

# Labor Selection, Turnover Costs and Optimal Monetary Policy\*

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## Abstract

We study optimal monetary policy and welfare properties of a DSGE model with a labor selection process, labor turnover costs and Nash bargained wages. We show that our model implies inefficiencies which cannot be offset in a standard wage bargaining regime. We also show that the inefficiencies rise with the magnitude of firing costs. As a result, in the optimal Ramsey plan, the optimal inflation volatility deviates from zero and is an increasing function of firing costs.

*JEL Codes:* E52, E24

*Keywords:* optimal monetary policy, hiring and firing costs, labor market frictions, policy trade-off.

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# 1 Introduction

There is a large literature showing that labor turnover costs can significantly affect the hiring and firing process and thus abate worker flows (see, e.g., Hopenhayn and Rogerson 1993 or Kugler and Saint-Paul 2004). Labor turnover may also have important implications for welfare analysis. Despite this, the role of labor turnover costs has been largely neglected in the analysis of optimal policy.<sup>1</sup> In this paper, we analyze the welfare effects of labor turnover costs with a particular focus on the design of optimal monetary policy.

We analyze this question within a DSGE model in which the labor market is characterized by a labor selection process, where firms select the most suitable workers, taking into consideration labor turnover costs. The model also features nominal rigidities, induced by adjustment costs on prices, as those are essential to address the real effects of monetary policy. A more detailed description of the labor market is as follows. Workers can apply only at one particular firm per discrete time period. Each worker-firm pair discerns its suitability, with the firm assessing whether there is a profit opportunity. Suitability is modeled based on an idiosyncratic productivity shock (in the form of operating costs), which is drawn when the two parties meet. This labor selection process produces worker flows which evolve endogenously and depend on the labor turnover costs.<sup>2</sup> An applicant is only hired when the future stream of discounted expected profits exceeds the hiring costs. An incumbent is fired when the future stream of discounted expected profits falls short of the firing costs. The wage process is modeled by considering two alternative bargaining arrangements: collective bargaining and individualistic bargaining. We show that in both cases Nash bargaining fails to allow decentralization of the efficient solution.<sup>3</sup>

Before proceeding with the design of optimal monetary policy, we discuss the role of labor turnover costs for the cyclical properties of labor market flows. The model outlined is indeed capable of generating strong amplification and persistence effects. The presence of idiosyncratic match specific shocks produces endogenous fluctuations in the job-finding rate. This amplifies

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<sup>1</sup>Some works have analyzed the business cycle implications of firing costs in search and matching models. See Costain et al. 2010 and Thomas and Zanetti 2009.

<sup>2</sup>This differs from a traditional search and matching model, which instead can be interpreted as a contact model (i.e., the matching function establishes a contact between workers and firms). See Brown et al. (2010).

<sup>3</sup>Collective bargaining is analyzed for two reasons. First, empirical studies (e.g. Cahuc and Carcillo 2011) show a positive correlation between firing costs and collective bargaining. Second, collective bargaining generates clear-cut analytical results for efficiency, which help to understand the nature of the monetary policy trade-off.

the response of worker flows to aggregate shocks.<sup>4</sup> Labor turnover costs tend to increase the retention rate, and to dampen the job-finding rate. As current employment becomes dependent on past employment, the model features additional persistence in response to shocks compared to a standard Walrasian labor market, much in line with the empirical evidence.

To analyze the welfare implications of labor turnover costs in the labor selection process, we first qualify the nature of the underlying distortion by comparing the competitive equilibrium (under flexible prices) with the constrained Pareto optimal allocation. Atomistic firms fail to internalize the effects of their decisions on aggregate unemployment, and thus on the pool of searching workers. This externality manifests itself in the difference between the efficient wage and the competitive wage: the first indeed responds to future productivity changes, while the second does not.<sup>5</sup> We also show that the inefficiency of the competitive economy increases with the level of firing costs.

To model the design of optimal monetary policy in this context, we use a public finance approach which mandates the use of state contingent instruments to correct distortions. For this reason, we employ the Ramsey approach<sup>6</sup> to analyze the design of optimal monetary policy. The optimal (Ramsey) plan is obtained by maximizing the discounted sum of agents' utilities subject to the constraints describing the competitive economy, therefore taking into account both short-run and long-run distortions. In our model, the Ramsey planner faces a tension between conflicting goals, stabilizing inflation versus stabilizing employment. On the one hand, he would like to close the inflation costs by setting inflation volatility to zero. On the other hand, it would be optimal to use inflation as a state contingent instrument to stabilize employment fluctuations. As a result of this trade-off, the optimal volatility of inflation deviates from zero. Since firing costs increase the inefficiency in the competitive economy, they also increase the optimal volatility of inflation.

Recently several authors have studied the design of optimal monetary policy in DSGE models with search and matching frictions (see, e.g., Blanchard and Galí 2010, Faia 2009 and Thomas 2008), a setting which allows to consider inefficient unemployment and congestion externalities

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<sup>4</sup>We find that the cyclical standard deviation of the job-finding rate and unemployment is several times larger than the one of output. See also Lechthaler et al. 2010.

<sup>5</sup>We show that neither collective nor standard individualistic Nash bargained wages can internalize this externality. Under collective bargaining there is also an additional distortion coming from the emergence of involuntary unemployment, as some workers would be willing to work at the current wages, but cannot. See also Christiano, Trabandt and Walentin 2010 on the importance of involuntary unemployment.

<sup>6</sup>See Atkinson and Stiglitz 1976, Lucas and Stokey 1983, Chari, Christiano and Kehoe 1991. More recently Adao et al. 2003, Khan, King, and Wolman 2003, Schmitt-Grohe and Uribe 2007 and Siu 2004.

when the Hosios 1990 condition is not met. None has considered optimal policy design for a model with labor selection and labor turnover costs. The analysis in this paper moves a step forward. In the search and matching model as well as in our model the wage loses the inter-temporal allocative role and atomistic firms fail to internalize the effects of their current hiring policy on the future pool of searching workers. Despite these similarities with other models featuring pecuniary externalities in the labor market, there are important differences which have implications for both the models' predictive power in terms of business cycle statistics and the implications for optimal monetary policy. In the search and matching model, over-hiring today affects the future probability that both firms and workers get in "contact" with each other (hence the labor market tightness). In the labor selection model, there is a composition effect: firms do not take into account the effects of their decisions on the future size of the pool of searching workers and thus distort the composition of employment (i.e., the mix of incumbent and newly hired workers). Our efficiency analysis also shows that in our model there is no simple rule (like the Hosios condition) that can restore efficiency (neither under collective nor under individualistic bargaining).

The rest of the paper proceeds as follows. Section 2 shows some stylized facts relating the dynamics of selected macro variables and labor turnover costs. Section 3 presents the model. Section 4 discusses the quantitative properties of the model. Section 5 analyzes the gap between efficient allocation and the competitive equilibrium. Section 6 presents the full-fledged Ramsey plan. Finally, section 7 concludes.

## 2 Labor Turnover Costs and Output Dynamics

To analyze the quantitative properties of our model it is useful to study the extent of labor turnover costs in the data and their effects on output dynamics. There is a vast literature<sup>7</sup> looking at the importance of labor turnover costs and employment protection legislation for unemployment and labor market flows. Evidence shows that hiring and firing costs tend to depress labor market flows, particularly in European economies, a phenomenon labeled as Eurosclerosis and captured by our model.

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<sup>7</sup>The literature dates back to Solow 1968, Sargent 1978, Nickell 1978, 1986. More recently, Bentolila and Bertola 1990 and Hopenhayn and Rogerson 1993 have shown that hiring and firing costs reduce labor turnover. Alvarez and Veracierto 2001 find that severance payments decrease aggregate productivity and output.

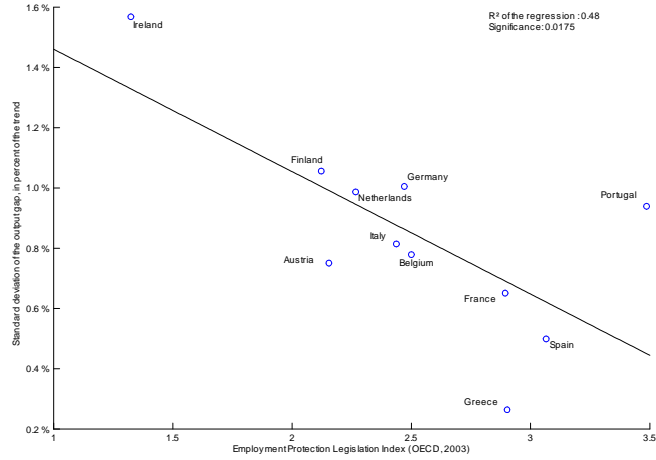


Figure 1: Output gap volatility and employment protection legislation.

