5th International Conference on Computational Harmonic Analysis

Vanderbilt University, Department of Mathematics
Nashville, TN, USA

May 19–23, 2014

Variable bandwidth in directional time-frequency analysis

Roza Aceska
Vanderbilt University
roza.aceska@gmail.com

We introduce the notion of variable bandwidth in time-frequency analysis with the additional element of direction. The directional short-time Fourier transform provides for a directionally sensitive time-frequency analysis. In addition, we define a directionally sensitive variable bandwidth weight on the time-frequency plane, which controls the frequency decay of a function (e.g. image) in a certain time interval and in a chosen direction.

Getting even more from less: Structure-exploiting sampling strategies for compressive imaging

Ben Adcock
Purdue University
adcock@purdue.edu
Coauthors: Anders C. Hansen, Clarice Poon and Bogdan Roman

It is well-known that natural images are not just sparse in wavelet bases and their various generalizations, but that there is a distinct structure to this sparsity. In this talk we present a method and a full theory for leveraging such structure within the context compressed sensing. This approach is based on recent theoretical developments which show that compressed sensing is possible under the substantially relaxed condition of asymptotic incoherence, as opposed to uniform incoherence. Structured sparsity can be exploited by sampling with asymptotically incoherent transforms according to a so-called multilevel random subsampling strategy. This approach improves on conventional compressed sensing strategies - typically based uniformly-incoherent random Gaussian and Bernoulli sensing matrices - in two important ways. First, one obtains a higher reconstruction quality, since uniformly incoherent matrices cannot exploit structured sparsity. Second, since it is a much weaker condition than uniform incoherence, one can easily find computationally-efficient sensing matrices that are asymptotically incoherent with wavelet bases. In particular, the Fourier and Hadamard transforms have this property, and unlike random matrices, both are highly efficient in terms of storage and computational cost. In the final part of the talk, we also compare this approach with other structure-exploiting algorithms for compressed sensing (typically based on wavelet tree or Gaussian mixture models) which seek to exploit structure by suitable modification of the reconstruction algorithm. We show that our approach of 'structured sampling', which uses the standard reconstruction algorithm of l^1 minimization, also outperforms these 'structured recovery' algorithms both in terms of speed and accuracy.

Interval Linear Optimization with Fuzzy Inequality Constraints

Ibraheem Alolyan
King Saud University
ialolyan@ksu.edu.sa

In many real-life situations, we come across problems with imprecise input values. Imprecisions are dealt with by various ways. One of them is interval based approach in which we model imprecise quantities by intervals, and suppose that the quantities may vary independently and simultaneously within their intervals. In most optimization problems, they are formulated using imprecise parameters. Such parameters can be considered as fuzzy intervals, and the optimization tasks with interval cost function are obtained. When realistic problems are formulated, a set of intervals may appear as coefficients in the objective function or the constraints of a linear programming problem. In this paper, we introduce a new method for solving linear optimization problems with interval parameters in the objective function and the inequality constraints, and we show the efficiency of the proposed method by presenting a numerical example.

Sampling and interpolation sets near the critical density in LCA groups

Jorge Abel Antezana
UNLP-CONICET
jaantezana@yahoo.com.ar
Coauthors: E. Agora, C. Cabrelli

In [3] Landau found necessary conditions satisfied by sampling and interpolation sets for Paley Wiener spaces PW_K, associated to a bounded set K of R^d. These conditions, given in terms of the so called lower and upper Beurling densities, D^- and D^+ respectively, are the following:

i. A sampling set L for PW_K satisfies $D^-(L) \geq |K|$;

ii. An interpolation set L for PW_K satisfies $D^+(L) \leq |K|$.

A natural question is whether or not there exist sampling and interpolation sets for PW_K with densities arbitrarily close to the critical density $|K|$. This is true, and it was proved by Marzo in [6] in R^d, adapting ideas of Lyubarskii and Seip [4] and Kohlenberg [2]. It also follows from a more recent work by Matei and Meyer [5].

Later on, in [1], Gröchenig, Kutyniok, and Seip suitably extended the concept of Beurling densities to the setting of locally compact abelian groups (LCA groups), and they proved a generalization of aforementioned Landau's theorem. However, in this setting, the authors left open the problem of existence of sampling and interpolation sets with densities arbitrarily close to the critical one. In this talk we will show that indeed, sampling and interpolation sets near the critical density do exist in the setting of LCA groups.

References

[1] Gröchenig K., Kutyniok G., Seip K., Landau's necessary density conditions for LCA groups, J. Funct. Anal. 255 (2008) 1831-1850.

[2] Kohlenberg A., Exact interpolation of band-limited functions, J. Appl. Phys., 24, no. 12 (1935) 1432-1436,

[3] H. Landau, Necessary density conditions for sampling and interpolation of certain entire functions, Acta Math. 117 (1967) 37-52.

[4] Lyubarskii Y. I., Seip K., Sampling and intepolating sequences for multiband-limited Functions and exponential bases on disconnected sets, J. Fourier Anal. Appl., 3, no. 5 (1997) 597-615.

[5] B. Matei, Y. Meyer, Simple quasicrystals are sets of stable sampling, Complex Var. Elliptic Equ. 55 (2010), 947-964.

[6] Marzo J., Riesz basis of exponentials for a union of cubes in R^d, preprint arXiv: math/0601288.

Gabor systems with randomization

Enrico Au-Yeung
Department of Mathematics, University of British Columbia
enricoauy@math.ubc.ca

The short-time Fourier transform and the corresponding inversion formula allow us to represent a function as a continuous superposition of time-frequency atoms given by translations and modulations of a fixed window function. A goal of time-frequency analysis is to find a discrete expansion of a function analogous to the continuous version. Recently, several authors have investigated Gabor frames, where the points in the time-frequency plane are irregular; these points do not form a lattice. Another direction of inquiry is to use several window functions instead of fixing one window function. Along this line of research, we introduce randomization to the Gabor system so that the window functions are random.

Finite extensions of Bessel sequences

Damir Bakic
University of Zagreb
bakic@math.hr
Coauthors: Tomislav Beric

We will characterize Bessel sequences in infinite-dimensional Hilbert spaces that can be extended to frames by adding finitely many vectors. We will also discuss finite-dimensional perturbations of Bessel sequences and some related topics.

Phase retrieval using Lipschitz continuous maps

Radu Balan
University of Maryland
rvbalan@math.umd.edu
Coauthors: Dongmian Zou, University of Maryland

In this note we prove that reconstruction from magnitudes of frame coefficients (the so called "phase retrieval problem") can be performed using Lipschitz continuous maps. Specifically we show that when the nonlinear analysis map $\alpha : \mathcal{H} \to \mathbb{R}^m$ is injective, with $(\alpha(x))_k = |\langle x, f_k\rangle|^2$, where $\{f_1, \ldots, f_m\}$ is a frame for the Hilbert space $\mathcal{H}$, then there exists a left inverse map $\omega : \mathbb{R}^m \to \mathcal{H}$ that is Lipschitz continuous. Additionally we obtain the Lipschitz constant of this inverse map in terms of the lower Lipschitz constant of α.

Noncommutative Zak transform and frame theory

Davide Barbieri
Universidad Autonoma de Madrid
davide.barbieri8@gmail.com
Coauthors: E. Hernndez, J. Parcet, V. Paternostro

Invariant subspaces with respect to several classes of unitary representations of noncommutative discrete groups can be effectively studied in spaces of operators by means of a generalized Zak transform. This turns out to define a surjective isometry onto an operator valued Hilbert space satisfying a quasi-periodicity condition. Relevant examples that can be treated are left shifts of countable subgroups, which generalize $\mathbb{Z}^n$ translations in $L^2(\mathbb{R}^n)$ to e.g. the corresponding situation in the Heisenberg group, or the quasiregular representation of a semidirect product. The main proofs will be given together with concrete examples.

Vector-valued ambiguity functions and balayage

John J. Benedetto
Norbert Wiener Center, University of Maryland
jjb@math.umd.edu

We solve an algebraic and an analytic problem related through the STFT.

For the algebraic problem, we define a vector-valued ambiguity function. The motivation is modelling of multi-sensor environments. The technology involves formulating and solving so-called frame multiplication problems in terms of representations of finite groups.

For the analytic problem, we use balayage to generalize Beurling's and Henry Landau's theory of Fourier frames to prove STFT and pseudo-differential operator frame inqualities.

The algebraic part is a collaboration with Travis Andrews and Jeff Donatelli. The analytic part is a collaboration with Enrico Au-Yeung.

Compression and signal processing using level-crossing sampling

Brigitte Bidegaray-Fesquet
CNRS, Grenoble, France
Brigitte.Bidegaray@imag.fr

An important issue in the design of mobile systems is to increase their autonomy. One way to achieve this is to use asynchronous event-driven architectures. These systems take typically samples each time a level is crossed. This leads to a reduced number of samples, compared to a Nyquist sampling but they are non-uniform. Dedicated signal processing algorithms have to be developed to deal with these samples. Each single computation is usually more complex than for uniform signal processing chains, but this is largely compensated by the drastic reduction of the number of samples. This form of signal compression is in particular studied for signals with known local Hlder regularity.

Dynamic equipartitioning of frame potentials: equiangular vs. equidistributed tight frames

Bernhard Bodmann
University of Houston
bgb@math.uh.edu
Coauthors: John Haas

Equiangular tight frames are optimal designs for many applications. However, their existence depends on dimensionality and size of the frame, and there does not seem to be a universal construction principle for all known examples. In this talk we present dynamic optimization principles in frame design as an alternative to traditional group-theoretic or combinatorial construction methods. The feasibility of dynamic frame design is based on a result due to Lojasiewicz which guarantees the convergence of the gradient descent on the manifold of Parseval frames for real analytic frame potentials. We introduce and discuss the class of equidistributed tight frames, which is more general than equiangular tight frames and is shown to have an abundance of examples in any dimension. Finally, equidistributed tight frames are characterized as the common stationary points for a family of real analytic frame potentials.

Toral eigenfuctions and their nodal sets

Jean Bourgain
Institute for Advanced Study
bourgain2010@gmail.com

The spectral properties of the torus are in some sense the easiest to explore because the eigenfunctions are explicit trigonometric polynomials.Nevertheless several main conjectures on their distribution and the behavior of their zero sets are still far from resolved.These are often problems at the interface of harmonic analysis and number theory.In the talk,some of those aspects and recent advances will be presented.We also briefly discuss the role of computer assisted numerics in the problem of nodal domain counting

A combinatorial characterization of tight fusion frames

Marcin Bownik
University of Oregon
mbownik@uoregon.edu
Coauthors: Kurt Luoto, Edward Richmond

In this talk we present a combinatorial characterization of tight fusion frame (TFF) sequences using Littlewood-Richardson skew tableaux. The equal rank case has been solved recently by Casazza, Fickus, Mixon, Wang, and Zhou. Our characterization does not have this limitation. We also develop some methods for generating TFF sequences. The basic technique is a majorization principle for TFF sequences combined with spatial and Naimark dualities. We use these methods and our characterization to give necessary and sufficient conditions which are satisfied by the first three highest ranks. We exhibit four classes of TFF sequences which have unique maximal elements with respect to majorization partial order. Finally, we give several examples illustrating our techniques including an example of tight fusion frame which can not be constructed by the existing spectral tetris techniques.

An algorithm for variable density sampling with block-constrained acquisition

Claire Boyer
Institut de Mathmatiques de Toulouse (IMT), France
claire.boyer@math.univ-toulouse.fr
Coauthors: Pierre Weiss (ITAV, France), Jérémie Bigot (ISAE, France)

Reducing acquisition time is of fundamental importance in various imaging modalities. The concept of variable density sampling provides an appealing framework to address this issue. It was justified recently from a theoretical point of view in the compressed sensing (CS) literature. Unfortunately, the sampling schemes suggested by current CS theories may not be relevant since they do not take the acquisition constraints into account (for example, continuity of the acquisition trajectory in Magnetic Resonance Imaging - MRI). In this paper, we propose a numerical method to perform variable density sampling with block constraints. Our main contribution is to propose a new way to draw the blocks in order to mimic CS strategies based on isolated measurements. The basic idea is to minimize a tailored dissimilarity measure between a probability distribution defined on the set of isolated measurements and a probability distribution defined on a set of blocks of measurements. This problem turns out to be convex and solvable in high dimension. Our second contribution is to define an efficient minimization algorithm based on Nesterov's accelerated gradient descent in metric spaces. We study carefully the choice of the metrics and of the prox function. We show that the optimal choice may depend on the type of blocks under consideration. Finally, we show that we can obtain better MRI reconstruction results using our sampling schemes than standard strategies such as equiangularly distributed radial lines.

Finding the right model for our data

Carlos Cabrelli
*University of Buenos Aires *
carlos.cabrelli@gmail.com

Finding the right model for given data Assume that we need to process a large amount of data that we know is low dimensional. A common practice in this case, is to impose some hypothesis on the data and use a standard model. For example we can assume that the data is band limited, then we can use an appropriate Paley-Wiener space. A more realistic approach is to select a big class of models (subspaces) and try to find the one that best fit our data. In this talk we will show that this is possible in many situations. In particular we illustrate this in the space $L^2(R^N)$, with shift invariant spaces, having extra-invariance, which provide a model for noisy data and give a way to control the jitter error.

Robust and efficient compressed sensing

Jameson Cahill
Duke University
jcahill@math.duke.edu
Coauthors: Peter Casazza and Dustin Mixon

In the design of measurement matrices for use in compressed sensing there are two properties which have received a lot of attention, namely, the null space property (NSP) and the restricted isometry property (RIP). The RIP is usually preferred as it can be used to give good stability and robustness guarantees, and it is also known to imply the NSP. However, these two properties are fundamentally different in that the NSP (as the name suggests) is only a property of the null space, whereas the RIP is an explicit property of the matrix itself. We show that, in fact, the RIP is in essence a property of the null space in the sense that having the same null space as a matrix with the RIP is just as good as being a matrix with the RIP. We also so that in some situations it is better to use a well conditioned matrix matrix with the same null space as a matrix with the RIP.

L^1 approximation via de Branges spaces and applications

Emanuel Carneiro
IMPA - Rio de Janeiro
carneiro@impa.br

Given a real-valued function, we address in this talk the problem of constructing optimal one-sided approximations of prescribed exponential type to this function. These approximations are optimal in the sense that they optimize a given weighted L^1 metric over the real line. We shall see how this construction is related to the theory of de Branges spaces of entire functions and comment a little on the interesting applications of these extremal functions, that include, for instance, new upper bounds for the pair correlation of zeros of the Riemann zeta-function.

Phase retrieval by projections

Peter G. Casazza
University of Missouri
casazzap@missouri.edu

Phaseless reconstruction has broad application to x-ray crystallography, electron microscopy, diffractive imaging, x-ray tomography and more. The mathematics of phase retrieval is a very active area of research at this time. Recently, in several areas of research such as crystal twinning, it has become necessary to do phaseless reconstruction from the norms of the projections of a signal onto subspaces. It was believed that norms of projections give much less information than inner products with vectors and so we would need many more projections to do phaseless than we can do with vectors. We will look at recent results on this problem which include the surprising results that we can do phaseless reconstruction with the same number of projections as we can do it with vectors. This now reverses the above problem to: Is it possible to do phaseless with fewer projections than we can do it with vectors?

The analysis of periodic point processes

Stephen D. Casey
American University
scasey.american@gmail.com

Our talk addresses the problems of extracting information from periodic point processes. These problems arise in numerous situations, from radar pulse repetition interval analysis to bit synchronization in communication systems. We divide our analysis into two cases - periodic processes created by a single source, and those processes created by several sources. We wish to extract the fundamental period of the generators, and, in the second case, to deinterleave the processes.

We first present very efficient algorithm for extracting the fundamental period from a set of sparse and noisy observations of a single source periodic process. The procedure is computationally straightforward, stable with respect to noise and converges quickly. Its use is justified by a theorem which shows that for a set of randomly chosen positive integers, the probability that they do not all share a common prime factor approaches one quickly as the cardinality of the set increases. The proof of this theorem rests on a probabilistic interpretation of the Riemann zeta function. We then build upon this procedure to deinterleave and then analyze data from multiple periodic processes. This relies both on the the probabilistic interpretation of the Riemann zeta function, the equidistribution theorem of Weyl, and Wiener's periodogram.

