CAN INSURERS PAY FOR THE "BIG ONE"?
MEASURING THE CAPACITY OF AN INSURANCE MARKET TO RESPOND TO CATASTROPHIC LOSSES

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INTRODUCTION

Recent catastrophic events such as the Northridge earthquake and Hurricane Andrew each cost the insurance industry in excess of $10 billion. While most insured losses were paid, each event did cause some insolvencies. These events illustrate the potential stress facing insurance markets. Andrew, which cost the insurance industry about $15.5 billion[1], would have been much more severe had its path veered slightly to hit Miami. Moreover, scenarios constructed by the insurance industry suggest the feasibility of a $76 billion hurricane in Florida, a $21 billion Northeast hurricane, a $72 billion California earthquake and a $101 billion New Madrid earthquake.[2] At first glance, it might appear that the insurance industry would be available to pay for such mega catastrophes. The U.S. property liability insurance industry′s surplus, essentially the value of its equity, is somewhat over $200 billion. The surplus is potentially available to pay for losses which exceed the reserves (established for their payment from premiums). However, the reality would be different; depending on the distribution of damage and the spread of coverage, many insurers would become insolvent.[3] Technically, this problem should be solved by the state operated insolvency guarantees which re-allocate defaulted liabilities amongst solvent insurers. But these only operate within small limits[4] and even this burden would stretch the already strained resources of surviving insurers. Thus, the prospect of a mega catastrophe brings the real threat of widespread insurance failures and unpaid insurance claims. Moreover, surviving insurers would be so depleted of surplus, and thus over-levered, they would have to reduce the future sale of

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[1]Source Property Claims Services of the American Insurance Services Group.

[2]These figures relate only to the insured damage. The total damage would be higher. For hurricane losses a substantial portion of total losses is likely to be insured. However, for earthquake losses. many properties are not insured and others carry high deductibles. Thus, for earthquake losses the total societal loss could be multiples of this estimate. These figures were produced in a study by Risk Management Solutions (2.5.95) though similar "ballpark" figures are being produced in other studies.

[3]Insurers can spread their liabilities to other insurers through reinsurance. In principle, the effects of catastrophes can be spread through the worldwide reinsurance market. In practice the available capacity of reinsurers is very limited. For example, it is unusual for any insurer to obtain more than a few $100m in catastrophe reinsurance, and a recent estimate puts the whole worldwide available reinsurance capacity at only $7.2 billion. (source H. N. Haag, Partner Reinsurance Company, presented at seminar Funding Catastrophe Protection, New York 11.1.95).

[4] Limits per loss and limits on the amount that can be assessed to any insurer.

all types of property liability insurance causing severe availability problems.[5]

These scenarios have led both state and federal governments to contemplate legislative solutions involving the government as a reinsurer and directly enlisting capital markets as providers of catastrophe capital. Both Florida and California have such proposals and the Natural Disaster Protection Act was introduced in Congress in 1993 with similar provisions. Moreover, the vulnerability of insurance markets had led to financial market innovations such as the catastrophe futures and options now traded on the CBOT. New instruments have appeared such as "Act of God Bonds" in which borrowers contract for some degree of debt forgiveness in the event of a predefined catastrophe. Another is the CatEPut in which re-capitalization can be achieved after a catastrophe by the firm exercising a put option purchased on its own stock. Also, in the absence of adequate reinsurance, insurers have sometimes swapped their catastrophe exposures.

Our purpose here is to derive a measure of the capacity of the insurance industry to respond to catastrophic events. We do not model specific events (a force 5 hurricane hitting Miami; or an 8.2. earthquake in San Francisco). Given appropriate technical (weather, seismic, etc) data, plus descriptions of insured properties for each insurer, one can estimate an insurer response for any given event. Such scenario analysis is now becoming feasible. However, there is a very large number (approaching infinite) of potential catastrophe scenarios and the data demands are enormous. Moreover, while such scenarios are valuable for local planning, they provide too much detail for assessing the efficiency of the insurance market in spreading its risk. Rather, we seek a more general response function. We estimate the distributional characteristics of catastrophic losses, and allocate such losses to individual insurers by use of correlations and financial data. The result is a function that defines the estimated deliverable insurance payments conditional on any given size of aggregate catastrophic loss. By default, it also estimates the liabilities that will be lost through insurer insolvencies.

Such a measure rests on two broad components; size and diversification (how much surplus is available and how effectively the riskiness of insurance losses is spread though the insurance market). The traditional instrument to spread risk between insurers is reinsurance. By buying and selling "options" on their portfolios with each other, and to specialized reinsurers, insurers can change the risk characteristics of their portfolios. In a paper that anticipated the capital asset pricing model, Borch, 1962, showed that the value maximizing trades would leave all insurers holding net of reinsurance portfolios defined solely on the market aggregate loss and that insurance would be priced solely on the correlation with this aggregate portfolio. We show that distribution of insurance liabilities that minimizes insolvencies, and thereby maximizes payments to policyholders, is similar to Borch′s equilibrium. However, this structure also provides a framework for measuring the available capacity of the industry to respond to major catastrophes.

II. DIVERSIFICATION AND THE MUTUALITY PRINCIPLE

(i) A Definition of Insurance Capacity

In this section, we examine a base case in which the liabilities in an insurance market are distributed amongst insurers so as to maximize payouts to policyholders for any loss scenario. This base case is useful for defining industry capacity and also provides a yardstick for measuring capacity. In the base case, insolvencies will be minimized for any given level of industry losses and thus actual payments to

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[5]See Gron (1994) and Froot, Scharfstein and Stein (1993).

policyholders will be maximized. The importance of this base case for later analysis is that it establishes a basic relationship between the capacity of the insurance industry to respond to catastrophic loss experience and the correlation structure of its liabilities. This leads to the next section in which we derive a measure of capacity which is parameterized by these correlations together with other firm and markets features.

