

Use of Max-Flow on FACTS devices.

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Abstract—FACTS devices can be used to mitigate cascading failures in a power grid by controlling the power flow in individual lines. Placement and control are significant issues. We present a procedure for determining whether a scenario can be mitigated using the concept of maximum flow. If it can be mitigated, we determine what placement and control setting will solve the scenario. This paper treats fourteen cascading failure scenarios and reports on the use of the max-flow algorithm both in determining the mitigation of each scenario and in finding FACTS settings that will mitigate the scenario.

Index Terms—Flexible AC Transmission System, FACTS placement, cascading failure, maximum flow, power system.

I. INTRODUCTION

The power grid of the United States is one of the largest interconnected networks in existence. The size of the network makes control and regulation a very difficult problem. One of the more devastating problems is the cascading failure. Flexible AC Transmission System (FACTS) devices have been used to regulate power across particularly important lines in an effort to prevent such failures. However, due to their cost, it is highly preferable to place as few devices as possible at the most effective locations.

The Maximum Flow algorithm has been recommended as a method of setting FACTS Devices [1]. However, not all cascading failures leave enough power flow in the system to avoid a complete outage. In section II we explain the problem in more detail. Section III gives a procedure for the use of a maximum flow algorithm, both in determining settings and for FACTS devices, as well as determining if there is a mitigation of an outage scenario. We present a static scenario analysis of fourteen outage scenarios in the IEEE 118 Bus system. In section IV we give the FACTS device locations and settings for the scenarios that can be mitigated. In section V we give a quick proof for each of the outage scenarios that can not be mitigated. Conclusion and Future Work sections are included in sections VI and VII.

II. BACKGROUND

A. Power Transmission cascading outages

A common problem in the power industry is that of limited capacity between generators and loads that has the potential to result in a cascading failure. This lack of capacity is usually a result of a transmission line fault resulting in line removal. If

a sufficient number of lines are removed, other lines become overloaded and eventually also fail. Eventually the entire power grid will fail. FACTS devices can be used on power lines to regulate power flow through a line. FACTS devices can be deployed at strategic locations in an effort to re-route power to avoid these outages. Unfortunately, it is only possible to control a few lines, due in part to cost of FACTS devices which limits the number that may be deployed. With FACTS devices placed appropriately, there are no more overloads on total power flow. With no overloads, no more lines will be removed due to an excess of power and the cascade is stopped before it results in power grid failure.

B. FACTS device model

The FACTS device model used in this paper is fairly simple. A "bogus" bus is inserted on the line where a FACTS device is to be placed. The original line should now run between this bogus bus and the original destination bus. A nearly infinite impedance is placed between the original source bus and the bogus bus to ensure mathematical solvability. The source bus should now sink the power that would flow over the line. The bogus bus will generate the same amount of power. This allows us to set the amount of power over a line. The rest of the power flows will adjust themselves accordingly during the solution of the load flow on the system.

C. Outage scenario identification

Fourteen cascading outage scenarios were identified using the method outlined in [2]. Cascading outages are created by removing one or more power lines. When these lines are removed, other lines overload and eventually trip off. Eventually the entire power system will fail. These scenarios are listed in IV and V with an evaluation of possible mitigations. For the purposes of this paper, we will be satisfied with finding any set of FACTS devices that results in a power system that has no total power overloads. If a scenario can be mitigated we say the scenario is *solved*.

D. The 118 bus system

All of the scenarios shown here are from the IEEE 118 bus system. The line capacities and FACTS device settings are given in MVA. Since the IEEE data does not give values line capacity, values were created to be plausible for a stressed power system. Line flow data is included when ever a line is mentioned. For lines that have a FACTS device on them the first number is the setting of the FACTS device; negative

This research supported in part by NSF MRI award CNS-0420869, in part by NSF CSR award CCF-0614633, in part by the UMR Intelligent Systems Center, and in part by UMR Opportunities for Undergraduate Research Experience (OURE).