Acceleration of nonlinear frame algorithm.

Wenjian Chen
Sun Yat-sen University & University of Central Florida
wenjianchen66@gmail.com
Coauthors: Qiyu Sun

We develop a polynomial acceleration technique to improve Van-Cittert iteration algorithm for solving localized nonlinear functional equations.

A null space property approach to compressed sensing with frames

Xuemei Chen
University of Maryland, College Park
xuemeic@math.umd.edu
Coauthors: Haichao Wang Rongrong Wang

An interesting topic in compressive sensing concerns problems of sensing and recovering signals with sparse representations in a dictionary. In this note, we study conditions of sensing matrices A for the ℓ^1-synthesis method to accurately recover sparse, or nearly sparse signals in a given dictionary D. In particular, we propose a dictionary based null space property (D-NSP) which, to the best of our knowledge, is the first sufficient and necessary condition for the success of the ℓ^1 recovery. This new property is then utilized to detect some of those dictionaries whose sparse families cannot be compressed universally. Moreover, when the dictionary is full spark, we show that AD being NSP, which is well-known to be only sufficient for stable recovery via ℓ^1-synthesis method, is indeed necessary as well. This is a joint work with Haichao Wang and Rongrong Wang.

Sampling in finite-dimensional reproducing kernel spaces

Cheng Cheng
University of Central Florida
cheng.cheng@knights.ucf.edu
Coauthors: Yingchun Jiang and Qiyu Sun

In this poster, we will discuss sampling and reconstruction of signals in a finite-dimensional reproducing kernel space.

The unitary extension principle on LCA groups

Ole Christensen
Technical University of Denmark
ochr@dtu.dk
Coauthors: Say Song Goh

We present an extension of the unitary extension principle by Ron & Shen to the setting of LCA groups. In the general setting this yields a tight frame for L^2 consisting of modulates of a collection of functions.

Hermite subdivision schemes and exponential polynomial generation

Mariantonia Cotronei
University of Reggio Calabria, Italy
mariantonia.cotronei@unirc.it
Coauthors: Costanza Conti (University of Firenze, Italy), Tomas Sauer (University of Passau, Germany)

Hermite subdivision schemes act on vector valued data interpreting their components as function values and associated consecutive derivatives. In this talk we address the problem of studying the exponential and polynomial preservation capability of such kind of schemes. The main tool for our investigation are convolution operators that annihilate the spaces generated by polynomial and exponential functions, which apparently is a general concept in the study of various types of subdivision operators. Based on these annihilators, we characterize the so-called spectral condition in terms of factorization of the subdivision operator and we then show how this factorization can be used to examine the convergence of the scheme.

Integrable wavelet transforms with abelian dilation groups

Brad Currey
Saint Louis University
curreybn@slu.edu
Coauthors: Hartmut Führ and Keith Taylor

We consider a class of semidirect products $G = \mathbb{R}^n \cdot H$, with H a suitably chosen abelian matrix group. The choice of H ensures that there is a wavelet inversion formula. Motivated in part by coorbit theory, we are looking for criteria to determine conditions under which there is a wavelet such that the associated reproducing kernel is integrable.

It is well-known that the existence of integrable wavelet coefficients is related to the question whether the unitary dual contains open compact sets. Our main general result reduces the latter problem to that of identifying compact open sets in the quotient space of all dual orbits of maximal dimension. This result is

applied to study integrability for certain families of dilation groups; in particular, we give a complete characterization valid for connected abelian matrix groups acting in dimension three.

Preimage Problem for Laplacian Eigenmaps with Applications to Data Integration and Recovery

Wojciech Czaja
University of Maryland College Park
wojtek@math.umd.edu
Coauthors: Alexander Cloninger and Timothy Doster

Nonlinear representation and data organization techniques became a vital part of machine learning and applied harmonic analysis, due to the their ability to solve many problems related to complex, high-dimensional, large and noisy data sets. Many of these techniques are defined by means of appropriately represented operators on data graphs, e.g., graph Laplacian or Schroedinger operators. This analysis leads to representation of data in a new feature space, which is the target space of the nonlinear process. On the one hand, such space may be very useful for direct comparison of heterogeneous sensing modalities. On the other hand, however, its non-physical nature may cause difficulties in many experimental applications. Our goal in this talk is to present an algorithm for fast, approximate inversion of Laplacian Eigenmaps - a leading and popular dimension reduction scheme. In our construction, we rely on Nystroem extension principle, L1 regularization, and multidimensional scaling. Furthermore, we shall combine this new preimage methodology together with the feature space rotations of Coifman and Hirn, to provide a unified methodology for data integration and recovery. We shall illustrate the usefulness of our algorithms with examples from remote sensing.

Frame properties of low autocorrelation stochastic waveforms

Somantika Datta
University of Idaho, Department of Mathematics, Moscow, ID, USA
sdatta@uidaho.edu

Waveforms with spike like autocorrelation are desirable in waveform design and are particularly useful in areas of radar and communications. In this work, stochastic waveforms are constructed whose expected autocorrelation can be made arbitrarily small outside the origin. Construction of stochastic frames for finite-dimensional spaces from such waveforms is addressed and their frame properties are studied using results from random matrix theory.

Spatio-temporal sampling sets for stable recovery of signals in evolutionary systems

Jacqueline Davis
Vanderbilt University
`jacqueline.t.davis@vanderbilt.edu`
Coauthors: Akram Aldroubi, Ilya Krishtal

Dynamical sampling is a new class of sampling problems in which an evolving signal is sampled at various times. The spatio-temporal problem in dynamical sampling asks the question: when do coarse spatial samplings of an evolving signal taken at varying times contain the same information as a finer spatial sampling taken at the earliest time? We show that under some conditions on the evolving system it is possible to compensate for spatial undersampling by taking additional time samples. In other words, spatial samples can be traded for time samples.

A Kantorovich Metric for Projection Valued Measures

Trubee Davison
University of Colorado
`Trubee.Davison@colorado.edu`

Given a compact metric space X, the collection of Borel probability measures on X can be made into a complete metric space via the Kantorovich metric. We generalize this well known result to projection valued measures. In particular, given a Hilbert space $\mathcal{H}$, consider the collection of projection valued measures from X into the projections on $\mathcal{H}$. We show that this collection can be made into complete metric space via a generalized Kantorovich metric. As an application, we use the Contraction Mapping Theorem on this complete metric space of projection valued measures to provide an alternative method for proving a fixed point result due to P. Jorgensen. This fixed point, which is a projection valued measure, arises from an iterated function system on X.

Structured measurements and ell_0 recovery in arbitrarily coherent dictionaries

Laurent Demanet
MIT
`laurent@math.mit.edu`
Coauthors: Nam Nguyen

We present a very simple algorithm, called the superset method, for sparse recovery from linear measurements of the form $Y = Adiag(x)B$. Recovery is not only possible in the regime of arbitrarily high dictionary coherence 1 - eps, but robust as long as the noise level is small in relation to eps. This type of

behavior does not in general hold for ell_1 minimization. The main application is to super-resolution from bandlimited samples, where A and B are partial Fourier matrices.

Image restoration: variational models, PDEs and wavelet frames

Bin Dong
Department of Mathematics, Program in Applied Mathematics, University of Arizona
dongbin@math.arizona.edu
Coauthors: Jian-Feng Cai, Qingtang Jiang, Stanley Osher, Zuowei Shen

Mathematics has become one of the main driving forces of the modern development of image restoration. There are several mathematical approaches that have been rather successful, including variational methods, partial differential equations (PDEs) based methods, and wavelet frame based methods. This talk is based on two of our recent papers that established connections between these popular methods. Our key observation is that: wavelet frame transform can be regarded as a discrete approximation of differential operators. However, the discrete approximation by wavelet frames is fundamentally different from finite difference approximate, due to their specific way of sampling derivatives. I will particularly discuss: (1) how some wavelet frame based image restoration models can be regarded as discrete approximation of a certain type of variational model through Gamma-convergence; (2) how to design iterative wavelet frame shrinkage that solves various types of nonlinear evolution PDEs, especially those used for image restoration; (3) through our theoretically analysis, how can we obtain new insights to all of these approaches, and combine the merits of them to create new and better models solving image restoration problems or other important and challenging problems in imaging science.

New Analysis of Manifold Embeddings and Signal Recovery from Compressive Measurements

Armin Eftekhari
Colorado School of Mines / SAMSI
armin.eftekhari@gmail.com
Coauthors: Michael B. Wakin

Compressive Sensing (CS) exploits the surprising fact that the information contained in a sparse signal can be preserved in a small number of compressive, often random linear measurements of that signal. Strong theoretical guarantees have been established concerning the embedding of a sparse signal family under a random measurement operator and on the accuracy to which sparse signals can be recovered from noisy compressive measurements. In this work, we address similar questions in the context of a different modeling framework. Instead of sparse models, we focus on the broad class of manifold models, which can arise in both parametric and non-parametric signal families. Using tools from the theory of empirical processes, we improve upon previous results concerning the embedding of low-dimensional manifolds under random measurement operators. We also establish both deterministic and probabilistic instance-optimal bounds in ℓ_2 for manifold-based signal recovery and parameter estimation from noisy compressive measurements. In line with analogous results for sparsity-based CS, we conclude that much stronger bounds are possible in the probabilistic setting. Our work supports the growing evidence that manifold-based models can be used with high accuracy in compressive signal processing.

Sparse Signal Recovery From Nonlinear Measurements

Yonina Eldar
Technion, Israel
yonina@ee.technion.ac.il

We consider an extension of compressed sensing to nonlinear measurements. We present and analyze several different optimality criteria for sparse recovery from nonlinear measurements which are based on the notions of stationarity and coordinatewise optimality and show that they can be used to develop efficient recovery algorithms. A special case that is of large interest in the area of optics is that of phase retrieval, in which one needs to recover an image given only its Fourier transform magnitude. We propose an efficient algorithm for phase retrieval and prove certain optimality properties of the proposed method. We also consider conditions on the number of measurements needed for stable phase retrieval and show that surprisingly the results coincide with those obtained in the linear measurement setting (up to constants). We demonstrate our algorithms and results on a variety of problems in optical imaging

Sparse approximations of spatially varying blur operators in the wavelet domain

Paul Escande
Département Mathématiques, Informatique, Automatique (DMIA) Institut Supérieur de l'Aéronautique et de l'Espace (ISAE) Toulouse, France
paul.escande@gmail.com
Coauthors: Jérémie Bigot (DMIA - ISAE - jeremie.bigot@isae.fr), Pierre Weiss (ITAV - CNRS - pierre.armand.weiss@gmail.com)

Restoration of images blurred by spatially varying PSFs is a problem met increasingly. One of the main difficulties is the computational burden caused by the huge dimensions of blur matrices. It prevents the use of naive approaches to perform matrix-vector multiplications. We study an original approach which consists of approximating blurring operators by sparse matrices in the wavelet domain. We provide theoretical complexity results and compare this approach to standard approximations as piecewise convolutions. We finish by showing that the sparsity pattern of the matrix can be pre-defined, which is central in tasks such as blind deconvolution.

Keywords: Image deblurring, spatially varying blur, operator approximations, wavelet transforms, sparse approximations, Calderon-Zygmund operators, structured sparsity.

Stable Recovery with Gauge Regularization

Jalal FADILI
CNRS-ENSICAEN-Univ. Caen
Jalal.Fadili@greyc.ensicaen.fr
Coauthors: Samuel Vaiter (CEREMADE CNRS-Univ. of Paris-Dauphine) Gabriel Peyr (CEREMADE CNRS-Univ. of Paris-Dauphine) Mohammad Golbabaee (CEREMADE CNRS-Univ. of Paris-Dauphine)

Regularization plays a pivotal role when facing the challenge of solving ill-posed inverse problems, where the number of observations is smaller than the ambient dimension of the object to be estimated. A line of recent work has studied regularization models with various types of low-dimensional structures. In such settings, the general approach is to solve a regularized optimization problem, which combines a data fidelity term and some regularization penalty that promotes the assumed lowdimensional/ simple structure. This paper provides a general framework to capture this low-dimensional structure through what we coin piecewise regular gauges. These are convex, non-negative, closed, bounded and positively homogenous functions that will promote objects living on low-dimensional subspaces. This class of regularizers encompasses many popular examples such as the ℓ_1 norm, $\ell_1 - \ell_2$ norm (group sparsity), as well as several others including the ℓ_∞ norm. We will show that the set of piecewise regular gauges is closed under addition and pre-composition by a linear operator, which allows to cover mixed regularization, and analysis-type priors (e.g. total variation, etc.). Our main results provide a unified sharp analysis of exact and robust recovery guarantees from partial measurements.

Mitigating the data-deluge by an adequate sampling for low-power systems

Laurent Fesquet
TIMA, Grenoble Institute of Technology
Laurent.Fesquet@imag.fr
Coauthors: Tugdual Le Pelletier, Taha Beyrouthy, Brigitte Bidegaray-Fesquet

Today, our digital society exchanges data flows as never it has been the case in the past. The amount of data is incredibly large and the future promises that not only human will exchange digital data but also technological equipment, robots, etc. We are close to open the door of the internet of things. This data orgy wastes a lot of energy and contributes to a non-ecological approach of our digital life. Indeed, Internet and the new technologies consume about 10It already exists design solutions to enhance the energetic performances of the electronic systems and circuits, of the computers and their mobile applications: a lot of techniques and also a lot of publications! Nevertheless, another way to reduce energy is to rethink the sampling techniques and digital processing chains. Considering that our digital life is dictated by the Shannon theory, we produce more digital data than expected, more than necessary. Indeed, useless data produce more computation, more storage, more communications and also more power consumption. If we disregard the Shannon theory, we can discover new sampling and processing techniques. A small set of ideas is given through examples such as filters, pattern recognition techniques, etc. but the Pandoras Box has to be opened to drastically reduce the useless data and mathematicians probably have a key role to play in this revolution.

This study is supported by the Persyval-lab project.

Constructing equi-isoclinic tight fusion frames

Matthew Fickus
Air Force Institute of Technology
Matthew.Fickus@gmail.com
Coauthors: John Jasper, Dustin G. Mixon, Jesse D. Peterson

A collection of equi-dimensional subspaces is an equi-isoclinic tight fusion frame (EITFF) for a given Euclidean space if it is a tight fusion frame for that space and if the principal angles between any pair of subspaces are all the same. When they exist, EITFFs are optimal packings in the Grassmannian. They are ideal for block-coherence-based compressed sensing, permitting reconstruction of sufficiently sparse signals via, for example, block orthogonal matching pursuit. The easiest way to construct an EITFF is to take a tensor product of an equiangular tight frame with an orthonormal basis. We discuss some new nontrivial generalizations of this construction, including one method that involves filter banks and another which uses harmonic frames.

Time-frequency decompositions in seismic data analysis

Sergey Fomel
The University of Texas at Austin
sergey.fomel@beg.utexas.edu

Seismic wave propagation in the Earth can be strongly affected by frequency attenuation. In addition, seismic signals exhibit variations in local slope, which can be described using non-stationary spectral analysis. For these reasons, time-frequency decompositions play an important role in seismic data processing, where they are used for signal enhancement and parameter estimation. I will describe both recent geophysical applications of time-frequency decompositions and recently developed time-frequency analysis techniques, such as regularized nonstationary regression.

Interpolation with complex B-splines

Brigitte Forster
University of Passau, Germany
brigitte.forster@uni-passau.de
Coauthors: Ramūnas Garunkštis (Vilnius University, Lithuania), Peter Massopust (Helmholtz Zentrum München, Germany) and Jörn Steuding (Würzburg University, Germany)

Cardinal B-splines of complex order or, for short, complex B-splines, are natural extensions of classical Curry-Schoenberg (polynomial) B-splines B_n, where the order $n \in \mathbb{N}$ is replaced by a complex number s. These complex B-splines inherit many of the important and interesting properties of the B_n. In this talk, we will concentrate on the interpolation property. Whereas for the fractional B-splines B_α, $\alpha > 1$, the interpolation property can be easily verified, this is not obvious for the complex case. In fact, this question is closely related to the growth conditions and the distribution of zeros of sums of Hurwitz zeta functions. In the talk, we give a positive answer to the question of interpolation with complex B-splines for a certain range of complex degrees. The complete characterization of the admissible degrees is still an open question.

