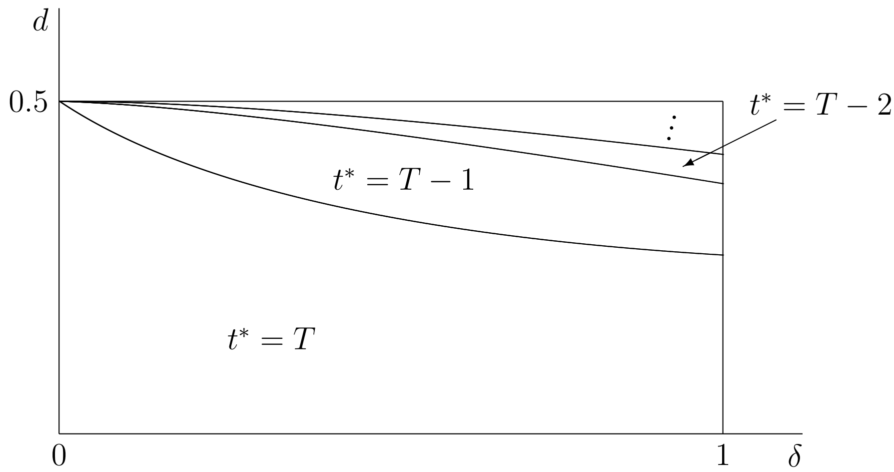

Fig. 1 Representative regions where $t^* = T$, $T - 1$ and $T - 2$

For example, if (δ, d) lies in the region for $t^* = T - 2$, which is above the locus of (δ, d) that satisfies $v(T - 1) = 1/2$ and below the one that satisfies $v(T - 2) = 1/2$, the proposer will not make any offer in periods $T-1$ and T. Therefore, if $T=2$, for example, the proposer postpones making an offer until period 3.

4 Efficiency Loss in Equilibrium

We have shown that if $t^*=0$, the equilibrium exhibits inefficient delay. Given $d \in (0,1/2)$, efficiency calls for an immediate agreement in the first period. Therefore, the efficiency loss when $t^*=0$ is

$$1-[2d(1-\delta^T)+\delta^T]=(1-\delta^T)(1-2d).$$

When $t^*>0$, on the other hand, the equilibrium is efficient. Thus, the equilibrium efficiency loss is given by

$$L(\delta,d,T) \equiv \begin{cases} (1-\delta^T)(1-2d) & \text{if } t^*=0 \\ 0 & \text{if } t^*>0. \end{cases} \tag{7}$$

The loss function $L(\delta,d,T)$ is decreasing in δ and d, and increasing in T whenever $t^*=0$. As δ or d decreases, or as T increases beyond a threshold, t^* becomes positive and the loss disappears. It follows from the definition of t^* that $t^*=0$ if and only if

