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Construction of Super Schwarzian Connection in Conformal Field Theories on Higher-Genus Super Riemann Surfaces

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In the global operator formalism of the superstring on higher-genus super Riemann surfaces we calculate the super Schwarzian connection in order to derive the super KN algebra. A new super differential is also proposed.

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§ 1. Introduction

Among several approaches to conformal field theories on higher-genus Riemann surfaces, the Krichever-Novikov (KN) formalism[1] appears to be simple and powerful, and is known as the global operator formalism. In this formalism the field operators are expressed in the meromorphic KN bases which are globally defined on higher-genus Riemann surfaces.[2] The Virasoro algebra is generalized to the KN algebra, which is possible to treat the Teichmüller deformations and the conformal deformations on the same footing.

An extension of the KN formalism to the superstring theory has been considered by many authors.[3] However, nobody has so far given the right central term of the super KN algebra, in which the presence of the super Schwarzian connection is necessary in order to render the central term coordinate-independent.

Contrary to the above global operator formalism, in our previous work[4] we have proposed the local operator formalism. In this formalism we start with the usual super Virasoro algebra or the usual operator product expansion of the super stress-energy tensor which is defined in a local region on the higher-genus Riemann surface Σ. Then it is analytically continued to the whole Σ by introducing the super Schwarzian connection R. This local operator method is simple and gives the right central term of the super KN algebra. But in this work we have not calculated an actual form of the R; we have only assumed its existence.

In this paper we use the global operator formalism of the superstring to calculate the super Schwarzian connection and then obtain the super KN algebra with the right central term in § 2. In § 3 we derive the transformation law of the super Schwarzian connection obtained above. The last section is devoted to concluding remarks.

§ 2. The super Schwarzian connection and the super KN algebra

We follow the superspace formulation of the super Riemann surface proposed in Ref. 5). Let Σ be a compact super Riemann surface of genus g with two distinguished points $P_\pm$ in generic position. In a local supercoordinate $\boldsymbol{z}=(z, \theta)$, the covariant derivative D is the square root of the ordinary derivative:

$$D=\frac{\partial}{\partial\theta}+\theta\frac{\partial}{\partial z}, \quad D^2=\frac{\partial}{\partial z}=\partial_z . \tag{2·1}$$

A superholomorphic function $f(\boldsymbol{z})$ consists of two ordinary holomorphic functions, $f(\boldsymbol{z})=f_0(z)+\theta f_1(z)$, with f_0 commuting with θ and f_1 anticommuting with θ. Under a superconformal transformation $\boldsymbol{z}\to\tilde{\boldsymbol{z}}(\boldsymbol{z})=(\tilde{z}(z, \theta), \tilde{\theta}(z, \theta))$, the covariant derivative D and the line element $ds\equiv dz+\theta d\theta$ transform according to the laws

$$D=(D\tilde{\theta})\tilde{D}, \quad ds=(D\tilde{\theta})^{-2}d\tilde{s} . \tag{2·2}$$

Then superconformal tensor fields $\phi(\boldsymbol{z})$ with weight h are defined by the covariant condition

$$ds^h\phi(\boldsymbol{z})=d\tilde{s}^h\tilde{\phi}(\tilde{\boldsymbol{z}}), \quad \phi(\boldsymbol{z})=(D\tilde{\theta})^{2h}\tilde{\phi}(\tilde{\boldsymbol{z}}). \tag{2·3}$$

In the following we use the coordinate $\boldsymbol{z}=(z, \theta)$ such that

$$z=\tau(Q)+i\sigma(Q)=\int_{Q_0}^{Q}dp , \tag{2·4}$$

where dp is a differential of the third kind defined on Σ with poles of the first order at the points $P_\pm$ with residues ± 1 and Q_0 an arbitrary initial point. The $\tau(Q)$ plays a role of the time parameter on Σ. The holomorphic part outside $P_\pm$ of the free superstring field can be written as

$$X^\mu(\boldsymbol{z})=x^\mu(z)+\theta\psi^\mu(z) . \tag{2·5}$$

The quantization of $x^\mu(z)$ and $\psi^\mu(z)$ is performed on the level line C_τ of the equal time