Exponentially decaying reconstruction error in one-bit compressive sensing

Simon Foucart
University of Georgia
foucart@math.uga.edu
Coauthors: R. Baraniuk, D. Needell, Y. Plan, and M. Wootters

In one-bit compressive sensing, s-sparse vectors $\mathbf{x} \in \mathbb{R}^n$ are acquired via extremely quantized linear measurements $y_i = \mathrm{sgn}\langle \mathbf{a}_i, \mathbf{x}\rangle$, $i = 1, \ldots, m$. Several procedures to reconstruct these sparse vectors have been shown to be effective when $\mathbf{a}_1, \ldots, \mathbf{a}_m$ are independent random vectors. They typically yield an error decaying polynomially in $\lambda := m/(s \log(n/s))$. This rate cannot be improved in the measurement framework described above. However, we show that a reconstruction error decaying exponentially in λ is achievable when thresholds $\tau_1, \ldots, \tau_m$ are chosen adaptively in the quantized measurements $y_i = \mathrm{sgn}(\langle \mathbf{a}_i, \mathbf{x}\rangle - \tau_i)$, $i = 1, \ldots, m$. Our procedure, which is robust to measurement error, is based on a simple recursive scheme involving either hard-thresholding or linear programming.

Coordinate systems of translates of a single function in $L_p(R)$

Daniel Freeman
St Louis University
dfreema7@slu.edu
Coauthors: Edward Odell (University of Texas), Thomas Schlumprecht (Texas A&M University), Andras Zsak (Peterhouse College, Cambridge University)

Wavelet coordinate systems are formed by starting with a single function in L_2 which is then translated and dilated to form a basis or a frame for L_2. We investigate what coordinate systems can be formed of just translations of a single function. Christensen, Deng, and Heil proved that a sequence of translations of a single function in L_2 cannot form a frame for L_2. In contrast to this, we show that for all $p > 2$, L_p has an unconditional Schauder frame formed by the integer translates of a single function in L_p. Furthermore, for all $1 \leq p$, L_p does not have an unconditional basis formed by translations of a single function in L_p.

Multiresolution Equivalence and Path-Connectedness

Veronika Furst
Fort Lewis College
furst_v@fortlewis.edu
Coauthors: Erich McAlister

An equivalence relation between multiresolution analyses was first introduced in 1996; an analogous definition for generalized multiresolution analyses was given in 2010. Both types of equivalence classes are path-connected in an operator-theoretic sense. Moreover, whenever two MRAs in $L^2(\mathbb{R})$ are equivalent, the GMRA path construction between their corresponding canonical GMRAs yields the natural analog of the MRA path.

Frames of multi-windowed exponentials on subsets of $\mathbb{R}^d$

Jean-Pierre Gabardo
McMaster University
gabardo@mcmaster.ca
Coauthors: Chun-Kit Lai

Given discrete subsets $\Lambda_j \subset \mathbb{R}^d$, $j = 1, \ldots, q$, consider the set of windowed exponentials $\bigcup_{j=1}^{q} \{g_j(x)e^{2\pi i\lambda\cdot x} : \lambda \in \Lambda_j\}$ on $L^2(\Omega)$. We show that a necessary and sufficient condition for the windows g_j to form a frame of windowed exponentials for $L^2(\Omega)$ with some set of frequencies Λ_j, $j = 1, \ldots, q$, is that $m \leq \max_{j\in J} |g_j| \leq M$ almost everywhere on Ω for some subset J of $\{1, \cdots, q\}$ and some positive constants m, M. If Ω is unbounded, we show that there is no frame of windowed exponentials if the Lebesgue measure of Ω is infinite. If Ω is unbounded but of finite measure, we give a sufficient condition for the existence of Fourier frames on $L^2(\Omega)$. At the same time, we also construct examples of unbounded sets with finite measure that have no tight exponential frame.

Nonuniform generalized sampling and stable recovery of multivariate signals

Milana Gataric
University of Cambridge
mg617@cam.ac.uk
Coauthors: Ben Adcock (Purdue University) Anders Hansen (University of Cambridge)

In this talk, we deal with the problem of multivariate signal recovery from a finite collection of pointwise samples of its Fourier transform taken nonuniformly. This problem is a major research topic, in both pure and applied mathematics, driven by numerous practical applications ranging from Magnetic Resonance Imaging (MRI) to Computed Tomography (CT), seismology and microscopy, where the samples are fixed and acquired on sampling schemes not necessarily Cartesian. We show that the desired reconstruction can be carried out stably, accurately and efficiently in any given finite-dimensional approximation subspace, within the so-called nonuniform generalized sampling framework. This is feasible under certain sufficient conditions on the sampling points — specifically, on their density and bandwidth — which are allowed to be highly nonuniform, i.e. the separation condition is not required. In particular, the sufficient density condition we provide is sharp and does not depend on the dimension. This density characterization of a sampling scheme fills an important gap in the existing nonuniform sampling theory literature, and it is based on our novel results on weighted Fourier frames which are improvement of existing results of Grochenig and Beurling. Moreover, in one dimension, we analyze the important case when the approximation space consists of compactly supported wavelets. We show that a linear scaling of the dimension of the space with the sampling bandwidth is both sufficient and necessary for stable recovery. Hence, up to constant factors, wavelets provide optimal bases for reconstruction.

Asymptotical analysis of inpainting via universal shearlet systems and clustered sparsity

Martin Genzel
Technische Universitt Berlin
genzel@math.tu-berlin.de
Coauthors: Gitta Kutyniok (Technische Universitt Berlin)

Inpainting is generally understood to be the process of recovering corrupted data signals. Problems with damaged or missing data often occur in the field of imaging science; just imagine, for example, seismic data measured by an array of sensors with a more or less large number of them failing due to electronic defects. This will result in an image with a bunch of small missing stripes. In the past decades, various algorithms have been developed to solve the problem of (image-)inpainting. However, most of the respective approaches are of empirical nature, and a theoretical foundation is often missing.

To meet this challenge, the novel concept of *clustered sparsity* has been developed recently and turned out to be a useful tool in the mathematical analysis of inpainting. The fundamental idea underlying this method is based on the assumption that a given signal in Hilbert space H possesses some "significant structure", which can be "efficiently" represented by an appropriate Parseval frame Φ for H.

In this presentation, an abstract inpainting framework is introduced which provides recovering algorithms as well as some suitable error estimates. Based on clustered sparsity, they are motivated by the ideas of *compressed sensing*. Concerning two dimensional images, these methods will be applied to a continuous model in $L^2(\mathbb{R}^2)$, which is governed by a line distribution. To capture this anisotropic feature, we will choose Φ to be an *universal shearlet system*. This novel construction of smooth band-limited Parseval frames particularly enables a uniform treatment of both shearlets and wavelets. Provided that the gaps of the corrupted image are appropriately bounded, the main result finally shows that one can indeed achieve *asymptotically perfect inpainting*. In this context, it turns out that, compared to classical wavelets, shearlets require much lower assumptions on the gap size.

AN ADAPTIVE MESHFREE SPECTRAL GRAPH WAVELET METHOD FOR PARTIAL DIFFERENTIAL EQUATIONS ON THE SPHERE

Kavita Goyal
Indian Institute of Technology Delhi
goyalkavita9@gmail.com
Coauthors: Mani Mehra

This paper proposes an adaptive meshfree spectral graph wavelet method to solve partial differential equations on the sphere. The method uses multiresolution analysis based on spectral graph wavelet for adaptivity. It uses radial basis functions for interpolation of functions and for approximation of the differential operators. The set of scattered node points is subject to dynamic changes at run time which leads to adaptivity. The beauty of the method lies in the fact that the same operator is used for the approximation of differential operators and for the construction of spectral graph wavelet. Initially, we have applied the method on spherical diffusion equation. The problem of pattern formation on the surface of the sphere (using Turing equations) is addressed to test the strength of the method. The numerical results show that the method can accurately capture the emergence of the localized patterns at all the scales and the node arrangement is accordingly adapted. The convergence of the method is verified. For each test problem, the CPU time taken by the proposed method is compared with the CPU time taken by a traditional method (spectral method using radial basis functions). It is observed that the adaptive meshfree spectral graph wavelet method is highly efficient.

The mystery of Gabor frames

Karlheinz Gröchenig
University of Vienna, Austria
karlheinz.groechenig@univie.ac.at

Gabor frames remain mysterious objects in time-frequency analysis. We will first review some structural results about Gabor frames over a lattice; this is the coarse structure of Gabor frames. For the fine structure one needs to understand which lattices generate a Gabor frame for a given window. The fine structure of Gabor frames is largely unknown, there are few results and many open conjectures. We will mention some recent results and formulate some explicit conjectures. .

Matrix Fourier multipliers for Parseval multi-wavelet frames

Deguang Han
University of Central Florida
deguang.han@ucf.edu

Matrix Fourier multipliers are matrices with L^∞-function entries that map Parseval multi-wavelet frames to Parseval multi-wavelet frames. Like Fourier wavelet multiplier, matrix Fourier multipliers can be used to derive new multi-wavelet frames and can help us better understand the basic theory on multi-wavelet frames. In this talk I will discuss a characterization of such matrix Fourier multipliers

Compressed sensing and analog inverse problems - the need for a new theory

Anders Hansen
University of Cambridge
ach70@cam.ac.uk
Coauthors: Ben Adcock, Clarice Poon, Bogdan Roman

Compressed sensing is based on the three pillars: sparsity, incoherence and uniform random subsampling. In addition, the concepts of uniform recovery and the Restricted Isometry Property (RIP) have had a great impact. Intriguingly, in an overwhelming number of analog inverse problems where compressed sensing is used or can be used (such as MRI, X-ray tomography, Electron microscopy, Reflection seismology etc.) these pillars are absent. Moreover, easy numerical tests reveal that with the successful sampling strategies used in practice one does not observe uniform recovery nor the RIP. In particular, none of the existing theory can explain the success of compressed sensing in a vast area where it is used. In this talk we will demonstrate how real world analog inverse problems are not sparse, yet asymptotically sparse, coherent, yet asymptotically incoherent, and moreover, that uniform random subsampling yields highly suboptimal results. In addition, we will present easy arguments explaining why uniform recovery and the RIP is not observed in practice. Finally, we will introduce a new theory that aligns with the actual implementation of compressed sensing that is used in applications. This theory is based on asymptotic sparsity, asymptotic incoherence and random sampling with different densities. This theory supports two intriguing phenomena observed in reality: 1. the success of compressed sensing is resolution dependent, 2. the optimal sampling strategy is signal structure dependent. The last point opens up for a whole new area of research, namely the quest for the optimal sampling strategies.

Different faces of the shearlet transform

Sören Häuser
TU Kaiserslautern, Germany
haeuser@mathematik.uni-kl.de
Coauthors: Stephan Dahlke (Philipps-Universität Marburg, Germany), Filippo de Mari (Università degli Studi di Genova, Italy), Ernesto de Vito (Università degli Studi di Genova, Italy), Gabriele Steidl (TU Kaiserslautern, Germany), Gerd Teschke (Hochschule Neubrandenburg, Germany)

Recently, shearlet groups have received much attention in connection with shearlet transforms applied for orientation sensitive image analysis and restoration. The square integrable representations of the shearlet groups provide not only the basis for the shearlet transforms but also for a very natural definition of scales of smoothness spaces, called shearlet coorbit spaces. The aim of this talk is twofold: first we discover isomorphisms between shearlet groups and other well-known groups, namely extended Heisenberg groups and subgroups of the symplectic group. Interestingly, the connected shearlet group with positive dilations has an isomorphic symplectic subgroup, while this is not true for the full shearlet group with all nonzero dilations.

Having understood the various group isomorphisms it is natural to ask for the relations between coorbit spaces of isomorphic groups with equivalent representations. These connections are examined in the second part of the talk. We describe how isomorphic groups with equivalent representations lead to isomorphic coorbit spaces. In particular we apply this result to square integrable representations of the connected shearlet groups and metaplectic representations of subgroups of the symplectic group. This implies the definition of metaplectic coorbit spaces.

Linear Independence of Time-Frequency Translates

Christopher Heil
Georgia Tech
heil@math.gatech.edu

The Linear Independence of Time-Frequency Translates Conjecture, also known as the HRT conjecture, states that any finite set of time-frequency translates of a given L^2 function must be linearly independent. This conjecture, which was first stated in print in 1996, remains open today. We will discuss this conjecture, its context, and the (frustratingly few) partial results that are currently available.

Riesz and Frame sequences generated by unitary actions of discrete groups

Eugenio Hernndez
Universidad Autnoma de Madrid
eugenio.hernandez@uam.es
Coauthors: Davide Barbieri and Javier Parcet

We characterize Riesz and frame sequencies which arise from the action of a countable discrete group Γ on a single element of a given Hilbert space H. As Γ might not be abelian, this is done in terms of an operator-valued bracket map taking values in the L^1-space associated to the group von Neumann algebra of Γ. Our result generalizes recent work for locally compact abelian groups of Hernández, Šikić, Weiss and Wilson. In many cases, the bracket map can be decribed in terms of a noncommutative form of the Zak transform.

Minimal $C^{1,1}$ extensions

Matthew Hirn
Ecole normale superieure
matthew.hirn@ens.fr
Coauthors: Ariel Herbert-Voss, Erwan Le Gruyer, Frederick McCollum

Consider the following Whitney type extension problem for $C^{1,1}(R^d)$. Suppose we are given a closed subdomain E of R^d, along with a 1st degree polynomial at each point in E that specifies a potential function value and a gradient. We refer to such information as a "1-field." Some questions one might ask are the following: When can we extend the 1-field to a function F in $C^{1,1}(R^d)$ such that F agrees with the specified function values and gradients on E? If such an F exists, how small can we make the Lipschitz constant of the gradient of F? The first question was answered by Whitney in 1934 as part of the more general Whitney Extension Theorem. The second question, however, was only recently answered in 2009 by Le Gruyer. In this talk we will discuss these results, and then explore some of the ramifications of them. In particular, we will generalize the concept of an absolutely minimal Lipschitz extension (AMLE) to 1-fields and more general extension problems, and prove the existence of quasi-AMLEs in this expanded framework. Additionally, we present a practical, efficient algorithm for computing AMLEs of 1-fields when the subdomain E is finite. This talk is based on joint work with Ariel Herbert-Voss, Erwan Le Gruyer, and Frederick McCollum.

Clifford-Fourier transforms and wavelets

Jeff Hogan
University of Newcastle, Australia
jeff.hogan@newcastle.edu.au
Coauthors: Andrew Morris (University of Newcastle, Australia)

We present some recent results in Clifford analysis aimed at the treatment of multi-channel signals. A covariant Clifford-Fourier transform is introduced and its kernel computed in even dimensions. A convolution theorem will be proved, and applications to the construction of Clifford-valued wavelets and multiwavelets explored.

Frame-based multi-scale Gaussian beams and parametrix construction for wave equations

Maarten V. de Hoop
Purdue University
mdehoop@purdue.edu
Coauthors: Michele Berra and Jos-Luis Romero

We construct frame-based multi-scale Gaussian beams following the dyadic parabolic decomposition of phase space. Using these, we generate a parametrix for wave equations. Our parametrix is the sum of purely Gaussian functions and solves Cauchy initial value problems. We establish an error estimate in terms of the scale content of the initial data. We then proceed with a construction of the corresponding approximate solution of the inhomogeneous wave equation with a boundary source, converging in the limit of fine scales also. This solution plays a role in many applications in electrical engineering and reflection seismology. We mention the connection with wave atoms, and related work by Laptev and Sigal (2000) and Bao et al. (2013).

