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The general relativistic MHD dynamo equation

M. Marklund[1,2] and C. A. Clarkson[3]
[1] *Department of Physics, Umeå University, SE–901 87 Umeå, Sweden*
[2] *Centre for Fundamental Physics, Rutherford Appleton Laboratory, Chilton Didcot, Oxfordshire, OX11 0QX, UK*
[3] *Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth, PO1 2EG, UK*
Email: mattias.marklund@physics.umu.se and chris.clarkson@port.ac.uk

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ABSTRACT
The magnetohydrodynamic dynamo equation is derived within general relativity, using the covariant $1+3$ approach, for a plasma with finite electric conductivity. This formalism allows for a clear division and interpretation of plasma and gravitational effects, and we have not restricted to a particular spacetime geometry. The results should be of interest in astrophysics and cosmology, and the formulation is well suited to gauge invariant perturbation theory. Moreover, the dynamo equation is presented in some specific limits. In particular, we consider the interaction of gravitational waves with magnetic fields, and present results for the evolution of the linearly growing electromagnetic induction field, as well as the diffusive damping of these fields.

Key words: MHD — Gravitation — Gravitational waves

1 INTRODUCTION

Magnetic fields play a vital role in many astrophysical systems and in cosmology, and the possible sources for the fields on different scales has been the focus of immense research over the years (Kronberg 1994; Han & Wielebinski 2002; Parker 1979; Zeldovich, Ruzmaikin & Sokoloff 1983; Beck et al. 1996). In particular, the origin of large scale cosmological fields is still not clearly understood. Although there are well understood mechanisms for amplifying a given seed field (Grasso & Rubinstein 2001; Widrow 2002), the explanation of the seed sources is more cumbersome, especially in the case of an inflationary scenario. A wide range of possible physical explanations for establishing a seed field of high enough field strength has been proposed in the literature (see, e.g., Harrison (1970); Hogan (1983); Vachaspati (1991); Gasperini, Giovanini & Veneziano (1995); Sigl, Jedamzik & Olinto (1997); Joyce & Shaposhnikov (1997); Davis et al. (2001); Calzetta & Kandus (2002); Dimopoulos et al. (2002); Betschart, Dunsby & Marklund (2004); Matarrese et al. (2004)). The main mechanism behind the amplification of the seed fields is the dynamo (Parker 1979; Zeldovich, Ruzmaikin & Sokoloff 1983; Beck et al. 1996), for which the fluid motion manages to amplify even very weak fields on very short time scales. The dynamo mechanism derives from the magnetohydrodynamic (MHD) approximation, and it has been analysed vigorously within, e.g., expanding cosmological models (for a selection, see Fennelly (1980); Sil, Banerjee & Chatterjee (1996); Gailis, Frankel & Dettmann (1995); Jedamzik, Katalinić & Olinto (1998); Brandenburg, Enqvist & Olesen (1996); Subramanian & Barrow (1998) and references therein).

Cosmic magnetic fields are observed on all but the largest scales. On the other hand, gravitational waves have yet to be directly detected. Although strong evidence exists in their favour (such as the PSR 1913+16 binary pulsar observations (Hulse & Taylor 1975)), the very weak interaction of gravitons with light and matter makes an earth based detection difficult. Currently, gravitational wave detectors, both of interferometer and bar type, are coming on-line, and are expected to start collecting important astronomical data, opening up a new window for observational astrophysics and cosmology. As gravitational wave information is gathered, the detailed study of typical wave signatures will be of major importance. Thus, knowledge of both direct gravitational wave signature, and indirect signatures, e.g., electromagnetic waves induced by gravity waves (Clarkson et al. 2004), will facilitate the study of gravitational waves, and is likely to bring new understanding to the observations.

The interaction of gravitational waves, and other general relativistic gravity effects, with electromagnetic fields and plasmas have attracted a broad interest over the last few years, because its possible application to both astrophysics and cosmology (e.g., Blandford & Znajek (1977); Thorne & Macdonald (1982); Macdonald & Thorne (1982); Marklund, Dunsby & Brodin (2000); Tsagas, Dunsby & Marklund (2003); Clarkson et al. (2004); Tomimatsu (2000); Tsagas & Barrow (1997); Tsagas & Maartens

(2000); Tsagas (2001)). Here, we will derive the general form of the dynamo equation, using the 1 + 3 covariant formalism (Ellis & van Elst 1998), and analyse its coupling to matter and spacetime geometry. This formalism allows for a clear cut interpretation of gravitational vs. purely MHD effects, and also presents the influence of the gravitational kinematics on the MHD modes for straightforward comparison. Specifically, we will consider the propagation of gravitational waves through a resistive magnetohydrodynamic plasma, and derive covariant equations for the propagation of a resulting magnetohydrodynamic wave. Here, a word of caution concerning the use of the phrase 'dynamo' is in place. While speaking of the dynamo equation, a general evolution equation for the magnetic field in diffusive media is assumed, while the dynamo mechanism is a particular application of this general equation. Indeed, since the dynamo mechanism is without question the most prominent feature of the general dynamo equation, we would like to stress that although we derive the general dynamo equation, we will not apply this to the dynamo mechanism. Instead, we will investigate what could appropriately be termed induction fields. The waves presented here represents linearly growing induction fields, and are thus not due to the proper dynamo mechanism which instead gives rise to rapid amplification of seed magnetic field due to the vortical fluid motion. Furthermore, the effects of a finite conductivity is presented. The equations are analysed numerically, and we discuss possible applications to astrophysical and cosmological systems.

2 BASIC EQUATIONS

When analysing low frequency magnetised plasma phenomena, MHD gives an accurate and computationally economical description. Specifically, a simple plasma model is obtained if the characteristic MHD time scale is much longer than both the plasma oscillation and plasma particle collision time scales, and the characteristic MHD length scale is much longer than the plasma Debye length and the gyro radius. These assumptions will make it possible to describe a two-component plasma in terms of a one-fluid description, as the electrons can be considered inertialess (see also, e.g., Servin & Brodin (2003); Moortgat & Kuijpers (2003, 2004); Papadopoulos et al. (2001); Papadopoulos, Vlahos & Esposito. (2002)). The one-fluid description means a tremendous computational simplification, especially for complicated geometries. Moreover, if the mean fluid velocity, the mean particle velocity, and the Alfvén velocity is much smaller than the velocity of light in vacuum, the description becomes non-relativistic and simplifies further.

2.1 Covariant theory

The 1+3 covariant approach relies on the introduction of a timelike vector field u^a: $u_a u^a = -1$, with which all tensorial objects are split into their invariant 'time' and 'space' parts: scalars, 3-vectors, and projected, symmetric, trace-free (PSTF) tensors (see Ellis & van Elst (1998) for a comprehensive review). This allows us to write the coupled Einstein-Maxwell equations in a relatively simple and intuitive fashion. Here, we introduce consistent 3-vector notation of Euclidean vector calculus, making the fully general relativistic equations relatively easy to read.

