Markovian Models for Harvested Energy in Wireless Communications

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***Abstract*—Energy harvesting provides a perpetual but unreliable energy source for powering communication devices without access to a fixed power supply. The harvested energy can be treated as a stochastic process due to the random nature of the energy source, such as solar energy and piezoelectric (PZ) energy. Typically, a stationary Markovian (SM) model for the harvested energy is used in the literature for analysis. In this paper, we propose a generalized Markovian (GM) model that captures the non-stationarity of Markovian model, by introducing an additional *scenario* parameter. Given the scenario parameter (that typically changes slowly), the harvested energy (that typically changes quickly) then follows a stationary Markovian model. Based on the empirical measurements, we use the Bayesian information criterion to quantify whether the generalized model can better model the harvested energy. We conclude that the PZ energy can be better modeled by the GM model, while it is sufficient to model the solar energy by a SM model.**

I. Introduction

Energy harvesting is a viable option as an perpetual energy source for powering communication devices in many practical scenarios. The devices may, for example, be located in remote locations without access to power supply and where it is inconvenient, dangerous, or even impossible to change the batteries. Examples of energy that can be harvested include solar energy, piezoelectric (PZ) energy and thermal energy.

The source of the harvested energy comes from the environment or from human activities. Typically, it is difficult to predict the time when the energy is available, as well as the amount available. To mitigate the unpredictability of the energy source, an energy buffer, such as a rechargeable battery or capacitor, collects any excess harvested energy that is not consumed. Energy for processing data and communication, when required, is then drawn from this energy buffer.

Nevertheless, the stored energy may still be exhausted due to the practically limited size of the energy buffer. In wireless communications powered by energy harvesters, this will result in an outage that leads to a temporary loss in communication. Moreover, it is desirable to send more bits when sufficient energy has been harvested. It remains challenging to optimally assign the amount of power or other resources in the face of an uncertain energy source.

To provide an in-depth study on the use of energy harvesters for communications, we need an analytically simple yet practically accurate model of the harvested energy. Several models on the harvested energy have been used in the literature [1]–[6], assuming a time slotted system. In [1], the harvested energy arrives in a known, deterministic manner.

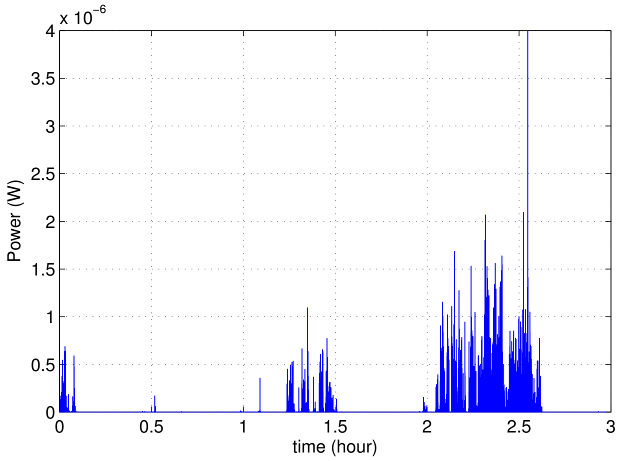

Fig. 1. Harvested PZ energy generated from natural wind over two hours.

In [2], the energy harvested is assumed to be unknown but the accumulated harvested energy can be bounded below and above by some linear functions. In [3], [4], detailed analysis on the throughput and delay performance was given assuming the harvested energy evolves independently over time. In [5], [6], the harvested energy is described by a stationary Markovian model. These models are analytically simple and are thus useful to provide insights for solving some key theoretical problems. However, the validity of the models have not been formally justified with empirical measurements and hence it is not known if the insights are still useful in practice.

