Some generalised comparative determiners

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Abstract. Functions denoted by specific comparative expressions called *generalised comparative determiners* are analysed. These expressions form verb arguments when applied to common nouns. They denote functions which take sets and a binary relation as argument and give a set as result. These functions are thus different from denotations of "ordinary" determiners. However, they do obey some similar constraints, properly generalised. It is shown that verbal arguments obtained from such generalised determiners extend the expressive power of NLs since functions that they denote are not just case extensions of type ⟨1⟩ quantifiers used to interpret "ordinary" determiner phrases.

1 Introduction

The basic meaning of the notion of determiner as established in modern formal linguistics is that of an expression which when applied to one (or more) common nouns (CNs) forms a noun phrase (NP) or rather a determiner phrase (DP). NPs are (syntactic) arguments of verb phrases and thus they can occur in various argumental positions, for instance as grammatical subjects, grammatical objects or various oblique objects. When a DP occurs in the direct object position (of a simple sentence of the form NP TVP DP) one can consider that the determiner which forms it denotes a function which takes a set (the denotation of the CN) and a binary relation (denotation of the TVP) and gives a set (the denotation of the TVP+DP) as result. In this paper I will discuss some functions of this type in cases where they are denoted not by "ordinary" determiners but by what can be called *generalised determiners*, that is expressions which take one or more common nouns as arguments and which give a verbal argument as result.

There are verbal arguments which cannot occur in every argumental position and thus are not strictly speaking NPs. Consider (1), (2) and (3):

(1) Leo washed himself.

(2) Leo washed himself and Lea.

(3) a. *Himself washed Leo.
b. *Himself and Lea washed them/themselves/Leo

In (1) *himself* occurs in direct object position and is thus similar to a an ordinary noun phrase. In addition it can be conjoined with an ordinary NP to make a Boolean complex NP which can occur in the direct object position, as shown in (2). However, neither *himself* alone nor its Boolean composition with an ordinary NP can occur in the subject position, as seen in (3a) and (3b).

The case of *himself* and of its Boolean compound indicated above is well known: it illustrates (nominal) anaphora. What is less well-known is the fact that there are expressions forming nominal anaphors in a way similar to the one in which "ordinary" DPs are formed: they can be formed by an application of an *anaphoric determiner* to a CN. To see this consider the following examples:

(4) Leo shaved every philosopher except himself.

(5) Lea admires most philosophers, including herself

.

In (4) the DP-like expression *every philosopher except himself* and in (5) the DP-like expression *most philosophers, including herself* are (complex) nominal anaphors. What is important for us is that they are formed by the application of (complex) determiner-like expressions *every..., except himself* and *most..., including herself*, respectively, to the common noun *philosophers*. These expressions are called anaphoric determiners and, obviously, anaphoric determiners are generalised determiners.

It is possible to mention many differences between functions denoted by ordinary determiners and functions denoted by anaphoric determiners (Zuber 2010a). In particular it is useful to consider that anaphoric determiners denote functions from sets (denotations of common nouns) to functions from binary relations (denotations of transitive verbs) to sets (denotations of verb phrases). Or, equivalently, anaphoric determiners denote functions which take a set (or sets) and a binary relation as argument and give a set as result.

Some languages, for instance Scandinavian, Slavic or Latin, have possessive anaphoric determiners which are morphologically simple, in contraposition to those given in (4) and (5). Slavic type anaphoric determiners and various properties of functions denoted by them are studied in Zuber 2010b and Zuber 2010c.

In his paper I am going to study another sub-class of generalised determiners, which I will call *comparative determiners*. I will in particular show that functions denoted by comparative determiners, though of the same type as functions denoted by anaphoric determiners, have different formal properties.

As an example of a comparative (generalised) determiner consider (6):

(6) Leo knows more philosophers than Lea.

The expression *more...than Lea* in (6) takes the common noun *philosophers* as argument and makes an expression which can be considered as "syntactic" argument of the verb *know*. In addition this argument cannot occur in the subject position. Consequently *more...than Lea* is a generalised determiner.

2 Formal preliminaries

The following technical and notational preliminary will be useful. Since we are interested basically in comparatives and proportional quantifiers, both of which are naturally interpreted only in finite universes, we will assume that our universe of discourse E is finite. Thus all sets (of individuals) considered are sub-sets of E. For any set A, $|A|$ is the cardinality of A and for any binary relation R the set aR is defined as follows: $aR = \{x : \langle a, x\rangle \in R\}$.