We document the effects of labor turnover costs on the business cycle by testing the relation between such costs and the volatility of the cyclical component of output. We focus on euro area data for two reasons. First, euro area countries have higher turnover costs than Anglo-Saxon countries. Second, the EMU period offers a unique experiment in which monetary policy was unaltered by regime shifts.

The data sample used covers the years from 1999 to mid-2008. This choice is motivated by the following reasons. First, it is motivated by our interest in studying the recent implications of labor market regulations for macroeconomic dynamics. Second, we need to isolate the dynamics of macro variables from policy regime shifts and thereby the choice to focus on the EMU period.<sup>8</sup> Figure 1 shows a negative and statistically significant relationship between labor turnover costs and the volatility of output. The volatility of real economic activity is calculated based on a quarterly output gap measure. The seasonally adjusted real GDP series (in 2000 prices) is taken from the International Financial Statistics. The output gap is calculated as percentage deviation of output from its trend, namely the Hodrick-Prescott filter with smoothing parameter  $\lambda = 1600$ . As a proxy for labor turnover costs, we use the employment protection legislation index (see OECD 2004),

<sup>8</sup>Weber et al. 2009 show in their VAR-estimations that the Euro-zone economy reacted differently to macroeconomic shocks between 1996 and 1999 than it did before and afterwards. Therefore, we do not include this interim period, during which capital markets already expected the introduction of the euro.

which is a weighted average of indicators capturing protection of regular workers against individual dismissals, requirements for collective dismissals and regulation of temporary employment. We choose this index because it is a more precise measure than alternative employment protection indicators.<sup>9</sup>

The negative correlation between the EPL index and output volatility remains robust also when we consider alternative specifications which include the GDP per capita, the interaction between employment protection and the gross replacement rate.<sup>10</sup> The relation above can be explained as follows. Higher labor turnover costs imply that the retention rates exceed job-finding rates, and thus current employment depends on past employment. The persistence in employment carries over to output. For the same observation period and the same sample of countries, we also find the expected positive relationship between the EPL and the first order autocorrelation coefficient of output (statistically significant at the 5 percent level).

In Section 4 we show that our model can replicate well both the above mentioned stylized facts and a number of business cycle statistics.

### 3 The Model

Our model grafts a labor market with labor selection, labor turnover costs, collective wage bargaining, and employed and unemployed workers into a New Keynesian framework with Rotemberg adjustment costs.<sup>11</sup> Households in our model make consumption decisions: due to insurance schemes such consumption decisions are independent from the employment history of the individual members of the family. Workers are heterogenous in terms of their operating costs. These assumptions allow endogenous hiring and firing decisions. Wages are set collectively with a Nash bargaining process between firms and incumbent workers. Firms in the production sector act in a monopolistic competitive fashion and face price adjustment costs a la Rotemberg 1982.

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<sup>9</sup>Compared with other indicators, such as the Employment Legislation Index in Botero et al. 2003, or the hiring and firing costs calculated by the World Bank in its “Doing Business” studies, the OECD’s indicator covers a larger range of relevant aspects of LTCs.

<sup>10</sup>Further evidence for the negative correlation between employment protection and employment/output volatility can be found in Abbritti and Weber (2009). They run a panel VAR for all OECD countries. See also Merkl and Schmitz (2011).

<sup>11</sup>See also Lechthaler, Merkl, and Snower 2010 for a prototype of this model economy.

### 3.1 Households

We assume that households have a standard utility function of the form:

$$W_t = \sum_{j=t}^{\infty} \beta^{j-t} E_t \frac{c_j^{1-\sigma}}{1-\sigma}, \quad (1)$$

where  $\beta$  is the household's discount factor,  $\sigma$  the elasticity of inter-temporal substitution,  $c$  a consumption aggregate (described below)<sup>12</sup> and  $E$  is the expectations operator.

As common in the literature (see Andolfatto 1996 or Merz 1995), we assume that each household consists of a large number of individuals, each individual supplies one unit of labor inelastically and shares all income with the other household members. This implies that consumption does not depend on a worker's employment status. Thus, the representative household maximizes its utility subject to the budget constraint:

$$Bo_t + c_t P_t = W_t N_t + BU_t + (1 + i_{t-1})Bo_{t-1} + \Pi_{a,t} - T_t, \quad (2)$$

where  $Bo$  are nominal holdings of one period discounted bonds,  $P$  is the aggregate price level,  $T$  are lump-sum tax payments,  $i$  is the nominal interest rate,  $\Pi_a$  are nominal aggregate profits, which are transferred in lump-sum manner,  $W$  is the nominal wage,  $N$  is the total household labor input,  $B$  are unemployment benefits, financed via a lump-sum tax, and  $U$  is the number of unemployed workers. Inter-temporal utility maximization yields the standard consumption Euler equation:

$$c_t = \beta E_t c_{t+1} \left( \frac{1 + i_t}{\pi_{t+1}} \right)^{-\frac{1}{\sigma}}, \quad (3)$$

where  $\pi_{t+1}$  is the expected inflation rate.

### 3.2 Production and the Labor Market

There are three types of firms. Firms producing intermediate goods employ labor, exhibit linear labor turnover costs (i.e., hiring and firing costs) and sell their homogenous products on a perfectly competitive market to the wholesale sector. Firms in the wholesale sector transform the intermediate goods into consumption goods and sell them under monopolistic competition to the

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<sup>12</sup>In what follows capital letters refer to nominal variables and small letters refer to real variables (i.e., de-trended by the price level).

retailers. They can change their price at any time but price adjustments are subject to a quadratic adjustment cost a la Rotemberg 1982. The retailers, in turn, aggregate the consumption goods and sell them under perfect competition to the households.

### 3.2.1 Intermediate Goods Producers and Employment Dynamics

Intermediate good firms hire labor to produce the intermediate good  $z$ . Their production function is:

$$z_t = a_t N_t, \tag{4}$$

where  $a$  is technology and  $N$  the number of employed workers. They sell the product at a relative price  $mc_t = P_{z,t}/P_t$ , which they take as given in a perfectly competitive environment, where  $P_z$  is the absolute price of the intermediate good and  $P$  is the economy's overall price level. The variable  $mc_t$  in this economy plays the role of marginal costs.

Each period every unemployed worker files an application at one particular firm and then draws a random operating cost realization  $\varepsilon$ , which follows a probability distribution  $g(\varepsilon_t)$ . The operating costs can be interpreted as a worker-firm pair specific idiosyncratic cost-shock. When the applicant draws a bad realization of the shock, he stays unemployed and applies at another firm the next period. Employed workers draw realizations from the same shock distribution and are fired when they draw a bad realization.

The firms learn the value of the operating costs of every worker at the beginning of a period and base their employment decisions on it, i.e., an unemployed worker with a favorable shock will be employed while an employed worker with a bad shock will be fired. Hiring and firing is not without costs, firms have to pay linear hiring costs,  $h$ , and linear firing costs,  $f$ , both measured in terms of the final consumption good. Wages are determined through Nash bargaining between incumbent workers and the firm (see the online Appendix for a different bargaining assumption). The bargaining process takes the form of a *right to manage*. This assumption leads to the following timing of events. First, the operating cost shock takes place and median workers and the intermediate goods firm bargain over the wage. Given the wage schedule, firms make their hiring and firing decisions. Thus, firms will only hire those workers who face low operating costs and fire those workers who face high operating costs.



The operating costs,  $\varepsilon$ , are measured in terms of the final consumption good. Moreover we define the hiring and the firing threshold respectively as  $v_{h,t}$  and  $v_{f,t}$ . In non-recursive form the value of an average incumbent can be expressed as the infinite sum of discounted future profits:

$$\Pi_t = E_t \left\{ \sum_{j=t}^{\infty} \Delta_{t,j} \left[ \left( a_j m c_j - w_j - \left( \frac{1}{1-\phi_j} \int_{-\infty}^{v_{f,j}} \varepsilon_j q(\varepsilon_j) d\varepsilon_j \right) \right) \prod_{k=t}^j (1 - \phi_k) \right. \right. \\ \left. \left. - \frac{\phi_j f}{1-\phi_{t-1}} \prod_{k=t}^j (1 - \phi_{k-1}) \right] \right\}, \quad (5)$$

where  $w$  is the real wage,  $\phi$  is the separation probability, and  $\Delta_{t,j}$  is the stochastic discount factor from period  $t$  to  $j$ . To simplify the value of an average incumbent, we rewrite it in recursive form as:

$$\Pi_t = (1 - \phi_t) \left( \left( a_t m c_t - w_t - \frac{1}{1 - \phi_t} \int_{-\infty}^{v_{f,t}} \varepsilon_t q(\varepsilon_t) d\varepsilon_t \right) + E_t \Delta_{t,t+1} \Pi_{t+1} \right) - \phi_t f. \quad (6)$$