Although some empirical measurements are available in the literature [7], the time variation of the harvested energy has so far not been analyzed. In our empirical measurements (see Appendix for details), the harvested energy may be a non-stationary Markovian process, as suggested in Fig. 1 when harvesting PZ energy due to wind. On the other hand, it may be difficult to predict if solar energy, as shown in Fig. 2, follows a non-stationary Markovian process. To provide an accurate description of the harvested energy, a more general model that captures the non-stationarity of the stochastic process is thus needed. An accurate model is also useful for improving the prediction of the harvested energy in [2], [8], which can result in better utilization of the system resources.

In this paper, we provide an analytically more general class of model for the harvested energy, with support from empirical measurements. Specifically, the harvested energy

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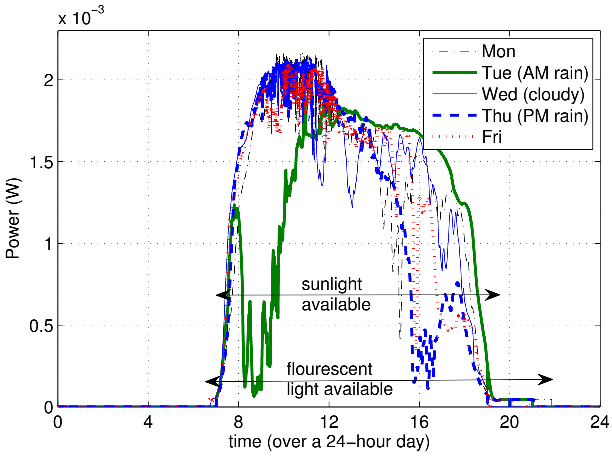
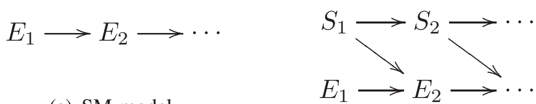


Fig. 2. Harvested solar energy over five days.



(a) SM model.

(b) GM model.

Fig. 3. The stationary Markovian (SM) model is commonly used in the literature, while scenario parameters $\{S_k\}$ are introduced in the generalized Markovian (GM) model. Here, the orders of all Markovian processes are one.

$\{E_1, E_2, \cdots\}$ is taken to be a non-stationary Markovian stochastic process over discrete time k. For analytical tractability, the process is assumed to be a stationary Markovian process when conditioned on a *scenario* parameter S_k, where S_k is another Markovian process. Fig. 3 illustrates the difference between a stationary Markovian (SM) model and the generalized Markovian (GM) model via a *causal diagram* [9], which is a directed graph that shows the causal relationship of the random variables. Intuitively, S_k characterizes different classes of energy-harvesting environments, such that in each class E_k is a stationary process.

In general, a good choice of S_k depends on the energy-harvesting technology and other system parameters. To allow the prediction of the harvested energy, a pragmatic choice is to take S_k as the past average harvested energy. We employ the Bayesian information criterion (BIC) to quantify if the harvested energy is better modeled by the GM model (with the above choice of S_k) or by the SM model. We conclude that the PZ energy can be better modeled by the GM model, while solar energy is usually better modeled by a SM model.

II. Theoretical Models

We made two assumptions throughout this paper.

- Time is slotted with discrete time index $k = 1, 2, \cdots, K$, where K can be infinitely large.
- The harvested energy $E_k \geq 0$ is in a discrete set $\mathcal{E}$. To this end, the harvested energy is quantized and we will discuss the effect of quantization subsequently.

We review the SM model which is commonly used to model the harvested energy, then we generalize to the GM model.

A. Stationary Markovian (SM) Model

The harvested energy is taken as a stochastic process with pmf $p(E_1^K)$. In general, we collect the harvested energy from time m to time n as vector $E_m^n = [E_m, \cdots, E_n]$, where $n \geq m$. If $n < m$, we define E_m^n as the null value. Other variables are denoted similarly.

For analytical tractability, it is common to assume that $\{E_k\}$ is an Lth-order stationary Markovian process, $L \geq 0$. That is, the joint pmf of E_1^K given L past harvested energy E_{1-L}^0 factorizes as

$$p(E_1^K | E_{1-L}^0) = \prod_{k=1}^{K} p(E_k | E_{k-L}^{k-1}). \quad (1)$$

This pmf specializes to several special cases in the literature.