Since the functions I will discuss are ofen related to quantifiers let me recall some basic notions concerning generalised quantifiers. Functions from sets (subsets of E) to truth-values are type $\langle 1\rangle$ quantifiers. Functions from pairs of sets to truth-values or binary relations between sets are type $\langle 1, 1\rangle$ quantifiers. They are denotations of unary determiners. Type $\langle 1, 1, 1\rangle$ quantifiers are ternary relations between sets. They are denotations of binary determiners. In fact for some syntactic reasons, one can distinguish two sub-classes of type $\langle 1, 1, 1\rangle$ quantifiers: (1) quantifiers whose type is noted $\langle\langle 1, 1\rangle 1\rangle$ and (2) quantifiers whose type is noted $\langle 1\langle 1, 1\rangle\rangle$. The first class corresponds to denotations of binary determiners which take two nominal arguments (as in *More students than teachers danced*) and the second class corresponds to denotations of binary determiners which take two predicative arguments (as in *More students danced than sang*).

We will also use the following notation for functions from sets or relations to sets. A type $\langle 1, 2 : 1\rangle$ function is a function having a set and a binary relation as argument and giving a set as result. A type $\langle 1, 1, 2 : 1\rangle$ function is a function which takes two sets and one binary relation as arguments and gives a set as a result. Such functions are denoted by unary and binary generalised determiners.

We are interested in the interpretation of sentences of the form NP TVP GDP where TVP is a transitive verb phrase and GDP is a generalised determiner phrase (an expression obtained by the application of a generalised determiner to a common noun). In such sentences NP is interpreted by a type $\langle 1\rangle$ quantifier, which is a set of sets, and $TVPs$ is interpreted by a binary relation. Concerning GDP there are two possibilities: if it is an "ordinary" DP, it is interpreted by a function which is an accusative extension of a type $\langle 1\rangle$ quantifier or, if it is not an DP it is interpreted by a function (from binary relations to sets) which is not an accusative extension of a type $\langle 1\rangle$ quantifier. An accusative extension Q_{acc} of a type $\langle 1\rangle$ quantifier Q is defined as follows (Keenan 1988):

Definition 1. *For each type $\langle 1\rangle$ quantifier Q, $Q_{acc}(R) = \{a : Q(aR) = 1\}$.*

Thus the accusative extension of a quantifier is a function from binary relations to sets induced by the quantifier in the way indicated in Definition 1. Accusative extensions of quantifiers permit one to compute directly denotations of verb phrases formed from transitive verb phrases and a noun phrase in the position of the direct object.

Accusative extensions of type $\langle 1\rangle$ quantifiers are specific type $\langle 2 : 1\rangle$ functions. They are specific because they satisfy the following accusative extension condition AE (Keenan and Westerstahl 1997):

Definition 2 (AE). *A type $\langle 2:1\rangle$ function F satisfies AE iff for R and S binary relations, and $a, b \in E$, if $aR = bS$ then $a \in F(R)$ iff $b \in F(S)$.*

Not all type $\langle 2:1\rangle$ functions satisfy AE. For instance the function $SELF$ defined as $SELF(R) = \{x : \langle x, x\rangle \in R\}$ which interprets the reflexive pronoun (as it occurs in (1) for instance) does not satisfy AEC. Similarly the type $\langle 2:1\rangle$ function $F(R) = MORE_{l,Ph}(R) = \{x : |xR \cap Ph| > |lR \cap Ph|\}$ (where l is the individual referred to by *Lea* and Ph is the set of philosophers), which is denoted by the verbal argument *more philosophers than Lea* in (6), does not satisfy AE.

The two functions above satisfy conditions weaker than AE. The function $SELF$ satisfies the following anaphor condition AC:

Definition 3 (AC). *A type $\langle 2:1\rangle$ function F satisfies the* anaphor condition *iff for R and S binary relations, and $a \in E$, if $aR = aS$ then $a \in F(R)$ iff $a \in F(S)$.*

The AC condition, sometimes called *predicate invariance* (Keenan and Westerstahl 1997), is obviously weaker than AE. The function $MORE_{l,Ph}$ above satisfies another weakening of AE, the so-called *argument invariance* condition AI:

Definition 4 (AI). *A type $\langle 2:1\rangle$ function F is argument invariant iff for any binary relation R and $a, b \in E$, if $aR = bR$ then $a \in F(R)$ iff $b \in F(R)$.*

The conditions AE, AC and AI concern type $\langle 2:1\rangle$ functions, considered here as being denoted by "full" verbal arguments. We are interested in denotations of generalised determiners, that is expressions forming verbal arguments when applied to common nouns. When such determiners are unary, that is when they apply to one common noun, they denote type $\langle 1, 2:1\rangle$ functions. The accusative extension condition for such functions is as follows:

Definition 5 (D1AE). *A type $\langle 1, 2:1\rangle$ function F satisfies D1AE iff for R and S binary relations, $X \subseteq E$ and $a, b \in E$, if $aR \cap X = bS \cap X$ then $a \in F(X, R)$ iff $b \in F(X, S)$.*

Denotations of ordinary determiners occurring in DPs which take direct object position satisfy D1AE. More precisely if D is a type $\langle 1, 1\rangle$ quantifier, then the function $F(X, R) = D(X)_{acc}(R)$ satisfies D1AE. Denotations of anaphoric determiners do not satisfy D1AE. For instance the function $F(X, R) = \{y : X \cap yR = \{y\}\}$ denoted by the anaphoric determiner *no... except himself/herself* does not satisfy D1AE.

