Australian Government

Department of Education,
Science and Training

INVESTIGATION OF EFFECTIVE MATHEMATICS TEACHING AND LEARNING IN AUSTRALIAN SECONDARY SCHOOLS

March 2004

Lawrence Ingvarson
Adrian Beavis
Alan Bishop
Ray Peck
Gerald Elsworth

This report has been prepared for the Australian Government Department of Education, Science and Training by

ACER

Australian Council for Educational Research

ISBN: 0 642 77468 4 (web version)

CONTENTS

FIGURES

TABLES

EXECUTIVE SUMMARY

This report describes a study into the factors associated with effective mathematics teaching.

The study aimed to assess a range of factors that were theorised to be associated with effective mathematics teaching. These included:

1. The knowledge, beliefs, understandings and practices of teachers;

2. Teacher qualifications and educational background; and

3. The extent and quality of professional development experienced by the teachers.

These factors, and others identified by a literature review were organised into a theory which in broad outline proposed that:

1. Effective teachers had a positive effect on a range of student outcomes;

2. Teacher practices in the classroom directly affect student outcomes;

3. Teacher practices in the classroom are directly affected by the capacity of the teacher – the quality and field(s) of their training and qualifications; and

4. Teacher capacity is directly affected – amplified or muted –by conditions within the school. In particular, the school's professional community was theorised to be of prime importance.

Data were collected from Principals, teachers and students. The Principals completed a questionnaire about enabling conditions in the school and the wider community in which the school was located. Teachers completed a questionnaire which provided information about their gender, educational background, their experiences of professional development and of teaching. Teachers were also asked to complete a set of classroom 'scenarios' that provided a measure of their mathematical content and pedagogical knowledge. These scenarios were designed so that teachers were required to complete mathematical tasks, and to identify and generalise student errors. These scenarios should be seen as a significant outcome of this study. Students were asked to complete a questionnaire and a *PATMaths* test. These were administered at two different times. The student questionnaire administered at Time 1 aimed to provide measures of various affective (and related) outcomes of mathematics teaching including: perceived effort, ability, task load, utility of mathematics, self efficacy, enjoyment, motivation, and the quality of their learning environment. The student questionnaire administered at Time 2 included the same set of items as at Time 1, but it also included an additional set of items asking them to report on aspects of their mathematics teacher's classroom practice. These practices were identified by reference to the standards developed by Monash University and the Australian Association of Mathematics Teachers (AAMT). Some of the variables constructed from these items proved to be strongly associated with effective mathematics teaching.

Interviews were also conducted at a small number of schools. Principals, Heads of Departments and teachers were interviewed.

A total of 50 schools agreed to participate in the study. Around half of these schools had participated in the Organisation for Economic Co-operation and Development's (OECD) Program for International Student Assessment (PISA) study in 2000 (Lokan, Greenwood, & Cresswell, 2001). Data from this study were used to identify those schools with a high average score in mathematical literacy, and those schools with a low average score. Thus, a wide spread of school contexts was sampled for the study. A total of 206 teachers provided data for the study. At Time 1, 7709 students (2663 at Year 8, 2562 at Year 9 and 2484 at Year 10) were surveyed and tested. At Time 2, 2684 (1123 at Year 8, 1028 at Year 9 and 533 at Year 10) were surveyed and tested. There was an average of 123 calendar days (not 'school' days) between Time 1 and Time 2.

The main findings of the study were:

1. Teaching practice (as defined by standards identified by *Monash University* and the *AAMT*) reported by students has a consistently important effect on affective outcomes of mathematics teaching;

2. Teacher knowledge and educational background is positively, but weakly related to teacher effectiveness. The more this education has to do with mathematical content and pedagogy, the more likely it is that teachers will be effective; and

3. The effectiveness of mathematics teaching in a school is related to the strength of professional community in the school's mathematics departments.

1 INTRODUCTION

In 2001 the Australian Government commissioned the Australian Council for Educational Research (ACER) to conduct an investigation of effective mathematics teaching and learning in Australian secondary schools (Years 7 to 10).

The purpose of the research project was to examine a range of factors including the knowledge, beliefs, understandings and practices of teachers of mathematics and their qualifications, professional development and relevant personal experiences, and how these impact on student learning outcomes in the high school years.

Main Stages in the Project

The study lasted 24 months and consisted of four main stages:

1. The literature review;

2. Assessments of student learning in mathematics;

3. A survey-based study which aimed to identify the key factors that contribute to effective teachers of mathematics, drawing upon information taken from a nationally representative sample of schools; and

4. Case studies which aimed to illuminate and explore issues surrounding effective mathematics teaching in schools. The case studies were designed to complement the survey-based part of the study.

This report describes each of these four stages of the study. It also provides: (a) a description of the theory guiding the study; (b) a list of the key research questions; (c) a description of the methods used to gather a wide range of data needed to address these questions; (d) the results taken from these data; and (e) an account of the implications of the findings for understanding teacher effectiveness.

Audience

The intended audience of this report are Directors General of education, universities, professional bodies, teachers, principals, and policy makers concerned with, in particular, mathematics teaching in Australian schools.

A preliminary description of the study’s theory

Since the research was concerned to identify the characteristics of effective mathematics teachers, it focuses upon teacher attributes and qualities. However, all teachers work within a context set both by system level requirements, the demands of the school in which they work and the characteristics of their students. Consequently, any theory of teacher effectiveness needs to consider these different contexts and the opportunities they offer (or the limitations that they impose).

Figure 1 shows that there are four main factors theorised to influence student outcomes in mathematics: (a) conditions in the school where the students are located – school enabling conditions; (b) the experience of teachers (especially in their professional development) – teacher enabling conditions; (c) the capacity of the teachers; and (d) what teachers do in their classroom – teacher practice. In Figure 1 these factors are ordered into a sequence such that factors displayed to the left generate outcomes to the right. For example, ‘school enabling conditions’ were theorised to shape ‘teacher enabling conditions’.

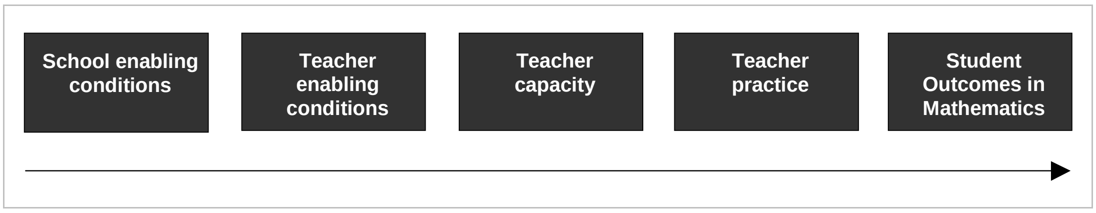


Figure 1 Graphical depiction of core elements of the theory guiding the research

Three influences on teacher practice are proposed in the theory:

1. The quality of the teachers in terms of background training, experience, subject matter qualifications and teacher education ('teacher capacity');

2. The quality and amount of professional development in which teachers have engaged over recent years ('teacher enabling conditions'); and

3. The quality of the professional community in which teachers work: for example, the ethos of the work group, opportunities for collegial work and leadership support within the school ('school enabling conditions').

The focus in this study was upon the teachers. The school context provides a starting point for understanding the support available to teachers. The student outcomes function as criteria for assessing the extent to which a teacher has been successful in improving student performance. Thus, the study was *not* concerned primarily with student outcomes, but rather with understanding what makes more or less effective teachers of mathematics.

A preliminary description of the method

The investigation of effective mathematics teaching was undertaken using a variety of instruments and techniques to try to capture both the context in which teaching was taking place as well as to provide reliable estimates of measurable processes and outcomes. To achieve this mix of data, a variety of data sources was used.

Sampling

A sample of schools was selected from the population of schools that participated in the OECD PISA study, for which data were collected in 2000. Around 230 schools participated in PISA for Australia. An analysis of the student achievement scores from these schools was undertaken using publicly available data. Those schools with the highest and those with the lowest school average on the PISA mathematics tests were selected and approached to participate in the study. (The estimation of the highest and lowest scoring schools controlled for the effect of a range of socio-economic background and school level factors. In this way the unique effect of the school on student achievement was identified.) This approach was designed to help identify schools where effective teaching was likely to be taking place, and those where it was less likely to be identified.

Sources of data

Survey instruments were sent to school principals (or, where principals preferred, to the Heads of mathematics departments), teachers and students. Students were also given a series of tests in mathematical ability.

A series of case studies were also conducted at selected schools. These case studies provided qualitative data for illustrating and illuminating issues.

Principal or Head of Mathematics Department Questionnaire

To obtain information at the school level a survey form was sent to each participating school in the study. Appendix 1 shows a copy of this instrument.

Teacher Questionnaire

To obtain information at the classroom level, a survey form was sent to six teachers of mathematics in each participating school in the study. Students from these teachers' classes also completed mathematics tests and questionnaires about their attitudes to mathematics (see below). There were two parts to the teacher questionnaire: Form A asked a range of questions about each teacher's background and their practice. Form B provided three scenarios focussing on teaching practice in mathematics. Appendix 1 shows a copy of these instruments.

Student tests and questionnaires

To obtain information at the student level, data were collected from up to six classes, usually two from each of Years 8, 9 and 10 in each participating school in the study. Students provided data collected at two time points. These data were designed to provide measures of change in a range of outcome variables, including their mathematics achievement.

Students completed either a Form 2B or 3B of the *PATMaths* test. Students were also asked to complete a questionnaire at each time they were tested. The first version of the questionnaire consisted of 52 questions. The second version of the questionnaire consisted of the same 52 questions, with an additional 34 questions. Appendix 1 shows a copy of the second version of this instrument.

Both versions of the student questionnaire provided measures of: (a) the perceived utility of studying mathematics; (b) student self-efficacy in mathematics; (c) enjoyment of mathematics; and (d) student motivation for mathematics. The second version also had questions asking about the student's experience of their mathematics teacher and classes based on the AAMT standards for exemplary teachers (1997).

Description of the structure of the report

This report aims to describe the results of this preliminary study into effective mathematics teaching in Australia. To achieve these aims, the report: (a) considers existing literature on effective mathematics teaching; (b) develops a theoretical framework which was used to guide the conduct of the study; (c) describes the methods used for the implementation of the study; (d) reports on the results of the statistical analyses conducted; (e) details the findings from a series of case studies undertaken as part of the study; and (f) provides a discussion of the findings and the light that they throw upon understanding effective mathematics teaching.

2 LITERATURE REVIEW

This literature review examined national and international research on factors contributing to effective mathematics teaching and learning in secondary schools (Years 7–10). It provided the basis for the development of a theoretical framework about factors affecting the quality of mathematics teaching and learning investigated later in the study. The brief for the review called for particular attention to be given to the knowledge, beliefs, understandings and practices of effective teachers, as well as the influence of qualifications, professional development and relevant personal experiences.

Structure of the literature review

The literature review begins by considering classroom factors affecting students' opportunity to learn and moves outwards to examine the influence of the wider school and system context on teacher effectiveness. It covers the following areas:

1. Student opportunity to learn, including aspects of: (a) teaching practices; (b) student activities; (c) student engagement and the learning environment; and (d) feedback.

2. Teacher and school level factors, including aspects of: (a) qualifications; (b) experience of professional development; (c) teaching capacity; (d) leadership; (e) program coherence; (f) resources; (g) the quality of the professional community in the school; (h) school characteristics; and (i) classroom characteristics.

3. System factors including: (a) policies on teacher qualifications; and (b) time allocated to mathematics teaching and learning.

Student Opportunity to Learn

The opportunity to learn mathematics effectively is dependent upon a wide range of factors, but among the most important are those which are related to activities and practices within the classroom. This is reflected in the focus of a number of important studies. For example, the *Effective Teachers of Numeracy Study* (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997) focussed strongly on the classroom and what happens there. Similarly, the *Third International Mathematics and Science Study* (TIMSS) focused on the classroom (Martin, Mullis, Gregory, Hoyle, & Shen, 2000).

Within the classroom, it is possible to discern a number of key elements. These include: (a) teaching practices; (b) the nature of student learning activities; (c) the amount and nature of engaged learning time experienced by students; (d) the learning environment; and (e) the scope and nature of the feedback given to students.

Teaching practices

Teaching practices are central to understanding what makes for effective teaching. Peterson's (1988, p. 21) list of effective teaching practices included:

1. a focus on meaning and understanding mathematics and on the learning task;

2. encouragement of student autonomy, independence, self-direction and persistence in learning; and

3. teaching of higher-level cognitive processes and strategies.

Hanna (1987) identified successful teaching strategies as requiring an organised approach to teaching, where material was taught until it was mastered. Brophy and Good (1986) reported similar findings. They argued that in classroom instruction three modes exist: (a) giving information; (b) soliciting information; and (c) providing feedback. In providing information, for example, an effective teacher requires an approach which structures the information so that the lesson forms a coherent whole; one which relates previous work to new material. To do this well requires clarity of presentation and good sequencing of information.

Student actions within and their reactions to what occurs in the classroom have also been identified as important determinants of teacher practice (Cooney, 1985; Pehkonen, 1997; Raymond & Leinenbach, 2000; Reid, 1997; Tomazos, 1997).

Student learning activities

Frameworks for effective teaching to support students' conceptual understanding need to emphasise the need for tasks that are mathematically challenging and significant (Askew, Brown, Denvir, & Rhodes, 2000; Fraivillig, 2001). The conceptual difficulty of the task and the teacher's use of this difficulty in 'pressing' the children to think, according to Kazemi (1998), results in activities where the mathematics drives the teaching and learning to the extent that, Kazemi claims, higher student achievement and conceptual understanding results. Groves and Doig (2002, submitted) contend that, in general, insufficient attention is being paid to "the critical role of conceptually focussed, robust tasks that can be used to support the development of sophisticated mathematical thinking".

Engagement

Student engagement, both its depth and extent, has come under scrutiny as a factor affecting student achievement (Nickerson, 1988; Stigler & Perry, 1988). Engagement is probably related to classroom environment. Gore (2000, p.128), for example, argues that the classroom environment needs to be supportive of learning, and this entails engaging students (as well as setting high expectations, encouraging students to be self-regulating, and articulating the criteria for the quality of students' work).

Feedback

In a major review of the research Hattie (1992) found that features related to teachers and teaching were the most important. In particular, the review showed that four attributes stood out as characteristics of effective teachers:

1. Feedback: expert teachers provide timely and useful feedback (as distinct from praise) to enhance student learning;

2. Challenge: expert teachers involve students in challenging tasks – challenging relative to the student's current level of achievement;

3. Structure: expert teachers structure classroom activities to allow an increased probability that feedback will occur. They also structure activities that permit students to engage in challenging tasks; and

4. Management: expert teachers are excellent managers of classrooms and they are able to engage and teach all students in the class, avoiding the temptation to 'teach to the middle'.

While these attributes appear to be generic, in reality, the capacity to implement them necessarily depends on a deep understanding of what is being taught. It is very difficult for teachers to run lively and effective classroom discussion, in which they respond to and build on students' ideas, and provide timely and appropriate feedback, without deep knowledge of the mathematics that is being taught and how students learn it. For example, Brophy (1990) pulls together a wide range of research that shows how dependent effective pedagogy is on deep knowledge of the content to be learned. Stein (2001) takes up this issue further in her review, pointing out that "it is now commonly accepted that a productive mathematics classroom is one where there is a great deal of talk" (p. 127). She means talk of a special kind – that allows students to grapple with ideas, and to take up positions and defend them. As she argues, effective mathematics teachers "can set up opportunities for mathematical argumentation in their classrooms by selecting tasks that have different solutions or that allow different positions to be taken and defended" (p. 129).

Finally, it should be noted that the *Second International Mathematics Study* (SIMS) was the last large-scale survey to publish results of its examination of 'teaching style', a facet of which was feedback (Robitaille (1993), cited in Bodin and Capponi (1996)). Feedback to students was seen to be one of the most important aspects of teaching style.

Teacher Factors

In posing the rhetorical question *What matters for teacher effectiveness?* Darling-Hammond and Ball (1998) answered that what matters is "teacher knowledge of subject matter, student learning and development, and teaching methods".

The characteristics of effective teachers that are considered here are those that are amenable to change and which have an impact. For this reason, the focus in the following sections is on initial teacher education and teacher learning through professional development.

Wenglinsky (2000a), who used multi-level structural equation modelling with data from the US *National Assessment of Educational Progress* (NAEP) program, found that:

1. Students' performance was higher when their teacher had majored or minored in the subject they were teaching;

2. Students performed at a higher level if their teacher received professional development in working with different student populations; and

3. Students whose teachers had received professional development in higher-order thinking skills outperform students whose teachers had not.

Findings that were not significant included:

1. Number of years teaching experience;

2. Whether the teacher had obtained a Masters degree or higher; and

3. Time spent on professional development over the last year on classroom management, cooperative learning, on-going forms of assessment, and interdisciplinary instruction.

Darling-Hammond and Ball (1998), in a review of research and practice in teacher education, found that reform strategies in schools that did not make "substantial efforts to improve teaching" were "less successful" (p. 23) than those that did. Further, they found that teachers who spent more time studying teaching were more effective overall, and "strikingly so in developing higher-order thinking skills" (p. 4).

Initial teacher education

Zeichner and Tabachnick (1981) found that pre-service teacher education typically has a weak effect on teachers' knowledge and beliefs. The authors point out that this may be due to the many hours of observations of other teachers made by many prospective teachers. They argue that this both instils traditional images of teaching and learning and shapes understanding of mathematics.

In Wenglinsky's (2000b) study of 40,000 prospective teachers, scores on the *Praxis* teacher licensing test were correlated with data on characteristics of 152 teacher education institutions that the teachers had attended, particularly whether they were 'accredited' by the National Council for Accreditation of Teacher Education (NCATE). The study showed that students graduating from NCATE accredited teacher education programs scored significantly higher on the Education Testing Service Praxis II tests of teacher knowledge than students from non-NCATE accredited institutions. Using this approach, it was possible to identify characteristics of the teacher-training institutions attended by effective teachers (where effective was measured by scores on the licensing examination). Askew's (1997) *Effective Teachers of Numeracy Study*, conducted in England, found that neither mathematical qualifications nor initial training were associated with teacher effectiveness. Preliminary results from the *Leverhulme Numeracy Research Programme* (Brown, 2000) confirm some of the key results of the *Effective Teachers of Numeracy Study*, including that higher teacher qualifications in mathematics appear to have no impact on a teacher's effectiveness.

Professional Standards

In some education systems, standards for teachers have been established to promote professional growth in education. These standards can also help to identify factors associated with effective teaching. The Ontario College of Teachers is the self-regulatory body for the teaching profession in Ontario (The Professional Affairs Department, 1999). In 1999, it produced *Standards of Practice for the Teaching Profession.* The goal of this document was to answer the question, *What does it mean to be a teacher?* The Ontario College of Teacher's answer was that it expected a teacher to demonstrate: (a) care for and commitment to students; (b) equity and respect in their treatment of students; and (c) encouragement of students' individuality and their contributions to society. In order to demonstrate professional knowledge, the College asserted that teachers must know the curriculum, the student, the subject matter, teaching practice, and education-related legislation. Professional knowledge of these factors, it was argued, would allow Ontario teachers to promote student learning and to be educational leaders who would collaborate with their colleagues and other professionals, parents, and the wider community.

The Department for Education and Employment in the United Kingdom set six standards for advanced teachers in that country (Department for Education and Employment, 1998). The Advanced Skills Teacher (AST) model requires excellence in: (a) student achievement; (b) subject knowledge; (c) planning; (d) teaching, managing pupils and maintaining discipline; (e) assessment; and (f) advising and supporting other teachers. Teachers are assessed through a portfolio, classroom observation, an interview, consultation with the teacher's principal, and references from colleagues. Under this model, excellent teachers must be up-to-date in their specialisation, fully understand the connections within their subject, and understand information communication technology (ICT). Evidence of their ability to plan is demonstrated by lesson plans that are assessed as having clear objectives and high expectations of the students. They are required to plan their teaching to build on students' current and previous work. The ability to teach is judged on the extent of flair, creativity, enthusiasm and challenge the teacher presents in class. In addition, the teacher is expected to use questioning and explanation skilfully, develop literacy, numeracy and ICT skills as appropriate, and be able to support students in need of additional assistance. They are also expected to maintain respect and discipline and be consistent and fair. Effective teachers are required to use assessment diagnostically, as an aid to planning, and to improve their teaching. They are expected to support and coach other teachers, to have highly developed interpersonal skills, be a role model for other staff in their personal and professional conduct, be effective managers of time, and be highly respected and able to motivate others. It is claimed that the students of excellent teachers demonstrate high outcomes in relation to prior attainment, are highly motivated and enthusiastic, exhibit high standards of discipline and behaviour, and, finally, have this reflected in a consistent record of parental involvement and satisfaction.

Martin (2001) describes the Western Australian standards, where advanced teachers are called 'Level 3 Teachers'. They are assessed on five competencies:

1. Level 3 teachers are expected to utilise innovative and/or exemplary teaching strategies which promote high levels of student participation and involvement. They should recognise and respect difference and diversity, and be sensitive to matters of gender, culture, class, disability, family circumstance and individual difference;

2. Level 3 teachers should employ consistent exemplary practice in developing and implementing student assessment and reporting processes. Assessment processes must be transparent to the students. These assessments need to be varied, inclusive of all students, provide useful feedback to students, be the basis of teaching modifications, be reported to parents, involve technology, and be consistent with policy;

3. Level 3 teachers must engage in self-development activities to critically reflect on their own teaching practice and on teacher leadership;

4. Level 3 teachers ought to enhance other teachers' professional knowledge and skills through conducting professional development sessions, mentoring, and supporting colleagues; and

5. Level 3 teachers are expected to provide high-level leadership in the school community. They should be involved in curriculum planning and management, and school policy formulation. They ought to have effective team building and negotiation skills, and be able to create an environment that values, encourages and respects diverse working styles.

The National Board for Professional Teaching Standards (NBPTS) (National Board for Professional Teaching Standards, 2000) offers a wide variety of certificates for excellent teachers which use standards-based performance assessments. All of the NBPTS Standards emphasise that accomplished teachers should be aware of what they are doing as they teach and why they are doing it. They are expected to be conscious of where they want student learning to go and how they want to help students get there. Furthermore, they must monitor progress towards these goals continuously and adjust their strategies and plans in light of feedback. Accomplished teachers are expected to set high and appropriate goals for student learning, connect worthwhile learning experiences to those goals, and articulate the connections between these goals and experiences. They must be able to analyse classroom interactions, student work products, and their own actions and plans in order to reflect on their practice and continually renew and reconstruct their goals and strategies.

Several major studies have confirmed the validity of the NBPTS (Bond, Smith, Baker, & Hattie, 2000; Silver, Mesa, Benken, & Mairs, 2002). According to Bond et al. (Bond, Jaeger, Smith, & Hattie, 2001), teachers who were certified by the NBPTS significantly outperformed peers who had sought but not achieved certification on 11 of 13 key dimensions of teaching expertise. Further, 74% of students taught by NBPTS teachers reflected a high level of comprehension of the concepts taught, compared to 29% in a control group. Silver et al. (2002) found that mathematics teachers who had been certified by the NBPTS used more challenging assessment activities that probed and promoted higher order thinking in students.

The quantity and depth of mathematics needed by teachers

Thompson (1992) claims that "the influence of teachers' conceptions [of mathematics] on their practice … cautions against reducing teacher preparation training on specific, well-defined skills and competencies", but the work of Askew *et al.* (1997, pp. 60-2), suggests that teachers with lower mathematical qualifications were more effective than teachers with higher levels of mathematical training. Ball et al. (2001) suggested that more coursework in mathematics is associated with a higher level of exposure to conventional teaching practices. These experiences may actually imbue teachers with pedagogical images and practices that hinder their teaching. For example, teachers with advanced mathematical training might not be able to 'unpack' mathematical content for students. Further, Darling-Hammond (2000), quoting Byrne (1983) who summarized the results of 30 studies relating teachers' subject matter knowledge to student achievement, claimed that the evidence is unclear. The results of the 30 studies were mixed, with 17 showing a positive relationship and 14 showing no relationship'. The report *Teacher Preparation Research: Current Knowledge Gaps and Recommendations*, prepared for the US Department of Education (Center for the Study of Teaching and Policy & Michigan State University (Wilson, 2001)), suggests that several possible explanations for this confusing finding bear further investigation, including the possibility that a teacher needs to understand subject matter knowledge from a pedagogical perspective. Monk (1994; 1994a) reported that teachers' content knowledge, measured by courses taken in mathematics during training, is positively related to student achievement but that there are diminishing returns in student achievement for teachers who studied more than a threshold level of five undergraduate courses in mathematics (Monk, 1994a, p. 130). Monk (1994) also found positive effects of mathematics education courses. Courses in undergraduate mathematics education contributed more to student achievement gains than did undergraduate mathematics courses.

In a major review of reviews on the topic, Wilson and Floden (2003, p. 14) concluded that:

> In the case of mathematics there appears to be a trend: teachers with mathematics or mathematics education degrees have students who demonstrate higher orders of achievement …. There might also be a threshold at which more mathematics knowledge does not help the teacher.

Several of the studies in the *Teacher Preparation Research* report (Center for the Study of Teaching and Policy & Michigan State University (Wilson, 2001)) showed a positive connection between teachers' subject matter preparation and higher student achievement in mathematics. However, undermining the view that the ideal preparation is a subject matter major, a number of studies reported complex and inconsistent results. The Report notes that there is some indication from research that teachers can acquire subject matter knowledge from various courses, including subject-specific academic coursework and study in an academic major. According to the Report, research suggests that changes in US mathematics teachers' subject matter preparation may be needed, given that pre-service teachers' knowledge of procedures and rules is often sound, while their knowledge of concepts and their reasoning skills may be weak. (This was found for both education and mathematics majors.) The Report argues that the solution is more complicated than simply requiring a major or more subject matter courses, a proposition supported by Ma's (1999) study. Ma documents substantial national differences in teachers' pedagogical content knowledge, contrasting the deep conceptual knowledge of Chinese primary school mathematics teachers with the shallow procedural understandings of US primary teachers.

Pedagogical content knowledge

Shulman (1987; 1986) argued that in addition to general pedagogical knowledge and content knowledge, teachers needed to make a link between the two. Berliner (1987, p. 92), too, has suggested that teaching has a special knowledge requirement, arguing:

> Classroom mathematics teaching … would seem to require that other domains of knowledge, besides content knowledge be brought to bear if the problems of classroom mathematics teaching are to be dealt with effectively. Some have argued that a personal knowledge of self is the key requirement (e.g. Lampert, 1984) and others have identified dozens of domains of knowledge that are drawn upon by teachers (e.g. Elbaz, 1981). This is a difficult conceptual problem when attempting to determine the sources of expertise in teaching. Our solution was simply to stipulate that there are two separate domains of knowledge that require blending in order for expertise in teaching to occur. These are 1. Subject matter knowledge, and 2. Knowledge of classroom organization and management, which we call pedagogical knowledge.

On this argument, pedagogical content knowledge assimilates aspects of teaching and learning with content. Romberg (1988, p. 228) discusses a range of studies, and comes to a similar conclusion saying:

> … what is clear is that the 'professional knowledge' of teaching should include at least three distinctly different but related categories:
>
> 1. Knowledge of the subject (mathematics) they are to teach to students and its relationship to other content (both within and outside of mathematics);
> 2. Knowledge of pedagogy, including an understanding of how students process, store, retain, and recall information and of how teachers contract with students for instruction, as well as including a knowledge of a variety of examples for each mathematical idea, knowledge of specific instructional techniques, and knowledge of instructional materials;
> 3. Knowledge of how to manage a complex instructional setting involving a large number of students, a variety of resources, space, and an increasingly complex instructional technology.