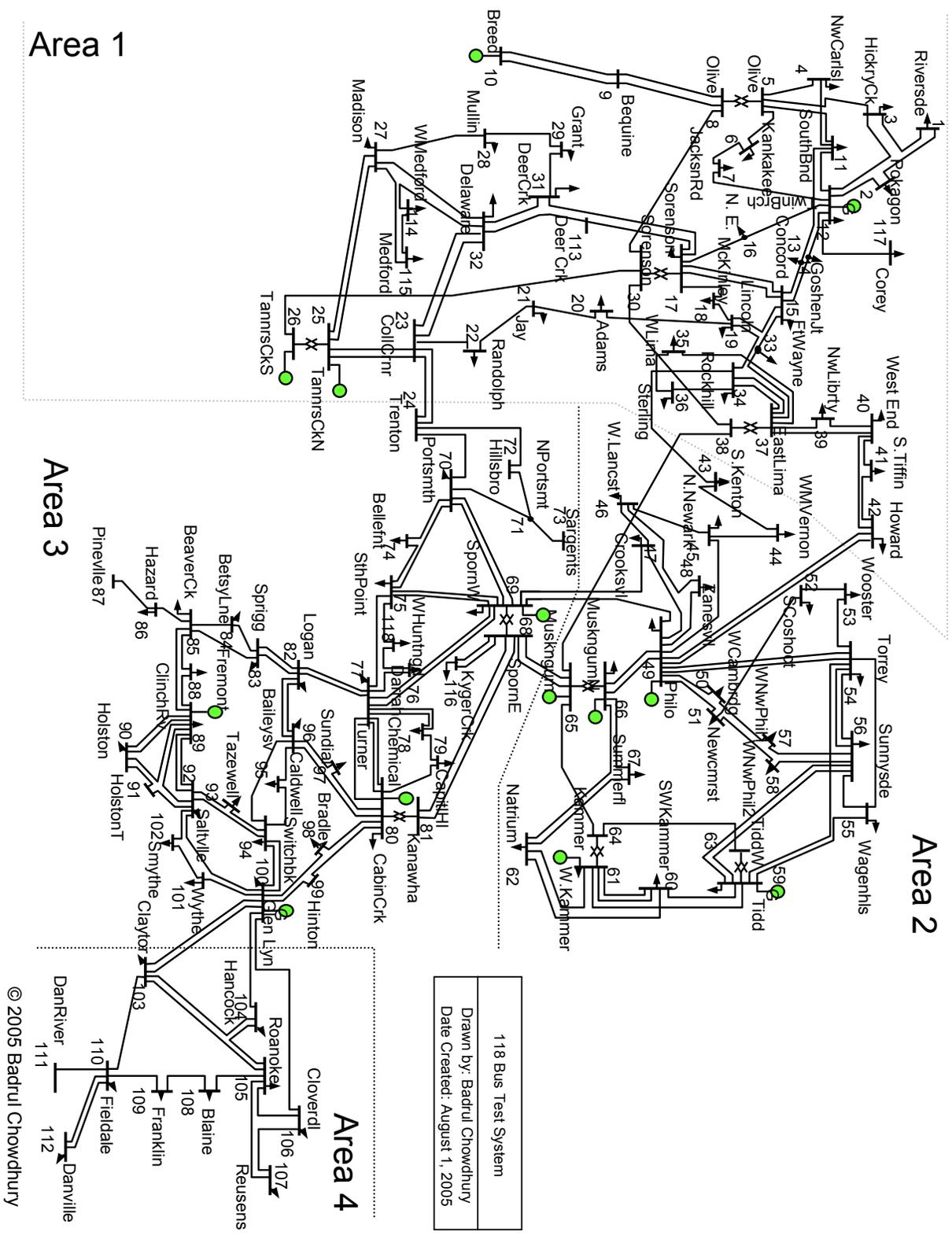


Fig. 1. 118 Bus System

numbers indicating power flow from the higher numbered bus to the lower one. The second is the line capacity. The complete set of data can be found on the UMR Facts Interaction Laboratory website (<http://filpower.umn.edu/>)

III. USE OF MAX-FLOW

A. Power system as a max-flow problem

A power system may be represented in terms of a graph as described in [3]. In this graph, the nodes represent buses within the power system. Each of these nodes has some net generation (source) or load (sink). The edges in the graph represent the power lines between buses. Each edge has a steady state value and a real power capacity. It is possible to change a multi-source multi-sink graph into a single-source single-sink graph as described in [4]. A new node defined as the common source is added to the graph. Generators can be represented as a line from this common source to the location of the generator. The capacity of these lines is equal to the generator that it represents. The same thing is done to create a common sink.

