Quantifying the Power Loss when Transmit Beamforming Relies on Finite Rate Feedback

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***Abstract*— Transmit beamforming achieves optimal performance in systems with multiple transmit-antennas and a single receive-antenna, from both the capacity and the received signal-to-noise ratio (SNR) perspectives, but ideally requires perfect channel knowledge at the transmitter. In practical systems where the feedback link can only convey a finite number of bits, transmit-beamformer designs have been extensively investigated using either the outage probability, or the average SNR, as the figure of merit. In this paper, we study the symbol error rate (SER) for transmit beamforming with finite-rate feedback, in a multi-input single-output (MISO) setting. We derive a SER lower bound, which is tight for good beamformer designs. Comparing this bound with the SER corresponding to the ideal case, we quantify the power loss due to the finite rate constraint, across the entire SNR range.**

***Index Terms*— Multi-antenna systems, transmit beamforming, finite rate feedback, power loss.**

I. Introduction

Multi-antenna diversity has by now been well established as an effective fading counter-measure for wireless communications. In certain application scenarios, e.g., cellular downlink, the number of receive antennas at the mobile is limited due to size and cost constraints, which motivates deployment of transmit-diversity systems. Our attention in this paper is thereby focused on downlink applications dealing with single receive- but multiple transmit-antennas.

To further improve system performance, the receiver can feed channel state information (CSI) back to the transmitter. With perfect CSI, transmit-beamforming achieves the optimal performance in the multi-input single-output (MISO) setting, in terms of maximizing both the received signal to noise ratio (SNR) and the mutual information between the channel input and output [17]. However, in practical wireless systems, the CSI at the transmitter suffers from imperfections originating from various sources, such as estimation errors, quantization effects, feedback delay and feedback errors. These considerations sparked recent research interest towards quantifying and exploiting imperfect (or partial) CSI in multi-antenna systems, e.g., [1], [2], [7]–[11], [13]–[17], [19], [21], [24], [26]–[28].

Partial CSI can be modeled in different ways. One approach is to describe it statistically based on feedback information. For example, CSI at the transmitter can be modeled as Gaussian distributed with local mean and local covariance [7], [10], [17], [24]. Under this model, two special forms of partial feedback have been extensively studied. One is the so-called mean feedback [8], [10], [15], [17], [24], [27], that assumes knowledge of the channel mean and models the channel covariance as spatially white. The other is termed covariance feedback, where the channel mean is set to zero, and the relative geometry of the propagation paths manifests itself in a generally nonwhite covariance matrix [8], [15], [19], [21], [24], [28].

Another class of CSI models imposes a bandwidth constraint on the feedback channel which is only able to communicate a finite number of bits per block [1], [9], [11], [13], [14], [16], [17]. In particular, finite-rate power control is investigated in [1] to reduce the outage probability that the mutual information falls below a certain rate. Transmit beamforming has been investigated based on the average SNR [14], [17], and also on the outage probability criterion [16]. Extension to unitary precoding for high-rate spatial multiplexing systems is pursued in [13]. Subject to finite-rate feedback, optimal transmission is also studied in [3], [11] to maximize the average channel capacity, while adaptive modulation together with transmit beamforming has been pursued in [26] to enhance the transmission rate.

For transmit-beamforming based on finite-rate feedback, a universal lower bound on the outage probability (applicable to all beamformers) is derived in [16]. Good beamformers are then constructed to attain this lower bound. On the other hand, beamforming vectors are designed in [14] to minimize the average SNR reduction due to finite-rate feedback. Albeit starting from different perspectives, the beamformer design criteria in [16] and [14] are equivalent, leading to the minimization of the maximum correlation between any pair of beamforming vectors. Interestingly, the beamformer design is linked to the line packing problem in Grassmannian manifold, where the

Manuscript received February 04, 2004, revised May 20, 2004. The work by S. Zhou is supported by UConn Research Foundation internal grant 445157. The work by G. B. Giannakis is prepared through collaborative participation in the Communications and Networks Consortium sponsored by the U. S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011. The U. S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon. Part of this work was presented at the 38th Annual Conferences on Information Sciences and Systems, Princeton, New Jersey, March 17-19, 2004. The associate editor coordinating the review of this paper and approving it for publication was Dr. T. Duman.

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Publisher Item Identifier

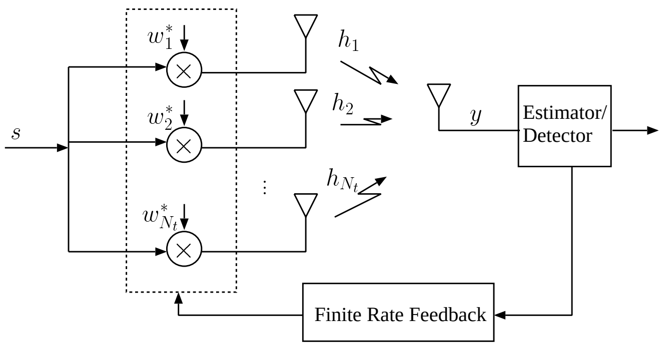

Fig. 1. The system model

minimum chordal distance between any pair of straight lines passing through the origin should be maximized [5].

Different from [16] and [14], we in this paper investigate the symbol error rate (SER) performance of transmit beamforming with finite rate feedback. We derive a universal SER lower bound, that is applicable to any beamformer design. This bound is tight for the good beamformers constructed in [14], [16], thus serving as a good performance indicator. Furthermore, comparing this lower bound with the SER corresponding to the perfect CSI case enables one to quantify the power loss due to the finite rate constraint. Our result is *valid across the entire SNR range*.