Optimal N-term approximation by linear splines on triangulations

Armin Iske
University of Hamburg, Germany
armin.iske@uni-hamburg.de
Coauthors: Laurent Demaret

Anisotropic triangulations provide efficient geometrical methods for sparse representations of bivariate functions from discrete data, in particular from image data. In previous work, we have proposed a locally adaptive method for efficient image approximation, called adaptive thinning, which relies on linear splines over anisotropic Delaunay triangulations. In this talk, we discuss asymptotically optimal N-term approximation rates for linear splines over anisotropic triangulations, where our analysis applies to three relevant classes of target functions: (a) piecewise linear horizon functions across Hölder smooth boundaries, (b) functions of $W^{\alpha,p}$ regularity, where $\alpha > 2/p - 1$, (c) piecewise regular horizon functions of $W^{\alpha,2}$ regularity, where $\alpha > 1$.

Fast computation of n-widths via greedy least-squares

Mark Iwen
Michigan State University
markiwen@math.msu.edu
Coauthors: Felix Krahmer

I will discuss fast and deterministic dimensionality reduction techniques for a family of subspace approximation problems. Let $S \subset \mathbf{R}^N$ be a given set of M points. The techniques discussed find an $O(n \log M)$-dimensional subspace that is guaranteed to always contain a near-best fit n-dimensional hyperplane $\mathcal{H}$ for S with respect to the cumulative projection error $\left(\sum_{\mathbf{x} \in S} \|\mathbf{x} - \Pi_{\mathcal{H}}\mathbf{x}\|_2^p\right)^{1/p}$, for any chosen $p > 2$. The deterministic algorithm runs in $O\left(MN^2\right)$-time, and can be randomized to run in only $O\left(MNn\right)$-time while maintaining its error guarantees with high probability. In the case $p = \infty$ the dimensionality reduction techniques can be combined with efficient algorithms for computing the John ellipsoid of a data set in order to produce an n-dimensional subspace whose maximum ℓ_2-distance to any point in the convex hull of P is minimized. The resulting algorithm remains $O\left(MNn\right)$-time. These methods allow the fast computation of tightly fitting bounding regions in large and high-dimensional data sets for use in, e.g., database indexing schemes.

When Buffon's needle problem helps in quantizing the Johnson-Lindenstrauss Lemma

Laurent Jacques
ISPGroup, ICTEAM/ELEN, University of Louvain (UCL), Belgium
`laurent.jacques@uclouvain.be`

In 1733, Georges-Louis Leclerc, Comte de Buffon in France, set the ground of geometric probability theory by defining an enlightening problem: What is the probability that a needle thrown randomly on a ground made of equispaced parallel strips lies on two of them?

In this presentation, we will show that the solution to this problem, and its generalization to N dimensions, allows us to discover a quantized form of the Johnson-Lindenstrauss (JL) Lemma, i.e., one that combines a linear dimensionality reduction procedure with a uniform quantization of precision $\delta > 0$.

In particular, given a finite set $\mathcal{S} \subset \mathbb{R}^N$ of S points and a distortion level $\epsilon > 0$, as soon as $M > M_0 = O(\epsilon^{-2} \log S)$, we can (randomly) construct a mapping from $(\mathcal{S}, \ell_2)$ to $((\delta\,\mathbb{Z})^M, \ell_1)$ that approximately preserves the pairwise distances between the points of $\mathcal{S}$.

Interestingly, compared to the common JL Lemma, the mapping is quasi-isometric and we observe both an additive and a multiplicative distortions on the embedded distances. These two distortions, however, decay as $O((\log S/M)^{1/2})$ when M increases. Moreover, for coarse quantization, i.e., for high δ compared to the set radius, the distortion is mainly additive, while for small δ the embedding tends to a Lipschitz isometric embedding.

We will also explain that there exists "almost" a quasi-isometric embedding of $(\mathcal{S}, \ell_2)$ in $((\delta\mathbb{Z})^M, \ell_2)$. This one involves a non-linear distortion of the ℓ_2-distance in $\mathcal{S}$ that vanishes for distant points in this set. Noticeably, the additive distortion in this case is slower and decays as $O((\log S/M)^{1/4})$.

Finally, the presentation will be illustrated by simple numerical simulations showing that the additive and the multiplicative errors behave as predicted when δ and M vary.

Wavelet techniques for p-exponent multifractal analysis

Stéphane Jaffard
University Paris Est
`jaffard@u-pec.fr`
Coauthors: Patrice Abry, Roberto Leonarduzzi, Clotilde Melot, Stéphane Roux, Maria Eugenia Torres, Herwig Wendt

The purpose of multifractal analysis is the construction of scaling functions that allow to estimate the fractal dimensions of the pointwise singularities of a signal. Independently of this theoretical interpretation, scaling functions are also used as an efficient tool for model selection and parameter estimation. However, up to now, this technique could only be applied for data which are locally bounded, an a priori hypothesis which is seldom met by real-life signals.

We will present an alternative of this method, which is obtained when the usual pointwise Hlder regularity is replaced by the p-exponent (a notion introduced by Calderon and Zygmund in the 1960s, in the context of PDEs). The a priori assumption now is that the data locally belong to L^p, an assumption which is much more frequently met by experimental data. We will present the mathematical background of these developments, and in particular how to derive the relevant wavelet-based scaling functions. We will also show applications to stochastic processes and real-life data (signals and images), for which the previous approach based on the Hölder exponent did not work.

Characterization of wavelets and MRA wavelets on local fields of positive characteristic

Qaiser Jahan
Indian Statistical Institute Kolkata, India.
`qaiserjahan01@gmail.com`
Coauthors: Biswaranjan Behera (Indian Statistical Institute Kolkata, India)

We provide a characterization of wavelets on local fields of positive characteristic based on results on affine and quasi affine frames. This result generalizes the characterization of wavelets on Euclidean spaces by means of two basic equations. We also give another characterization of wavelets. Further, all wavelets which are associated with a multiresolution analysis on such a local field are also characterized.

Correspondence between Wavelet Frame Shrinkage and Nonlinear Diffusion

Qingtang Jiang
University of Missouri - St. Louis
`jiangq@umsl.edu`
Coauthors: Bin Dong and Zuowei Shen

In this talk we will discuss the correspondence between the wavelet frame shrinkage and nonlinear diffusion. The connection will provide new and inspiring interpretations of these both approaches for image restoration. In particular, the wavelet frame shrinkage algorithms that are commonly used in image restoration, such as the iterative soft-thresholding algorithms, lead to new types of nonlinear PDEs that have not been considered in the literature. On the other hand, the nonlinear PDE based approach also provides new insights into the desirable choices of adaptive thresholds for wavelet frame shrinkage and enable us to design better wavelet frame algorithms for image restoration.

Cardinal splines in piecewise constant tension

Masaru Kamada
Ibaraki University
m.kamada@mx.ibaraki.ac.jp

The cubic spline gives the smoothest interpolation of data in the sense that it minimizes the integral of its squared second derivative. The linear spline gives the shortest polygonal interpolation. The spline in tension was devised as a generalization of those two splines. It minimizes the integral of a weighted sum of the squared first and second derivatives. The weight on the first derivative is called tension because its increase makes the spline in tension approach the shortest linear spline while retaining smoothness like the cubic spline. The tension has been fixed as a single constant over the entire domain since its first appearance in 1966.

In this talk, we shall look at the spline in tension as the output of a linear dynamical system model with a series of delta functions input. In exchange of restricting the sampling points to be uniform, we can place a different tension in each sampling interval because the control theory allows for time-varying dynamics. Solving the dead beat control problem of the system, we will eventually have a locally supported basis for the space of cardinal splines in piecewise constant tension.

An application is an adaptive image interpolation algorithm made up of the univariate spline interpolations in the horizontal and vertical directions. Varying the tension in proportion to an index of sharp change in brightness, we obtain image interpolation results with less ringing artifacts compared to those by the cubic spline interpolation.

We will also discuss a similar model for bivariate splines in piecewise constant tension which may generalize the bicubic spline interpolation.

Total Activation: Generalized Total Variation Regularization for Functional Magnetic Resonance Imaging Data Analysis

F. Isik Karahanoglu
Ecole Polytechnique Federale de Lausanne, University of Geneva
isik.karahanoglu@epfl.ch
Coauthors: Dimitri Van De Ville

Total Variation (TV) regularization is an edge-preserving method extensively used in image and signal processing for denoising, restoration, and deconvolution. TV regularization implicitly assumes a generating system that consists of piecewise constant signals. We extend the notion of TV in the sense that the underlying generating system is represented as the combination of weighted and shifted Green's function of a general differential operator, L, instead of the first order differential operator, D, in TV regularization. We cast a denoising problem whose regularization term combines sparsity constraint and discrete differential filter associated to the inverse of the underlying system; i.e., "analysis" formulation. Further tailoring the operator enables to handle different driving signals; i.e., n-th order polynomials.

Employing 1-D generalized TV, we develop a novel spatio-tempoal deconvolution scheme, for which we coin the term Total Activation (TA), for functional magnetic resonance imaging (fMRI) data analysis. Specifically, we tune the general differential operator, L, to invert the transfer function of fMRI system; i.e., hemodynamic response function. TA aims to recover the underlying activity-inducing signals, which are closely related to neuronal activity, without any restrictions on the timing or duration of the activations. That allows for detection of spontaneous brain activity and observation of the non-stationary dynamics. The variational formulation is convex and contains a data-fitting term and two regularizers for the different dimensions of the data. First, temporal regularization identifies the "innovation" signal (which is spike-type) as the sparse driver of the hemodynamic system. Spatial regularization is incorporated using mixed-norm based on anatomical priors of brain regions; i.e., activities in the same brain regions are favored to be coherent. We employ the efficient generalized forward-backward splitting algorithm, which is a fast iterative shrinkage algorithm that alternates between temporal and spatial domain solutions until convergence to the final estimate of the underlying activity-inducing signal is reached. We show that TA is able to recover the underlying activations in fMRI experiment using visual stimuli. Moreover, we present that TA disentangles meaningful brain networks driven by short transitions during resting-state fMRI.

Prolate spheroidal wave functions: spectral analysis of the associated differential and sinc kernel operators with some related applications

Abderrazek Karoui
University of Carthage, Faculty of Sciences of Bizerte, Tunisia
abderrazek.karoui@fsb.rnu.tn
Coauthors: Aline Bonami, MAPMO, University of Orleans, France.

For fixed real number $c > 0$, the prolate spheroidal wave functions (PSWFs) $(\psi_{n,c})_{n\geq 0}$ form a basis with remarkable properties for the space of band-limited functions with bandwidth c. They have been largely studied and used in various classical signal processing applications as well as in some emerging and promising subjects and area such as the performance of MIMO transmissions in wireless network, random matrices and compressed sensing.

Note that the PSWFs were first known as the bounded eigenfunctions of the Sturm-Liouville operator's L_c defined on $C^2([-1,1])$ by $L_c(\psi) = -(1-x^2)\frac{d^2\psi}{d\,x^2} + 2x\frac{d\psi}{d\,x} + c^2x^2\psi$. A breakthrough in the subject of the PSWFs is due to D. Slepian, H. Landau and H. Pollack, who have shown that the $\psi_{n,c}$ are also the eigenfunctions of the integral operators F_c and $Q_c = \frac{c}{2\pi} F_c^* F_c$, defined on $L^2([-1,1])$ by

$$F_c(f)(x) = \int_{-1}^{1} e^{i\,c\,x\,y} f(y)\,dy, \quad Q_c(\psi)(x) = \int_{-1}^{1} \frac{\sin c(x-y)}{\pi(x-y)}\,\psi(y)\,dy. \qquad (1)$$

As a result, the PSWFs exhibit the unique properties to form an orthogonal basis of $L^2([-1,1])$, and an orthonormal basis of $B_c = \{f \in L^2(\mathbf{R}), \text{ Support } \widehat{f} \subset [-c,c]\}$, the Paley-Wiener space of $c-$band-limited functions. We let $(\lambda_n(c))_n$ denotes the infinite sequence of the eigenvalues of Q_c, arranged in the decreasing order. Most of the above mentioned applications of the PSWFs heavily rely on their explicit analytic properties as well as on the precise behaviour and decay rate of the corresponding eigenvalues $(\lambda_n(c))_{n\geq 0}$. Although, there exists a rich literature on the numerical computation and asymptotic behaviours of the $\psi_{n,c}$ and $\lambda_n(c)$, very little is know about their explicit estimates.

The main purpose of this talk is to give a description of the recent results we have recently obtained regarding the problem of the accurate explicit estimates of the PSWFs and their associated eigenvalues. To this end, we first give some new and useful bounds of the $\psi_{n,c}$ and the eigenvalues $\chi_n(c)$ of the differential operator L_c. Then, under the condition that $q = c^2/\chi_n(c) < 1$, we prove that $\psi_{n,c}$ is uniformly approximated on $[-1,1]$ by a single function involving the function J_0 : the Bessel function of first kind and order zero. A special interest is devoted to the computation of normalisation constant appearing in this uniform approximation of the $\psi_{n,c}$ so that the $L^2([-1,1])-$norm of this later equals 1. As an important consequence of this uniform approximation, we prove that under the condition $q = c^2/\chi_n(c) < 1$, the exact term of the exponential decay rate of the $\lambda_n(c)$ is given by

$$\widetilde{\lambda}_n(c) = \frac{1}{2}\exp\left(-\frac{\pi^2}{2} n \int_{\Phi\left(\frac{2c}{\pi n}\right)}^{1} \frac{1}{t(\mathbf{E}(t))^2}\,dt\right). \qquad (2)$$

Here, $\mathbf{E}(x) = \displaystyle\int_0^1 \sqrt{\frac{1-x^2t^2}{1-t^2}}\,dt$ is the elliptic Legendre integral of the second kind and $\Phi = \Psi^{-1}$, where $\Psi(x) = \dfrac{x}{\mathbf{E}(x)}$, $0 \leq x \leq 1$. Also, we give the quality of approximation by the PSWFs in the spaces of band-limited, almost band-limited functions as well as on the Sobolev spaces $H^s([-1,1]), s > 0$. The different results of this talk, are illustrated by some numerical examples.

Sparse approximations with α-molecules

Sandra Keiper
TU Berlin
keiper@math.tu-berlin.de
Coauthors: Philipp Grohs, Gitta Kutyniok, Martin Schfer

Cartoon-like image are a good known model to describe natural images. They are typically defined to be functions in $\mathbb{R}^2$ which are Hölder continuous onto two areas separated by a α-Hölder continuous boundary curve. One can show that the error of the N-term approximation for such images cannot exceed $N^{-\alpha}$. Also well known is that shearlets and curvelets are well suited for such anisotropic structures. In a recent paper we have shown that we can abstract many anisotropic representation systems, like shearlets but also wavelets, to the notion of α-molecules. This allows us to deduce general results like optimally sparse approximations. So we have shown that the shearlet transform associated to the coefficient α as well as the curvelet transform provide almost optimally sparse approximations. Since shearlets are more suitable for the digital realm we use shearlets for implementation.

Wiener algebra and Gabor frames with infinite support

Yeonhyang Kim
Central Michigan University
kim4y@cmich.edu

A frame in a Hilbert space H is a sequence of vectors (f_i) for which there exist constants $0 < A \leq B < \infty$; such that for all f in H, $A||f||^2 \leq \sum |\langle f, f_i \rangle|^2 \leq B||f||^2$. Due to some desirable properties, frames with infinite support are of interest in applications and have been extensively studied. In this paper, we explore the question of when we can generate a frame with infinite support for L^2-space using frame techniques and the Wiener algebra.

Self-similar operator on a Bernoulli convolution

Keri Kornelson
University of Oklahoma
kkornelson@ou.edu
Coauthors: Palle Jorgensen, Karen Shuman

Some, but not all, Bernoulli convolution measures are known to be spectral, in the sense that there exists a Fourier basis for the corresponding L2 Hilbert space. The support of the Bernoulli convolutions are self-similar fractal subsets of the real line. We describe an operator mapping between Fourier bases on one such spectral measure and find that it, too contains self-similarity properties. In this talk, we will describe the properties of this "operator-fractal".