Anaphoric functions satisfy the following condition (Zuber 2010c):

Definition 6 (D1AC). *A type $\langle 1, 2:1\rangle$ function F satisfies D1AC (*anaphor condition for unary determiners*) iff for R and S binary relations $X \subseteq E$, and $a \in E$, if $aR \cap X = aS \cap X$ then $a \in F(X, R)$ iff $a \in F(X, S)$.*

In this article we are interested in functions denoted by comparative generalised determiners. As we will see they do not satisfy D1AC. The condition which characterises such functions is as follows:

Definition 7 (D1AI). *A type $\langle 1,2:1\rangle$ function F satisfies D1AI (*argument invariance for unary determiners*) iff for any binary relation R, $X \subseteq E$ and $a, b \in E$, if $aR \cap X = bR \cap X$ then $a \in F(R)$ iff $b \in F(R)$.*

The following property gives a justification of condition D1AI:

Proposition 1. *If the function F of type $\langle 1,2:1\rangle$ satisfies D1AI then the function G^A of type $\langle 2:1\rangle$ defined as $G^A(R) = F(A,R)$ satisfies AI.*

What Proposition 1 informally says is that functions satisfying D1AI are those from which we get functions satisfying AI when fixing their set argument.

Functions from binary relations and sets to sets which satisfy D1AI have the following obvious property:

Proposition 2. *If a function F from binary relations to sets satisfies D1AI then for any $X, Y \subseteq E$ one has $F(X, E \times Y) = \emptyset$ or $F(X, E \times Y) = E$.*

The conditions specified above are related to the fact that "proper" anaphors, anaphoric determiners or comparative generalised determiners cannot occur in subject position. We have seen that such expressions are close, in some sense, to expressions denoting various quantifiers. As we will see functions denoted by generalised comparative determiners, in addition to property of argument invariance satisfy a natural generalisation of the conservativity property characteristic of "ordinary" determiners.

Conservativity and related properties (intersectivity, etc) are properties of quantifiers. It is possible, however to naturally generalise this notion to functions of the type studied here. We have the following definition (Zuber 2010a):

Definition 8. *A function F of type $\langle 1,2:1\rangle$ is conservative iff $F(X,R) = F(X,(E \times X) \cap R)$*

The following property gives plausibility to the above definition of generalised conservativity (Zuber 2010b):

Proposition 3. *Let D be a type $\langle 1,1\rangle$ quantifier and F a type $\langle 1,2:1\rangle$ function defined as: $F(X,R) = D(X)_{acc}(R)$. Then F is conservative iff D is conservative.*

It is easy to check that the anaphoric function denoted by the anaphoric determiner *no... except himself* and the comparative function denoted by the generalised determiner *more...than Lea* (both mentioned above) are conservative. As is well established conservative quantifiers have various important subclasses, such as classes of intersective, cardinal, etc. quantifiers (cf. Keenan and Westerstahl 1997). It is also possible to generalise the notions of intersective, co-intersective and cardinal quantifiers in such a way that they apply to type $\langle 1,2:1\rangle$ functions. Thus we have the following definitions (Zuber 2010a):

Definition 9. *A type $\langle 1,2:1\rangle$ function is intersective (resp. co-intersective) iff $F(X_1,R_1) = F(X_2,R_2)$ whenever $(E \times X_1) \cap R_1 = (E \times X_2) \cap R_2$ (resp. $(E \times X_1) \cap R'_1 = (E \times X_2) \cap R'_2$).*

The following proposition, similar to Proposition 1, can be considered as justifying the above definition:

Proposition 4. *Let D be a type $\langle 1,1\rangle$ quantifier and F a type $\langle 1,2:1\rangle$ function defined as: $F(X,R)=D(X)_{acc}(R)$. Then F is intersective (resp. co-intersective) iff D is intersective (resp. co-intersective).*

It is easy to see that both functions mentioned above are intersective.

Concerning co-intersective functions it is easy to show that the function $EVERY(X)$-BUT-$SELF(R)$ defined in (7), and denoted by the anaphoric determiner *every... but himself*, is co-intersective:

$$EVERY(X)\text{-}BUT\text{-}SELF(R)=\{x: X\cap xR'=\{x\}\} \qquad (7)$$

It is also possible to generalise other sub-properties of conservativity. Consider so-called cardinal quantifiers. A type $\langle 1,1\rangle$ quantifier F is cardinal iff $F(X_1)(Y_1)=F(X_2)(Y_2)$ whenever $|X_1\cap Y_1|=|X_2\cap Y_2|$; numerals denote cardinal quantifiers. Type $\langle 1,2:1\rangle$ cardinal functions are defined as follows:

Definition 10. *A type $\langle 1,2:1\rangle$ function is cardinal iff $F(X_1,R_1)=F(X_2,R_2)$ whenever $\forall y(|X_1\cap yR_1|=|X_2\cap yR_2|)$*

Obviously, cardinal, intersective and cardinal functions are conservative.