B. Max-flow as a heuristic

Unfortunately, max-flow is only a heuristic for solving this problem. Since max-flow can not account for reactive power, it can not give any guarantees about the flow, only a prediction. Even the real-power limits on each line are only an estimate for the maximum amount of real power that can safely flow through the line. We are able to prove some scenarios to be unsolvable by this heuristic. Other scenarios we have found solutions for and have verified these scenarios using the loadflow experiments.

C. Procedure for determining unsolvable scenarios using max flow

Max flow is used to determine if it is possible to use FACTS devices to mitigate overloads. The procedure is to run max flow and check the added power sink lines. If one of the lines is not filled to its original steady-state capacity (indicating that a node is not getting all of the power it needs) there will be a cut-set of the power system that uses more power than there is capacity to feed it. This area can be found by checking for regular lines that are filled to capacity. The area that doesn't get all of the power it needs will be surrounded by these lines.

The basic proof goes back to Kirchoff's Current Law. If you draw a circle on the graph of the power grid, the net power flowing in is equal to the net power sunk in the area. Since max-flow would fill any remaining capacity between the common source and the common sink, there must not be any path remaining. If (as shown in Figure 2) if the net draw of Area B is greater than the capacity of the lines feeding it, at least one node in Area B will not get the power it needs. For these scenarios (explained in section V) the lines (and their capacities) that form a cut-set of the graph will be given.

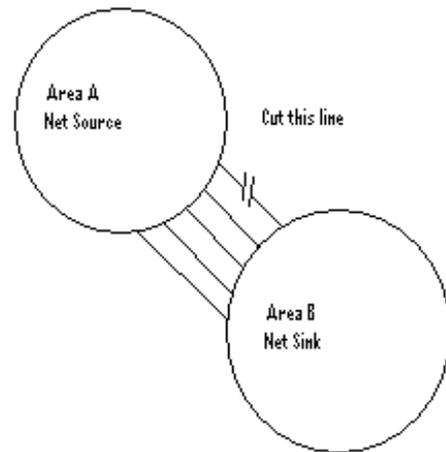


Fig. 2. Example

D. Procedure for setting FACTS devices with Max Flow

Since max-flow will attempt to supply power to every node that needs it, using the max-flow values to set FACTS devices should draw or limit power as needed to balance the power flow.

For scenarios that were not proved to be unsolvable, we started adding FACTS devices. For the first two, the method was simple. Add a device with the max-flow value as its setting on the most overloaded line. We kept adding devices to the highest overloaded line until none were left. This will limit the power over that line to prevent it from overloading, while not limiting so far as to overload other lines.

For the scenarios in sections IV-C and IV-D, this method did not work as well. Since max-flow only accounts for real power, it under-estimates the power flow across lines that are near to a generator. This causes max-flow values to be too restrictive. Furthermore, max-flow does not account for the direction power will tend to flow. This can cause max-flow to attempt to force power over high-impedance lines when low-impedance alternatives may be a better option.

IV. SOLVABLE LINE OUTAGE SCENARIOS

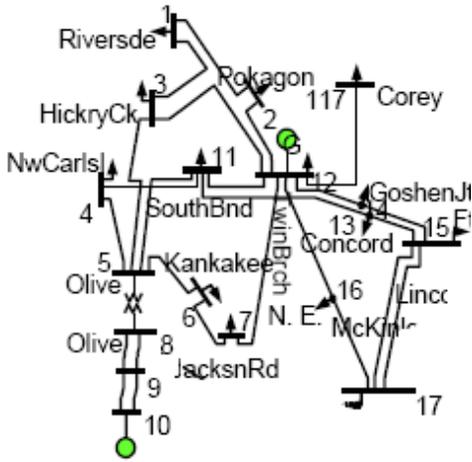
There were four scenarios that were able to be solved. For each of these scenarios, the locations and settings of FACTS devices are given in the format: line(FACTS setting, Line capacity). When a FACTS setting is negative, this indicates that power is flowing backwards (i.e. from bus 15 to bus 13). Also note that overloads are only considered when the total capacity is overloaded.

A. Line 4-5

It appears that it might be possible to mitigate this outage if it is caught before line 5-11 goes out. After that line leaves, there will not be enough capacity to handle transferring power away from the generator at bus 10.