- If each pmf is replaced by a Dirac-delta function, $\{E_k\}$ becomes deterministic (rather than random) as in [1].
- If $L = 0$, $\{E_k\}$ becomes i.i.d. as in [3], [4].
- If $L = 1$, $\{E_k\}$ becomes a first-order stationary Markovian process as in [5], [6], see also Fig. 3(a).

B. Generalized Markovian (GM) Model

We propose to generalize the SM model by introducing another stationary Markovian process $\{S_k\}$. We call S_k the scenario parameter. The joint pmf, given L past harvested energy E_{1-L}^0 and L' past scenario parameters $S_{1-L'}^0$, where $L \geq 0$ and $L' \geq 0$, is assumed to be given by

$$\begin{aligned} & p(E_1^K, S_1^K | E_{1-L}^0, S_{1-L'}^0) \\ &= \prod_{k=1}^{K} p(E_k | E_{k-L}^{k-1}, S_{k-L'}^{k-1}) p(S_k | S_{k-L'}^{k-1}). \end{aligned} \quad (2)$$

If S_k is a fixed constant, then (2) effectively reduces to (1). However, (1) cannot reduce to (2) due to the presence of the scenario parameters for $L' \geq 1$. Thus, the GM model is indeed more general than the SM model.

1) Interpretation of Scenario Parameters: To develop intuition on relevant choices of S_k later, we give the motivation of the scenario parameter. In our context, we interpret S_k as a long-term, slowly-varying parameter that depends on the harvesting environment. The GM model is likely to be useful if S_k is chosen such that given different S_k, the short-term statistics of E_k follow distinctly different stationary Markovian processes (each with different state transition probabilities). For example, S_k may indicate if it is day or night when solar energy is harvested, or indicate if the wind is strong or weak when the PZ energy is extracted from the wind energy.

Another interpretation can be obtained by studying the pmf. For simplicity, consider $L = 1$ and $L' = 1$. Then (2) becomes

$$p(E_1^K, S_1^K | E_0, S_0) = \prod_{k=1}^{K} p(E_k | E_{k-1}, S_{k-1}) p(S_k | S_{k-1}). \quad (3)$$

This pmf is illustrated in Fig. 3(b) via a directed graph, known as a *causal diagram* [9]. From Fig. 3(a), we can interpret that in the SM model, the immediate past harvested energy are taken to be the key "cause" of the present harvested energy.

However, if the immediate past scenario parameter also plays an important role as a "cause", then the GM in Fig. 3(b) can be a better model.

2) Choice of Scenario Parameters: For simplicity, we assume $L' = 1$ subsequently. Allowing for $L' \geq 1$ in general, and optimizing for L', can only result in a better GM model.

An ideal scenario parameter should satisfy these properties.

- *Relevant*: S_k accurately reflects the scenario that relates to significant change of the environment.
- *Readily available*: S_k can be measured or calculated by the communication device that uses the energy harvester.
- *Generic*: S_k is independent of the energy-harvesting technology.

The first property has been justified based on the interpretation of the S_k earlier. The second property is desirable since if the communication device knows S_k, it can perform online estimation of the harvested energy based on the GM model, and then exploits this knowledge to compute energy-efficient schemes for communications. An example of a scenario parameter that is *not* readily available is the current weather condition or the current time: although it is available during empirical measurements, it is difficult or impossible for the communication device to obtain this information. The third property is clearly desirable since it allows the same model to be used generally, while the difference of the energy-harvesting technology and other related system parameters can be captured by the transition probabilities of the model.