In the next section we will discuss various examples of comparative generalised determiners which denote functions having one of the properties defined in definitions 2, 3 and 4.

3 Some unary comparative determiners

In this section I will basically discuss numerical comparative (and superlative) generalised determiners, that is, semantically, quantifiers and more generally functions applying to sets and relations involving (numerical) comparisons of cardinalities of various sets. In that way the results will be clear and at the same time directly extensible to the case of adjectival comparison. However, I will discuss adjectival comparative (superlative) constructions only briefly since they give rise to generalised determiners only in a special case.

Let me first clarify the distinction between what have been sometimes called *comparative quantifiers* and comparative functions in which we are interested here and which are not comparative quantifiers (or their case extensions).

Numerical comparisons make it clear that various entities can be compared in comparative constructions. To show this, let me present two numerical comparative constructions in semantics of which binary (comparative) quantifiers but not comparative type $\langle 1,2:1\rangle$ functions are involved. In (8) we have comparative constructions involving what are now known as binary determiners (Keenan and Mos 1985, Beghelli 1994, Zuber 2009):

(8) a. More students than priests dance
b. More students dance than talk

In (8a) we have a binary determiner denoting a type $\langle\langle 1,1\rangle 1\rangle$ quantifier and in (8b) a binary determiner denoting a type $\langle 1\langle 1,1\rangle\rangle$ quantifier. Their semantics is given in (9) and (10) respectively:

$$MORE(S)\,THAN(P)(D) = |S \cap D| > |P \cap D| \tag{9}$$
$$MORE(S)(D)\,THAN(T) = |S \cap D| > |S \cap T| \tag{10}$$

We know that DPs formed with binary determiners occur in object position:

(11) Leo met more students than priests.

One can say that in (11) we have a binary generalised determiner which denotes a type $\langle 1,1,2:1\rangle$ function. But clearly this is not the case of a genuine generalised determiner: the DP it forms can occur in subject position (as in (8a), the function it denotes is just the accusative extension of the function given in (9a). Later on we will see that there are genuine generalised determiners which denote type $\langle 1,1,2:1\rangle$ functions.

Consider example (12), similar to the one in (6) used above to illustrate the case of genuine generalised determiners:

(12) Leo met more students than Lea.

When comparing (12) with (11) we see that different things are compared in these sentences: in (11) we compare properties that one individual has whereas in (12) we compare individuals which share one (non trivial) property.

In (12) we have a genuine generalised determiner, *more...than Lea*. It is genuine because the function it denotes is not an accusative extension of a type $\langle 1,1\rangle$ quantifier. Informally this can be seen by the following reasoning in which condition D1AE is used. Suppose the set of Xs that Bill met is the same as the set of Xs that Leo met. It does not follow from this that the sentence *Bill met more Xs than Lea* has the same truth value as the sentence *Leo met more Xs than Lea.*

The function which is denoted by the generalised determiner *more...than Lea* is given in (13):

(13) $F(X,R) = MORE_l(X,R) = \{y : |yR \cap X| > |lR \cap X|\}$, where l is the individual referred to by *Lea*

It is obvious that the function in (13) satisfies the argument invariance for unary determiners. Indeed, suppose that (14a) holds. Then clearly (14b) holds as well:

(14)a. $aR \cap X = bR \cap X$
b. $a \in MORE_l(X, R)$ iff $b \in MORE_l(X, R)$

It is also easy to show that function $MORE_l$ is conservative. We have to show that $MORE_l(X, R) = MORE_l(X, (E \times X) \cap R)$. But this is obvious given the semantics in (12) and the fact that $a((E \times X) \cap R) = X \cap aR$.

In fact, the function $MORE_l$ has a stronger property than conservativity: it is cardinal (in the sense of definition 4 above). To show this we have to prove that (15) holds if (16) holds:

(15) $MORE_l(X_1, R_1) = MORE_l(X_2, R_2)$
(16) $\forall y(|X_1 \cap yR_1| = |X_2 \cap yR_2|)$.
.

We have the following chain of equivalent statements: $x \in MORE_l(X_1, R_1)$ iff (given (12)) $|xR_1 \cap X_1| > |lR_1 \cap X_1|$ iff (given (16)) $|xR_2 \cap X_2| > |lR_2 \cap X_2|$ iff $x \in MORE_l(X_2, R_2)$. Thus $MORE_l$ is a cardinal type $\langle 1, 2 : 1 \rangle$ function.