It is well known that in a market in which risk bearing is costly to firms but where transacting between firms is costless, the Pareto optimal risk sharing arrangement is one in which the industry "mutualizes" its risk in the sense that all insurers hold the same net (after reinsurance) liability portfolio. This result, due to Borch (1962), is identical to (and preceded) the capital asset pricing model. According to the Borch, the Pareto optimal reinsurance arrangement is one in which each insurer holds a net (after reinsurance) portfolio which is a proportionate claim on L (this is equivalent to the CAPM proposition that each investor will hold the market portfolio). This implies that all insurers′ portfolios are perfectly correlated after reinsurance transactions have been exploited. After all possibilities for diversification through reinsurance are exhausted, insurers will hold the same loss portfolio though the scale may differ. The aggregate loss for the market is $\sum L_i \equiv L$. The riskiness of the aggregate portfolio will depend on the total number of individual policies insured, "n", and on their correlations. If the number of policyholders is very large and the policy correlations are low then, by the law of large numbers, L will have little risk ($\sigma(L/n) \rightarrow 0$ as "n"$\rightarrow\infty$). But with small "n" and/or correlation, L will have higher risk.

To address the implications of limited liability, first consider the terminal value of equity, T_i, of an insurer, i, in a simple one period model:

(1) $T_i = MAX\{(P_i + Q^o{}_i)(1+r)-\alpha_i L;\ 0\}$

where Q^o is opening surplus and is premium income net of expenses. For simplicity, assume that the market is competitive, thus $P_i = E(L_i)/(1+r)$, and a simple risk free investment rate r. Denoting $Q_i = Q^o{}_i(1+r)$, terminal equity is re-stated as:

(1') $T_i = MAX\{E(L_i) - L_i + Q_i;\ 0\}$

Now consider the implications of limited liability for policyholders. The amount which insurer "i" can pay to policyholders, L_i^p , is the minimum of the face value of its liability of its financial resources which, in this model are the sum of surplus and premiums $Q_i + E(L)$.

(2) $L_i^p = MIN\{L_i\ ;\ Q_i + E(L_i)\}$

If there is a bad draw from the loss distribution, i.e., a catastrophic loss, the ability of the insurer to pay the unexpected loss L_i-E(L) depends on the surplus Q_i. If we scale up this problem, then the ability of the market to respond to unexpected losses depends on the total industry surplus, but also on how the liabilities and surplus are distributed across insurers. We will use this concept to define and measure market capacity.

If we compare this limited liability world with Borch's equilibrium, there is an apparently stark contrast. In Borch's world, insurers are risk averse and will gain from risk sharing reinsurance transactions amongst each other. In the simple version of limited liability, insurers own a put option on the value $E(L_i)-L_i+Q_i$ where the striking price is $E(L_i)+Q_i$ and the value of this option will increase as variance of the underlying asset increases. Thus, apparently, insurers would not engage in risk reducing reinsurance transactions. We can add more structure to resolve this difference by allowing premium rates to depend on insurer risk.[6] This is additional structure is not necessary for our present task, but it does focus our attention on the what the payouts to policyholders would be when insurers are perfectly diversified as shown by Borch.

Consider a Borch equilibrium in which each insurer, "i" holds a share α_i of L and assume that each insurer's surplus is scaled to its share of aggregate loss. The first implication is that the aggregate terminal equity of insurers will be the difference between the "unexpected industry loss E(L)-L, and the industry equity $\sum Q_i$ as shown in equation 3a below. The second implication is that the industry's whole surplus will be available to meet unexpected losses. Thus, the amount of aggregate losses that will be paid to policyholders, L^p, will be the minimum of the face value of losses L and the industry's total resources $E(L)+Q_i$, as shown in equation 3b.

$$(3a) \qquad \sum T_i = MAX\left\{\sum\{\alpha_i E(L)-\alpha_i L+Q_i\};\ 0\right\} = MAX\left\{E(L)-L+\sum Q_i\};\ 0\right\}$$

$$(3b) \qquad \sum L_i^P = MIN\left\{L;\ E(L)+\sum Q_i\right\}$$

Currently, the U.S. property casualty insurance industry's surplus is about $230 billion. In these circumstances, the whole surplus would become available to pay for the loss. In effect, with perfect diversification, the industry acts as a single firm. No one firm would become insolvent until the entire industry surplus is exhausted and, at this point, all firms would simultaneously become insolvent. This equilibrium distributes industry liabilities and resources in a way that maximizes payouts to policyholders.

Definition: For any configuration of losses for which insurers are liable, the capacity of the insurance market is the proportion of those liabilities that is deliverable given the financial resources of firms on whom the losses fall and given all arrangements (such as reinsurance, guarantee funds, etc) for re-allocating those losses amongst insurers.

In the equilibrium considered, all industry surplus would be accessible by policyholders.