A marginal worker is hired whenever the current and future expected discounted profits are larger than the hiring costs. Note that the expected profit has to be indexed by  $t + 1$ :

$$h = a_t m c_t - w_t - v_{h,t} + E_t (\Delta_{t,t+1} \Pi_{t+1}). \quad (7)$$

Unemployed workers whose operating cost is lower than this value get a job, while those whose operating cost is higher remain unemployed. The resulting hiring probability is given by:

$$\eta_t = \int_{-\infty}^{v_{h,t}} g(\varepsilon_t) d\varepsilon_t. \quad (8)$$

Similarly, the firm will fire a worker if current losses are higher than the firing cost. Again, a zero profit condition defines the firing threshold as follows:

$$-f = a_t m c_t - w_t - v_{f,t} + E_t (\Delta_{t,t+1} \Pi_{t+1}), \quad (9)$$

and the separation rate is defined as:

$$\phi_t = \int_{v_{f,t}}^{\infty} g(\varepsilon_t) d\varepsilon_t. \quad (10)$$

The change in employment ( $N_t - N_{t-1}$ ) is the difference between hirings from the unemployment pool ( $\eta U_{t-1}$ ) and firings from the employment pool ( $\phi N_{t-1}$ ), where  $U_{t-1}$  and  $N_{t-1}$  are the aggregate

unemployment and employment levels:  $N_t - N_{t-1} = \eta U_{t-1} - \phi N_{t-1}$ . Letting  $(n_t = N_t/L_t)$  be the employment rate, we assume a constant workforce,  $L_t$ , and normalize it to one. Therefore, the employment dynamics read as follows:

$$n_t = n_{t-1}(1 - \phi_t - \eta_t) + \eta_t. \quad (11)$$

The unemployment rate is simply  $u_t = 1 - n_t$ . From equation 11 it is immediately clear that present employment depends on past employment in presence of labor turnover costs. If there are no labor turnover costs the hiring rate equals the retention rate ( $\eta = 1 - \phi$ ) and equation 11 collapses to  $n_t = \eta_t$ . However, as already discussed, labor turnover costs drive a wedge between hiring rate and retention rate, the terms no longer cancel and, thus, employment depends on past employment (the persistence is larger with larger hiring and firing costs).

### 3.2.2 Wage Bargaining

For simplicity, let the real wage  $w_t$  be the outcome of a Nash bargain between the median worker<sup>13</sup> with operating cost  $\varepsilon^I$  and her firm. The median worker faces no risk of dismissal at the negotiated wage. The wage is renegotiated in each period  $t$ . Under bargaining agreement, the median worker receives the real wage  $w_t$  and the firm receives the expected profit  $(a_t m c_t - w_t)$  in each period  $t$ . Under disagreement, the worker's fallback income is  $b$ , assumed for simplicity to be equal to the real unemployment benefit.<sup>14</sup> The firm's fallback position is  $-k$ , where  $k$  is the cost for the firm in case of disagreement.<sup>15</sup> This cost might arise because of lost production or due to a potential strike. Assuming that disagreement in the current period does not affect future surpluses, workers' surplus is  $(w_t - b)$  while the firm's surplus is  $a_t^I m c_t - w_t - \varepsilon^I + k$ , where  $\varepsilon^I$  are the operating costs of the median worker. Consequently, the Nash-product is:

$$\Theta = (w_t - b)^\gamma (a_t^I m c_t - w_t - \varepsilon^I + k)^{1-\gamma}, \quad (12)$$

<sup>13</sup>For simplicity, we allow the median worker to bargain over wages. Our bargaining process has similarities with the one in Hall and Milgrom 2008. There are, however, important differences. In our case the wage is fixed in the cross-section but not over time. Individualistic bargaining would not change the main implications of the model. See online Appendix A for details.

<sup>14</sup>For realism, we set a positive value of  $b$  in all our quantitative exercises. But we set it to zero in the comparison between social planner and competitive economy.

<sup>15</sup>The assumption that the fall-back option is not the outside option is consistent with the bargaining literature (see, e.g., Binmore, Rubinstein and Wolinsky, 1986).

where  $\gamma \in (0, 1)$  represents the bargaining strength of the worker relative to the firm. Maximizing the Nash-product with respect to the real wage, yields the following simple equation:

$$w_t = \gamma (a_t m c_t - \varepsilon_t^I + k) + (1 - \gamma) b. \quad (13)$$

Note that the collectively bargained wage depends upon the median worker's idiosyncratic productivity: this induces involuntary unemployment as workers with lower operating costs would be willing to work at lower wages, but cannot be hired at the collectively bargained wages. In online Appendix A, we will discuss also the case of individual bargained wages.

### 3.3 Wholesale Sector and Retail Sector

Firms in the wholesale sector are distributed on the unit interval and indexed by  $i$ . They produce a differentiated good  $y_{i,t}$  using the linear production technology  $y_{i,t} = z_{i,t}$ , where  $z_{i,t}$  is their demand for intermediate goods. They sell their goods under monopolistic competition to the retailers who use the differentiated goods to produce the final consumption good according to the Dixit-Stiglitz-aggregator  $y_t = \left( \int y_{i,t}^\nu di \right)^{\frac{1}{\nu}}$ , where  $\nu$  is the demand elasticity. The index above is associated with the price index  $P_t = \left( \int P_{i,t}^{1-\nu} di \right)^{\frac{1}{1-\nu}}$  (where  $P_{i,t}$  and  $y_{i,t}$  denote the firm specific price and output level respectively). From the cost minimization problem of the aggregating firm, we obtain the optimal demand function for differentiated products:

$$y_{i,t} = y_t \left( \frac{P_{i,t}}{P_t} \right)^{-\nu}. \quad (14)$$

Firms in the wholesale-sector can change their prices every period, facing quadratic price adjustment costs a la Rotemberg. They maximize the following profit function:

$$\tilde{\Pi}_{W,t} = E_t \sum_{j=t}^{\infty} \Delta_{t,j} \left[ \frac{P_{i,j}}{P_j} y_{i,j} - m c_j y_{i,j} - \frac{\Psi}{2} \left( \frac{P_{i,j}}{P_{i,j-1}} - 1 \right)^2 y_j \right], \quad (15)$$

where  $\Psi$  is a parameter measuring the extent of price adjustment costs, which are due whenever there are price changes (i.e., whenever  $P_{i,j}/P_{i,j-1}$  is different from 1). Taking the derivative with respect to the price yields, after some manipulations, the following expectational Phillips curve:

$$0 = (1 - \nu) + \nu m c_t - \Psi (\pi_t - 1) \pi_t + E_t \{ \Delta_{t,t+1} \Psi (\pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \pi_{t+1} \}. \quad (16)$$

### 3.4 Aggregation

To solve for the equilibrium, we start by deriving aggregate real profits of intermediate firms which are given by revenues minus wage payments, operating costs and labor turnover costs:

$$\tilde{\Pi}_I = mc_t a_t n_t - w_t n_t - (1 - \phi_t) n_{t-1} \Xi_t^i - (1 - n_{t-1}) \eta_t \Xi_t^e - n_{t-1} \phi_t f - (1 - n_{t-1}) \eta_t h, \quad (17)$$

where  $\Xi_t^i$  is the expected value of operating costs for incumbent workers, conditional on not being fired and  $\Xi_t^e$  is the expected value of operating costs for entrants, conditional on being hired, defined by:

$$\Xi_t^e = \frac{\int_{-\infty}^{v_{h,t}} \varepsilon_t g(\varepsilon_t) d\varepsilon_t}{\eta_t}, \quad (18)$$

$$\Xi_t^i = \frac{\int_{-\infty}^{v_{f,t}} \varepsilon_t g(\varepsilon_t) d\varepsilon_t}{1 - \phi_t}. \quad (19)$$

The real profits ( $\tilde{\Pi}_W$ ) of the wholesale sector are given by:

$$\tilde{\Pi}_W = y_t - mc_t a_t n_t - \frac{\Psi}{2} (\pi_t - 1)^2 y_t. \quad (20)$$

Retailers earn zero-profits. Aggregate real profits in this economy are therefore given by:

$$\begin{aligned} \tilde{\Pi}_{a,t} &= y_t - w_t n_t - n_{t-1} \phi_t f - (1 - n_{t-1}) \eta_t h - (1 - \phi_t) n_{t-1} \Xi_t^i - \\ &\quad (1 - n_{t-1}) \eta_t \Xi_t^e - \frac{\Psi}{2} (\pi_t - 1)^2 y_t. \end{aligned} \quad (21)$$

The latter can be substituted into the budget constraint, (2), and after imposing equilibrium in the bond market and using the government budget constraint ( $g_t + B_t u_t = T_t$ ) we obtain the following resource constraint:

$$\begin{aligned} g_t + c_t &= y_t - n_{t-1} \phi_t f - (1 - n_{t-1}) \eta_t h - (1 - \phi_t) n_{t-1} \Xi_t^i - \\ &\quad (1 - n_{t-1}) \eta_t \Xi_t^e - \frac{\Psi}{2} (\pi_t - 1)^2 y_t. \end{aligned} \quad (22)$$

Note that final aggregate demand includes government expenditure,  $g_t$ , which follows an exogenous AR(1) process.