In an arbitrary curved spacetime, in units such that $c = 1$ and $G = 1/8\pi$, Maxwell's equations take the form (Tsagas & Barrow 1997; Ellis & van Elst 1998; Marklund et al. 2003)

$$\dot{B}^{\langle a\rangle} + \mathrm{curl}\, E^a = -\left(\tfrac{2}{3}\Theta h^{ab} - \sigma^{ab} + \epsilon^{abc}\omega_c\right) B_b - \epsilon^{abc}\dot{u}_b E_c, \tag{1}$$

$$-\dot{E}^{\langle a\rangle} + \mathrm{curl}\, B^a = \mu_0 j^{\langle a\rangle} + \left(\tfrac{2}{3}\Theta h^{ab} - \sigma^{ab} + \epsilon^{abc}\omega_c\right) E_b - \epsilon^{abc}\dot{u}_b B_c, \tag{2}$$

$$\mathrm{D}_a E^a = 2\omega_a B^a, \tag{3}$$

$$\mathrm{D}_a B^a = -2\omega_a E^a, \tag{4}$$

where $\mathrm{curl}\, X^a \equiv \epsilon^{abc}\mathrm{D}_b X_c$ and $\dot{X}^{\langle a\rangle} \equiv h^{ab}\dot{X}_b$ for any spatial vector X^a, and $h_{ab} = g_{ab} + u_a u_b$ is the metric on the local rest space orthogonal to the observer four-velocity u^a. Moreover, we have assumed quasi-neutrality, i.e., the total charge density satisfies $\rho \approx 0$ since the electrons and ions follow the same motion. Furthermore, we will assume that Ohm's law in the form

$$j^{\langle a\rangle} = \frac{1}{\mu_0\lambda}\left(E^a + \epsilon^{abc}v_b B_c\right), \tag{5}$$

holds, where $j^{\langle a\rangle} \equiv h^{ab}j_b$ is the three-current, $(\mu_0\lambda)^{-1}$ is the electric conductivity, λ is the magnetic diffusivity, and v^a is the plasma three-velocity. We note that high conductivity implies small magnetic diffusivity. The case of non-ideal MHD in curved spacetimes has been treated sparsely in the literature (with some exceptions (Fennelly 1980; Jedamzik, Katalinić & Olinto 1998)), and the possible effects of a finite conductivity due to gravity–electromagnetic field or fluid couplings has therefore largely gone unnoticed.

Thus, taking the curl of the curl of B^a and using the commutator relation

$$\mathrm{curl}\,(\mathrm{curl}\, B)^a = -\mathrm{D}^2 B^a + \mathrm{D}^a(\mathrm{D}_b B^b) + 2\epsilon^{abc}\dot{B}_{\langle b\rangle}\omega_c + \mathsf{R}^{ab}B_b, \tag{6}$$

where R_{ab} is defined by the relation

$$\mathsf{R}_{bd} \equiv \tfrac{2}{3}\left(\mathscr{E}+\Lambda-\tfrac{1}{3}\Theta^2-\sigma^2+\omega^2\right)h_{bd}+\Pi_{bd}-\dot{\sigma}_{\langle bd\rangle}+\mathrm{D}_{\langle b}\dot{u}_{d\rangle} -\tfrac{1}{3}\Theta\left(\sigma_{bd}-\omega_{bd}\right)-\dot{u}_{\langle b}\dot{u}_{d\rangle}-2\left(\sigma_{\langle b}{}^{k}\sigma_{d\rangle k}+\omega_{\langle b}\omega_{d\rangle}\right)-2\sigma_{k[b}\omega^{k}{}_{d]}, \tag{7}$$

where $\mathscr{E}=\mathscr{E}_{\text{fluid}}+\frac{1}{2}(E^2+B^2)$ is the total energy density, and $\Pi^{ab}=\Pi^{ab}_{\text{fluid}}-(E^{\langle a}E^{b\rangle}+B^{\langle a}B^{b\rangle})$ is the total anisotropic pressure. Taking the curl of Eq. (2), using Eq. (1) and (5) to replace E^a by B^a and $j^{\langle a\rangle}$, we obtain

$$\mathrm{curl}\,(\mathrm{curl}\,B)^a=\lambda^{-1}\epsilon^{abc}\epsilon_{cde}\mathrm{D}_b(v^dB^e)-\left(\tfrac{2}{3}\Theta+\lambda^{-1}\right)\left[\dot{B}^{\langle a\rangle}+\left(\tfrac{2}{3}\Theta h^{ab}-\sigma^{ab}+\epsilon^{abc}\omega_c\right)B_b-\lambda\mu_0\epsilon^{abc}\dot{u}_bj_{\langle c\rangle}+\epsilon^{abc}\epsilon_{cde}\dot{u}_bv^dB^e\right] -\tfrac{2}{3}\mu_0\lambda\epsilon^{abc}j_{\langle b\rangle}\mathrm{D}_c\Theta+\tfrac{2}{3}\epsilon^{abc}\epsilon_{bde}v^dB^e\mathrm{D}_c\Theta-\epsilon^{abc}\mathrm{D}_b\left[\sigma_{cd}\left(\mu_0\lambda j^{\langle d\rangle}-\epsilon^{def}v_eB_f\right)\right]-\epsilon^{abc}\mathrm{D}_b\left[\epsilon_{cde}\omega^d\left(\mu_0\lambda j^{\langle e\rangle}-\epsilon^{efg}v_fB_g\right)\right] -\epsilon^{abc}\mathrm{D}_b\left(\epsilon_{cde}\dot{u}^dB^e\right)+\mathrm{curl}\,\dot{E}^{\langle a\rangle}. \tag{8}$$

Furthermore, the last term in Eq. (8), due to the displacement current $\dot{E}^{\langle a\rangle}$, may be written

$$\mathrm{curl}\,\dot{E}^{\langle a\rangle}=-\ddot{B}^{\langle a\rangle}+\Xi^a \tag{9}$$

where

$$\Xi^a\equiv-\left(\tfrac{2}{3}\dot{\Theta}h^{ab}-\dot{\sigma}^{\langle ab\rangle}+\epsilon^{abc}\dot{\omega}_{\langle c\rangle}\right)B_b-\left(\Theta h^{ab}-\sigma^{ab}+\epsilon^{abc}\omega_c\right)\dot{B}_{\langle b\rangle}-\tfrac{1}{3}\Theta\left(\tfrac{2}{3}\Theta h^{ab}-\sigma^{ab}+\epsilon^{abc}\omega_c\right)B_b -\epsilon^{abc}\dot{u}_b\left[\mathrm{curl}\,B_c-\mu_0j_{\langle c\rangle}+\epsilon_{cde}\dot{u}^dB^e-\left(\tfrac{1}{3}\Theta h_{cd}-\sigma_{cd}+\epsilon_{cde}\omega^e\right)E^d\right]+\tfrac{1}{2}\epsilon^{abc}E_bq_c+\left[2\dot{u}^{\langle a}\omega^{b\rangle}+\mathrm{D}^{\langle a}\omega^{b\rangle}-(\mathrm{curl}\,\sigma)^{ab}\right]E_b -\epsilon^{abc}\ddot{u}_{\langle b\rangle}E_c+\epsilon^{abc}\left(\sigma_{bd}+\epsilon_{bde}\omega^e\right)\mathrm{D}^dE_c \tag{10}$$