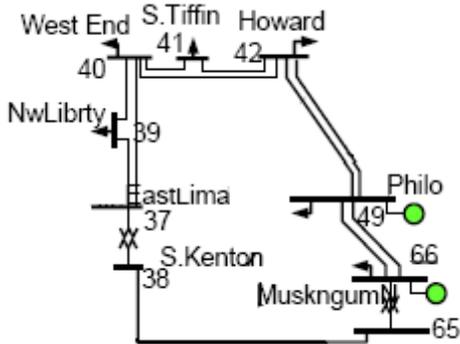
Two FACTS devices are needed to have a stable system with no overloads. One FACTS device on each of 5-11 (1.158,

1.158) and 7-12 (0.2098, 0.5557) appear to mitigate the outage. These values are pulled directly from the max-flow results.



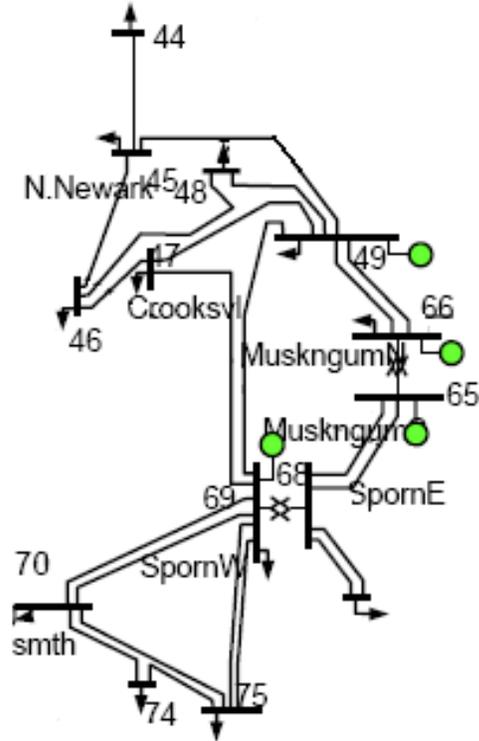
B. Line 37-39

This outage may be mitigated by installing FACTS devices on line 37-40 (0.6615, 0.6615). Line 37-40 is the line that gets most of the flow when 37-39 gets removed.



C. Line 47-69

This scenario is an example of one that can not be solved by max-flow. Attempting to use max-flow left total power overloads on some lines, even ones that had FACTS devices on them. This is most likely due to max-flow's inability to account for reactive power and FACTS devices not managing reactive power. Instead, this scenario was solved by setting FACTS devices to a value just below the real power capacity of each line. If a line was overloaded on total power, the FACTS device setting was lowered. This was one of the most trial-and-error scenarios to solve, but the following FACTS device placements and settings will remove all total power overloads: 47-49 (-0.215, 0.216), 48-49 (-0.520, 0.5266).

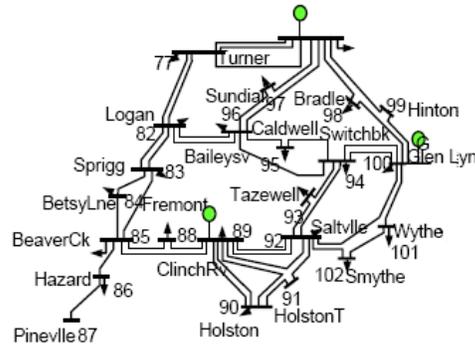


D. Line 89-92

Because line 89-92 has two circuits, this scenario has two basic cases. If both circuits go out, there is not enough capacity leaving the area to handle the power generated at bus 89. Lines 82-83 and 91-92 will overload.

In the event that only one line goes out, the situation changes. If Circuit 1 (low impedance) goes out, a facts device on line 82-83 (-0.49, 0.7109) and one on 91-92 (-0.0855, 0.1425) will be sufficient to solve the system.

If it is the high impedance side (Circuit 1) that goes out, there are no total power overloads. This situation does not require FACTS devices to solve.