Inspection of the resource constraint shows that the presence of hiring and firing costs as well as of price adjustment costs induces a waste of resources. Indeed, setting hiring and firing costs to zero would increase output by an amount equal to  $n_t \phi_t f + (1 - n_t) \eta_t h$ , while setting inflation equal to zero would increase output by an amount equal to  $\frac{\Psi}{2} (\pi_t - 1)^2 y_t$ .

## 4 Dynamic Properties of the Model

Before turning to the design of optimal monetary policy, we outline the calibration of the model and study the quantitative properties of the model under a Taylor rule.

### 4.1 Model Calibration

The calibration is summarized in table 1 below.

*Preferences.* The discount rate,  $\beta$ , is set to 0.99, consistently with an annual interest rate of 4 percent. The intertemporal elasticity of substitution,  $\sigma$  is set to 2. The elasticity of substitution between different product types,  $\nu$ , is set to 10 (see, e.g., Galí 2008).

*Pricing.* Since direct estimates of the parameter of price-adjustment costs,  $\Psi$ , are not available, we follow much of the literature and perform an indirect calibration. Up to a first order approximation, a model with Rotemberg adjustment costs is observationally equivalent to a model with Calvo staggering. The log-linearized Phillips curve indeed becomes equivalent across the two models if  $\Psi = \frac{\theta\nu}{(1-\theta)(1-\beta\theta)}$ , where  $(1-\theta)$  is the probability that a firm can reset its price in the Calvo-model. Hence, for given elasticity of substitution across varieties, we calibrate the parameter  $\Psi$  so as to get an average contract duration in the Calvo model of four quarters: this value is in line with microeconomic evidence for Europe (see Alvarez et al. 2006) and it corresponds to the value most widely used in the macro literature.

*Labor market.* The bargaining power of workers,  $\gamma$ , is set to a benchmark value of 0.5. Taking continental Europe as reference point, the firing costs are set to 60 percent ( $f = 0.6$ ) of the annual productivity which amounts to approximately 66 percent of the annual wage<sup>16</sup> and the hiring costs are set to 10 percent ( $h = 0.1$ ) of annual productivity (see Chen and Funke 2003). Unemployment benefits are set to 65 percent of the level of productivity ( $b = 0.65$ ). This implies that in steady state the wage replacement rate is roughly 71 percent, a value which is in line with evidence for continental European countries (see OECD 2007). Operating costs are assumed to follow a

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<sup>16</sup>For the period from 1975 to 1986 Bentolila and Bertola 1990 calculate firing costs of 92 percent, 75 percent and 108 percent of the respective annual wage in France, Germany and Italy respectively. The OECD 2004 reports that many European countries have reduced their job security legislation somewhat from the late 1980 to 2003 (in terms of the overall employment protection legislation strictness). Therefore, we consider  $f = 0.6$  to be a realistic number for continental European countries.

logistic distribution<sup>17</sup> with zero mean (i.e. the median incumbent worker has  $\varepsilon^I = 0$ ). The scaling parameter of the distribution and the payments under disagreement,  $k$ , are chosen in such a way that the resulting labor market flow rates match the empirical hiring and firing rates described further below. This yields a scale parameter of 0.53 and payments under disagreement slightly below 0.28. We calibrate our flow rates using the Kaplan-Meier functions for West Germany<sup>18</sup>. Wilke’s 2005 Kaplan-Meier functions indicate that about 20 percent of the unemployed leave their status after one quarter. The steady state quarterly value for the firing rate was calibrated at 2% as this leads to a steady state value for the unemployment rate of 9%. This is roughly in line with Wilke’s estimated yearly risk of unemployment. The numbers assigned to the job flows are in line with the values reported by the OECD outlook 2004 also for other continental European countries.<sup>19</sup>. Based on this report we set the quarterly job-finding rate,  $\eta$ , at a value of 0.20 and the quarterly firing rate,  $\phi$ , at a value of 0.02.

Table 1: Parameters of the Numerical Model

Parameter	Description	Value	Source
$\beta$	Subjective discount factor	0.99	Standard value
$\sigma$	Consumption utility	2	Intertemp. elasticity of subst.
$\varepsilon$	Elasticity of subst.	10	Gali 2008
$\Psi$	Price adjustment cost	116.5	Equivalent to $\theta = 0.75$
$a$	Productivity	1	Normalization
$\gamma$	Workers’ bargaining power	0.5	Standard value
$f$	Firing cost	0.6	Bentolila and Bertola 1990
$h$	Hiring cost	0.1	Chen and Funke 2003
$b$	Unemployment benefits	0.65	OECD 2007
$E(\varepsilon)$	Expected value of op. costs	0	Normalization
$sd$	Distr. scaling parameter	0.53	To match the flow rates
$k$	Payments under disagreement	0.28	To match the flow rates

*Shocks.* We parameterize the shock processes in line with evidence for industrialized countries. Productivity shocks follow an  $AR(1)$  process. The autocorrelation is set to 0.95 and the standard error of the shock is 0.008. Government consumption evolves according to the following exogenous

<sup>17</sup>The logistic distribution approaches the normal distribution, but features a neater expression for the cumulative density function.

<sup>18</sup>We choose the Kaplan-Meier functions for Germany, as it is the largest continental European country.

<sup>19</sup>The numbers of the OECD outlook are not directly applicable to our model, since they are on a monthly basis. We therefore adjust them using the method described in Shimer 2012.

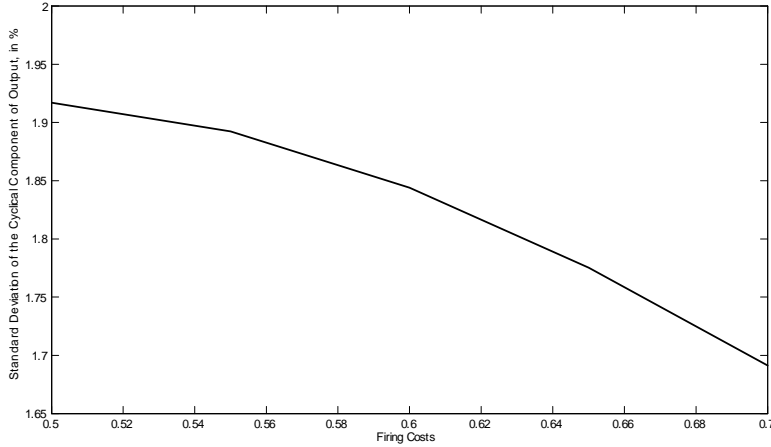


Figure 2: Changes in the volatility of output with respect to changes in the firing costs, under Taylor rules and both, productivity and government expenditure shocks.

process,  $\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \varepsilon_t^g$ , where the steady-state share of government consumption,  $g$ , is set so that  $\frac{g}{y} = 0.25$  and  $\varepsilon_t^g$  is an i.i.d. shock with standard deviation  $\sigma_g$ . Following much of the DSGE literature we set  $\sigma_g = 0.0074$  and  $\rho_g = 0.9$  (e.g., Perotti 2004).

## 4.2 Quantitative Properties of the Competitive Economy

In this section we study the quantitative implications of the model. We assume that monetary policy follows a Taylor rule with a weight 1.5 on inflation. First, we assess the model's ability to replicate the negative relation, outlined in section 2, between turnover costs and output volatility. To make our model results comparable to the empirical ones, we compute the standard deviation of the model's HP-filtered output for joint productivity and government spending shocks under different firing cost levels, leaving all other parameters constant. Figure 2 shows that the model also generates a negative relation between output volatility and firing costs. Interestingly, this is in contrast with the relation between output volatility and firing costs found in previous studies using the search and matching model: Silva and Toledo 2009 show that firing costs in the standard search and matching model amplify, rather than dampen, labor market volatilities.

To complete the assessment of the empirical validity of our model, we compare a series of euro area statistics with the model equivalent. Table 2 reports the standard deviations and the

autocorrelations for a number of selected variables for the four largest euro area countries, namely France, Germany, Italy and Spain, and for the EMU period, Q1 1997-Q1 2010. Quarterly data for GDP is taken from Eurostat and data for employment, the job-finding rate and the separation rate is taken from the OECD dataset. The data are HP-filtered with a quarterly smoothing parameter of 1600.

Table 2: Selected statistics for main euro area countries.

	Standard Deviations*				Autocorrelation	
	$y$	$u$	$\eta$	$\phi$	$y$	$u$
France	1.00	6.45	6.89	7.11	0.90	0.95
Germany	1.56	7.06	10.73	13.08	0.84	0.95
Italy	1.36	5.02	12.91	8.91	0.88	0.99
Spain	1.32	12.02	12.15	13.20	0.93	0.97
* In percentage points						

Table 3 shows equivalent statistics from the model, which have also been computed using Hodrick Prescott filtered series with the same smoothing parameter as used for the data.

