

# An Improved Layered MIMO Detection Algorithm With Near-Optimal Performance

A. Alimohammad, S. F. Fard, B. F. Cockburn

This letter presents a new layered symbol detection algorithm for multiple-input multiple-output (MIMO) systems. In this scheme, the layers are divided into two groups and detected differently. For the layer with the smallest post-detection signal-to-noise ratio, an exhaustive search is performed over the signal constellation; for the remaining layers the conventional iterative vertical Bell laboratories layered space-time (V-BLAST) technique is utilized. The proposed algorithm also uses a different symbol detection ordering than that recommended in the original V-BLAST algorithm. Simulation results show that the error rate performance of the proposed detection algorithm approaches closely that of an optimal maximum likelihood detector with no reduction in the symbol detection throughput, while its computational complexity is less than  $|\mathcal{Q}|$  times that of V-BLAST, where  $|\mathcal{Q}|$  is the cardinality of signal constellation  $\mathcal{Q}$ .

*Layered Space-Time Architecture:* The vertical Bell laboratories layered space-time (V-BLAST) algorithm is one of the most promising detection techniques for the practical implementation of multiple-input multiple-output (MIMO) receivers [1]. In a spatial multiplexing MIMO system with  $n_T$  transmitting antennas and  $n_R$  receiving antennas, the high-rate data stream is demultiplexed into  $n_T$  equal-capacity parallel sub-streams forming a sequence of symbol vectors  $\mathbf{s}$  that is transmitted simultaneously over a richly-scattered wireless channel without the need for additional frequency bandwidth or total transmit power. The  $n_R$ -dimensional vector  $\mathbf{r}$  of received symbols can be written in the standard baseband discrete-time model as  $\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}$ , where  $\mathbf{n}$  denotes an additive white Gaussian noise vector comprising statistically-independent, normally-distributed complex variables with equal variance  $\sigma_n^2$ . The  $n_R \times n_T$  channel matrix can be represented as  $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_{n_T}]$ , where  $\mathbf{h}_k$  is the column vector of complex transfer gains from the  $k$ -th transmitter antenna to each of the  $n_R$  receiver antennas.

At the receiver, assuming that the channel matrix  $\mathbf{H}$  is known (or estimated perfectly), a linear filtering matrix, such as the one based on the minimum mean-square error (MMSE) criterion [2], can be computed as  $\mathbf{G} = (\mathbf{H}^H \mathbf{H} + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I}_{n_T})^{-1} \mathbf{H}^H$ , where  $\mathbf{H}^H$  denotes the Hermitian of  $\mathbf{H}$  and the signal-to-noise ratio (SNR)  $\sigma_s^2/\sigma_n^2$  is determined (or estimated perfectly) at the receiver. V-BLAST detects each layer (symbol) separately by using an iterative decision feedback approach. For each layer  $k$ , first an interference nulling step tries to reduce the amount of interference towards  $s_k$  by multiplying the received signal  $\mathbf{r}$  by a nulling vector  $\mathbf{g}_k$ , where  $\mathbf{g}_k$  is the  $k$ -th row of  $\mathbf{G}$ . Second, symbol  $s_k$  from the  $k$ -th transmitter antenna is detected using the slicer function  $\mathcal{Q}(\cdot)$ , which returns the nearest symbol in the signal constellation  $\mathcal{Q}$  to the estimated symbol  $\hat{s}_k$ . Finally, the interference on the  $n_T - 1$  other signals due to  $s_k$  can be subtracted from the received signal. V-BLAST proceeds iteratively through the above three steps until all  $n_T$  transmitted symbols are recovered.

Note that since all  $n_T$  components of  $\mathbf{s}$  utilize the same constellation  $\mathcal{Q}$ , the weakest layer  $k$ , i.e., the layer with the smallest post-detection SNR  $E\{|s_k|^2\}/(\sigma_n^2\|\mathbf{g}_k\|^2)$  will dominate the error performance of the system. Thus it was recommended in [1] that the detection algorithm start with the layer with the strongest post-detection SNR (i.e., corresponding to the row  $\mathbf{g}_k$  of  $\mathbf{G}$  with the minimum norm) and then proceed successively to detect the symbol of the layer with the next weakest SNR. As shown in Algorithm 1, after estimating and canceling  $s_k$ ,  $\mathbf{h}_k$  is zeroed and hence  $\mathbf{G}$  must use a deflated version  $\mathbf{H}_{\bar{k}}$  of  $\mathbf{H}$  in the next iteration, where  $\mathbf{H}_{\bar{k}}$  denotes the matrix obtained by zeroing column  $k$  of  $\mathbf{H}$ . The notation  $O(i)$  denotes the layer  $k$ , where  $k \in \{1, \dots, n_T\}$ , that is to be detected at step  $i$ . Note that under the assumption of quasi-stationary block-fading channels, the channel variation is negligible over a coherence period and it changes independently from one period to another. Therefore, nulling vectors need be computed only once for every block of received symbols.