The rest of this paper is organized as follows. We present the system model in Section II, derive the SER lower bound in Section III and quantify the power loss in Section IV. We then collect numerical results in Section V and draw concluding remarks in Section VI.

Notation: Bold upper and lower case letters denote matrices and column vectors, respectively; $||\cdot||$ denotes vector norm; $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^{\mathcal{H}}$ stand for conjugate, transpose, and Hermitian transpose, respectively; $E\{\cdot\}$ denotes expectation; $\mathbf{I}_N$ stands for an identity matrix of size N, and $\mathbf{0}_{M\times N}$ for an $M\times N$ all zero matrix; $\mathcal{C}^N$ stands for the N-dimensional complex vector space; $\mathcal{CN}(\mathbf{b},\mathbf{\Sigma})$ denotes the complex Gaussian distribution with mean $\mathbf{b}$ and covariance $\mathbf{\Sigma}$.

II. System Model

Fig. 1 depicts the considered multi-input single-output (MISO) system with N_t transmit-antennas and a single receive-antenna. Let h_μ denote the channel coefficient between the μ-th transmit antenna and the single receive antenna, and collect h_μ's into the channel vector $\mathbf{h} := [h_1,\ldots,h_{N_t}]^T$. As in [14], [16], we assume that h_μ's are independently and identically distributed (i.i.d.) according to a complex Gaussian distribution with zero mean and unit variance; i.e.,

$$\mathbf{h}\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_{N_t}). \tag{1}$$

The extension to correlated channels needs further investigation and goes beyond this paper's scope.

We adopt transmit-beamforming without temporal power control [14], [16]. Specifically, each information symbol s is multiplied by a beamforming vector $\mathbf{w}^*$, where

$$\mathbf{w} := [w_1, w_2, \ldots, w_{N_t}]^T \tag{2}$$

has unit norm $\|\mathbf{w}\| = 1$. The N_t entries of the vector $s\mathbf{w}^*$ are transmitted simultaneously from N_t antennas, which leads to a received sample:

$$y = \mathbf{w}^{\mathcal{H}}\mathbf{h}s + \eta, \tag{3}$$

where η is the additive complex Gaussian noise with zero mean and variance N_0. With E_s denoting the average symbol energy, the instantaneous SNR in (3) is

$$\gamma = |\mathbf{w}^{\mathcal{H}}\mathbf{h}|^2 E_s/N_0. \tag{4}$$

As in [14], [16], we suppose that the receiver is able to feed a finite number of (say B) bits back to the transmitter. We assume that the feedback link is error-free and delay-free. The feedback bits will be used to select the beamforming vector. With B bits, the transmitter has a total of $N = 2^B$ beamforming vectors to choose from. Let us denote these vectors as $\mathbf{w}_1,\mathbf{w}_2,\ldots,\mathbf{w}_N$, and collect them into a matrix (the codebook of beamforming vectors):

$$\mathbf{W} := [\mathbf{w}_1,\mathbf{w}_2,\ldots,\mathbf{w}_N]. \tag{5}$$

The beamforming vector is selected as follows. The receiver is assumed to have perfect knowledge of $\mathbf{h}$, and chooses the beamforming vector to maximize the instantaneous SNR; i.e.,

$$\mathbf{w}_{\rm opt} = \arg\max_{\mathbf{w}\in\{\mathbf{w}_i\}_{i=1}^N} |\mathbf{w}^{\mathcal{H}}\mathbf{h}|^2. \tag{6}$$

The index of $\mathbf{w}_{\rm opt}$ is coded into B feedback bits. After receiving the B feedback bits, the transmitter switches to the optimal beamforming vector $\mathbf{w}_{\rm opt}$.

With the selection process in (6), the instantaneous SNR in (4) becomes

$$\gamma = \max_{1\le i\le N}\{|\mathbf{w}_i^{\mathcal{H}}\mathbf{h}|^2\}E_s/N_0, \tag{7}$$

which indicates that system performance depends critically on the design of the beamforming vectors $\{\mathbf{w}_i\}_{i=1}^N$.

A. Good Beamformers

The codebook optimization on $\{\mathbf{w}_i\}_{i=1}^N$ has been throughly investigated in [16] and [14], where [16] uses the outage probability as figure of merit, and [14] the *average* receive-SNR. Albeit starting from different perspectives, these two criteria lead to the same beamformer designs. Specifically, with i.i.d. channels in (1), it turns out that a good beamformer should minimize the maximum correlation between any pair of beamforming vectors; i.e.,

$$\mathbf{W}_{\rm opt} = \min_{\mathbf{W}\in\mathcal{C}^{N_t\times N}}\max_{1\le i<j\le N}|\mathbf{w}_i^{\mathcal{H}}\mathbf{w}_j|. \tag{8}$$

In [14], the beamformer design problem is explicitly linked to the Grassmannian line packing problem in [5]. Specifically, $\mathbf{w}_i$ can be viewed as coordinates of a point on the surface of a hypersphere with unit radius centered around origin. This point dictates a straight line in a complex space $\mathcal{C}^{N_t}$ that passes through the origin. The two lines generated by $\mathbf{w}_i$ and $\mathbf{w}_j$ have a distance defined as:

$$\begin{aligned} d(\mathbf{w}_i,\mathbf{w}_j) &:= \sin(\theta_{i,j}) \\ &= \sqrt{1-|\mathbf{w}_i^{\mathcal{H}}\mathbf{w}_j|^2}, \end{aligned} \tag{9}$$

where $\theta_{i,j}$ denotes the angle between these two lines. The distance $d(\mathbf{w}_i, \mathbf{w}_j)$ is known as "chordal distance" [5]. So the beamformer design in (8) is equivalent to:

$$\mathbf{W}_{\text{opt}} = \max_{\mathbf{W}\in\mathcal{C}^{N_t\times N}} \min_{1\leq i<j\leq N} d(\mathbf{w}_i, \mathbf{w}_j). \quad (10)$$

For illustration purposes, we next list the code design examples from [14], which will also be used in Section V.