(ii) Distributional Assumptions

Consider each insurer's aggregate loss as the sum of its catastrophe exposure and its idiosyncratic risk. Part of the individual insurer loss, d_i, is idiosyncratic and diversifiable; i.e., $COV(d_i, d_j = 0$ for all $i \neq j$). The remaining part of the insurer's loss is catastrophe risk in the sense that all insurers are exposed to highly correlated losses, L_U, from events such as earthquakes. The proportion of the total pool of catastrophe losses written by insurer "i" is c_i. Thus, the insurer's loss is:

$$(4) \qquad L_i = c_i L_U + d_i$$

Given $\sum L_i$ must equal the aggrgegate industry losses, $L \equiv L_U + D$; (where $D \equiv \sum_i d_i$ is the total industry diversifiable losses), then$\sum c_i = 1$. The essential characteristic of diversifiable risk is that it will tend to zero if a large enough number of policies is insured. To provide a rationale for a reinsurance market, we assume that any individual insurer holding n_i policies is insufficiently diversified to secure this risk elimination, but the

[6]See Doherty and Tinic 1982.

total insurance market having $\sum_i n_i \equiv n$ policies does effectively eliminate risk; i.e.;

(5) $\sigma(D/n) \approx 0$; $\sigma(d_i/n_i) \not\approx 0$; $\sigma(c_i L_U/n_i) \not\approx 0$; $\sigma(L_U/n) \not\approx 0$

The first item in (5) says that diversifiable risk can be substantially eliminated by diversification across the marketplace. The second item says that each individual insurer′s endowment of potentially diversifiable exposures is not sufficient to eliminate this risk (i.e., it does not have sufficient policies to exploit the law of large numbers). The third and fourth parts of (5) assert that the risk in L_U is not diversifiable (i.e., policies are positively correlated). The third assumption is particularly important in providing a rationale for insurance. By definition of d_i and c_iL_u, the former can be reduced through further risk spreading whereas the latter cannot.

(iii) Necessary Conditions for Capacity Maximization

We now show the following result which shows that a necessary condition for risk sharing behavior that is optimal from the shareholders' view will involve insurers holding portfolios that

PROPOSITION: *A necessary condition for the average industry capacity per policyholder, $\sum_i E(L_i^p/n)$, to be maximized is that all firms hold a net of reinsurance portfolio which is proportional to L_U and D.*

The proposition requires that all insurers hold portfolios of the form $\alpha_i L = \alpha_i L_U + k_i D$ where α_i and k_i are firm specific constants.[7] Suppose that this were not true, then at least one insurer would hold a portfolio containing some idiosyncratic risk;

i.e. $\alpha_i L_U + d_i$ where $d_i \neq k_i D$.

Since $D \equiv \sum_i d_i$, the existence of one insurer holding $\alpha_i L_U + d_i$ implies that all other insurers must hold in total

$$(1-\alpha_i)L_U + D - d_i \quad = \quad \sum_{j\neq i} \alpha_j L_U + D - d_i$$

which cannot be of the form

$$\sum_{j\neq i} \alpha_j L_U + \sum_{j\neq i} k_j D$$

[7]It will be noticed that the reinsurance structure that maximizes industry capacity ($\alpha_i L = \alpha_i L_U + k_i D$ for all i) is of similar structure to the Pareto optimal reinsurance market identified by Borch (1962). The similarity is more pronounced when it is noticed that, since D is diversifiable, the value of k_i makes little difference to the availability of surplus to pay catastrophic losses. Thus, one can consider the special case in which $\alpha_i = k_i$. However, even for this special case, our result and that of Borch are not necessary identical. While, in both results, insurers′ loss portfolios are defined solely on L, we rely on a maximization of aggregate dollar surplus whereas Borch relied on expected utility maximizing trade between risk averse insurers. The non-linearity in our results comes from the truncating effects of insolvency whereas non-linearity in Borch′s reinsurance structure comes from the parameters of the various insurers utility functions.

since $d_i \neq k_i D$ and $D \equiv \sum_i d_i$.[8] Thus, at least one other insurer must hold a portfolio of the form $\alpha_j L_U + d_j$ where $d_j \neq k_j D$. Of the universe of insurers "M" we define a subset "m_1" having such "undiversified" portfolios $\alpha_i L_U + d_i$ and subset "m_2" having "diversified" portfolios of the form $\alpha_j L_U + k_j D$. Since

$$(1-\sum_{j\in m2} k_j)D \quad = \quad \sum_{i\in m1} d_i$$

then the following mutual exchange is possible. All type m_1 insurers pool their diversifiable risk which leads to an aggregate m_1 diversifiable liability of $(1-\sum_{j\in m2} k_j)D$. Now define a set of weights k_i' and apportion this aggregate liability over m_1 insurers such that each assumes a liability of :

$$k_i'(1-\sum_{j\in m2} k_j)D$$

$$= k_i D \qquad \text{since } k_i' \equiv k_i (1/(1-\sum_{j\in m2} k_j)) \text{ and } \sum k_i' = 1.$$

These conditions ensure that $\sum k_i = 1$ (i.e. that diversifiable risk D is fully allocated over all insurers). Since the only requirement placed on k_i' is that it sum to unity, these weights can be chosen such that the $E(d_i) = E(k_i D)$. Thus, these transactions will leave all m_2 insurers unaffected and will leave the expected face value of liability of all m_1 unchanged. However, since $\sigma(d_i/n_i) > 0$; and $\sigma(k_i D/n_i) \Rightarrow 0$, these transactions are mean preserving, and risk reducing, for all m_1 insurers. Now since the payable loss on any insurer is a short position in a put option, its value will increase as its standard deviation is reduced. Consequently, these transactions will leave $E(L_i^p /n)$, where L_i^p is defined by (3b), unchanged for all m_2 insurers but increased for all m_1 insurers. As a result, aggregate available industry capacity $\sum_i E(L_i^p /n)$ will be increased. Q.E.D.

The proposition shows the necessary conditions for capacity maximization. The sufficient conditions concern the relationship between the liability allocation, $|\alpha_i|$, and the and the distribution of surplus, $|Q_i|$, across insurers. The effect of surplus will become important in the capacity measures derived in the next section.