Factorization results in matrix Banach algebras

Ilya Krishtal
Northern Illinois University
krishtal@math.niu.edu
Coauthors: Thomas Strohmer, Tim Wertz

We present conditions for a matrix in a Banach algebra that admits an LU-factorization in $B(\ell^2)$ to admit an LU-factorization in the Banach algebra. A few other classical factorizations follow.

α-Molecules: Wavelets, Shearlets, and beyond

Gitta Kutyniok
Technische Universitt Berlin
kutyniok@math.tu-berlin.de
Coauthors: Philipp Grohs (ETH Zurich), Sandra Keiper (Technische Universität Berlin), and Martin Schäfer (Technische Universität Berlin)

The area of applied harmonic analysis provides a variety of multiscale systems such as wavelets, curvelets, shearlets, or ridgelets. A distinct property of each of those systems is the fact that it sparsely approximates a particular class of functions. Some of these systems even share similar approximation properties such as curvelets and shearlets which both optimally sparsely approximate functions governed by curvilinear features, a fact that is usually proven on a case-by-case basis for each different construction. The recently developed framework of parabolic molecules, which includes all known anisotropic frame constructions based on parabolic scaling, provides a unified concept for a sparse approximation results of such systems.

In this talk we will introduce the novel concept of α-molecules which allows for a unified framework encompassing most multiscale systems from the area of applied harmonic analysis with the parameter α serving as a measure for the degree of anisotropy. The main result essentially states that the cross-Gramian of two systems with the same degree of anisotropy exhibits a strong off-diagonal decay. One main consequence we will discuss is that all such systems then share similar approximation properties, and desirable approximation properties of one can be deduced for virtually any other system with the same degree of anisotropy.

A nonlinear derivative defined in the discrete domain and its applications

Olivier LALIGANT
Le2i Lab., University of Burgundy
olivier.laligant@u-bourgogne.fr

A nonlinear derivative is directly defined in the discrete domain. This derivative is motivated by the asymmetry of pattern in discrete signal as step or edge in 2D signal. Thanks to the special definition (in the discrete domain) of this derivative, pattern can be detected in a univocal way. This derivative is the only one able to perfectly detect and localize ideal edges in image. Beside this fundamental benefit, the derivative has the nice property to reduce noise. Applications to edge detection, noise reduction and noise estimation are described and their performances are studied.

Shearlet-based analysis of singularities and applications to fluorescent image analysis of neuronal cultures

Demetrio Labate
University of Houston
dlabate@math.uh.edu
Coauthors: Kanghui Guo, Burcin Ozcan, Manos Papadakis

During the last decade, a new generation of multiscale representations has emerged - most notably the shearlet representation - offering a very powerful framework for microlocal analysis. Using the shearlet transform, in particular, it is possible to derive a precise geometric characterization of the set of singularities of a large class of multidimensional functions and distributions. These properties provide the theoretical underpinning for several innovative algorithms for image processing and feature extraction. We will show an application of these ideas to the automated extraction of morphological features from fluorescent images of neuronal cultures.

The Two Weight Inequality for the Cauchy transform for $\mathbb{R}$ to $\mathbb{C}_+$

Michael T Lacey
207 Mimosa Dr
lacey@math.gatech.edu
Coauthors: E. Sawyer, C.-Y. Shen, Uriate-Tuero and B. Wick

We characterize those pairs of weights, on on the real line, and the other on the complex plane, such that the Cauchy transform is bounded from L^2 of the first weight, to L^2 of the second. The characterization is in terms of a joint Poisson A_2 condition, and a suite of testing inequalities. This verifes a conjecture of Nazarov-Treil-Volberg.

Applications of the inequality include a question of Donald Sarason on the composition of Toeplitz operators, and another of William Cohn, on the characterization of Carleson measures for model spaces.

On prolate shift frames and Shannon sampling

Joseph Laeky
New Mexico State University
jlakey@nmsu.edu
Coauthors: Jeffrey A. Hogan

Prolate spheroidal wave functions, for fixed time and bandwidth parameters, are eigenfunctions of time-limiting and band limiting, and they form an orthonormal basis for the Paley-Wiener space of the given bandwidth. We show that, when suitably normalized, certain collections of their lattice shifts also form frames for the Paley-Wiener space. A collection of prolate shifts can also provide a Riesz basis when normalized such that there is one prolate-shift per unit time-bandwidth. We use such shifts to help explain that, at least for certain subspaces of the Paley-Wiener space, finite subseries of the Shannon sampling expansion provide effective approximations of the full Shannon sampling expansion.

Spectral measures associated with Factorization of Lebesgue measures

Chun-Kit Lai
McMaster University
cklai@math.mcmaster.ca
Coauthors: Jean-Pierre Gabardo

Let Q be a fundamental domain of some full-rank lattice in $\mathbb{R}^d$ and let μ and ν be two positive Borel measures on $\mathbb{R}^d$ such that the convolution $\mu * \nu$ is a multiple of χ_Q. We consider the problem as to whether or not both measures must be spectral (i.e. each of their respective associated L^2 space admits an orthogonal basis of exponentials) and we show that this is the case when $Q = [0,1]^d$. This theorem yields a large class of examples of spectral measures which are either absolutely continuous, singularly continuous or purely discrete spectral measures. In addition, we propose a generalized Fuglede's Conjecture for spectral measures on $\mathbb{R}^1$ and we show that it implies the classical Fuglede's Conjecture on $\mathbb{R}^1$.

Diagonalizing the finite Zak transform and the finite Balian-Low theorem

Mark Lammers
UNCW
lammersm@uncw.edu

We give a diagonalization of the matrix representation of the finite Zak transform and use it to produce a fractional finite Zak transform. We proceed to use the finite Zak transform to provide a conjecture for a quantitative finite Balian-Low theorem. More specifically, we propose a minimzer of the Heisenberg sum which still yields an orthonormal basis under time-frequency shits in the finite setting.

Oversampling of wavelet frames for real dilations

Jakob Lemvig
Technical University of Denmark
jakle@dtu.dk
Coauthors: Marcin Bownik

Oversampling of wavelet frames has been a subject of extensive study by several researchers dating back to the early 1990s. The first oversampling results are due to Chui and Shi (1994), who proved that oversampling by odd factors preserves tightness of dyadic affine frames. This is now the central result of the subject known as the Second Oversampling Theorem. In this talk, we generalize the Second Oversampling Theorem for wavelet frames and dual wavelet frames in higher dimensions from the setting of integer dilations to real dilations.

Robust and Fast Subspace Recovery

Gilad Lerman
University of Minnesota
lerman@umn.edu
Coauthors: Work 1: Teng Zhang; Work 2: Michael McCoy, Joel Tropp, Teng Zhang; Work 3: Matthew Coudron; Work 4: John Goes, Teng Zhang, Raman Arora; Work 5: Tyler Maunu

Consider a dataset of vector-valued observations that consists of a modest number of noisy inliers, which are explained well by a low-dimensional subspace, along with a large number of outliers, which have no linear structure. We first describe a convex optimization problem that can reliably fit a low-dimensional model to this type of data. When the inliers are contained in a low-dimensional subspace we provide a rigorous theory that describes when this optimization can recover the subspace exactly. We present an efficient algorithm for solving this optimization problem, whose computational cost is comparable to that of the non-truncated SVD. We also show that the sample complexity of the proposed subspace recovery is of the same order as PCA subspace recovery and we consequently obtain some nontrivial robustness to noise. At last, we discuss modifications of this convex strategy to obtain some computational advantages.

Fast thresholding algorithms with feedbacks for sparse signal recovery

Shidong Li
Department of Mathematics
shidong@sfsu.edu
Coauthors: Tiebin Mi and Shidong Li

We provide another framework of fast iterative algorithms based on null space tuning, thresholding, and feedbacks for sparse signal recovery arising in sparse representation and compressed sensing. Several algorithms with different feedbacks are derived. Convergence results are also provided. The core algorithm is shown to converge in finite many steps under a (preconditioned) restricted isometry condition. Numerical studies about the effectiveness and the speed of the algorithms are also presented. The algorithms are seen as particularly effective for large scale problems.

Sampling and Inference for Spatiotemporal Single-Photon Imaging

Yue M. Lu
Harvard University
yuelu@seas.harvard.edu

In this talk, we will present some of our recent work on spatiotemporal single-photon sensors (SPS). In particular, we will present the performance bounds of the SPS in acquiring light intensity fields; on time-sequential adaptive sensing schemes that allow one to push the imaging capabilities of SPS systems beyond the nominal limit imposed by current hardware; and on new image formation algorithms that can efficiently "decode" the massive bitstreams generated by the SPS.

A Statistical Problem from Spectroscopy, whose Solution Hints of Compressive Sensing

Bradley Lucier
Purdue University
lucier@math.purdue.edu
Coauthors: Gregery T. Buzzard (Purdue University), Dor Ben-Amotz, Purdue University (Purdue University)

A number of Raman spectroscopy instruments incorporating micromirror arrays have been built in Dor Ben-Amotz's laboratory in the Chemistry Department at Purdue.

A question arises: How best to set up the mirrors to distinguish among a given set of chemical species.

The answer comes from a non-standard problem in Optimal Design of Experiments, a subfield of statistics.

We'll present an overview of the problem, the mathematical challenges, and some experimental results.

Multiscale Geometric Methods for Statistical Learning and Data in High-Dimensions

Mauro Maggioni
Duke University
mauro.maggioni@duke.edu
Coauthors: M. Crosskey, S. Gerber, D. Lawlor, S. Minsker, N. Strawn

We discuss a family of ideas, algorithms, and results for analyzing various new and classical problems in the analysis of high-dimensional data sets. These methods rely on the idea of performing suitable multiscale geometric decompositions of the data, and exploiting such a decomposition to perform a variety of tasks in signal processing and statistical learning. In particular, we discuss the problem of dictionary learning, where one is interested in constructing, given a training set of signals, a set of vectors (dictionary) such that the signals admit a sparse representation in terms of the dictionary vectors. We discuss a multiscale geometric construction of such dictionaries, its computational cost and online versions, and finite sample guarantees on its quality. We then generalize part of this construction to other tasks, such as learning an estimator for the probability measure generating the data, again with fast algorithms with finite sample guarantees, and for learning certain types of stochastic dynamical systems in high-dimensions.

Multiscale High-Dimensional Neural Networks Approximations

Stphane Mallat
Ecole Normale Suprieure, Paris
`stephane.mallat@ens.fr`
Coauthors: Xu Chen, Xiu-Yuan Cheng, Matthew Hirn

Learning problems are high-dimensional approximations which brutally face the curse of dimensionality. Supervised and unsupervised learning require some form of dimensionality reduction, which amounts to computing invariants. Remarkable results have recently been obtained by highly non-linear deep neural networks. We show that such approximation networks can be constructed with scale separation strategies, using iterated wavelet tranforms. For complex classification problems, multiscale wavelet operators must be learned from data. Classification applications will be shown on images, sounds and unstructured data.

Coorbit spaces with voice in a Fréchet space

Filippo De Mari
University of Genova, Italy
`demari@dima.unige.it`
Coauthors: Dahlke, Univ. Marburg, Germany De Vito, Univ. Genova, Italy Labate, Univ. Houston, USA Steidl, Univ. Kaiserslautern, Germany Teschke, Univ. Neubrandenburg. Germany Vigogna, Univ. Genova, Italy.

This is joint work with Dahlke, De Vito, Labate, Steidl, Teschke and Vigogna. We set up a new general coorbit space theory for non integrable kernels. Given a unitary representation π of a group G on a Hilbert space H, we assume that it satisfies $\|f\|^2 = \int_G |Vf(x)|^2\,dx$, whenever $f \in H$, where V is the voice transform associated to an admissible vector $u \in H$, namely $Vf(x) = \langle f, \pi(x)u\rangle_{\mathcal{H}}$, and we suppose that the kernel $K = Vu$ belongs to T, a rather general Fréchet space of functions on the group that is required to satisfy a number of axioms. This replaces the standard assumption $K \in L^1(G)$ and works even when π is not irreducible.

This framework applies to the classical since kernel and to non-integrable wavelets such as the Shannon wavelet. Some of the most interesting examples come from reproducing representations of triangular subgroups of $Sp(2, \mathbb{R})$, including the so-called *Schrödingerlets*. In all these cases the natural Fréchet space is $T = \bigcap_{p\in(1,+\infty)} L^p(G)$. This theory is succesful in the sense that it provides a workable sobstitute for the standard integrability condition on the kernel, it contains the classical coorbit space theory even for non irreducible representations, it applies to several interesting examples and it is compatible with the recent theory developed by Christensen and Ólafsson.

Some optimal design problems for finite frames

Pedro Massey
Univ. Nacional de La Plata and IAM-CONICET, Argentina
pedromassey@gmail.com
Coauthors: Mariano Ruiz and Demetrio Stojanoff (Univ. Nacional de La Plata and IAM-CONICET, Argentina)

In this talk we will consider the following two optimal design problems in finite frame theory:

- Given a redundant frame $\mathcal{F} = \{f_i\}_{i=1}^n$ in $\mathbb{C}^d$, we compute the spectral and geometrical structure of minimizers of the frame potential among the dual frames $\mathcal{G} = \{g_i\}_{i=1}^n$ for $\mathcal{F}$ such that $\sum_{i=1}^n \|g_i\|^2 \geq t$ and such that $\|T_{\mathcal{F}^\#} - T_{\mathcal{G}}\| \leq \epsilon$, where $T_{\mathcal{G}}$ denotes the synthesis operator of $\mathcal{G}$ and $\mathcal{F}^\#$ denotes the canonical dual of $\mathcal{F}$.

- Given a frame $\mathcal{F} = \{f_i\}_{i=1}^n$ in $\mathbb{C}^d$, we compute the spectral and geometrical structure of the minimizers of the frame potential among all coherent perturbations $V \cdot \mathcal{F} = \{V\, f_i\}_{i=1}^n$ where V is any invertible operator V such that $\|V^*V - I\| \leq \delta$ and $\det(V^*V) \geq s$.

The motivation of these problems is the search of numerical stable encoding-decoding schemes based on (perturbations) of $\mathcal{F}$. Our approach relies in Lidskii's type inequalities (both additive and multiplicative) from matrix analysis theory. It turns out that the matrix models behind these two (seemingly unrelated) problems are intimately connected. The talk is based on joint work with Mariano Ruiz and Demetrio Stojanoff.

Directional time-frequency analysis via continuous frames

Peter Massopust
Technische Universitt Mnchen
massopust@ma.tum.de
Coauthors: Ole Christensen and Brigitte Forster

Using the concept of ridge functions, Grafakos and Sansing obtained directionally sensitive time-frequency decompositions in $L^2(\mathbb{R}^n)$ based on Gabor systems in $L^2(\mathbb{R})$. We generalize their result by showing that similar results hold starting with general frames for $L^2(\mathbb{R})$, both in the setting of discrete frames and continuous frames. This allows to apply the theory for several other classes of frames, e.g., wavelet frames and shift-invariant systems. We will consider applications to the Meyer wavelet and complex B-splines. In the special case of wavelet systems, we show how to discretize the representations using ε-nets.

Stable sampling and Fourier Multipliers

Basarab Matei
LAGA Paris XIII
matei@math.univ-paris13.fr
Coauthors: Y. Meyer, J. Ortega-Cerd

We study the relationship between stable sampling se- quences for bandlimited functions in $L^p(R^n)$ and the Fourier multipliers in L^p. In the case that the sequence is a lattice and the spectrum is a fundamental domain for the lattice the connection is complete. In the case of irregular sequences there is still a partial relationship.

Frames of translates on the Heisenberg group

Azita Mayeli
City University of New York
amayeli@qcc.cuny.edu

Frames generated by translates of functions play an important role in sampling theory and wavelet theory. In this talk, we will give a complete characterization of frames and Riesz bases of translations on the Heisenberg group analogues to the characterization in the Euclidean case. More precisely, we will give a necessary and sufficient condition for Plancherel transform of a function whose translations form a frame (resp. Riesz basis) for its closed linear span.