Pedagogical content knowledge, particularly an understanding of students' mathematical thinking, was taken as the basis for a teacher development program, *Cognitively Guided Instruction* (CGI), by Carpenter and Fennema in the early 1990s. The CGI program was based on research about the development of students' understanding of number 'standards' (National Council of Teachers of Mathematics (1989; 1995)). This approach requires that, teachers have a good knowledge of students' thinking and solution strategies when working on mathematical problems. Vacc and Bright (1999) have shown there is a positive correlation between children's achievement and a teacher's use of a CGI approach particularly for problem solving, solution of addition and subtraction word problems and recall of number facts.

Professional development

Hawley and Valli (1999) provide a set of nine "design principles for effective professional development", that reflects the shift from form to substance that has taken place over recent years. A useful review of relevant research and examples of programs based on those principles accompanies each principle. Their first principle emphasises that: *the content of professional development focuses on what students are to learn and how to address the* *different problems students may have in learning the material*. The second principle is based on research indicating that collaborative examination by teachers of their students' work in relation to standards has significant effects of student achievement: *professional development should be based on analyses of the differences between (a) actual student performance and (b) goals and standards for student learning*. The third flows from the first two principles and a growing confidence in an expanding knowledge base for teaching. If the first two principles are in operation, they help to define an agenda for a teacher's professional development. They imply a radical idea for the teaching profession; that professional development should be based in part on what teachers need – rather than want – to learn: *professional development should involve teachers in the identification of what they need to learn and in the development of the learning experiences in which they are involved*.

Kennedy (1998) found ten research studies over the previous 20 years that specifically examined the impact of professional development programs on student learning. These programs varied in terms of their impact on student learning and the permanence of the effects on teacher practices. Kennedy teases out the presumed links between teacher and student learning for each program (the program 'logic') and the factors that might explain why the strength of the links varies between programs. Some programs focused on training teachers in generic teaching behaviours, or methods like co-operative group work. Others focussed mainly on providing teachers with research-based knowledge, say, about how student understanding of number develops, but left teachers to use their teaching experience to devise appropriate teaching methods.

As she looked across the ten programs, Kennedy (1998) found that differences in program form did not account for differences in effects on student achievement. These forms included total contact hours, distributed time, in-class visitations, and whole-school approaches. Instead, what distinguished the most successful professional learning programs was the way each engaged teachers in the content of what was to be taught and provided research-based knowledge about how students learn that subject matter. The more successful professional development programs that focussed first on promoting specific pedagogical practices were more likely to fade with time, because they did not deepen teachers' understanding about the content and how students learn it.

For example, one of the most successful programs in Kennedy's (1998) review was the CGI program. Developed after extensive research in the 1980s, the CGI program enhances teachers' understanding of research on the development of children's conception of whole number operations involving single-digit and multi-digit numbers. The focus throughout the program is on feedback about children's mathematical thinking. A key characteristic of the program was that teachers listened to their students and built on what they already knew. Instead of advocating particular teaching methods or materials, the CGI team provided issues for teachers to consider as they planned their teaching programs. Several other Australian studies now confirm Kennedy's (1998) findings (e.g. Bobis, 2001; Clarke, 2001).

Why is the content of professional development so important? One reason is that this kind of learning is 'generative'. It enhances teachers' professional knowledge and understanding (Franke, 2002).

A number of studies have shown that professional development for teachers in the United States has been ineffective. Indeed, it has been claimed that it was: "intellectually superficial, disconnected from deep issues of curriculum and learning, fragmented, and non-cumulative" (Ball et al., 2001). Little and McLaughlin (1993) also argued for a similar view. They saw professional development programs as updating teachers' knowledge rather than as providing an opportunity for sustained learning about issues to do with curriculum, students or teaching. They argued that, in the United States, professional development has not been connected to developmental learning for several reasons. These reasons include:

1. Teaching being widely seen as only needing ‘common sense’ – there is no need for further professional learning;

2. Teaching being perceived as a career in which sustained learning is not required for adequate performance;

3. There being no coherent infrastructure for professional development, that is, there is no overall agency or group responsible for organising professional development.

More recently, however, Varella (2000) and Franke (2002) have shown that long-term professional development had positive effects on teachers.

Teacher capacity

The capacity of teachers to be effective is associated with four key facets: (a) their expertise; (b) their conception of what it means to be a teacher and a teacher of mathematics; (c) their beliefs about teaching; and (d) their teaching experience.

<u>Expert teachers</u>

Berliner et al. (1988) reviewed the study of expert teachers and identified some common characteristics (and how they differed from novices). These characteristics were:

1. Experts often made inferences about objects and events while novices usually held more ‘literal views’.

2. Experts often classified problems to be solved at a relatively high conceptual level while novices usually classified problems using ‘surface’ characteristics.

3. Experts tended to have faster and more accurate ‘pattern recognition capabilities’ compared to novices.

4. Experts seemed to take longer to examine the problem and to build a problem representation.

5. Experts built different ‘problem representations’ compared with novices.

6. Experts showed greater self-regulatory capabilities than novices.

7. Expertise was built up slowly, and with much practice.

Other studies of expert teachers in mathematics flesh out some of the characteristics identified by Berliner (Berliner, 1987). For example, Askew et al’s (1997) study of effective teachers of numeracy found that the effective teachers had “coherent beliefs and understandings” and that these “underpinned their teaching of numeracy”. Effective teachers thought that discussion of concepts and images was important in exemplifying the teacher's network of knowledge and skills, and in revealing pupils' thinking, and that it was the teacher's responsibility to intervene to assist the pupil to become more efficient in the use of calculating strategies. Askew et al. (1997) also found that teachers who gave priority to pupils acquiring a collection of standard arithmetical methods, over establishing understanding and connection, produced lower numeracy gains. Lower numeracy gains were also produced by teachers who gave priority to the use of practical equipment rather than developing effective methods, and delayed the introduction of more abstract ideas.

Askew et al. (1997) also found that teachers' beliefs and understandings of the mathematical and pedagogical purposes of classroom practices were more important than their actual practice, and that having an A-level or a degree in mathematics was not associated with being highly effective. In a similar vein, Leinhardt and Greeno (1986) found that expert teachers draw on a richer and deeper knowledge structure than novice teachers. Novices had less well developed 'schemata', due, probably, to having had less experience. The schemata of expert teachers, Leinhardt and Greeno argued, allowed them to decide which issue merited attention. This finding is also similar to that reported by Borko and Livingstone (1989) who noted that expert teachers had 'scripts for change' stored mentally, and these could be quickly accessed and implemented.

Planning was identified by Sternberg and Horvath (1995) as one of the key dimensions underlying the expertise of experienced teachers. They also saw expert teachers as knowledgeable, able to adapt to practical constraints, and able to perform their teaching tasks rapidly with little or no cognitive effort. Askew and Wiliam (1995) placed emphasis on the problem solving orientation of the experienced teacher noting that:

> … the more teachers knew about their pupils' mathematical knowledge, the better the pupils were at word problem-solving. Knowledgeable teachers questioned their pupils about problem-solving processes and listened to their responses, while less knowledgeable teachers tended to explain problem solving processes to pupils or just observe their pupils' solutions (p.21).

Berliner (1987) proposed five stages in teacher development: (a) novice; (b) advanced beginner; (c) competent; (d) proficient; and (e) expert. In a later paper, Berliner et al. (1988, p.79) elaborated this claim noting: "we might expect experienced/expert teachers of mathematics to be more likely than novices or postulants (i.e. student teachers) to focus on events that have instructional significance; to 'read' the patterns of classrooms quicker and more accurately; to agree among themselves about what is and is not going on; to be more work orientated in their views of classes; and to make more assumptions and hypotheses about classroom phenomena".

Teacher conceptions

Fennema and Franke (1992) proposed that *knowledge of mathematics, knowledge of pedagogy, knowledge of students' cognitions* and *beliefs* are the key components of teachers' knowledge. They argued that teachers' knowledge develops in specific contexts and that, therefore, it should not be studied in isolation. In considering teachers' knowledge, Ball (1991), Fennema et al. (1996), and Lloyd and Wilson (1998) have shown that teachers' specific *subject-matter knowledge* plays a critical role in their teaching practice. Because of the close conceptual connections between beliefs and knowledge, Thompson (1992, p. 132) argued that it was not useful for researchers to distinguish between teachers' knowledge and beliefs. She suggested it was better to focus on teachers' *conceptions*, that is, on mental structures encompassing beliefs, concepts, meanings, rules, mental images and preferences concerning the discipline of mathematics.

Using TIMSS data from 12 countries, Philippou and Christou (1999) investigated teachers' conceptions of mathematics and the teaching and learning of mathematics. They argued that differences in teachers' instructional emphases may be explained by differences in their conceptions of mathematics and of teaching of mathematics, and, further, that pupils' achievement and teachers' conceptions were directly related. The results of this study suggested that there was consistency between teachers' conceptions of mathematics and teachers' conceptions about the teaching of mathematics.

Andrews and Hatch (1999) showed that although teachers simultaneously hold a variety of, not necessarily consistent, conceptions of mathematics and its teaching, for most, there were dominant conceptions of mathematics which manifested in similar beliefs about teaching. This was consistent with the findings from a study carried out in Finland (Pehkonen, 1997). Middleton conducted a survey of 490 secondary school teachers from 11 urban sites in the United States. Eighty eight per cent of the surveyed teachers were frequent participants in the *Urban Mathematics Collaborative* project. This project was initiated in 1984 with the aims of improving mathematics education in urban schools, and of identifying new models of meeting the professional needs of high school teachers. The data indicated that teachers viewed mathematics primarily as requiring thinking in a logical, scientific, inquisitive manner, and as involving knowledge used to develop understanding. Most teachers seemed to hold an eclectic view of mathematics, yet despite this, Middleton concluded that: "… conceptions of the nature of mathematics were found to be related to teachers' conceptions of mathematics teaching."

These studies suggest that teachers simultaneously hold a variety of conceptions of mathematics but tend to have one view which dominates. There also appears to be a general agreement that teachers' dominant pedagogical beliefs tend to be consistent with their dominant perspectives on mathematics (Andrews & Hatch, 1999).

In contrast to the findings of the quantitative research described above, qualitative studies (Cooney, 1985; Raymond, 1997; Thompson, 1984) have been carried out on teachers' actual practice. These show a rather different picture. They have identified not only inconsistencies between teachers' conceptions of mathematics and their conceptions of the teaching of mathematics but also inconsistencies between teachers' conceptions of mathematics teaching and their practices.

Teacher beliefs

Teacher belief-systems appear to have important effects on student outcomes (Thompson, 1992). The *Effective Teachers of Numeracy Study* (Askew et al., 1997) examined beliefs about teaching and mathematics held by teachers. This study of effective teachers (identified by a high average gain over one school year in pupils' scores compared with other classes from the same year group) found that they had the key beliefs related to: (a) the meaning of numeracy; (b) "the relationship between teaching and pupil's learning of numeracy"; and (c) "presentation and intervention strategies" (Askew et al., 1997, p. 1). Teachers in the study were interviewed about their educational orientations to teaching, mathematics and styles of interaction with students. The results of these interviews suggested that there were three types of orientation that teachers adopted in the teaching of mathematics: *Connectionist*, *Transmission*, and *Discovery*. All but one of the highly effective teachers were classified as 'connectionist'. That is, these effective teachers saw mathematics as richly connected and adopted classroom strategies which help students to make links. Teachers holding other orientations were all classified as being only moderately effective. These findings have been supported by Ma (1999) who used the concept of 'connectedness' – together with three other properties of understanding, 'basic ideas', 'multiple representations' and 'longitudinal coherence' – to describe the teaching focus of teachers who have a 'profound understanding of fundamental mathematics' (p.118).

Teaching experience

Martin et al.'s (2000) study of school effectiveness, using TIMSS data found that teaching experience, was not a significant factor in student achievement except in Singapore and the United States. However, as Abbott-Chapman et al. (1990) note, more experienced teachers tend to be promoted out of the classroom and into administrative positions. Teacher experience may, it could therefore be argued, manifest itself in other contexts within the school.

School Factors

Student learning is, typically, affected most directly by the quality of opportunities for learning that individual teachers can provide. However, the quality of teaching is in turn affected by a wide variety of conditions at the school level. Workplace conditions can exert a powerful influence over the quality of teaching in two main ways: (a) when they help to attract and retain quality people into the profession; and (b) when they energise teachers and reward their accomplishments (Darling-Hammond, 2000).

Schools differ in their capacity to ensure that all mathematics teachers are well qualified in mathematics and with specific training in the teaching of mathematics. Ingersoll (1996), for example, documented the extent of this problem in the United States. School systems and sectors also differ in the career structures they provide and the incentives they provide for professional development. As a result, schools differ in their capacity to compete for good teachers and the best of recent graduates (Ingersoll, 1999). No Australian research was found which examined relationships between these aspects of school capacity and student learning outcomes in mathematics. However, recent research identifies a group of important factors that can be loosely classified as school or organisational capacity. Newman (2000) defines school capacity as “the collective power of the school staff to improve student achievement school-wide”. Newman identifies four dimensions of school capacity: (a) leadership; (b) professional community; (c) program coherence; and (d) teaching resources.

Which aspects of school organisation matter most to effective teaching? There is a long tradition of research, for example, examining the relationship between the organisational context of schools and the quality of teaching and student learning. Gameron, Secada and Marrett’s (1998) review of this literature suggested that the relationships between school ‘restructuring’ and student learning were weak, except where organisational resources (time, leadership, collaboration, administrative support, knowledge and skills) were deployed in ways that promote professional interaction and development. Peterson, McArthy and Elmore’s (1996) research casts doubt on the capacity of new management structures to benefit classroom practice. They (1996, p. 149) argued that teaching is the most important factor in effectiveness, and that while school structures can provide opportunities, these structures, of themselves, do not directly contribute to this teaching.

Leadership

Hill et al. (1996) have shown, in the *Victorian Quality Schools Project*, that 26.5% of the between-school variation in student achievement was due to leadership support for teachers’ practices. They commented that:

> *Leadership Support* is overwhelmingly important in establishing a positive working environment for teachers … [it] has significant direct and/or indirect effects on **all** measured aspects of teachers’ *Affect* and perceptions of their *work environmen*t (p. 578) *(all emphases in the original)*.

While leadership is important in schools, it is probably limited in the extent to which it can influence teaching practices within classrooms. Elmore (2000), for example, argued that, in the United States, this is in part due to the fact that teachers generally ‘do their own thing’ in the classroom, beyond the direct control of the school leadership.

Program coherence

Program coherence is a measure of integration of the different elements in the school as an organisation. Newman et al. (2000) developed the concept of ‘program coherence’ as a dimension of school capacity. They defined program coherence as “the extent to which the school's programs for student and staff learning are coordinated, focused on clear learning goals, and sustained over a period of time" (p. 263). One indicator of program coherence, for example, is the extent to which teacher professional development is linked to attaining the school's goals for improved student learning outcomes.

Resources

A meta-analysis of sixty primary research studies examining a variety of school inputs and their effect on student achievement was conducted by Greenwald, Hedges and Laine (1996). Levels of expenditure on students were found to be strongly related to student achievement, as was being in small schools or small classrooms.

Professional community

Professional community refers to the collegial relationships that exist between teachers within a school (or a department in larger schools). This may involve the ethos of the work group, opportunities for collegial work and a sense of professionalism. Being and feeling part of an educational team is also part of the notion of professional community. While this appears to be a teacher-related factor, as part of the school environment it necessarily involves the support of the school leadership. Aspects of a professional community include the continuing development of members' expertise, mechanisms for its development and maintenance and the level of collegiality it supports or engenders.

The isolated condition of teachers' work is a barrier to professional learning and productive forms of professional peer review and accountability (Lortie, 1975). Goodlad (1984) called it 'autonomous isolation'. Related to this is the concept of 'loose-coupling' (Weick, 1976). This refers to the relative impenetrability of teachers' work to external attempts to define and shape it. Weick attributed loose coupling to the weak knowledge base that underpins teaching and a professional culture with its wide tolerance of personal style over evidence-based practice.

The existence of an active, accountable professional community within and across schools is important for effective teacher development and high quality teaching (Little & McLaughlin, 1993; Louis, Kruse, & Marks, 1996). Louis, Kruse and Marks (1996) identified the key elements of professional community as: (a) shared norms and values; (b) a collective focus on student learning; (c) collaboration; (d) 'de-privatised' practice; and (e) reflective dialogue. Darling-Hammond (1992) points out that teachers in strong professional communities accept a mutual obligation to review their practices in the light of profession-defined standards. Fullan (2001) argued for the central importance of collegiality among teachers for the implementation of change – significant change depends on frequent opportunities to converse about concrete examples of practice. Minstrell (1999) argued that expertise is not developed "by listening to lectures about content, about learning, or about pedagogy … [it] requires significant desire and time … the guidance of a mentor, interact[ions] within a community of colleagues."

The elements of a school's professional community can be extended to the wider profession and include a teacher's membership of professional organisations (for example, the Australian Association of Mathematics Teachers), the extent of professional reading and, more recently, participation in on-line 'chat' groups.

Mathematics departments and schools probably differ widely in the frequency, quality and specificity of conversations that teachers have about their practice. Ball and Cohen (1999) emphasised the importance of making practice itself the site for learning, while Little (1982) identified how norms of collegiality and experimentation characterised more effective schools. Collegiality necessarily involves personal contact. As Fullan (2001, p.124) explained:

> Significant educational change consists of changes in beliefs, teaching style, and materials, which can come about only through a process of personal development in a social context. … There is no getting around the primacy of personal contact.

On this view, dialogue draws out existing beliefs and assumptions and exposes them to feedback from (trusted) colleagues.

School characteristics

The identification of demographic features of schools is critical to understanding what factors affect variation in teacher effectiveness. These factors may include the language background of the students, the socio-economic status of the school catchment area, the proportion of Indigenous students, and the geographic location of the school. (See, for example, Askew et al., 1997; Hill et al., 1996; Martin et al., 2000.)

Classroom characteristics

In their review of school effectiveness research, Muijs and Reynolds (2000, p. 276) reported that "classroom climate is … [a] significant [factor]" in student achievement. Classroom climate is seen as a businesslike environment that is nevertheless relaxed and supportive of pupils, with high teacher expectations.

System-level Factors

System-level factors operate at the state or school sector level. They shape conditions for teaching and learning in schools; for example, levels of resources for school education, curriculum and assessment policies, and investments in teacher education and on-going professional development.

There have been few studies examining variations in policies and resources across the Australian states and territories and their relationship to student learning outcomes. International studies of student achievement such as TIMSS (Lokan et al., 2001) or PISA (OECD, 1999) can be useful for identifying conditions at the national school system level that may be related to variations in student achievement. For example, PISA data shows that in some European countries with 'tracked systems' – those which distribute students across sectors according to likely labour market destinations – tracks have a strong effect on student achievement.

The United States is well placed to examine the effects of system factors on student outcomes because national databases exist such as the *Schools and Staffing Survey* and NAEP. Darling-Hammond (2000) found that variation in mathematics achievement on NAEP tests across US states was attributable more to variations in policies affecting teacher quality than factors such as student demographic characteristics, class size, overall spending levels, or teacher salaries. The major policy variations affecting teacher quality were state requirements for teachers to be licensed and to have a major or minor in mathematics from university. She claimed that:

> … the effects of well-prepared teachers on student achievement can be stronger than the influences of student background factors, such as poverty, language background, and minority status. And, while smaller class sizes appear to contribute to student learning … the gains are most likely to be realised when they are accompanied by the hiring of well-qualified teachers (p. 39).

Several studies confirm that improving teacher quality is the most effective option for policy seeking to improve student learning outcomes (Darling-Hammond & Ball, 1998). Wenglinsky (2002) used NAEP data to examine factors affecting student learning in eighth-grade mathematics. Factors included student background, teacher quality (measured using such variables as having a major in mathematics, professional development, classroom practices) and class size. Wenglinsky concluded that the quality of the teacher was the most important factor shaping student learning.

Monk (1994; 1994a) examined the effects of subject matter preparation of secondary mathematics and science teachers on student performance gains. Teacher content preparation was positively related to student learning gains. The effects of content knowledge were strengthened when accompanied by coursework in pedagogy, leading Monk (1994; 1994a, p. 142) to state that "it would appear that a good grasp of one's subject area is a necessary but not sufficient condition for effective teaching". (It should be noted, however, that Monk also argued that it was too risky to make generalisations about the significance of teacher subject matter knowledge.)

The major review of research on factors affecting student learning outcomes, *What Matters Most* (National Commission on Teaching and America's Future, 1996), came to the conclusion that the most important influence on what students learn is what their teachers do. The report's recommendations for systemic reform called for: (a) the development of a new infrastructure for professional learning organised around standards for teaching; (b) new career paths that reward teachers for evidence of professional development; and (c) a focus on creating school conditions that enable teachers to teach well.

Policies on teacher qualifications

Teacher qualifications are of two varieties: qualifications in mathematics and qualifications in mathematics education ('methods courses'). There appears to be considerable diversity in the content and the amount of time given to mathematics in education courses. In Australia, policy on teachers' qualifications to teach mathematics in secondary schools varies between the states. This further encourages diversity in the academic backgrounds of teachers of mathematics. One consequence of this diversity may be that some teachers end up less-well qualified in mathematics than is needed in order for them to be effective. There is some evidence of this problem from the United States. Darling-Hammond (1996) in *What Matters Most*, reported:

> … nearly one-fourth … of all secondary teachers do not have even a minor in their main teaching field. This is true for more than 30% of mathematics teachers.… Fifty-six percent of high school students taking physical sciences are taught by out-of-field teachers, as are 27% of those taking mathematics. (p. 196)

Ma (1999. p. 145), citing the National Center for Research on Teacher Education, suggested that most US teacher education programs focus mainly on pedagogical knowledge rather than teaching fundamental mathematics for profound understanding. This was, Ma argued, a major impediment to reform. Darling-Hammond and Ball (1998) argued that teacher qualifications play an important part in teacher effectiveness in mathematics. Citing Ferguson's (1991) study, they reported that:

> … every additional dollar spent on more highly qualified teachers netted greater increases in student achievement than any other less instructionally-focused uses of school resources. The effects were so strong, and the variations in teacher expertise were so great, that, after controlling for socio-economic status, the large disparities in achievement between black and white students were almost entirely explained by differences in teachers' qualifications. (p. 2)

While the *Effective Teachers of Numeracy Study* (Askew et al., 1997) suggested that neither mathematical qualifications nor initial training were factors strongly correlated with effectiveness, the Australian and American research cited above suggests that there is a threshold below which a teacher will not be sufficiently qualified in mathematics to be effective. (See, for example, Monk (1994; 1994a) and Darling-Hammond and Ball (1998).)

Time allocated to mathematics

The amount of time devoted to mathematics classes has been the subject of international scrutiny for some time. For example, the *Second International Mathematics Study* (SIMS) surveyed principals and teachers on this aspect of teaching practice. More recent data on the amount of time allocated to mathematics in Australian secondary schools was collected by TIMSS. Principals (or Heads of mathematics departments where appropriate) responded to questions seeking this information as part of 'opportunity to learn' within the School Questionnaire. The data from the TIMSS Australian junior secondary school population showed that 80 percent of schools taught their students for between 186 and 205 days per year. An analysis of instructional time, on a weekly basis, showed that about 30 per cent of schools devoted 221 – 240 minutes per week to mathematics, and a slightly smaller percentage 181 – 200 minutes per week. Of the remaining schools, about 14 percent devoted less than 180 minutes per week and about 16 percent more than 260 minutes per week (Lokan, Ford, & Greenwood, 1996). Lokan et al. (1996) commented that:

> … there are quite large differences around the country in the amount of school time devoted to mathematics and science. … [and although the] data have not yet been analysed with respect to amount of instructional time … the findings in earlier IEA studies that such differences are usually associated with achievement differences… [suggest that] schools or systems may wish to review their priorities with respect to mathematics and science in relation to other learning areas. (p. 207)

Conclusion

This literature review examined national and international research on factors contributing to effective mathematics teaching and learning in secondary schools (Years 7–10). The review found evidence that each of the following was important for understanding teacher effectiveness, especially for the teaching of mathematics:

1. Knowledge: teacher knowledge of mathematics and how to teach it is important, but equally important is how that knowledge is organised and applied by the teacher (Ball et al., 2001; Fennema & Franke, 1992).

2. Beliefs: teacher beliefs about teaching and mathematics appear to be important in shaping practices (Askew et al., 2000; Askew et al., 1997), but teaching practice is not always consistent with beliefs (Cooney, 1985; Raymond, 1997; Thompson, 1992).

3. Understandings: The quantity and quality of knowledge needed by a mathematics teacher is widely seen to be important, but what is the optimum amount and quality remains contested (Askew et al., 1997; Thompson, 1984). The *Teacher Preparation Research* report (Center for the Study of Teaching and Policy & Michigan State University (Wilson, 2001)), argues, for example, that much more research needs to be done before strong conclusions can be drawn, including research about how much subject matter knowledge and what type prospective teachers need in order to ensure student learning, and the relationship between components of pedagogical preparation and teacher effectiveness. Distinction was often drawn between pedagogical and

content knowledge, but the relationship between them is complex (Shulman, 1987), suggesting that treating them separately may lead to oversimplification.

4. Practices: What teachers do in classrooms is important, and knowing what practices are most effective is also important. One approach to examining and assessing these practices may be to address the question of what constitutes expertise. Another is to consider how experts differ from novices (Berliner, 1987; Berliner et al., 1988). Yet another may be to use standards developed to rate teachers.

5. Qualifications: the data on the influence of qualifications is contradictory but Wenglinsky (2000a) showed in a methodologically sound study they probably are important. The same study showed also that postgraduate qualifications at Masters or higher level, or having a major in mathematics, was not associated with teacher effectiveness.

6. Professional development: the data on the relationship between professional development and effective teaching suggests that if it concerns higher order thinking there will be a positive association with effectiveness, however much professional development is seen as ineffective (Ball, 2000; Ball & Cohen, 1999; Little & McLaughlin, 1993).

7. Personal experiences: The number of years teaching is probably not associated with teaching effectiveness (Martin et al., 2000; Wenglinsky, 2000a).

The following was also identified in the review as likely to be associated with effective mathematics teaching: (a) school leadership (Hill et al., 1996), although the extent to which it can affect classroom learning may be limited (Peterson et al., 1996); (b) program coherence (Newman F. et al., 1996); (c) resources (Greenwald et al., 1996); (d) the existence of a professional community in the school (Little, 1982; Little & McLaughlin, 1993; Louis, Kruse et al., 1996; Louis, Marks, & Kruse, 1996; McLaughlin & Talbert, 2001); (e) characteristics of students and the classroom; and (f) system level factors.

3 THEORETICAL FRAMEWORK FOR THE RESEARCH

This chapter provides a statement of the theory that was used to guide the design of the study, and subsequently, to shape decisions about what data to collect and how to analyse them. At the heart of this theory is the graphical depiction, shown as Figure 2. This figure provides a summary of the main factors that appear to influence student achievement in mathematics. These factors were identified from the literature review.

An overview of the theory

Figure 2 describes the relations theorised to exist between factors associated with effective mathematics teaching. It can be seen that there are five broad groupings of factors, shown by the boxes of 'reversed' text across the top of the Figure. Four of these broad groupings represent independent variables, and the fifth – *Learning Outcomes* – represents the dependent variable. Below these dark boxes the theory is depicted in more detail using 'unreversed' boxes. In Figure 2, those elements to the left are theorised to be causally prior to those elements depicted to the right. For example, learning outcomes are seen as arising from the students' opportunity to learn. Many of the boxes in Figure 2 are connected by arrows. The arrows indicate that the variable from which an arrow comes, causes a change in the variable at which the arrow points. Double-headed arrows indicate a reciprocal effect is theorised to exist between the two variables. In reality, the relations between these variables will be much more complex than can be depicted here. This model thus represents a compromise between the complexity of the real world and a parsimonious account that captures the main features of that world.