Subsequently, we focus on a pragmatic choice of the scenario parameter that satisfy the properties of being readily available and generic. Specifically, let the scenario parameter be given by past-N-sample average of the harvested energy:

$$S_k = \frac{1}{N} \sum_{i=k-N+1}^{k} E_i. \tag{4}$$

This scenario parameter tracks the long-term average of E_k and can be useful to track harvested energy that occurs in bursts of high values, such as in Fig. 1. To quantify if this scenario parameter is also relevant, we employ the Bayesian information criterion (BIC) [10] in the next section. The methodology to be used can be applied to other choices of scenario parameters, besides (4).

III. Model Comparison by BIC

We first give a general description of the BIC, then we apply the BIC calculation to compare the SM and GM models.

Suppose we have collected n samples from empirical measurements, which are assumed to be modeled by some statistical model $\mathcal{M}$ with m unknown statistical parameters[1] $\boldsymbol{\theta}(\mathcal{M}) = [\theta_1, \cdots, \theta_m] \in \Theta(\mathcal{M})$. The BIC is defined as [10]

$$\text{BIC}(\mathcal{M}) = -2L_{\max}(\mathcal{M}) + m \log n \tag{5}$$

where $L_{\max}(\mathcal{M}) = \max_{\theta \in \Theta(\mathcal{M})} L(\theta)$ and $L(\theta)$ is the log-likelihood of the empirical measurements given parameter θ. The BIC is fairly easy to calculate and applies to any assumed model. Thus, the assumed model need not be the true model.

Given two or more competing models for the same set of n samples, the model with the smallest BIC is chosen[2]. The BIC is weakly consistent, i.e., the chosen model is the best one in terms of the smallest Kullback-Leibler (KL) distance with probability one for large n. Moreover, the model with the smallest m is chosen if there is more than one model with the smallest KL distance. More properties and comparisons of BIC with other information criterion are in [11].

A. SM Model

We consider the derivation of the BIC in (5), based on the SM model in (1). For simplicity, we consider a first-order SM model, i.e., $L = 1$. Similar derivations can be made for $L \geq 1$.

Suppose the harvested energy comes from the discrete state space $\mathcal{E} = \{e_1, \cdots, e_{|\mathcal{E}|}\}$. For notational convenience, we enumerate the state space as $\mathcal{E} = \{1, \cdots, |\mathcal{E}|\}$ subsequently. From the empirical measurements, suppose n_{ij} samples are observed with $E_k = j$ given $E_{k-1} = i$ for $k = 1, \cdots, n$. Clearly, $\sum_{(i,j)\in\mathcal{E}^2} n_{ij} = n$. From (1) with $L = 1$, we get

$$p(E_1^K | E_0) = \prod_{k=1}^{K} p(E_k | E_{k-1}) = \prod_{i=1}^{|\mathcal{E}|} \prod_{j=1}^{|\mathcal{E}|} p_{ij}^{n_{ij}}, \tag{6}$$

where $p_{ij} \triangleq p(E_k = j | E_{k-1} = i)$ is independent of k, since the process is stationary.

The probabilities sum to one according to $\sum_{j=1}^{|\mathcal{E}|} p_{ij} = 1$ for all i. Without loss of optimality, we fix $p_{i1} = 1 - \sum_{j=2}^{|\mathcal{E}|} p_{ij}$ for all i (other choices are possible). The remaining $m \triangleq |\mathcal{E}|^2 - |\mathcal{E}|$ parameters that we wish to estimate are then $\boldsymbol{\theta} \triangleq [p_{12}, \cdots, p_{|\mathcal{E}|2}, \cdots, p_{1|\mathcal{E}|}, \cdots, p_{|\mathcal{E}||\mathcal{E}|}, \}$, which can vary independently over the space of $\Theta \triangleq [0, 1]^m$. The log-likelihood given parameter $\boldsymbol{\theta} \in \Theta$ is then

$$L(\boldsymbol{\theta}) = \log p(E_1^K | E_0) = \sum_{i=1}^{|\mathcal{E}|} \sum_{j=1}^{|\mathcal{E}|} n_{ij} \log p_{ij}. \tag{7}$$

It can be shown that the maximum (log-)likelihood estimate (MLE) is $p_{ij}^{\star} = \arg\max_{\boldsymbol{\theta}\in\Theta} L(\boldsymbol{\theta}) = n_{ij}/n$. From (5),

$$\text{BIC} = -2 \sum_{i=1}^{|\mathcal{E}|} \sum_{j=1}^{|\mathcal{E}|} n_{ij} \log(n_{ij}/n) + (|\mathcal{E}|^2 - |\mathcal{E}|) \log n. \tag{8}$$

The BIC for Markovian order $L > 1$ can be obtained similarly.