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**Algorithm 1** MMSE V-BLAST algorithm.

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 $\hat{\mathbf{H}} = \mathbf{H};$ 
for ( $i = 1; i \leq n_T; i++$ ) do
   $\mathbf{G} = (\hat{\mathbf{H}}^H \hat{\mathbf{H}} + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I})^{-1} \hat{\mathbf{H}}^H;$ 
   $O(i) = k = \min_j \|\mathbf{g}_j\|^2; \{ \text{Ordering} \}$ 
   $\mathbf{g}_i = \mathbf{G}(k, :);$ 
   $\hat{\mathbf{H}} = \hat{\mathbf{H}}_{\bar{k}};$ 
end for
for (every received symbol vector  $\mathbf{r}$  in a block) do
  for ( $i = 1; i \leq n_T$ ) do
     $k = O(i);$ 
     $\hat{s}_k = \mathbf{g}_k^H \mathbf{r}; \{ \text{Nulling} \}$ 
     $s_k = \mathcal{Q}(\hat{s}_k); \{ \text{Slicing} \}$ 
    if ( $i < n_T$ ) then
       $\mathbf{r} = \mathbf{r} - \mathbf{h}_k s_k; \{ \text{Cancellation} \}$ 
    end if
  end for
return  $\mathbf{s};$ 
end for

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It is well-known that there is a substantial gap between the performance provided by the optimal maximum likelihood (ML) criterion and that of the suboptimal V-BLAST algorithm [3]. This is due to the symbol cancellation steps where unreliable decisions cause non-negligible error propagation. While ML detection provides optimal performance, its computational complexity is high and likely to be prohibitive for moderate numbers of antennas and/or high-order modulations, making it impractical for implementation. Thus there remains a strong motivation for a computationally-efficient suboptimal detector that is able to achieve both near-ML performance and spatial multiplexing gain [3]. While various modifications of the V-BLAST algorithm have been proposed to improve its performance [4]–[6], they are unable to approach optimal performance with reasonable computational complexity.

*Proposed Layered Detection Algorithm:* Our detection scheme is motivated by the observation that the performance of the V-BLAST detector is especially limited by the most ill-conditioned sub-channel (i.e.,  $\mathbf{h}_k$  with the minimum norm). Thus, accurate detection of the weakest layer has a significant impact on the error rate performance

of the MIMO system. Instead of the “best-first” V-BLAST cancelation approach, which is widely-used and generally believed to be the optimum ordering scheme for the layered architecture, we propose to start with the worst sub-channel and to detect the weakest layer optimally using an exhaustive search over all possible transmitted symbols from constellation  $\mathcal{Q}$ . A conventional V-BLAST detector is then applied to the remaining  $n_T - 1$  layers.

The pseudo-code of our proposed detection scheme is shown in Algorithm 2. After calculating the nulling vectors and determining their associated ordering, the detection process starts by canceling the contribution of a first tentative symbol  $s_k^j \in \mathcal{Q}$  from the “worst” layer  $k$  of the received signal  $\mathbf{r}$ , where  $j = \{1, \dots, |\mathcal{Q}|\}$  and  $|\mathcal{Q}|$  is the cardinality of signal constellation. After detecting the remaining  $n_T - 1$  symbols using V-BLAST, an error metric  $\xi_j = \|\mathbf{H}\mathbf{s}_j - \mathbf{r}\|^2$  is computed for  $s_k^j$ , where  $\mathbf{s}_j = [s_1^j, s_2^j, \dots, s_{n_T}^j]^T$  is the detected symbol vector. This iterative process is then repeated for each of the  $|\mathcal{Q}| - 1$  other tentative symbols in the constellation  $\mathcal{Q}$ . Then the algorithm chooses the symbol vector  $\mathbf{s}_j$  with the smallest error metric  $\xi_j$  as the detected symbol vector. We will refer to the detection process of symbol vector  $\mathbf{s}_j$  for each tentative symbol  $s_k^j$  as a sub-detector.

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**Algorithm 2** The proposed detection algorithm.