Table 3: Selected statistics from the model.

	Productivity Shocks	Gov. Sp. Shocks
St. dev. $y$	1.75	0.09
St. dev. $u$	7.74	0.89
St. dev. $\eta$	7.73	0.92
St. dev. $\phi$	9.47	1.13
Autocorr. $y$	0.81	0.90
Autocorr. $u$	0.90	0.90

Our model is able to produce a sizable standard deviation of the unemployment rate, the job-finding rate and the separation rate (relative to output). This is again in contrast with the standard search and matching model which, as noticed by Shimer 2005, produces volatilities for unemployment and worker flows that are significantly lower than the ones found in the data. The reason for which our model can generate sizable worker flow volatilities is as follows. Since an aggregate productivity shock increases the expected present value of a worker, firms are willing to hire workers with larger operating costs,  $\varepsilon$ . The endogenous adjustment of the hiring and firing thresholds amplifies the dynamic response to shocks. As a result, the volatility of the job-finding rate and of the separation rate is several times larger than the volatility of the underlying



productivity shock (or of output volatility).

In addition, our model always generates a strong negative correlation between the job-finding and the separation rate (in line with the data), as the hiring and firing thresholds are tightly connected (see equations 7 and 9). In contrast to that, search and matching models with endogenous separations typically generate a positive correlation between the job-finding rate and the separation rate (see, e.g., Krause and Lubik 2007).

A comparison between tables 2 and 3 shows that the model also generates high persistence in unemployment and output. Persistent employment and output dynamics are consistent with the empirical evidence and represent an important stylized fact of the business cycle. It has to be noted that employment and output are a lot more persistent than the underlying aggregate shocks.<sup>20</sup> As explained above, employment persistence, namely its dependence on past dynamics, arises due to the time-varying gap between the retention rate and the job finding rate. When hiring and firing costs increase, this gap also increases, making employment more persistent.<sup>21</sup>

## 5 Social Planner Solution and Competitive Economy

**Definition 1.** For a given nominal interest rate  $\{i_t\}_{t=0}^{\infty}$  and for a given set of the exogenous processes  $\{a_t, g_t\}_{t=0}^{\infty}$  a determinate competitive equilibrium for the distorted competitive economy is a sequence of allocations and prices  $\{c_t, \pi_t, mc_t, v_{h,t}, \eta_t, \Xi_t^e(v_{h,t}), v_{f,t}, \phi_t, \Xi_t^i(v_{f,t}), w_t, y_t, n_t\}_{t=0}^{\infty}$  which, for given initial  $B_0$  satisfies equations 3, 7, 8, 18, 9, 10, 19, 11, 13, 16, 22 and the production function  $y_t = a_t n_t$ .

To highlight the nature of the distortion in the model, we proceed first by comparing the solution of the competitive economy with the constrained pareto optimal allocation.<sup>22</sup> Such a comparison allows us to highlight the externality that arises when atomistic agents fail to internalize the aggregate consequences of their decisions. To highlight the role of labor market frictions, we focus on an economy with flexible prices and assume that the distortion stemming from monopolistic

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<sup>20</sup>The HP-filtered productivity (government spending) time series has an autocorrelation coefficient of 0.71 (0.69). The autocorrelation coefficients of unemployment and output are 10 to 20 percentage points larger, i.e. the model generates endogenous persistence.

<sup>21</sup>For a further discussion of the empirical validity of the labor selection model see Lechthaler et al. 2010.

<sup>22</sup>In the constrained pareto optimal allocation the planner maximizes agents' utility under technological and resource constraints and by facing the same labor market restrictions, namely hiring and firing costs, as the competitive agents.

competition is offset by an appropriate subsidy such that the steady state marginal costs equal one,  $mc = 1$ . Furthermore, for simplicity we assume that government expenditure and unemployment benefits are set to zero.

The constrained efficient allocation is obtained by a social planner who maximizes agents' utility under the resource constraint and the evolution of employment. The social planner chooses the set of processes  $\{c_t, n_t, v_{h,t}, v_{f,t}\}_{t=0}^{\infty}$  to maximize:

$$U_t = \sum_{j=t}^{\infty} \beta^{j-t} E_t U(c_j), \quad (23)$$

subject to

$$c_t = a_t n_t - n_{t-1} f \phi(v_{f,t}^{PE}) - (1 - n_{t-1}) h \eta(v_{h,t}^{PE}) - (1 - \phi(v_{f,t}^{PE})) n_{t-1} \Xi_t^i(v_{f,t}^{PE}) - (1 - n_{t-1}) \eta(v_{h,t}^{PE}) \Xi_t^e(v_{h,t}^{PE}), \quad (24)$$

and

$$n_t = n_{t-1} (1 - \phi(v_{f,t}^{PE}) - \eta(v_{h,t}^{PE})) + \eta(v_{h,t}^{PE}), \quad (25)$$

where  $PE$  denotes the planner economy.

We define  $\lambda_t^{PE}$  as the Lagrange multiplier on constraint 24 and  $\mu_t^{PE}$  as the Lagrange multiplier on constraint 25. After some manipulation of the first order conditions, the value of a worker reads as follows:

$$\begin{aligned} \mu_t^{PE} = & a_t - E_t \Delta_{t,t+1} [\phi(v_{f,t+1}^{PE}) f - h \eta(v_{h,t+1}^{PE}) + (1 - \phi(v_{f,t+1}^{PE})) \Xi_{t+1}^i(v_{f,t+1}^{PE})] + \\ & + E_t \Delta_{t,t+1} [\eta(v_{h,t+1}^{PE}) \Xi_{t+1}^e(v_{h,t+1}^{PE})] + E_t \Delta_{t,t+1} [1 - \phi(v_{f,t+1}^{PE}) - \eta(v_{h,t+1}^{PE})] \mu_{t+1}^{PE}, \end{aligned} \quad (26)$$

and the first order conditions for the threshold values are

$$v_{h,t}^{PE} = \mu_t^{PE} - h, \quad (27)$$

$$v_{f,t}^{PE} = \mu_t^{PE} + f. \quad (28)$$

The current (shadow) value of a worker,  $\mu_t^{PE}$ , depends positively on aggregate productivity,  $a_t$ , and on a number of future expected terms. Those future variables affect the current marginal value of a worker as follows. Expected firing costs,  $\phi(v_{f,t+1}^{PE}) f$ , affect negatively the current value of a

worker: for a given firing rate,  $\phi(v_{f,t+1}^{PE})$ , an increase in employment increases the number of separated matches. Hence an increase in employment increases expected firing costs. Expected hiring costs,  $\eta(v_{h,t+1}^{PE})h$ , affect positively the current value of a worker. For given hiring rate,  $\eta(v_{h,t+1}^{PE})$ , higher employment reduces the number of future hirings, making the planner save an amount equivalent to  $\eta(v_{h,t+1}^{PE})h$ . This, in turn, increases the current value of a worker. Furthermore, the current value of a worker is affected negatively by the expected operating costs of incumbent workers,  $(1 - \phi(v_{f,t+1}^{PE}))\Xi_{t+1}^i(v_{f,t+1})$ , and positively by the expected operating costs of new entrants,  $\eta(v_{h,t+1}^{PE})\Xi_{t+1}^e(v_{h,t+1})$ . For given expected operating costs per incumbent worker,  $\Xi_{t+1}^i(v_{f,t+1})$ , an increase in the number of retained incumbent workers increases the overall operating costs. Similarly, for given expected operating costs per entrant,  $\Xi_{t+1}^e(v_{h,t+1})$ , a decrease in the number of hired workers (implied by the lower number of unemployed workers) reduces the overall operating costs. Finally, the current value also depends on the discounted future value of the worker,  $\mu_{t+1}^{PE}$ , multiplied by the difference between the retention rate and the hiring rate,  $1 - \phi(v_{f,t+1}^{PE}) - \eta(v_{h,t+1}^{PE})$ . A worker is more valuable when he is more likely to be retained, as this reduces future hiring and firing costs.

In the competitive economy,<sup>23</sup> a firm maximizes its profits:

$$Max_{\{n_t, v_{f,t}, v_{h,t}\}} \Pi_t = E_t \sum_{j=t}^{\infty} \Delta_{t,j} \left[ \begin{array}{l} a_t n_t - w_t n_t - n_{t-1} \phi(v_{f,t}^{CE}) f - s_t h \eta(v_{h,t}^{CE}) - \\ (1 - \phi(v_{f,t}^{CE})) n_{t-1} \Xi_t^i(v_{f,t}^{CE}) - s_t \eta(v_{h,t}^{CE}) \Xi_t^e(v_{h,t}^{CE}) \end{array} \right], \quad (29)$$

subject to

$$n_t = n_{t-1}(1 - \phi(v_{f,t}^{CE})) + \eta(v_{h,t}^{CE}) s_t, \quad (30)$$

where  $w_t$  is the bargained wage (either under collective bargaining or under individualistic bargaining)<sup>24</sup> and  $s_t$  is the number of applicants at a particular firm, which is taken as exogenous by atomistic firms. After taking the first order condition with respect to  $n_t$  and rearranging, we obtain the marginal value of a worker in the competitive economy:

$$\mu_t^{CE} = a_t - w_t - E_t \Delta_{t,t+1} [\phi(v_{f,t+1}^{CE}) f + (1 - \phi(v_{f,t+1}^{CE})) \Xi_{t+1}^i(v_{f,t+1})] + E_t \Delta_{t,t+1} \mu_{t+1}^{CE} (1 - \phi(v_{f,t+1}^{CE})), \quad (31)$$

<sup>23</sup>To make analytical results comparable between the social planner solution and the competitive equilibrium we outline the maximization problem of a competitive firm.