B. GM Model

We consider the derivation of the BIC in (5), based on the GM in (2). For simplicity, we assume $L = L' = 1$. Similar derivations can be made for $L, L' \geq 1$.

Suppose the scenario parameter comes from the discrete state space $\mathcal{E}' = \{1, \cdots, |\mathcal{E}'|\}$. From the empirical measurements, suppose there are n_{aij} samples observed where the energy harvested is $E_k = j$ given $E_{k-1} = i$ and $S_{k-1} = a$

[1]In our case, $\boldsymbol{\theta}(\mathcal{M})$ are the transition probabilities of the Markovian model.

[2]In our case, the competing models are the SM and the GM models.

for $k=1,\cdots,n$; also suppose there are n'_{ab} samples observed where the scenario parameter is $S_k=b$ given $S_{k-1}=a$ for $k=1,\cdots,n$. By a similar derivation as for the SM model, the log-likelihood given parameter $\boldsymbol{\theta}\in\Theta$ is

$$L(\boldsymbol{\theta})=\sum_{a=1}^{|\mathcal{E}'|}\left(\sum_{i=1}^{|\mathcal{E}|}\sum_{j=1}^{|\mathcal{E}|}n_{aij}\log p_{aij}+\sum_{b=1}^{|\mathcal{E}'|}n'_{ab}\log q_{ab}\right). \quad (9)$$

Similarly, the MLE is $p^\star_{aij}=n_{aij}/n$ and $q^\star_{ab}=n'_{ab}/n$. The BIC can then be calculated from (5). The BIC for Markovian order $L>1$ can be obtained similarly.

IV. BIC BASED ON EMPIRICAL MEASUREMENTS

Empirical measurements using commercially available energy harvesters are performed to harvest PZ and solar energy. The measurement setup in Fig. 6 is described in the Appendix.

A. Design Parameters

We motivate the choice of the key design parameters.

1) Quantizer for E_k: The measurements are captured at fixed intervals and converted to harvested energy $\widetilde{E}_k$ (i.e., the unquantized version) over discrete time k. The digital multimeter used for the measurements has high precision (see Appendix). Thus, $\widetilde{E}_k$ is assumed to take any real non-negative values. To calculate the transition probability, we quantize each $\widetilde{E}_k\in\mathbb{R}^+$ into $E_k\in\mathcal{E}\subseteq\mathbb{R}^+$, where $\mathcal{E}$ is a finite set.

Typically the quantizer is fixed for a given scenario and not a design choice. To ensure a fair comparison, the same quantizer is used when comparing the SM and GM models. We consider two quantizers, corresponding to a coarsely-quantized case and a finely-quantized case respectively:

- A two-state quantizer: set E_k as some constant if $\widetilde{E}_k$ exceeds some quantization threshold, otherwise $E_k=0$.
- A twenty-state quantizer: the quantization thresholds are fixed as uniform steps between the maximum and the minimum of the unquantized samples $\{\widetilde{E}_k\}$.

Some energy harvesters, e.g. the solar energy harvester in [12], harvest either an approximately constant amount of energy (if $\widetilde{E}_k$ is large enough) or no energy at all in our empirical measurements. This justifies the use of the two-state quantizer in such cases. The twenty-state quantizer approximates the availability of arbitrary amount of harvested energy.