Memoryless scalar quantization (MSQ) for random frames

Kateryna Melnykova
University of British Columbia
melnykova@math.ubc.ca
Coauthors: Ozgur Yilmaz

Frames are known to provide stable and robust discrete signal representations. In general, these representations are in terms of frame coefficients that generally take on a continuous range of values. To be amenable to digital storage, transmission, and processing, it is crucial that the amplitudes of the frame coefficients are discretized or "quantized". The simplest quantization method in this setting is memoryless scalar quantization (MSQ) where one essentially rounds off each frame coefficient separately. For error analysis of MSQ, the engineering literature often relies on the "White Noise Hypothesis" (WNH) that says that, for a fixed frame and random signal, the coefficient quantization errors associated with each coefficient are i.i.d. random variables. Nonetheless, the WNH is shown to be not rigorous and is not completely valid, at least in certain cases. In this talk, we will focus on Gaussian random frames and estimate the MSQ error rigorously. Specifically, we will show that the result essentially coincides with the error bound predicted by the WNH.

Simple n-dimensional wavelet sets

Kathy Merrill
Colorado College
kmerrill@coloradocollege.edu

New examples of wavelet sets that are finite unions of convex polytopes will be presented for both scalar and matrix dilation in $L^2(R^n)$, for all $n \geq 2$. Several examples with all faces parallel to coordinate planes will be included.

Nonlinear Dimensionality Reduction: The Inverse Map.

Francois Meyer
University of Colorado at Boulder
fmeyer@colorado.edu
Coauthors: Nathan Monnig and Bengt Fornberg

Nonlinear dimensionality reduction embeddings computed from datasets do not provide a mechanism to compute the inverse map. In this talk, we address the problem of computing a stable inverse map to such a general bi-Lipschitz map. The approach relies on radial basis functions (RBFs) to interpolate the inverse map everywhere on the low-dimensional image of the forward map. We demonstrate that the scale-free cubic RBF kernel performs better than the Gaussian kernel: it does not suffer from ill-conditioning, and does not require the choice of a scale. The proposed construction is shown to be similar to the Nystrom extension of the eigenvectors of the symmetric normalized graph Laplacian matrix. Based on this observation, we provide a new interpretation of the Nystrom extension with suggestions for improvement.

Representation of functions on big data: graphs and trees

H. N. Mhaskar
California Institute of Technology & Claremont Graduate University
hmhaska@gmail.com
Coauthors: C. K. Chui, F. Filbir

Many current problems dealing with big data can be cast efficiently as function approximation on graphs. The information in the graph structure can often be reorganized in the form of a tree; for example, using clustering techniques. The objective of this paper is to develop a new system of orthogonal functions on weighted trees. The system is local, easily implementable, and allows for scalable approximations without saturation. A novelty of our orthogonal system is that the Fourier projections are uniformly bounded in the supremum norm. We describe in detail a construction of wavelet–like representations and estimate the degree of approximation of functions on the trees.

Accuracy, Sum Rules and Crystal Wavelets

Ursula Molter
FCEyN, Universidad de Buenos Aires
umolter@dm.uba.ar
Coauthors: Maria del Carmen Moure, Alejandro Quintero

The scaling function of a crystal Multiresolution Analysis can be identified with a vector-scaling function for a Multiresolution Analysis associated to a lattice. Therefore, the well known techniques for this case can be applied. (Eg: determining the accuracy, approximation properties, etc.) However, the structure that the underlying crystal group provides, enables us to simplify these properties - and for example allows to find *sum rules* for multiwavelets, which before were only available for single functions. The group structure is also helpful for the actual construction of crystal scaling functions. Further, when trying to build the crystal wavelet basis, much fewer calculations have to be performed by taking advantage of the tight structure imposed by the crystal group.

On structural decompositions of finite frames

Sivaram K. Narayan
Department of Mathematics, Central Michigan University, Mount Pleasant, MI 48859
sivaram.narayan@cmich.edu
Coauthors: Alice Chan, Martin S. Copenhaver, Logan Stokols, Allison Theobold.

A *frame* in an n-dimensional Hilbert space H_n is a possibly redundant collection of vectors $\{f_i\}_{i\in I}$ that span the space. A *tight* frame is a generalization of an orthonormal basis. A frame $\{f_i\}_{i\in I}$ is said to be *scalable* if there exist nonnegative scalars $\{c_i\}_{i\in I}$ such that $\{c_i f_i\}_{i\in I}$ is a tight frame. In this report we study the combinatorial structure of frames and their decomposition into tight or scalable subsets by using partially-ordered sets (posets). We define the *factor poset* of a frame $\{f_i\}_{i\in I}$ to be a collection of subsets of I ordered by inclusion so that nonempty $J \subseteq I$ is in the factor poset if and only if $\{f_j\}_{j\in J}$ is a tight frame for H_n. We prove conditions which factor posets satisfy and use these to study the *inverse factor poset problem*, which inquires when there exists a frame whose factor poset is some given poset P. We determine a necessary condition for solving the inverse factor poset problem in H_n which is also sufficient for H_2. We then turn our attention to scalable frames and present partial results regarding when a frame can be scaled to have a given factor poset.

Combining Riesz bases in higher dimensions

Shahaf Nitzan
Kent State University
shahaf.n.h@gmail.com
Coauthors: Gady Kozma

In a previous work, Joint with Gady Kozma, we proved that every finite union of intervals admits a Riesz basis of exponentials. In this talk we will discuss an extension of this result to higher dimensions: Every finite union of rectangles, with edges parallel to the exes, admits a Riesz basis of exponentials.

Approximating scalable frames

Kasso Okoudjou
University of Maryland
kasso@math.umd.edu
Coauthors: Xuemei Chen, Gitta Kutyniok, Friedrich Philipp, and Rongrong Wang

A frame for a finite dimensional real Euclidean space is scalable if its vectors can be rescaled in such away that the resulting system becomes a tight frame. In the first part of this talk we shall present some quantitative measures of scalability of a frame. The second part of the talk will focus on the best approximations of non scalable frames by scalable ones. The results we present in this second part use in an essential way the previously introduced measures of scalability.

Coorbits, function spaces and atomic decomposition

Gestur Olafsson
Louisiana State University
olafsson@math.lsu.edu
Coauthors: J. Christensen, K.-H. Grochenig, A. Mayeli

Function spaces often come with natural symmetries and group action. This is one of the starting point in the Coorbit theory of Feichtinger and Grochenig. The theory was initiated in the late 1980's and then developed further by several authors. We will give an overview over the ideas behind the coorbit theory, discuss new developments and examples.

Using p-adic MRA's to analyze equivalence bimodules between noncommutative solenoids

Judith A. Packer
University of Colorado, Boulder
packer@colorado.edu

Let p be a prime number, and consider a noncommutative solenoid $C^*(\mathbb{Z}[\frac{1}{p}]\times\mathbb{Z}[\frac{1}{p}], \Psi_\alpha) = \mathcal{A}_\alpha$ where Ψ_α is a multiplier on $\mathbb{Z}[\frac{1}{p}] \times \mathbb{Z}[\frac{1}{p}]$. The speaker together with F. Latrémolière constructed a Morita equivalence bimodule between $\mathcal{A}_\alpha$ and $\mathcal{A}_\beta$ for a different multiplier Ψ_β on $\mathbb{Z}[\frac{1}{p}] \times \mathbb{Z}[\frac{1}{p}]$ by using a Heisenberg equivalence bimodule construction due to M. Rieffel. This bimodule involved completing continuous functionw with compact support on the locally compact abelian group $M = [\mathbb{Q}_p \times \mathbb{R}]$. This talk will use p-adic multiresolution analyses of Shelkovich and Skopina to construct the corresponding Hilbert module Ξ between $\mathcal{A}_\alpha$ and $\mathcal{A}_\beta$ as a projective multiresolution structure.

A Novel Look at Extension Principles

Manos Papadakis
University of Houston
mpapadak@math.uh.edu
Coauthors: Nikolaos Atreas and Theodoros Stavropoulos

Extension Principles address a fundamental problem in wavelet construction applicable to digital data processing. When the integer translates of a refinable function do not form a frame for their closed linear span but form only a Bessel family the construction of affine wavelet frames with desirable spatial localization cannot be carried out as in the classical multiresolution theory of Mallat and Meyer. In this case Extension Principles provide the complete answer to this problem and prescribe an efficient design strategy for stable wavelet filters with symmetry and antisymmetry properties and small support, because refinable functions can be designed to have arbitrarily smooth Fourier transforms, compact support and nice symmetry properties in the spatial domain. We will present the most recent work on Extension Principles including results on extension principles on distributional refinable functions. Our talk will highlight how any pair of homogeneous dual multiwavelet affine frames of $L_2(\mathbb{R}^s)$ gives rise to a pair of inhomogeneous dual affine multiwavelet frames constructed from a pair of refinable function vectors and vice versa. We will also show how this equivalence between these two types of affine wavelet frames extend the Mixed Oblique Extension Principle.

Linear combinations of frame generators in systems of translates

Victoria Paternostro
Technische Universitt Berlin
paternostro@math.tu-berlin.de
Coauthors: Carlos Cabrelli (University of Buenos Aires) Carolina Mosquera (University of Buenos Aires)

A finitely generated shift invariant space V is a closed subspace of $L^2(\mathbb{R}^d)$ that is generated by the integer translates of a finite number of functions. A set of frame generators for V is a set of functions whose integer translates form a frame for V. In this talk we give necessary and sufficient conditions in order that a minimal set of frame generators can be obtained by taking linear combinations of the given frame generators. Surprisingly the results are very different to the recently studied case when the property to be a frame is not required.

Greedy algorithm for subspace clustering from corrupted and incomplete data

Alexander Petukhov
University of Georgia
petukhov@hotmail.com
Coauthors: I.Kozlov

We describe the Fast Greedy Sparse Subspace Clustering (FGSSC) algorithm providing an efficient method for clustering data belonging to a few low-dimensional linear or affine subspaces. The main difference of our algorithm from predecessors is its ability to work with noisy data having a high rate of erasures (missed entries at the known locations) and errors (corrupted entries at unknown locations).

The algorithm has significant advantage over predecessor on synthetic models as well as for the Extended Yale B dataset of facial images. In particular, the face recognition misclassification rate turned out to be 6–20 times lower than for the SSC algorithm. FGSSC algorithm is able to perform clustering of corrupted data efficiently even when the sum of subspace dimensions significantly exceeds the dimension of the ambient space.

A Balian-Low theorem for subspaces of $L^2(\mathbb{R})$

Götz Pfander
Jacobs University
g.pfander@jacobs-university.de
Coauthors: Carlos Cabrelli, Ursula Molter

The Balian-Low theorem states that the window function φ of a Gabor Riesz basis $(\varphi, a\mathbb{Z}\times b\mathbb{Z})$ for $L^2(\mathbb{R}$ cannot be well-localized in time and in frequency. We extend this result to Gabor Riesz bases of subspaces of $L^2(\mathbb{R})$ that are invariant under a time-frequency shift λ that lies outside the lattice a $a\mathbb{Z} \times b\mathbb{Z}$.

Adaptive Stable Reconstruction of M-Sparse Vectors from Fourier Data

Gerlind Plonka
University of Goettingen
plonka@math.uni-goettingen.de

We propose a deterministic stable algorithm for sparse vector reconstruction from Fourier data. Particularly, if the vector N-dimensional vector $\mathbf{x}$ is M-sparse, we need at most $\min\{M \log(N), N\}$ Fourier values in order to recover $\mathbf{x}$. The algorithm works iteratively and does not incorporate any a priori knowledge on the sparsity M of $\mathbf{x}$. Each iteration step only involves the solution of a linear system of size at most M. If we are allowed to choose the Fourier samples for reconstruction adaptively at each iteration level then we can develop a strategy to ensure that the coefficient matrix in the linear system is well-conditioned.

The proposed method can be generalized to functions that can be sparsely represented in a finite basis.

Our idea is compared with the Prony method for vector reconstruction, where M-sparse vectors can theoretically be recovered by $2M$ Fourier data. However, the Prony method is particularly for larger M severely ill-conditioned. Further, we compare the approach with compressive sensing algorithms, where the Fourier data are chosen randomly, and where usually at least $\mathcal{O}(M \log N)$ data are needed to obtain a correct reconstruction with high probability. The talk is based on joint work with Shai Dekel.

Adaptive total variation regularization in image processing

Surya Prasath
University of Missouri-Columbia
prasaths@missouri.edu

We consider an adaptive version of the well-known total variation (TV) based regularization model for image restoration in image processing. Adaptive TV based schemes try to avoid the staircasing artifacts [Nikolova, Local strong homogeneity of a regularized estimator, SIAM J. Applied Math. 2000] associated with TV regularization based energy minimization models. An algorithm based on a modification of the split Bregman technique proposed by [Goldstein and Osher, The split Bregman algorithm for L^1 regularized problems, SIAM J. Imaging Sci. 2009], can be used for solving the adaptive case. Convergence analysis of such an alternating scheme is proved using the Fenchel duality and a recent result on the weak convergence of Douglas-Rachford splitting method [Svaiter, On weak convergence of the Douglas-Rachford method, SIAM J. Control Optim. 2011]. We demonstrate comparative experimental results using the modified split Bregman, dual minimization and additive operator splitting for the gradient descent scheme for the TV diffusion equation to highlight the efficiency of adaptive TV based schemes for image denoising, restoration and decomposition problems.

Interpolation via compressive sensing

Holger Rauhut
RWTH Aachen University
rauhut@mathc.rwth-aachen.de
Coauthors: Rachel Ward, Christoph Schwab

Compressive sensing predicts that sparse vectors can be recovered from incomplete linear (randomized) measurements via efficient algorithms such as ℓ_1-minimization. This principle can be easily adapted to interpolation problems where the functions of interest are known to be well-approximated by a sparse expansion in terms of an orthogonal function system such as the trigonometric system. In certain situations of interest, however, functions are not only sparse but also smooth. It turns out that a good model of smooth approximately sparse functions is provided by a norm ball in a weighted ℓ_p-space on the expansion coefficients with $p < 1$. In this context, one passes to weighted ℓ_1-minimization as recovery method. We will present theoretical estimates on the achievable approximation rates. Numerical experiments show the effectiveness of the new approach. We also present an application to uncertainty quantification, i.e., the numerical solution of parametric partial differential equations.

Big Data, Sigtetics IBC, and the Holomorphic Characterization of Quantum Imaging Communication Channels

Domingo Rodriguez
University of Puerto Rico, USA
domingo@ece.uprm.edu

This work formulates a type of quantum imaging communication channel, in the context of theoretical quantum detection and estimation operations performed in configuration or phase space, following some features of the work of Masanori Ohya on information dynamics. A quantum channel is described in this work as a linear mapping between complex separable Hilbert spaces. A quantum communication channel, in turn, is described as a class of quantum channels, with an associated subclass of imaging channels. This work also discusses an information-based complexity (IBC) approach, following the fundamental work of Joseph F. Traub, for estimating a type of cost incurred when trying to model this class of imaging channels under a discrete-time, discrete-frequency computational structure. It also tries to estimate the complexity associated with the processing of big data generated through approximate solutions for potential quantum sensing applications. Finally, this work explores how the use of signal and tensor analytics (SIGTETICS) in a Toeplitz algebra computational framework contributes to a discussion of an interpretation of certain aspects of the Segal-Bargmann holomorphic representation, following the works of Brian C. Hall and Peter Woit.

The optimal transport transform and its application to pattern recognition in image databases

Gustavo K. Rohde
Carnegie Mellon University
gustavo.rohde@gmail.com

Numerous applications in science and technology require pattern recognition to be performed in signal and image databases. The predominant scenario in these applications is that the dimension of available signals is usually much larger than the number of data samples available. In observing the variations encountered in signal and image databases, it is important to note that these can be explained by changes in the signal intensities, as well as their locations. Inspired by the continuity equation, we describe a new signal processing framework that adopts a Lagrangian point of view (as opposed to the usual linear Eulerian one) using concepts from optimal transport theory. Based on a linearized version of the optimal transport metric we have derived a new nonlinear signal analysis and synthesis framework (i.e. a transform) that can be useful in analyzing the structure of signal variations encountered in image databases. Preliminary theoretical and experimental evidence pointing to potential increase in classification accuracies in discrimination problems will be shown. Examples using image databases of faces and cells will be shown. Applications in cancer detection (cytopathology) from images of cells will be presented.