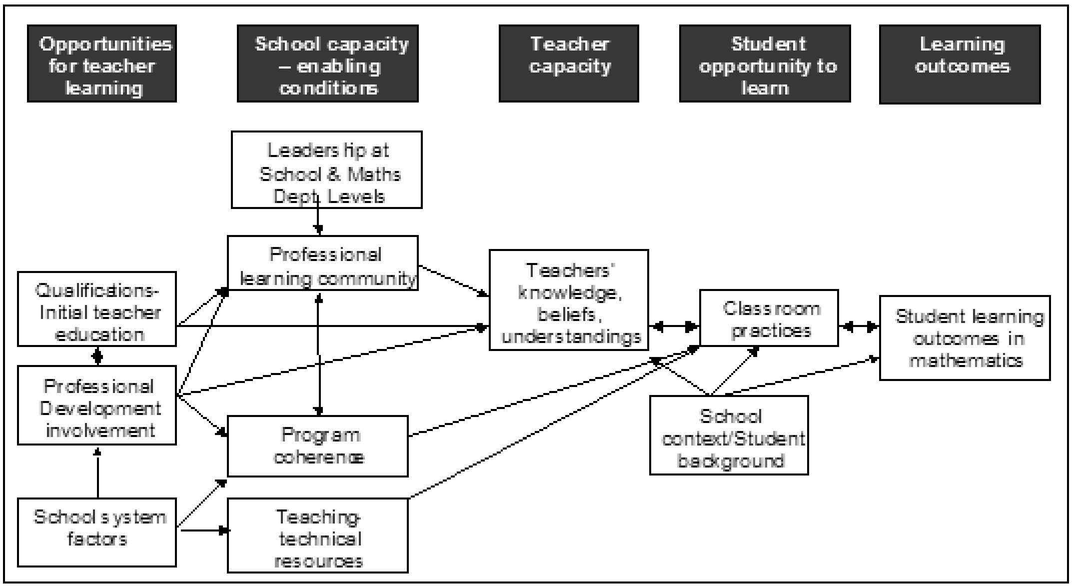


Figure 2 Summary of the main factors influencing student achievement in mathematics

Overall, the theory advances the view that the quality of classroom practices and activities is of central importance in determining student opportunity to learn and, as a consequence, student learning outcomes. The quality of this opportunity to learn is, in turn, shaped primarily by teacher capacity. A range of factors shape teacher capacity. Among these are teachers' own opportunities for learning about the content they teach and how to help students learn that content. Indicators of these opportunities were the nature and depth of knowledge that mathematics teachers gained both in their initial university training and through continuing professional development activity.

As the literature review indicates, many researchers draw attention to school capacity as well as teacher capacity. The theory includes several aspects of school capacity such as: (a) the quality of leadership and administrative support at both mathematics department and school levels; (b) the strength of professional community in the mathematics department, as indicated through joint work, shared values, and accountability; (c) the degree of coherence in the curriculum program; and (d) material resources such as computer software and hardware.

School system level policies and practices also indirectly shape the quality of mathematics teaching at the school level, but were beyond the scope of this study. They are nevertheless important. For example, long term studies (e.g. Dolton, Chevalier, & McIntosh, 2001) point to three main factors affecting the capacity of the teaching profession to recruit and retain high quality mathematics graduates: (a) relative starting salaries; (b) workplace conditions; and (c) long term career prospects in teaching.

Elaboration of the theory

This section describes in more detail the key relationships between each of the elements of the theory.

Opportunities for teacher learning

The opportunities that teachers have for learning were seen to be defined by three factors: (a) the qualifications and the content of their initial training; (b) the level of professional development involvement that they experienced; and (c) system level factors. It is likely that that there is a reciprocal causal relationship between the first two of these factors – the richer the educational background of the teacher, the more likely they will seek an environment which will encourage on-going professional development. School system factors, by setting the macro-context, are likely to influence opportunities for professional development involvement. These relations can be seen marked on Figure 2 as the arrow from *School system* factors to *Professional development involvement*, and the reciprocal arrow between *Professional development involvement* and *Qualifications and initial teacher education*.

School capacity – enabling conditions

The enabling conditions provided by the school were seen to be described by four factors: (a) leadership at both the school and mathematics department level; (b) the quality of the professional learning community in the school; (c) program coherence within the school; and (d) the quality and quantity of teaching resources in the school.

Leadership at the school was seen as directly influencing the quality of the professional community in the school. It was theorised that there would be a reciprocal relationship between the quality of the professional community in the school and the coherence of the school's program. The stronger the professional community, the more likely it would be that there was coherence in the school's program. These relations can be seen marked on Figure 2 by the arrow from *Leadership* to *Professional community*, and the reciprocal arrow from *Professional community* to *Program coherence*. The resources needed for teaching were seen as having their major effects on student opportunity to learn.

Professional development involvement was seen to influence the professional learning community at the school and its program coherence. That is, the more involvement there was in professional development, the more likely that there would be a professional learning community in the school, and that its program would be coherent. Professional community was also seen to be influenced by the qualifications and the initial training of the teachers. School system factors were seen to influence the coherence of the school program, and the technical resources available in the school. Each of these effects is marked on Figure 2.

Teacher capacity

Teacher capacity was seen as being constituted by the teacher's knowledge, beliefs and understandings of teaching in general, and of teaching mathematics in particular. Teacher capacity was seen as shaped by the professional community in the school, their qualifications and initial teacher training, and the involvement of the teacher in professional development. The more a teacher had exposure to these environments and opportunities to learn, it was theorised, the greater their capacity to be an effective teacher. School and student background characteristics were also theorised to influence teacher capacity (for example, teaching non-English speaking background students could influence the effectiveness of a teacher). Each of these effects is marked on Figure 2.

Student opportunity to learn

Many factors can influence student opportunity to learn, but for this study, with its focus on effective mathematics teaching, it was the classroom practices of teachers which were seen as the most important of these factors. These classroom practices were seen to be in part shaped by the school in general, and the family background characteristics of students. They were theorised to be reciprocally associated with teacher capacity, and shaped by the influence of teaching technical resources and the program coherence. Again, these effects are marked on Figure 2.

Learning outcomes

Student learning outcomes relate to their achievement in mathematics. For this study, these outcomes were understood as both their knowledge and their attitudes to learning mathematics. Learning outcomes were seen to be reciprocally related to classroom practices. On this view, the more successfully students learn, the more likely it is that the teacher will adopt practices that encourage further successful learning. Learning outcomes were also seen to be affected by the school context and the family background of the students. These effects are marked on Figure 2.

Conclusion

The theory proposes that the direct effects on student outcomes are the classroom practices of teachers, and these are reciprocally related to the teacher's knowledge, beliefs and understandings. These in turn are shaped by the school and their educational and professional development experiences, as well as school system factors.

4 METHOD

The study involved three main stages:

1. The selection of a sample of schools and the selection of a sample of teachers of Years 7 to 10 mathematics from within these schools;

2. The collection and analysis of classroom and school level data on factors that impact on student learning outcomes in mathematics; and

3. The conduct of case studies with a small number of teachers and schools.

A brief description of the methods used to complete each of these three stages follows.

Selection of samples

Schools were sampled, and from within schools, teachers and their mathematics classes were sampled. This section describes the sampling procedures used and some of the limitations of the achieved sample.

Sampling schools

To maximise the variation in teaching practices, schools were sampled from the set of schools that had participated in the Program for International Student Assessment (PISA) study conducted in 2000. This study was conducted by the Organisation for Economic Co-Operation and Development (OECD). The data from this study were publicly available. They provided a range of family background and school context measures, as well as a measure of mathematics literacy (OECD, 1999). Some 230 schools in Australia participated in PISA 2000 with 35, 15-year-old students being sampled at random within each school. The modal Year level for 15-year-olds across Australia is Year 10 and so the sample corresponded to the age at which most students would have completed the junior secondary years. The primary focus of the PISA study in 2000 was on reading but information was also collected about mathematics performance. Mathematics performance data were collected through a matrix sampling of domains related to general mathematical literacy. The sampling of items was used to generate an average measure of mathematics performance for each school.

The schools were sampled such that those which had the highest and the lowest average PISA mathematics achievement scores were selected. These schools were identified by using a multi-level modelling procedure. This procedure separated out individual and background effects on mathematics literacy scores, thus permitting the unique contribution of the schools to be identified. A total of 36 schools was identified. Of these, there were 18 schools in which student performance was ‘above’, and 18 schools in which student performance was ‘below’, what would have been expected on the basis of the social characteristics of its students and the social context of the school. For this purpose ‘above expectation’ was defined as in the top third and ‘below expectation’ was defined as in the bottom third of schools. This process was intended to provide a set of schools between which there was a wide range of mathematics performance.

Sampling teachers and classes

Schools that agreed to participate in the study were asked to forward a list of all teachers of Years 7 to 10 mathematics in their school. Six teachers were then randomly selected by research staff from this list using a random number table. This sample was stratified by year level so that from each school there were two teachers at Years 8, 9 and 10. These teachers were then asked to nominate a class they taught at this year level and to administer the tests and questionnaires. In this way teachers were linked to specific students, and these, together, were linked to their school.

The achieved sample

The final number of schools participating in the study is described in Table 1. It had been intended that only high and low achieving schools that had participated in the PISA study would be sampled. Twenty three schools which had participated in the Third International Mathematics and Science Study (TIMSS), selected using the same procedures for the selection of PISA schools, were added to increase the number of schools participating in the study.

Table 1 Number of schools participating (PISA versus non-PISA)

		Number in study
PISA School	*Low*	*10*
	High	*17*
Total PISA schools		27
Non-PISA schools		23
Total all schools		**50**

Of these 50 schools, 14 came from New South Wales and the Australian Capital Territory, eight came from Victoria, five came from Queensland, eight came from South Australia, six came from Western Australia, came five from Tasmania and four came from the Northern Territory.

The collection and analysis of student, classroom and school level data

Data were collected from school principals, teachers and students. There were two waves of data collection from students. The number of calendar days (not 'school days') between these tests ranged from a minimum of 11 to a maximum of 179 days. The mean number of days between the waves was 123 (SD = 32.1). There was only one wave of data collection for the teachers and school principal.

Information about mathematics achievement was collected using the recently revised *PATMaths* test – Forms 2B and 3B (ACER, 1998). It was used to obtain measures of growth in mathematics achievement. Student responses to mathematics teaching were gathered using a questionnaire developed as part of the *Victorian Quality Schools* project (Hill et al., 1996), and from other sources. This instrument provided information about the attitudes of students to their mathematics learning and about their perceptions of mathematics teaching.

For teachers, a self-report questionnaire was developed to gather data about their approaches to mathematics teaching. This questionnaire was based on the work of Cohen & Hill (2000) and the National Board of Professional Teaching Standards (2002). Information was also gathered about teachers' education and their participation in professional development.

Information about the context in which mathematics teaching was organised within the school was gathered by means of a questionnaire completed either by the Principal or the Head of the mathematics department in the school. Information was obtained about time allocations to mathematics, class organisation (including ability grouping) and a range of other factors that might influence how mathematics was taught at the school.

Principal and school level data

Fifty completed survey forms were returned from schools completed either by the Principal (n = 16) or the Head of the mathematics department (n = 34) in the school.

This questionnaire asked about the school's sector, geographic location, size, the proportion of students who were receiving the Educational Maintenance Allowance, the proportion who were from an Indigenous background, the proportion who were from a non-English speaking background, whether the school streamed mathematics classes, had single sex classes, the number of teachers in the school who taught mathematics and their qualifications and their years of service. There was also a 35-item inventory designed to tap factors perceived to be influencing the effectiveness of mathematics teaching and learning for Years 7 to 10 in the school.

The level of missing data from the principal questionnaire was low, with most items having only one or two valid responses missing. The highest levels of missing data occurred for those questions concerning the background of students. Up to seven respondents did not provide data for these questions.

The data from the Principal's questionnaire were merged with the student and teacher data (using the school ID number as the link between the various data sets). After this merge was completed, and after cases were removed because of data missing from other data files (in particular, student data), data from a total of 43 principal questionnaires were left in the final version of the full data file.

Teacher level data

Completed survey forms were received from 206 teachers. Table 2 shows the distribution of teachers across Year levels. The distribution was fairly even across each of the years. A total of 24 teachers did not identify a year level on the survey form, but a year level was matched from the student data. In the final version of the merged data there were 182 teacher questionnaires used, with 64 from Year 8, 58 from Year 9 and 60 from Year 10.

Table 2 Distribution of teachers across year levels

	Year Level	Frequency	Percent	Valid Percent
Valid	8	64	29.6	33.5
	9	58	27.7	31.3
	10	60	31.1	35.2
	Total	182	88.3	100.0
Missing		24	11.7	
Total		206	100.0	

The teacher questionnaire asked about the teacher's gender, level of education, the fields of study of their education, the number of years teaching, whether they were a member of the Australian Association of Mathematics Teachers (AAMT), the amount, frequency and type of professional development they had had over the previous three years, the nature of this professional development, the context in which they taught, and their teaching practices. The questionnaire also asked about a range of teaching activities and strategies and asked the extent to which the teacher agreed with these approaches, and the extent to which they had been able to implement them.

The level of missing data for the teacher questionnaire was low, with most items having less than 2% missing data. For the items to do with professional development, the school context and teaching practice, from which a number of new variables used in the analyses were constructed, the SPSS PC *EM* algorithm was used to impute missing values. Before this procedure was run, the amount of (listwise) missing data for these variables was typically low (<5%).

Student level data

Two waves of data were collected from the students. In the first wave students were asked to complete a *PATMaths* test and a questionnaire. For the second wave they were again asked to complete each of these instruments. Data from the second wave was matched to individual student data from the first wave. This was done so that a measure of growth in mathematics achievement of the students could be obtained. The number of calendar days (not 'school days') between these tests ranged from a minimum of 11 to a maximum of 179 days. The mean number of days between the waves was 123 (*S.D.* = 32.1).

Table 3 Distribution of students across year levels at Time 1 and Time 2

Year Level	**Time 1**		**Time 2**	
	n	**%**	**n**	**%**
8	2663	34.5	1127	41.9
9	2562	33.2	1028	38.2
10	2484	32.2	533	19.8
Total	7709		2688	

Table 3 shows the number of students at each year level who responded to at least one instrument at Time 1 and at Time 2. It can be seen that at Time 1 that students were distributed fairly evenly across Years 8 to 10, but that at Time 2, Year 10 students were under-represented.

The student data were primarily collected to provide data to construct indicators of effective teaching. The research was not concerned to understand student achievement or student attitudes towards mathematics except to the extent that it informed the research about the students' classroom teacher. Therefore, student data were only of use if their mathematics teacher had also provided data for the study. Similarly, it was important to match the student data and teacher data to the school. Once all sets of data were merged, the final data set consisted of 4348 cases (at the student level). Of these 46% were female, 52% were male and there was 2% missing data. These students came from 182 classes drawn from 43 schools. There were on average 93 students from each school, and an average of 24 students from each class. The final number of students available to the study, as distributed across year levels is shown in Table 4. It will be observed that there are more cases, for example, at Year 10 (n = 1411) in Table 4, than for Time 2 in Table 3 (n = 533). This occurs because cases were retained from Time 1 that did not match cases at Time 2 (and vice-versa) to increase the number of cases available for statistical procedures needed to construct some of the variables used in the analyses.

Table 4 Distribution of students across year levels in the final version of the data

Year Level	N	%
Year 8	1564	36.0
Year 9	1350	31.0
Year 10	1411	32.5
	4325	99.5
Missing	23	0.5
Total	4348	100.0

The case studies

To complement the quantitative data by developing an understanding the context in which mathematics is taught in schools, a series of six case studies were conducted. The schools selected for these studies covered a range of geographic locations (rural and urban, different states of Australia), sectors and average achievement levels in mathematics as measured by school average PISA mathematics literacy scores.

School principals, Heads of Department and mathematics teachers were interviewed face-to-face, and the data taken from these interviews were then matched to other data from the study to help: (a) illustrate the variety of contexts in which Mathematics may be taught in schools; (b) enrich theorising about this teaching and the factors shaping it; and (c) assist in interpreting the results taken from the analysis of the quantitative data.

5 RESULTS: ASSESSING PROFESSIONAL KNOWLEDGE

An important aim of this study was to examine relationships between mathematics teachers' professional knowledge and student learning outcomes. This chapter provides a description of a method developed for assessing this knowledge.

Teacher professional knowledge

For the purpose of this study, two aspects of teacher professional knowledge were investigated: (a) knowledge of mathematics; and (b) knowledge of pedagogy specific to teaching mathematics. These were consistent with the categories *Knowledge of Mathematics* and *Knowledge of Student's Learning of Mathematics*, as defined in the AAMT *Standards for Excellence in Teaching Mathematics in Australian Schools*. These standards, read in part, as follows:

> 1.2 Knowledge… of mathematics.
>
> Excellent teachers of mathematics have a sound, coherent knowledge of the mathematics appropriate to the student level they teach, and which is situated in their knowledge and understanding of the broader mathematics curriculum. They understand how mathematics is represented and communicated, and why mathematics is taught. They are confident and competent users of mathematics who understand connections within mathematics, between mathematics and other subject areas, and how mathematics is related to society.
>
> 1.3 Knowledge… of students' learning of mathematics.
>
> Excellent teachers of mathematics have rich knowledge of how students learn mathematics. They have an understanding of current theories relevant to the learning of mathematics. They have knowledge of the mathematical development of students including learning sequences, appropriate representations, models and language. They are aware of a range of effective strategies and techniques for: teaching and learning mathematics; promoting enjoyment of learning and positive attitudes to mathematics; utilising information and communication technologies; encouraging and enabling parental involvement; and for being an effective role model for students and the community in the ways they deal with mathematics. (Australian Association of Mathematics Teachers, 2002)

The notion of 'pedagogical content knowledge', first introduced by Shulman et al. (1987; 1986) in the mid-1980s was intended to capture the kind of knowledge effective teachers gain about how to help students learn in specific content areas. It includes, for example, familiarity with topics children find interesting or difficult, the representations most useful for teaching a specific content idea, and learners' typical errors and misconceptions. Shulman et al. (1987) at Stanford University in the late 1980s developed methods for gaining access to this subject-specific kind of professional knowledge.

Hill et al. (2002) developed items to measure teachers' Content Knowledge and 'Familiarity with Student Thinking' also with the intention of eventually developing a scale of teacher pedagogical content knowledge. Three different 'genres' of 'student thinking items' were identified. This work is still in progress. These researchers have been experimenting with cases, scenarios and other kinds of situational prompts to probe teacher knowledge about the subject matter they teach. The present research draws on this research and attempts to take it further.

Development of the classroom scenarios

Four 'Classroom Scenarios' were developed and trialled. Three of these were used: (a) *Tom's Troubles*; (b) *Polygon Patterns*; and (c) *Coin Toss*. They are reproduced below. (See Figure 3 (*Tom's Troubles*), Figure 4 (*Polygon Patterns*) and Figure 1 (*Coin Toss*). Each scenario, except *Coin Toss* is based around evidence of work that students have produced.

Within each scenario, items (questions) were developed that would meet the following specifications:

1. A balance with regard to knowledge of mathematics and pedagogical content knowledge;

2. Balance and appropriateness with regard to the Year 8 – 10 mathematics curriculum as defined in *Mathematics – a curriculum profile for Australian schools*, (Curriculum Corporation, 1994);

3. Realistic classroom situations that set the items in meaningful contexts; and

4. An achievable completion time.

Items measuring *Knowledge of Mathematics* mainly required teachers to do some mathematics. Those measuring *Knowledge of Pedagogy* mainly required teachers to respond to some mathematics done by students such as:

- identify a question that might help students to 'confront and determine their faulty practice' (*Polygon Patterns*, Parts 1 and 2)

- identify and generalise student errors. (*Tom's Troubles*, Parts B and C; *Polygon Patterns*, Parts 1 and 2)

Table 5 Categorisation of classroom scenarios by knowledge type and content strand

Classroom scenario	Content strands	Knowledge of content	Knowledge of pedagogy	Total
Tom's Troubles (Parts A, B & C)	Number & Algebra	Part A (4)	Part B (4) & Part C (3)	11
Polygon Patterns (Parts 1, 2 & 3)	Space & Number	Part 1 (3)* & Part 3 (3)	Part 1 (3)* & Part 2 (3)	9*
Coin Toss (Parts A & B)	Chance & Measurement	Part A (2) & Part B (2)		4
Total		12.5	11.5	24

**Note: Part 1 of Polygon Patterns represented both types of knowledge and scores were therefore shared equally between* Knowledge of Content *and* Knowledge of Pedagogy.

While it can be seen that score points were shared between the two types of Knowledge, this was not so well achieved across the Content Strands. The Number strand was allocated more points than the other Strands. This was an attribute of the Scenarios selected for the study.

'Model' responses to each of the 8 items were developed and allocated scores. Unlike the three-category Scoring Rubrics developed by the *National Board for Professional Teaching Standards* in the USA, who use a 0, 1, 2 scale for their assessment centre tasks, the scores used in this study ranged from 3 to 5 categories depending on the task. Analysis of results indicated that 7 of the 8 scores were robust in that they indicated real differences in item difficulty. Only one code (that for *Tom's Troubles*, Part A) was collapsed from 0, 1, 2, 3, 4 to 0, 1, 2, 3 because no teacher was awarded a score of 2, making the category redundant.

The Rubrics were modified or refined in the light of teacher responses. The final form of the Rubrics appears in Appendix 2.

Tom's Troubles

Last week I had spent two lessons revising the Laws of Integers and was pleased with how well the class had retained the concepts. As we had also covered algebraic notation and expressions in Term 1, I decided that this week we would revise and link both topics with some substitution and evaluation problems but using integers.

My philosophy in regard to calculators was to encourage students to use them when appropriate. We had spent one lesson talking about how to enter and use the negative sign for example. Mostly though I expected them to 'do things in their heads' as much as possible.

Tom was a conscientious student of 'average ability' who lacked some confidence. Last night while correcting his homework, I noticed that he had given several incorrect responses. I was concerned because I thought his understanding was better than this. Unfortunately he had shown no working.
Here is the list of Tom's four homework problems together with his answers.

1. Evaluate $n^2 + 20$ if n = -3 — Tom's answer 11

2. Evaluate $3x^2$ if x = 2 — Tom's answer 36

3. Evaluate $5(a - b)^2$ if a = 3 and b = 5 — Tom's answer 20

4. Evaluate $\frac{2t^2 + 20}{5}$ when t = - 4 — Tom's answer -60

Part A:
What are the correct answers to Tom's homework questions.

Part B:
For each of Tom's homework questions, explain briefly how Tom probably arrived at his answers.

Part C:
Write a report (less than 150 words) generalising the misunderstandings, or practices, that could explain Tom's responses.

Figure 3 Teacher scenario *Tom's Troubles*

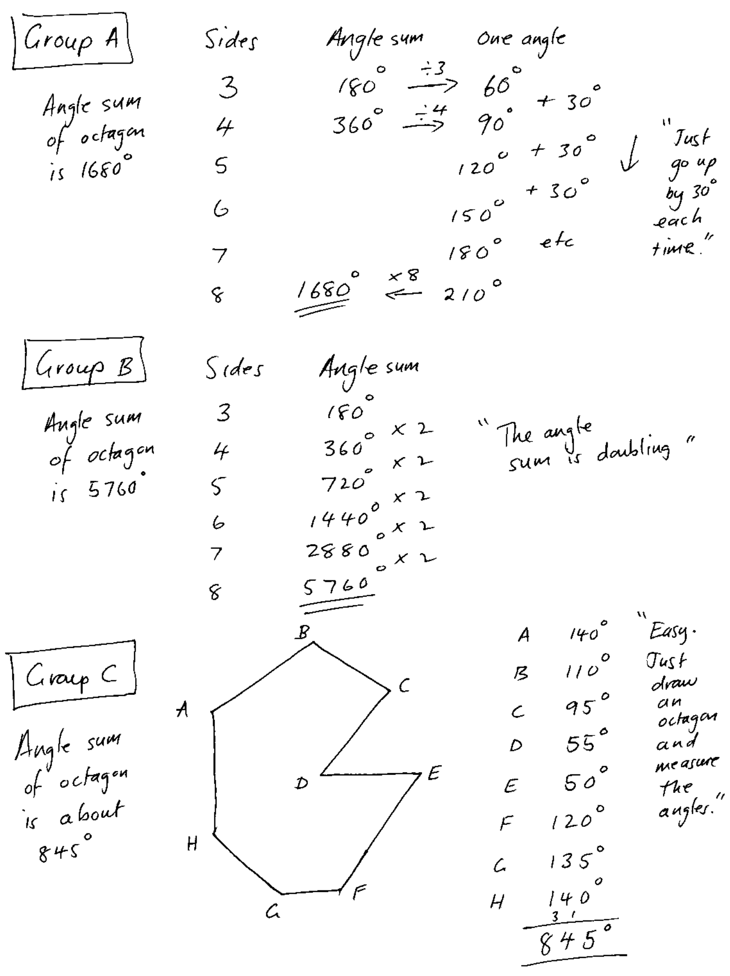
Polygon Patterns

A Year 8 class was set the following problem Geometry question as a problem solving activity.
"What is the sum of the interior angles of an octagon?"

The students were familiar with the names and properties of angles and polygons and were assigned to groups. After about 15 minutes, most groups had made what they felt was good progress and were ready to report their findings and conjectures to the class for discussion.

The following reports were given by three groups on the whiteboard before the lesson ended.

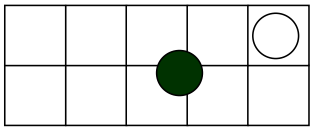
STUDENT WORK



1. For each group with an incorrect answer briefly explain where have they made their error(s)?
2. Briefly explain what question you would ask each group that has made an error to help them confront and determine their faulty practice.
3. In the next lesson, Group C said that they now believed their answer was incorrect because the octagon that they had drawn was concave rather than convex. They felt that the sum of the interior angles of a concave octagon would differ from that of a convex octagon. Is this belief true or false? Explain in about 50 words and with suitable diagram(s), if appropriate.

Figure 4 Teacher scenario *Polygon Patterns*

Coin Toss



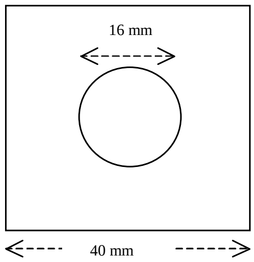
In a Probability unit, the students have been attempting to apply their knowledge to games of chance.
Coin Toss is a popular carnival game where participants attempt to land a coin within a square so that the coin does not cross the sides of the any square.
In the diagram shown, the white coin is a winning throw but the black one is not.
To determine an empirical estimate of the chance of winning and create some discussion you divide the class into two groups and allow each group to determine their own experiment al procedure.
Group A decides that each member should throw the coin until they have a win and record the number of throws taken.
Group B decides that each member of the group should have 10 throws each and record the number of wins that they have. Their r esults and methods follow.
Group A: (11 students)
Results 3, 12, 17, 5, 10, 9, 12, 14, 25, 8, 13
Working $\left(\frac{1}{3}+\frac{1}{12}+\frac{1}{17}+\frac{1}{5}+\frac{1}{10}+\frac{1}{9}+\frac{1}{12}+\frac{1}{14}+\frac{1}{25}+\frac{1}{8}+\frac{1}{13}\right)\div 11\approx 0.117$

Group B: (10 students)
Results 0, 1, 0, 0, 1, 2, 0, 0, 1, 0
Working (1 + 1 + 2 + 1) ÷ 100 = 0.05

Part A:
Are both these methods valid ways of obtaining an empirical estimate of the probability of winning the game? Explain.

Part B:
When investigating the theoretical probability of winning the game, with a 16 mm coin and 40 mm squares, three models are proposed by the class . Which model would you support and why?