COROLLARY: When the necessary conditions for maximization of capacity per policyholder $\sum_i E(L_i^p /n)$ are satisfied, all insurers will hold net of reinsurance portfolios L_i that are perfectly correlated with aggregate industry losses, L.

Note that $COV(L_i ; L) = E\{\{c_i\{L_U - E(L_U)\} + \{d_i - E(d_i)\}\}\{L - E(L)\}\}$ which can be simplified to $E\{\{c_i\{L_U - E(L_U)\}\}\{L - E(L)\}\}$ since d_i is independent of L by assumption. Using $COV(D,L)=0$ and $L = L_U + D$, we can write; $\sigma^2(L) = E\{\{L_U - E(L_U)\}\{L - E(L)\}\}$. Thus, $COV(L_i ; L) = c_i \sigma^2(L)$. Proof of the corollary follows immediately from the proposition noting that $COV(L_i ; L) = c_i \sigma^2(L)$ and that α_i and k_i are

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[8] To see this, consider that all other insurers did hold portfolios of the form $\sum_{j\neq i} \alpha_j L_U + \sum_{j\neq i} k_j D$. Thus the total of the diversifiable risk portfolios of all insurers would be:

(a). $D = \sum_{j\neq i} k_j D + d_i$

This can be re-stated as

(b). $D = \sum_{j\neq i} k_j D + k_i D - k_i D + d_i \quad = \quad \sum_j k_j D + (d_i - k_i D)$

which is equal to

(c). $D = D + (d_i - k_i D)$

since $\sum_j k_j = 1$. However, since $d_i \neq k_i D$; then (c), and therefore (a), is contradicted.

constants. Q.E.D.

The corollary shows that each insurer must hold a net portfolio which is perfectly correlated with the aggregate insurable loss L to maximize capacity. This will provide a yardstick for measuring capacity. Since $\alpha_i L = \alpha_i L_U + k_i D$ maximizes capacity for a given initial industry surplus Q, and since this result is characterized by perfect correlation between all L_i and L, then it seems that the actual correlations will provide a measure of capacity utilization.

Various frictions can frustrate the conditions described in the proposition and corollary. In addition to factors that limit firm size, reinsurance and other insurer hedges are costly. Froot and O'Connell 1996, recently estimated the cost of catastrophe reinsurance from the complete set of contracts brokered by the largest reinsurance broker. The transaction cost, (Price-Expected Loss)/Expected loss), ranges between about 10% and 140% from 1970-1995. In the last decade of the series, the average transaction cost is about 65%. Several explanations can be given for this high cost including diverging estimates of expected losses [9], moral hazard and excessive rent taking. Another explanation for incomplete diversification lies in the prospect that shareholders may seek to expropriate wealth from policyholders by choosing a high risk financial structure (Myers 1977 and Doherty and Tinic 1982). This expropriation will be mitigated by reputation effects and where the policyholders and/or their agents can monitor the financial condition and reinsurance purchases of their insurers. We now examine the relationship between capacity, correlations between insurer loss distributions and the financial structure of insurers.

CORRELATIONS AND CAPACITY UTILIZATION FOR A GIVEN CATASTROPHIC LOSS

Our task is to estimate the ability of the industry to respond to an abnormal loss experience defined by equation (3b). This the industry response conditional on industry losses of any given size "L".

The response function is illustrated in Figure 1. The horizontal axis measures possible values for aggregate insurance industry losses, and the vertical axis measures the *expected* payout of all firms combined. Consider just two possible loss scenarios; first an earthquake that causes an industry loss of $30 billion over and above the expected loss E(L) and, second a combination of a Florida hurricane of $20 billion and automobile losses that are $10 billion above expected. Notice both scenarios lead to revealed industry losses that are $30 billion above expected value (denoted E(L)+30). But the scenarios would impact different insurers and the could lead to different levels of insolvency depending the distribution of coverage across insurers. For example, the expected payout in the first scenario might be "W" which is very low because much of the California earthquake coverage is from local insurers that are poorly diversified and poorly capitalized. However, the second scenario might be spread more evenly over firms and the payout is shown as "Y". Points W and Y are the conditional responses which are described below in equations 7-9.These are only two of many potential configurations that could result in industry losses of $30 billion above expected value. The average of all possible payouts for all feasible scenarios which sum to $30 billion above expected loss is denoted "X". This value, X, is the conditional response, i.e., the expected payout of the industry conditional on the industry loss being expected value plus $30 billion. The locus of all such conditional payouts is the response function which is shown as OZ. Notice that OZ lies at or below the 45° line and, we postulate, will diverge from the 45° line as loss realizations increase. The divergence implies that insolvencies will increase dis-proportionately with losses as more and more insurers are stressed and that failures are

[9] Though loss estimated were calculated by independent catastrophe modeling firms.

passed through the market via reinsurance thus causing "knock on" insolvencies.