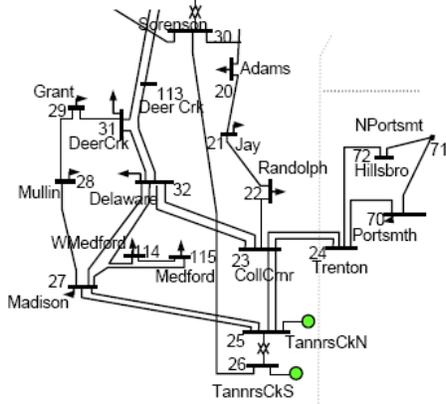


V. UNSOLVABLE LINE OUTAGE SCENARIOS

There were ten scenarios that were unsolvable. For each of these scenarios, we define an area of the grid that does not have enough capacity to either bring in all the power that area needs, or to distribute all the power it generates. The list of lines that enter or leave this area are given in the format: line (capacity).

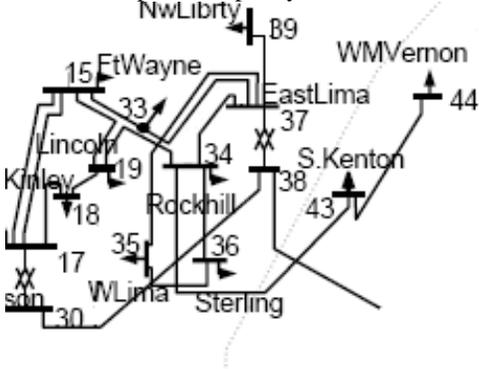
A. Line 26-30

Removing line 26-30 leaves the only output for the generator at bus 26 going through transformer 25-26 (capacity of 2.032). The transformer does not have enough capacity to handle the output (3.14), so the generator will have to back down. Further more, line 26-30 was a main power trunk up to the northern half of area 1. Without that power source, this area is deprived of power. Generation will have to be increased elsewhere. There is no combination of FACTS devices that will save this scenario due to lack of line capacity.



B. Line 34-37

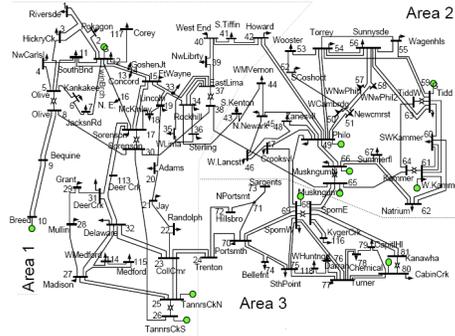
After removing Line 34-37, an area designated by lines 19-34 (0.1889), 35-36 (0.0333), and 43-44 (0.377) can be established. This area contains buses 34, 36, and 43. While there is enough total capacity in the lines to handle the total draw of these buses it does not appear to be any set of FACTS devices that will mitigate this failure. The cause lies with the real power vs. total power ratio on lines 19-34 and 35-36. Each of these lines has a significantly lower real capacity in relation to its total capacity. The real power draw of these buses is 1.07 while the real power capacity of the lines is only about 0.60.



C. Line 38-65

There is a net lack of capacity of approximately .3 going from areas 2 and 3 to area 1. This may be solved by a combination of routing power with FACTS devices and modifying generation/load. However, simply re-routing power will not be enough. There is not enough capacity going from areas 2 and 3 to area 1.

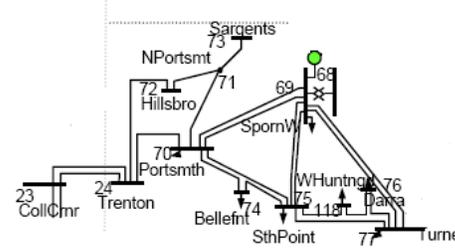
Note: This was not determined by adding up all of the buses in Area 1, but by measuring the net flow in to Area 1 in a steady state (no outage) system. However, there should be no difference in the calculations.



D. Line 69-70

There is not enough capacity to re-route power through lines 70-75 and 74-75. This leaves us with routing power north through area 2 and then reversing flow on the line 23-24 to re-supply busses 24, 70, 71, 72, and 73. There is not enough capacity between area 2 and area 3 to handle this extra load. Also, there is not enough capacity between area 1 and area 2 to handle this extra load. It is possible that FACTS devices combined with modifying generation may help handle this outage, particularly increasing generation at buses 25 and 26.