Asymptotics for optimal spectrograms

Jose Luis Romero
University of Vienna
jose.luis.romero@univie.ac.at
Coauthors: Luis Daniel Abreu, Karlheinz Groechenig

We consider the problem of optimizing the concentration of the spectrogram of a function within a given target region and obtain asymptotics for the time-frequency profile of the corresponding solutions. The main result shows that these solutions are organized in such a way that they almost tile the target time-frequency spectrum.

Parseval quasi-dual frames

Mariano Ruiz
Univ. Nac. de La Plata and Inst. Arg. de Matemtica -CONICET
mruiz@mate.unlp.edu.ar
Coauthors: Pedro Massey and Gustavo Corach

Let $\{f_i\}_{i\in I}$ and $\{g_i\}_{i\in I}$ be a pair of frames for a Hilbert space H, with synthesis operators F and X respectively . In this talk, we discuss some results concerning the computation of the infimum of the operator norm of $FX^* - I$, where $\{g_i\}_{i\in I}$ is any Parseval frame. In some sense, this quantity measures the (normalized) worst-case error in the reconstruction of vectors when analyzed with the Parseval frame and synthesized with $\{f_i\}_{i\in I}$. In case that the infimum is attained on a Parseval frame $\{g_i\}_{i\in I}$, we call it a quasi-dual of $\{f_i\}_{i\in I}$. This talk is based on a joint work with Pedro Massey (UNLP-IAM, Argentina) and Gustavo Corach (UBA-IAM, Argentina).

Random encoding of quantized finite frame expansions

Rayan Saab
Mathematics Department, UCSD
rsaab@ucsd.edu
Coauthors: Mark Iwen

Frames generalize the notion of bases and provide a useful tool for modeling the measurement (or sampling) process in several modern signal processing applications. In the digital era, such a measurement process is typically followed by a quantization, or digitization step. One family of quantization methods, popular for its robustness to errors and ability to act progressively on the measurements is Sigma-Delta quantization. In this talk, we show that a simple post-processing step consisting of a discrete random embedding of the Sigma-Delta bit-stream yields near-optimal rate distortion performance with high probability, while allowing efficient reconstruction. Our result holds for a wide variety of frames, including smooth frames and random frames.

The framework of α-molecules

Martin Schfer
Technische Universitt Berlin
schaefer@math.tu-berlin.de
Coauthors: Philipp Grohs (Eidgenssische Technische Hochschule Zrich), Sandra Keiper (Technische Universitt Berlin), Gitta Kutyniok (Technische Universitt Berlin)

Since classical wavelet systems are not optimally suited to represent multivariate data if anisotropic structures are involved, such as edges or rays in images for example, many new representation systems beyond wavelets have been developed over the last decade. The considered model situation are functions with singularities along lower dimensional embedded manifolds, with the aim to provide optimally sparse approximation of these objects. Some of the most well-known nowadays termed directional representation systems are ridgelets, curvelets, and shearlets, to name just a few.

The great variety of new systems motivated the search for a common framework with the ability to simultaneously establish general results, concerning for example approximation properties. The concept of parabolic molecules, introduced recently by two of the collaborators, was a first step in this direction. It can unify shear-based and rotation-based constructions, such as classical shearlets and curvelets, under one roof and allows to investigate their approximation behavior simultaneously.

However, since the framework is confined to parabolic scaling, it cannot comprise systems based on different scaling laws, like ridgelets, wavelets, or newer hybrid constructions. This motivated the generalization of the concept to α-molecules, where a more general anisotropic scaling law, specified by the parameter α, is utilized. This framework, which we will present in this talk, is general enough to comprise all the aforementioned constructions, and at the same time still specific enough to capture their main features and properties. As a main result, we will show, that the framework of α-molecules can be used to identify large classes of representation systems with the same sparse approximation behavior.

1-Bit Compressive Sensing: Models and Algorithms

Lixin Shen
Syracuse University
lshen03@syr.edu

One-bit compressive sensing models an extreme quantization in signal processing and recently came to the forefront of the field of compressive sensing. A number of efficient algorithms were developed recently for this problem. In this talk, we shall propose a new model and an algorithm for restoring sparse signals from their 1-bit measurements.

Multiresolution analyses through low-pass filter on local fields of positive characteristic

Niraj K. Shukla
Indian Institute of Technology Indore, Indore, India
o.nirajshukla@gmail.com
Coauthors: Aparna Vyas

The concepts of a first-stage wavelet basis can be generalized to a countable subset of a local field having positive characteristic by using a prime element of such a field. In this paper, we provide a characterization of first-stage discrete wavelet system on a countable subset of a local field of positive characteristic. Further, with the help of a first-stage wavelet basis, we get some results on refinement equation and refinement coefficients which provides a construction of multiresolution analysis on local field of positive characteristic.

Linear independence of the system of translates and uniqueness of trigonometric series

Ivana Slamic
University of Rijeka, Croatia
islamic@math.uniri.hr

Given a square integrable function ψ, various properties of the corresponding system of integer translates can be expressed in terms of the periodization function p_ψ. Various non-redundancy notions can be considered, and some of them have already been characterized. Namely, we know that ℓ^2-linear independence of the system is equivalent to periodization function being positive almost everywhere, whilst the system is minimal if and only if $1/p_\psi$ belongs to L^1.

We have considered the problem of ℓ^p-linear independence, where $p \neq 2$, in order to find neccessary and sufficient conditions in terms of p_ψ. First, considering $p < 2$ case, we have established the connection with ℓ^p-sets of uniqueness – the sets of positive Lebesgue measure that do not support functions with Fourier coefficients in ℓ^p and were first introduced by Y.Katznelson.

Here, we present the results on the case where $p > 2$. The results on ℓ^2-linear independence and minimality were leading us naturally to several conjectures, which appeared to be false. We present a method of construction of linearly dependent systems which can be considered as counterexamples to such conjectures. Various results concerning properties of the partial sums of trigonometric null series play part in this construction. On the other hand, a new sufficient condition can be given in terms of sets of uniqueness for trigonometric series, despite such sets can be found amongst sets of Lebesgue measure zero. Moreover, adding an assumption that concerns Rajchman's multiplication theorem, we can give a characterization of ℓ^p-linear independence.

Admissible frames and Fourier frame approximation

Guohui Song
Clarkson University
gsong@clarkson.edu
Coauthors: Anne Gelb

Frames can be seen as a generalization of basis with over-completeness. Localized frames have obtained popularity and demonstrated robustness and efficiency in extensive literature of sampling. There are still some applications in which the data are sampled with weakly-localized frames. We will introduce admissible frames that could be used to obtain stable and accurate reconstruction with sampling data from weakly-localized frames. In particular, we apply it to Fourier frames and present fast and stable algorithms for Fourier frame approximation.

Nonnegativity as an obstruction to spanning structure

Anneliese H. Spaeth
Huntingdon College
anneliese.h.spaeth@alumni.vanderbilt.edu
Coauthors: Alexander M. Powell

Many obstructions to spanning structure are widely studied, including time-frequency localization and translation invariance. We examine the obstruction of pointwise nonnegativity of functions in $L^p(\mathbb{R})$ to spanning structure in those spaces. Some structures for which we have answered the existence question include monotone bases, unconditional bases, unconditional quasibases, Riesz bases, frames, Markushevich bases, and conditional quasibases.

Linear independence of time-frequency shifts of functions with fast decay

Darrin Speegle
Saint Louis University
speegled@slu.edu
Coauthors: Marcin Bownik

The HRT conjecture states that time-frequency shifts of L^2 functions are linearly independent.

In this talk, we establish the linear independence of time-frequency translates for functions f having one sided decay $\lim_{x\to\infty} |f(x)|e^{cx\log x} = 0$ for all $c > 0$. Generalizations to higher dimensions are also considered.

Total frame potential and its applications in data clustering

Tobias Springer
TU Dortmund
tobias.springer@tu-dortmund.de
Coauthors: Joachim Stöckler, Katja Ickstadt

For statistical analysis of microarray gene expression data, the clustering of short time series is an important objective in order to identify subsets of genes sharing a temporal expression pattern. An established method, the Short Time Series Expression Miner (STEM) by Ernst et al. (2005), assigns time series data to the closest of suitably selected prototypes followed by the selection of significant clusters and eventual grouping.

For the clustering of normalized d-dimensional data $Y = \{y_j\}_{j=1,\ldots,N}$ we propose to minimize the penalized frame potential

$$\mathrm{F}_\alpha(\Theta) = \frac{1}{\alpha}\mathrm{TFP}(\Theta) - \sum_{\ell=1}^{m} \max_{j=1,\ldots,N} \langle y_j, \theta_\ell \rangle \tag{3}$$

for $\alpha > 0$. The functional contains the "total frame potential" TFP of Finite Unit Norm Tight Frames (FUNTFs), see Benedetto and Fickus (2003), and includes a data-driven component for the selection of prototypes. We show that the solution of the corresponding constrained optimization problem is naturally connected to the spherical Dirichlet cells

$$D_j = \{v \in \mathbb{R}^d : \|v\|_2 = 1,\, \langle y_j, v\rangle = \max_{k=1,\ldots,N} \langle y_k, v\rangle\}$$

of the given normalized data. Furthermore, the minimizers of F_α are, given that $\alpha > 0$, in the interior of the Dirichlet cells and the objective function F_α is differentiable in the minimum with the extremal condition

$$4TT^*T + 2T\Lambda = \alpha Y_s$$

where T, $Y_s \in \mathbb{R}^d$ have normalized columns and $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_m)$ contains the Lagrange multipliers.

The general problem is closely related to the search for point configurations on the unit sphere like in Tammes' (1930) or Thomson's Problem (1904). Moreover, the minimization of (**??**) (subject to the constraint that the solution is normalized) contains connections to problems in matrix completion (see e.g. Candès and Tao (2009) or Mazumder, Hastie and Tibshirani (2010)).

The idea of using the frame potential in combination with a data-dependent term for optimization was originally proposed by Benedetto, Czaja and Ehler (2010) for finding sparse representations.

A Bayesian approach to the multivariate change-point problem in the wavelet domain

Robert Steward
St. Louis University
rstewa12@slu.edu

We look at the statistical change-point problem in the multivariate setting. In particular, suppose we observe some stochastic process that undergoes a shift to the process mean at an unknown time. We propose a multivariate method for predicting this change-point location by conducting a Bayesian analysis on the empirical wavelet detail coefficients of the original time series. We show that if the mean function of our time series is expressed as a multivariate step function, then our Bayesian-wavelet method performs comparably with classical methods such as maximum likelihood estimation (MLE). The advantage to our method is seen in its ability to adapt to more general situations such as piecewise smooth mean functions.

Gabor Frames, Sampling Matrices, and Total Positivity

Joachim Stöckler
Technische Universität Dortmund, Germany
joachim.stoeckler@math.tu-dortmund.de

Many properties of Gabor Frames $\mathcal{G}(g,\Lambda)$ on a regular time-frequency lattice $\Lambda = \alpha\mathbb{Z} \times \beta\mathbb{Z}$ can be expressed in terms of the matrices

$$P(x) = (g(x + \alpha j - k/\beta))_{j,k\in\mathbb{Z}}, \qquad x \in \mathbb{R},$$

thanks to the results by Ron and Shen in the 1990's and their fiberization technique for shift-invariant systems. In particular, frame bounds for $\mathcal{G}(g,\Lambda)$ are directly obtained from uniform operator bounds of $P(x)$ and operator bounds of the Moore-Penrose pseudo-inverses $P^\dagger(x)$. We call matrices of the more general form $P = (g(x_j - y_k))_{j,k\in\mathbb{Z}}$ sampling matrices.

In my talk, I explain the relation of density results for Gabor frames and sampling matrices. The necessary condition $\alpha\beta \le 1$ for Gabor frames, in general, is closely related to density results for sampling in shift-invariant spaces. For special window functions g, namely if g is a totally positive function of finite type, the sufficient condition $\alpha\beta < 1$ was found by K. Gröchenig and the author in [1], and was related to a sampling density for g.

In recent work with T. Kloos, we use the close connection of totally positive functions and exponential B-splines in order to study properties of the Gabor frame $\mathcal{G}(g,\Lambda)$. We show that Zak transforms of g have only one zero in their fundamental domain of quasi-periodicity and, for a special example, we give good estimates for the lower frame-bound near the critical density.

1. K. Gröchenig, J. Stöckler, Gabor frames and totally positive functions, Duke Math. J. 162(2013), 1003-1031.

Invertible frame multipliers and their inverses

Diana Stoeva
Acoustics Research Institute, Austrian Academy of Sciences
dstoeva@kfs.oeaw.ac.at
Coauthors: Peter Balazs (Acoustics Research Institute, Austrian Academy of Sciences)

Frame multipliers are operators which combine analysis with one frame, multiplication with a bounded scalar sequence (called the symbol), and synthesis with (possibly) another frame. They are not only interesting from a theoretical point of view, but also important for applications. In signal processing, multipliers are a particular way to implement time-variant filters. They are used for example in denoising and psychoacoustical modeling.

In this talk we concentrate on invertible frame multipliers. When the two frames are Riesz bases and the symbol is semi-normalized, one can invert the corresponding multiplier using a multiplier with the reciprocal symbol and the canonical duals of the given Riesz bases. Here we extend the class of Riesz bases to a larger class of frames, allowing the same way of inversion (using the reciprocal symbol and the canonical duals). For the general case of overcomplete frames, where such way of inverting might not work, we give a formula for the inverse multiplier using any dual of one of the frames and a unique dual of the other one. Further, we consider the case of Gabor multipliers. We give necessary and sufficient conditions for an invertible operator on L^2 (and its inverse) to be represented as a Gabor frame multiplier with a constant symbol.

Discrete lines and geometric compressed sensing

Irena Stojanoska
Technische Universitt Berlin
stojanoska@math.tu-berlin.de
Coauthors: Gitta Kutyniok (Technische Universitt Berlin)

Compressed sensing is a novel methodology in data processing which takes advantage of the fact that most signals admit a sparse representation. In such a case it then allows to recover the signal from considerably less measurements than those required by traditional methods.

We are interested in exploiting additional information about the signal, namely having sparse geometric structure. One goal in this setting is to improve the existing compressed sensing results over those where no structure is assumed. Another goal is to broaden the range of applications of the methodology of compressed sensing.

In this work we present signals consisting of unions of discrete lines as the simplest case of geometric sparsity. We discuss their properties and the application of compressed sensing to such signal models. Our results include construction of a unit norm tight frame from the set of discrete lines; a weaker version of the restricted isometry property of our measurement matrix for sparse vectors

with random support, as well as application of compressed sensing in separation of discrete points and lines.

The abc-problem for Gabor systems

Qiyu Sun
University of Central Florida
qiyu.sun@ucf.edu
Coauthors: Xin-rong Dai

A long standing problem in Gabor theory is to identify ideal window functions on intervals of length c and time-frequency shift lattices $aZ \times bZ$ such that the corresponding Gabor system is a frame for the space of all square-integrable functions on the real line. In this talk, I will present our answer to the above *abc*-problem for Gabor systems.

Triangular subgroups of $\mathfrak{Sp}(n, R)$ and reproducing formulae

Anita Tabacco
Politecnico di Torino
anita.tabacco@polito.it
Coauthors: Elena Cordero

We consider the (extended) metaplectic representation of the semidirect product $\mathcal{G} = H^d \rtimes Sp(d, R)$ between the Heisenberg group and the symplectic group. Subgroups $H = \Sigma \rtimes D$, with Σ being a $d \times d$ symmetric matrix and D a closed subgroup of $GL(d, R)$, are our main concern. We shall give a general setting for the reproducibility of such groups. As a byproduct, the extended metaplectic representation restricted to some classes of such subgroups is either the Schrödinger representation of R^{2d} or the wavelet representation of $R^d \rtimes D$, with D closed subgroup of $GL(d, R)$. Finally, we shall provide new examples of reproducing groups of the type $H = \Sigma \rtimes D$, in dimension $d = 2$.