It is useful to start with the average surplus per policy available to pay unexpected claims of any insurer "i":

(6) $E(T_i/n_i) = (1/n_i) \int^{z_i} [E(L_i) - L_i + Q_i] f(L_i) dL_i$ where $z_i = E(L_i)+Q_i$

To derive the conditional response function note that the aggregate industry terminal equity, conditional on , industry losses being L, is:

(7) $\sum_i E(T_i|L) = \sum_i \int^{z_i} [E(L_i) - L_i + Q_i] f(L_i|L) dL_i$

where: $z_i = E(L_i) + Q_i$. This value is shown in Figure 1 as the wedge between $E(L)+\sum Q_i$ and the response function OZ. Thus, the response function can be defined as $R|L \equiv E(L)+\sum Q_i - \sum_i E(T_i|L)$. To estimate the function, it is necessary to make distributional assumptions about L. Using the truncated normal distribution and using the properties of conditional moments (See Hogg and Craig p 71-72), this becomes:

$$E(T_i \mid Q_{i0}, L) = (P + Q_{i0} - m)\, N\left[\frac{P + Q_{i0} - \mu_{L_i|L}}{\sigma_{L_i|L}}\right] + \sigma_{L_i|L} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{P + Q_{i0} - \mu_{L_i|L}}{\sigma_{L_i|L}}\right)^2} \qquad \textbf{(1)}$$

$$\textit{where} \quad \mu_{L_i|L} = \mu_i + \frac{\rho_i \sigma_i}{\sigma_L}(L - \mu_L) \quad , \textit{ and } \quad \sigma^2_{L_i|L} = \sigma_i (1 - \rho_i^2)$$

where ρ_i is the correlation coefficient between L_i and L and μ is used to denote expectation. Not surprisingly, this formulation resembles option pricing models. However, insurance losses tend to be skewed and a more plausible actuarial distribution for both firm and industry losses is the log normal. We will use this form to estimate industry capacity:

$$E(T_i \mid Q_{i0}, L) = (P + Q_{i0})\, N(C_i) - e^{D_i} N\left(C_i - \xi_i \sqrt{1 - \gamma_i^2}\right) \qquad \textbf{(2)}$$

where:

$$C_i = \frac{\ln(P + Q_{i0}) - \nu_i - \frac{\xi_i \gamma_i}{\xi_L}(\ln L - \nu_L)}{\xi_i \sqrt{1 - \gamma_i^2}} \qquad \textbf{(3)}$$

Notice that $R|L = f(E(L_i); E(L); \sigma(L_i); \sigma(L); r_i; Q_i : L)$. Thus, we can measure the capacity utilization of the industry for any industry loss L, as a function of two industry variables $\{E(L), \sigma(L)\}$ and of four firm variables $\{E(L_i), \sigma(L_i), r_i, Q_i\}$.

ESTIMATION METHODOLOGY

$$D_i = \nu_i + \frac{\xi_i \gamma_i}{\xi_L}(\ln L - \nu_L) + \frac{\xi_i^2 (1 - \gamma_i^2)}{2} \quad \textbf{(4)}$$

This section discusses the methodology used in this paper to estimate the capacity of the property-liability insurance industry. We discuss parameter estimation techniques. Finally, we outline our sample selection procedures and the approaches used to control for the effects of important factors such as reinsurance transactions and industry ownership structure that may have an impact on capacity.

Parameter Estimation

We discuss parameter estimation for the lognormal case. Parameters are estimated using time series data on a sample of companies and the industry. Data are assumed to be available over a period of length T. We define L_{it} = the observed losses of a given firm in year t, t = 1, 2, . . . , T. To detrend the data and estimate the lognormal parameters for each company, we conduct the ordinary least squares regressions:

$$\begin{aligned} ln(L_{it}) &= \alpha_i + \beta_i t + \epsilon_{it} \\ ln(L_t) &= \alpha_y + \beta_y t + \omega_t \end{aligned} \quad \textbf{(5)}$$

We use the sample period, t = 1, 2, . . ., T, to estimate the model and then predict capacity for year T+1. Accordingly, our estimate of $\nu_i = a_i + b_i(T+1)$ and our estimate of $\nu_L = a_L + b_L(T+1)$, where a_i, b_i, a_L, and b_L are the estimated values of α_i, β_i, α_L, and β_L. We estimate γ_i as the empirical correlation coefficient between the error vectors e_{it} and w_t (the estimated values of ϵ_{it} and ω_t) and denote the estimate as g_i. Likewise, ξ_i and ξ_L are estimated as the sample standard deviations of e_{it} and w_t, with the estimates denoted as z_i and z_L.

The estimated parameters are inserted into equations (7) and (8) to derive the predicted values of ending surplus for the normal and lognormal cases, respectively. We also need estimates of P_i and Q_{i0}. For Q_{i0} we use the surplus of the company at the end of period T, and for P_i we use the lognormal expected value, $P_i = \exp[\alpha_i + \beta_i(T+1) + z_i^2/2]$. Thus, expenses are ignored in estimating capacity. This makes sense because we are estimating the ability of the companies in the sample to pay claims, and expenses are netted out before claims are paid.

Sample Selection and Modeling Approach

The data for the study are taken from the regulatory annual statements filed by insurers with the National Association of Insurance Commissioners (NAIC). For years prior to 1992, our annual statement data come from the A.M. Best Company. For years after 1992, the data are obtained directly from the NAIC.

One of the objectives of the study is to estimate changes in market efficiency and industry capacity over time. Accordingly, we selected the longest time period for which data were available to us, 1977-1995. We analyze the data for the entire sample period and for three sub-periods of approximately equal length, 1977-1982, 1983-1988, and 1989-1995. The six/seven year length of our sample sub-periods is appropriate because insurance underwriting profits are cyclical and the period of the cycle has been estimated as between six and seven years in length (Cummins and Outreville, 1987). Thus, each of our sub-periods approximately covers one complete cycle.

One objective of the study is to provide evidence on the effects of reinsurance on the market capacity to bear loss. To estimate the effects of reinsurance, we calculate capacity using two definitions of incurred losses: (1) direct losses incurred, and (2) net losses incurred, defined as direct losses incurred plus reinsurance assumed and minus reinsurance ceded. Direct losses incurred are defined as losses paid or owed directly to policyholders, while net losses incurred reflect the netting out of reinsurance transactions. We anticipate that capacity will be higher under the net loss definition than under the direct loss definition, because the former takes reinsurance into account. Reinsurance is expected to smooth out spikes in the losses of individual insurers, because they receive reimbursement from reinsurers to compensate for unusually large losses.

