Advanced Materials Research *Online: 2011-05-31*
ISSN: 1662-8985, Vols. 255-260, pp 2272-2275
doi:10.4028/www.scientific.net/AMR.255-260.2272

Affine Reconstruction Based on Parallel Plane and Infinity Point

Yue ZHAO [a], Xiaohua HU[b]

Institute of mathematics and statistics, Yunnan University, KunMing 650091, China

[a]zhao6685@yahoo.com, [b]hxh2199@163.com

Key words: computer vision; affine reconstruction; infinite homography; point at infinity.

Abstract. Affine reconstruction is to restore the affine shape of the object. Generally, there are two ways of achieving, one is to determine the plane at infinity, another is to determine the plane homography. Using the homography which had determined the plane at infinity achieve affine reconstruction. In this paper, firstly give out the homography of infinity plane and the algorithm of affine reconstruction, then proved: if the scene contains a set of parallel planes and a infinity point, the homography of infinity plane can be obtained and affine reconstruction can be linearly got in the scene. Computer simulation and real experiments show that the linear affine reconstruction algorithm is correct, and the approach has a good precision.

Introduction

In computer vision, three-dimensional (3-D) reconstruction is the restoration of the coordinates of space points in world coordinate system (WCS), using corresponding points of two or more images, and restore the 3-D structural information of the object. It is an important research field in computer vision[1]. Basically, the affine reconstruction is to get the express of the plane at infinity under the projective space or identify the plane at infinity, and reconstruct object 3-D surface under different of affine transformation. Pollefeys[2] proposed to use modeling of constraints to determine the plane at infinity. In the 1990s, as the 3-D reconstruction developed, the reconstruction process can be turned into a problem of nonlinear optimization[2-4]. If the homography of plane is given, the 3-D reconstruction process can be turned into linear process. So, it is important to solve the approach of homography of plane at infinity. Because the projection of plane at infinity is unknown in image plane, the homography cannot be calculated like the ordinary plane. Therefore, this paper is based on using the parallel plane to determine the plane at infinity, and select a group of parallel planes to obtain their corresponding homography and reach to the constraints on the homography matrix of plane at infinity[5]. Through deduction of equation of a set of parallel plane and their homography of point at infinity, give out an approach of homography at infinity using a set of parallel plane and points at infinity, then solve the coordinates in WCS using the principle of triangulation [6].

Homography Matrix at Infinity and Affine Reconstruction

Homography Matrix. Let Π be any plane which does not across the center of camera, the images are I and I' with two cameras. Let X be any point on the Π, the images under two cameras are m , m'. As there are matrixes H_1, H_2 between spatial plane Π and two image planes (H_1, H_2 are invertible matrixes), $m = H_1 X$, $m' = H_2 X$. The plane Π is not through the center of the camera, so H_1, H_2 are 2D projective transformation from the plane Π to its corresponding images. There is a transformation between m and m' ,so

$$H = H_2 H_1^{-1}, \tag{1}$$

therefore $m' = Hm$.

The matrix H accomplishes the transformation from the first image to the second, and H is invertible matrix. So H is called homography matrix of two images[7].

Homography Matrix of Plane at Infinity. Let π_∞ be any infinity plane of the space, m and m' are image matching points of M on the plane π_∞ . If it presences invertible matrix H_∞ , then $\lambda m' = H_\infty m$, $\lambda \neq 0$ .The H_∞ is called two-view homography of infinity plane[8], denoted by: $H_\infty = KRK^{-1}$; K is the camera intrinsic parameters, R is rotation matrix.

Affine Reconstruction. The derivation of homography matrix of infinity plane is given by using a set of parallel planes and a pair of infinity points.

Proposition. If the space contains a set of parallel planes π_1, π_2 and a pair of infinity points p_1, p_1', the homography matrix of infinity plane can be obtained, and the object can be affine reconstruction linearly.

Proof. Let π_1, π_2 be the parallel planes and not coincide, their homography matrix are H^1, H^2, then there exits nonzero constants s_1, s_2 with

$$s_1 H^1 = H_\infty + K\frac{tn^T}{d_1}K^{-1}, \tag{2}$$

$$s_2 H^2 = H_\infty + K\frac{tn^T}{d_2}K^{-1}. \tag{3}$$

Because of $e' \approx Kt$, $\alpha = K^{-T}n$, there exits nonzero λ ,so that $e' = \lambda Kt$, let Eq. 2 and Eq. 3 make the subtraction $\frac{s_1}{s_2}H^1 + e'(\frac{\lambda(d_1-d_2)}{d_1 d_2}\alpha)^T = H^2$ $s_1H^1 - s_2H^2 = \lambda(\frac{d_1-d_2}{d_1d_2})e'\alpha^T$, so

$$\frac{s_1}{s_2}H^1 + e'(\frac{\lambda(d_1-d_2)}{d_1 d_2}\alpha)^T = H^2. \tag{4}$$