The above example suggests various questions concerning the number, the variety of patterns and the specificity of properties of generalised comparative determiners and of the functions they denote. For instance one would like to know, roughly speaking, whether there are many other patterns of generalised comparative determiners which denote cardinal functions, whether there are generalised determiners which denote intersective non cardinal functions, or which denote conservative non intersective functions, etc. of course such questions cannot be fully answered here and, in addition some of them should be made more precise. In what follows I will try to give partial answers to some of these questions.

Observe first that in the above example we can replace the expression *more* by *less* or *the same number (of)*. Obviously the generalised determiners thus obtained will still denote cardinal functions.

We can still make generalised determiners more complex by adding explicitly numerals to the determiners discussed above. Consider the example in (17). It is probably ambiguous with the readings indicated in (18a) and (18b):

(17) Leo read 5 more books than Lea.
(18)a. Leo read exactly 5 more books than Lea.
b. Leo read at least 5 more books than Lea.

In the above examples we have the generalised comparative determiners *5 more... than Lea* and *exactly 5 more... than Lea*. They denote type $\langle 1, 2 : 1 \rangle$ functions which are instances of functions given in (19a) and (19b) respectively:

(19)a. $EXACTLY\text{-}MORE_{n,i}(X, R) = \{x : |xR \cap X| = |iR \cap X| + n\}$
b. $AT\text{-}LEAST\text{-}MORE_{n,i}(X, R) = \{x : |xR \cap X| > |iR \cap X| + n\}$

It is easy to show that functions in (19a) and (19b) are cardinal and satisfy condition D1AI. Moreover, the above examples show that there is an infinite number of generalised determiners denoting cardinal functions.

It is still possible to make comparative generalised determiners more complex by making Boolean compounds of them. Such Boolean complex determiners are given in the following examples:

(20) Leo read more books than Lea but less than 17.

(21) Leo read more books than Lea and more than Bill.

(22) Leo read less books than Lea but more than Bill.

Again, functions involved in the semantics of above examples are cardinal functions. This follows from the observation that functions discussed here form Boolean algebras, the fact that I will not comment in more detail.

Recall that cardinal functions are intersective ones. One may wonder whether there are generalised comparative determiners denoting intersective non cardinal functions. I think that a good example of such a determiner is *the same...as Lea* as it occurs in (23). The function denoted by this determiner is an instance of the function given in (24):

(23) Leo read the same books as Lea.

(24) $THE\text{-}SAME_i(X, R) = \{y : yR \cap X = iR \cap X\}$.

The function in (24) is intersective. We have to show that (26) follows from (25). This is true because (25) entails the two equalities given in (27):

(25) $(E \times X_1) \cap R_1 = (E \times X_2) \cap R_2$

(26) $\{y : yR_1 \cap X_1 = iR_1 \cap X_1\}$ iff $\{y : yR_2 \cap X_2 = iR_2 \cap X_2\}$

(27) (i) $yR_1 \cap X_1 = yR_2 \cap X_2$, (ii) $iR_1 \cap X_1 = iR_2 \cap X_2$

We get the needed result by replacing in (26) the equal parts indicated in (27).

It is also easy to show that function $THE\text{-}SAME_i$ given in (24) does not satisfy D1AE and that it satisfies D1AI.

In quite the same way one can show that the generalised determiner *different... than Lea*, as it occurs in (28), denotes an intersective comparative function:

(28) Leo knows different languages than Lea.

One can consider that (28) is ambiguous with possible readings in which the set of languages known by Leo and by Lea are either "just" different or they are disjoint. These two readings can be distinguished by two different functions $DIFFERENT_i$ indicated in (29a) and (29b) respectively:

(29)a. $DIFFERENT_i(X, R) = \{y : yR \cap X \neq iR \cap X\}$
b. $DIFFERENT_i(X, R) = \{y : yR \cap X \cap iR \cap = \emptyset\}$

Both functions, the one in (29a) and the one in (29b), are intersective.

All the examples of generalised determiners presented above contain a complementizer *than* or *as*. We discussed basically the cases when these complementizers are followed by a proper name. In fact this is not necessary: they can be followed by virtually any NP. Thus we can have generalised determiners like *more... than most students, the same... as some philosophers, different ... than the ten teachers*, etc. The presentation of semantics of such determiners necessitates the generalisation of the notation aR to the notation $Q_{nom}(R)$, where Q is a type $\langle 1 \rangle$ quantifier, a denotation of an NP. $Q_{nom}(R)$, the nominative extension of Q is defined as follows:

(30) $Q_{nom}(R) = \{x : Q(Rx) = 1\}$, where $Rx = \{y : \langle y, x \rangle \in R\}$

The nominal extension of a type $\langle 1 \rangle$ quantifier is different from its accusative extension. The difference can be illustrated by the following example. Let $Q = MOST(S)$. Then $MOST(S)_{nom}(R)$ is the set of objects to which most Ss are in the relation R whereas $MOST(S)_{acc}(R)$ is the set of objects which are in the relation R to most Ss.