by commuting spatial and time like covariant derivatives and using Maxwell's equations. The commutation of derivatives introduces curvature effects into the expression. Here $q^a=q^a_{\text{fluid}}+\epsilon^{abc}E_bB_c$ is the energy flux due to fluid and electromagnetic (i.e., Poynting flux) contributions. The latter derives from the gravitational self-interaction of the electromagnetic field, and can in many cases safely be neglected. We note that all terms in Ξ^a defined by Eq. (10) are curvature contributions. The displacement current is normally neglected in the MHD approximation, since it corresponds to high frequency phenomena. Here we see that the presence of gravity alters this interpretation of the displacement current. We will be interested in low frequency phenomena, and we therefore assume $\mathrm{curl}\,\dot{E}^{\langle a\rangle}\approx\Xi^a$.

Thus, equating (6) with the expression (8), we obtain the general relativistic dynamo equation (see also Tsagas (2004) for the source free general relativistic electromagnetic wave equations)

$$\dot{B}^{\langle a\rangle}-\epsilon^{abc}\epsilon_{cde}\mathrm{D}_b(v^dB^e)-\lambda\mathrm{D}^2B^a=-\tfrac{2}{3}\lambda\Theta\dot{B}^{\langle a\rangle}+2\lambda\epsilon^{abc}\omega_b\dot{B}_{\langle c\rangle}+2\lambda\mathrm{D}^a\left[\omega_b\left(\mu_0\lambda j^{\langle b\rangle}-\epsilon^{bcd}v_cB_d\right)\right]-\lambda\mathsf{R}^{ba}B_b -\left(1+\tfrac{2}{3}\lambda\Theta\right)\left[\left(\tfrac{2}{3}\Theta h^{ab}-\sigma^{ab}+\epsilon^{abc}\omega_c\right)B_b-\mu_0\lambda\epsilon^{abc}\dot{u}_bj_{\langle c\rangle}+\epsilon^{abc}\epsilon_{cde}\dot{u}_bv^dB^e\right] -\tfrac{2}{3}\mu_0\lambda^2\epsilon^{abc}j_{\langle b\rangle}\mathrm{D}_c\Theta+\tfrac{2}{3}\lambda\epsilon^{abc}\epsilon_{bde}v^dB^e\mathrm{D}_c\Theta-\lambda\epsilon^{abc}\mathrm{D}_b\left[\sigma_{cd}\left(\mu_0\lambda j^{\langle d\rangle}-\epsilon^{def}v_eB_f\right)\right] -\lambda\epsilon^{abc}\mathrm{D}_b\left[\epsilon_{cde}\omega^d\left(\mu_0\lambda j^{\langle e\rangle}-\epsilon^{efg}v_fB_g\right)\right]-\lambda\epsilon^{abc}\mathrm{D}_b\left(\epsilon_{cde}\dot{u}^dB^e\right)+\lambda\Xi^a \tag{11}$$

where we have used Maxwell's equation (4) and Ohm's law (5). The terms on the right hand of the dynamo equation (11) represents the influence of general relativistic gravity, while the left hand side gives the normal dynamo action from the fluid vorticity $\epsilon_{abc}\mathrm{D}^bv^c$.

Sometimes it may be advantageous to keep the electric field in the gravitational right hand side of Eq. (11), in which case we have

$$\dot{B}^{\langle a\rangle}-\epsilon^{abc}\epsilon_{cde}\mathrm{D}_b(v^dB^e)-\lambda\mathrm{D}^2B^a=-\tfrac{2}{3}\lambda\Theta\dot{B}^{\langle a\rangle}+2\lambda\epsilon^{abc}\omega_b\dot{B}_{\langle c\rangle}+2\lambda\mathrm{D}^a\left(\omega_bE^b\right)-\lambda\mathsf{R}^{ba}B_b -\left(1+\tfrac{2}{3}\lambda\Theta\right)\left[\left(\tfrac{2}{3}\Theta h^{ab}-\sigma^{ab}+\epsilon^{abc}\omega_c\right)B_b-\epsilon^{abc}\dot{u}_bE_c\right]-\tfrac{2}{3}\epsilon^{abc}E_b\mathrm{D}_c\Theta-\lambda\epsilon^{abc}\mathrm{D}_b\left(\sigma_{cd}E^d\right) -\lambda\epsilon^{abc}\mathrm{D}_b\left(\epsilon_{cde}\omega^dE^e\right)-\lambda\epsilon^{abc}\mathrm{D}_b\left(\epsilon_{cde}\dot{u}^dB^e\right)+\lambda\Xi^a. \tag{12}$$

2.2 Three-dimensional vector notation

The general relativistic dynamo equation may also be formulated in terms of three-dimensional vector notation. For every spacelike vector X^a or PSTF tensor Y^{ab} we denote the corresponding three-dimensional vector or tensor according to $\vec{X}$ and $\bar{\bar{Y}}$. With these definitions, Eq. (12) takes the form

$$\dot{\vec{B}}-\vec{\nabla}\times(\vec{v}\times\vec{B})-\lambda\nabla^2\vec{B}=-\tfrac{2}{3}\lambda\Theta\dot{\vec{B}}+2\lambda\vec{\omega}\times\dot{\vec{B}}+2\lambda\vec{\nabla}(\vec{\omega}\cdot\vec{E})-\lambda\vec{B}\cdot\bar{\bar{\mathsf{R}}}-\left(1+\tfrac{2}{3}\lambda\Theta\right)\left(\tfrac{2}{3}\Theta\vec{B}-\bar{\bar{\sigma}}\cdot\vec{B}+\vec{B}\times\vec{\omega}-\vec{a}\times\vec{E}\right) -\tfrac{2}{3}\lambda\vec{E}\times\vec{\nabla}\Theta-\lambda\vec{\nabla}\times(\bar{\bar{\sigma}}\cdot\vec{E})-\lambda\vec{\nabla}\times(\vec{\omega}\times\vec{E})-\lambda\vec{\nabla}\times(\vec{a}\times\vec{B})+\lambda\vec{\Xi}, \tag{13}$$

where we have introduced the three-dimensional acceleration vector $\vec{a}$ corresponding to $\dot{u}^a$, and $\vec{\nabla}$ corresponds to D^a. Dot and cross products are defined in the obvious way. Furthermore, the terms due to the displacement current takes the form