Model 1

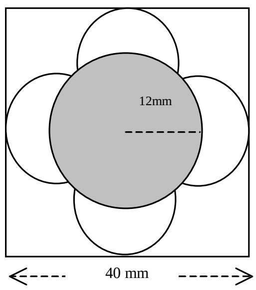


The probability of winning is the ratio of the area of the coin to the area of the square.

$$\frac{\pi\times 8^2}{40^2}\approx 0.126$$

(Not to scale)

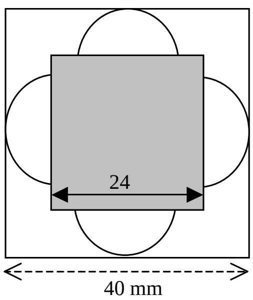
Model 2



The probability of winning is the ratio of the area of the circle (formed by the centre of the coin as it moves around just touching the four sides of the square) to that of the square

$$\frac{\pi\times 12^2}{40^2}\approx 0.283$$

Model 3



The probability of winning is the ratio of the area of the square (formed by the centre of the coin as it moves around just touching the four sides of the square) to that of the square

$$\frac{\pi\times 24^2}{40^2}\approx 0.36$$

Which model would you support and why?

Figure 5 Teacher scenario *Coin Toss*

Administration of the scenarios

The scenarios were sent to teachers as part of a questionnaire. They were not, therefore, administered under controlled conditions, so some caution is needed when considering the importance of the results reported below. It is possible, for example, that teachers collaborated with each other when completing the scenarios. (A check of this possibility by examining how similar answers were among teachers from the same school, suggested that this was a rare occurrence.)

Results

Responses by the teachers

Of the teachers who completed and submitted Part A of the teacher questionnaire, 89% also completed Part B (the scenarios). This was an encouraging response. Of the 21 who did not, four objected or submitted facetious responses, five claimed insufficient time, three were not returned, one clearly collaborated with colleagues, and eight others made insufficient attempts (less than 3 of the 8 items) or sent the Part B survey back uncompleted. Of the 21 who did not complete Part B sufficiently 16 were male teachers.

Summary statistics

Summary statistics for the teachers' scores are shown in Table 6

Table 6 Summary statistics of teacher scores on mathematical problem scenarios

	Maximum score possible	Mean	Median	Range	Inter-quartile range	Std Dev
Knowledge of Content	12.5	9.2	9.5	0.0 – 12.5	7.5 – 11.5	2.49
Knowledge of Pedagogy	11.5	8.0	8.5	1.0 – 11.5	7.0 – 9.5	1.98
Total Knowledge	24.0	17.2	18.0	4.0 – 24.0	15.0 – 20.0	3.95

The distributions of teacher scores over both types of Knowledge were different. For *Knowledge of Content* there was a large number of teachers clustering together at the upper end (negative skew), whereas for *Knowledge of Pedagogy* this was not the case and the scores were more 'normally' distributed.

Analysis of relationships between measures of teacher knowledge and other variables

<u>Gender</u>

Around 55% of the teachers in the analysis were male and 45% were female. It can be seen from Table 7 that while there was no difference between the overall mean achievement of male and female teachers, male teachers achieved slightly better on the Knowledge of Content items whereas female teachers achieved slightly better on the Knowledge of Pedagogy items. These differences were not statistically significant ($P > 0.05$).

Table 7 Mean teacher knowledge scores by gender

Scores by gender	Number	Knowledge of content	Knowledge of pedagogy	Total knowledge
Male	90	9.5	7.8	17.3
Female	74	8.9	8.3	17.2

Years teaching

Comparisons based upon the number of years teaching mathematics were also made. Table 8 shows the mean scores for teachers by the number of years they had been teaching.

Table 8 Mean teacher knowledge scores by years teaching mathematics

Years teaching mathematics	Number of teachers	Knowledge of content	Knowledge of pedagogy	Total knowledge
1 to 5	21	8.4	7.5	15.9
6 to 10	27	8.9	7.9	16.8
11 to 15	24	8.8	7.7	16.5
16 to 20	21	9.3	8.1	17.4
21 to 25	21	9.5	8.0	17.5
26 to 30	26	9.7	8.1	17.8
>30	19	9.9	9.0	18.8

It is interesting to note the trend towards higher scores as the number of years teaching mathematics increases. However this trend is weak. (A measure of the strength of this trend is given by Pearson's *r*, which ranges in values from +1 to –1, where 0 is no trend.[1] Pearson's correlation between *Years Teaching* mathematics and *Total Knowledge* is weakly positive at 0.2, although still statistically significant.)

Initial teacher education

There was a statistically significant relationship between whether teachers had received training in the teaching of mathematics in their initial teacher education program and:

- *Pedagogical Knowledge* (r = 0.36; P<.001) – this is a moderately strong association, and;

- *Total Knowledge* (r = 0.25; P<.001) – this is also a moderately strong association

This should be regarded as a key finding from this study..

[1] While there are no fixed rules concerning how to interpret the value of a correlation co-efficient, a rough guide, in the context of typical social scientific research is as follows: 0-0.19 weak; 0.2-0.39 moderate; 0.4-0.59 strong and 0.6 and greater very strong.

Mathematics background

There were statistically significant though weak, relationships between whether teachers had majored in mathematics in their undergraduate degree and *Knowledge of Pedagogy* (r = 0.16; P<0.054) and *Total Knowledge* (r = 0.17; P<.04).

Teachers who said their main teaching subject was mathematics also did significantly better on the test of *Pedagogical* and *Content Knowledge* (r = 0.17; P < 0.03). These relationships did not hold up for teachers who only had a minor or less in mathematics in their first degree.

These should be regarded as key findings from this study.

Other findings

There were also weak, but statistically significant, correlations between teachers' *Pedagogical* and *Content Knowledge* in mathematics and: (a) student scores on the *PATMaths* test; (b) student-reported enjoyment of mathematics; (c) student ratings of their teachers' pedagogical skills, and; (d) student ratings of the quality of the learning environment in their mathematics classes.

Conclusion

The high proportion of teachers who completed these assessments in a questionnaire format was a positive outcome.

The use of these scenarios provides some evidence that:

- teachers who had received training in the teaching of mathematics in their initial teacher education program had higher levels of *Pedagogical Knowledge* and *Total Knowledge*

- teachers who had majored in mathematics in their undergraduate degree had higher levels of *Knowledge of Pedagogy* and *Total Knowledge* (r = 0.17; P<.04)

- teachers who said their main teaching subject was mathematics had higher levels of *Pedagogical* and *Content Knowledge*

- teachers who scored higher on *Pedagogical* and *Content Knowledge* in mathematics tended produce higher levels of student achievement, and have students who tended to report higher levels of enjoyment of mathematics, gave higher ratings of their teachers' pedagogical skills, and of the quality of the learning environment in their mathematics classes.

6 RESULTS: EFFECTIVE MATHEMATICS TEACHERS (1)

This section of the report describes and investigates the data and relations within these data, in a largely descriptive way. This approach is designed to give a sense of the possible strength of relations between teacher practices and knowledge and the growth in a range student outcomes related to the teaching of mathematics. This growth can be taken as an indicator of teacher effectiveness.

The first section provides simple summary statistics concerning the extent of growth between Time 1 and Time 2 for each of nine student outcome variables. The second section describes the results of a series of multiple regression analyses that were undertaken to investigate those factors which contributed to changes in student outcomes, and hence were associated with effective teaching of mathematics. The third section examines data the data using cross-tabulations designed to contrast ‘high’ and ‘low’ scoring PISA schools as a way of identifying effective teaching practices.

Descriptive statistics concerning change over time in student scores

An examination of Table 9 shows that, on average, there was very little change measured in a wide range of outcome variables. For example, it can be seen that the mean PATMaths score at Time 1 was 59.8 and that this had reduced to 59.0 at Time 2. Results such as these made it difficult to detect the impact of teachers and hence to identify the characteristics of those who were most effective.

Table 9 Mean scores at Time 1 and Time 2 of student outcome variables (showing also standard deviation of scores)

	Mean	***SD***
PATMaths score Test 1	59.8	7.72
PATMaths score Test 2	59.0	8.90
Student's Perceived Effort in Mathematics Time 1	2.6	0.56
Student's Perceived Effort in Mathematics Time 2	2.6	0.55
Student's Perceived Mathematical Ability Time 1	2.8	0.44
Student's Perceived Mathematical Ability Time 2	2.6	0.44
Student's Perceived Learning Environment Time 1	2.6	0.56
Student’s Perceived Learning Environment Time 2	2.6	0.58
Student's Perceived Task Load in Mathematics Time 1	2.7	0.36
Student's Perceived Task Load in Mathematics Time 2	2.6	0.36
Student Perceived Utility of Mathematics Time 1	3.0	0.74
Student Perceived Utility of Mathematics Time 2	2.3	0.79
Student Self Efficacy in Mathematics Time 1	2.4	0.50
Student Self Efficacy in Mathematics Time 2	3.2	0.53
Student Enjoyment of Mathematics Time 1	2.9	0.68
Student Enjoyment of Mathematics Time 2	2.3	0.68
Student Motivation in Mathematics Time 1	2.3	0.54
Student Motivation in Mathematics Time 2	3.2	0.56

Multivariate analysis – investigations of change in student outcomes

Overview of the analyses

The results reported in this section were obtained by using multiple regression. This is a procedure which, as Kerlinger and Pedhazur (1973, p. 3) note:

> … is a method of analysing the collective and separate contributions of two or more independent variables … to the variation of a dependent variable.

In this section of the study, a series of multiple regression analyses are conducted to provide a measure of the extent to which there has been change in a range of student outcome variables.

In the analyses reported here, the dependent (outcome) variable was always taken from Time 2. However it was not the level of a variable at Time 2 of itself which was of interest, but rather the extent to which there had been a change between Time 1 and Time 2. It was therefore necessary to take account of the effect of the level at which the students started. The regression equation, therefore, always included a matching variable at Time 1 so that the proportion of variance seen in the outcome due to the initial starting condition was statistically controlled. For example, when considering say, the effect of teacher pedagogical knowledge on change in student motivation, the regression equation took the form:

Student motivation Time 2 = Student motivation Time 1 + Teacher Pedagogical knowledge (+ error)[*]

This procedure, in effect gives the correlation, in the above example, between teacher pedagogical knowledge and the *change* in student motivation.

Constraints

The regression analyses were designed to give a sense of possible connexions between teacher practices and knowledge and student outcomes. However, these analyses are conducted under a range of constraints that mean caution should be exercised in attaching substantive significance to the findings. The constraints under which these analyses were conducted include:

1. The data are multilevel in structure (students within classes within schools) and this is not taken into account in these analyses. This means that, among other problems, there is an increased risk of falsely rejecting the null hypothesis – that claiming that there is a change when in fact there was no real change – a Type 1 error (Rowe, 2003);

2. The number of cases available for analyses was often reduced because of low response rates at Time 2;

3. The scales used in some of these analyses require improvement;

4. There is no control for the effect of family background; and

5. Preliminary results (see Table 9) suggest that was little evidence of growth among the students, making detection of this growth difficult.

[*] For ease of explanation, the intercept term has been omitted from this equation.

Reporting the results

There are three parts of the results of the regression analyses that are reported here: (a) whether the independent variable of interest makes a statistically significant contribution to explaining the dependent variable; (b) what proportion of variance it explains in the dependent variable, and; (c) whether it is associated with a positive or negative change in the dependent variable. The criterion used to establish statistical significance was the conventional value of 0.05. The proportion of variance explained is given as a percentage.

Growth in mathematics achievement – PATMaths

Table 10 lists those variables that were associated with growth in a *PATMaths* score, and the proportion of variance that each variable accounted for in this growth.

Table 10 summarises the results of 24 different regression analyses – one for each row of the table. Each row of the table presents the contribution of an independent variable to the measured growth in PATMaths scores between Time 1 and Time 2. For example, in the third row of the table it can be seen that having a mathematics major in undergraduate study accounted for 1.9% of the variance in the growth in observed PATMaths scores. Each independent variable listed in Table 10 has the question number from the student questionnaire from which these data were taken. Some independent variables were constructed by adding together different variables. Where this occurs, the source of each item is shown, for example, the variable *Collegiality in the school* was constructed by aggregating the scores from Questions 22a, b and d.

Many of the independent variables used in these analyses had small effects on the dependent variable and are sometimes negative. A negative association means that a decrease in the value of the variable is associated with an increase in growth of the *PATMaths* score. It was expected that all independent variables (apart from the sex of the teacher) would have a positive effect on the *PATMaths* score.

Table 10 Proportion of variance explained by a range of independent variables in the change in *PATMaths* scores between Time 1 and Time 2, also showing direction of the effect

Independent variable	Proportion of variance	Direction of effect
Sex of teacher (Q1)	0.8%	Female teachers have a positive effect
Level of teacher's education (Q2)	1.2%	+
Mathematics major in undergraduate study (Q4)	1.9%	–
Mathematics major in post graduate study (Q6)	0.9%	–
Mathematics is main teaching subject (Q8)	0.6%	–
Years teaching (Q9)	0.1%	+
Years teaching mathematics (Q10)	0.3%	+
Member of AAMT (Q14)	0.3%	+

Independent variable	**Proportion of variance**	**Direction of effect**
Index of PD Support (Q15a-f)	1.0%	+
PD Experience – School support & feedback (Q19j-m)	0.4%	+
PD Experience – Freq of Feedback (Q20a,b)	0.4%	+
PD Experience – Content focus (Q21a-c)	0.2%	–
Collegiality in school (Q22a,b,d)	0.5%	+
Collegiality by visits in school (Q22c,e)	0.3%	+
Computer resources in school (Q23abce)	0.6%	+
Other resources in school (Q23dfhi)	2.3%	+
Influence of the Mandated Curriculum on teacher practice (Q25abe)	0.3%	+
Influence of profession on teacher practice(Q25f-i)	1.0%	+
Professional Community in the School (Q26a-e)	1.2%	+
Innovation in the school (Q27a-df-h)	0.5%	+
Leadership in the school (Q28a-h)	0.5%	+
Teaching Practices (Q29a-d,gh)	0.6%	+
Teacher content knowledge	0.5%	+
Teacher pedagogical knowledge	0.3%	+

An examination of Table 10 shows that all variables displayed had weak associations with growth in *PATMaths* achievement scores. Having mathematics as part of the teacher's undergraduate study, as a major in post graduate study and as a main teaching subject were all associated negatively with growth in *PATMaths* scores. It is not clear why this should be so. However, positive professional development experiences by teachers were related to growth in *PATMaths* scores. Teacher mathematical content and pedagogical knowledge was also associated with growth in *PATMaths* scores.

Change in attitudes to mathematics achievement

There was a range of scales designed to measure various aspects of students' attitudes towards mathematics. These, all based upon reports by the student, included:

- Perceived effort in mathematics;

- Perceived mathematical ability;
- Perceived task load involved in the study of mathematics;
- Perceived environment;
- Perceived utility of mathematics to the student;
- Reported mathematical self-efficacy of the student;
- Reported enjoyment of mathematics by the student; and
- Reported level of motivation for the study of mathematics by the student.

Each of these variables was examined to identify those variables associated with their growth in the students. The full suite of variables used to examine growth in *PATMaths* scores was used, and while there were a number of these variables where weak effects were observed, it was clear that the most important variables across nearly the whole set of these student attitude variables was the students' perceptions of their teachers practices. For this reason this section of the report focuses upon these variables.

Teaching practices as perceived by the students were reported at Time 2 only (Questions 53 to 86 on the student questionnaire). These practices consisted of: (a) knowledge of mathematics; (b) personal attributes of the teacher; (c) assessment and feedback (d) effective pedagogy; (e) knowledge of students; and (f) the quality of the student learning environment. Table 11 shows the proportion of variance explained in growth towards a positive attitude by students. All associations between the independent and dependent variables were positive.

Table 11 The proportion of variance explained by a set of independent variables measuring aspects of teacher practice on growth in various student attitudes to mathematics

	Teacher practices (Independent Variables)					
DVs	**Knowledge of maths***	**Personal attributes**	**Assessment and feedback**	**Effective pedagogy**	**Knowledge of students***	**Learning environ-ment**
Effort*	1.4%	1.5%	1.2%	1.8%	1.3%	0.7%
Ability**	1.0%	1.3%	0.4%	1.5%	1.6%	*ns*
Task load**	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*
Environ-ment**	13.0%	21.6%	12.9%	20.5%	18.2%	12.5%
Utility	4.6%	6.3%	5.6%	7.2%	6.1%	3.1%
Self-efficacy	0.2%	*ns*	*ns*	*ns*	*ns*	*ns*
Enjoyment	9.6%	9.9%	7.4%	10.9%	9.0%	4.4%
Motivation	4.2%	7.1%	3.5%	4.6%	5.5%	3.0%

** This scale had low reliability (Cronbach's alpha >0.5 < 0.7)*
*** These scales had very low reliability (Cronbach's alpha < 0.5)*
ns = not statistically significant

An examination of Table 11 shows that the attitude most affected by this set of teacher practice variables is the student's views of the quality of their classroom environment. They have virtually no detectable effect on student self-efficacy in mathematics. Enjoyment is another attitude which appears amenable to change. For both the classroom environment and the enjoyment variables, the strongest effect in their growth appears to be a teacher who is perceived to have positive personal attributes and good pedagogical skills. Thus, it may be concluded, from these data that an effective mathematics teacher is one who is liked by the students and who uses an effective pedagogy. The low reliability of the classroom environment scale suggests that this conclusion should be treated cautiously for this variable. This conclusion is stronger when based upon the more reliable *Enjoyment of mathematics* scale.

Differences between 'high' and 'low' PISA schools

In this section of the report cross tabulations are used to examine differences between schools. These schools are grouped according to whether they had, on average, high or low scores on mathematical literary as measured by the PISA assessments. The extent to which indicators of effective teaching were present in high scoring PISA schools and absent in low scoring PISA schools is taken as an indication that they are associated with effective teaching.

The immediate work group, the subject department, and the quality of teaching

Mathematics teachers, like most teachers, usually practice in isolation, but the working relationships they have with colleagues can play an important part in their effectiveness. Over time, the culture of work groups may precipitate powerful norms that shape member attitudes, values and behaviour, as social psychologists have reported since the Hawthorne studies of the 1930s. In the case of schools, the culture of the subject department may shape attitudes to students and expectations for their success. The department culture may promote a climate of openness and experimentation in teaching, or one of privacy and low expectations for peer review and accountability. When these norms of collegiality and experimentation are strong and shared, as Little (1982) found in her work, they can significantly enhance the quality of professional learning on the job and so, also, student learning outcomes.

It became apparent early in the study, after visiting schools and talking to teachers and principals for the case study component, that mathematics departments differed greatly in their level of 'connectedness' between teachers; the extent to which they engaged in joint work and operated as a coordinated unit. Heads of mathematics departments also differed markedly in terms of the resources at their disposal to promote professional development among their staff, and their capacity to exercise leadership in developing coherent mathematics programs. They also differed in terms of the discretion they had over staffing and processes for teacher accountability. In some schools the very notion of the 'mathematics department' seemed to be under threat, or to be regarded as an anachronism and a barrier to curriculum and pedagogical innovation. One school principal, for example, pointed out that,

> Many teachers in Years 8 to 10 only teach one or two maths classes – it's not their first area. And the Maths coordinator was appointed at the end of Term 2 as an add on to his role as a science coordinator.

The case studies suggested that the Head of Department role had retained much the same status in independent schools as thirty years ago, but had declined in terms of salary, authority and status in other schools. There was, however, little Australian research literature on the changing nature of subject departments and the differing role of subject department heads in different school systems, or the effects of these changes on the quality of curriculum programs and student learning opportunities. So, this section is designed to explore the relationship between the immediate work group, the subject department, and the quality of teaching.

Method

Preliminary interviews were conducted with a small number of principals and department heads early in the study about factors that helped or hindered their ability to provide effective mathematics programs for students in Years 7 to 10. Most of these principals and department heads had strong views on the topic. The factors they saw as affecting their ability to provide effective mathematics programs fell into the following categories: (a) the culture of the mathematics department; (b) difficulties in attracting and retaining staff; (c) the constraints on innovation caused by centralised curriculum and external examinations; (d) the attitudes of experienced mathematics teachers toward change; (e) parental attitudes; (f) resources; and (g) lack of opportunities for professional development.

A questionnaire was developed based on these interviews. It consisted of a series of statements that principals and department heads could respond to, indicating the extent to which they agreed or disagreed on a four point Likert-type scale. These are the questions making up the second section of the Principal's questionnaire. These questions were dichotomised so that a score of 1 or 2 was recoded to 1, and a score of 3 or 4 was recoded to 2, where 1 was labelled 'Low' and 2 was labelled 'High'. These data were then combined in cross tabulations and *Fisher's Exact* test applied (as $n > 8$ and < 50) (Langley, 1971, p. 292) to test for evidence of association.

Fisher's Exact test is used when the sample size is less than 50 cases. As it is used here, it provides an estimate of the probability of the observed frequencies having arisen if there were no association between a dependent variable (say, level of professional community in a school) and whether the school was a high or low scoring PISA school. Where the probability is low (less than 0.05 is one conventional cut off point) it may be concluded that there is an association between, keeping with the example, whether there is a high or low level of professional community in a school and whether it is a 'high' or 'low' PISA school.

Professional community

The literature review referred to research on professional community at the school level and its influence on teacher capacity. Louis, Kruse and Marks (1996) identified the key elements of professional community as: (a) shared norms and values; (b) a collective focus on student learning; (c) collaboration; (d) de-privatised practice; and (e) reflective dialogue. Darling-Hammond (1992) points out strong professional communities are also high on accountability. Teachers in such communities accept a mutual obligation to review their practices in the light of profession-defined standards. Weick and McDaniel (1993) drew attention to the need for teachers to have time for professional dialogue if they were to cope adequately with the value-laden, non-routine nature of decisions that have to be made about practice. It would be expected, therefore, that schools which had strong professional community would, on average, produce stronger student outcomes than those without such a community.

Table 12 shows that 16 of 17 principals and department heads in high scoring PISA schools said the level of accountability among teachers in the mathematics department was high, compared with five of 10 principals in low scoring PISA schools. On these data, it is likely that there is an association between PISA ranking and the level of accountability among teachers within mathematics departments. High levels of professional accountability appear to be associated with high average levels of student achievement in schools.

Table 12 Level of professional accountability among teachers within the mathematics department of 'high' and 'low' PISA schools

School PISA rank	Level of accountability		Total
	Low	High	
Low	5 (50%)	5 (50%)	10 (100%)
High	1 (6%)	16 (94%)	17 (100%)
Total	6 (22%)	21 (78%)	27 (100%)

P=0.015

Similarly, there was an association between levels of consensus among staff about standards for teaching and learning mathematics and whether the school had a high or low PISA score. Table 13 shows that all 17 high scoring PISA schools scored highly on this variable, while five of 10 low scoring PISA schools scored highly on it. High levels of consensus about standards for quality teaching and learning in mathematics appear to be associated with high average levels of student achievement in schools.

Table 13 Consensus about standards for quality teaching and learning in mathematics

School PISA rank	Level of consensus		Total
	Low	High	
Low	5 (50%)	5 (50%)	10 (100%)
High	0 (0%)	17 (100%)	17 (100%)
Total	5 (18%)	22 (82%)	27 (100%)

P=0.003

Principals and department heads also responded in a similar way when asked to report on teacher expectations of student success in mathematics in their schools. Table 14 shows these relations. High levels of teacher expectation that students will have success in mathematics appear to be associated with high average levels of student achievement in schools.

Table 14 Expectations of student success among mathematics teachers

School PISA rank	Level of expectations		Total
	Low	High	
Low	5 (50%)	5 (50%)	10 (100%)
High	1 (6%)	16 (94%)	17 (100%)
Total	6 (22%)	21 (78%)	27 (100%)

P=0.015

Consistent with the data on teacher retention, and reports of 'out of field' teaching described in the case studies, Table 15 indicates that low PISA schools appear to have had more difficulty in finding mathematics teachers who can provide leadership. As one principal stated:

> Most teachers in the maths department are reluctant to take on leadership – those who do take it on do so to limit change – the traditional approach to maths is very strong.

Table 15 shows that 6 of 10 school principals from low PISA schools, compared with 14 of 17 from high PISA schools could point to a number of teachers capable of providing leadership in the mathematics department.

Table 15 Availability of teachers who can provide leadership at the level of the mathematics department in the school

	Level of availability		
School PISA rank	**Low**	**High**	**Total**
Low	6 (60%)	4 (40%)	10 (100%)
High	3 (18%)	14 (82%)	17 (100%)
Total	9 (33%)	18 (67%)	27 (100%)

P=0.034

The following set of tables (Table 16 to Table 19) relates to other aspects of professional community. While the differences are not statistically significant, the tables point towards the possibility that there is a consistent difference between high and low PISA schools in reported levels of: collegiality (Table 16); willingness to work collaboratively (Table 17); openness about practice (Table 18); and time for teachers to plan and review their work together (Table 19).

It is important to recognise that it is premature to make claims about the direction of causality here. It is possible that high PISA schools are more able to attract the kinds of teachers who seek workplaces consistent with their preference for collegiality and accountability. Nevertheless, these findings suggest that it could be worthwhile investigating reasons for possible relationships between professional community, teachers' practice and student learning outcomes.

Table 16 Levels of collegiality and professional community among teachers within the mathematics department

	Level of collegiality …		
School PISA rank	**Low**	**High**	**Total**
Low	3 (30%)	7 (70%)	10 (100%)
High	1 (6%)	16 (94%)	17 (100%)
Total	4 (15%)	23 (85%)	27 (100%)

P=0.13 (that is there is a 1 in 7.7 chance that there is an association in this table. The conventional chance is set at 1 in 20)

Table 17 Willingness of mathematics teachers to work collaboratively as part of a team teaching approach

School PISA rank	Level of willingness …		Total
	Low	High	
Low	5 (50%)	5 (50%)	10 (100%)
High	4 (23%)	13 (77%)	17 (100%)
Total	9 (33%)	18 (67%)	27 (100%)

P=0.16 (that is, there is a 1 in 6.3 chance that there is an association in this table.)

Table 18 A culture of openness and collegiality among teachers of mathematics

School PISA rank	Level of openness …		Total
	Low	High	
Low	4 (40%)	6 (60%)	10 (100%)
High	2 (12%)	15 (88%)	17 (100%)
Total	6 (22%)	21 (78%)	27 (100%)

P=0.11 (that is, there is a 1 in 9.1 chance that there is an association in this table.)

Table 19 Time for teachers to plan and review their programs together

School PISA rank	Amount of time to plan …		Total
	Low	High	
Low	8 (80%)	2 (20%)	10 (100%)
High	9 (53%)	8 (47%)	17 (100%)
Total	17 (63%)	10 (37%)	27 (100%

P=0.16 (that is, there is a 1 in 6.3 chance that there is an association in this table.)

Attracting and retaining effective mathematics teachers

The case studies suggested that attracting and retaining effective mathematics teachers was a major issue for schools. As one principal wrote:

> I suppose our greatest problem is attracting quality maths teachers and keeping them. We have been fortunate over the past four to five years of having mostly well qualified teachers of mathematics. However, this year three experienced teachers leave us as well as two experienced science teachers. We have had extreme difficulty in finding suitable replacements and will probably start 2003 at least two specialist Maths teachers down.

Table 20 shows that 6 of 10 department heads in low PISA schools said they had difficulty in retaining well-qualified teachers of mathematics, while only 3 of 17 heads in high PISA schools said they had a similar problem. High levels of difficulty in retaining well qualified teachers of mathematics appear to be associated with low average levels of student achievement in schools.