That Eq. 4 can be converted to

$$xH^1 + e'y^T = H^2, \tag{5}$$

then x, y can be obtained by linear equations, under the difference of nonzero constant, α can be solved linearly[9,10]. Other, P, P' are the corresponding infinity points, their coordinates are $P = (u, v, 1)$ and $P' = (u', v', 1)$, then P, P' meet $P' = H_\infty P$; Also, $s_1H^1 = H_\infty + K\frac{tn^T}{d_1}K^{-1} = H_\infty + \lambda e'y^T$, so $H_\infty = s_1H^1 - \lambda e'y^T$, therefore:

$$H_\infty = s_1H^1 - \lambda e'y^T, \tag{6}$$

s_1 and λ can be denoted linearly by a pair of infinity points.

Let camera be $p = (I, 0)$ and $p' = (H_\infty, e')$, then $\left[p = (I,0), p' = (H_\infty, e')\right]$ is an affine reconstruction. X is any point of the space, its world coordinate is $X = (X, Y, Z, 1)^T$, its image coordinates are $m = (u, v, 1)^T$ and $m' = (u', v', 1)^T$ under the camera P , P'. According to $\lambda_1 m = pX$, $\lambda_2 m' = p'X$ have

$$\lambda_1 m = pX, \lambda_1\begin{pmatrix}u\\v\\1\end{pmatrix} = \begin{pmatrix}p_{11}, p_{12}, p_{13}, p_{14}\\ p_{21}, p_{22}, p_{23}, p_{24}\\ p_{31}, p_{32}, p_{33}, p_{34}\end{pmatrix}\begin{pmatrix}X\\Y\\Z\\1\end{pmatrix}, \tag{7}$$

$$\lambda_2 m' = p'X, \lambda_2\begin{pmatrix}u\\v\\1\end{pmatrix} = \begin{pmatrix}p'_{11}, p'_{12}, p'_{13}, p'_{14}\\ p'_{21}, p'_{22}, p'_{23}, p'_{24}\\ p'_{31}, p'_{32}, p'_{33}, p'_{34}\end{pmatrix}\begin{pmatrix}X\\Y\\Z\\1\end{pmatrix}. \tag{8}$$

Let Eq. 7 and Eq. 8 convert to linear equations: $AX = B$, that $A = \begin{pmatrix} p_{31}u - p_{11}, p_{32}u - p_{12}, p_{33}u - p_{13} \\ p_{31}v - p_{21}, p_{32}v - p_{22}, p_{33}v - p_{23} \\ p'_{31}u' - p'_{11}, p'_{32}u' - p'_{12}, p'_{33}u' - p'_{13} \\ p'_{31}v' - p'_{21}, p'_{32}v' - p'_{22}, p'_{33}v' - p'_{23} \end{pmatrix}$,

$X = (X, Y, Z)^T$, $B = \begin{pmatrix} p_{14} - vp_{34} \\ p_{24} - vp_{34} \\ p'_{14} - vp'_{34} \\ p'_{24} - vp'_{34} \end{pmatrix}$. Solving the linear equations, the world coordinate of X can be obtained [11]. Under the above proposition's proof process, the algorithm of the affine reconstruction is established as follows

Step1: Input the scene contains parallel planes π_1, π_2, through Eq. 1, calculate the corresponding homography matrix H^1, H^2 respectively;

Step2: According to Eq. 5 the homography matrix of parallel planes π_1, π_2 to solve α;

Step3: According to Eq. 6 the infinity points to get infinity homography matrix H_∞;

Step4: Output construction of affine reconstruction matrix, Using Eq. 7,8, the world coordinate of the space point can be given by the theory of triangulation.

Simulation Experiment

In simulation experiment, we choose the positive tri-prism as the subject of an experiment. The simulated camera has the following property: $K = [1000, 0.2, 640; 0, 800, 480; 0, 0, 1]$, $R_1 = [10, 33, 69; 37, 45, 27; 53, 45, 21]$, $T_1 = [20; 40; 70]$, $R_2 = [10, 30, 69; 30, 45, 76; 32, 65, 39]$, $T_2 = [50; 30; 60]$, take the first image of the object under the (R_1, T_1) and the second image of the object under the (R_2, T_2), the result of the simulation experiment as follows: homography of infinity plane $H_\infty = \begin{pmatrix} -0.280882 & -1.340078 & 755.533748 \\ -0.669964 & -0.050004 & -104.732430 \\ -0.000835 & -0.000401 & 1 \end{pmatrix}$. The result of affine reconstruction as the Fig. 1, vary the noise level from 0.2 pixels to 2.0 pixels .For each noise level, 50 independent trials are preformed and the result shown as the Fig. 2.