$$\vec{\Xi} = -\left(\tfrac{2}{3}\dot{\Theta}\vec{B} - \dot{\bar{\bar{\sigma}}}\cdot\vec{B} + \vec{B}\times\dot{\vec{\omega}}\right) - \left(\tfrac{2}{3}\Theta\dot{\vec{B}} - \bar{\bar{\sigma}}\cdot\dot{\vec{B}} + \dot{\vec{B}}\times\vec{\omega}\right) - \dot{\vec{a}}\times\vec{E} + \tfrac{1}{3}\Theta\vec{\nabla}\times\vec{E} + (\bar{\bar{\sigma}}\cdot\vec{\nabla})\times\vec{E} - (\vec{\omega}\times\vec{\nabla})\times\vec{E}$$
$$-\vec{a}\times\left[\vec{\nabla}\times\vec{B} - \mu_0\vec{j} + \vec{a}\times\vec{B} - \left(\tfrac{2}{3}\Theta\vec{E} - \bar{\bar{\sigma}}\cdot\vec{E} + \vec{E}\times\vec{\omega}\right)\right] + \tfrac{1}{2}\vec{E}\times\vec{q} - \bar{\bar{H}}\cdot\vec{E}, \tag{14}$$

where we have introduced the magnetic part of the Weyl tensor $H^{ab} = -2\dot{u}^{\langle a}\omega^{b\rangle} - \mathrm{D}^{\langle a}\omega^{b\rangle} + (\mathrm{curl}\,\sigma)^{ab}$, which signifies the presence of gravitational waves or frame dragging effects.

2.3 The equations of motion

The dynamo equation contains the centre of mass fluid velocity $v^a \equiv (\mathscr{E}_{(e)}v^a_{(e)} + \mathscr{E}_{(i)}v^a_{(i)})/(\mathscr{E}_{(e)} + \mathscr{E}_{(i)})$ and the current $j^{\langle a\rangle} \equiv \rho_{(e)}v^a_{(e)} + \rho_{(i)}v^a_{(i)}$, e (i) denoting the electrons (ions). Using quasi-neutrality, the evolution of the total fluid energy density (dropping the index denoting the fluid) $\mathscr{E} \equiv \mathscr{E}_{(e)} + \mathscr{E}_{(i)}$ and the centre of mass velocity is given by

$$\dot{\mathscr{E}} + \mathrm{D}_a(\mathscr{E}v^a) = -\left(\Theta + \dot{u}_a v^a\right)\mathscr{E}, \tag{15}$$

and

$$\mathscr{E}\left(\dot{v}^{\langle a\rangle} + v^b\mathrm{D}_b v^a\right) = -\left(\tfrac{1}{3}\Theta v^a + \dot{u}^a + \sigma^a{}_b v^b + \epsilon^{abc}\omega_b v_c\right)\mathscr{E} + \epsilon^{abc}j_b B_c, \tag{16}$$

respectively, in the cold plasma limit. Moreover, the current j^a can be expressed in terms of the magnetic field via Maxwell's equation (2) and Ohm's law (5).

We note that the Eqs. (15) and (16) can easily be generalised to incorporate anisotropic and/or viscous effects (see, e.g., Subramanian & Barrow (1998)).

3 SPECIAL CASES

In order to extract some information from the rather complicated relativistic dynamo equation, we shall present some special cases.

3.1 Infinite conductivity

The large bulk of literature on general relativistic MHD has treated the case of ideal MHD, i.e., infinite conductivity. As shown below, the general dynamo equation greatly simplifies when this assumption is made.

As $\sigma \to \infty$, the diffusion of the magnetic field lines decreases, i.e., $\lambda \to 0$, and we are left with

$$\dot{B}^{\langle a\rangle} - \epsilon^{abc}\epsilon_{cde}\mathrm{D}_b(v^d B^e) = -\left(\tfrac{2}{3}\Theta h^{ab} - \sigma^{ab} + \epsilon^{abc}\omega_c\right)B_b - \epsilon^{abc}\epsilon_{cde}\dot{u}_b v^d B^e, \tag{17}$$

or

$$\dot{\vec{B}} - \vec{\nabla}\times(\vec{v}\times\vec{B}) = -\left[\tfrac{2}{3}\Theta\vec{B} - \bar{\bar{\sigma}}\cdot\vec{B} + \vec{B}\times\vec{\omega} + \vec{a}\times\left(\vec{v}\times\vec{B}\right)\right], \tag{18}$$

which of course also can be found directly from Maxwell's equations and the vanishing of the Lorentz force. Infinite conductivity is a common assumption in cosmology, for example.

3.2 Effects of expansion/collapse

As in the case of infinite conductivity, the case of curved spacetime MHD has largely resorted to analysing the effects of expansion/collapse, at which the gravitational–electromagnetic field coupling is kept at a minimum. It also gives a good description of some fundamental aspects of cosmological MHD phenomena.

The simplest case where the nontrivial effects of expansion occur is in homogeneous and isotropic fluid background. The homogeneity and isotropy significantly simplifies the Einstein equations, and the spacetime is described in terms of the scalar quantities $\mathscr{E}$, p, Θ, and Λ. Therefore, equations (11) and (13) simplify to

$$\dot{B}^a - \epsilon^{abc}\epsilon_{cde}\mathrm{D}_b(v^d B^e) - \lambda\mathrm{D}^2 B^a = -\tfrac{2}{3}\lambda\Theta\dot{B}^a - \lambda\mathsf{R}^{ba}B_b - \left(1 + \tfrac{2}{3}\lambda\Theta\right)\tfrac{2}{3}\Theta B^a + \lambda\Xi^a, \tag{19}$$

and

$$\dot{\vec{B}} - \vec{\nabla}\times(\vec{v}\times\vec{B}) - \lambda\nabla^2\vec{B} = -\tfrac{2}{3}\lambda\Theta\dot{\vec{B}} - \lambda\vec{B}\cdot\bar{\bar{\mathsf{R}}} - \left(1 + \tfrac{2}{3}\lambda\Theta\right)\tfrac{2}{3}\Theta\vec{B} + \lambda\vec{\Xi}, \tag{20}$$

respectively, where $\mathsf{R}_{ab} = \tfrac{2}{3}(\mathscr{E} + \Lambda - \tfrac{1}{3}\Theta^2)h_{ab}$ is the Ricci three-curvature. The effects of the displacement current becomes

$$\Xi^a = -\tfrac{2}{3}\dot{\Theta}B^a - \Theta\dot{B}^a - \tfrac{2}{9}\Theta^2 B^a, \tag{21}$$

i.e.,

$$\vec{\Xi} = -\tfrac{2}{3}\dot{\Theta}\vec{B} - \Theta\dot{\vec{B}} - \tfrac{2}{9}\Theta^2\vec{B} \tag{22}$$

Here we have neglected the gravitational self-interaction of the electromagnetic field. Thus, from Eq. (19) and (20), together with (21) and (22), we obtain

$$\dot{B}^a - \epsilon^{abc}\epsilon_{cde}\mathrm{D}_b(v^dB^e) - \lambda\mathrm{D}^2B^a = -\tfrac{1}{3}\Theta(2B^a + 5\lambda\dot{B}^a) - \tfrac{1}{3}\lambda\left(\mathscr{E} - 3p + 4\Lambda + \tfrac{2}{3}\Theta^2\right)B^a, \tag{23}$$

and

$$\dot{\vec{B}} - \vec{\nabla}\times(\vec{v}\times\vec{B}) - \lambda\nabla^2\vec{B} = -\tfrac{1}{3}\Theta(2\vec{B} + 5\lambda\dot{\vec{B}}) - \tfrac{1}{3}\lambda\left(\mathscr{E} - 3p + 4\Lambda + \tfrac{2}{3}\Theta^2\right)\vec{B}, \tag{24}$$

respectively. Thus, we see that for collapsing solutions ($\Theta < 0$), there will be a growing magnetic field.