Table 20 Difficulty in retaining well qualified teachers of mathematics

School PISA rank	Level of difficulty in retaining …		Total
	Low	High	
Low	4 (40%)	6 (60%)	10 (100%)
High	14 (82%)	3 (18%)	17 (100%)
Total	18 (67%)	9 (33%)	27 (100%)

P=0.03

Out of field teaching

In the case studies, several principals and department heads indicated concern about the extent to which they had to deploy staff trained in other subject areas to teach mathematics. This poses a threat to student learning outcomes in a variety of ways. One, mentioned by the principals and department heads, was that students may pass through secondary school without encountering teachers with an enthusiasm for their subject and a drive to pass that enthusiasm on to others. Table 21 suggests there may be a greater likelihood of students in high PISA schools encountering teachers who love mathematics and who pass on this enthusiasm to their students. (Note that the associations seen in Table 21 are statistically significant at only around the 0.10 level. This finding should be treated as indicative.)

Table 21 Proportion of teachers of mathematics who love their subject and pass their enthusiasm on to students

School PISA rank	Proportion who love maths …		Total
	Low	High	
Low	5 (50%)	5 (50%)	10 (100%)
High	3 (18%)	14 (82%)	17 (100%)
Total	8 (30%)	19 (70%)	27 (100%)

P=0.09 (that is, there is a 1 in 11.1 chance that there is an association in this table.)

Teacher attitudes

In the case study interviews, some principals expressed a degree of frustration with some of their mathematics teachers whom they perceived to be too “set in their ways”. A survey item was administered to Principals which sought further information about this problem. Table 22 shows that there was some evidence of differences between the high and low PISA schools in terms of the perceived willingness of specialist mathematics teachers in their ability to see learning from the students’ point of view.

Of the 10 department heads of low scoring PISA schools, 6 said they had some teachers who had difficulties seeing learning from the students’ point of view, while only 5 of 17 department heads in high PISA schools had this view. (Note that the associations seen in are statistically significant at only the 0.10 level. This finding should be treated as indicative.)

Table 22 Difficulties some mathematics teachers have in seeing learning from the students' point of view

School PISA rank	Level of willingness …		Total
	Low	High	
Low	6 (67%)	4 (33%)	10 (100%)
High	5 (31%)	12 (69%)	17 (100%)
Total	11 (44%)	16 (56%)	27 (100%)

P=0.10 (that is, there is a 1 in 10 chance that there is an association in this table.)

Parent and student attitudes

Table 23 shows that principals and department heads in low scoring PISA schools were more likely to report low levels of parental support for students. All principals and department heads from high PISA schools indicated parental support for students to do well in mathematics, while 6 of 10 from low PISA schools reported this support. High levels of parental support for students to do well in mathematics appear to be associated with high average levels of student achievement in schools.

Table 23 Parental support for students to do well in mathematics

School PISA rank	Level of parental support …		Total
	Low	High	
Low	4 (40%)	6 (60%)	10 (100%)
High	0 (0%)	17 (100%)	17 (100%)
Total	4 (15%)	23 (85%)	27 (100%)

P=0.05

Summary

The data from this small survey of principals and heads of mathematics departments suggests further investigation is warranted into the relationship between departmental and school characteristics and student outcomes. Importantly, there were differences found between high and low PISA schools in levels of professional community. Related, perhaps, to these differences were significant difference in: (a) the ability of high and low PISA schools to retain effective teachers; (b) levels of out of field teaching; (c) teacher attitudes to change; and (d) perceived parental attitudes.

8 RESULTS: EFFECTIVE MATHEMATICS TEACHERS (2)

The previous section examined bi-variate relations, focussing upon a range of potentially interesting associations that could give insight into, or understanding of, effective mathematics teaching. These analyses did not take account of the hierarchical structure of the data. In this study, student responses were nested within classes and classes were nested within schools. The analysis now takes account of this hierarchical structure as part of a more systematic, multivariate examination of teacher effectiveness in mathematics.

The data

The data set consisted of 1290 student responses. Of these students, 51% were females, 39% were at Year 8, 39% were at Year 9 and 22% were at Year 10. These students came from 116 different classes and from 32 different schools. There were thus an average of around 11 students from each class and 40 students from each school. Of the 32 schools, 22 were government, four were Catholic and six were independent schools. There were six low scoring PISA schools, and 15 high scoring PISA schools. The balance was made up of schools which had not participated in PISA.

The data set consisted of only those cases for which there were matching student, teacher and principal data. In other words, only those cases were included in the data file for which there was a school questionnaire, a teacher questionnaire and student questionnaire (plus *PATMaths* test results). It was still possible for there to be missing data, for example, when a respondent omitted to complete a question in a questionnaire. All cases had *PATMaths* scores from Time 1 and Time 2.

Missing data

Missing data may compromise analyses and it is possible under some conditions to reconstruct these using a variety of procedures. This was done with the data used for the analyses reported here. Basically these procedures seek to match the known characteristics of a case with similar cases and then replace the missing data with the most probable value based upon those matched cases without the missing data.

Missing data points were imputed by two methods using LISREL 8.5. Data missing from continuous variables were imputed using the EM algorithm from other variables and cases within the set. (No 'external' or 'matching' variables were used.) Data missing on nominal and ordinal variables were imputed employing the 'similar response pattern' method, again using only variables from within the set. Imputation was successful for all but 4 cases providing a data set for analysis of 1290 cases. (Note: after imputation, one variable had distributional characteristics that made it unsuitable to use in multi-level modelling – *the proportion of teachers with a minor sequence in mathematics at university*. This variable was not used in subsequent analyses.)

The multilevel analysis

Background

The analysis reported in this section was a multi-level analysis. A multivariate analysis seeks to simultaneously examine the effects of a set of independent variables on a dependent variable. In this study, the dependent variable was an indicator of teacher effectiveness. There is a wide range of multivariate techniques available. The method used here is based upon linear regression. This procedure permits the unique contribution of each independent variable to be estimated. Thus, in the results discussed below, the effect of any one variable in the model is reported net of the effects of other variables in the model. As will be seen, however, this method of analysis can encounter difficulties if two or more independent variables are correlated with each other. Where they are correlated, the analysis faces the problem of high co-linearity. Co-linearity may wash out the effects of the correlated variables because they make little unique contributions to the data. It may also produce unstable solutions which limit the interpretation of results. (The second is the more common problem.) Co-linearity, in fact, proved to be a major problem in the analysis of the data reported here.

Analysis of the data was conducted using *MLWin*. The first aim of the analysis was to examine how variance in the dependent variable was distributed across levels within the data. These levels were the student, the teacher (or class), and the school. If a substantial proportion was distributed at either the class or school level, then multi-level modelling is recommended (Rowe, 2003). Once it was established whether multi-level modelling is required, the regression analysis proceeded. There is a variety of approaches to the conduct of these analyses, and decisions about which is the best way forms part of the art of data analysis. These decisions are also shaped by the characteristics of the data, limitations imposed by the method, the theory driving the study and the research questions being addressed.

Approach to the analysis

To provide some degree of protection against multiple significance test effects, and, particularly, to provide a check on the likelihood of multi-co-linearity and the occurrence of 'suppression' effects (both between and within) conceptually similar clusters of variables ('blocks'), a 'block-wise' entry approach was used. If a large raft of variables is used in analysis, then there is an increased risk that a certain proportion of associations will be found to be significant as a function of random error. It is necessary to guard against this risk, and by entering variables in groups, this risk is reduced. Suppression occurs when a variable, not significant in a bi-variate analysis ‘becomes’ significant in a multi-variate analysis or when the sign of the relationship changes between the bi-variate and multi-variate analyses.

Explanatory (independent) variables were clustered into six blocks according to the theory guiding the research. These blocks were:

- Block 1: the Time 1 'pre-' measure of the response (dependent) variable under investigation (if available);

- Block 2: student and related characteristics (sex, year level, age as at Time 1, and the number of days between the tests);

- Block 3: school characteristics (computer resources, other resources, innovation levels, leadership quality, sector, size, proportion of students receiving Educational Maintenance Allowance, proportion of NESB students, teacher qualifications in mathematics, time spent teaching mathematics and, principal’s perceptions of resources, teacher quality, collegiality and family support in the school);

- Block 4: teacher professional development experiences (support, amount, range, type, the extent to which it related to collegiality, the influence of the mandated curriculum and the profession, and the extent to which there is a professional community in the school);

- Block 5: teacher characteristics (pedagogical and content knowledge, sex educational level and content, years teaching and teaching mathematics and whether a member of the AAMT); and

- Block 6: teaching practices (Student perception of teacher mathematics knowledge, teacher personal attributes, teacher assessment and feedback, teacher's pedagogy, knowledge of students, the learning environment, and the teacher's self report of teaching practice).

Each 'block' was tested for significance, as a block, when entered. This was done using a chi-square test based on the differences between the deviance (-2*loglikelihood ratio) for each model. If the block was significant, individual variables within the block were then assessed for significance, one at a time, over and above the previous variables in the model. This strategy was necessitated by the considerable degree of multi-co-linearity which was found to exist within many blocks. Those variables that were individually significant were then included in the model, and the next block entered.

The dependent variables

There were a nine dependent variables, each taken to be an indicator of effective mathematics teaching. These indicators were: (a) mathematics achievement as measured by growth in *PATMaths* scores; (b) effort; (c) perceived mathematics ability; (d) quality of the learning environment; (e) task load; (f) utility of mathematics; (g) mathematical self efficacy; (h) enjoyment; and (i) motivation. The higher the growth on one of these indicators, the more effective the teacher was taken to be. By examining the factors which contributed to a higher score, the intent was to identify those which were most strongly associated with effective teaching.

PATMaths

The level of mathematical achievement of the student was seen as an important educational outcome, and hence a useful indicator of effective teaching.

The variance *PATMaths* at Time 2 was partitioned across the three levels as follows: School, 19.9%; Class, 34.1%, and; Student 45.9%. This indicated that multi-level analysis was necessary (for correct parameter estimation and significance testing). Table 24 shows the results of this analysis. The block number refers to each of the blocks described in the above section of the report titled *Approach to the Analysis*. The '+' sign indicates that the variable was statistically significant and was positively associated with *PATMaths* score at Time 2.

The 'final model' accounted for 30.0% of the response variance. The *PATMaths* score at Time 1 accounted for 29.5% so the additional variables added only 0.5%. This is very little improvement. This increase is so small that a substantive interpretation of these results seems unwarranted.

Table 24 Results of multilevel analysis examining *PATMaths* scores at Time 2

Block No.	Response	*PATMaths* Time 2
1	*PATMaths* score Time 1	+
5	Teacher content knowledge	+
6	Student perception of teacher personal attributes	+
6	Student perception of teacher assessment and feedback	+
6	Student perception of teacher's knowledge of students	+
6	Student perception of the learning environment	+
6	Teacher self report of teaching practice	+

Effort

The effort a student made in studying mathematics was seen as an important educational outcome, and hence a useful indicator of effective teaching. These data were based upon student reports of effort required and may therefore be susceptible to compliance effects. The effort scale had poor reliability (Cronbach's alpha = 0.60, where the criterion for acceptable reliability using Cronbach's alpha is around 0.7 and above. Cronbach's alpha provides a measure of the internal consistency of a scale, that is the extent to which it appears to unidimensional.)

The variance in effort at Time 2 was partitioned across the three levels as follows: School, 7.4%; Class, 0%, and; Student 92.3%. This indicated that multi-level analysis was probably unnecessary, however analyses were still conducted using this method. This was done as multi-level analysis was necessary for some of the other affective outcomes, so to retain coherence, it was used for all. Table 25 shows the results of this analysis.

The 'final model' accounted for 22.6% of the response variance. Effort at Time 1 accounted for 19.6% so the additional variables added 3.0%. This is little improvement. On this evidence, the factors (weakly) positively associated with a growth in student effort in mathematics (as reported by the students) were: the time allocated by school to study mathematics, schools being in the non-government sector, and student perceptions of their teacher's mathematical knowledge, personal attributes, assessment and feedback, the quality of their pedagogy, knowledge of the students, and learning environment. Family support as reported by the principal was weakly negatively associated with student effort. This negative relationship may arise because parents with children not performing well, may take a more active role than those whose children are performing satisfactorily at school.

Table 25 Results of multilevel analysis examining reported effort in mathematics by students at Time 2

Block No.	Response	Effort Time 2
1	Effort Time 1	+
3	Government v other sectors (Government =1, Others = 0)	–
3	Time allocated by school to study mathematics	+
3	Family support as reported by the principal	–
6	Student perception of teacher mathematics knowledge	+
6	Student perception of teacher personal attributes	+
6	Student perception of teacher assessment and feedback	+
6	Student perception of teacher's pedagogy	+
6	Student perception of teacher's knowledge of students	+
6	Student perception of the learning environment	+

Ability

The perceptions that students have of their ability in mathematics was seen as an important educational outcome because it was likely to influence achievement as well as other outcomes, for example, enjoyment of mathematics. Again, these data were based upon student self reports. The ability scale had very poor reliability (Cronbach's alpha = 0.18).

The variance in reported mathematics ability at Time 2 was partitioned across the three levels as follows: School, 3.0%; Class, 2.5%, and; Student 94.5%. This indicated that multi-level analysis was unnecessary, however analysis were still conducted using this method. Table 26 shows the results of this analysis.

The 'final model' accounted for 25.9% of the response variance. Ability at Time 1 accounted for 21.6% so the additional variables added 4.3%. This is little improvement. On this evidence, the factors (weakly) positively associated with a growth in perceptions of student ability in mathematics (as reported by the students) were: the sex of the student (being male), the time allocated by school to study mathematics, schools being in the non-government sector, and student perceptions of their teacher's mathematical knowledge, personal attributes, the quality of their pedagogy, knowledge of the students, and learning environment. Family support as reported by the principal was weakly negatively associated with student perceptions of their ability.

Table 26 Results of multilevel analysis examining reported ability in mathematics by students at Time 2

Block No.	Response	Ability Time 2
1	Ability Time 1	+
2	Sex of the student (1 = male, 0 = female)	+
3	Time allocated by school to study mathematics	+
3	Family support as reported by the principal	–
6	Student perception of teacher mathematics knowledge	+
6	Student perception of teacher personal attributes	+
6	Student perception of teacher's pedagogy	+
6	Student perception of teacher's knowledge of students	+
6	Student perception of the learning environment	+

<u>Learning environment</u>

The perceived learning environment that a student had in mathematics was seen as an important educational outcome: the better the environment as perceived by the student, the better the learning that is likely to occur. The data measuring the learning environment were based upon student self reports. The variable was a scale made up of items from the first part of the student questionnaire and concerns a wide range of contexts in which the student learns. It is based upon Questions 4, 7, 9, 13, 21, 28, 29 and 35 of the student questionnaire which ask about learning at home and in the classroom. This variable is different from the other learning environment variable which is based upon students' reports of their teacher's practice given in the last part of the student questionnaire (which was used only at Time 2). The learning environment scale had very poor reliability (Cronbach's alpha = 0.31).

The variance in reported the quality of the learning environment at Time 2 was partitioned across the three levels as follows: School, 0%; Class, 14.8%, and; Student 85.2%. This indicated that multi-level analysis was necessary. Table 27 shows the results of this analysis.

The 'final model' accounted for 42.4% of the response variance. Perceptions of the quality of the learning environment at Time 1 accounted for 15.9% so the additional variables have added 26.5%. This is a considerable improvement and an interesting result, although the poor quality of the scale needs to be also considered. (Its poor reliability means it may be loaded with measurement error and the results therefore an artefact of this error.) On this evidence, the factors positively associated with a growth in perceptions of the quality of the learning environment (as reported by the students) were: student perceptions of their teacher's mathematical knowledge, personal attributes, the quality of their pedagogy, knowledge of the students, and learning environment.

Table 27 Results of multilevel analysis examining reported mathematics learning environment by students at Time 2

Block No.	Response	Learning environment at Time 2
1	Learning environment at Time 1	+
6	Student perception of teacher mathematics knowledge	+
6	Student perception of teacher personal attributes	+
6	Student perception of teacher's pedagogy	+
6	Student perception of teacher's knowledge of students	+
6	Student perception of the learning environment	+
6	Teacher self report of teaching practice	+

Task load

The perceived task load that a student had in mathematics was seen as an important educational outcome. That is, the more the student saw themselves as being involved in, and challenged by, the work the more likely they would be to achieve positive results. Once again, these data were based upon student self reports, and so are limited because of the risk of compliance effects. This variable was a scale made up of items asking about the ease, difficulty, familiarity and interest that students experienced in tackling mathematics. The scale had very poor reliability (Cronbach's alpha = 0.45).

Table 28 Results of multilevel analysis examining reported task load of mathematics by students at Time 2

Block No.	Response	Task load at Time 2
1	Quality of learning environment at Time 1	+
6	Student perception of teacher mathematics knowledge	–
6	Student perception of teacher personal attributes	–
6	Student perception of teacher assessment and feedback	–

The variance in reported task load at Time 2 was partitioned across the three levels as follows: School, 1%; Class, 0%, and; Student 99%. This indicated that multi-level analysis was unnecessary, but nevertheless was undertaken. Table 28 shows the results of this analysis.

The 'final model' accounted for 9.9% of the response variance. Perceptions of the task load at Time 1 accounted for 8.9% so the additional variables added 1.0%. This is a very small improvement to the model. There were no factors in this model that were positively associated with a growth in perceptions of task load. A substantive interpretation of the negative associations between some of the student perception (Block 6) variables and task load seems unwarranted given how little of the variance (1%) is explained by these variables.

Utility

The perceived utility of mathematics was seen as an important educational outcome. The more useful students see mathematics, the more likely they are be to be motivated and perhaps engaged, leading to higher achievement. These data were based upon student self reports. The scale was based upon four items from the student questionnaire asking about how useful mathematics was for them now and in the future. The utility scale had satisfactory reliability (Cronbach's alpha = 0.84).

The variance in the reported utility of mathematics at Time 2 was partitioned across the three levels as follows: School, 3.7%; Class, 3.7%, and; Student 92.7%. This indicated that multi-level analysis was probably necessary. Table 29 shows the results of this analysis.

The 'final model' accounted for 22.1% of the response variance. Perceptions of the utility of mathematics at Time 1 accounted for 20.8% so the additional variables added 1.3%. This is a very small improvement to the model. On this evidence, the factors weakly positively associated with a growth in perceptions of the utility of mathematics were: student perceptions of their teacher's mathematical knowledge, personal attributes, feedback and assessment, the quality of their pedagogy, knowledge of the students, and learning environment. Year level of the student was weakly negatively associated with student perceptions of the utility of mathematics. Students in higher year levels were slightly less likely to report increases in the utility of mathematics.

Table 29 Results of multilevel analysis examining the reported utility of mathematics by students at Time 2

Block No.	Response	Utility at Time 2
1	Utility at Time 1	+
2	Year level	–
6	Student perception of teacher mathematics knowledge	+
6	Student perception of teacher personal attributes	+
6	Student perception of teacher assessment and feedback	+
6	Student perception of teacher's pedagogy	+
6	Student perception of teacher's knowledge of students	+
6	Student perception of the learning environment	+

Mathematical self-efficacy

Perceived mathematical self-efficacy was seen to be an important educational outcome. The more efficacious the students, the more likely they would be to be engaged and positive in their achievement. These data were based upon student self reports. The scale was based upon four items from the student questionnaire asking about how confident and well they could learn and do mathematics. The self efficacy scale had satisfactory reliability (Cronbach's alpha = 0.78).

The variance in reported mathematical self-efficacy at Time 2 was partitioned across the three levels as follows: School, 1.9%; Class, 6.4%, and; Student 91.7%. This indicated that multi-level analysis was probably necessary. Table 30 shows the results of this analysis. The 'final model' accounted for 17.4% of the response variance. Perceptions of the utility of mathematics at Time 1 accounted for 17.4% so the additional variables added nothing to the model. On this evidence, there were no factors in this model which accounted for growth in self-efficacy.

Table 30 Results of multilevel analysis examining the reported mathematical self-efficacy by students at Time 2

Block No.	Response	Self-efficacy at Time 2
1	Self-efficacy at Time 1	+

Enjoyment

The perceived enjoyment of mathematics was seen as an intrinsically important educational outcome. These data were based upon student self reports, so may be biased by compliance effects. The scale was based upon items from the student questionnaire asking about how self-confidence, enjoyment of mathematics and school, motivation, and excitement at doing mathematics. The enjoyment scale had satisfactory reliability (Cronbach's alpha = 0.83).

The variance in reported enjoyment of mathematics at Time 2 was partitioned across the three levels as follows: School, 2.8%; Class, 6.1%, and; Student 91.1%. This indicated that multi-level analysis was probably necessary. Table 31 shows the results of this analysis.

Table 31 Results of multilevel analysis examining the reported enjoyment of mathematics by students at Time 2

Block No.	Response	Enjoyment at Time 2
1	Enjoyment at Time 1	+
6	Student perception of teacher mathematics knowledge	+
6	Student perception of teacher personal attributes	+
6	Student perception of teacher assessment and feedback	+
6	Student perception of teacher's pedagogy	+
6	Student perception of teacher's knowledge of students	+
6	Student perception of the learning environment	+

The 'final model' accounted for 45.0% of the response variance. Perceptions of the utility of mathematics at Time 1 accounted for 30.2% so the additional variables added 14.8%. This is a notable improvement to the model. On this evidence, the factors positively associated with a growth in enjoyment of mathematics (as reported by the students) were: student perceptions of their teacher's mathematical knowledge, personal attributes, feedback and assessment, the quality of their pedagogy, knowledge of the students, and learning environment.

Motivation

The motivation to do mathematics was seen as an important educational outcome. The more motivated students are about studying mathematics, the more likely they will be engaged and achieve well. These data were based upon student self reports. The scale was based upon items from the student questionnaire asking about how motivated and keen the students were about mathematics, as well as how much they enjoyed and found worthwhile the study of mathematics. The effort scale had satisfactory reliability (Cronbach's alpha = 0.79).

The variance in the reported utility of mathematics at Time 2 was partitioned across the three levels as follows: School, 4.3%; Class, 3.1%, and; Student 92.6%. This indicated that multi-level analysis was probably necessary. Table 32 shows the results of this analysis.

The 'final model' accounted for 29.0% of the response variance. Motivation at Time 1accounted for 20.8% so the additional variables added 8.2%. This is a modest improvement to the model. On this evidence, the factors positively associated with a growth in motivation (as reported by the students) were: student perceptions of their teacher's mathematical knowledge, personal attributes, feedback and assessment, the quality of their pedagogy, knowledge of the students, learning environment. Year level of the student was weakly negatively associated with student perceptions of their ability. Students in higher year levels were slightly less likely to report increases in motivation.

Table 32 Results of multilevel analysis examining the reported motivation to study mathematics by students at Time 2

Block No.	Response	Motivation at Time 2
1	Motivation at Time 1	+
2	Year level	–
6	Student perception of teacher mathematics knowledge	+
6	Student perception of teacher personal attributes	+
6	Student perception of teacher assessment and feedback	+
6	Student perception of teacher's pedagogy	+
6	Student perception of teacher's knowledge of students	+
6	Student perception of the learning environment	+

Conclusion

The multi-level modelling reported here was undertaken in the context of problems caused by co-linearity in the data. One substantive interpretation of this is that the principals, teachers and students – each providing a different perspective – had a high level of agreement about what they were seeing and reporting upon. In other words, while this agreement caused considerable problems for the analysis, it does suggest that there is a broad consensus at the three levels (principal, teacher and student) about what these groups were seeing in their schools. This would suggest that for future investigation the knowledge of this convergence of views be taken into account in the design of instruments and the research more generally. One approach might be to adopt multi-level measures of the key factors associated with teacher effectiveness.

The multi-level analyses reported above, for a range of indicators of teacher effectiveness, share the following general characteristics:

1. The strongest predictor of an outcome at Time 2 was always the strength of that variable at Time 1;

2. None of the Block 4 variables (teacher professional development) contributed to the models;

3. Block 2 (student characteristics), Block 3 (school characteristics) and Block 5 (teacher characteristics) variables occasionally contributed to the models; and

4. Block 6 variables (teaching practices) were consistently the most important predictors of student outcomes, and of these predictors, student perceptions of teacher practice appear to have the strongest effect.

Table 33 shows the proportion of variance that was accounted for in the full model for each of the dependent variables and the proportion of the variance contributed by all variables in the model, excluding the Time 1 measure of the dependent variable. It can be seen that typically, Blocks 2 to 6 contributed very little to understanding the variance in these outcomes with the exception of the perceived learning environment and enjoyment of mathematics.

Table 33 Summary of multi-level results

Dependent variable	% of variance in full model	% of variance contributed by Blocks 2-6
PATMaths	29.5	0.5
*Effort	22.6	3.0
**Ability	25.9	4.3
**Learning Environment	42.4	26.5
** Task load	9.9	1.0
Utility	22.1	1.3
Self-efficacy	17.4	0
Enjoyment	45.0	14.8
Motivation	29.0	8.2

** Cronbach’s alpha >0.5<0.7, ** Cronbach’s alpha <0.5*

There are a number of possible explanations for these results. First, the problem of co-linearity in the data may have masked effects. Second, the measures used may be too insensitive to capture the change. Third, there may have been insufficient time for change to develop between Time 1 and Time 2. Fourth, there may be strong compliance effects as many of these variables are based upon student reports of their perceptions of teacher practice. Fifth, it may that these results are pointing to the truth, in which case self-efficacy appears particularly resistant to change, whereas perceptions of the learning environment and enjoyment of mathematics appears susceptible to change.

The focus of the study was, however, upon effective mathematics teaching. Only for *PATMaths* was there a teacher level variable (Block 3) which contributed to explaining variation in growth of student’s mathematical achievement. This was the variable measuring teacher content knowledge of mathematics. The Block 6 variables, which are measures of teacher practice described by students, are consistently the most powerful predictors of these dependent variables. This suggests that what teachers do in classrooms is the strongest predictor of growth in positive mathematical outcomes. However, these data do not permit identification of which factors contribute to identifying why students perceive their mathematics classes in these ways.

9 RESULTS FROM THE CASE STUDIES: THE MATHEMATICS DEPARTMENT AND EFFECTIVE TEACHING

A series of case studies were conducted for the study. These were designed to contextualise the work of the individual teachers and explore the professional community generated in their mathematics departments. This environment appears to be a crucial factor in supporting effective teachers.

A key aim of these case studies was to provide illustrations of the diversity of professional communities that can be found in schools.

Six case studies of school mathematics departments are presented below. The data for these studies were drawn from a wide variety of sources in the study. These included: (a) interviews with the Head of Department, several teachers, and in two cases, the Principal; (b) surveys of teachers; (c) surveys of Principals and Department Heads; (d) student attitude surveys and achievement tests; and (e) information taken from data collected by the PISA project (OECD, 2001).

The quotations used below come from the different staff who were interviewed at the schools. These quotations, and the other data referred to in these case studies, were not intended to represent the full range of views or data obtained from each school. Rather, the intention was to abstract out the essential qualities of each case and so highlight the particular strengths of each professional community.

Case 1: *Wilson's School*: creating a mathematics learning community

Wilson's School is an independent, co-educational school in a state capital city with a strong mathematical reputation. All students have personal laptop computers. The average PISA score for students from this school was above average. The school exists on two sites and has a Head of Mathematics at each site. Both Heads are involved in staff selection. The school has a mixed mathematics staff with mainly female teachers in the middle years, and with a few men in the senior years who, according to one teacher: "see themselves more as mathematicians than maths teachers, although they all trained as maths teachers". Training and staff development, especially in mathematics, are seen as the key to success. As one staff member noted:

> I think you need people who are fully maths trained…Research says that your best maths teachers have got to be at the junior level, because that's make or break time for the kids…You have to have a full understanding of maths to teach at Year 7.