$$C_\tau=\{Q\in\Sigma:\ \tau(Q)=\tau\in\boldsymbol{R}\} . \tag{2·6}$$

As for the quantization of the bosonic part we use the result of the previous paper,[6] in which the univalence conditions of the bosonic string around $P_\pm$ and homology cycles are considered weak constraints. As for the quantization of the fermionic part we expand ψ^μ in the KN bases of 1/2-form $\{h^r(Q)\}$[7]

$$\psi^\mu(z)=\psi_r{}^\mu h^r(z) , \tag{2·7}$$

where $h^r(Q)=h^r(z)dz^{1/2}$ and $h^r(z)$ behaves like near $P_\pm$

$$h^r(z)\cong z_\pm{}^{\mp r-(1/2)}+O(z_\pm) . \tag{2·8}$$

Here r takes integer values for the Ramond sector and half-integer values for the Neveu-Schwarz sector. The coefficients $\psi_r{}^\mu$ are subject to the anticommutation relation

$$\{\psi_r{}^\mu, \psi_s{}^\nu\}=\eta^{\mu\nu}\delta_{r+s,0} . \tag{2·9}$$

The holomorphic super stress-energy tensor outside $P_\pm$ of the free superstring field is given by

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$$T(z)=:\frac{1}{2}DX^{\mu}(z)\partial X_{\mu}(z):. \qquad (2\cdot10)$$

Let us calculate the operator product expansion $T(z_1)T(z_2)$. For this purpose we consider the contraction $\langle T(z_1)T(z_2)\rangle$, which is defined by $\langle T(z_1)T(z_2)\rangle=T(z_1)T(z_2)-:T(z_1)T(z_2):$. Owing to Wick's theorem it reduces to

$$\begin{aligned}4\langle T(z_1)T(z_2)\rangle&=\langle :DX^{\mu}(z_1)\partial X_{\mu}(z_1)::DX^{\nu}(z_2)\partial X_{\nu}(z_2):\rangle\\&=\langle DX^{\mu}(z_1)\partial X^{\nu}(z_2)\rangle:\partial X_{\mu}(z_1)DX_{\nu}(z_2):-\langle\partial X^{\mu}(z_1)DX^{\nu}(z_2)\rangle:DX_{\mu}(z_1)\partial X_{\nu}(z_2):\\&\quad+\langle DX^{\mu}(z_1)DX^{\nu}(z_2)\rangle:\partial X_{\mu}(z_1)\partial X_{\nu}(z_2):+\langle\partial X^{\mu}(z_1)\partial X^{\nu}(z_2)\rangle:DX_{\mu}(z_1)DX_{\nu}(z_2):\\&\quad+\langle DX^{\mu}(z_1)\partial X^{\nu}(z_2)\rangle\langle\partial X_{\mu}(z_1)\partial X_{\nu}(z_2)\rangle\\&\quad+\langle DX^{\mu}(z_1)DX^{\nu}(z_2)\rangle\langle\partial X_{\mu}(z_1)\partial X_{\nu}(z_2)\rangle. \end{aligned}\qquad (2\cdot11)$$

The contraction $\langle X^{\mu}(z_1)X^{\nu}(z_2)\rangle$ is given by

$$\langle X^{\mu}(z_1)X^{\nu}(z_2)\rangle=\langle x^{\mu}(z_1)x^{\nu}(z_2)\rangle-\theta_1\theta_2\langle\psi^{\mu}(z_1)\psi^{\nu}(z_2)\rangle\,, \qquad (2\cdot12)$$

where

$$\langle x^{\mu}(z_1)x^{\nu}(z_2)\rangle=\eta^{\mu\nu}[\ln(z_1-z_2)+\text{regular term}]\,, \qquad (2\cdot13)$$

$$\langle\psi^{\mu}(z_1)\psi^{\nu}(z_2)\rangle=\eta^{\mu\nu}\left[\frac{1}{z_1-z_2}+\text{regular term}\right]. \qquad (2\cdot14)$$