2) Markovian Order for E_k: A model is considered better if its BIC is smaller. Since each model can have different Markovian orders L, for each model we first optimize L so as to minimize the BIC. Specifically, let $\mathrm{BIC}^\star(\mathcal{M})$ be the BIC minimized over L for the $\mathcal{M}=\mathrm{SM}$ or $\mathcal{M}=\mathrm{GM}$ model. If $\mathrm{BIC}^\star(\mathrm{SM})<\mathrm{BIC}^\star(\mathrm{GM})$, the SM model (with the optimal L) is a better model. Otherwise, the GM model is a better.

3) Quantizer and Markovian Order for S_k: We use the scenario parameter S_k in (4), which is relevant only for the GM model. For simplicity a two-state quantizer and a Markovian order of $L'=1$ is used for S_k. We will focus on the above (simple) choice to show that there exist energy harvesting scenarios where a GM model is better. Nevertheless, an exhaustive search for the optimal quantizer and L' may still result in a lower BIC for the GM model.

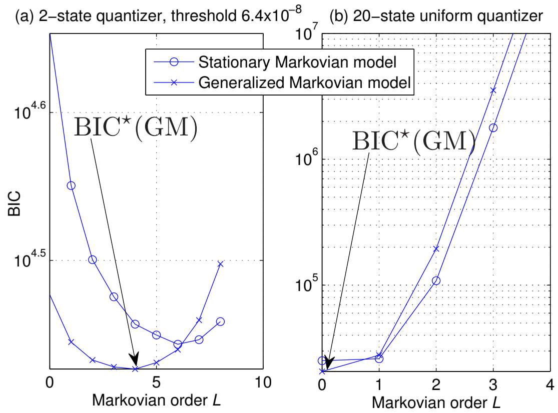


Fig. 4. BIC for harvested PZ energy. Different quantizer for E_k is used, while S_k is always quantized into two states with quantizer threshold 2×10^{-8}. Here, the minimum BIC is obtained with the GM model.

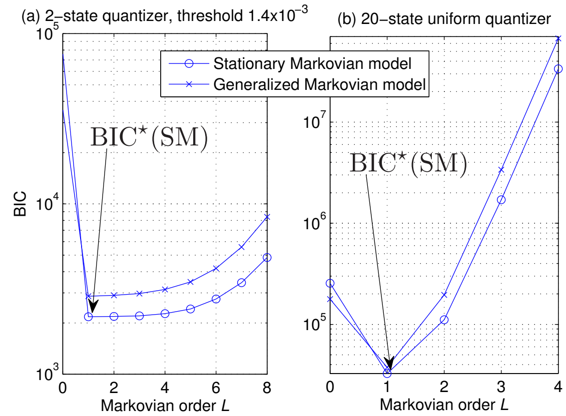


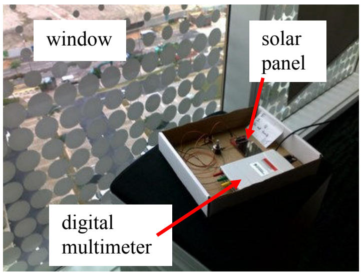
Fig. 5. BIC for harvested solar energy. Different quantizer for E_k is used, while S_k is always quantized into two states with quantizer threshold 5×10^{-4}. The minimum BIC is obtained with the SM model.

B. BIC for Harvested PZ and Solar Energy

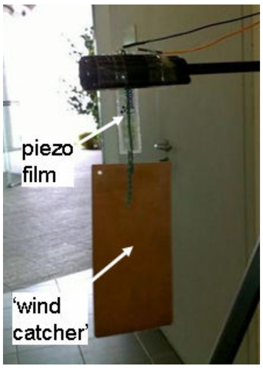
Based on the design parameters, we obtain the BIC for harvested PZ and solar energy separately.