<sup>24</sup>See the online Appendix for a formal derivation of this case.

while the cutoffs are given by:<sup>25</sup>

$$v_{h,t}^{CE} = \mu_t^{CE} - h, \quad (32)$$

$$v_{f,t}^{CE} = \mu_t^{CE} + f. \quad (33)$$

The comparison of the equations characterizing the competitive equilibrium and the planner solution highlights similarities and differences. First, the marginal value of a worker in the competitive economy is reduced by the presence of wages which depress firms' profitability and hiring. Second, a firm in the competitive economy does not take into account the effects of its decisions on the pool of future applicants,  $s_t$ . In the presence of labor turnover costs hiring decisions lead to long term employment relations. Hence, by hiring a worker today the firm reduces the potential pool of applicants in the next period. Atomistic firms do not internalize this type of negative externality, which tends to induce over-hiring in the current period. This generates a composition effect: Compared to the first best solution, there are fewer entrants, who have relatively low operating costs (due to the cost of hiring), but more incumbents, who have relatively high operating costs (due to the cost of firing). Thus, the composition of employment (between incumbents and entrants) is distorted, as atomistic firms fail to internalize the effects of their hiring behavior on the size of the pool of searching workers.

The two effects just discussed run in opposite directions. Thus, it is possible to imagine a wage rule under which the two effects offset each other:<sup>26</sup> this would equalize the solution under the competitive equilibrium and the planner economy. However, in our model such a wage rule cannot be implemented through Nash bargaining processes. This divergence becomes evident by comparing the competitive (Nash bargained) wages and the efficient ones. We derive the efficient wage norm by equating the marginal value of a worker in the competitive and in the planner solution, namely by setting  $v_{f,t}^{CE} = v_{f,t}^{PE}$ . Using equations 26 and 31, we derive the efficient wage as:

$$w_t^* = E_t \Delta_{t,t+1} \eta (v_{h,t+1}^{CE}) [\mu_{t+1}^{CE} - h - \Xi_{t+1}^e (v_{h,t+1}^{CE})]. \quad (34)$$

The competitive (Nash bargained) wage reads as follows (under the assumptions of  $mc = 1$  and

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<sup>25</sup>This is an alternative formulation of equations 9 and 7.

<sup>26</sup>Hosios 1990 shows that such a wage rule exists for a search and matching model.

$b = 0$ ):

$$w_t = \gamma (a_t - \varepsilon_t^I + s). \quad (35)$$

The main difference between the two wages lies in the fact that the efficient wage does not depend on any contemporaneous variables, but reacts to future hirings and to the future marginal value of a worker. The opposite is true for the competitive wage, which is insensitive to future variables. The reason for this discrepancy lies in the fact that atomistic firms do not internalize the effects of their current hiring decisions on the future pool of applicants and on the future hiring prospects, hence the competitive wage remains insensitive to future shocks and fluctuations in future variables. The following lemmas formalize this line of arguments.

**Lemma 1.** The elasticities of the efficient wage with respect to current and future aggregate productivity are governed by the following equations:

$$\xi_{w_t^*, a_t} = \frac{\partial w_t^*}{\partial a_t} \frac{a_t}{w_t^*} = 0 \quad (36)$$

$$\xi_{w_t^*, a_{t+1}} = \frac{\partial w_t^*}{\partial E_t a_{t+1}} \frac{E_t a_{t+1}}{w_t^*} = E_t \Delta_{t,t+1} \frac{\eta(v_{h,t+1}^{CE}) a_{t+1}}{w_t^*} > 0 \quad (37)$$

**Proof.** See online Appendix B.

The lemma above makes explicit the fact that efficient wages depend solely on expected future realizations of productivity and of other endogenous variables. The planner, contrary to atomistic firms, internalizes the fact that current hiring and retentions affect the pool of future applicants. When choosing the optimal path of employment, he optimally balances current and future hirings and firings, by taking into account future realizations of productivity. The difference in the dynamic behavior between competitive Nash bargained wages and the efficient ones is therefore a manifestation of the aggregate externality operating in our model. The next lemma follows as a corollary.

**Lemma 2.** The wage in the competitive economy cannot replicate the efficient wage under standard Nash bargaining. Thus, the competitive economy is not constrained pareto-efficient.

It must be stressed that the above result does not hinge on our assumption that wages are bargained collectively. In online Appendix A we show that also under standard individualistic Nash bargaining wages fail to replicate the efficient one.

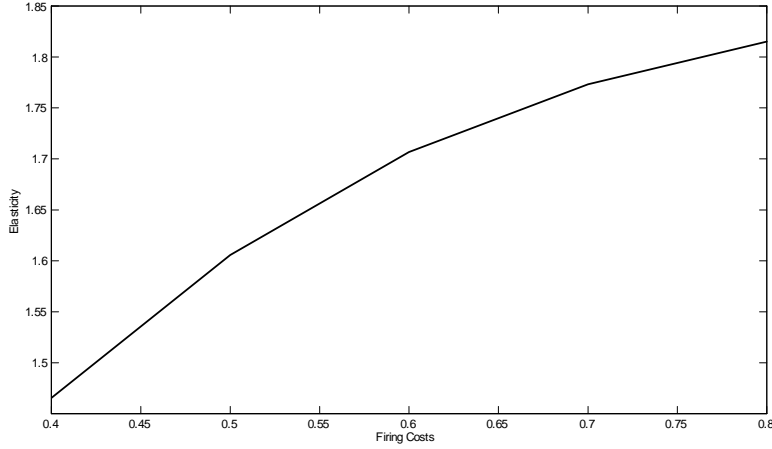


Figure 3: Average Elasticity of the Wage with Respect to Next Period's Productivity.

Thus far, we have shown that the competitive economy is not constrained pareto efficient. To better qualify the nature of the distortion in our model, it is instructive to analyze how it is affected by turnover costs. We do so in the following Lemma.

**Lemma 3.** The higher firing costs, the higher is the elasticity of the efficient wage with respect to future productivity shocks (see Figure 3 for an illustration).

**Proof.** See online Appendix B.

Intuitively, turnover costs produce long term contractual relations and their increase makes the expected survival rate of a match longer. The planner responds to this by rendering efficient wages more responsive to future productivity.

Note that since the elasticity of the competitive wage to expected future shocks and to expected future realizations of the endogenous variables is zero, it is by construction insensitive to firing costs. Hence, when the elasticity of efficient wages to future firing costs increases, this generates a larger divergence between the dynamic behavior of the competitive and the planner economy: such a difference in turn measures the extent of the inefficiency. Further below in the numerical results of the Ramsey plan, we show that the optimal volatility of inflation increases when firing costs increase: this is consistent with the arguments arising from Lemma 3. When the inefficiency increases, the Ramsey planner is more prone to use state contingent movements in inflation to fight

its consequences.

To rejoin the above results to the analysis of the optimal monetary policy design discussed in the next section, it is instructive to discuss how a monetary authority, endowed solely with inflation, can affect the competitive economy so as to get closer to the efficient equilibrium. As discussed above, the efficient wage is more responsive to future shocks: this implies that efficient employment is more stable compared to the competitive one. The Ramsey monetary authority should therefore aim at stabilizing the employment fluctuations in the competitive economy. Consider the equation characterizing firms' hiring decision:

$$h + v_{h,t} = a_t m c_t - w_t + E_t(\Delta_{t,t+1} \Pi_{t+1}). \quad (38)$$

The monetary authority can use state contingent movements in inflation, by increasing its volatility. This stabilizes fluctuations of current and future profits (the right hand side of the above equation), fluctuations of hiring and firing thresholds and therefore of employment.