3.3 The effects of rotation

In a rotating space time, i.e., $\omega^a \neq 0$, Einstein's equations shows that the vorticity has as its source the acceleration $\dot{u}^a$. Gödel's universe does not require this, as it is spacetime homogeneous, but for the sake of completeness we will keep both spacetime acceleration and vorticity, while neglecting the expansion and shear (thus only considering only rigid rotation), in the equations presented below. They read

$$\begin{aligned}\dot{B}^{\langle a\rangle} &- \epsilon^{abc}\epsilon_{cde}\mathrm{D}_b(v^dB^e) - \lambda\mathrm{D}^2B^a = 2\lambda\epsilon^{abc}\omega_b\dot{B}_{\langle c\rangle} + 2\lambda\mathrm{D}^a\left[\omega_b\left(\mu_0\lambda j^{\langle b\rangle} - \epsilon^{bcd}v_cB_d\right)\right] - \lambda\mathsf{R}^{ba}B_b - \lambda\epsilon^{abc}\mathrm{D}_b\left(\epsilon_{cde}\dot{u}^dB^e\right)\\ &-\left[-\epsilon^{abc}\omega_bB_c - \mu_0\lambda\epsilon^{abc}\dot{u}_bj_{\langle c\rangle} + \epsilon^{abc}\epsilon_{cde}\dot{u}_bv^dB^e\right] - \lambda\epsilon^{abc}\mathrm{D}_b\left[\epsilon_{cde}\omega^d\left(\mu_0\lambda j^{\langle e\rangle} - \epsilon^{efg}v_fB_g\right)\right] + \lambda\Xi^a\end{aligned} \tag{25}$$

or

$$\begin{aligned}\dot{\vec{B}} &- \vec{\nabla}\times(\vec{v}\times\vec{B}) - \lambda\nabla^2\vec{B} = 2\lambda\,\vec{\omega}\times\dot{\vec{B}} + 2\lambda\vec{\nabla}\left[\vec{\omega}\cdot\left(\mu_0\lambda\vec{j} - \vec{v}\times\vec{B}\right)\right] - \lambda\vec{B}\cdot\bar{\bar{\mathsf{R}}} - \left[\vec{B}\times\vec{\omega} - \vec{a}\times\left(\mu_0\lambda\vec{j} - \vec{v}\times\vec{B}\right)\right]\\ &-\lambda\vec{\nabla}\times\left[\vec{\omega}\times\left(\mu_0\lambda\vec{j} - \vec{v}\times\vec{B}\right)\right] - \lambda\vec{\nabla}\times\left(\vec{a}\times\vec{B}\right) + \lambda\vec{\Xi}.\end{aligned} \tag{26}$$

We note that the contribution Ξ^a from the displacement current is only slightly simplified compared to the generic case. Thus, as in many problems of general relativity, rotation of spacetime gives rise to highly complex equations, and the dynamo equation is no exception.

4 GRAVITATIONAL WAVES

Linear gravitational waves on a homogeneous background can be covariantly defined as tensor perturbations satisfying $\mathrm{D}_a\sigma^{ab} = \mathrm{D}_aE^{ab} = \mathrm{D}_aH^{ab} = 0$, thus being transverse and traceless. In order to clearly see the effects of gravitational waves on the induction of electromagnetic fields, we consider this perturbation on a Minkowski background. In this case, the dynamo equation becomes

$$\dot{B}^{\langle a\rangle} - \epsilon^{abc}\epsilon_{cde}\mathrm{D}_b(v^dB^e) - \lambda\mathrm{D}^2B^a = -\lambda\mathsf{R}^{ba}B_b + \sigma^{ab}B_b - \lambda\epsilon^{abc}\mathrm{D}_b(\sigma_{cd}E^d) + \lambda\Xi^a, \tag{27}$$

or

$$\dot{\vec{B}} - \vec{\nabla}\times(\vec{v}\times\vec{B}) - \lambda\nabla^2\vec{B} = -\lambda\vec{B}\cdot\bar{\bar{\mathsf{R}}} + \bar{\bar{\sigma}}\cdot\vec{B} - \lambda\vec{\nabla}\times(\bar{\bar{\sigma}}\cdot\vec{E}) + \lambda\vec{\Xi}, \tag{28}$$

where we have neglected all products of gravitational variables.

The contribution from Ξ^a becomes

$$\Xi^a = \dot{\sigma}^{\langle ab\rangle}B_b + \sigma^{ab}\dot{B}_{\langle b\rangle} + \epsilon^{abc}\sigma_{bd}\mathrm{D}^dE_c - (\mathrm{curl}\,\sigma)^{ab}E_b. \tag{29}$$

Equation (7) gives an expression for the Ricci three-curvature in terms of the shear of the perturbation. Thus, using (29), Eqs. (27) and (28) give us

$$\dot{B}^{\langle a\rangle} - \epsilon^{abc}\epsilon_{cde}\mathrm{D}_b(v^dB^e) - \lambda\mathrm{D}^2B^a = \sigma^{ab}B_b + \lambda\left[2\dot{\sigma}^{\langle ab\rangle}B_b - \epsilon^{abc}\mathrm{D}_b(\sigma_{cd}E^d) + \sigma^{ab}\dot{B}_{\langle b\rangle} + \epsilon^{abc}\sigma_{bd}\mathrm{D}^dE_c - H^{ab}E_b\right], \tag{30}$$

or

$$\dot{\vec{B}} - \vec{\nabla}\times(\vec{v}\times\vec{B}) - \lambda\nabla^2\vec{B} = \bar{\bar{\sigma}}\cdot\vec{B} + \lambda\left[2\dot{\bar{\bar{\sigma}}}\cdot\vec{B} - \vec{\nabla}\times(\bar{\bar{\sigma}}\cdot\vec{E}) + \bar{\bar{\sigma}}\cdot\dot{\vec{B}} + (\bar{\bar{\sigma}}\cdot\vec{\nabla})\times\vec{E} - \bar{\bar{H}}\cdot\vec{E}\right], \tag{31}$$

respectively, where $H^{ab} = (\mathrm{curl}\,\sigma)^{ab}$. The terms on the rhs describe the effect of the curvature, and given a GW they will drive the magnetic field evolution. We see that the finite electric conductivity gives rise to new couplings between GWs and magnetic fields.