$$v(1)=(1-\delta^T)d+\frac{\delta^T}{1+\delta}>\frac{1}{2}. \tag{8}$$

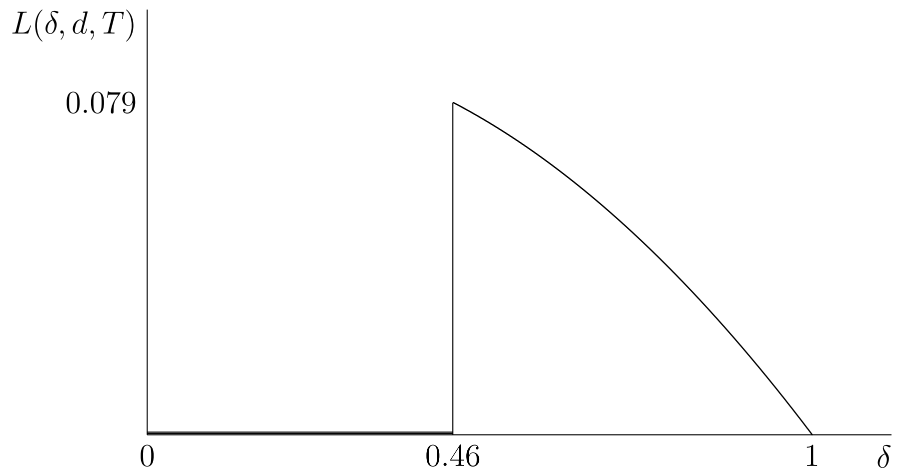

Fig. 2 $L(\delta,d,T)$ when $d=0.45$ and $T=2$

Figure 2 depicts the efficiency loss with respect to δ when $d = 0.45$ and $T = 2$. As Figure 2 illustrates, when $\delta < 0.46$, the proposer is too impatient to wait until period 3 to materialize his large first-mover advantage; hence there is no efficiency loss as two players reach an agreement in the first period. When $\delta \geq 0.46$, the subgame perfect equilibrium becomes inefficient since player 1 will delay making an offer until period 3. As δ further increases, efficiency loss decreases since players put more weights on their future payoffs. The efficiency loss $L(\delta, d, T)$ depends on d and T in similar fashions.

In this paper, we have provided a simple bargaining model with perfect information to demonstrate the possibility of a unique, inefficient, subgame perfect equilibrium. The mechanism to support such a unique, inefficient, equilibrium does not rely on the existence of multiple equilibrium outcomes, which enables us to construct a punishment scheme. As we have shown, the first-mover advantage, or the proposer's advantage, plays an essential role in our analysis. The smaller the disagreement payoffs, the larger is the first-mover advantage. Therefore, in the case where the disagreement payoffs sharply drop in the future, the current proposer has a large incentive to wait until that time to materialize his first-mover advantage. Indeed, he may do so if he is allowed to postpone making an offer without losing the right to propose.

With the basic idea illustrated in our simple model, one can easily extend our results to a more general model where players' disagreement payoffs vary deterministically over time. Consider, for example, the bargaining environment in which a certain number of periods of high disagreement payoffs and a certain number of periods of low disagreement payoffs cyclically emerge. Then our results suggest that the proposer may not make any offer in a phase of high disagreement payoff if the disagreement payoffs sharply drop in the near future.

References

Avery, C. and P. B. Zemsky (1994), "Money Burning and Multiple Equilibria in Bargaining," *Games and Economic Behavior,* **7**, 154–168.

Busch, L-A. and Q. Wen (1995), "Perfect Equilibria in a Negotiation Model," *Econometrica,* **63**, 545–565.

Busch, L-A., S. Shi and Q. Wen (1998), "Bargaining with Surplus Destruction." *Canadian Journal of Economics*, **31**, 915–932.

Chatterjee, K. and L. Samuelson (1990), "Perfect Equilibria in Simultaneous-Offers Bargaining," *International Journal of Game Theory*, **19**, 237–267.

Fernandez, R. and J. Glazer (1991), "Striking for a Bargain Between Two Completely Informed Agents," *American Economic Review,* **81**, 240–252.

Furusawa, T. and Q. Wen (2003), "Bargaining with Stochastic Disagreement Payoffs," forthcoming in *International Journal of Game Theory.*

Haller, H. and S. Holden (1990), "A Letter to the Editor on Wage Bargaining," *Journal of Economic Theory,* **52**, 232–236.

Houba, H. (1994), "The Policy Bargaining Model." *Journal of Mathematical Economics,* **28**, 1–27.

Jehiel, P. and B. Moldovanu (1995), "Negative Externalities May Cause Delay in Negotiation," *Econometrica,* **63**, 1321–1336.

Kambe, S. (1999), "When is There a Unique Equilibrium in Less Structured Bargaining?" *Japanese Economic Review*, **50**, 321–342.

Manzini, P. (1999), "Strategic Bargaining with Destructive Power," *Economics Letters*, **65**, 315–322.

Perry, M. and P. Reny (1993), "A Non-cooperative Bargaining Model with Strategically Timed Offers," *Journal of Economic Theory*, **59**, 50–77.

Rubinstein, A. (1982), "Perfect Equilibrium in a Bargaining Model," *Econometrica*, **50**, 97–109.

Sákovics, J. (1993), "Delay in Bargaining Games with Complete Information," *Journal of Economic Theory*, **59**, 78–95.