In (2·13) we have used the result obtained in a previous work.[8] In the standard single-valued representation both contractions can be written by the prime forms and theta functions. In such representations the spin structures are taken into account in (2·14).[9] However, the forms of (2·13) and (2·14) are enough for us in the following calculations of operator products. Hence we have

$$\langle X^{\mu}(z_1)X^{\nu}(z_2)\rangle\equiv-\eta^{\mu\nu}G(z_1,z_2)=-\eta^{\mu\nu}[-\ln z_{12}+\bar{G}(z_1,z_2)]\,, \qquad (2\cdot15)$$

where $z_{12}=z_1-z_2-\theta_1\theta_2$ and $\bar{G}(z_1,z_2)$ is the regular term outside $P_{\pm}$ and symmetric with respect to z_1 and z_2. From (2·15) it follows that

$$\begin{aligned}\langle DX^{\mu}(z_1)\partial X^{\nu}(z_2)\rangle&=\eta^{\mu\nu}\left[\frac{\theta_{12}}{z_{12}^2}-D_1\partial_2\bar{G}(z_1,z_2)\right],\\ \langle\partial X^{\mu}(z_1)DX^{\nu}(z_2)\rangle&=\eta^{\mu\nu}\left[-\frac{\theta_{12}}{z_{12}^2}-\partial_1D_2\bar{G}(z_1,z_2)\right],\\ \langle DX^{\mu}(z_1)DX^{\nu}(z_2)\rangle&=\eta^{\mu\nu}\left[\frac{1}{z_{12}}-D_1D_2\bar{G}(z_1,z_2)\right],\\ \langle\partial X^{\mu}(z_1)\partial X^{\nu}(z_2)\rangle&=\eta^{\mu\nu}\left[\frac{1}{z_{12}^2}-\partial_1\partial_2\bar{G}(z_1,z_2)\right],\end{aligned}\qquad (2\cdot16)$$

where $\theta_{12}=\theta_1-\theta_2$. By use of (2·16) and the formula

$$f(z_1)=\sum\frac{1}{n!}(z_{12})^n\partial_2{}^n(1+\theta_{12}D_2)f(z_2)\,, \qquad (2\cdot17)$$

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the first four terms in (2·11) reduce to

$$\frac{\theta_{12}}{z_{12}^2} 6T(z_2)+\frac{1}{z_{12}}2D_2T(z_2)+\frac{\theta_{12}}{z_{12}}4\partial_2T(z_2)+\cdots . \tag{2·18}$$

The last two terms in (2·11) become

$$\hat{c}\left[\frac{1}{z_{12}^3}+\frac{\theta_{12}}{z_{12}^2}\left(-D_1\partial_2\bar{G}-\partial_1D_2\bar{G}\right)+\frac{1}{z_{12}^2}(-D_1D_2\bar{G})+\frac{1}{z_{12}}(-\partial_1\partial_2\bar{G})\right], \tag{2·19}$$

where $\hat{c}=\eta^{\mu\nu}\eta_{\mu\nu}$ is the number of space-time dimensions.

In order to calculate (2·19) let us define an antisymmetric function $l(z_1, z_2)$ by

$$l(z_1, z_2)=D_1D_2\bar{G}(z_1, z_2) . \tag{2·20}$$

The super Schwarzian connection $R(z)$ is then defined by

$$R(z_2)=\lim_{1\to2}D_1l(z_1, z_2)=(D_1l)_2 , \tag{2·21}$$

which is holomorphic outside $P_\pm$ and transforms according to the law

$$R(z)=\tilde{R}(\tilde{z})(D\tilde{\theta})^3-\frac{1}{2}S(\tilde{z}, z) , \tag{2·22}$$

where $S(\tilde{z}, z)$ is the super Schwarzian derivative

$$S(\tilde{z}, z)=\frac{D^4\tilde{\theta}}{D\tilde{\theta}}-2\frac{D^3\tilde{\theta}}{D\tilde{\theta}}\frac{D^2\tilde{\theta}}{D\tilde{\theta}} . \tag{2·23}$$