4.1 Gravitational waves in a constant magnetic field

The case of a GW passing through an initially constant magnetic field will help illustrate these equations. We treat all disturbances ∇B in the magnetic field as well as the gravitational wave variables as first order perturbations, and we therefore neglect all terms in the equations which are a product of derivatives of the magnetic field with GW terms (σ_{ab}), and terms $\mathcal{O}([\nabla B]^2)$. We may also neglect disturbances in the energy density of the fluid. With these approximations, we find that the momentum conservation equation (16) becomes

$$\mathscr{E}\dot{v}^a = \epsilon^{abc}j_b B_c, \tag{32}$$

while the current is given by Eq. (2):

$$j^a = \mu_0^{-1}\,\mathrm{curl}\,B^a. \tag{33}$$

The dynamo equation (30) simplifies to

$$\dot{B}^a - \epsilon^{abc}\epsilon_{cde}B^e\mathrm{D}_b v^d - \lambda\mathrm{D}^2B^a = 2\lambda\dot{\sigma}^{ab}B_b + \sigma^{ab}B_b. \tag{34}$$

We may decouple Eqs. (32)–(34) by taking the time derivative of Eq. (34):

$$\ddot{B}^a - \lambda\mathrm{D}^2\dot{B}^a + \frac{1}{\mu_0\mathscr{E}}B^b\left[-B^a\mathrm{D}^2B_b + B^c(\mathrm{D}^a\mathrm{D}_bB_c - \mathrm{D}_b\mathrm{D}_cB^a)\right] = 2\lambda\ddot{\sigma}^{ab}B_b + \dot{\sigma}^{ab}B_b \tag{35}$$

where (neglecting backreaction from the electromagnetic field) the shear perturbation is determined by the source-free wave equation

$$\ddot{\sigma}^{ab} - \mathrm{D}^2\sigma^{ab} = 0. \tag{36}$$

We split the perturbed magnetic field parallel and perpendicular to the 'background' magnetic field according to

$$B^a = B_0\left[(1+\mathscr{B})e^a + \mathscr{B}^a\right], \quad \mathscr{B}^a \equiv N^{ab}B_b/B_0, \quad \text{and} \quad \mathscr{B} \equiv \left(e_aB^a - 1\right)/B_0. \tag{37}$$

where $N_{ab} = h_{ab} - e_ae_b$ and e^a is a spacelike unit vector: $e_ae^a = 1$ (see Clarkson & Barrett (2003) for a detailed discussion of this split). Here, B_0 is a constant, denoting the magnitude of the magnetic field when the GW is zero (in which case $B^a = B_0e^a$). The perturbations in the static magnetic field are given by $\mathscr{B}$, parallel to the background field, and $\mathscr{B}^a$ perpendicular to it. We neglect all products of $\mathscr{B}$ and $\mathscr{B}^a$, and terms of the form $\mathscr{B}\sigma$ (note that $\nabla e = \mathcal{O}(\sigma)$), since these are of higher order.

The shear perturbation can be decomposed according to

$$\sigma_{ab} = \Sigma_{ab} + 2\Sigma_{(a}e_{b)} + \left(e_ae_b - \tfrac{1}{2}N_{ab}\right)\Sigma \tag{38}$$

by introducing the gravitational wave variables

$$\Sigma \equiv \sigma^{ab}e_ae_b, \quad \Sigma_a \equiv N_{ab}\sigma^{bc}e_c, \quad \text{and} \quad \Sigma_{ab} \equiv \left(N_{(a}{}^cN_{b)}{}^d - \tfrac{1}{2}N_{ab}N^{cd}\right)\sigma_{cd}, \tag{39}$$

and the operators

$$\mathrm{D}_\parallel \equiv e^a\mathrm{D}_a = \frac{\mathrm{d}}{\mathrm{d}z} \quad \text{and} \quad \mathrm{D}_\perp^a \equiv N^{ab}\mathrm{D}_b, \tag{40}$$

allow us to split the spatial derivative along, and perpendicular to, the unperturbed magnetic field.[1]

Using these definitions, and the frame choice[2] $\dot{e}^a = 0 = \mathrm{D}_\perp^a e_a$, we may split Eq. (35) into a longitudinal scalar equation along e^a:

$$\ddot{\mathscr{B}} - \lambda\mathrm{D}^2\dot{\mathscr{B}} - C_A^2\mathrm{D}^2\mathscr{B} = 2\lambda\ddot{\Sigma} + \dot{\Sigma}, \tag{41}$$

where $C_A \equiv (\epsilon_0B_0^2/\mathscr{E})^{1/2}$ is the Alfvén velocity of the MHD plasma, and a transverse vector equation perpendicular to e^a:

$$\ddot{\mathscr{B}}^a - \lambda\mathrm{D}^2\dot{\mathscr{B}}^a - C_A^2\left(\mathrm{D}_\parallel^2\mathscr{B}^a - \mathrm{D}_\perp^a\mathrm{D}_\parallel\mathscr{B}\right) = 2\lambda\ddot{\Sigma}^a + \dot{\Sigma}^a. \tag{42}$$

We also have the $\mathrm{D}_aB^a = 0$ constraint, which takes the simple form $\mathrm{D}_\parallel\mathscr{B} + \mathrm{D}_\perp^a\mathscr{B}_a = 0$ using our new variables. Neglecting backreaction, gravitational waves propagating along the background magnetic field will not affect the magnetic field evolution, due to the condition of vanishing divergence $\mathrm{D}_a\sigma^{ab} = 0 = e_a\sigma^{ab}$. This can also be seen from Eq. (38), where (if e^a represents the propagation direction of the gravitational wave) Σ_{ab} is the only non-vanishing part of the decomposition of σ_{ab}, thus removing the coupling to the magnetic field via Σ and Σ^a. On the other hand, including the backreaction from the magnetic field on the gravitational wave will in general make the gravitational wave non-transverse, by the introduction of small scale tidal forces as well as large scale curvature effects, but this issue is left for future research.

Next we introduce spatial harmonics for the perturbed variables (see Clarkson & Barrett (2003)). Define the scalar harmonics

$$D_\perp^2Q_{(k_\perp)} = -k_\perp^2Q_{(k_\perp)}, \tag{43}$$

[1] Note that the definition of $\mathrm{D}_\perp^a$ for tensors has projection with N_{ab} on each free index (Clarkson & Barrett 2003).
[2] We refer to Clarkson & Barrett (2003) for a full discussion of frame choice vs. true GW degrees of freedom.

where $k_\perp$ is the harmonic number for the part of the fields perpendicular to e^a. We can then define vector harmonics of even and odd parity,

$$Q^a_{(k_\perp)} = (k_\perp)^{-1} D^a_\perp Q_{(k_\perp)} \quad \bar{Q}^a_{(k_\perp)} = (k_\perp)^{-1} \epsilon^a{}_b D^b_\perp Q_{(k_\perp)}, \tag{44}$$

respectively, where $\epsilon_{ab} = \epsilon_{abc} e^c = -\epsilon_{ba}$ is the volume element on two-surfaces orthogonal to e^a. These vector harmonics obey $D^2_\perp Q^a_{(k_\perp)} = -k^2_\perp Q^a_{(k_\perp)}$ (similarly for $\bar{Q}^a_{(k_\perp)}$).