Example codes from [14]:

1) $N_t = 2$ and $N = 4$ ($B = 2$ feedback bits). The matrix $\mathbf{W}$ is constructed as:

$$\mathbf{W}^T = \begin{bmatrix} -0.1612-0.7348j & -0.5135-0.4128j \\ -0.0787-0.3192j & -0.2506+0.9106j \\ -0.2399+0.5985j & -0.7641-0.0212j \\ -0.9541 & 0.2996 \end{bmatrix}. \quad (11)$$

The achieved maximum correlation is

$$\max_{1\leq i<j\leq N} |\mathbf{w}_i^{\mathcal{H}}\mathbf{w}_j| = 0.57735,$$

and the achieved minimum chordal distance is

$$\min_{1\leq i<j\leq N} d(\mathbf{w}_i, \mathbf{w}_j) = \sqrt{0.6713} = \sin(55^\circ).$$

2) $N_t = 2$ and $N = 8$ ($B = 3$ feedback bits). The matrix $\mathbf{W}$ is constructed as:

$$\mathbf{W}^T = \begin{bmatrix} 0.8393-0.2939j & -0.1677+0.4256j \\ -0.3427+0.9161j & 0.0498+0.2019j \\ -0.2065+0.3371j & 0.9166+0.0600j \\ 0.3478-0.3351j & 0.2584+0.8366j \\ 0.1049+0.6820j & 0.6537+0.3106j \\ 0.0347-0.2716j & 0.0935-0.9572j \\ -0.7457+0.1181j & -0.4533-0.4719j \\ -0.7983+0.3232j & 0.5000+0.0906j \end{bmatrix}. \quad (12)$$

The achieved maximum correlation is

$$\max_{1\leq i<j\leq N} |\mathbf{w}_i^{\mathcal{H}}\mathbf{w}_j| = 0.84152,$$

and the achieved minimum chordal distance is

$$\min_{1\leq i<j\leq N} d(\mathbf{w}_i, \mathbf{w}_j) = \sqrt{0.2918} = \sin(32^\circ).$$

Remark 1 (links with selection combining): The beamformer design in (8) is challenging only when the codebook size $N > N_t$. When $N \leq N_t$, $\mathbf{W}$ can be chosen as N columns of an arbitrary $N_t \times N_t$ unitary matrix, which leads to the minimal chordal distance as $1 = \sin(90^\circ)$. The resulting beamforming system has identical performance as a selection combining (SC) system with N diversity branches [14], [16]. Hence, the system error performance can be readily evaluated using the closed-form expression for SC schemes in [4, eq. (36)]; see also [20, eq. (9.338)] for extensions to generalized selection combining (GSC), and [23, eq. (8)] for transmit-SC combined with receive-MRC (maximum ratio combining) in systems with multiple antennas at both the transmitter and the receiver.

III. Performance Analysis

Our objective in this section is to evaluate the average performance for beamformed transmissions with finite-rate feedback. Towards this objective, we assume that s is drawn from a PSK (phase shift keying) constellation; similar derivations can be carried out for other constellations, as detailed in Remark 2 later on. Conditioned on the instantaneous SNR γ, the symbol error rate (SER) is [20, eq. (8.22)]:

$$\text{SER}(\gamma) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \exp\left(-\frac{g_{\text{PSK}}\gamma}{\sin^2\theta}\right) d\theta, \quad (13)$$

where M is the constellation size, and

$$g_{\text{PSK}} := \sin^2(\pi/M) \quad (14)$$

is a constellation-dependent constant. With $\mathbf{h}$ a random vector, the average SER is expressed as:

$$\overline{\text{SER}} = E_{\mathbf{h}}\{\text{SER}(\gamma)\}. \quad (15)$$

Our *objective* is to find either exact or (tight) approximate expressions for $\overline{\text{SER}}$.

A. SER formulation

We define a normalized channel vector as:

$$\tilde{\mathbf{h}} = \frac{\mathbf{h}}{\|\mathbf{h}\|}, \quad (16)$$

such that $\mathbf{h} = \|\mathbf{h}\| \cdot \tilde{\mathbf{h}}$ and $\|\tilde{\mathbf{h}}\| = 1$. We then decompose the SNR in (7) as:

$$\begin{aligned} \gamma &= \|\mathbf{h}\|^2 \frac{E_s}{N_0} \max_i \left| \mathbf{w}_i^{\mathcal{H}} \cdot \frac{\mathbf{h}}{\|\mathbf{h}\|} \right|^2 \\ &= \|\mathbf{h}\|^2 \frac{E_s}{N_0} \left[1 - \min_i d^2(\mathbf{w}_i, \tilde{\mathbf{h}})\right], \end{aligned} \quad (17)$$

where $d(\cdot)$ is the chordal distance in (9). To simplify notation, we define the average transmit SNR $\overline{\gamma} := E_s/N_0$, and two random variables:

$$\gamma_h := \|\mathbf{h}\|^2, \quad (18)$$

$$Z := \min_i d^2(\mathbf{w}_i, \tilde{\mathbf{h}}), \quad (19)$$

such that

$$\gamma = \gamma_h(1-Z)\overline{\gamma}. \quad (20)$$

Based on (1), γ_h is Gamma distributed with parameter N_t and mean $E\{\|\mathbf{h}\|^2\} = N_t$; hence, its probability density function (pdf) is [20]:

$$p(\gamma_h) = \frac{\gamma_h^{N_t-1}}{\Gamma(N_t)} e^{-\gamma_h}, \quad (21)$$

where $\Gamma(\cdot)$ is the Gamma function. On the other hand, Z is a random variable within the interval $[0,1]$, i.e., $Z \in [0,1]$. Let $p(z)$ and $F_Z(z)$ denote the pdf and cdf (cumulative distribution function) of Z. Due to the i.i.d. assumption in (1), γ_h is independent of $\tilde{\mathbf{h}}$ [16]; thus, γ_h and Z are independent.