Nonlinear sparse reconstructions in Hilbert spaces

Tang, Wai-Shing
National University of Singapore
`mattws@nus.edu.sg`

We consider a new framework of nonlinear sparse reconstruction in a Hilbert space setting, which may be thought of as nonlinear extension of finite-dimensional sparse recovery problems to infinite-dimensional spaces. We establish exponential convergence of the iterative hard thresholding algorithm to reconstruct sparse signals in a union of closed linear subspaces of a Hilbert space from their nonlinear observations. We also propose an optimization framework in a Banach space setting and use it to stably reconstruct signals in a union of closed linear subspaces of a Hilbert space.

This research is joint work with Qiyu Sun.

Greedy algorithms in compressed sensing

Vladimir Temlyakov
University of South Carolina
`temlyakovv@gmail.com`

We study sparse representations and sparse approximations with respect to incoherent dictionaries. We address the problem of designing and analyzing greedy methods of approximation. A key question in this regard is: How to measure efficiency of a specific algorithm? Answering this question we prove the Lebesgue-type inequalities for algorithms under consideration. A very important new ingredient of the talk is that we perform our analysis in a Banach space instead of a Hilbert space. It is known that in many numerical problems users are satisfied with a Hilbert space setting and do not consider a more general setting in a Banach space. There are known arguments that justify interest in Banach spaces. In this talk we give one more argument in favor of consideration of greedy approximation in Banach spaces. We introduce a concept of M-coherent dictionary in a Banach space which is a generalization of the corresponding concept in a Hilbert space. We analyze the Quasi-Orthogonal Greedy Algorithm (QOGA), which is a generalization of the Orthogonal Greedy Algorithm (Orthogonal Matching Pursuit) for Banach spaces. It is known that the QOGA recovers exactly S-sparse signals after S iterations provided $S < (1 + 1/M)/2$. This result is well known for the Orthogonal Greedy Algorithm in Hilbert spaces. The following question is of great importance: Are there dictionaries in $\mathbb{R}^n$ such that their coherence in ℓ_p^n is less than their coherence in ℓ_2^n for some $p \in (1, \infty)$? We show that the answer to the above question is yes. Thus, for such dictionaries, replacing the Hilbert space ℓ_2^n by a Banach space ℓ_p^n we improve an upper bound for sparsity that guarantees an exact recovery of a signal.

Sparse stochastic processes and operator-like wavelet expansions

Michael Unser
EPFL, Lausanne, Switzerland
Michael.Unser@epfl.ch

We introduce an extended family of continuous-domain sparse processes that are specified by a generic (non-Gaussian) innovation model or, equivalently, as solutions of linear stochastic differential equations driven by white Lvy noise. We present the functional tools for their characterization. We show that their probability distributions are infinitely divisible, which induces two distinct types of behaviorGaussian vs. sparseat the exclusion of any other. This is the key to proving that the non-Gaussian members of the family admit a sparse representation in a matched wavelet basis.

We use the characteristic form of these processes to deduce their transform-domain statistics and to precisely assess residual dependencies. These ideas are illustrated with examples of sparse processes for which operator-like wavelets outperform the classical KLT (or DCT) and result in an independent component analysis. Finally, for the case of self-similar processes, we show that the wavelet-domain probability laws are ruled by a diffusion-like equation that describes their evolution across scale.

Democracy of shearlet bases with applications to approximation and interpolation

Daniel Vera
University of Houston
danielv@math.uh.edu

Shearlets are based on wavelets with composite dilation and inherit important features from wavelets. Shearlets provide (near) optimal approximation for the class of so-called cartoon-like images. Moreover, there are distribution spaces associated to them and there exist embeddings between these and classical (dyadic isotropic) inhomogeneous spaces. We prove that the shearlets are democratic bases for the shear anisotropic inhomogeneous Besov and Triebel-Lizorkin spaces (i.e. they verify the p-Temlyakov property also known as p-space property) for certain parameters. Then, we prove embeddings (or characterizations) between approximation spaces and discrete weighted Lorentz spaces (in the framework of shearlet systems) and prove (that these embeddings are equivalent to) Jackson and Bernstein type inequalities. This allows us to find (real) interpolation spaces.

Non integrable representations and coorbit space theory

Stefano Vigogna
Universit degli Studi di Genova, Dipartimento di Matematica
vigogna@dima.unige.it
Coauthors: Stephan Dahlke (Marburg), Filippo De Mari (Genova), Ernesto De Vito (Genova), Demetrio Labate (Houston), Gabriele Steidl (Kaiserslautern), Gerd Teschke (Neubrandenburg).

For a unitary representation π of a locally compact group G (with its Haar measure) on a Hilbert space $\mathcal{H}$, the voice transform $V_u : \mathcal{H} \to L^\infty(G)$ with respect to a fixed vector $u \in \mathcal{H}$ is defined by $V_u v(x) := \langle v, \pi(x)u \rangle$. The representation π is called reproducing if V_u is an isometry of $\mathcal{H}$ into $L^2(G)$. In signal analysis, reproducing representations perform very efficient reconstruction procedures, as it is well established in many applications, notably for wavelets and shearlets. Coorbit space theory allows to extend the reproducing properties to a whole family of Banach spaces of functions and distributions, providing a characterization of smoothness spaces in terms of the decay of the voice transform. Classically, this is possible under two basic assumptions: the reproducing representation has to be irreducible, and the reproducing kernel $V_u u$ has to be integrable. I will show some examples of non irreducible representations with non integrable kernels which motivate the interest for a generalization of the classical coorbit theory. As a particularly interesting case, I will exhibit a new reproducing representation, whose admissible vectors are the so-called Schrödingerlets. I will sketch the basic ideas that allow to overcome the classical obstructions and suggest how a more general theory can naturally arise. The general theory will be described in detail in De Mari's talk.

Phase Retrieval Using Bandlimited Window Functions

Adityavikram Viswanathan
Michigan State University
aditya@math.msu.edu

Phase Retrieval refers to the problem of reconstructing a signal from its intensity measurements. It arises in many applications such as x-ray crystallography and diffraction imaging, where the detectors only capture intensity information about the underlying physical process. As is well known, the phase encapsulates vital information about the underlying signal, which makes signal recovery from such measurements extremely challenging.

In this work, we introduce a novel and efficient computational framework for reconstructing signals from their magnitude measurements. Through the use of window functions (or masks), we estimate phase differences between pairs of magnitude measurements. The band-limited design of these window functions allows us to restrict this computation to a small subset of all possible phase difference pairs. We then solve an angular synchronization problem to recover the unknown phases. Theoretical and numerical results demonstrating the accuracy and efficiency of this reconstruction framework will be presented.

Compressive Sensing Based MIMO Radar

Haichao Wang
University of California, Davis
wanghaichao0501@gmail.com
Coauthors: Thomas Strohmer

We derive a theoretical framework for the recoverability of targets in the azimuth-range-Doppler domain using tools developed in the area of compressive sensing. We will also consider the case that the targets are not assumed to be on the grids.

A one stage reconstruction method for Sigma-Delta quantization in compressed sensing

Rongrong Wang
University of British Columbia
rongwang@math.ubc.ca
Coauthors: Ozgur Yilmaz

We introduce an alternative reconstruction method for signals whose compressed samples are quantized via a Sigma-Delta quantizer. The method is based on solving a convex optimization problem, and unlike the previous approaches, it is stable and robust and admits an unconditional convergence to the true solution no matter whether the support is recovered or not. As a consequence, with an appropriate choice of quantization order, the method can achieve a root exponential error decay with respect to the oversampling rate. Finally, our theory applies to "fine" Sigma-Delta quantizers and "coarse" Sigma-Delta quantizers, e.g., 1-bit per measurement.

Sub-linear Algorithm for Recovering Sparse Fourier Series

Yang Wang
Michigan State University
ywang@math.msu.edu
Coauthors: Andrew Christlieb, David Lawlor

We present new deterministic algorithms for the sparse Fourier transform problem, in which we seek to identify $k \leq N$ significant Fourier coefficients from a signal of bandwidth N. Previous deterministic algorithms exhibit quadratic runtime scaling, while our algorithms scales linearly with k in the average case in the noiseless setting. We also present a multi-scale algorithm for noisy signals which proves to be extremely robust and efficient. This multi-scale algorithm is based on the beta-expansion scheme for robust A/D conversion. We also present the first efficient algorithm for ultra-high dimensions signals.

Density Conditions for Sampling in de Branges Spaces

Eric Weber
Iowa State University
esweber@iastate.edu
Coauthors: Sa'ud al-Sa'di, Hashemite University, Jordan

We consider the problems of sampling and interpolation in de Branges spacesHilbert spaces of entire functions which are square integrable on the real line with respect to some weight function and satisfy some growth conditions. The class of de Branges spaces considered are those whose weight function has a phase function which is bounded below. For this class, we prove that the Homogeneous Approximation Property holds for the reproducing kernel. As a consequence, necessary conditions for sampling and interpolating sequences are shown, which generalize some well-known sampling and interpolation results in the Paley-Wiener space.

Beyond harmonic analysis with Fourier-like transforms

Cameron Williams
University of Waterloo
cameronlwilliams@gmail.com
Coauthors: Bernhard G. Bodmann (University of Houston) Donald J. Kouri (University of Houston)

This talk presents a family of transforms which share many properties with the Fourier transform. We prove that these transforms are isometries on $L^2(\mathbb{R})$ and enjoy the same scaling property. The transforms can be chosen to leave functions of Gaussian or super-Gaussian type invariant. We also establish short-time analogs of these transforms.

Reconstruction of structured functions from sparse Fourier data

Marius Wischerhoff
Institute for Numerical and Applied Mathematics, University of Gttingen, Germany
m.wischerhoff@math.uni-goettingen.de
Coauthors: Gerlind Plonka

In this talk we will use the Prony method for parameter estimation of structured functions to reconstruct some structured, real-valued functions by a small number of Fourier samples.

First, we will consider the case of non-uniform translates of radial functions. In particular, we will show that linear combinations of N shifted versions of a radial function, where the shifts and the coefficients are real-valued, can be uniquely recovered from $3N + 1$ Fourier samples which are taken from three lines through the origin in the Fourier domain. Further, we will show that this approach can be generalized to the case of d-variate functions with $d > 2$.

In the second part of the talk, we will turn to the reconstruction of polygonal shapes, i.e., we will consider characteristic functions $f = \mathbf{1}_D$ of a domain D in the real plane, where D is a polygon with N vertices. We will show that the polygon D can be uniquely recovered from $3N$ samples of the Fourier transform of f, where these samples are taken from three lines through the origin in the Fourier space.

Kernel-based Approximation Methods for Stchastic Partial Differential Equations

Qi Ye
Department of Mathematics, Syracuse University
qiye@syr.edu

In this talk, we will show how to solve the high-dimensional stochastic partial differential equations by the kernel-based approximaiton methods. We extend the classical kernel-based approaches for deterministic data to stochastic data. The kernel-based methods are meshfree methods, and the kernel-based estimators are also feasible for high-dimensional domains or complicated boundary conditions. The kernel-based approximate solutions of the stochastic equations are constructed by the suitable positive definite kernels, e.g., compact support kernels (Wendland functions) and Sobolev-spline kernels (Matérn functions).

Quantization of compressed sensing measurements: stability, robustness, and more

Ozgur Yilmaz
The University of British Columbia
oyilmaz@math.ubc.ca

In this talk, we will discuss how to efficiently quantize compressive samples of sparse or compressible signals. Our focus will be Sigma-Delta quantization. Sigma-Delta methods are typically used to quantize redundant expansions, e.g., oversampled bandlimited functions or frame expansions. We have recently shown that these quantizers also provide superior approximations when used in the compressed sensing setting by establishing a link between compressed sensing quantization and frame quantization. Our original result was only valid in the case of exactly sparse signals with no noise and with Gaussian measurement matrices. I will summarize recent developments which remove these restrictive requirements and further enable us to obtain approximation rates that are better than inverse polynomial in the number of measurements. The recent results are joint work with Rongrong Wang and Rayan Saab.

Two-distance tight frames

Wei-Hsuan Yu
University of Maryland, College park
u690604@gmail.com
Coauthors: Alexander Barg, Kasso Okoudjou

A set of S of unit vectors in n-dimensional Euclidean space is called spherical two-distance sets, if there are two numbers a and b so that the inner products of distinct vectors of S either a or b. When $a + b \neq 0$, we derive new structural properties of the Gram matrix of a two-distance set that also forms a tight frame for $\mathbb{R}^n$. In addition, we establish a one-to-one correspondence between two-distance tight frames with certain strongly regular graph. This allows us to use the adjacent matrix of these strongly regular graphs to construct two-distance tight frames. Several new examples are obtained along this characterization.

Edge detection of brain tumor using curvelet transform

Noore Zahra
Sharda University
noor_zahra_india@yahoo.co.in
Coauthors: Shahin Naz, Manisha Rajoriya

Medical images has lots of curves, phase information and textured information. Edges are the most prominent feature for images. Edges defined the boundaries between different textures it shows discontinuities in image intensity from one pixel to other. In medical imaging detecting and enhancing the boundaries between cavities is an important task and curvelet transform is proved as as a best tool for edge detection in the recent few years. The curvelet transform is a higher dimensional generalization of the wavelet transform designed to represent images at different scales and different angles. Curvelet transform capture efficiently edges in an image; it is a good candidate for multiscale edge enhancement. In this paper curvelet transform is used to find the edges in tumor. The idea is to modify the curvelet coefficients of the input image in order to enhance its edges.

On the relationship between the Mittag-Leffler transform and the fractional Fourier transform

Ahmed Zayed
DePaul University
azayed@condor.depaul.edu

There are different extensions of the Fourier transform to fractional orders. The most common extension in applications is based on the observation that the Hermite functions are eigenfunctions of the Fourier transform. This extension of the Fourier transform is related to a class of integral transforms known as linear canonical transforms which play an important role in optics and signal processing.

On the other hand, there is another extension of the Fourier transform to fractional orders based on the Mittag-Leffler transform. This extension is related to fractional derivatives and Taylor's series of fractional order.

In this talk we examine the connection between these extensions of the Fourier transform and the sampling expansions associated with them.

Tensor product complex tight framelet: construction and application

Zhenpeng Zhao
University of Alberta
zzhao7@ualberta.ca
Coauthors: Bin Han, and Qun Mo

In this talk, we shall address the construction of tensor product complex tight framelets with increasing directionality with respect to the real-valued wavelets. The applications of tensor product complex tight framelets on increasing the directionality of initial stage filters in dual tree complex wavelet transform and image denoisnig with mixtures of Gaussian scale mixture model will also be discussed. We shall show that tensor product complex tight framelets have superior performance in image denoising, compared with undecimated wavelet transform, and etc.

Mathematical analysis for information theoretic learning

Ding-Xuan Zhou
City University of Hong Kong
mazhou@cityu.edu.hk

Information theoretic learning is a learning framework that uses descriptors from information theory (entropy and divergence) estimated directly from data to substitute the conventional statistical descriptors of variance and covariance. Minimum error entropy (MEE) is a principle for designing supervised learning algorithms that falls into the information theoretic learning framework. MEE algorithms have been applied successfully in various fields for more than a decade, and can deal with problems involving non-Gaussian noise for which the classical least squares method is not ideal. In this talk we consider empirical MEE learning algorithms in a regression setting. Statistical consistency of an MEE algorithm in an empirical risk minimization framework is presented in details, including error entropy consistency and regression consistency for homoskedastic models and heteroskedastic models. Fourier analysis plays a crucial rule in our analysis. Regularization schemes in reproducing kernel Hilbert spaces are also discussed.

Smooth affine shear tight frames

Xiaosheng Zhuang
City Univ. of Hong Kong
xzhuang7@cityu.edu.hk
Coauthors: Bin Han

A directional wavelet tight frame is generated by isotropic dilations and translations of directional wavelet generators, while an affine shear tight frame is generated by anisotropic dilations, shears, and translations of shearlet generators. We shall show that these two tight frames are actually connected in the sense that the affine shear tight frame can be obtained from a directional wavelet tight frame through sub-samplings. Consequently, the affine shear tight frame indeed has an underlying filter bank systems from the MRA structure of the directional wavelet tight frame.