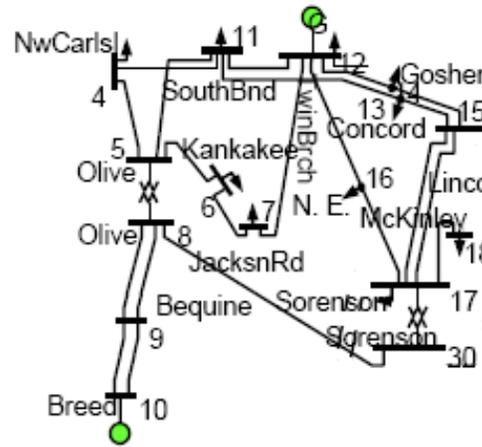
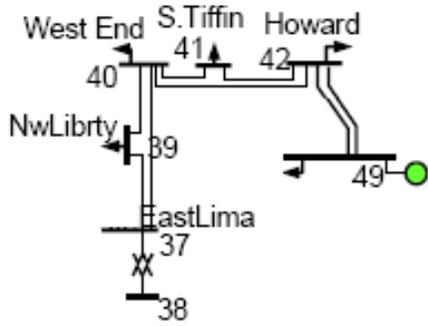
An area defined by lines 23-24 (0.186743), 74-75 (0.785414), and 70-75 (0.006772) does not have enough flow in to the area once line 69-70 is removed. The total capacity is much closer. Several of these lines have a much higher total capacity than real capacity. The real power capacity feeding this area is 0.978929. The net real power load is 1.65.



E. Line 42-49

There are two main effects of removing this line. First, it is the primary power feed to buses 41 and 42. Second, it is one of several lines that carries power from area 2 to area 1. In the first case, lines 40-41 (0.349) and 40-42 (0.400) do not have enough capacity to handle the load of those buses 41 and 42 after the loss of line 42-49. The real load of those buses is 1.33.

In the second case, there is a net flow of power from areas 2 and 3 to area 1. This flow occurs over lines 42-49, 38-65, 34-43, and 23-24. Removing line 42-49 does not leave enough capacity over the remaining lines to handle this net flow.

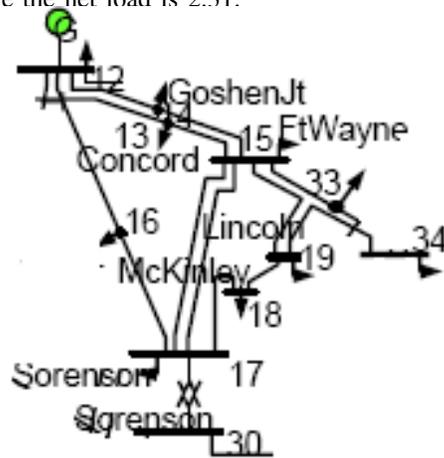
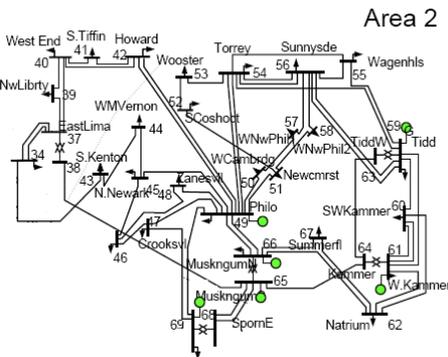


F. Line 64-65

This ends up killing power to most of Area 2. Line 64-65 is a trunk in to the eastern half of Area 2. Without this line, power must flow in along the lower lines. Combined with the very low capacity of transformer 65-66, there is simply not enough capacity left to supply this area. The lines that feed this area are 34-43 (0.097), 37-39 (0.8245), 37-40 (0.6615), 47-69 (0.8797), 49-69 (0.7312), and 65-66 (0.1903). The list of affected buses is bus 39 through bus 67 except for bus 65. Real power capacity feeding this area is 3.3842. Real power load of this area is 3.62.

H. Transformer 17-30

The area containing buses 15, 16, 17, 18, and 19 lacks enough capacity to handle the load. The lines feeding this area are 12-16 (0.3797), 13-15 (0.0559), 14-15 (0.1421), 17-31 (0.1982), 17-113 (0.0921), 19-20 (0.3203), 15-33 (0.3657), and 19-34 (0.1889). Net capacity entering the area is 1.72, while the net load is 2.31.