Using the definitions of $\{\gamma_h, Z, \overline{\gamma}\}$, we can write (15) explicitly as

$$\overline{\text{SER}} = \int_{\gamma_h=0}^{\infty}\int_{z=0}^{1} \text{SER}(\gamma_h(1-z)\overline{\gamma}) p(\gamma_h) p(z) d\gamma_h dz. \quad (22)$$

Substituting (13), (21) into (22), and utilizing the fact that the moment generating function of γ_h is $E\{e^{s\gamma_h}\} = (1-s)^{-N_t}$ [20], we obtain:

$$\overline{\text{SER}} = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \int_0^1 \left[1 + \frac{g_{\text{PSK}}\overline{\gamma}}{\sin^2\theta}(1-z)\right]^{-N_t} dF_Z(z)d\theta. \tag{23}$$

Notice that $F_Z(z)$ depends on the particular beamformer design $\mathbf{W}$. To obtain the exact SER in (23), one has to find the exact $F_Z(z)$ for each particular beamformer. Unfortunately, this task is difficult to accomplish. We next develop a SER lower bound, that is applicable to all beamformers. Our approach here is analogous to [16], where a lower bound on the outage probability is derived.

B. SER Lower Bound

To find a SER lower bound, we look for a cdf function $\tilde{F}_Z(z)$, such that:

$$F_Z(z) \le \tilde{F}_Z(z), \qquad 0 \le z \le 1. \tag{24}$$

Replacing $F_Z(z)$ in (23) by $\tilde{F}_Z(z)$, we define a SER lower bound as:

$$\overline{\text{SER}}_{\text{lb}} := \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \int_0^1 \left[1 + \frac{g_{\text{PSK}}\overline{\gamma}}{\sin^2\theta}(1-z)\right]^{-N_t} d\tilde{F}(z)d\theta. \tag{25}$$

The proof that $\overline{\text{SER}} \ge \overline{\text{SER}}_{\text{lb}}$ is given in Appendix A.

We next derive such a function $\tilde{F}_Z(z)$ under the geometrical framework presented in [16]. Around each beam-vector $\mathbf{w}_i$, we define a spherical cap on the surface of the hypersphere,

$$S_i(z) = \{\tilde{\mathbf{h}} : d^2(\mathbf{w}_i, \tilde{\mathbf{h}}) \le z\},$$

where $0 \le z \le 1$. Define $A\{S_i(z)\}$ as the area of the cap $S_i(z)$. Then $A\{S_i(1)\}$ is the whole surface area of the unit hypersphere. According to [16, Lemma 2], the surface area of the spherical cap $S_i(z)$ is:

$$A\{S_i(z)\} = \frac{2\pi^{N_t} z^{N_t-1}}{(N_t-1)!}. \tag{26}$$

Notice that $F_Z(z) = \Pr(\min_i d^2(\mathbf{w}_i, \tilde{\mathbf{h}}) \le z)$ equals

$$F_Z(z) = \Pr(d^2(\mathbf{w}_1, \tilde{\mathbf{h}}) \le z, \text{ or } d^2(\mathbf{w}_2, \tilde{\mathbf{h}}) \le z, \ldots, \text{ or } d^2(\mathbf{w}_{N_t}, \tilde{\mathbf{h}}) \le z), \tag{27}$$

which can be interpreted as the probability that $\tilde{\mathbf{h}}$ falls in the union of the regions $\{S_i(z)\}_{i=1}^N$ (denoted by $\cup_{i=1}^N S_i(z)$). Based on the key observation that $\tilde{\mathbf{h}}$ is uniformly distributed on the surface of the unit hypersphere, when $\mathbf{h}$ is i.i.d. [16], we can find $F_Z(z)$ as:

$$F_Z(z) = \frac{A\{\cup_{i=1}^N S_i(z)\}}{A\{S_i(1)\}}. \tag{28}$$

Due to the union operation, it is true that:

$$A\{\cup_{i=1}^N S_i(z)\} \le \sum_{i=1}^N A\{S_i(z)\}. \tag{29}$$

Based on the area computation of (26), we obtain:

$$F_Z(z) \le \frac{\sum_{i=1}^N A\{S_i(z)\}}{A\{S_i(1)\}} = N z^{N_t-1}. \tag{30}$$

Taking into account that $F_Z(z) \le 1$, we define an upper bound $\tilde{F}_Z(z)$ as

$$\tilde{F}_Z(z) = \begin{cases} N z^{N_t-1}, & 0 \le z < (1/N)^{1/(N_t-1)} \\ 1, & z \ge (1/N)^{1/(N_t-1)} \end{cases}. \tag{31}$$

Substituting (31) into (25), we obtain the SER lower bound in (32) at the top of the next page, where the integration of z can be simply carried out by defining a new variable $t = 1/z$.

Notice that the lower bound is independent of any particular beamformer design. A good beamformer shall try to come as close as possible to this lower bound. This confirms the design guidelines in (8) and (10), as follows.