The transformation law (2·22) and the periodicity of R on Σ will be derived in § 3. In terms of $R(z_2)$ and $l(z_1, z_2)$ the bracket in (2·19) reduces to

$$\begin{aligned}&\frac{1}{z_{12}^3}+\frac{\theta_{12}}{z_{12}^2}[-2R(z_2)-2z_{12}(D_1{}^3l)_2+z_{12}D_2(D_1{}^2l)_2]\\&+\frac{1}{z_{12}^2}[-\theta_{12}R(z_2)-z_{12}(D_1{}^2l)_2-z_{12}\theta_{12}(D_1{}^3l)_2]\\&+\frac{1}{z_{12}}[-D_2R(z_2)+(D_1{}^2l)_2+\theta_{12}D_2(D_1{}^2l)_2-\theta_{12}(D_1{}^3l)_2]+\cdots\\&=\frac{1}{z_{12}^3}+\frac{\theta_{12}}{z_{12}^2}[-3R(z_2)]+\frac{1}{z_{12}}[-D_2R(z_2)]+\frac{\theta_{12}}{z_{12}}[-4(D_1{}^3l)_2+2D_2(D_1{}^2l)_2]+\cdots .\end{aligned} \tag{2·24}$$

One can prove in the Appendix that the last bracket in (2·24) is related to the super Schwarzian connection R, i.e.,

$$(D_1{}^3l)_2-\frac{1}{2}D_2(D_1{}^2l)_2=\frac{1}{2}\partial_2R(z_2) . \tag{2·25}$$

Collecting all results of (2·18), (2·24) and (2·25) we have the operator product expansion

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$$T(z_1)T(z_2)=\hat{c}\left[\frac{1}{4z_{12}^3}+\frac{\theta_{12}}{z_{12}^2}\left(-\frac{3}{4}R_2\right)+\frac{1}{z_{12}}\left(-\frac{1}{4}D_2R_2\right)+\frac{\theta_{12}}{z_{12}}\left(-\frac{1}{2}\partial_2 R\right)\right]$$
$$+\frac{\theta_{12}}{z_{12}^2}\left(\frac{3}{2}T_2\right)+\frac{1}{z_{12}}\left(\frac{1}{2}D_2T_2\right)+\frac{\theta_{12}}{z_{12}}\partial_2 T_2+\cdots. \qquad (2\cdot26)$$

This equation is equivalent to the super KN algebra

$$[L_V, L_W]=L_{W\wedge V}+\frac{\hat{c}}{8}\chi_{VW}, \qquad (2\cdot27)$$

where

$$L_V=\frac{1}{2\pi i}\oint_{C_\tau}dzd\theta V(z)T(z), \qquad (2\cdot28)$$

$$V\wedge W=V\partial W-W\partial V+\frac{1}{2}(DW)(DV), \qquad (2\cdot29)$$

$$\chi_{VW}=\frac{1}{2\pi i}\oint_{C_\tau}dzd\theta[WD\partial^2 V+V\wedge WR(z)] \qquad (2\cdot30)$$

and $V(z)$ ($W(z)$) being the superconformal vector field with weight $h=-1$. This coincides with the result obtained by the local method in the previous work,[4] in which decompositions of (2·27) into those of the Neveu-Schwarz-Ramond types were considered.

§ 3. The transformation law of the super Schwarzian connection

In this section we would like to show that the super Schwarzian connection $R(z)$ difined by (2·21) is certainly subject to the transformation law (2·22). We first define a function $F(z_1, z_2)$ by

$$F(z_1, z_2)=\ln\frac{\tilde{z}_{12}}{z_{12}}, \qquad (3\cdot1)$$

where $\tilde{z}_{12}=\tilde{z}_1-\tilde{z}_2-\tilde{\theta}_1\tilde{\theta}_2$ and $z=(z, \theta)\to\tilde{z}=(\tilde{z}(z, \theta), \tilde{\theta}(z, \theta))$ is any superconformal transformation. Then we have