To see the usefulness of the notation $Q_{nom}(R)$ consider the following example in (31). The generalised determiner *the same... as most students* occurring in (31) denotes the function $THE\text{-}SAME_{MOST(S)}$ given in (32) :

(31) Leo knows the same languages as most students.

(32) $THE\text{-}SAME_{MOST(S)}(X, R) = \{y : yR \cap X = MOST(S)_{nom}(R) \cap X\}$

The extension of the complements of *than* (or of *as*) enriches, though somewhat trivially, all the classes of generalised comparative determiners we have considered. There is, however one class of comparative functions formally defined in the preceding section, which do not seem to be denoted in natural languages: these are co-intersective functions.

Some superlative constructions also give rise to generalised comparative determiners. It is enough for our purpose to consider that superlatives correspond to specific conjunctions of comparatives: roughly *the oldest man* is the man who is older than m_1, and older than m_2... and older than m_n. So roughly speaking, to get a superlative we form a conjunction in which conjuncts are the complements of *then*. We will indicate implicitly such a conjunction by the expression *than anybody else* or *anything else*. Consider now (33a), its semantics in (33b) and its superlative counterpart in (34):

(33)a. More teachers danced than sang.
b. $MORE(T)(D)THAN(S) = |T \cap D| > |T \cap S|$

(34) More teachers danced than did anything else.

By applying the above idea of conjunction of comparatives we get (35):

(35) $MOST(T)(D) = 1$ iff $\forall X(X \cap D = \emptyset) \rightarrow |T \cap D| > |T \cap X|$

The equivalences in (36) and (37) are easy to be proved. From them and (35) follow the two equivalences in (38):

(36) For all sets X and Y, $|X \cap Y| > |X' \cap Y|$ iff $\forall Z(X \cap Z = \emptyset) \rightarrow |X \cap Y| > |Z \cap Y|$
(37) For any set X, Y, $|X| = |X \cap Y| + |X \cap Y'|$
(38) $MOST(T)(D) = 1$ iff $|T \cap D| > |T \cap D'|$ iff $2 \times |T \cap D| > |T|$

Thus the superlative associated with the comparative construction in which a binary determiner occurs is the "classical" determiner *most (of)* denoting $MOST$. It is not a genuine generalised determiner. However, a "generalised superlative determiner" can be associated with the comparative construction in (39). Its superlative counterpart is given in (40a) and its denotation is given in (40b):

(39) Leo read more books than Lea.
(40)a. Leo read more books than anybody else.
b. $NSUP(X, R) = \{x : \forall y(y \neq x \rightarrow |xR \cap X| > |yR \cap X|\}$

The function $NSUP$ is denoted by the following generalised determiners, supposedly equivalent, *the most, more than anybody else* or *the greatest number of.* Interestingly, this function is not anaphoric: using fact 2, one shows that it does not satisfy D1AC. Informally, suppose that Leo studies precisely those languages that he knows. It does not follow from this that *Leo studies the greatest number of languages* is equivalent to *Leo knows the greatest number of languages.*

To conclude this section I give an example of a non-numerical comparative and superlative constructions which contain a generalised determiner. Consider (41) which contains the determiner *an older... than Lea*: Sentence (41) has two readings: absolute, in (42a) and relative, in (42b):

(41) Leo hugged an older woman than Lea.
(42)a. Leo hugged a woman older than Lea.
b. Leo hugged a woman that was older than a woman hugged by Lea.

When the determiner is used in a DP in subject position, as in (43), the relative reading disappears:

(43) An older woman than Lea was dancing.

Any gradable adjective introduces a (total) relation; in the above case we have the relation O corresponding to *be older than.* Consequently in the semantics of (41) two type $\langle 1, 2 : 1 \rangle$ functions are involved:

(44)a. $F_{O,l}(X,R) = \{x : \exists y(y \in xR \cap X \wedge \langle y,l\rangle \in O\}$
b. $F_{O,l}(X,R) = \{x : \exists y,z(y \in xR \cap X \wedge z \in lR \cap X \wedge \langle y,z\rangle \in O\}$

Only the function in (44b) is a genuine comparative type $\langle 1,2:1\rangle$ function.

There may be some problems with the above example since it involves some unicity conditions. The absolute and relative readings are better seen in superlative constructions (the classical paper concerning this problem is Szabolcsi 1986: (45) is ambigous with the two meanings given respectively in (46a), absolute reading, and (46b), relative reading:

(45) Leo hugged the oldest woman

(46)a. Leo hugged a woman who was older than any other woman
b. Leo hugged a woman who was older than any other woman hugged by anybody else.

Clearly the different readings of the superlatives in (46a) and (46b) are related to the different comparatives from which they originate. Thus an explanation of these different readings is easy to conceive along the lines here proposed. In particular one observes that when the superlative occurs in subject position it can have only the absolute reading as in (47):

(47) The oldest woman lives in Japan.