Let z_o denote the critical value where all the spherical caps $S_1(z_o)$, $S_2(z_o)$, $\ldots, S_N(z_o)$ do not overlap. For a given beamformer matrix $\mathbf{W}$, the value of z_o is available by re-interpreting the result of [16, Lemma 5] as:

$$\begin{aligned} z_o &= 1 - \frac{1 + \max\limits_{1\le i<j\le N_t} |\mathbf{w}_i^{\mathcal{H}}\mathbf{w}_j|}{2} \\ &= \frac{1 - \max\limits_{1\le i<j\le N_t} |\mathbf{w}_i^{\mathcal{H}}\mathbf{w}_j|}{2}. \end{aligned} \tag{33}$$

For $z < z_o$, since all the spherical caps do not overlap, we have

$$F_Z(z) = \tilde{F}_Z(z) = N z^{N_t-1}, \qquad z \le z_o. \tag{34}$$

Hence, $\tilde{F}_Z(z) \ne F_Z(z)$ only when $z > z_o$. To minimize the difference between $\tilde{F}_Z(z)$ and $F_Z(z)$, a good beamformer design tries to maximize z_o as much as possible. If z_o reaches the maximum of $(1/N)^{1/(N_t-1)}$, the true SER in (23) would coincide with the SER lower bound of (32); e.g., in the case of $N_t = 2$ and $N = 2$. Therefore, the maximum correlation $\max_{1\le i<j\le N_t} |\mathbf{w}_i^{\mathcal{H}}\mathbf{w}_j|$ should be minimized, agreeing with (8) that was obtained based on either the outage probability, or the average SNR, criterion.

Numerical results show that the universal lower bound in (32) is tight for good beamformer designs. This is consistent with [16], where the lower bound on outage probability is tight.

Remark 2 (extension to arbitrary constellations): For brevity, we derived the SER lower bound in (32) for the PSK constellation. However, the same procedure can be applied to arbitrary two-dimensional signal constellations. Specifically, we first need to express the conditional SER (conditioned on the instantaneous SNR γ) as the sum of a small number of finite integrals as described in [6] (notice that the SER for PSK in (12) is a special case with only one finite integral). Then, we average the conditional SER over the random variable γ, and obtain a lower bound following the same derivation to reach (32). For example, applying this procedure with a square QAM (quadrature amplitude modulation) constellation (the desired finite-integral form of the conditional SER is available in e.g., [20, eq. (8.12)]), we obtain the SER lower bound for QAM

$$\begin{aligned}\overline{\text{SER}}_{\text{lb}} &= \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\int_0^{(1/N)^{1/(N_t-1)}}\left[1+\frac{g_{\text{PSK}}\overline{\gamma}}{\sin^2\theta}(1-z)\right]^{-N_t} N(N_t-1)z^{N_t-2}dzd\theta \\ &= \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\left(1+\frac{g_{\text{PSK}}\overline{\gamma}}{\sin^2\theta}\right)^{-1}\left[1+\left[1-\left(\frac{1}{N}\right)^{\frac{1}{N_t-1}}\right]\frac{g_{\text{PSK}}\overline{\gamma}}{\sin^2\theta}\right]^{1-N_t} d\theta,\end{aligned} \tag{32}$$

$$\begin{aligned}\overline{\text{SER}}_{\text{lb}}^{\text{QAM}} &= \frac{4}{\pi}\frac{\sqrt{M}-1}{M}\int_0^{\frac{\pi}{4}}\left(1+\frac{g_{\text{QAM}}\overline{\gamma}}{\sin^2\theta}\right)^{-1}\left[1+\left[1-\left(\frac{1}{N}\right)^{\frac{1}{N_t-1}}\right]\frac{g_{\text{QAM}}\overline{\gamma}}{\sin^2\theta}\right]^{1-N_t} d\theta \\ &+ \frac{4}{\pi}\frac{\sqrt{M}-1}{\sqrt{M}}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(1+\frac{g_{\text{QAM}}\overline{\gamma}}{\sin^2\theta}\right)^{-1}\left[1+\left[1-\left(\frac{1}{N}\right)^{\frac{1}{N_t-1}}\right]\frac{g_{\text{QAM}}\overline{\gamma}}{\sin^2\theta}\right]^{1-N_t} d\theta\end{aligned} \tag{35}$$

in (35) at the top of this page, where M is the constellation size, and

$$g_{\text{QAM}} = \frac{1.5}{M-1}. \tag{36}$$

As one can easily verify from the development in the ensuing section, the power loss due to finite rate (as opposed to ideal) feedback for *arbitrary* two-dimensional constellations will be identical to that for PSK.

IV. Quantification of Power Loss Due to Finite-Rate Feedback

When N goes to infinity, the performance of beamformed transmissions with finite-rate feedback should approach the ideal case with perfect channel knowledge at the transmitter. With perfect CSI, $Z = 0$ in (23) and one can derive the SER for PSK as:

$$\overline{\text{SER}}_{\text{perfect CSI}} = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\left(1+\frac{g_{\text{PSK}}\overline{\gamma}}{\sin^2\theta}\right)^{-N_t} d\theta. \tag{37}$$

Therefore, the lower bound in (32) is tight with respect to N, since it coincides with (37) when $N = \infty$.

Comparing (32) with N finite against (37) with $N = \infty$, we will be able to quantify the power loss due to the finite-rate constraint. Towards this objective, we assume that:

AS0): the SER lower bound can be achieved by some good beamformer designs.