Using these definitions, we may expand the magnetic field variables as

$$\mathscr{B} = \sum_{k_\perp} \mathscr{B}^S_{(k_\perp)} Q_{(k_\perp)}, \tag{45}$$

$$\mathscr{B}^a = \sum_{k_\perp} \mathscr{B}^V_{(k_\perp)} Q^a_{(k_\perp)} + \bar{\mathscr{B}}^V_{(k_\perp)} \bar{Q}^a_{(k_\perp)}, \tag{46}$$

and similarly for the GW shear variables

$$\Sigma = \sum_{k_\perp} \Sigma^S_{(k_\perp)} Q_{(k_\perp)}, \quad \text{and} \quad \Sigma^a = \sum_{k_\perp} \Sigma^V_{(k_\perp)} Q^a_{(k_\perp)} + \bar{\Sigma}^V_{(k_\perp)} \bar{Q}^a_{(k_\perp)}. \tag{47}$$

Using these harmonic expansions, Eq. (41) can now be written as a partial differential equation for each $k_\perp$, using $' = \partial_z$:

$$\ddot{\mathscr{B}}_S - C^2_A \mathscr{B}''_S - \lambda \dot{\mathscr{B}}''_S + k^2_\perp \lambda \dot{\mathscr{B}}_S + k^2_\perp C^2_A \mathscr{B}_S = 2\lambda \ddot{\Sigma}_S + \dot{\Sigma}_S, \tag{48}$$

while the even part of (42) may be derived from $k_\perp \mathscr{B}_V = \mathscr{B}'_S$ (the div-B constraint), and is in fact the same equation, but with S replaced by V; these give the dynamics of the 'even sector', while for the odd sector we have

$$\ddot{\bar{\mathscr{B}}}_V - C^2_A \bar{\mathscr{B}}''_V - \lambda \dot{\bar{\mathscr{B}}}''_V + k^2_\perp \lambda \dot{\bar{\mathscr{B}}}_V + k^2_\perp C^2_A \bar{\mathscr{B}}_V = 2\lambda \ddot{\bar{\Sigma}}_V + \dot{\bar{\Sigma}}_V. \tag{49}$$

The GW variables obey $\ddot{\Sigma} - D^2_\parallel \Sigma_S + k^2_\perp \Sigma_S = 0$, and similarly for Σ_V and $\bar{\Sigma}_V$; $\mathrm{D}^b \sigma_{ab} = 0$ implies $k_\perp \Sigma_V = \mathrm{D}_\parallel \Sigma_S$. We have dropped $k_\perp$ subscripts, with the understanding that these equations apply for each value of $k_\perp$. We may now introduce harmonics for ∂_z and ∂_t in the usual way if desired.

4.2 Integration

As a means of understanding the effect of conductivity on a magnetic field in a GW spacetime we shall consider the case of a Gaussian pulse of GW travelling in the positive z-direction through a static magnetic field such that at $t = 0$,

$$\Sigma^{(k)}_V = \Sigma e^{-z^2/L^2} \cos(\omega z), \tag{50}$$

$$\dot{\Sigma}^{(k)}_V = \Sigma e^{-z^2/L^2} \left[\omega \sin(\omega z) + 2\frac{z}{L} \cos(\omega z)\right], \tag{51}$$

where we set $\Sigma = -1$ without loss of generality (alternatively, this could be $\bar{\Sigma}_V$ because the decoupled equation for $\bar{\mathscr{B}}_V$ is identical to that of $\mathscr{B}_V$). This means that $\mathscr{B}$ is measured in units of the GW amplitude in the plots below. We will now integrate Eq: (48) for a variety of situations assuming that at $t = 0$, $\mathscr{B}_S = \mathscr{B}_V = 0$ – i.e., the interaction switches on at $t = 0$. We take $\omega = \pi/3$, $k_\perp = 0$ (so $\mathscr{B}_S = 0$) and $L = 8$, below, all in units where $c = 1$.

The qualitative features of finite conductivity are shown in figures 1 and 2.

In Fig. 1 we consider the case of $C_A = 1$, that is waves in a very tenuous plasma, giving rise to resonant magnetic field amplification. The figures shown apply equally to $\mathscr{B}_V$ or $\bar{\mathscr{B}}_V$. In the case of infinite conductivity, shown on the left, we have a linearly growing magnetic field pulse, which is continually sourced by the GW as it travels through the background field. When conductivity is present, however, this amplification is inhibited in that the induced magnetic field can only grow to a certain strength (compare the scales of the two plots) before a saturation amplitude is met, at an amplitude considerably less than if the magnetic diffusivity is zero.

If the magnetic field is strong, or the fluid dense, then the Alfvén velocity can be small, resulting in weak non-resonant amplification of GW. The plot on the left of Fig. 2 shows the modes with $C_A = 0.5$ when $\lambda = 0$. Dispersion shows up here as a trail of small amplitude waves, trailing behind the initial pulse, due to the mismatch in the GW and plasma dispersion relations. The plot on the right demonstrates what happens when the conductivity is finite. As in the resonant case the forward driven pulse is damped and can only reach a certain amplitude, which is considerably less than when $C_A = 1$. The trailing part of the wave has nothing to sustain it, so diffuses and rapidly decays away.

5 DISCUSSION

Finite conductivity can alter the behaviour of conducting fluids, plasmas and electromagnetic fields in a vast variety of ways. When spacetime curvature may be neglected these effects are well understood in a significant number of physical situations, ranging from the early universe, to astrophysical and laboratory plasmas. When spacetime curvature effects are present and