Ownership structure in the insurance industry also may have an effect on market capacity. Many insurance firms are organized as *insurance groups*, consisting of several individual companies under common ownership. Under U.S. corporation law, the owners of the group hold a valuable option, namely, the option to allow a financially troubled subsidiary to fail. The claimants against the insolvent subsidiary cannot reach the assets of other insurers in the group unless they succeed in "piercing the corporate veil," which usually requires showing that the owners engaged in fraud or some other abnormal activity (Easterbrook and Fischel, 1985). Although the owners may decide to rescue a failing subsidiary to protect reputational or franchise value, they are under no legal obligation to do so.

Thus, capacity is likely to be overestimated if an insurer is defined a group with the premiums and surplus of the group equal to the sum of the premiums and surplus of the members of the group. To control for the potential bias caused by insurance groups and also to provide information on the effects of grouping on capacity, we estimate capacity using two definitions of the insurance industry: (1) the industry is defined as consisting of insurance groups and unaffiliated single companies; and (2) alternatively, the industry is defined as consisting of companies that are members of groups (affiliated companies) and unaffiliated single companies. Because of the default option, definition (1) is likely to lead to overestimates of capacity. However, because of the possibility that the parent organization may choose to come to the assistance of a subsidiary that is facing financial difficulties, definition (2) may tend to understate capacity. Thus, the capacity estimates provided by the two industry definition establish upper and lower bounds on the capacity of the industry to pay claims.

CONCLUSION: PRELIMINARY RESULTS AND WORK IN PROGRESS

Figure 2 is a preliminary estimate of the response function. This is based on net business and thus reflects the effects of reinsurance assumed and ceded. The response function shows the estimated response (vertical axis) for different potential levels of 1995 industry losses (horizontal axis) where parameters were estimated from data from 1977-1995. The thick (denoted "raw data") is the basic result. There are several reasons to be cautious about this response function and we will mention these reservations before commenting on the apparent capacity.

The first cause for concern is that the process of estimating conditional distributions from correlations coefficients between the L_i and L and the distributional parameters of L, does not guarantee that firm losses will sum to industry losses. Moreover, missing data and inconsistent conventions for treating group and subsidiary data further compound the “adding up” problem. One effect of this is seen in the data where, for losses in the region of $250-270 billion, the apparent response exceeds industry losses. This should not happen and we are currently working on this issue. A related problem is that we have used “raw” correlation coefficients based on historical loss experience. There is considerable noise in these estimates and thus the raw values probably are poor proxies for the degree of diversification achieved by different firms. A quick and dirty fix for this problem is to truncate some of the extreme value (in this case eliminate the negative values). This alone makes the response function more well behaved in the sense that it does not penetrate the 45° line frontier and it increases the estimated response for larger loss scenarios. This is shown as the thinner line in Figure 2 denoted “Correl Adj”. A preferred approach is to use fitted correlations by regressing the raw correlations on firm specific variables that reflect the degree of firm diversification such as the concentration of each firms book by line and geography. This will be done in the next round and in the meantime we cannot isolate the directional error in our tentative results arising from this problem.

A second concern is that the NAIC data conventions for allocating surplus in group situations differs from that of Best’s, and the former is thought to overestimate surplus (we think by about $30 billion). This too needs correcting and the results shown over-estimate the response function. A third major issue is that we have not yet reflected potential re-allocations of insolvent firms’ liabilities through the state solvency guarantee schemes. This inclusion of the guarantee schemes will increase the estimates industry response.

We have various other concerns which are work in progress. Subject to these major reservations we can offer a very guarded comment on our results. The expected industry losses for 1995 are $198 billion. Examining both the raw response function and the adjusted function show that the industry appears to be very robust for losses up to about $280 billion. If this result were believable, it would appear that the industry could respond to some of the major scenarios outlines in the introductory paragraph such as an Andrew hitting Miami or a repeat of the 1906 earthquake. However, our analysis is not yet ripe enough to have confidence in this conclusion.

Our continuing work will address these and related problems of the model and data. We hope shortly to have much improved estimates of the response function. Moreover, we hope to be able to isolate the effects of reinsurance and guarantee schemes in improving diversification and therefore the industry’s response. Moreover, we will propose a measure of market efficiency which measures what degree of potential diversification potential is being used. This measure should be very useful in monitoring changes in a market which is widely though to be cyclical and to be subject to re-current crises.

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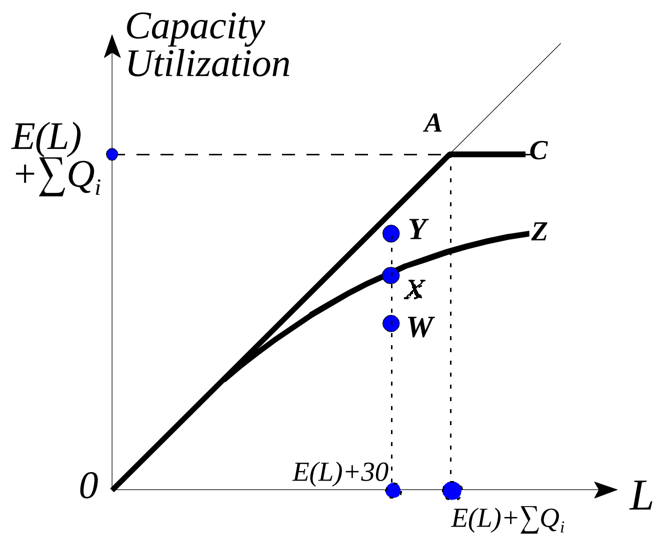
Figure 1: Capacity Utilization



Note: The line OAC represents maximum capacity utilization. The line 0Z = $E(L) + \sum Q - \sum E(T_i | L)$ represents estimated capacity utilization.