The performance of the beamformers in AS0) will depend on N_t, N, and $\overline{\gamma}$; for this reason, we explicitly express the SER as $\overline{\text{SER}}_{\text{lb}}(N_t, N, \overline{\gamma})$. On the other hand, the SER with perfect CSI depends only on N_t and $\overline{\gamma}$, denoted as $\overline{\text{SER}}_{\text{perfect CSI}}(N_t, \overline{\gamma})$. Due to the finite-rate constraint,

$$\begin{aligned}\overline{\text{SER}}_{\text{lb}}(N_t, N, \overline{\gamma}) &\geq \overline{\text{SER}}_{\text{lb}}(N_t, \infty, \overline{\gamma}) \\ &= \overline{\text{SER}}_{\text{perfect CSI}}(N_t, \overline{\gamma}).\end{aligned} \tag{38}$$

To compensate for this performance loss, one has to increase the transmission power. Suppose that when we increase the average SNR from $\overline{\gamma}$ to $\overline{\gamma}_{\text{new}}$, we arrive at:

$$\overline{\text{SER}}_{\text{lb}}(N_t, N, \overline{\gamma}_{\text{new}}) = \overline{\text{SER}}_{\text{lb}}(N_t, \infty, \overline{\gamma}). \tag{39}$$

We define the difference between $\overline{\gamma}_{\text{new}}$ and $\overline{\gamma}$ as the power loss (in decibels) due to the finite-rate constraint as:

$$L(N_t, N, \overline{\gamma}) = 10\log_{10}\overline{\gamma}_{\text{new}} - 10\log_{10}\overline{\gamma}. \tag{40}$$

Notice that the power loss in (40) is a function of $\overline{\gamma}$, and thus varies over the entire SNR range.

We have the following results on $L(N_t, N, \overline{\gamma})$:

Proposition 1 *Under AS0), the power loss for transmit-beamforming due to the finite-rate feedback constraint satisfies:*

$$L(N_t, N, \infty) = 10\log_{10}\left[1-\left(\frac{1}{N}\right)^{\frac{1}{N_t-1}}\right]^{\frac{1}{N_t}-1}, \tag{41}$$

$$L(N_t, N, \overline{\gamma}) \leq L(N_t, N, \infty). \tag{42}$$

Proof: The power loss at high SNR ($\overline{\gamma} \to \infty$) can be easily found. At high SNR, we simplify (32) as:

$$\overline{\text{SER}}_{\text{lb}}(N_t, N, \overline{\gamma}) \approx (G_c \cdot \overline{\gamma})^{-G_d}, \tag{43}$$

where $G_d = N_t$ is termed as the *diversity gain*, and

$$\begin{aligned}G_c = g_{\text{PSK}}&\left[\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}(\sin\theta)^{2N_t}d\theta\right]^{-\frac{1}{N_t}} \\ &\times\left[1-\left(\frac{1}{N}\right)^{\frac{1}{N_t-1}}\right]^{1-\frac{1}{N_t}}\end{aligned} \tag{44}$$

is referred to as the *coding gain* (see e.g., [22], [25]). Eq. (43) implies that the SER versus average SNR curve in fading channels is well approximated by a straight line at high SNR, when plotted on a log-log scale. The diversity gain G_d determines the slope of the curve, while G_c (in decibels) determines the shift of the curve in SNR relative to a benchmark SER curve of ($\overline{\gamma}^{-G_d}$). Substituting (43) into (39), we obtain $L(N_t, N, \infty)$ in (41).

For the power loss at arbitrary SNR, we prove (42) in Appendix B. ■

Proposition 1 testifies that the power loss across the entire SNR range is bounded to be less than or equal to $L(N_t, N, \infty)$.

TABLE I
THE POWER LOSS (IN DECIBELS) DUE TO THE FINITE-RATE CONSTRAINT

	$N=2$	$N=4$	$N=8$	$N=16$	$N=32$	$N=64$
$N_t=2$	1.51	0.62	0.29	0.14	0.07	0.03
$N_t=3$	-	2.01	1.26	0.83	0.56	0.39
$N_t=4$	-	3.24	2.26	1.65	1.23	0.94
$N_t=6$	-	-	3.90	3.09	2.51	2.07
$N_t=8$	-	-	5.16	4.25	3.57	3.05

TABLE II
THE MINIMUM NUMBER OF FEEDBACK BITS B FOR POWER LOSS WITHIN 1dB

N_t	2	3	4	5	6	7	8
$N \geq$	3	12	55	256	1220	5851	28170
$B = \lceil \log_2 N \rceil$	2	4	6	8	11	13	15

In other words, the distance (or, the horizontal shift), between $\overline{\text{SER}}_{\text{perfect CSI}}(N_t, \overline{\gamma})$ and $\overline{\text{SER}}_{\text{lb}}(N_t, N, \overline{\gamma})$ is *no larger* than $L(N_t, N, \infty)$ decibels across the *entire* SNR range.

When $N = 2^B$ is large, we use $\ln(1+x) \approx x$ for small x to obtain:

$$L(N_t, N, \infty) \approx \frac{10}{\ln 10}\left(1 - \frac{1}{N_t}\right) 2^{-\frac{B}{N_t - 1}}. \tag{45}$$

Hence, the power loss in decibels due to the finite-rate constraint decays exponentially with the number of feedback bits B for large B. This implies that B does not need to be very large for the system performance to be close to the optimal, as confirmed by numerical results.

On the other hand, if a system requires the power loss to be within L_0 decibels relative to the perfect CSI case, we can easily identify from (41) the least number of beamforming vectors needed as:

$$N \geq \left(1 - 10^{\frac{L_0}{10} \cdot \frac{N_t}{1 - N_t}}\right)^{-(N_t - 1)}. \tag{46}$$

Notice that only high SNR analysis is carried out in [16] based on the outage probability. Moreover, the distortion measure defined in [16, eq. (52)] does not translate to power loss. For a given reduction on the average capacity, or, the average SNR, the minimum N is also calculated in [14]. However, the calculation in [14] relies on a *loose* bound on the minimum chordal distance in line packing, which affects the accuracy.