There is a great deal of sharing of ideas with immediate colleagues teaching similar years, considerable email communication, and frequent informal meetings. The organisation of the staff creates a flat managerial structure with all staff involved in decision-making, planning and assessment: "there are more leaders than before…giving more people a say in what is happening."

Years 7 and 8 are 'blocked' for mathematics classes to allow for the mixing and team-teaching of classes, but as Year 9 is not a conventional year (with a special outdoor/adventure educational program) the mathematics teachers in that year work as a team and design their own curriculum. The overall mathematics curriculum is seen to be a general framework. As one teacher observed: "it works as a guide not a bible". A variety of texts and materials are selected, created, used, and evaluated by the staff.

The staff interviewed at *Wilson's School* see their goal as creating a mathematics learning community, which they called 'a Mathematics Network'. One expression of this Network is the extra, after-school mathematics problem solving sessions run by staff two days a week, sometimes based around the *Mathematics Olympiads*[∇]. Another is the way ideas are shared between staff. This is preferred to formal professional development. Staff take responsibility for leading developmental activities with other staff, or they may also engage someone from outside to work with the staff. For example, a recent initiative was a *Mathematician-in-Residence* program, which involved a postgraduate student working with the staff to create problem-solving units. These units were designed to engage the students in teams applying mathematics to real life situations, for example, bridge-building. All of these projects involved an excursion, supported financially by the school and parents. The school also hired an independent evaluator who conducted formative and summative evaluations of the initiative. Not all staff participate in these initiatives. (This is particularly true of those who see themselves more as 'pure mathematicians'.)

Not all professional development at *Wilson's School* is informal. There is compulsory professional development every Wednesday afternoon for all mathematics teachers. There are also professional development sessions at other times as well, for example, special seminars on graphic calculators. The teachers see this professional development as "functional and practical". Professional development in mathematics is recognised throughout the school as fundamental, and attendance at mathematics education conferences is supported. Accountability is high, assessment is frequent and used critically. Parent feedback occurs frequently and is taken very seriously, especially if it is about what the parents feel is an under-performing teacher. As one teacher observed:

> Ultimately we want our students to achieve well, and we meet them at lunch-times, we ring up parents, we follow through, we give them feedback…we get to know their parents very well.

Professional creativity is also encouraged and respected. According to one teacher at *Wilson's School*: "… you are really able to take a few risks and know that it won't be a major disaster". This risk taking is in part possible because of the achievement levels of the students coming into the school, but it also seemed to be related to the level of teacher confidence in and commitment to the mathematics learning community that they have been able to create.

So what is creating a professional community about, as seen from the perspective of this mathematics department? The following appear to be the main elements of this process:

1. A focus on the learning of mathematics as the key goal, for staff and students;

2. The sharing of ideas for teaching mathematics;

3. The quality of mathematics training and development of the staff;

4. Teamwork in teaching and in professional development;

5. Using outside experts as facilitators;

6. Regular compulsory mathematical professional development;

7. Accountability for the quality of mathematics learning; and

---

[∇] The *Mathematics Olympiad* is an internationally organised competition designed for high achieving students to represent their country. The focus of the competition is upon mathematical problem solving.

8. Creativity and risk-taking.

Case 2: *Richmond School*: making the tough decisions

Richmond School is a co-educational, government school in a small country town six hours drive from the state capital city. This school was above average on the PISA test. With three classes at each of Years 7, 8 and 9, it has a large percentage of students receiving the Education Maintenance Allowance, and a small, yet significant, number of Indigenous students. Less than ten students progress through to Year 12 studies, but, as was noted by one of *Richmond's* teachers: "it is important to the Richmond community that the school maintains a Year 11 and 12".

One of the major challenges for the Principal and the Head of Department at *Richmond School* is the transient nature of some of the teachers, together with the difficulty of attracting well qualified teachers to the school. As the Head observed:

> …this year three experienced teachers [of mathematics] leave us as well as two experienced science teachers. We have had extreme difficulty in finding suitable replacements and will probably start 2003 at least two specialist maths teachers down.

Only half the current teachers in the department have any initial training in mathematics teaching.

In order to be as effective as the school has been, as judged by its above average PISA score, some influential practices have been established by the Head of Department and the Principal. One of the most important of these practices is the use of streaming. The school receives students with a wide range of prior achievements. Streaming is designed to deal with this circumstance. Streaming into three classes takes place at Years 7, 8 and 9. It is based on *PATMaths* tests, and national and state profile levels. One of these streams is for remedial work.

The goal, according to the Head of Department is "to take the students to the best level we can". In order to do this, and because he felt that the state curriculum was inappropriate for many of the students, three years ago the Head of Department restructured the whole school course. He rejected the textbooks that were in use because they were seen to be too closely linked with the state structure. These have been replaced with the use of several 'alternative' mathematics programs and materials. The Head of Department also tried to set up one room as a specialist maths room, but, as he said, "it's hard to keep it in that standard because most rooms get trashed eventually". However the students do use graphics calculators, spreadsheets, and internet computer materials.

Staff meetings in the mathematics department were seen to be unproductive, largely because many staff taught multiple subjects. The Head of Department now makes decisions after informal discussions with the Principal and with individual staff. The Head noted: "I just give the information to the teachers". This is seen to be both easier and more effective by the Head.

One difficulty of adopting an autocratic approach to departmental management is the clashes that sometimes occur with reluctant staff. The support of the Principal is then critical. As the Principal commented:

> "I see [the Head of Department] as a maverick, but I can also see that the work he is doing in the school is fantastic. He probably works all of the periods with

> the kids and lunchtimes as well. He is a maverick, he will rub me up the wrong way and he will tell me to ‘get stuffed’. But I can put up with that …

The Head of Department sees a strong need for valid and up-to-date information about student and teacher progress. Consequently, the Head insists on regular testing, and uses *Maths Mate* test profiles and the *PATMaths* tests for regular student evaluation against state and national norms. He also uses *PATMaths* for teacher evaluation. He argued:

> we look at the change in the *PATMaths* scores and the change in peer percentile, relative to their peers. I did it on a class basis and it was quite clear which classes were going backwards and which classes weren’t, and it was very strongly linked to the teacher, who it was and what they did.

This practice has been continuing for four years. One consequence has been, according to the Head of Department:

> A lot of the people who got strong negatives were moved into other faculty areas and interests, because they didn’t have a strong maths background.

What then are the main lessons to come from this departmental situation, where some firm decisions have had to be made? The following are suggested:

1. Consideration needs to be given to how best to group students, with streaming used, based on appropriate diagnostic measures;

2. With mixed cultural backgrounds, socio-economic and achievement levels amongst the students, adoption of state curricula may not always be appropriate;

3. Assessment may play a crucial role for Head of Department decision-making, with state and national norms used for evaluation purposes;

4. An autocratic Head of Department, who may be achieving positive learning outcomes for students, will usually need the support of the Principal;

5. In small schools the Head of Department may need to work closely with individual staff; and

6. Creative evaluation and adoption of appropriate texts are crucial for achieving positive learning outcomes for students and monitoring their progress.

Case 3: *Charleston Grammar*: growing a professional community

Charleston Grammar is a boys independent school of some 1400 students with a strong reputation in mathematics. It has extensive Information Technology resources. The school scored well above average on the PISA tests. One third of the mathematics staff is involved as examiners in the state-wide end-of-secondary school tests. There is no separate mathematics staff room, but the Head of Department sees the faculty as “more of a support and interest group.” The staff is mixed in ages. Some older more experienced staff who “didn’t want to change, or compromise” have left since the current Head of Department arrived at the school about eight years ago. These teachers have tended to be replaced by experienced teachers from the state system, some of whom had retired at 55.

The Head of Department observed that it had been difficult to create the right level of collegiality, but that his strategies for doing so were now working well. The staff collaborate in order to keep each other up to date with the mathematics. He said.

> We all thrive off each other. One person will go to a meeting (like the annual teachers' maths conference)…and I will pick their brains or we will organise a meeting for the teachers to demonstrate this or that, and there will be prepared notes for the others...

There are no weekly meetings, but there are many informal ones, mainly focussed on mathematics. "We have to interact because of the common tests in the school." Also one afternoon after school, once a term, the staff have a meeting followed by refreshments. It is then that they discuss new texts, calculator programs, and it is also used as a time for sharing interests. The Head of Department commented:

> We're a consistent, coherent group with basically a program in mind, that we can execute, and there are no slackers …. those who can't hack it don't want to be in it. You have to build the ethos…(before) they had individual successes. I like to feel that these days, it's more of a group success.

Syllabuses are jointly and informally planned and are kept linked at Years 9 to 12 to the state examination syllabus. There have been adjustments to the curriculum, largely because of the state requirements. Textbooks are used but are seen by teachers as not being 'overused':

> We use the textbook for the fundamentals and for homework…they tend to be lacking in extension work…If we come across good materials we'll distribute single copies to each teacher, or if we think it's really good we'll do multiple copies.

According to the Head of Department, the staff "try to produce something that is going to challenge the kids and get them focused, a puzzle, a problem." The main goal is for students to be stretched.

Streaming is used at *Charleston*, but the classes all cover the same topics. There is no acceleration of the faster students but extension work is used. Students can get help after school two days a week, with staff rostered for that duty. There is testing every four to five weeks, with all marks going to the Head of Department for evaluation:

> We get a greater sense of achievement from the boys, they feel that they are being tested, challenged a lot, and they feel 'Gee the pressure is always on' but they seem to thrive on it.

Both tests and projects are used for evaluation. The school encourages competition, but within streams.

Staff are evaluated in their performance, and are allocated to classes by the Head of Department. There is considerable movement amongst the teachers across year levels.

The main findings to come from examining *Charleston* about growing a professional community in the mathematics department include:

1. The Head of Department needs to know the staff well;

2. Review and, if necessary, change the teaching allocations each semester so that status is not seen as a function of the levels or ability of students taught;

3. Keep the focus on the quality of the mathematics, through informal meetings, staff discussions, email exchanges and so on;

4. Encourage the exchange of ideas and information about mathematical topics and their teaching;

5. Use regular testing to both challenge the students and evaluate staff performance;

6. Pay detailed attention to the state syllabus requirements;

7. Involve the staff in the state examining process; and

8. Stream classes and monitor student progress.

Case 4: *Salisbury College*: maximizing effectiveness through staff diversity

Salisbury College is an all-girls, Year 7 to 12, systemic Catholic school in a pleasant suburb, with a mainly Anglo-Celtic culture amongst the 1200 girls. The school scored above average on the PISA test. About 10% of the students receive financial concessions. A somewhat higher proportion have a non-English speaking background. Of the 13 mathematics staff, six are specialists, two of whom have Masters degrees. The mathematics coordinator and the assistant coordinator receive a reduced teaching load.

The mathematics department has its own small staff room, where the staff frequently meet informally. They also have regular formal meetings. They have computers in the staff room for their own use with a connection to the Worldwide Web, together with site licences for various software packages.

There is considerable staff diversity at *Salisbury College*. This is well utilised. How it is utilised is illustrated by the staff allocations. These relate strongly to the student organisation, based on streamed classes from Year 7. For example, in Year 8 there are three bands with three classes at Level 1, four in Level 2, and two at Level 3. In the high achieving classes of Level 1, the staff agreed to take 32 or more students per class so that there would be less than 15 in the Level 3 classes. This enables the teachers to give more help to the Level 3 students. A staff Team leader was appointed for each Level, at each Year, and this teacher is responsible for coordinating the detailed unit content, assessment and teaching approaches for that Level group of classes. This is seen as a positive approach by the Head of Department who said:

> It takes the pressure off the Head of Department, which is a big pressure in a department of our size, but it is also a training ground for other teachers…

Not only are the staff diverse in their backgrounds, they also teach in different ways. The strategy of giving all the staff some responsibility for planning, and assessing, within their Teams guaranteed some structure and security whilst allowing for diversity to be recognized and utilized. As the Head of Department observed:

> "There is a group of three teachers and one becomes the administrator, but not a planner. You stick together and work through what your classes can manage … but not in terms of methodology … we all have our individual differences.

There are no accelerated classes at the school, the staff preferring to use an enrichment approach. Textbooks are used along with a variety of other materials. They also use a variety of assessment approaches, including rich tasks, group projects, and multiple choice questions, with well described criteria, rubrics, and marking schemes.

The mathematics department has concentrated on building a good mathematics department by finding ways to utilize the diversity of its staff, by:

1. Creating a flat administration so that each teacher in the department has a level of responsibility;

2. Recognising the need for teams to jointly plan the topics to be taught, timelines, and assessments;

3. Recognising that joint planning does not mean adopting identical teaching approaches;

4. Arranging for a well equipped mathematics staff room available to teachers all the time; and

5. Requiring the mathematics Coordinator to accept the diversity of staff as a benefit to be exploited, not a problem to be solved.

Case 5: *Herringbone School*: recognising independent professionalism

Herringbone is a selective and well-equipped government high school situated in a semi-rural setting just outside a capital city. The school scored above average on the PISA mathematics test. The students come from mainly middle to low income families with about a third of the students having a non-English speaking background.

The staff are also selected for this school, and because of this are thought by the Head of Department, and by themselves, to be at the peak of their careers. Their appointment to this school indicates high levels of competence and effectiveness This means that they are prepared to act independently in their teaching. The school appears to have a friendly and cooperative mathematics staff, with their own staff room. This encourages communication, and informal sharing of ideas.

The staff have an air of congeniality that comes in part from having their students among the state's highest mathematics achievers. There is an *esprit de corps* among the staff, but little collegial interaction on classroom issues. Everyone is treated as a competent professional and so left alone to do their job. The Head of Department highly rated the staff's love of mathematics and their ability to pass this enthusiasm on to their students.

The state syllabus statement is the basis for the mathematics program, and the main goal of the mathematics department is to maintain student results at the top of Year 12 state results. Consequently textbooks and other materials are selected for their effectiveness in supporting the state syllabus. Assessment is also jointly planned, and results monitored. The task at the junior levels is to prepare the students for mathematics courses at Years 11 and 12, and beyond. The staff have very high expectations for the students' achievements.

Students are first selected on the basis of the state administered tests, and then they are streamed, 'but not severely' because of school entry criteria. The higher achievers are kept within their age cohort and not accelerated into higher year-level mathematics. There is, however, a great deal of use of enrichment materials. There is a Mathematics Club after school, which helps prepare students for mathematics competitions and for participating in the *Mathematics Olympiads*. The staff are also rostered for work after hours with the boarders.

There is regular testing at the end of each chapter of the textbooks at the end of each term and at the end of each year. These tests are constructed by the Year level staff as a group. The focus is on procedural and factual aspects. As was noted by one of the staff: "understanding is important, but being able to do it is foremost!". Test results are evaluated against the state's test results. The test results do not appear to affect the classroom practices of the teachers. One teacher characterised the prevailing view among the staff thus: "Students are the ones who fail here not the teachers."

This then is the picture from the mathematics department at *Herringbone College*. It is one that believes in recognising independent professionalism among its staff. It does this through:

1. Being able to select its staff as well as its students;
2. Having clarity in its goal and purpose, in relation to state-wide assessments;
3. Monitoring of student achievement against state norms;
4. Focusing on the love of mathematics and its practices;
5. Conducting extra-curricular mathematical activities; and
6. Having high expectations of its students.

Case 6: *Green Park School*: a difficult environment

Green Park school provides the opportunity to examine the challenges confronting a Head of Department who is struggling to develop her department in a school where the students are not generally achieving highly. It is a government secondary school, situated in a state capital metropolitan area with more than 1200 students. The school scored below average on the PISA test. It once was a Technical School, and seems to be still thought of in that way by many parents. This view tends to be reinforced by the more elderly and traditional male teachers working at the school. But the school is apparently also seen by many in the community as one of the best in the area. It is popular with many parents, according to staff interviewed at the school, largely because it has worked hard to overcome its former image and because it gets some high end-of-school state examination results. More than 40% of its students are receiving the Education Maintenance Allowance and about 10% are from non-English speaking backgrounds.

The staff is a very stable one. As the Head of Department noted:

> We have a lot of long-term staff in this school … and a lot of them live in the town, and half a dozen or more have sent their children to this school, so they've not only made the choice to teach here, and they don't want to leave, they also have chosen to send their children to this school … a fairly positive indication of how highly most people consider this school…. Now whether it's because we actually enjoy being here … but there are certainly a lot of people that choose to actually be here and stay here, and wouldn't consider moving.

The staff have a variety of levels of qualifications in mathematics.

The Head of Department is enthusiastic and hardworking but has a difficult task to develop her department. The main difficulty is that, like all Heads of Department in her school, she only receives a three period allowance to act as the Head of Department. She runs a department of some 20 teachers. She is responsible for all the staff allocations, ordering all resources, planning the curriculum, organising professional development in the department as well having to deal with student discipline in the department (before handing particularly problematic cases to the relevant Head of Year). Despite this load, she still maintains a strong commitment to the teaching of mathematics. She said:

> I've made the choice, I've been here 17 years and I'm not interested in being a disciplinarian at a sub-school level … the maths curriculum is my first love, and that's what I'm interested in.

The time allowance problem is a reflection of the relatively low standing of the Heads of Department positions in the school. This is further reinforced by the lack of a money allowance. This applies to all Heads of Department, and contrasts sharply with the status and resources available to Year level coordinators and other staff involved in administration. The Head of Department sees a particular problem also in that nearly all of the senior staff are male. She noted:

> The people applying for the year level administrators were male and the people applying for the Heads of Department jobs were female, and I suspect that that was one of the big reasons why they get the money and the Heads of Department don't … It never used to be like that when I first started at the school.

The state department of education was planning, at the time of the case study, to give extra money to the school as part of the numeracy development in the state. The Head of Department had some plans for how best to use that money. She reported that:

> One of the things we're planning to do is to give the teachers the time allowance to meet once a fortnight, and then once a fortnight they will also then intervene in someone else's class, which could be team teaching, it could be taking out the slow learners from two or three classrooms – put them in here, do something with them, or put them into another classroom and do some specialised work to improve their numeracy skills. It's up to us how we implement that extra time, but that's what we're planning at the moment.

So the best use of the money was seen as giving the teachers time off their teaching so that they can engage in professional development. But the Head of Department recognises that could be difficult to implement. Previous experience with after-school meetings suggest some teachers will not attend. She observed:

> They see [it] as a waste of their time … Even if it was in a lunchtime or their free session, it would still be considered their time, when they could be doing other things – marking, correcting, writing curriculum, whatever. And also they probably think that they're doing a good job … they see it more as a support for new teachers … they're older teachers, they don't need to know any new tricks.

She also reported that some engaged in non-school activities during free time. This raises the question of teacher performance appraisal at this school, and how it might lead to better staff professional development. At *Green Park*, each senior staff member interviews about ten people when it comes to determining increments. However the process appears flawed with the Head of Department reporting it is not always treated seriously, rarely if ever involving classroom observation, and guided more by principles of 'mateship' than professional standards. However the Head feels powerless to change this situation. She said:

> I think we're probably more into mateship, looking after each other, we don't want to 'dob', we're all Australians, we're not going to 'dob' the person in who's not doing their job properly. We're going to cover for them, if we can. We're going to encourage … we're not going to actually dock their pay because of it.

The relationship between the students and the mathematics teachers is also not good. The Head said of it:

> [The students are] very narrow-minded … a lot of our kids have that same attitude to teachers. Teachers are dirt. You don't have to do what a teacher says… especially maths teachers. "My parents never understood maths, so we don't have to …" it's stupid anyway.

One of the effects of this has been a pressure to lower standards in mathematics. This in part occurs because the failure of the students is attributed by the school administration to the teachers. It is further compounded by the effect of the new system of curriculum standards which the state has implemented which are not seen to be reasonable for the school. Despite these pressures, the staff are keen to maintain standards. The Head of Department said:

> [The mathematics department] is trying desperately to keep standards in maths, because it is important, because if they don't have it, come Year 12, they are going to compete against everyone in the state.

What then does this Head of Department see as her major problems?

1. A stable staff who don't want to change and who do not see any reason to change;
2. Lack of time allowance for administering and managing a large department;
3. Lack of financial allowance and reward for the amount of work done;
4. Lack of professional respect by the senior management staff at the school for the work of Heads of subject departments;
5. Priority being given to Year coordination rather than subject department development;
6. Limited financial opportunities for professional development for her staff;
7. Limited desire by her staff for any professional development;
8. Uncommitted, and in some cases unprofessional, behaviour by some staff;
9. Unhelpful and rudimentary professional appraisal procedures by senior staff;
10. Poor relationships between students and certain mathematics staff; and
11. Pressure to lower standards in relation to the State-wide standards, by overly rewarding low standard work by students.

Discussion

There are some clear and consistent findings from these case studies which are also supported by some of the survey result (reported earlier). The findings suggest that mathematics departments which have strong professional communities have more successful student outcomes. These schools *enabled* heads of mathematics departments to exercise leadership in building a vigorous and mutually accountable professional team and considerable authority and autonomy in developing coherent mathematics programs adapted to the needs of students.

The teachers in the mathematics departments in these case study schools knew their students well and tailored their programs to meet their learning needs and stage of development in mathematical understanding. Teachers in these departments were enthusiastic about mathematics and fed off each other's enthusiasm. They did not like formal staff meetings usually, but they were working collaboratively a great deal of the time.

These findings challenge some conventional views about the so-called 'balkanisation' of the school curriculum caused by subject-based departments. There was no evidence of incompatibility in these schools between effective schools and effective departments (when these departments operated in the ways described above). In fact, these findings could be taken to suggest that strong professional learning teams at the mathematics department level are a necessary foundation for effective schools.

There have been considerable changes over the past thirty years in internal management structures and career structures within schools. In some sectors, these changes have often had negative effects on the status and responsibilities of subject department heads in relation to other positions of responsibility within secondary schools. While it used to be true that becoming the head of a subject department was a career step carrying considerable weight in terms of salary and status, 'subject coordination' now brings little of either in some school systems. The findings from this study give reason to reconsider this trend. The nature of the immediate work group appears to play an important role in teachers' effectiveness in meeting student needs and, therefore, job satisfaction.

In summary, the case studies indicate that there could be value in giving attention to the role of the mathematics department and the adequacy of current methods for preparing teachers for transition into the role of Department Head, or Subject Coordinator.

10 DISCUSSION AND CONCLUSION

This was a study of effective mathematics teaching in Australian secondary schools (Years 8 to 10). It was designed to examine a range of factors including: (a) the knowledge, beliefs, understandings and practices of mathematics teachers; (b) teacher qualifications; and (c) the importance of professional development and other experiences of teachers.

The study was guided by findings taken from a literature review, and by a theory that posited the likely key factors that needed to be considered in understanding effective mathematics teaching. This theory saw the key to understanding effective mathematics teaching as the practices of teachers in their classroom. What teachers do, as Darling-Hammond (1996) cryptically noted, 'counts'. But there is a number of factors that influence what and how much a teacher can do in a classroom. These, it was theorised, included: (a) school-based constraints and opportunities; (b) teacher background characteristics including their education and experience; and (c) teacher enabling conditions, especially the quality of professional development.

Prior to investigating this theory, it was necessary to establish what would count as valid and reliable indicators of effective teaching. It would be by using these indicators that effective teachers could be identified, their characteristics described, and weighted for their importance. Student outcomes were judged to be the best indicators since, if teachers were having an effect, it ought to be observable in their students. As a result, a range of student-level indicators was developed. These were: their growth in mathematics achievement, and their growth in their perceptions of their level of effort, ability, self-efficacy, enjoyment, motivation, the task load and the quality of their learning environment.

The study used a wide range of data collection strategies: (a) interviews with Principals and teachers; (b) questionnaires to Principals, teachers and students; (c) tests for students; and (d) classroom scenarios to assess teachers' mathematical and pedagogical knowledge.

Table 34 and Table 35 provide an overview of some of the findings taken from the study. Table 34 shows the key factors theorised to be associated with effective mathematics teaching, and what the literature review suggested would be the importance of each factor. Typically, the results of the study are consistent with the main findings found in the literature.

Table 34 Summary of findings for the main factors theorized to be associated with effective teaching

Factor	Effect of factor
Knowledge	*Literature review suggested:* teacher content and pedagogical knowledge is important. *This study found:* weak positive effects of content and pedagogical content knowledge on growth of mathematics achievement.
Beliefs and understandings	*Literature review suggested:* beliefs shape practice, but are not always consistent with it, while understandings – both the quality and quantity – are important, but the optimum balance is not known. *This study found:* belief in student success by teachers was positively, though weakly, associated with growth in mathematics achievement. A love of mathematics by teachers was weakly associated with schools having a high average level of mathematics achievement. Similarly (weak) associations were found for belief in collegiality, for openness about practice and a willingness to work collaboratively with other teachers.
Practices	*Literature review suggested:* practices are important, and the study of experts (compared with novices) may help clarify their importance. *This study found:* teacher practices were consistently the most powerful predictors of mathematic achievement growth, and with growth in affective student outcomes. This is consistent with Darling Hammond's (1996) view that what teachers do "counts".
Qualifications	*Literature review suggested:* the evidence about the effect of qualifications is contradictory. *This study found:* contradictory evidence. There was no effect of qualifications found in the multi-level analysis.
Professional development	*Literature review suggested:* much professional development appears to be ineffective, however if concerned with higher order thinking it is more likely to be effective. *This study found:* contradictory evidence, with no effect found in the multi-level analyses, but some weak effects of professional development observed in the, methodologically less rigorous, bi-variate analyses.
Personal experiences	*Literature review suggested:* years teaching is not associated with teacher effectiveness. *This study found:* weak positive effects associated with being a female teacher, years of practice and membership of the AAMT, but none was found in the multi-level analyses.

Table 35 shows the factors that accounted for most of the variance in an indicator variable, net of the effect of other variables in the model. The matching variable at Time 1 is excluded from this table. The matching variable at Time 1 has no substantive explanatory role to play in understanding effective mathematics teaching. This is because it was the change in, not the initial level of, this variable that was used to index effective teaching.

Table 35 Most important factors associated with various indices of effective mathematics teaching showing also proportion of variance in growth between Time 1 and Time 2

Indicator	Strongest predictors of growth
PATMaths	Non computer resources in the school (2.3%)
	Mathematics major in undergraduate study (1.9%)
	Level of teacher's education (1.2%)
	Quality of the professional community in the mathematics department (1.2%)
Effort*	Teacher practice – effective pedagogy (1.8%)
Perceived ability**	Teacher practice – knowledge of students (1.6%)
Perceived quality of the learning environment**	Teacher practice – personal attributes (21.6%)
	Teacher practice – effective pedagogy (20.5%)
	Teacher practice – knowledge of students (18.2%)
Perceived task load**	Nil
Perceived utility of mathematics	Teacher practice – effective pedagogy (7.2%)
Perceived self-efficacy in mathematics	Nil
Enjoyment	Teacher practice – effective pedagogy (10.9%)
	Teacher practice – personal attributes (9.9%)
	Teacher practice – knowledge of mathematics (9.6%)
Motivation	Teacher practice – personal attributes (7.1%)
	Teacher practice – knowledge of students (5.5%)

** Cronbach's alpha >0.5 < 0.7, ** Cronbach's alpha < 0.5*

Table 35 shows that growth in affective student outcomes – for example, enjoyment and motivation – is most commonly influenced by aspects of the teacher's practice, while growth in mathematics achievement is most strongly influenced by (non-computer) resources in the school, the teacher's educational background and the quality of the professional community in the mathematics department.

A second approach used by the study was to take the average PISA score of a school as an indicator of effective mathematics teaching, or as evidence of an effective mathematics department in the school. Using a series of cross tabular analyses, there was evidence that professional accountability, consensus about standards for teaching mathematics and expectations of student success were more likely to be found in mathematics departments of high scoring PISA schools. While it is not clear what way the causal arrow is pointing in these analyses, the importance of the mathematics department was highlighted by the case studies. This suggests that many of the successes and problems encountered in teaching mathematics effectively could be traced to the functioning (or dys-functioning) of the mathematics department within the school.