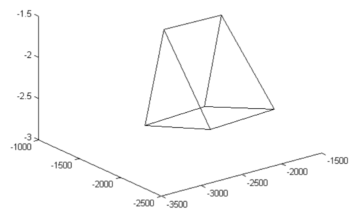

Fig.1 The result of affine reconstruction

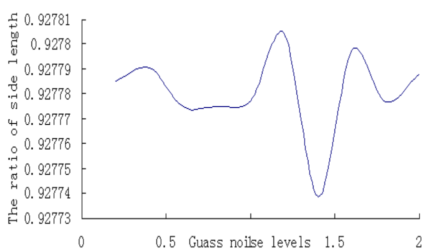

Fig. 2 The length of the curve compared with the noise

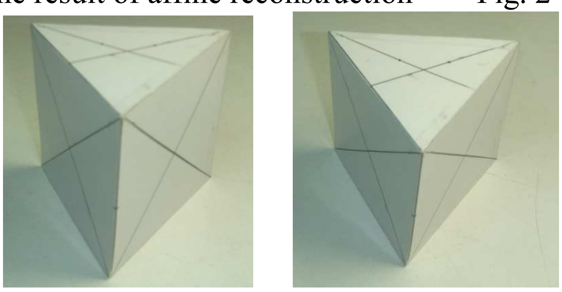

Fig.3 Experimental pictures

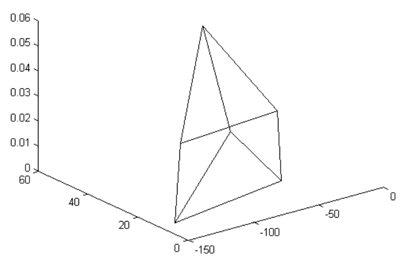

Fig. 4 The result of affine reconstruction

Real Data

In the real experiment, let the camera do the motions of a rigid body, take two images as Fig. 3, homography of infinity plane $H_\infty = \begin{pmatrix} -1.435366 & 0.034831 & 1288.842128 \\ -0.385459 & -0.588098 & 3661.992579 \\ -0.000280 & -0.000032 & 1 \end{pmatrix}$.

Summary

This paper mainly discusses the camera intrinsic parameters unchanged using the information of parallel planes and infinity points in scenes, gives the algorithm of affine reconstruction linear, and proofs the above conclusions by mathematical methods. At the same time, this approach is suit for the space object which contains a parallel plane and infinite points, further, the object in the scene can be affine reconstruction linear. At last, Computer simulation and real data validate the accuracy and robustness of this method, the approach of this paper gains considerable flexibility.

Acknowledgment

This work is supported by the Scientific Research Foundation of Yunnan Education Department (2010Y245) and the Teaching Administration Reform Item of Institute of Mathematics and Statistics of Yunnan University.

References

[1] Fauferas O.What can been seen in three dimensions with an uncalibrated stereo. (In Proc ECCV92, Santa Margherita Ligure, Springer-Verlag, Italy 1992),p.563-578.

[2] Pollefeys M.,Gool Van L.,Osterlinck A. The modulus constraint: A New Constraint for self-calibration, In: Proceedings of International conference on Pattern Recognition, Vienna Austria, Vol.31-42 (1996), p.349-353.

[3] Pollefeys M.Koch R.Gool VL. Self-calibration and metric reconstruction in spite of varying and unknown internal camera parameters. In:(Proceedings of International conference on Computer Vision.Bombay, 1998), p.90-95.

[4] Wu FC.HU ZY.A new theory and algorithm of linear camera self-calibration. Chinese Journal of Computers, 24(11) (2001), p.1121-1135.

[5] Sun-Fengmei, Hu-Zhangyi. Some properties about Homography matrix constraint on the intrinsic parameters.Journal of Computer-Aided and computer Graphics. 19(5) (2007), p. 647 – 650.

[6] Ma-Songde,Zhang-Zhenyou.Computer Vision. Computing theory and Algorithm basics.(Science Press, BeiJing 1998), p. 89-92.

[7] Hartley R,Zisserman A.Multiple View Geometry In Computer Vision. (Cambridge Cambridge University Press, U K 2000).

[8] Zhao-Weimin, Liang-Dong. Stratified reconstruction of three-dimensional objects based on two images. Computer Engineering and Applications. 36 (2003), p.78-80.

[9] Wu-Fucao, Hu-Zhangyi. Determine homography matrix of infinite plane linearly and camera calibration . Acta automatica sinica. 28(4) (2002), p.487-496.

[10] Sun-Fengmei, Wu-Fucao, Hu-Zhangyi. Determine homography matrix of infinite plane based on projection of parallel planes. Journal of Software. 14(5) (2003), p. 935- 946.

[11] Wu-Fucao ect. Mathematical Methods of Computer Vision. (Science Press, BeiJing 2008), p.63-75, p.104-105.