## 6 Optimal Ramsey Policy

### 6.1 Ramsey Setup

The optimal monetary policy plan is determined by a monetary authority that maximizes the discounted sum of agents' utilities given the constraints of the competitive economy. The next task is to select the relations that represent the relevant constraints in the planner's optimal policy problem. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations, in the spirit of the primal approach described in Lucas and Stokey 1983.<sup>27</sup>

$$a_t m c_t - w_t - v_{h,t} + E_t(\Delta_{t,t+1} \Pi_{t+1}) = h \quad (39)$$

$$a_t m c_t - w_t - v_{f,t} + E_t(\Delta_{t,t+1} \Pi_{t+1}) = -f \quad (40)$$

$$n_t = n_{t-1}(1 - \phi_t - \eta_t) + \eta_t \quad (41)$$

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<sup>27</sup>There is a fundamental difference, though, between that classic approach and the one followed here, which stems from the impossibility, in the presence of sticky prices and other frictions, of reducing the planner's problem to a maximization only subject to a single implementability constraint. Khan, King and Wolman 2003 adopt a similar structure to analyze optimal monetary policy in a closed economy with price stickiness and monetary frictions.

$$w_t = \gamma (a_t m c_t - \varepsilon_t^I + k) + (1 - \gamma) b \quad (42)$$

$$0 = (1 - \nu) + \nu m c_t - \Psi (\pi_t - 1) \pi_t + E_t \{ \Delta_{t,t+1} \Psi (\pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \pi_{t+1} \} \quad (43)$$

$$\begin{aligned} g_t + c_t &= y_t - n_{t-1} \phi_t f - (1 - n_{t-1}) \eta_t h - (1 - \phi_t) n_{t-1} a_t \Xi_t^i - \\ &\quad (1 - n_{t-1}) \eta_t \Xi_t^e - \frac{\Psi}{2} (\pi_t - 1)^2 y_t \end{aligned} \quad (44)$$

The government resource constraint does not need to be included among the equilibrium conditions as fiscal policy is passive (lump sum taxation).

**Definition 2.** Let  $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^\infty$  represent the sequence of Lagrange multipliers on the constraints (39), (40), (41), (42), (43), (44). Then for a given stochastic process  $\{a_t, g_t\}_{t=0}^\infty$ , plans for the control variables  $\Xi_t^n \equiv \{c_t, n_t, w_t, m c_t, \pi_t, v_{h,t}, v_{f,t}\}_{t=0}^\infty$  and for the co-state variables  $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}\}_{t=0}^\infty$  represent a first best constrained allocation if they solve the following maximization problem:

$$\text{Min}_{\{\Lambda_t^n\}_{t=0}^\infty} \text{Max}_{\{\Xi_t^n\}_{t=0}^\infty} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \quad (45)$$

subject to (39), (40), (41), (42), (43), (44).

As a result of constraints (39), (40) and (43) exhibiting future expectations of control variables, the maximization problem as spelled out in (45) is intrinsically non-recursive. As first emphasized in Kydland and Prescott 1980, a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) co-state variables. Such variables, that we denote  $\chi_{1,t}$ ,  $\chi_{2,t}$  and  $\chi_{3,t}$  for (39), (40) and (43) respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan. Another aspect concerns the specification of the law of motion of these Lagrange multipliers. For this case, both constraints feature a simple one period expectation, the same co-state variables have to obey the laws of motion:

$$\frac{\chi_{1,t+1}}{\beta} = \lambda_{1,t}, \quad \frac{\chi_{2,t+1}}{\beta} = \lambda_{2,t}, \quad \frac{\chi_{3,t+1}}{\beta} = \lambda_{5,t}. \quad (46)$$

Using the new co-state variables so far described, we amplify the state space of the Ramsey allocation to be  $\{a_t, g_t, \chi_{1,t}, \chi_{2,t}, \chi_{3,t}\}_{t=0}^\infty$  and we define a new saddle point problem which is recursive in the new state space. Consistently with a *timeless perspective*, we set the values of the three co-state variables at time zero equal to their solution in the steady state.



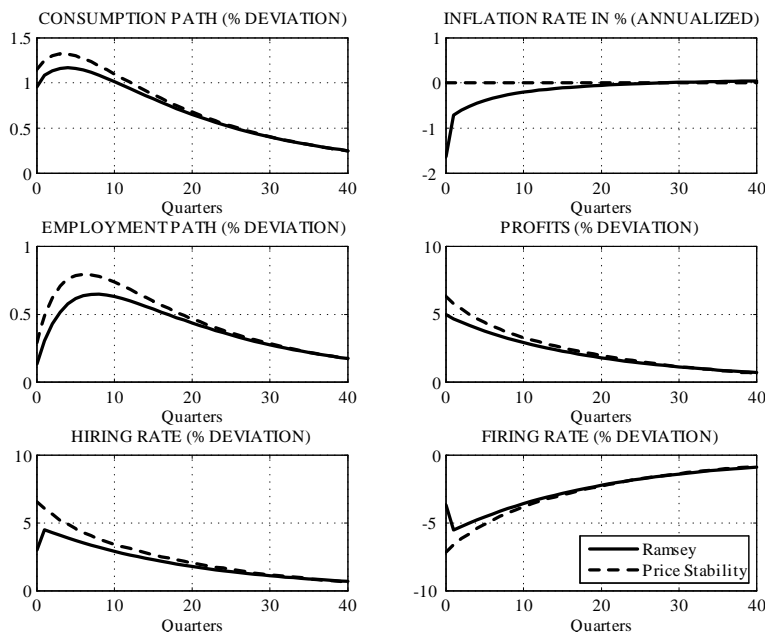


Figure 4: Impulse responses to productivity shocks, Ramsey vs. price-stability.

## 6.2 Response to Shocks and Optimal Volatility of Inflation

To compute responses of the optimal plan to shocks we resort on first order approximations of the first order conditions of the Lagrangian problem described in definition 2. Technically, we compute the stationary allocation that characterizes the deterministic steady state of the first order conditions to the Ramsey plan. We then compute a second order approximation<sup>28</sup> of the respective policy functions in the neighborhood of the same steady state. This amounts to implicitly assuming that the economy has been evolving and policy has been conducted around such a steady state already for a long period of time (under timeless perspective).

Figure 4 shows impulse response functions of the Ramsey plan (solid line) to positive productivity shocks. The Ramsey plan is compared to the competitive equilibrium under a zero inflation policy (dashed line). In response to an increase in productivity consumption, output and employment increase. The hiring and firing thresholds increase, implying an increase (reduction) in the mass of hirings (firings). The monetary authority in this context has a trade-off between stabilizing

<sup>28</sup>Second order approximation methods have the particular advantage of accounting for the effects of volatility of variables on the mean levels of the same. See Schmitt-Grohe and Uribe (2007) among others.

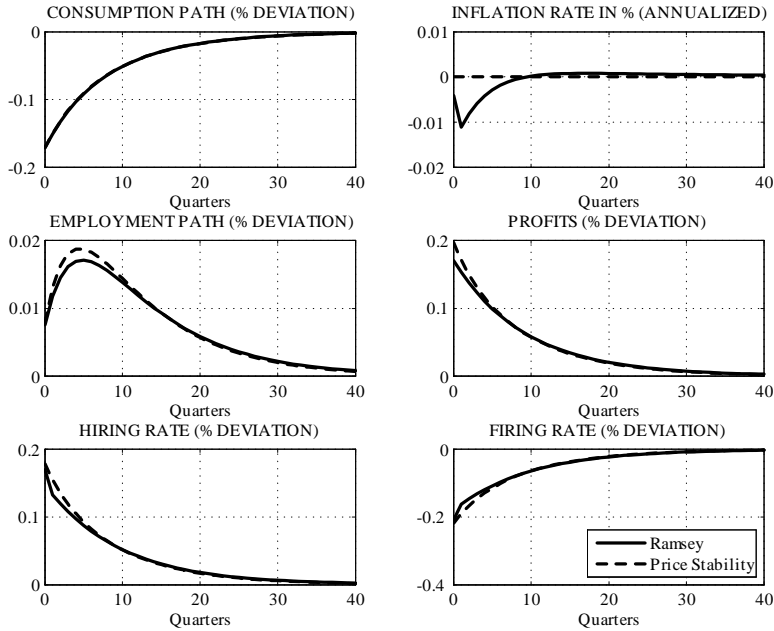


Figure 5: Impulse responses to government expenditure shocks, Ramsey vs. price-stability.

inflation and reducing inefficient unemployment fluctuations. As explained in section 5, the Ramsey planner uses state contingent movements in inflation to dampen fluctuations in firms' profits (the profits in figure 4 are defined as the right hand side of equation 38): the latter induces a dampening in the fluctuations of hiring/firing and employment. Overall, fluctuations of all real variables are smaller in the Ramsey plan relatively to the zero inflation policy. Importantly and contrary to traditional New Keynesian models, deviations from price stability arise in this model even in response to productivity shocks.