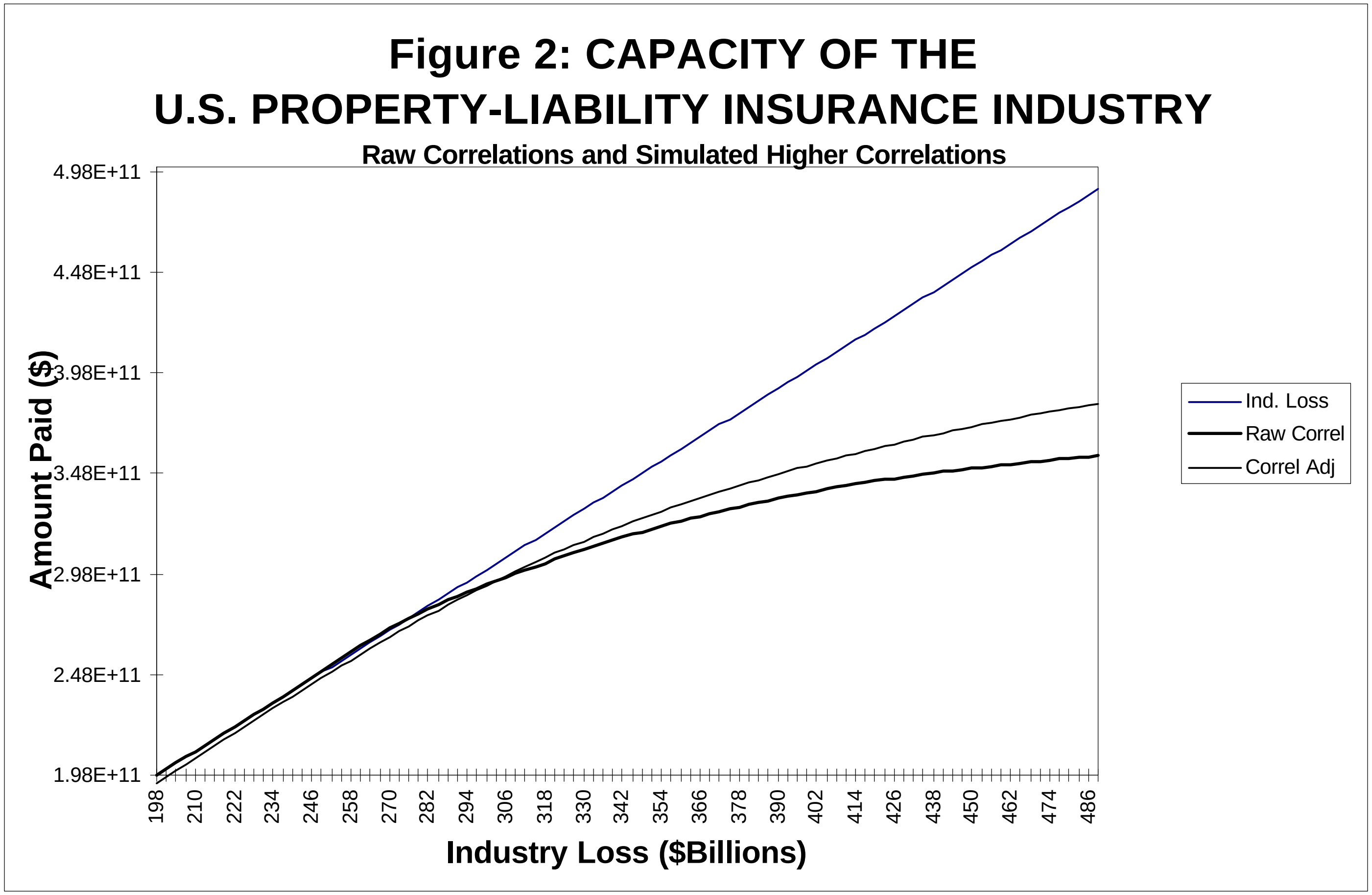


Figure 2: CAPACITY OF THE
U.S. PROPERTY-LIABILITY INSURANCE INDUSTRY
Raw Correlations and Simulated Higher Correlations
Amount Paid ($)
4.98E+11
4.48E+11
3.98E+11
3.48E+11
2.98E+11
2.48E+11
1.98E+11
198
210
222
234
246
258
270
282
294
306
318
330
342
354
366
378
390
402
414
426
438
450
462
474
486
Industry Loss ($Billions)
Ind. Loss
Raw Correl
Correl Adj

APPENDIX

The Model

To estimate the capacity of the industry, we would like to estimate the payments made by individual insurers conditional on the total industry loss. We make use of some general results relating to conditional probability distributions to provide an overview of the problem and then propose specific models based on the normal and lognormal distributions.