Finally, recall that Proposition 1 holds under AS0). For practically adopted beamformers with AS0) not exactly satisfied, the power loss computation is only *approximate*. Notice that without AS0), only *asymptotic* analysis (with $N \to \infty$) may be possible, as in [16].

V. NUMERICAL RESULTS

In this section, we collect some numerical results.

Case 1: We compute the power loss in (41) for various cases and list the results in Table I, where we are interested in non-trivial configurations with $N \geq N_t$.

On the other hand, if we want to limit the power loss to be within 1dB relative to the perfect CSI case, the minimum number of beamforming vectors N can be computed from (46), which leads to the minimum number of feedback bits B; the results are listed in Table II. We clearly see that as N_t increases, the needed number of feedback bits increases considerably in order to bring the performance close to the optimum.

Case 2: We now compare the SER lower bound in (32) with the actual SER of (15), which is obtained via Monte Carlo simulations. We use 4-PSK constellations in all cases unless otherwise specified. Fig. 2 depicts the results with $N_t = 2$. The beamformer codebooks for $N = 4, 8$ are listed in Section II-A. The codebook with $N = 2$ is $\mathbf{W} = \mathbf{I}_2$, which corresponds to the selection diversity. As shown in Fig. 2, the SER lower bound for the $N_t = 2$ case is almost identical to the actual SER, even with small N.

Case 3: We now test the SER lower bound with the actual SER for the $N_t = 3$ case. We use the beamformers listed in [12] with $N = 4, 8, 16$. As shown in Fig. 3, the bound is tight for the given beamformers, although the difference between the bound and the actual SER increases relative to the $N_t = 2$ case.

Case 4: We now test the SER lower bound with the actual SER for the $N_t = 4$ case. We use the beamformers listed in [12] with $N = 16, 64$. The codebook with $N = 4$ is $\mathbf{W} = \mathbf{I}_4$, corresponding to selection combining. As shown in Fig. 4, the bound is also tight for the given beamformers, with the difference between the bound and the actual SER being larger than those corresponding to $N_t = 2, 3$. The increasing difference could be due to either of the following two reasons, or both: i) the bound becomes less tight for large N_t; ii) the beamformer currently found for large N_t is not as close to the optimum as in the $N_t = 2$ case.

Case 5: We now test the SER lower bound in (35) for QAM constellations. Fig. 5 depicts the results for 16-QAM with the same beamformers as in Case 4 where $N_t = 4$. The same observations can be obtained for QAM constellations, as for PSK.

Figs. 2, 3, 4 and 5 show that the distance between the SER lower bound and the SER with perfect CSI only slightly increases as the average SNR increases, yet it stays always

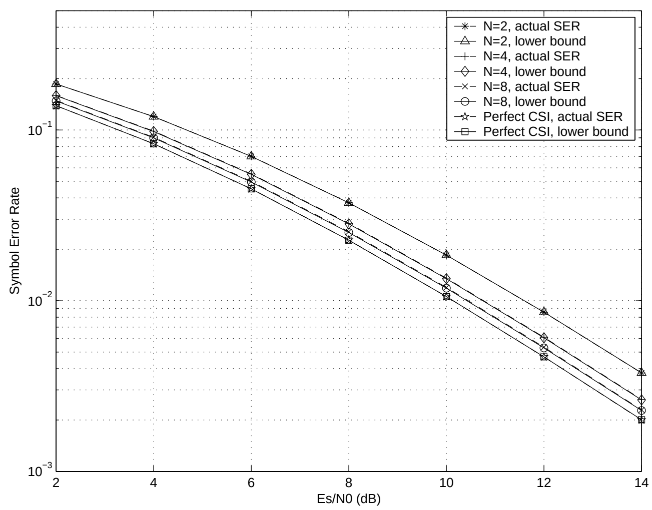

Fig. 2. The actual SER versus the lower bound ($N_t = 2$, QPSK)

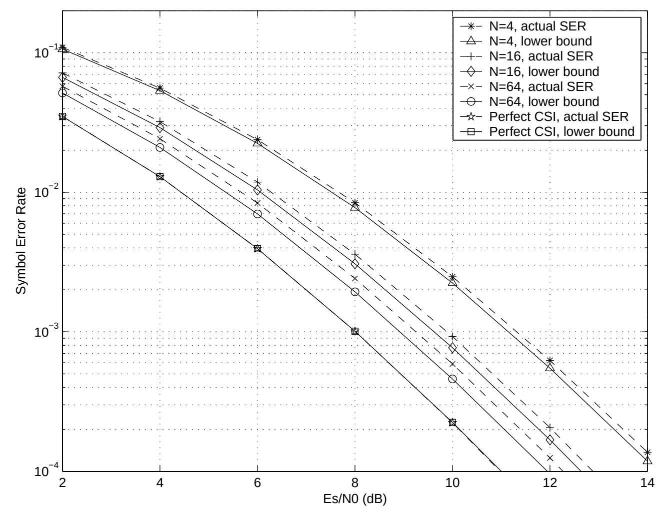

Fig. 4. The actual SER versus the lower bound ($N_t = 4$, QPSK)