$$D_1D_2F(z_1, z_2)=\frac{(D_1\tilde{\theta}_1)(D_2\tilde{\theta}_2)}{\tilde{z}_{12}}-\frac{1}{z_{12}}$$
$$=\frac{1}{z_{12}}\frac{1+\theta_{12}\hat{D}_2+z_{12}\hat{D}_3+z_{12}\theta_{12}\hat{D}_4+\cdots}{1+\theta_{12}\hat{D}_2+z_{12}\hat{D}_3+z_{12}\theta_{12}(\hat{D}_4/2+\hat{D}_2\hat{D}_3)+\cdots}-\frac{1}{z_{12}}$$
$$=\theta_{12}\frac{1}{2}(\hat{D}_4-2\hat{D}_2\hat{D}_3)+O(z_{12}), \qquad (3\cdot2)$$

where

$$\hat{D}_2=\frac{D_2{}^2\tilde{\theta}_2}{D_2\tilde{\theta}_2}, \quad \hat{D}_3=\frac{D_2{}^3\tilde{\theta}_2}{D_2\tilde{\theta}_2}, \quad \hat{D}_4=\frac{D_2{}^4\tilde{\theta}_2}{D_2\tilde{\theta}_2}$$

and

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$$S(\tilde{z}_2, z_2) \equiv \hat{D}_4 - 2\hat{D}_2\hat{D}_3 \tag{3·3}$$

is the super Schwarzian derivative.

Recall the function $G(z_1, z_2)$ defined by (2·15), which is the scalar function, i.e., $G(z_1, z_2) = \tilde{G}(\tilde{z}_1, \tilde{z}_2)$. Then it follows that

$$D_1 D_2 G(z_1, z_2) = -\frac{1}{z_{12}} + l(z_1, z_2)\,, \quad l(z_1, z_2) \equiv D_1 D_2 \bar{G}(z_1, z_2)\,. \tag{3·4}$$

Similary

$$\tilde{D}_1 \tilde{D}_2 \tilde{G}(\tilde{z}_1, \tilde{z}_2) = -\frac{1}{\tilde{z}_{12}} + \tilde{l}(\tilde{z}_1, \tilde{z}_2)\,. \tag{3·5}$$

Multiplying both sides of Eq. (3·5) by $(D_1\tilde{\theta}_1)(D_2\tilde{\theta}_2)$, we have

$$\begin{aligned}(D_1\tilde{\theta}_1)(D_2\tilde{\theta}_2)\tilde{D}_1\tilde{D}_2\tilde{G}(\tilde{z}_1, \tilde{z}_2) &= D_1 D_2 G(z_1, z_2) \\ &= -\frac{(D_1\tilde{\theta}_1)(D_2\tilde{\theta}_2)}{\tilde{z}_{12}} + (D_1\tilde{\theta}_1)(D_2\tilde{\theta}_2)\,\tilde{l}(\tilde{z}_1, \tilde{z}_2)\,.\end{aligned} \tag{3·6}$$

Subtracting (3·6) from (3·4) we get

$$l(z_1, z_2) - \tilde{l}(\tilde{z}_1, \tilde{z}_2)(D_1\tilde{\theta}_1)(D_2\tilde{\theta}_2) = \frac{1}{z_{12}} - \frac{(D_1\tilde{\theta}_1)(D_2\tilde{\theta}_2)}{\tilde{z}_{12}} = -\theta_{12}\frac{1}{2}S(\tilde{z}_2, z_2) - O(z_{12})\,, \tag{3·7}$$

hence

$$\begin{aligned}D_1 l(z_1, z_2) - [D_1\tilde{l}(\tilde{z}_1, \tilde{z}_2)](D_1\tilde{\theta}_1)(D_2\tilde{\theta}_2) - \tilde{l}(\tilde{z}_1, \tilde{z}_2)(D_1{}^2\tilde{\theta}_1)(D_2\tilde{\theta}_2) \\ = -\frac{1}{2}S(\tilde{z}_2, z_2) + D_1 O(z_{12})\,.\end{aligned} \tag{3·8}$$