The above observation suggests that superlatives with relative readings are not related to an accusative extension of (the denotation of) any noun phrase, and thus in particular the superlative itself cannot be considered as an "ordinary" noun phrase denoting a type $\langle 1\rangle$ quantifier. This is indeed the case since it does not satisfy the AE. We show this informally, just using English examples, by showing that the corresponding "comparative-anaphoric" form of superlative, the one given in (46b) does not satisfy AE. Suppose that (48) holds:

(48) The persons that Leo hugged are the same as those that Bill kissed.

In (48) we have an instance of the conditional part of the AE condition. One observes now that, given (48), the sentence in (49a) needs not hold the same truth value as the one in (49b):

(49)a. Leo hugged a woman who is older than any other woman hugged by anybody else
b. Bill kissed a woman who is older than any other woman kissed by anybody else.

On the other hand the function interpreting the absolute reading of the superlative does satisfy the AE condition : from (48) follows the identity of truth values between (50a) and (50b) :

(50)a. Leo hugged a woman who is older than any other woman.
b. Bill kissed a woman who is older than any other woman.

The generalised determiner we have in (45) corresponds to the expression *the oldest* when it forms the superlative DP which occurs in the object position gives rise to the relative reading. It denotes the function in (51):

(51) $F_{O,a}(X, R) = \{x : \exists y(y \in xR \cap X \wedge \forall z(z \neq y \wedge z \in aR \cap X) \rightarrow \langle y, z\rangle O\}$

In the next section I discuss some binary comparative determiners.

4 Some binary generalised determiners

Recall that natural languages have binary or even n-ary determiners, that is expressions which take two or n common nouns to form a DP. As Keenan and Moss 1985 noted, n-ary determiners can be easily obtained by the conjunctions of common nouns in the "syntactic" scope of an unary determiner:

(52) Most students, teachers and priests were sleeping.

This sentence probably means that most students and most teachers and most priests were sleeping and not that most individuals which are students, teachers and priests ("at the same time") were sleeping. Under this reading the determiner *most..and...and...* is a ternary determiner.

Deterniners taking many common nouns as arguments as illustrated by the above example have an obvious property: their denotations are Booleanly reducibly to a conjunction of denotations of unary determiners (see Keenan and Moss 1985). It has been observed, however that natural languages have also binary determiners whose denotations are not reducible in that sense (Keenan and Moss 1985, Beghelli 1994). For instance, the quantifier denoted by the binary determiner in (10) above is not Booleanly reducible (cf. Beghelli 1994).

Of course binary determiners occurring in DPs in object position can be considered as generalised binary determiners: if D_2 is a type $\langle\langle 1, 1\rangle 1\rangle$ quantifier then the function $F((X_1, X_2, R) = D_2(X_1, X_2)_{acc}(R)$ is a well-defined type $\langle 1, 1, 2 : 1\rangle$ function. In what follows I indicate, however, that natural languages have genuine generalised binary, or even n-ary, comparative determiners. Though the notion of Boolean reducibility of n-ary determiners will not be made more precise, it will be intuitively clear that such determiners can be either Boolean reducible or Boolean irreducible. Moreover, functions denoted by these determiners have similar properties to the functions denoted by unary determiners: they are "at least" conservative and satisfy the condition of argument invariance.

We need first to define various properties of functions denoted by binary determiners, similar to those which have functions denoted by unary generalised determiners. Such properties are well-defined for "ordinary" binary or n-ary determiners (cf. Keenan and Moss 1985, Beghelli 1994, Zuber 2005, Zuber 2009). I give here some such definitions for type $\langle 1,1,2:1\rangle$ functions. For conservativity we have the definition11 and the proposition 5 (Zuber 2010a):

Definition 11. *A type $\langle 1,1,2:1\rangle$ function F is conservative iff for any $X_1, X_2 \subseteq E$ and any binary relations R_1 and R_2, if $E \times X_1 \cap R_1 = E \times X_1 \cap R_2$ and $E \times X_2 \cap R_1 = E \times X_2 \cap R_2$ then $F(X_1, X_2, R_1) = F(X_1, X_2, R_2)$.*

Proposition 5. *A type $\langle 1,1,2:1\rangle$ function F is conservative iff for any $X_1, X_2 \subseteq E$ and binary relation R one has $F(X_1, X_2, R) = F(X_1, X_2, (E \times (X_1 \cup X_2)) \cap R)$.*

Since many of the determiners we will present denote cardinal functions, here is the corresponding definition:

Definition 12. *A type $\langle 1,1,2:1\rangle$ function is cardinal iff $F(X_1, Y_1, R_1) = F(X_2 Y_2, R_2)$ whenever $\forall y(|X_1 \cap yR_1| = |X_2 \cap yR_2|)$ and $\forall x(|Y_1 \cap xR_1| = |Y_2 \cap xR_2|)$*