The most thorough examination of the quantitative data was done in a series of multi-level analyses. These examined growth in each of the eight affective student outcomes and the growth in mathematics achievement as measured using *PATMaths* scores. The theoretical framework developed for the study drove these analyses. For all analyses – whether with mathematics achievement or an affective outcome – various aspects of students' perceptions of teacher practices were the most important independent variables used in these analyses. Teacher content and pedagogical content knowledge was also important for growth in mathematics achievement.

The multi-level analyses, for the range of indicators of teacher effectiveness, shared the following general characteristics:

1. The strongest predictor of an outcome at Time 2 was always the strength of that variable at Time 1;

2. None of the variables measuring teacher professional development contributed to predicting or explaining effective mathematics teaching. That is, on these data, professional development does not appear to uniquely contribute to effective teaching;

3. Student characteristics, school characteristics and teacher characteristics were not very important in predicting or influencing levels of teacher effectiveness; and

4. Teacher practice variables, as reported by the students, were consistently the most important predictors of student outcomes related to effective teaching. Of these predictors, student perceptions of teacher practice have the strongest effect.

These results broadly confirm the theory underlying this study. Teacher practices – as defined by the standards proposed by the Monash University and AAMT teams – are associated with growth in mathematics achievement and in students having a more positive affective orientation towards mathematics. Further, the case studies suggested that the quality of the professional community, as shaped by the mathematics department in the school, is associated with effective teaching practices. There was also evidence that both pedagogical and mathematical content knowledge was related to effective mathematics teaching.

Overall, the study points to the fact that what teachers do in a classroom in large measure determines the quality of student learning.

REFERENCES

Abbott-Chapman, J., Hughes, P., Holloway, G., & Wyld, C. (1990). *Identifying the Qualities and Characteristics of the "Effective" Teacher*. Hobart: Youth Education Studies Centre, University of Tasmania.

ACER. (1998). *Progressive Achievement Tests in Mathematics*. Camberwell: ACER Press.

Andrews, P., & Hatch, G. (1999). A new look at secondary teachers' conceptions of mathematics and its teaching. *British Educational Research Journal, 25*(2), pp. 203 - 223.

Askew, M., Brown, M., Denvir, H., & Rhodes, V. (2000). *Describing primary mathematics lessons observed in the Leverhulme Numeracy Research Programme: A qualitative framework.* Paper presented at the Twenty-fourth Conference of the International Group for the Psychology of Mathematics Education, Hiroshima, Japan.

Askew, M., Brown, M., Rhodes, V., Johnson, D., & William, D. (1997). *Effective Teachers of Numeracy. Final Report*. London, UK: King's College.

Askew, M., & William, D. (1995). *Recent research in mathematics education 5-16*. London: HMSO.

Australian Association of Mathematics Teachers. (1997). *Numeracy = everyone's business: Report of the Numeracy Education Strategy Development Conference*. Adelaide: Author.

Australian Association of Mathematics Teachers. (2002). *Standards for Excellences in Teaching Mathematics in Australian Schools*. Adelaide: Author.

Ball, D. (2000). Bridging practices. Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education, 51*(3), pp. 241 - 247.

Ball, D., & Cohen, D. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of Policy and practice*. San Francisco: Jossey-Bass.

Ball, D. L. (1991). Research on teaching mathematics: making subject-matter knowledge part of the equation. In J. E. Brophy (Ed.), *Advances in research on teaching: Teachers' subject matter knowledge and classroom instruction.* (pp. 1 - 48). Greenwich, CT: JAI Press.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (pp. 433 - 456). Washington, DC: AERA.

Berliner, D. C. (1987). In pursuit of expert pedagogues. *Educational Researcher, 15*(7), pp. 5 - 13.

Berliner, D. L., Stein, P., Sabers, D., Brown, C. P., Cushing, K., & Pinnegar, S. (1988). Implications of research on pedagogical expertise and experience for mathematics teaching. In D. Grouws & T. Cooney & D. Jones (Eds.), *Perspectives on research on effective mathematics teaching* (Vol. 1, pp. 67 - 95). Reston, Va: Lawrence Erlbaum Associates for the National Council of Teachers of Mathematics.

Bobis, J. (2001). *The effect of Count Me In Too on Year 3 Basic Skills Test results*: NSW Department of Education and Training.

Bodin, A., & Capponi, B. (1996). Junior Secondary School Practices. In A. J. Bishop & K. Clements & C. Keitel & J. Kilpatrick & C. Laborde (Eds.), *International Handbook of Mathematics Education Part 1* (Vol. 4, pp. 564 - 614). Dordrecht: Kluwer.

Bond, L., Jaeger, R. M., Smith, T., & Hattie, J. (2001). *Accomplished Teaching Validation Study*. Greensboro: University of North Carolina at Greensboro.

Bond, L., Smith, T., Baker, W. K., & Hattie, J. (2000). *The certification system of the National Board for Professional Teaching Standards: A construct and consequential validity study*. Greenshore, NC: The University of North Carolina.

Borko, H., & Livingston, C. (1989). Cognition and improvisation: Differences in mathematics instruction by expert and novice teachers. *American Educational Research Journal, 26*(4), pp. 473 - 498.

Brophy, J. E. (Ed.). (1990). *Teachers' knowledge of subject matter as it relates to their teaching practice* (Vol. 2). Greenwich, Conn.: JAI Press.

Brophy, J. E., & Good, T. L. (1986). Teacher behavior and student achievement. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching* (3rd ed., pp. 328 - 375). New York: Macmillan Publishing Company.

Brown, M. (2000, June 4). *Effective teaching of numeracy.* Paper presented at the High Expectations - Outstanding Achievement. The Early Years of Schooling P - 4 Conference, Melbourne.

Byrne, C. J. (1983). Teacher knowledge and teacher effectiveness: A literature review, theoretical analysis and discussion of research strategy. Paper presented at the meeting of the Northwestern Educational Research Association, Ellenville, NY.

Center for the Study of Teaching and Policy, & Michigan State University (Wilson, S. M., Foden, R. E and Ferrini-Mundy J.),. (2001). *Teacher Preparation Research: Current Knowledge Gaps and Recommendations*.

Clarke, D. (2001). *Understanding, assessing and developing young children's' mathematical thinking: research as a powerful tool for professional growth. Keynote address to the annual conference of the Mathematical Education Research Group of Australasia*.

Cohen, D., & Hill, H. (2000). Instructional policy and classroom performance: the mathematics reform in California. *Teachers College Record, 102*(2), 294-343.

Cooney, T. J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education, 16*, pp. 324 - 336.

Curriculum Corporation. (1994). *Mathematics-a curriculum profile for Australian schools*. Melbourne: Author.

Darling-Hammond, L. (1992). Creating standards of practice and delivery for learner-centred schools. *Stanford Law and Policy Review, Winter.*

Darling-Hammond, L. (1996). What matters most. *Phi Delta Kappan, 78*(3), 193 - 201.

Darling-Hammond, L. (2000). Teacher quality and student achievement: A review of state policy evidence. *Education Policy Analysis Archives, 8*(1), pp. 1 - 44.

Darling-Hammond, L., & Ball, D. L. (1998). *Teaching for high standards: What policy-makers need to know and be able to do*. Philadelphia, PA: Consortium for Policy Research in Education.

Department for Education and Employment. (1998). *Advanced Skills Teachers: Information Supplement 2*. London: HMSO.

Dolton, P., Chevalier, A., & McIntosh, S. (2001). *Recruiting and retaining teachers in the UK: An analysis of graduate occupational choice from the 1960s to the 1990s.*: Centre for Economic Performance, London School of Economics.

Elbaz, F. (1981). The teacher's "practical knowledge": Report of a case study. *Curriculum inquiry, 11*, pp. 43 - 71.

Elmore, R. F. (2000). *Building a new structure for school leadership*. Washington, DC: The Albert Shanker Institute.

Fennema, E., Carpenter, T., Franke, M., Levi, L., Jacobs, V., & Empson, S. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Reserach in Mathematics Education., 27*(4), pp. 403 - 434.

Fennema, E., & Franke, M. (1992). Teachers' knowledge and its impact. In D. A. Grows (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147 - 164). New York: MacMillan.

Ferguson, R. (1991). Paying for public education: New evidence on how and why money matters. *Harvard Journal of Legislation, 28*, 465 - 498.

Fraivillig, J. (2001). Strategies for advancing children's mathematical thinking. *Teaching Children Mathematics, 7*(8), 454 - 459.

Franke, M. (2002). Designing professional development to support generative growth in teachers with different knowledge, skills and identities. Paper presented at the annual meeting of the American Educational Research Association, New Orleans.

Fullan, M. (2001). *The new meaning of educational change* (3rd Edition ed.). New York: Teachers College Press.

Gameron, A., Secada, W. G., & Marrett, C. B. (1998). The organisational context of teaching and learning: Changing theoretical perspectives. In M. T. Hallinan (Ed.), *Handbook of Sociology of Education*.

Goodlad, J. (1984). *A Place called school: Prospects for the future*. New York: McGraw-Hill.

Gore, J. M. (2000). Beyond our differences: A reassembling of what matters in teacher education. *Journal of Teacher Education, 52*(2), pp. 124 - 134.

Greenwald, R., Hedges, L. V., & Laine, R. (1996). The effects of school resources in student achievement. *Review of Educational Research., 66*(3), pp. 361 - 396.

Groves, S., & Doig, B. (2002, submitted). Developing conceptual understanding: The role of the task in communities of mathematical inquiry.

Hanna, G. (1987). *Instructional strategies and achievement in grade 8.* Paper presented at the 11th International Conference for the Psychology of Mathematics Education, Montreal.

Hattie, J. (1992). Towards a model of schooling: a synthesis of meta-analyses. *Australian Journal of Education, 36*, 5-13.

Hawley, W., & Valli, L. (1999). The essentials of effective professional development: a new consensus. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the Learning Profession. Handbook of policy and practice* (pp. 151-180). San Francisco: Jossey Bass.

Hill, H., Schilling, S., Phelps, G., & Ball, D. (2002). *Developing measures of teacher content knowledge for teaching. Paper presented to the American Educational Research Association Conference, New Orleans*.

Hill, P. W., Rowe, K. J., Holmes-Smith, P., & Russell, V. J. (1996). *The Victorian Quality Schools Project: A study of school and teacher effectiveness.* (Volume 1). Melbourne: Centre for Applied Educational Research, University of Melbourne.

Ingersoll, R. (1996). *Out of field teaching and educational equality*. Washington, DC: National Centre for Educational Statistics.

Ingersoll, R. (1999). The problem of underqualified teachers in American secondary schools. *Educational Researcher, 28*(2), pp. 26 - 37.

Kazemi, E. (1998). Discourse that promotes conceptual understanding. *Teaching Children Mathematics, 4*(7), 410 - 414.

Kennedy, M. (1998). *Form and substance in in-service teacher education* (Vol. 13). Arlington VA: National Science Foundation.

Kerlinger, F. N., & Pedhazur, E. J. (1973). *Multiple Regression in Behavioural Research*. New York: Holt, Rinehart and Winston.

Lampert, M. (1984). Teaching about thinking and thinking about teaching. *Journal of curriculum studies, 16*, pp. 1 - 18.

Langley, R. (1971). *Practical Statistics Simply Explained* (Revised ed.). New York: Dover Publications.

Leinhardt, G., & Greeno, J. G. (1986). The cognitive skill of teaching. *Journal of educational psychology, 78*(2), pp. 75 - 95.

Little, J. (1982). Norms of collegiality and experimentation: Workplace conditions of school success. *American Educational Research Journal, 19,* pp. 325 - 340.

Little, J., & McLaughlin, M. (Eds.). (1993). *Teachers' work: Individuals, colleagues and context*. New York: Teachers College Press.

Lloyd, G., & Wilson, M. (1998). Supporting innovation: The impact of a teacher's conceptions of function on his implementation of a reform curriculum. *Journal for Research in Mathematics Education, 29*(3), pp. 248 - 274.

Lokan, J., Ford, P., & Greenwood, L. (1996). *Maths & Science on the line: Australian Junior Secondary Students' Performance in the Third International Mathematics and Science Study*. Melbourne: Australian Council for Educational Research.

Lokan, L., Greenwood, L., & Cresswell, J. (2001). *15-up and Counting, Reading, Writing, Reasoning … How Literate are Australia's students? The PISA 2000 Survey of Students' Reading, Mathematical and Scientific Literacy Skills*. Melbourne: ACER.

Lortie, D. (1975). *Schoolteacher: A sociological study*. Chicago: University of Chicago Press.

Louis, K. S., Kruse, S., & Marks, H. (1996). Schoolwide professional community. In F. M. Newman (Ed.), *Authentic achievement: Restructuring schools for intellectual quality*. San Francisco: Jossey-Bass.

Louis, K. S., Marks, H., & Kruse, S. (1996). Teachers' professional community in restructuring schools. *American Educational Research Journal, 33*(4), pp. 757 - 798.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: teachers' Understanding of Fundamental mathematics in China and the United States.* Mawwah, NJ: Lawrence Erlbaum Associates.

Martin, G. (2001). *Competency Framework For Teachers*. Perth: Education Department of Western Australia.

Martin, M. O., Mullis, I. V. S., Gregory, K. D., Hoyle, C., & Shen, C. (2000). *Effective Schools in Science and Mathematics*. Chestnut Hill, MA: International Study Centre, Boston College.

McLaughlin, M., & Talbert, J. (2001). *Professional communities and the work of high school teaching*. Chicago: University of Chicago Press.

Minstrell, J. (1999). Expertise in Teaching. In R. Sternberg & J. Horvath (Eds.), *Tacit Knowledge in Professional Practice*. Mahwah: Erlbaum.

Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review, 13*(2), pp. 125 - 145.

Monk, D. H. (1994a). Subject matter preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review, 13*(2), pp. 125 - 145.

Muijs, D., & Reynolds, D. (2000). School effectiveness and teacher effectiveness in mathematics: Some preliminary findings from the evaluation of the Mathematics Enhancement Programme (Primary). *School Effectiveness and School Improvement, 11*(3), pp. 273 - 303.

National Board for Professional Teaching Standards. (2000). The Professional Standard; The Official newsletter of the National Board for Professional Teaching Standards (Vol. 1): National Board for Professional Teaching Standards.

National Board for Professional Teaching Standards. (2002). *Home Page*. Available: http://www.NBPTS.org/ [2002, 07 January].

National Commission on Teaching and America's Future. (1996). *What Matters Most*. Washington, DC: National Commission on Teaching and America's Future.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Virginia: The Council.

National Council of Teachers of Mathematics. (1995). *Assessment Standards for School Mathematics*. Reston, Virginia: The Council.

Newman F. et al. (1996). *School restructuring and student learning*. San Francisco: Jossey-Bass.

Newman, F. M., King, M. B., & Youngs, P. (2000). Professional development that addresses school capacity: Lessons from urban elementary schools. *American Journal of Education, 108*(4), pp. 259 - 299.

Nickerson, R. (1988). On improving thinking through instruction. In E. Rothkopf (Ed.), *Review of Research in Education* (Vol. 15, pp. 3 - 57). Washington, DC: American Educational Research Association.

Organisation for Economic Co-operation and Development. (1999). *Measuring Student Knowledge and Skills: A New Framework for Assessment*. Paris: OECD.

Organisation for Economic Co-operation and Development. (2001). *Knowledge and Skills for Life: First results form PISA 2000 (Education and Skills)*. Paris.

Pehkonen, E. (1997). Teachers' conceptions of mathematics teaching. Paper presented at the Current State of Research on Mathematical Beliefs Conference, Helsinki.

Peterson, P. L. (1988). Teaching for higher-order thinking in mathematics: the challenge for the next decade. In D. D. Grouws & T. T.Cooney & D. Jones (Eds.), *Perspectives on research on effective mathematics teaching* (Vol. 1, pp. pp. 2 - 26). Reston, Va.: Lawrence Erlbaum Associates for the National Council of Teachers of Mathematics.

Peterson, P. L., McArthy, S. J., & Elmore, R. (1996). Learning form school restructuring. *American Educational Research Journal, 3*(1), pp. 119 - 154.

Philipou, G., & Christou, C. (1999). Teachers' conceptions of mathematics and students' achievement: A cross-cultural study based on results from TIMSS. *Studies in Educational Evaluation, 25*, pp. 379 - 398.

Raymond, A. (1997). Inconsistency between a Beginning Elementary School Teacher's Mathematics Beliefs and Teaching Practice. *Journal for Research in Mathematics Education, 28*(5), pp. 550 - 576.

Raymond, A., & Leinenbach, M. (2000). Collaborative Action Research on the Learning and Teaching of Algebra: A Story of One. *Educational Studies in Mathematics, 41*(3), pp. 283 - 307.

Reid, D. (1997). *Constraints and opportunities in teaching proving.* Paper presented at the 21st Annual meeting of the International Group for the Psychology of Mathematics Education, Lahti, Finland.

Robitaille. (1993). Characteristics of Schools, Teachers, and Students. In L. Burstein (Ed.), *The IEA Study of Mathematics III, Student Growth and Classroom Processes* (pp. 29 - 57). Oxford: Pergamon Press.

Romberg, T. A. (1988). Can teachers become professionals? In D. D. Grouws & T. T.Cooney & D. Jones (Eds.), *Perspectives on research on effective mathematics teaching* (Vol. 1, pp. 224 - 244). Reston, Va.: Lawrence Erlbaum Associates for the National Council of Teachers of Mathematics.

Rowe, K. J. (2003). *Practical Multilevel Analysis with Mlwin and LISREL*. Camberwell: ACER.

Shulman, L. (1987). Knowledge and Teaching. *Harvard Educational Review, 57*(1), pp. 1 - 22.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), pp. 4 - 14.

Silver, E., Mesa, V., Benken, B. M., & Mairs, A. (2002). *Putting it together: What do the NBPTS data tell us about teaching and assessing mathematics for understanding in the middle grades?* New Orleans: Paper presented at the annual meeting of the American Educational Research Association.

Stein, M. K. (2001). Teaching and learning mathematics: How instruction can foster the knowing and understanding of number. In J. E. Brophy (Ed.), *Subject-specific instructional methods and activities* (Vol. 8). Amsterdam: JAI An Imprint of Elsevier Science.

Sternberg, R. J., & Horvarth, J. A. (1995). A prototype view of expert teaching. *Educational Researcher*, *26*(6), pp. 9- 17.

Stigler, J. W., & Perry, M. (1988). Cross-cultural studies of mathematics teaching and learning. In D. Grouws & T. Cooney (Eds.), *Perspectives on research on effective mathematics teaching* (pp. 194 - 223). Hillsdale, NJ: Lawrence Erlbaum.

The Professional Affairs Department. (1999). *Standards of Practice for The Teaching Profession*. Toronto, Ontario: Ontario College of Teachers.

Thompson, A. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*(2), pp. 105 - 127.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127 - 146). New York: Macmillan Publishing Company.

Tomazos, D. (1997). *Investigating change in a primary mathematics classroom: valuing the students' perspective.* Paper presented at the 21st Annual meeting of the International Group for the Psychology of Mathematics Education, Lahti, Finland.

Vacc, N., & Bright, G. (1999). Elementary pre-service teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education, 30*(1), 89 - 110.

Varella, G. F. (2000). Science teachers at the top of their game: What is teacher expertise? *Clearing house, 74*(1), pp. 43 - 50.

Weick, K. E. (1976). Educational Organisations as loosely-coupled systems. *Administrative Science Quarterly, 21*(1), pp. 1 - 19.

Weick, K. E., & McDaniel, R. R. (1993). How professional organisations work: Implications for school organisation and management. In T. Sergiovanni & J. H. Moore (Eds.), *Schooling for tomorrow: Directing reform to issues that count* (pp. 330 - 355). Boston, MA: Allyn & Bacon.

Wenglinsky, H. (2000a). *How teaching matters: Bringing the classroom back into the discussions of teacher quality*. Princeton, NJ: Educational Testing Service.

Wenglinsky, H. (2000b). *Teaching the Teachers*. Princeton, NJ: Educational Testing Service.

Wenglinsky, H. (2002). *How school matters: The link between teacher classroom practices and student academic performance* (Vol 10). Education Policy Analysis Archives. Available: http://epaa.asu.edu/epaa/v10n12/ [2002, 14 February].

Wilson, S. M., & Floden, R. E. (2003). *Creating effective teachers: concise answers to hard questions*. Washington: American Association of Colleges of Teacher Education.

Zeichner, K., & Tabachnik, B. R. (1981). Are the effects of teacher education washed out by school experience? *Journal of Teacher Education, 32*(3), pp. 7 - 11.

APPENDIX 1

This appendix contains copies of the questionnaires and a specimen copy of a Rich Assessment Task booklet. Copies of the *PATMaths* test cannot be reproduced for copyright reasons. Form B of the teacher questionnaire is not displayed here as the contents of it are displayed in the body of the report.

The questionnaires listed include:

1. The Principal or school questionnaire

2. The teacher questionnaire (Form A)

3. The student questionnaire (Time 2 version). The Time 1 version was the same as the Time 2 version except that it did not have questions 53 – 86 included.

The Principal or school questionnaire

EFFECTIVE MATHEMATICS TEACHING AND LEARNING PROJECT

PRINCIPAL/HEAD OF DEPARTMENT SURVEY

School: ________________________________

THIS STUDY HAS BEEN COMMISSIONED BY THE COMMONWEALTH GOVERNMENT DEPARTMENT OF EDUCATION, SCIENCE AND TECHNOLOGY

All responses to this questionnaire will remain confidential.

ACER

AUSTRALIAN COUNCIL FOR EDUCATIONAL RESEARCH

INTRODUCTION

INVESTIGATION OF EFFECTIVE MATHEMATICS TEACHING AND LEARNING IN AUSTRALIAN SECONDARY SCHOOLS

THANK YOU FOR AGREEING TO PARTICIPATE IN THIS RESEARCH PROJECT COMMISSIONED BY THE COMMONWEALTH GOVERNMENT!

THE MAIN PURPOSE OF THIS RESEARCH PROJECT IS TO IDENTIFY, FROM A RANGE OF POSSIBLE INFLUENCES, THOSE THAT APPEAR TO BE HAVING A STRONG IMPACT ON STUDENT LEARNING OUTCOMES IN MATHEMATICS IN YEARS 7 TO 10.

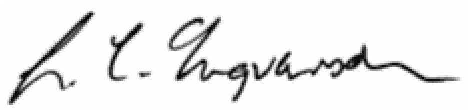
THE PURPOSE OF THIS QUESTIONNAIRE IS TO ASCERTAIN YOUR PERCEPTIONS OF FACTORS AFFECTING THE CAPACITY OF YOUR SCHOOL TO PROVIDE QUALITY MATHEMATICS PROGRAMS FOR STUDENTS IN YEARS 7-10. THE STATEMENTS IN THE QUESTIONNAIRE ARE BASED ON INTERVIEWS WE HAVE CONDUCTED WITH SCHOOL PRINCIPALS ABOUT MATHEMATICS IN THEIR SCHOOLS.

AS YOU KNOW, THIS QUESTIONNAIRE IS ONE OF SEVERAL METHODS WE ARE USING IN YOUR SCHOOL TO GATHER INFORMATION FOR THIS RESEARCH PROJECT. OTHERS INCLUDE A SURVEY OF A SAMPLE OF MATHEMATICS TEACHERS AND THEIR STUDENTS AND A SET OF TESTS OF STUDENT LEARNING IN MATHEMATICS.

THIS QUESTIONNAIRE SHOULD BE COMPLETED BY EITHER THE PRINCIPAL, OR THE HEAD OF THE MATHS DEPARTMENT, OR BOTH.

WE WOULD BE VERY GRATEFUL IF YOU COULD SPARE SOME OF YOUR VALUABLE TIME TO COMPLETE THE QUESTIONNAIRE.

THANKS AGAIN.



LAWRENCE INGVARSON ON BEHALF OF THE PROJECT TEAM

Head of the Teaching and Learning Division
Australian Council for Educational Research
19 Prospect Hill Road (Private Bag 55)
Camberwell
Australia 3124

Tel no: 61 3 9277 5612
Fax no: 61 3 9277 5514
E-mail: ingvarson@acer.edu.au

www.acer.edu.au

SECTION 1: Principal/Head of Department Survey

Name(s) and position: ______________________________

School: ______________________________

Q 1 School sector:

To which school sector does your school belong?

Government ❑ Catholic ❑ Independent ❑

Q 2 Region:

My school is in:

a state capital metropolitan area: ❑
a regional city or country town: ❑
other (please specify): ______________

Q 3 Size of your school:

At the beginning of the 2002 school year, how many students were enrolled at your school (approximately)? ________

Q 4 What percentage of the students in the following year levels are in receipt of Educational Maintenance Allowance (EMA)?

Year Level	**Percent**
7	
8	
9	
10	

Q 5 What percentage of the students in the following year levels are of Indigenous status?

Year Level	**Percent**
7	
8	
9	
10	

Q 6 What percentage of the students in the following year levels are considered to be of Non-English speaking background (NESB)?

Year Level	**Percent**
7	
8	
9	
10	

Q 7 Are the maths classes streamed? Yes No (Please circle)
Please indicate the basis of the streaming (eg, maths achievement, reading ability)

____

____

____

____

Q 8 Does your school have single sex maths classes? Yes No (Please circle)

Q 9 If you have single sex classes are they for Males only Females only (Please circle)

Q 10 If you have mixed sex classes what is the approximate percentage of males, on average, in the classes?

Year Level	**Percentage of males**
7	
8	
9	
10	

Q 11 How many teachers teach mathematics in Years 7-10 in your school?

____

Q 12 How many of the teachers of mathematics in Year 7-10 have initial training in teaching methods specific to the teaching of mathematics? ________

Q 13 How many of the teachers of mathematics in Years 7-10 have a university:

major (three year sequence) in mathematics? ________

minor (two year sequence)) in mathematics? ________

none ________

Q 14 What is the average size of Years 7-10 mathematics classes

(approximately)? ________

Q 15 What is the approximate amount of time allocated to mathematics classes per week for:

	Minutes
Year 7	________
Year 8	________
Year 9	________
Year 10	________

Q 16 What is the usual length of a single maths period in your school?

Year Level	**Period (minutes)**
7	
8	
9	
10	

SECTION 2: Principal/Head of Department Survey

FACTORS INFLUENCING THE EFFECTIVENESS OF MATHEMATICS TEACHING AND LEARNING FOR STUDENTS IN YEARS 7 TO 10

The following list of factors is based on interviews with principals, senior school administrators and co-ordinators of mathematics departments. Each factor pair ranges from highly positive to highly negative in their influence on effectiveness.

Please indicate the **importance** of each of these factors to the effectiveness of mathematics teaching and learning in Years 7-10 *in your school* by ticking one appropriate box per pair.

Factor	Influence High			High	Factor
1. Complying with state curriculum and standards frameworks.	❑	❑	❑	❑	Not complying with state curriculum and standards frameworks.
2. Preparing students for maths courses at Years 11-12, and beyond.	❑	❑	❑	❑	Not preparing students for maths courses at Years 11-12, and beyond.
3. Lack of freedom for individual teachers to design mathematics curricula suited to the needs and interests of their students at Years 7 to 10	❑	❑	❑	❑	Freedom for individual teachers to design mathematics curricula suited to the needs and interests of their students at Years 7 to 10
4. No consistency or coherence in mathematics programs within and across Years 7 to 10	❑	❑	❑	❑	Consistent and coherent mathematics programs within and across Years 7 to 10
5. Inability to attract well-qualified teachers of mathematics.	❑	❑	❑	❑	Ability to attract well-qualified teachers of mathematics.
6. Inability to retain well-qualified teachers of mathematics.	❑	❑	❑	❑	Ability to retain well-qualified teachers of mathematics.
7. Teachers not trained in methods of teaching mathematics, but teaching mathematics.	❑	❑	❑	❑	Teachers teaching mathematics who are trained in methods of teaching mathematics.
8. Teachers of mathematics who are not enthusiastic about mathematics.	❑	❑	❑	❑	Teachers of mathematics who are enthusiastic about mathematics.
9. A lack of specialist mathematics teachers willing to teach in Years 7-10.	❑	❑	❑	❑	Specialist mathematics teachers who are willing to teach in Years 7-10.
10. Specialist mathematics teachers who do not adapt their teaching practices to the needs of students at Years 7 to 10.	❑	❑	❑	❑	Specialist mathematics teachers who adapt their teaching practices to the needs of students at Years 7 to 10.
11. Teachers of maths who emphasise the need to cover content in a sequential manner.	❑	❑	❑	❑	Teachers of maths who do not feel the need to cover content in a sequential manner.
12. Difficulties some maths teachers have in seeing learning from the students' point of view.	❑	❑	❑	❑	The ease some maths teachers have in seeing learning from the students' point of view.