In this equation we take the limit $z_1 \to z_2$, so that

$$D_1 l(z_1, z_2) \to R(z_2)\,,$$

$$[D_1\tilde{l}(\tilde{z}_1, \tilde{z}_2)](D_1\tilde{\theta}_1)(D_2\tilde{\theta}_2) = (D_1\tilde{\theta}_1)\tilde{D}_1\tilde{l}(\tilde{z}_1, \tilde{z}_2)(D_1\tilde{\theta}_1)(D_2\tilde{\theta}_2) \to \tilde{R}(\tilde{z}_2)(D_2\tilde{\theta}_2)^3\,,$$

$$\tilde{l}(\tilde{z}_1, \tilde{z}_2) \to 0\,,$$

$$D_1 O(z_{12}) \to 0\,.$$

Therefore, we have

$$R(z_2) = \tilde{R}(\tilde{z}_2)(D_2\tilde{\theta}_2)^3 - \frac{1}{2}S(\tilde{z}_2, z_2)\,. \tag{3·9}$$

Ths is nothing but the transformation law of the super Schwarzian connection (2·22).

Note that the super Schwarzian derivative $S(\tilde{z}, z)$ becomes zero when $z = (z, \theta) \to \tilde{z} = (\tilde{z}, \tilde{\theta})$ is the super Möbius transformation[10]

$$\tilde{z} = \frac{az+b}{cz+d} + \theta\frac{\gamma z + \delta}{(cz+d)^2}\,, \quad ad - bc = 1\,,$$

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$$\tilde{\theta}=\frac{\gamma z+\delta}{cz+d}+\theta\frac{1+\delta\gamma/2}{cz+d}. \tag{3·10}$$

This shows that $R(z)\ ds^{3/2}$ is periodic on Σ.

§ 4. Concluding remarks

We have constructed the super Schwarzian connection $R(z)$ in the superstring theory on the higher-genus Riemann surface. We have found that the operator product expansion for the super stress-energy tensor contains the $R(z)$. Then by use of this $R(z)$ the super KN algebra has been derived. We have given the transformation law of the super Schwarzian connection obtained above. Owing to this law the cocycle χ_{VW} defined by (2·30) is coordinate-independent.[4)]

Let us remark upon transformation property of the super stress-energy tensor $T(z)$. The operator product expansion (2·26) for $T(z)$ can be rewritten as

$$\boldsymbol{T}(z_1)\boldsymbol{T}(z_2)=\frac{\hat{c}}{4}\frac{1}{z_{12}^3}+\frac{3}{2}\frac{\theta_{12}}{z_{12}^2}\boldsymbol{T}(z_2)+\frac{1}{2}\frac{1}{z_{12}}D_2\boldsymbol{T}(z_2)+\frac{\theta_{12}}{z_{12}}\partial_2\boldsymbol{T}(z_2)+\cdots, \tag{4·1}$$

where

$$\boldsymbol{T}(z)\equiv T(z)-\frac{\hat{c}}{2}R(z). \tag{4·2}$$

This is the same form as in the genus-zero case. The effect of a finite superconformal transformation $z\to\tilde{z}$ on $\boldsymbol{T}(z)$ is computed by requiring the expansion (4·1) to hold in both z and $\tilde{z}$ and is known to be

$$\boldsymbol{T}(z)=\tilde{\boldsymbol{T}}(\tilde{z})(D\tilde{\theta})^3+\frac{\hat{c}}{4}S(\tilde{z},z). \tag{4·3}$$

Recalling the transformation law (3·9) of $R(z)$ we find the transformation law of $T(z)$ without the anomalous term[11)]

$$T(z)=\tilde{T}(\tilde{z})(D\tilde{\theta})^3. \tag{4·4}$$

Namely, our $T(z)$ is the primary superconformal field with weight $h=3/2$. This makes the KN operator L_V defined by (2·28) coordinate-independent.