Figure 5 shows impulse responses of the Ramsey plan (solid line) in response to government expenditure shocks and in comparison to the zero inflation policy (dashed line). An increase in government expenditure crowds out consumption demand. However, because of the increase in aggregate demand employment increases, the mass of firings shrinks and the mass of hirings rises. Once again deviations from price stability arise. This result is consistent with past literature (see Khan, King and Wolman 2003), which has shown that shocks to government spending cause fluctuations in the ratio between aggregate demand and output which prevent the implementability of the flexible price allocation with constant mark-ups. Also consistently with previous studies,

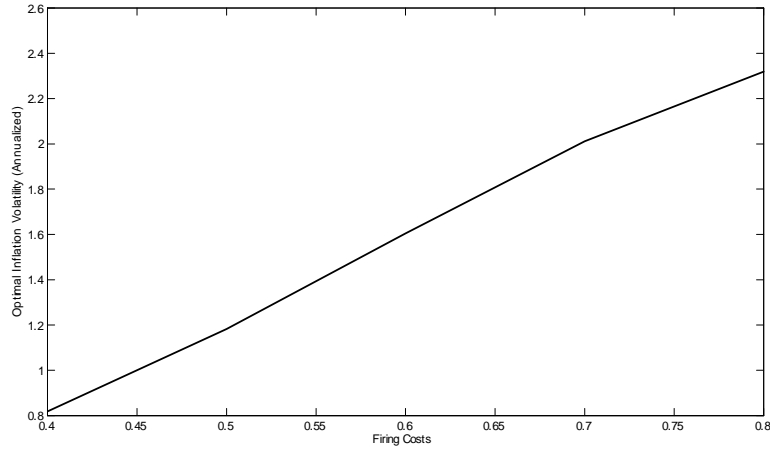


Figure 6: Optimal inflation volatility in response to the two shocks

deviations from zero inflation are rather small under this shock and consequently the differences between the Ramsey policy and the zero inflation policy are rather minor.

An interesting question is whether the policy maker favors larger movements in inflation when the extent of the distortion becomes larger. As discussed in Lemma 3 and demonstrated in figure 3, larger labor turnover costs lead to larger inefficiencies, hence they increase the incentives of the policy maker to deviate from the zero inflation policy. This is confirmed by figure 6 which shows that the optimal volatility of inflation, in response to both productivity and government expenditure shocks, increases when firing costs increase.

### 6.3 Comparison with a Model of Wage Rigidity

Monetary trade-offs also emerge in a model with wage rigidities along the lines of Erceg et al. 2000 (EHL hereafter). It is therefore instructive to compare the two models in terms of implications for business cycle dynamics and the design of optimal monetary policy. To do so, we construct and simulate a model with Rotemberg adjustment costs in both prices and wages and an endogenous labor supply (i.e. with disutility of labor), which we label the wage rigidity (WR) model,<sup>29</sup> but

<sup>29</sup>Note that EHL use Calvo staggering instead of Rotemberg adjustment costs. We use Rotemberg adjustment costs to get a closer comparison between the two models. In contrast to EHL, we do not use subsidies to remove the monopolistic distortions. However, the introduction of subsidies would only lead to some quantitative changes (i.e., leave our main conclusions unaffected).

without labor selection and labor turnover costs. The parameters common across the two models are set as before. For the WR model, we set the Frisch elasticity of labor supply,  $\varphi$ , to 1 and the wage rigidity parameter to four quarters. In the optimal monetary policy analysis, we consider average contracts in the range from two to six quarters.<sup>30</sup> We start by comparing the business cycle properties of the two models and their relation to the data.

Table 4: Business cycle statistics in data for Germany (1980-2004), FLM and EHL (all variables are HP-filtered with  $\lambda = 1600$ ).

	Data	FLM (Prod. / Gov. Spending)	EHL (Prod. / Gov. Spending)
St. dev. $w$ / St. dev. $y$	0.89	0.70 / 1.79	0.03 / 0.04
St. dev. $u$ / St. dev. $y$	8.50	4.41 / 10.00	-
St. dev. $\eta$ / St. dev. $y$	7.48	4.41 / 10.32	-
St. dev. $\phi$ / St. dev. $y$	8.00	5.40 / 12.64	-
Autocorr. $y$	0.72	0.81 / 0.90	0.70 / 0.73
Autocorr. $\pi$	0.79	0.89 / 0.55	0.70 / 0.75
Autocorr. $u$	0.95	0.90 / 0.90	-
Corr. ( $\pi, y$ )	0.21	-0.68 / 0.65	-1.00 / 0.93

Table 4 compares business cycle statistics for HP filtered series from Germany from 1980-2004<sup>31</sup> with simulated data from EHL and our model. We have chosen Germany for this table because it is Europe's largest economy and the country for which we have the longest possible and most comprehensive labor market statistics. Our model performs better than EHL along several dimensions. First, it generates predictions for job findings, separations and unemployment well in line with the data (EHL is silent about this). The volatilities of those variables in our model are quite high, as in the data. Second, the wage volatility in our model is also close to the one in the data. On the contrary, the wage volatility in EHL is excessively smooth. In our model wages are compressed in the cross-section, but they do not feature any rigidity over time. Interestingly in our model wages are even more volatile than the underlying productivity shocks, which is due to the fluctuations in marginal costs. Third, in contrast to EHL, in our model macro and labor market

<sup>30</sup>To calibrate the parameter for the cost of wage adjustment, we equate the log-linear version of the Phillips curve for wages under the Rotemberg and Calvo set-ups. This leads to the following expression for the parameter on the cost of wage adjustment:  $\Psi^w = \frac{\theta^w \nu (1 + \varphi \nu)}{(1 - \theta^w)(1 - \beta \theta^w)}$ , where  $\nu$  is the elasticity across different varieties of labor services and  $\theta^w$  represents the probability of adjusting wages.

<sup>31</sup>The labor market data for Germany is taken from Gartner et al. 2012. Flows are constructed from the IAB-employment sample. HP smoothing parameter is set to  $\lambda = 1600$ . Information on job-finding and separation rates is only available until 2004 due to a structural break in the data. Wages are overall gross wages divided by working hours, deflated by the GDP deflator (Source: German Statistical Office).

variables are highly persistent: autocorrelations are indeed high, particularly those of output and unemployment. Traditional medium-scale DSGE models resort to a number of additional model devices (habit persistence, variable capital utilization, etc.) to re-produce the empirically relevant macro persistence. In our model this persistence comes genuinely from the mechanisms at work in the labor market.

Table 5: Optimal standard deviation of inflation

	WR					FLM				
	contract duration*					firing costs**				
	2	3	4	5	6	40	50	60	70	80
$\nu = 10$	0.90	1.47	2.03	2.63	3.28	0.82	1.18	1.60	2.01	2.31
$\nu = 4$	0.60	1.01	1.47	1.97	2.53	1.19	1.64	2.12	2.58	2.92
	* average contract duration in quarters					** firing costs in percent of productivity				

The two models can be compared also in terms of their implications for optimal monetary policy. We calculate the optimal volatility of inflation using the Ramsey approach also for the WR model and compare it with our model. Table 5 illustrates the results. Under an elasticity of substitution of  $\nu = 10$ , the two models yield fairly similar results. Notice that for the WR model the parameter  $\nu$  represents both, the elasticity across different product varieties and across different labor services. Under our benchmark calibration for the firing costs, namely 60% of output, the optimal inflation volatility for our model is 1.6%. For the WR model, the optimal volatility of inflation is 2% when the average contract duration is four quarters. For the WR model, the optimal inflation volatility increases with the degree of wage rigidity, while in our model it increases with the level of firing costs. Interestingly the results are also affected by the degree of varieties substitution (and labor services for the WR model). With a value of  $\nu = 4$  (as in EHL 2000), the optimal volatility of inflation falls in the WR model, while it increases in our model. For this parameter specification, the policy trade offs are amplified in our model and dampened in the EHL. This difference also highlights important differences in the shock transmission between the two models. In our model the elasticity of substitution affects nominal rigidities: when it falls, market competition falls and so do the sensitivity of prices to shocks and the welfare cost of inflation. Since inflation is less costly, the policy maker trades off inflation with output volatility. In the WR model a fall in  $\nu$  dampens fluctuations in the wage mark-up, hence the welfare costs

of real wage rigidities. The incentives of the policy maker to correct labor market distortions are lower and the optimal volatility of inflation falls.

In closing, some observations are worth on the micro-foundations related to each of the two models. In our model, it is possible to connect labor turnover costs to institutional parameters (e.g. employment protection legislation) or primitive parameters (e.g. training costs). By contrast, the sources of wage rigidities are not fully understood yet and cannot easily be connected to one specific institution.

## **7 Conclusions**

The design of optimal monetary policy is derived in a DSGE model with sticky prices, labor selection, labor turnover costs and Nash bargained wages. The type of labor market frictions considered gives rise to non-trivial trade-offs for the monetary authority. Optimal policy features deviations from price stability and those deviations are larger the larger the size of firing costs. From a theoretical point of view, our analysis shows that the case for price stability can be challenged if one considers a model with a significant role for labor turnover costs.

A natural extension of this analysis is to consider the role of labor turnover costs in a DSGE model for a currency area model. Euro area countries face significant differences in terms of labor market institutions, particularly turnover costs and employment protection indices. An analysis of those differences could shed light on the differential response of output and inflation to common monetary policy actions.

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