1) PZ Energy: Fig. 4 shows the BIC for (a) two-state quantizer with quantization threshold 64 nW and (b) twenty-state quantizer with uniform quantization. In both cases, S_k is quantized into two states with quantizer threshold 20 nW. From Fig. 4(a), since $\mathrm{BIC}^\star(\mathrm{GM})$ is the smallest, we conclude that the (fourth-order) GM model is better than any of the SM model. The same conclusion applies from Fig. 4(b), where we conclude the (zeroth-order, or i.i.d.) GM model is better.

In our numerical results (not shown here), if the quantization threshold for E_k is changed, it is usually possible to optimize the quantization threshold for S_k such that the GM model is still better. The cases when the GM model is *not* better occurs only when the quantization threshold for E_k is too large, say greater than 500 nW. Intuitively, this is because in such an extreme case, most of the E_k samples are less than the quantization threshold. Thus E_k is typically zero and the scenario parameter S_k is less useful for predicting E_k. In this case, the (more complicated) GM model is also less useful.



(a) Solar energy is harvested near a window in a typical office building.



(b) PZ energy is produced when wind blows a "wind catcher" that bends the PZ material.

Fig. 6. Measurement setups for measuring the energy harvested.

From the BIC viewpoint, for harvested PZ energy, the GM model is thus a more appropriate model if reasonable quantization thresholds are used on E_k.

2) Solar Energy: Fig. 5 shows the BIC for (a) two-state quantizer with quantization threshold 1.4 mW and (b) twenty-state quantizer with uniform quantization. In both cases, S_k is quantized into two states with quantizer threshold 0.5 mW. From Fig. 4, since $\mathrm{BIC}^{\star}(\mathrm{SM})$ with $L = 1$ is the smallest, we conclude that the first-order SM model is better than any of the SM model. We have optimized the quantization threshold for S_k, however, the BIC for the SM model remains smaller.

Contrary to the case of PZ energy, the SM model is thus sufficient for harvested solar energy. This lends some support to the research that have made use of the first-order SM model to develop wireless communications strategies [5], [6], assuming the energy is harvested from solar energy.

V. Conclusion

Motivated by empirical measurements, we have proposed a generalized Markovian (GM) model for the harvested energy, as a generalization of the conventional stationary Markovian (SM) model. We have proposed to use the BIC as a metric to gauge the suitability of the models, due to the wide applicability of the BIC in many applications. From the viewpoint of BIC, we have established that for harvested PZ energy, which is more erratic and occurs in burst, the GM model is better. For harvested solar energy, however, a first-order SM model, which is currently widely used in the literature, is sufficient. As future work, we will gauge the accuracy of the Markovian models via specific problems in wireless communications.

Appendix

A. Measurement Setup for Harvesting Solar Energy

We used the solar energy harvesting panel module that is part of the solar energy harvesting development kit from Texas Instrument [12]. The solar panel was placed next to a glass window in a typical office building as shown in Fig. 6(a), where it harvested energy both from the sun and indoor florescent light. A load resistance was connected to the output of the panel. The voltage delivered across the load was recorded by a digital multimeter, with a resolution of 10 μV, every thirty seconds over twenty-two days. Hence, about $n = 65,000$ samples were collected, which were then converted to the harvested energy delivered to the load. Typical measurement results over a 24-hour period for five days are shown in Fig. 2. The constant power level of around 0.05 mW after the 19th hour, if present, was produced mainly by fluorescent light.

B. Measurement Setup for Harvesting PZ Energy

We used a PZ film element available from Measurement Specialties [13]. The measurement was carried out at a open space in the roof floor of a high building, set up as shown in Fig. 6(b). The PZ film was clamped vertically and attached via a string to a plastic sheet or a "wind catcher". The wind will cause the plastic sheet, and hence the PZ film, to oscillate back and forth. The oscillation depends on the strength and duration of the wind, causing the harvested energy to appear erratic in amount and frequency. A load resistor is connected at the output of the PZ film. The voltage across the load was recorded every 0.2 s over a three-hour period by a digital multimeter with a resolution of 1 mV and converted to energy. Hence, about $n = 11,000$ samples were collected. The harvested energy is obtained as shown in Fig. 1.

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