In general, let L_i = the loss paid by company i, where $E(L_i) = \mu_i$, $Var(L_i) = \sigma_i^2$, and i = 1, 2, . . ., N, for a sample of N firms. Define the total losses of the industry as the sum of the losses of the individual companies, i.e.,

$$L = \sum_{i=1}^{N} L_i \tag{6}$$

where L = the total losses of the industry. The usual formulas for the mean and variance of a sum of random variables apply, so that

$$E(L) = \sum_{i=1}^{N} \mu_i$$
$$Var(L) = \sum_{i=1}^{N} \sigma_i^2 + 2 \sum_{i<j}\sum \sigma_{ij} \tag{7}$$

where $\sigma_{ij} = Cov(L_i,L_j)$. To conserve parameters and focus on the industry loss, we rewrite the variance of the industry loss as follows:

$$Var(L) = \sum_{i=1}^{N} Cov(L_i, L) = \sum_{i=1}^{N} Cov(L_i, \sum_{i=1}^{N} L_i) \tag{8}$$

The well-known formula for the conditional expected value (see Hogg and Craig, p. 94) can be used to obtain:

$$E(L_i | L) = \mu_i + \frac{\rho_i \sigma_i}{\sigma_L}(L - \mu_L) = \mu_i + \beta_i (L - \mu_L)$$
$$where \quad \beta_i = \frac{\rho_i \sigma_i}{\sigma_L}(L - \mu_L) = \frac{Cov(L_i, L)}{\sigma_L^2} \tag{9}$$

Summing over the N firms in the industry, we find that $\Sigma_i E(L_i|L) = L$, because Var(L) = $\Sigma_i Cov(L_i,L)$ and $\mu_L = \Sigma_i \mu_i$. Thus, we can use these formulas to allocate any given industry loss among the firms in the sample.

The above results are not distribution dependent, i.e., we did not have to assume that losses follow any particular probability distribution in order to obtain the results. To calculate the capacity of the industry, however, it is helpful to have a distributional assumption. We develop the model under two assumptions: (1) the distribution of the L_i is multivariate normal and (2) the distribution of the L_i is multivariate lognormal. We develop the model in general and then specify the formulas for the normal and lognormal cases.

Insurers are assumed to begin the period with premiums, P, and beginning equity (surplus), Q_0. The insurer is assumed to pay claims up to the point where these resources are exhausted and to declare bankruptcy and default if the claims exceed its resources. The expected equity of the insurer at the end of the period, conditional on an industry loss of L, is given by:

$$E(T_i \mid Q_{i0}, L) = \int_0^{P_i + Q_{i0}} (P_i + S_{i0} - L_i)\, f(L_i \mid L)\, dL_i \qquad \textbf{(10)}$$

where $f(L_i|L)$ = the distribution of the losses of a given insurer (L_i), conditional on the losses of the industry, L.

In the case where the L_i are jointly normally distributed, the conditional distribution in (5) is given by:

$$f(L_i \mid L) = \frac{1}{\sqrt{2\pi}\,\sigma_i \sqrt{1-\rho_i^2}}\, e^{\frac{1}{1-\rho_i^2}\left[\frac{L_i - \mu_i}{\sigma_i} - \rho_i \frac{L - \mu_L}{\sigma_L}\right]^2} \qquad \textbf{(11)}$$

So $E(L_i|L) = \mu_i + (\rho_i\sigma_i/\sigma_L(L - \mu_L)$. Inserting equation (6) into equation (5) and simplifying, we obtain the expression for the expected ending surplus under the assumption of multivariate normality:

$$E(T_i \mid Q_{i0}, L) = (P + Q_{i0} - m)\, N\left[\frac{P + Q_{i0} - \mu_{L_i|L}}{\sigma_{L_i|L}}\right] + \sigma_{L_i|L}\, \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{P + Q_{i0} - \mu_{L_i|L}}{\sigma_{L_i|L}}\right)^2} \qquad \textbf{(12)}$$

$$where \quad \mu_{L_i|L} = \mu_i + \frac{\rho_i\,\sigma_i}{\sigma_L}(L - \mu_L) \quad , \; and \quad \sigma^2_{L_i|L} = \sigma_i(1 - \rho_i^2)$$

.N[·] = the standard normal distribution function.

Using an analogous approach for the case where L_i and L are jointly lognormal, we obtain:

$$E(T_i \mid Q_{i0}, L) = (P + Q_{i0})\, N(C_i) - e^{D_i}\, N(C_i - \xi_i\sqrt{1 - \gamma_i^2}) \qquad \textbf{(13)}$$

where

$$C_i = \frac{\ln(P + Q_{i0}) - \nu_i - \frac{\xi_i \gamma_i}{\xi_L}(\ln L - \nu_L)}{\xi_i \sqrt{1 - \gamma_i^2}} \quad \textbf{(14)}$$

$$D_i = \nu_i + \frac{\xi_i \gamma_i}{\xi_L}(\ln L - \nu_L) + \frac{\xi_i^2 (1 - \gamma_i^2)}{2} \quad \textbf{(15)}$$

and υ_i, υ_L = the location parameters of the joint lognormal distribution of L_i and L,

ξ_i, ξ_L = the dispersion parameters of the joint lognormal distribution of L_i and L, and

γ_i = the correlation coefficient between $\ln(L_i)$ and ln(L).

In the lognormal case, we have the complication that L_i and L cannot be jointly lognormal if the L_i, i = 1, . . . , N, are jointly lognormal, because sums of lognormals are not lognormal. Hence, the formula (equation (8)) is only approximate in this case.

The normal and lognormal models yield estimates of the expected end-of-period surplus of the insurers in the sample. However, our ultimate objective is to estimate the amount of claims paid. The amount of claims paid can easily be estimated using the following relationship:

$$E(L_i \mid Q_{i0}, L) = P_i + Q_{i0} - E(T_i \mid Q_{i0}, L) \quad \textbf{(16)}$$

That is, the expected payment equals the resources at the beginning of the period minus the expected amount of surplus at the end of the period.