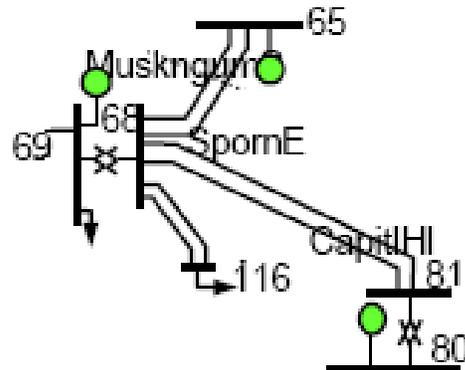


G. Transformer 5-8

After the loss of transformer 5-8, there is not enough capacity in lines 16-17 (0.397), 13-15 (0.559), 14-15 (0.142) to feed the northwest corner of the grid. The real power draw of that area is 3.25. In addition, line 8-30 does not have enough capacity to handle the output from the generator at bus 10. That generator will have to back down to avoid further overloads.

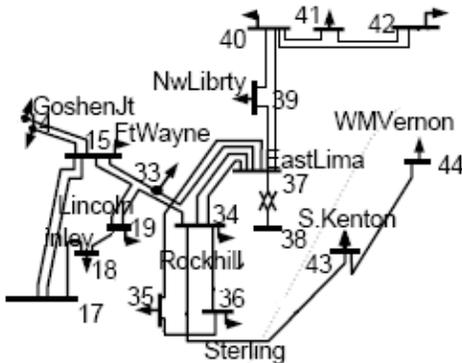
I. Transformer 68-69

Due to the low real power capacity of lines 65-68 (0.3187) and 68-81 (0.6654), there is not enough power to supply buses 68 and 116. The needed real power capacity is 1.84, but there is only 0.9842 available.



J. Transformer 37-38

Establishing an area containing buses 34, 35, 36, 37, 39, and 40 results in a net draw of 2.578. However, the maximum flow in to this area through lines 15-33 (0.3657), 19-34 (0.1889), 40-41 (0.3494), 40-42 (0.4002), and 34-43 (0.0971) is only 1.4011. The net draw of this area is 1.70. With out the transformer 37-38, there is not enough flow in to this area to handle the draw. This outage also creates problems in other areas of the grid, namely transformer 65-66.



VI. CONCLUSION

This paper established a method to analyze cascading outage scenarios using the max-flow algorithm. We applied this analysis to 14 outage scenarios on the IEEE 118 bus system. Four were found to be solvable with no overloaded lines. Two more appear to be close to solvable as they have very little overload. The remaining eight are reasonably guaranteed to have an unacceptable overload, regardless of what is done with FACTS devices. In most cases, there is simply not another

route that can be used to transfer power in to an area. We suspect that a larger, more realistic, graph may provide a better test system for further analysis.

VII. FUTURE WORK

Work continues in performing similar testing on larger power systems. Several side notes are worth mentioning. The max-flow algorithm does not work well for FACTS devices placed near generators. Since the current model works with real power only, max-flow will not allow enough power to properly supply the loads. If a modified version of max-flow is created to handle reactive power as well, this problem should be eliminated.

It was also discovered that max-flow has another, more immediate use. Assuming a stable power system, max-flow should allow the maximum power to flow through all the sink "lines". If this is not the case, there must be some cut across the system where there is not enough capacity to transfer the needed power. This is exactly the scenario that results in an un-solvable power system.

REFERENCES

- [1] A. Armbruster, M. Gosnell, B. McMillin, and M. Crow, "The maximum flow algorithm applied to the placement and distributed steady-state control of FACTS devices," in *Proceedings of the 37th Annual North American Power Symposium (NAPS)*, October 2005, pp. 77-83. [Online]. Available: citeseer.ist.psu.edu/armbruster05maximum.html
- [2] B. H. Chowdhury and S. Baravc, "Creating cascading failure scenarios in interconnected power systems," in *IEEE Power Engineering Society General Meeting*, June 2006.
- [3] A. Armbruster, M. Gosnell, B. McMillin, and M. L. Crow, "Power transmission control using distributed max flow," in *COMPSAC*. IEEE Computer Society, 2005, pp. 256-263. [Online]. Available: <http://doi.ieeecomputersociety.org/10.1109/COMPSAC.2005.121>
- [4] P. Elias, A. Feinstein, and C. Shannon, "A note on the maximum flow through a network," *IRE Transactions on Information Theory*, vol. 2, no. 4, pp. 117-119, 1956.