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 $\hat{\mathbf{H}} = \mathbf{H};$ 
for ( $i = 1; i \leq n_T; i++$ ) do
   $\mathbf{G} = (\hat{\mathbf{H}}^H \hat{\mathbf{H}} + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I})^{-1} \hat{\mathbf{H}}^H;$ 
  if  $i > 1$  then
     $O(i) = k = \min_j \|\mathbf{g}_j\|^2;$ 
  else
     $O(i) = k = \max_j \|\mathbf{g}_j\|^2;$ 
  end if
   $\mathbf{g}_i = \mathbf{G}(k, :);$ 
   $\hat{\mathbf{H}} = \hat{\mathbf{H}}_{\bar{k}};$ 
end for
for (every received symbol vector  $\mathbf{r}$  in a block) do
  for ( $j = 1; j \leq |\mathcal{Q}|; j++$ ) do
     $\hat{\mathbf{r}} = \mathbf{r};$ 
     $k = O(1);$  {Get weakest layer}
     $\hat{\mathbf{r}} = \hat{\mathbf{r}} - \mathbf{h}_k s_k^j;$  {Cancel tentative symbol}
    for ( $i = 2; i \leq n_T; i++$ ) do
       $k = O(i);$  {Get strongest layer to be detected}
       $\hat{s}_k = \mathbf{g}_k \hat{\mathbf{r}};$   $s_k = \mathcal{Q}(\hat{s}_k);$  {Nulling and slicing}
       $s_k^j = s_k;$ 
      if ( $i < n_T$ ) then
         $\hat{\mathbf{r}} = \hat{\mathbf{r}} - \mathbf{h}_k s_k;$  {Cancelation}
      end if
    end for
     $\xi_j = \|\mathbf{H}\mathbf{s}_j - \mathbf{r}\|^2;$ 
  end for
   $\ell = \min_j (\xi_j);$ 
  return  $\mathbf{s}_\ell;$ 
end for

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Figure 1 shows the symbol error rate (SER) of alternative detection schemes for an uncoded  $4 \times 4$  MIMO system utilizing 16-QAM modulation over a Rayleigh fading channel. This figure shows that the performance of the proposed detection technique approaches closely that of an optimal ML detector.

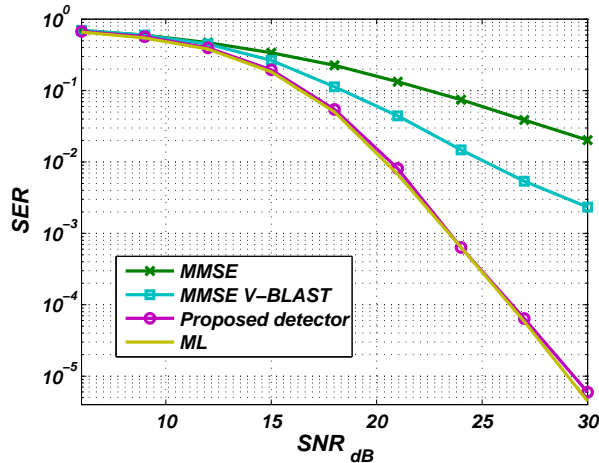


Fig. 1. Symbol error rate of alternative detection schemes for a  $4 \times 4$  16-QAM MIMO system over a Rayleigh fading channel.

*Computational Complexity Comparison:* V-BLAST requires  $n_T$  nulling steps (i.e., vector multiplications),  $n_T$  slicing steps (i.e., symbol comparisons) and  $n_T - 1$  cancelation steps (i.e., vector-symbol multiplications and vector subtractions) to detect every transmitted symbol vector. The computational complexity of our proposed scheme is increased roughly by  $|\mathcal{Q}|$  compared to V-BLAST, as each of the  $|\mathcal{Q}|$  sub-detectors requires one fewer nulling and slicing operations for the worst layer. The number of nulling, slicing and cancelation steps using V-BLAST and our proposed detector are compared in Table I. It is important to note that  $|\mathcal{Q}|$  sub-detectors in the proposed scheme

	V-BLAST	Proposed detector
Nulling	$n_T$	$ \mathcal{Q} (n_T - 1)$
Slicing	$n_T$	$ \mathcal{Q} (n_T - 1)$
Cancelation	$n_T - 1$	$ \mathcal{Q} (n_T - 2)$

TABLE I

THE NUMBER OF NULLING, SLICING AND CANCELATION STEPS USING V-BLAST AND OUR PROPOSED DETECTOR

can operate independently and, therefore, a  $|\mathcal{Q}|$ -fold parallel implementation of sub-detectors provides the same symbol detection throughput as in V-BLAST technique. For a compact implementation, one could implement only one instance of a sub-detector and time multiplex it among all  $|\mathcal{Q}|$  sub-detectors at the expense of lowering the symbol detection throughput.

*Conclusions:* It is widely appreciated that the layered space-time architecture of V-BLAST is capable of realizing the extraordinary spectral efficiencies of MIMO systems over rich-scattering wireless channels. However, the error propagation due to imperfect decision feedback represents a bottleneck in achievable performance. This letter presented an improved detection algorithm that provides near-optimal error rate performance with no impact on the symbol detection throughput given  $|\mathcal{Q}|$ -fold parallel implementation of sub-detectors, where  $|\mathcal{Q}|$  denotes the cardinality of the signal constellation. For an implementation with maximum symbol detection throughput, the computational complexity is slightly less than  $|\mathcal{Q}|$  times that of V-BLAST; however, for a more compact design one could implement only one instance of the sub-detector and time multiplex it, which would lower the symbol

detection throughput roughly by  $|Q|$ .

### Authors' Affiliations:

A. Alimohammad, S. F. Fard, and B. F. Cockburn are with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada.

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