Finally we would like to reconsider the definition (2·28) of L_V. Since the $d\theta$ here is equivalent to $\partial/\partial\theta$, the $dzd\theta$ is not appropriate for the super differential. We propose a new super differential

$$\delta_z\equiv dz\partial_\theta+d_z\theta=dsD+d\theta, \tag{4·5}$$

where d_z is a total derivative operator

$$d_z\equiv dz\partial_z+d\theta\partial_\theta=\delta_z D. \tag{4·6}$$

Under the superconformal transformation the δ_z transforms according to the law

$$\delta_z=\tilde{\delta}_z(D\tilde{\theta}). \tag{4·7}$$

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The super KN operator L_V should be defined at first by using the δ_z as follows:

$$L_V = \frac{1}{2\pi i}\oint_{C_\tau} \delta_z V(z)T(z)\,, \tag{4·8}$$

which reduces to (2·28), because $V(z)T(z)$ is holomorphic and univalent around P_+. The same is true for the definition (2·30) of χ_{VW}. The details of the new super differential δ_z will be reported elsewhere.

Appendix

We would like to show the relation (2·25). Since the function $l(z_1, z_2)$ is defined by $l(z_1, z_2) = D_1 D_2 \bar{G}(z_1, z_2)$, (2·20), it is antisymmetric with respect to z_1 and z_2. Hence $l(z_1, z_2) = l(1, 2)$ has a form, by considering the form of (2·12),

$$l(1,2) = \sum_{m,n} A_{[mn]}(z_1{}^m z_2{}^n - z_1{}^n z_2{}^m) + \theta_1\theta_2 \sum_{m,n} B_{(mn)}(z_1{}^m z_2{}^n + z_1{}^n z_2{}^m)\,, \tag{A·1}$$

where $A_{[mn]}$ is antisymmetric and $B_{(mn)}$ is symmetric with respect to m and n. Therefore, we have

$$D_1 l(1,2) = \theta_1 \sum_{m,n} A_{[mn]}(m z_1{}^{m-1} z_2{}^n - n z_1{}^{n-1} z_2{}^m) + \theta_2 \sum_{m,n} B_{(mn)}(z_1{}^m z_2{}^n + z_1{}^n z_2{}^m)\,,$$

$$D_1{}^2 l(1,2) = \sum A_{[mn]}(m z_1{}^{m-1} z_2{}^n - n z_1{}^{n-1} z_2{}^m) + \theta_1\theta_2 \sum B_{(mn)}(m z_1{}^{m-1} z_2{}^n + n z_1{}^{n-1} z_2{}^m)\,,$$

$$D_1{}^3 l(1,2) = \theta_1 \sum A_{[mn]}(m(m-1) z_1{}^{m-2} z_2{}^n - n(n-1) z_1{}^{n-2} z_2{}^m)$$
$$+ \theta_2 \sum B_{(mn)}(m z_1{}^{m-1} z_2 + n z_1{}^{n-1} z_2{}^m)$$

and

$$R_2 \equiv (D_1 l(1,2))_2 = \theta_2 \sum A_{[mn]}(m-n) z_2{}^{m+n-1} + \theta_2 \sum B_{(mn)} 2 z_2{}^{m+n}\,, \tag{A·2}$$

$$(D_1{}^3 l(1,2))_2 = \theta_2 \sum A_{[mn]}[m(m-1) - n(n-1)] z_2{}^{m+n-2} + \theta_2 \sum B_{(mn)}(m+n) z_2{}^{m+n-1}\,, \tag{A·3}$$

$$(D_1{}^2 l(1,2))_2 = \sum A_{[mn]}(m-n) z_2{}^{m+n-1}\,, \tag{A·4}$$

$$\frac{1}{2} D_2 (D_1{}^2 l(1,2))_2 = \frac{1}{2}\theta_2 \sum A_{[mn]}(m-n)(m+n-1) z_2{}^{m+n-2}\,. \tag{A·5}$$

Subtracting (A·5) from (A·3) we get

$$(D_1{}^3 l(1,2))_2 - \frac{1}{2} D_2 (D_1{}^2 l(1,2))_2$$
$$= \frac{1}{2}\theta_2 \sum A_{[mn]}(m-n)(m+n-1) z_2{}^{m+n-2} + \theta_2 \sum B_{(mn)}(m+n) z_2{}^{m+n-1}\,.$$

The right-hand side terms are equal to $\partial_2 R_2/2$, as is seen from (A·2).

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