Factor	Influence High			High	Factor
13. Not enough teachers of maths who love the subject and pass their enthusiasm on to students.	❑	❑	❑	❑	Enough teachers of maths who love the subject and pass their enthusiasm on to students.
14. Teachers who believe they have no need for professional development.	❑	❑	❑	❑	Teachers who believe they have a need for professional development.
15. Lack of professional development opportunities tailored to the needs of teachers of mathematics at Years 7 to 10.	❑	❑	❑	❑	Professional development opportunities tailored to the needs of teachers of mathematics at Years 7 to 10.
16. Limited opportunities for effective professional development with the capacity to improve classroom practice.	❑	❑	❑	❑	Opportunities for effective professional development with the capacity to improve classroom practice.
17. Low levels of collegiality and professional community among teachers within the maths department.	❑	❑	❑	❑	High levels of collegiality and professional community among teachers within the maths department.
18. Low levels of professional accountability among teachers within the maths department.	❑	❑	❑	❑	High levels of professional accountability among teachers within the maths department.
19. Lack of time for teachers to plan and review their programs together.	❑	❑	❑	❑	Sufficient time for teachers to plan and review their programs together.
20. Shortage of teachers who can provide leadership at the level of the maths department in the school.	❑	❑	❑	❑	Availability of teachers who can provide leadership at the level of the maths department in the school.
21. A culture of privacy and individualism among teachers of maths.	❑	❑	❑	❑	A culture of openness and collegiality among teachers of maths.
22. Lack of consensus about standards for quality teaching and learning in maths	❑	❑	❑	❑	Consensus about standards for quality teaching and learning in maths
23. Low expectations of student success among teachers of maths.	❑	❑	❑	❑	High expectations of student success among teachers of maths.

Factor	Influence High			High	Factor
24. The ageing of the maths teacher workforce.	❑	❑	❑	❑	The availability of younger maths teacher.
25. Limited capacity for senior management to provide incentives for teachers to engage in effective professional development.	❑	❑	❑	❑	Extensive capacity for senior management to provide incentives for teachers to engage in effective professional development.
26. A reluctance among teachers of maths to take on new approaches to teaching maths.	❑	❑	❑	❑	A readiness among teachers of maths to take on new approaches to teaching maths.
27. The negative attitudes students have toward maths as a compulsory subject.	❑	❑	❑	❑	The positive attitudes students have toward maths as a compulsory subject.
28. The lack of interest students have in maths.	❑	❑	❑	❑	The high interest students have in maths.
29. The lack of parental support for students to do well in maths.	❑	❑	❑	❑	Parental support for students to do well in maths.
30. Limited availability of computers for student use.	❑	❑	❑	❑	High availability of computers for student use.
31. Limited availability of quality computer software for maths teaching	❑	❑	❑	❑	High availability of quality computer software for maths teaching
32. Lack of appropriate curriculum materials and resources for teachers of maths	❑	❑	❑	❑	Wealth of appropriate curriculum materials and resources for teachers of maths
33. Difficulties maths teachers have in working collaboratively as part of a team teaching approach	❑	❑	❑	❑	The ease that maths teachers have in working collaboratively as part of a team teaching approach
34. Lack of support by other departments (e.g. science, technology) for changes to the maths curriculum 7-10.	❑	❑	❑	❑	Support by other departments (e.g. science, technology) for changes to the maths curriculum 7-10.
35. Insufficient financial resources for developing maths teaching in your school.	❑	❑	❑	❑	Sufficient financial resources for developing maths teaching in your school.

PLEASE USE THE REMAINING SPACE TO COMMENT ON ANY OTHER FACTORS THAT YOU BELIEVE INFLUENCE THE EFFECTIVENESS OF MATHEMATICS TEACHING AND LEARNING, AT YOUR SCHOOL, FOR STUDENTS IN YEARS 7 TO 10:

The teacher questionnaire (Form A)

Note that the formatting of the teacher questionnaire has not been retained for this appendix.

EFFECTIVE MATHEMATICS TEACHING AND LEARNING

TEACHER SURVEY
PART A

Name: ______________________________

School: _____________________________

THIS STUDY HAS BEEN COMMISSIONED BY THE COMMONWEALTH GOVERNMENT DEPARTMENT OF EDUCATION, SCIENCE AND TECHNOLOGY

All responses to this questionnaire will remain confidential.

ACER

AUSTRALIAN COUNCIL FOR EDUCATIONAL RESEARCH

INTRODUCTION

INVESTIGATION OF EFFECTIVE MATHEMATICS TEACHING AND LEARNING IN AUSTRALIAN SECONDARY SCHOOLS

THANK YOU FOR HELPING US WITH THIS RESEARCH PROJECT COMMISSIONED BY THE COMMONWEALTH GOVERNMENT!

THE MAIN PURPOSE OF THIS RESEARCH PROJECT IS TO IDENTIFY, FROM A RANGE OF POSSIBLE INFLUENCES, THOSE THAT APPEAR TO BE HAVING A STRONG IMPACT ON STUDENT LEARNING OUTCOMES IN MATHEMATICS IN YEARS 7 TO 10. WE HOPE THAT THIS RESEARCH WILL BE USEFUL TO POLICY MAKERS AS THEY CONSIDER OPTIONS TO SUPPORT MATHEMATICS TEACHING IN THE FUTURE.

AS YOU KNOW, THIS QUESTIONNAIRE IS ONE OF SEVERAL METHODS WE ARE USING IN YOUR SCHOOL TO GATHER INFORMATION FOR THIS RESEARCH PROJECT. OTHERS INCLUDE A SURVEY OF PRINCIPALS AND HEADS OF MATHEMATICS DEPARTMENTS, AND TESTS GIVEN TO STUDENTS. THIS QUESTIONNAIRE WILL HELP US GAUGE THE IMPORTANCE OF A RANGE OF INFLUENCES ON MATHEMATICS TEACHING.

THE QUESTIONNAIRE IS IN THREE BROAD SECTIONS:

1: BACKGROUND INFORMATION AND YOUR INITIAL PREPARATION FOR TEACHING
2 YOUR PROFESSIONAL DEVELOPMENT EXPERIENCES
3: YOUR OWN TEACHING PRACTICES.

WE WOULD BE VERY GRATEFUL IF YOU COULD SPARE APPROXIMATELY 15 MINUTES TO COMPLETE THE QUESTIONNAIRE BELOW.

THANKS AGAIN.

LAWRENCE INGVARSON ON BEHALF OF THE PROJECT TEAM

Head of the Teaching and Learning Division
AUSTRALIAN COUNCIL FOR EDUCATIONAL RESEARCH

19 Prospect Hill Road (Private Bag 55)
Camberwell
Australia 3124

Tel no: 61 3 9277 5612
Fax no: 61 3 9277 5514
E-mail: ingvarson@acer.edu.au

www.acer.edu.au

SECTION A: BACKGROUND INFORMATION

1 Are you Male ❑ or Female ❑

2 What is the highest level of formal education you have completed? (Please tick one)

Secondary school with 3 or 4 years of teacher training ❑

Bachelor degree or equivalent with no teacher training ❑

Bachelor degree or equivalent with teacher training (e.g. Dip. Ed.) ❑

Masters or doctoral degree with no teacher training ❑

Masters or doctoral degree with teacher training ❑

3 In what subject areas and year levels were you trained to teach in your initial teacher education program?

Subjects	**Year level**

4 What was your undergraduate major field of study? ____________________

5 What was your undergraduate minor field of study? ____________________

6 What was your major field of study at post graduate level? ______________

7 Was your teacher education course:

End on (eg Dip Ed) ❑

Concurrent (eg B Ed) ❑

Double degree (eg B Sc/Dip Ed) ❑

8 My main teaching subject is: (Please tick one)

Mathematics	❑	Health and Physical Education	❑
Science	❑	Technology	❑
English	❑	Religious studies	❑
The Arts	❑	LOTE	❑

To answer the next two questions treat years teaching part time as equivalent to full time and only count the years when you were working as a teacher. Do not, for example, count years on family leave.

9 How many years have been teaching? ____________ year(s)

10 How many years have you been teaching mathematics? ________ year(s)

11 Are you teaching mathematics in a single-sex class? Yes ❑ No ❑

12 If Yes to Question 11, is it a class of Males ❑ or Females ❑

13 Are you, or have you ever, taught Year 11 or 12? Yes ❑ No ❑

SECTION B: PROFESSIONAL DEVELOPMENT

Are you a member of the Australian Association of Teachers of Mathematics and/or a state/territory maths teachers association? Yes ❑ No ❑

For the professional development in which you participated in the last **12 months**, did you receive any of the following types of support? (Please tick the appropriate box)

Release time from teaching (i.e. your regular teaching responsibilities were temporarily taken over by another teacher). Yes ❑ No ❑

Scheduled time during the teaching year for professional development.

Yes ❑ No ❑

Financial support for professional development activities or programs that took place outside regular working hours. Yes ❑ No ❑

Full or partial reimbursement for course fees Yes ❑ No ❑

Reimbursement for conference or workshop fees (eg maths subject association conference). Yes ❑ No ❑

Reimbursement for travel and/or daily expenses. Yes ❑ No ❑

Over the past **THREE years**, how many university courses have you taken in mathematics or mathematics education? (Please tick one box)

0 ❑ 1 ❑ 2 ❑ 3 ❑ 4 ❑ More than 4 ❑

Over the past **THREE years**, have you participated in professional development activities or taken courses in any of the following? (Please tick all that apply)

Use of hand-held technology (eg graphing calculators) ❑

Use of other technology (eg computers, motion detectors) ❑

Methods of teaching mathematics ❑

Co-operative group methods of teaching ❑

Interdisciplinary teaching ❑

Teaching higher-order thinking skills ❑

Teaching students from different cultural backgrounds ❑

Teaching students with limited English proficiency ❑

Teaching students with special needs (e.g. visually impaired; gifted and talented) ❑

Outcomes or standards-based teaching ❑

Classroom management and organisation ❑

Other professional issues ❑

None of the above ❑

In the past **THREE years**, have you participated in the following activities? (Please tick all that apply).

University courses in mathematics. ❑

University courses in mathematics education. ❑

Observational visits of other mathematics teachers in other schools. ❑

Individual or collaborative research on a topic related to your teaching. ❑

Regularly scheduled collaboration, related to teaching maths, with other teachers. ❑

Mentoring, or peer observation and coaching. ❑

Participation in a teacher network (e.g. one organised by an outside agency or over the Internet). ❑

Attending workshops, conferences or training provided by your state branch of the Australian Association of Teachers of Mathematics (AAMT) or the AAMT itself. ❑

Workshops, conferences, or training, in which you were the presenter. ❑

In the past **THREE years**, to what extent have your PD experiences:

	Not at all	**To a minor extent**	**To a moderate extent**	**To a major extent**
provided models to illustrate new maths teaching practices?	❑	❑	❑	❑
provided opportunities to talk about and share maths ideas with colleagues in your school?	❑	❑	❑	❑
led you to collaborate with colleagues in examining or reviewing students' maths work?	❑	❑	❑	❑
led you to collaborate with colleagues in examining or reviewing *your own* students' maths work?	❑	❑	❑	❑
engaged you in analysing your own students' maths achievement, in relation to learning outcomes?	❑	❑	❑	❑
engaged you in analysing other students' maths achievement, in relation to learning outcomes?	❑	❑	❑	❑
engaged you in actively reflecting on your maths teaching practice?	❑	❑	❑	❑
engaged you in identifying specific areas of your maths teaching practice that you needed to develop?	❑	❑	❑	❑
given you opportunities to test new maths teaching practices for yourself?	❑	❑	❑	❑
enabled you to gain feedback about your own maths teaching from colleagues or other maths teachers?	❑	❑	❑	❑
provided time to consolidate and master new things that you were expected to learn?	❑	❑	❑	❑
related to other programs designed to improve maths learning in your school?	❑	❑	❑	❑
made provision for follow-up or continuing assistance in your school, or classroom, to help you implement changes in your maths practices?	❑	❑	❑	❑

In the past **THREE years**:

	Never	Once	2-3 times	Often
how frequently is your maths teaching observed by other maths teachers in your school?	❑	❑	❑	❑
how frequently do you receive feedback on your maths teaching from other maths teachers in your school?	❑	❑	❑	❑

To what extent, over the past **five years**, have you undertaken professional development programs with a focus on:

	Not at all	To a minor extent	To a moderate extent	To a major extent
deepening your understanding of the maths content that you teach?	❑	❑	❑	❑
knowledge about how students learn the maths content that you teach?	❑	❑	❑	❑
methods of teaching the maths content that you teach?	❑	❑	❑	❑

SECTION D: SCHOOL CONTEXT

How often would you have the following types of interactions with other teachers of maths?

	Rarely	Monthly	Weekly	Almost daily
Discussions about how to teach a maths concept.	❑	❑	❑	❑
Working together on preparing maths teaching materials.	❑	❑	❑	❑
Visits to other teachers' classrooms to observe their maths teaching.	❑	❑	❑	❑
Examination of student work in maths.	❑	❑	❑	❑
Informal observations of my maths classroom by other maths teachers.	❑	❑	❑	❑

To what extent are the following statements about conditions for maths teaching true for your classroom and school?

	Not at all	To a minor extent	To some extent	To a major extent
Computers are readily available for student use.	❑	❑	❑	❑
Quality computer software is available for maths teaching and learning.	❑	❑	❑	❑
Quality educational and planning software is available for your own use at home.	❑	❑	❑	❑
Calculators are readily available for student use.	❑	❑	❑	❑
Adequate training is provided for teachers on the use of computers and software.	❑	❑	❑	❑
Quality concrete materials are available for maths teaching.	❑	❑	❑	❑
Adequate training is provided in the use of concrete materials for maths teaching.	❑	❑	❑	❑
School administrators are supportive.	❑	❑	❑	❑
Support is readily available from school system people external to the school.	❑	❑	❑	❑

To what extent is each of the following a problem for maths teaching in your school as a whole?

	Not at all	To a minor extent	To some extent	To a major extent
Student interest in mathematics.	❑	❑	❑	❑
Student reading abilities.	❑	❑	❑	❑
Student absences.	❑	❑	❑	❑
Teacher interest in maths.	❑	❑	❑	❑
Teacher preparation to teach maths.	❑	❑	❑	❑
Time to teach maths.	❑	❑	❑	❑
Opportunities for teachers to share ideas.	❑	❑	❑	❑
Opportunities for professional development.	❑	❑	❑	❑
Interruptions for announcements, assemblies, and other school activities.	❑	❑	❑	❑
Large classes.	❑	❑	❑	❑
Maintaining discipline.	❑	❑	❑	❑
Sufficient parental support.	❑	❑	❑	❑

How much does each of the following influence what you teach in your Years 7-10 maths classes?

	Not at all	To a minor extent	To some extent	To a major extent
The maths curriculum framework in your state or territory.	❑	❑	❑	❑
State maths tests.	❑	❑	❑	❑
Textbooks.	❑	❑	❑	❑
Guidelines set by the maths department in your school.	❑	❑	❑	❑
State education department/employer guidelines.	❑	❑	❑	❑
Professional teaching standards (eg AAMT).	❑	❑	❑	❑
Your own maths content background.	❑	❑	❑	❑
Your understanding of what motivates your students.	❑	❑	❑	❑
Parents and community.	❑	❑	❑	❑

To what extent do you agree or disagree with each of the following statements?

	Strongly agree	Agree	Dis-agree	Strongly disagree
You can count on most staff members to help out anywhere, anytime--even though it may not be part of their official duties.	❑	❑	❑	❑
Teachers in this school are continually learning and seeking new ideas.	❑	❑	❑	❑
There is a great deal of co-operative effort among staff members.	❑	❑	❑	❑
Staff members maintain high standards.	❑	❑	❑	❑
This school seems like a big family, everyone is so close and cordial.	❑	❑	❑	❑

Please indicate how strongly you agree or disagree with each of the following statements.

	Strongly agree	Agree	Dis-agree	Strongly disagree
In this school, we solve problems; we don't just talk about them.	❑	❑	❑	❑
My job provides me with professional stimulation and growth.	❑	❑	❑	❑
In this school, I am encouraged to experiment with my teaching.	❑	❑	❑	❑
The principal is interested in innovation and new ideas.	❑	❑	❑	❑
I can get good advice from other maths teachers in this school when I have a maths teaching problem.	❑	❑	❑	❑
I feel that I have many opportunities to learn new things in my present job.	❑	❑	❑	❑
I feel supported by colleagues to try out new ideas.	❑	❑	❑	❑
Most maths teachers in this school are learning and seeking new ideas.	❑	❑	❑	❑
The maths staff seldom evaluates its programs and activities	❑	❑	❑	❑

Please indicate how strongly you agree or disagree with each of the following statements.

	Strongly agree	Agree	Dis-agree	Strongly disagree
The principal deals effectively with pressures from outside the school that might interfere with my maths teaching.	❑	❑	❑	❑
The principal sets priorities, makes plans, and sees that they are carried out.	❑	❑	❑	❑
Staff are recognized for a job well done.	❑	❑	❑	❑
Staff are involved in making decisions that affect them	❑	❑	❑	❑
The principal knows what kind of school he/she wants and has communicated it to the staff.	❑	❑	❑	❑
The school's administration knows the problems faced by the staff.	❑	❑	❑	❑
The school administration's behaviour toward the staff is supportive and encouraging.	❑	❑	❑	❑
The principal lets staff know what is expected of them.	❑	❑	❑	❑
The principal does a good job of getting maths resources for this school.	❑	❑	❑	❑

SECTION E: Mathematics Teaching Practices

In teaching maths to students in Years 7/8 to 10 classes, how often do you usually ask them to do the following?

	Never	**Some-times**	**Often**	**Always**
Explain the reasoning behind an idea	❑	❑	❑	❑
Work on problems for which there is no immediately obvious method of solution.	❑	❑	❑	❑
Represent and analyse relationships using graphs, charts or tables.	❑	❑	❑	❑
Write equations and functions to represent relationships.	❑	❑	❑	❑
Practice computational skills.	❑	❑	❑	❑
Work in small groups to come up with a joint solution or approach to a problem or task.	❑	❑	❑	❑
Relate what they are learning in mathematics to their daily lives.	❑	❑	❑	❑
Decide on the own procedures for solving a complex problem and then discuss their procedures and results.	❑	❑	❑	❑

Note the final section of the questionnaire is not reproduced here as no data were reported from it.

Student Questionnaire (Time 2 version)

Confidential Student Questionnaire

School Name:

Student Name: Family Name Given Name

Date of Birth: / / 19
Day Month Year

INSTRUCTIONS:

- Use pencil only, preferably 2B
- Do **not** use any pens or ball-point pens
- Erase mistakes fully
- Make no stray marks

USE 2B PENCIL ONLY

Please MARK LIKE THIS: *NOT LIKE THIS:*

In this booklet you will find questions about you and school mathematics.
Please read each question carefully and answer as accurately as you can by marking a bubble.
If you make a mistake, erase your error and mark the correct bubble. In this booklet, there are no 'right' or 'wrong' answers. Your answers should be the ones that are 'right' for you.
You may ask for help if you do not understand something or are not sure how to answer a question.

SAMPLE QUESTION

Imagine that this happened to you:

	A part of your mathematics textbook was missing. This was because:	Strongly Agree	Agree	Disagree	Strongly Disagree
1.	you didn't count all the pages.	SA	A	D	SD
2.	your teacher didn't tell you to check.	SA	A	D	SD

Look at the first reason given: "You didn't count all the pages."
If you agree with that reason, then mark either **SA = Strongly Agree or A = Agree.**

If you disagree with the reason, then mark either **D = Disagree or SD = Strongly Disagree.**

Now mark how you feel about the other reason, using **SA, A, D, or SD.**

DO NOT FOLD OR DEFACE THIS SHEET IN ANY WAY

ACER

Imagine that this happened to you:

	A part of your mathematics homework was wrong. This was because:	Strongly Agree	Agree	Disagree	Strongly Disagree
1.	you didn't understand the topic very well.	SA	A	D	SD
2.	you were careless about completing it.	SA	A	D	SD
3.	the part marked wrong included a step that was difficult.	SA	A	D	SD
4.	there had been no-one at home to ask to help you.	SA	A	D	SD

Imagine that this happened to you:

	You got the results that you wanted in mathematics. This was because:	Strongly Agree	Agree	Disagree	Strongly Disagree
5.	the work covered in class was easy.	SA	A	D	SD
6.	you spent a lot of time studying maths.	SA	A	D	SD
7.	the teacher is good at explaining maths.	SA	A	D	SD
8.	you are good at maths.	SA	A	D	SD

Imagine that this happened to you:

	You had trouble with some of the problems in your maths homework. This was because:	Strongly Agree	Agree	Disagree	Strongly Disagree
9.	there was not time to get help from your teacher.	SA	A	D	SD
10.	you don't think in the logical way that maths needs.	SA	A	D	SD
11.	you didn't take enough time to do each problem properly.	SA	A	D	SD
12.	the problems were very difficult.	SA	A	D	SD

Imagine that this happened to you:

	You have not been able to keep up with the rest of the class in maths. This is because:	Strongly Agree	Agree	Disagree	Strongly Disagree
13.	students sitting near you wouldn't work.	SA	A	D	SD
14.	you haven't spent much time working on maths.	SA	A	D	SD
15.	the topics were difficult.	SA	A	D	SD
16.	you have always had a difficult time in maths classes.	SA	A	D	SD

Imagine that this happened to you:

	You have been able to complete the last few maths problems easily. This was because:	Strongly Agree	Agree	Disagree	Strongly Disagree
17.	the problems were more interesting.	SA	A	D	SD
18.	the effort that you put in made it easier.	SA	A	D	SD
19.	you are a very able maths student.	SA	A	D	SD
20.	you were lucky to have friends to help you.	SA	A	D	SD

Imagine that this happened to you:

	You were able to understand a difficult topic in maths. This was because:	Strongly Agree	Agree	Disagree	Strongly Disagree
21.	the way the teacher presented the topic helped.	SA	A	D	SD
22.	your ability is more obvious when you are challenged.	SA	A	D	SD
23.	you put in extra time studying the topic.	SA	A	D	SD
24.	the topic was easy because you had done a topic like it before.	SA	A	D	SD

PLEASE DO NOT WRITE IN THIS AREA

2

Imagine that this happened to you:

	You got a low mark in a maths test. This was because:	Strongly Agree	Agree	Disagree	Strongly Disagree
25.	you are not the best student in maths.	SA	A	D	SD
26.	you studied, but not hard enough.	SA	A	D	SD
27.	there were types of questions that you had never seen before.	SA	A	D	SD
28.	the teacher had spent very little time on the maths in class.	SA	A	D	SD

Imagine that this happened to you:

	You have understood most of the work in maths this year. This is because:	Strongly Agree	Agree	Disagree	Strongly Disagree
29.	the teacher made learning maths interesting.	SA	A	D	SD
30.	you are just good at maths.	SA	A	D	SD
31.	you spent hours of extra time studying maths.	SA	A	D	SD
32.	the maths is pretty easy.	SA	A	D	SD

Imagine that this happened to you:

	You made a mistake solving a maths problem that the teacher asked you to do on the board. This was because:	Strongly Agree	Agree	Disagree	Strongly Disagree
33.	it was a very hard problem.	SA	A	D	SD
34.	you didn't understand the topic being asked about.	SA	A	D	SD
35.	you were unlucky.	SA	A	D	SD
36.	you hadn't been working hard in class.	SA	A	D	SD

Now mark your feelings about these statements.
If you agree with the statement, then mark either **SA = Strongly Agree or A = Agree.**
If you disagree with the statement, then mark either **D = Disagree or SD = Strongly Disagree.**

		Strongly Agree	Agree	Disagree	Strongly Disagree
37.	I have a lot of self-confidence when it comes to maths.	SA	A	D	SD
38.	I enjoy the maths I do at school.	SA	A	D	SD
39.	I'm motivated to want to learn maths at my school.	SA	A	D	SD
40.	I learn maths at school that will be useful to me when I leave school.	SA	A	D	SD
41.	I'm no good at maths.	SA	A	D	SD
42.	Learning maths is fun at my school.	SA	A	D	SD
43.	I'm keen to do well at school maths.	SA	A	D	SD
44.	The maths I learn at school will help me get a job when I leave school.	SA	A	D	SD
45.	I get excited about the maths I do at school.	SA	A	D	SD
46.	I'm not the type to do well at maths.	SA	A	D	SD
47.	I think it is worth trying hard at maths.	SA	A	D	SD
48.	The maths I learn at school is useful to me now.	SA	A	D	SD
49.	I enjoy being at my school.	SA	A	D	SD
50.	I want to get good maths results.	SA	A	D	SD
51.	I'm sure that I can learn maths.	SA	A	D	SD
52.	The maths I learn at school will be useful to me in the future.	SA	A	D	SD

PLEASE DO NOT WRITE IN THIS AREA

3

As you read the statements listed below, please think about your current mathematics class.

Please read each statement carefully and indicate the extent to which you agree or disagree with the statement by marking the appropriate bubble.

My maths teacher		Strongly Agree	Agree	Disagree	Strongly Disagree
53.	knows me well.				
54.	doesn't expect everyone to be able to do the maths.				
55.	doesn't expect everyone to do the maths the same way.				
56.	knows how to explain the maths to me.				
57.	shows me how maths helps me in real life.				
58.	makes me feel I can learn maths.				
59.	uses a variety of methods for teaching maths.				
60.	uses computers in maths classes to help me learn.				
61.	is enthusiastic about teaching maths.				
62.	helps me to think for myself in maths.				
63.	tries to help everyone to do well at maths.				

Please read each statement carefully and indicate how often it occurs by marking the appropriate bubble.

My maths teacher		Always	Often	Rarely	Never
64.	makes me feel it's OK to ask for help when I don't understand.				
65.	gets me to work with other students in class.				
66.	helps everyone equally.				
67.	makes me feel curious and want to know more about maths.				
68.	is well organised.				
69.	makes me think really hard sometimes.				
70.	gives me the chance to be creative.				
71.	helps me when I need it.				
72.	keeps me up to date with how I'm doing in maths.				
73.	gives me helpful comments on my work.				
74.	helps me to think about why I get something wrong.				

Please read each statement carefully and indicate how often it occurs by marking the appropriate bubble.

In teaching maths my teacher		Always	Often	Rarely	Never
75.	makes links between different areas of maths.				
76.	promotes maths as being enjoyable.				
77.	sets high standards for my maths work.				
78.	encourages me to be positive about maths.				
79.	helps me with my maths outside class time.				
80.	involves my parents in my maths learning.				
81	allows me to extend myself in maths.				
82.	takes everyone's interests and needs into account in maths lessons.				
83.	uses assessment that is fair.				
84.	uses a range of assessment methods.				
85.	gives me useful feedback on my maths work.				
86.	gives my parents useful feedback on my maths progress.				

4