Finally, the condition of argument invariance for type $\langle 1,1,2:1\rangle$ functions is formulated as follows:

Definition 13 (D2AI). *A function F of type $\langle 1,1,2:1\rangle$ satisfies* argument invariance condition for binary determiners *(D2AI) iff for any $a, b \in E$, $X, Y \subseteq E$ and R a binary relation, if $a((E \times X) \cap R) = b((E \times X) \cap R)$ and $a((E \times Y) \cap R) = b((E \times Y) \cap R)$ then $a \in F(X, Y, R)$ iff $b \in F(X, Y, R)$.*

Let us see now some examples of binary generalised determiners which denote conservative argument invariant functions. Consider sentence (53): one of its readings is given in (54a). In (53) we have a binary generalised determiner which denotes an instance of the type $\langle 1,12:1\rangle$ function given in (54b):

(53) Leo read more books and articles than Lea.

(54)a. Leo read more books than Lea and more articles than Lea.

b. $MORE_{i,j}(X, Y, R) = \{y : |yR \cap X > |iR \cap X| \wedge |yR \cap Y > |jR \cap Y|\}$

It is easy to show that function $MORE_{i,j}$ is cardinal and thus conservative. It is also argument invariant. The fact that (34a) is equivalent to (34b) qand that we have a conjunction in (35) shows that this function is Booleanly reducible. Furthermore, since the number of CNs which can occur as conjuncts in (34a) is not limited, example (34a) shows how to construct n-ary comparative determiners.

A binary generalised determiner denoting the non-reducible type $\langle 1,1,2:1\rangle$ function is given in (55a); the corresponding function is given in (55b):

(55)a. Leo read more books than Lea articles.

b. $MORE_{2,i}(X, Y, R) = \{y : |yR \cap X| > |iR \cap Y|\}$

Using the method similar to the one used in connection with the example (24) above one shows that the function in (55b) is cardinal and argument invariant.

We get similar examples be replacing in the above examples *more* by *less* or *the same number of*. Similarly we can make some Boolean compounds as in (56), where we also have a generalised binary determiner denoting a cardinal function:

(56) Leo read more books than Lea articles but less than 17 altogether.

Consider finally (57a) and the function in (57b) which is denoted by the generalised determiner *three times more... than Lea...*:

(57)a. Leo read at least three times more books than Lea articles

b. $MORE_{3\times,i}(X,Y,R) = \{y : |yR \cap X| \geq 3 \times |iR \cap Y|\}$

The type $\langle 1,1,2:1\rangle$ function in (57b) is cardinal.

5 Conclusive remarks

By analogy with anaphoric determiners I distinguished a subclass of generalised determiners called *comparative determiners*. A generalised determiner is an expression which when applied to one or more common nouns forms a generalised DP, that is an expression which can serve as argument of a verb phrase. A genuine generalised DP is an expression which cannot serve as the grammatical subject of a sentence. Anaphoric determiners (studied in Zuber 2010a, 2010b) are genuine generalised determiners because when applied to a common noun they form nominal anaphors which are verbal arguments which cannot occur in subject position. Since I was basically interested in the logical properties of comparative determiners, no attempt has been made to justify their category syntactically. From the logical point of view it appears that their denotations have many striking similarities with denotations of "ordinary" determiners: for instance, they are conservative in a naturally generalised sense. Furthermore, as with ordinary determiners, there are not only unary generalised determiners but also n-ary ones.

Semantically, comparative DPs, in the same way as nominal anaphor, cannot be interpreted by functions corresponding to generalised (type $\langle 1\rangle$) quantifiers denoted by ordinary NPs. Formally this amounts to saying that denotations comparative DPs do not satisfy the specific invariant condition for type $\langle 1\rangle$ quantifiers given in called accusative extension condition. They satisfy, however the strictly weaker condition given of argument invariance. Consequently one can say that comparative (generalised) DPs essentially augment the expressive power of, say, English, since the expressive power of English would be less than it is if the only noun phrases we need were ones interpretable as subjects of main clause intransitive verbs. The reason is that such DPs must be interpreted by functions from relations to sets which lie outside the class of generalised quantifiers as classically defined, that is type $\langle 2:1\rangle$ functions which are not extensions of type $\langle 1\rangle$ quantifiers.

It might be interesting to notice the analogy with nominal anaphors. Keenan (1987, 1988, 2007) shows that something similar is true because of the existence of nominal anaphors. More specifically, anaphors like *himself, herself* (considered as the second nominal argument of transitive verbs) also must be interpreted by functions which do not satisfy the AE, and thus the generalised type $\langle 1\rangle$ quantifiers are not enough for their interpretation. Anaphoric functions interpreting nominal anaphors satisfy another weakening of the AE, the condition AC.

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