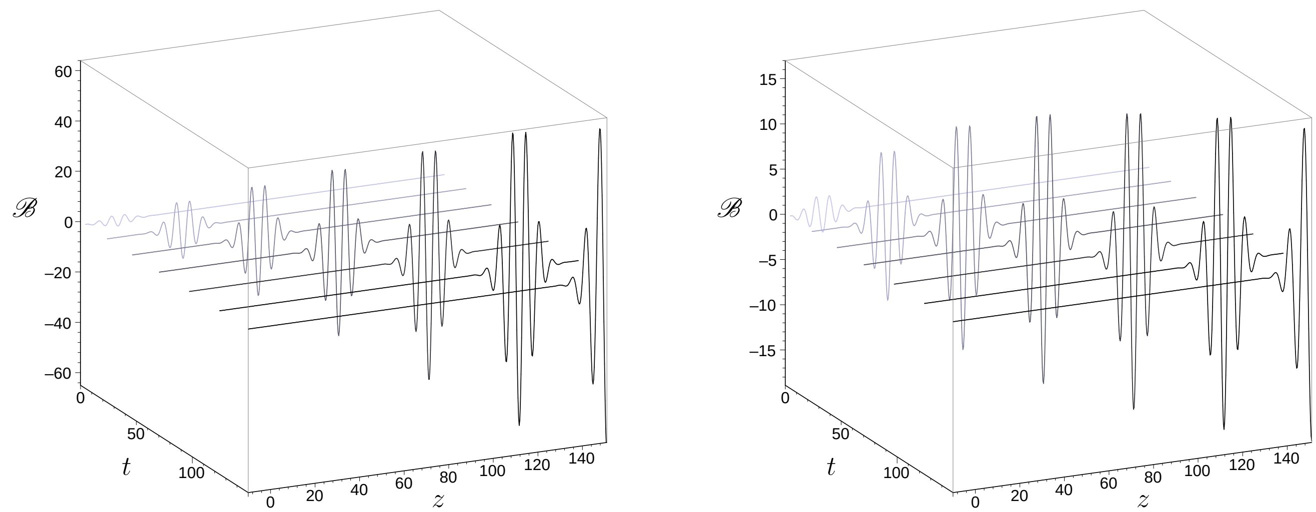

Figure 1. The effect of λ on long wavelength modes, with $C_A = 1$. The plot on the left has $\lambda = 0$ showing linear growth of the magnetic field as the GW passes through the background field. On the right, we have the same situation but with finite conductivity, $\lambda = 0.05$. While there is some early amplification of the magnetic field, this quickly saturates, with late time behaviour just a brief disturbance as the GW passes. Note the difference of the vertical scale between the two plots.

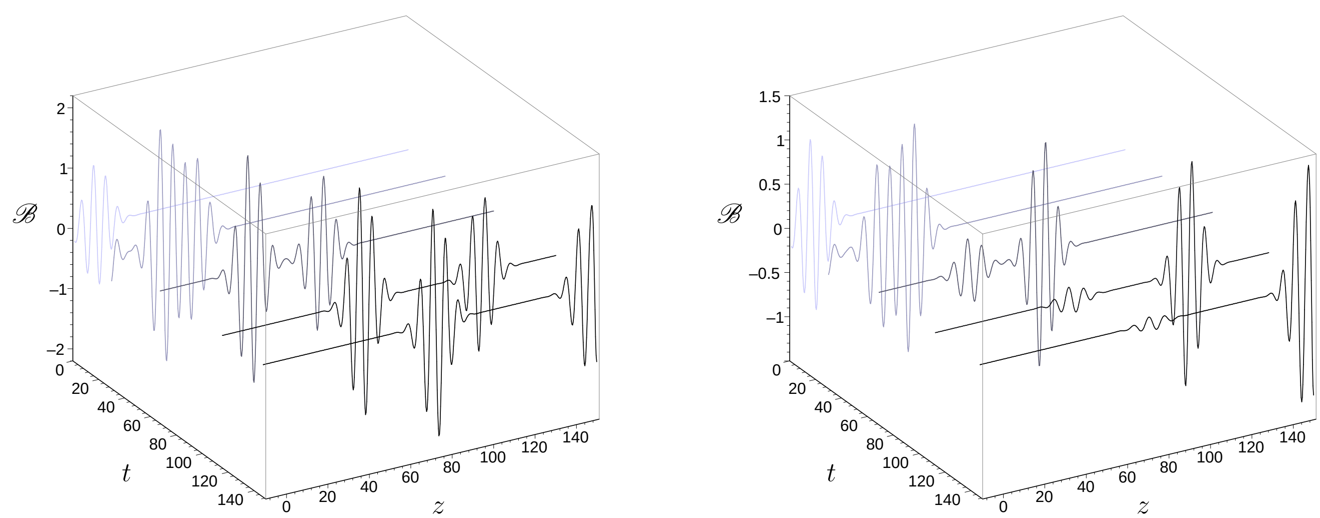

Figure 2. The effect of λ on slow Alfvén waves, with $C_A = 0.5$. The plot on the left has $\lambda = 0$ showing little growth of the magnetic field as the GW passes through the background field, trailed by a slow wave. The dispersion takes the form of a slower wave trailing the oscillations induced by the GW. On the right, we have the same scenario but with finite conductivity, $\lambda = 0.05$. The trailing Alfvén waves are dissipated very quickly. The main pulse, as in the previous plot, is from a direct forcing from the GW.

significant, however, very little is known about the role played by finite conductivity in astrophysics and cosmology. We present here, for the first time, the dynamo equation in its full generality in an arbitrary curved spacetime.

The MHD approximation, describing low frequency charged fluid phenomena, allows us to neglect the displacement current in Ampère's Law, from which we may derive a diffusion equation for the magnetic field – the dynamo equation (11) or (12). For easy comparison with non-general relativistic results we have utilised the intuitive 1+3 covariant approach to relativistic analysis (Ellis & van Elst 1998). This approach splits spacetime into space and time in a covariant way by use of a timelike vector field, which then also plays the role of the convective derivative, but now on an arbitrary curved manifold. Spacetime dynamics is then covariantly described by the use of invariantly defined scalars, 3-vectors, and PSTF tensors, made up from the Riemann curvature tensor, and the dynamical quantities associated with u^a. These quantities feed into Maxwell's equations in a non-trivial way, making the resultant electromagnetic field experience 'gravitational currents' of a very different nature from flat space physics.

In the relativistic dynamo equation we have derived, these gravitational currents come in a huge variety of terms which are very difficult to analyse in general. We have presented in some simplifying assumptions which are often used in spacetime modelling, in order to better gain an intuition about the kinds of effects conductivity coupled to gravity can produce.

As a specific example of analysing the relativistic dynamo equation, we considered in detail the case of a plane GW interacting with a static magnetic field. We believe this may play an significant role around compact objects, where magnetic fields and gravity waves can interact strongly. By introducing covariant vector harmonics adapted to the magnetic field we reduced the dynamo equation to two fairly simple coupled PDEs with a constraint, and one decoupled PDE, with the space dimension along the background field. These PDEs for the induced magnetic field are sourced by the GW in an intuitive fashion, and may be analysed by standard means. We integrated these equations numerically for a Gaussian pulse of radiation passing through the background static magnetic field; the summary of this integration is presented in the figures. In the case of infinite conductivity, there is linear amplification of the magnetic field which mimics the GW; this is followed by a trail of oscillations resulting from waves in the magnetic field travelling with the Alfvén speed. When magnetic diffusivity is present, these trailing modes diffuse quickly, and the overall amplification of the magnetic field is inhibited; a ripple in the magnetic field is forced as the GW passes through.

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