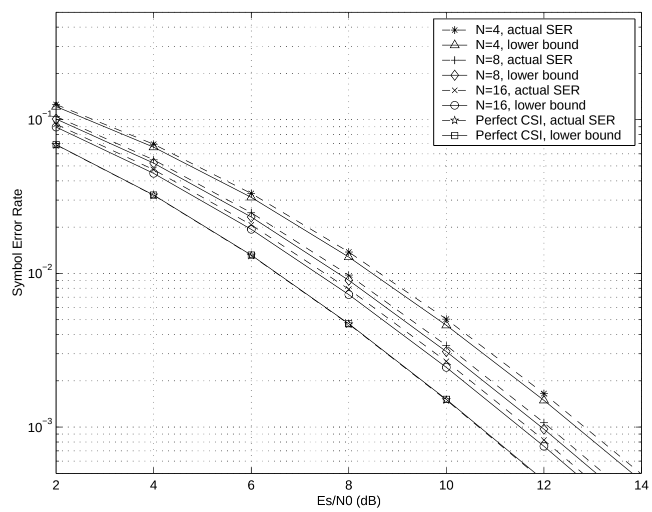

Fig. 3. The actual SER versus the lower bound ($N_t = 3$, QPSK)

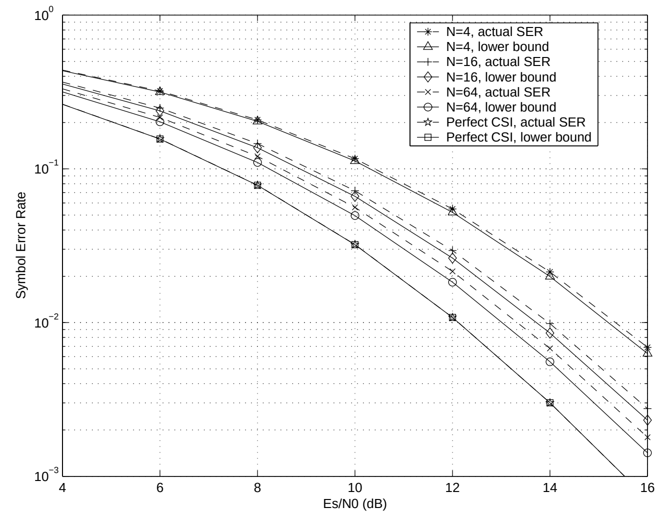

Fig. 5. The actual SER versus the lower bound ($N_t = 4$, 16-QAM)

bounded by the maximum power loss listed in Table I. This confirms (42), and more importantly, shows that "high" SNR analysis is often accurate even in the low and medium SNR range [25].

VI. CONCLUSIONS

In this paper, we developed a symbol error rate (SER) lower bound for transmit-beamforming systems with finite-rate feedback. This bound applies to all beamformer designs and is tight for well-constructed beamformers. Comparing this bound with the SER corresponding to the ideal case with perfect channel knowledge, we quantified the power loss due to the finite rate constraint across the entire SNR range.

Performance analysis in this paper will facilitate future work on adaptive modulation in beamformed transmissions with finite-rate feedback. Especially interesting is the applicability of these results to low or medium SNRs, since the target error rate for an uncoded adaptive system is usually not very low and the selected transmission mode will not operate in a high SNR regime.

APPENDIX

A. *Proof of* $\overline{\mathrm{SER}} \geq \overline{\mathrm{SER}}_{\mathrm{lb}}$

For notational convenience, let us define

$$f(\theta, z, \overline{\gamma}) := [1 + g_{\mathrm{PSK}}\overline{\gamma}(1-z)/\sin^2\theta]^{-N_t}. \quad (47)$$

Based on (23) and (25), we have

$$\overline{\mathrm{SER}} - \overline{\mathrm{SER}}_{\mathrm{lb}} = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{\pi}} \underbrace{\int_0^1 f(\theta, z, \overline{\gamma})[dF_Z(z) - \tilde{d}F_Z(z)]}_{:=\varphi(\theta,\overline{\gamma})} d\theta. \quad (48)$$

Using integration by parts, we can simplify the function $\varphi(\theta, \overline{\gamma})$ in (48) as

$$\varphi(\theta, \overline{\gamma}) = f(\theta, z, \overline{\gamma})[F_Z(z) - \tilde{F}_Z(z)]\big|_0^1 + \int_0^1 -[F_Z(z) - \tilde{F}_Z(z)]\frac{df(\theta, z, \overline{\gamma})}{dz}dz. \quad (49)$$

The first term in (49) is 0. Because $f(\theta, z, \overline{\gamma})$ is an increasing function of z, and $-[F_Z(z) - \tilde{F}_Z(z)] \geq 0$ due to (24), the second term in (49) is always non-negative for any θ. Therefore $\varphi(\theta, \overline{\gamma}) \geq 0$, which leads to $\overline{\mathrm{SER}} - \overline{\mathrm{SER}}_{\mathrm{lb}} \geq 0$ in (48).

B. Proof of (42)

Let us define $\xi(N_t, N, \overline{\gamma}) := \overline{\gamma}_{\mathrm{new}}/\overline{\gamma}$, so that

$$L(N_t, N, \overline{\gamma}) = 10 \log_{10} \xi(N_t, N, \overline{\gamma}). \tag{50}$$

Eq. (39) implies that:

$$\overline{\mathrm{SER}}_{\text{perfect CSI}}(N_t, \overline{\gamma}) = \overline{\mathrm{SER}}_{\mathrm{lb}}(N_t, N, \overline{\gamma} \cdot \xi(N_t, N, \overline{\gamma})). \tag{51}$$

Notice that $\overline{\mathrm{SER}}_{\mathrm{lb}}$ is a decreasing function of the average SNR $\overline{\gamma}$. In order to prove

$$\xi(N_t, N, \overline{\gamma}) \leq \xi(N_t, N, \infty), \tag{52}$$

we need to show:

$$\overline{\mathrm{SER}}_{\text{perfect CSI}}(N_t, \overline{\gamma}) \geq \overline{\mathrm{SER}}_{\mathrm{lb}}(N_t, N, \overline{\gamma} \cdot \xi(N_t, N, \infty)). \tag{53}$$

Define $\beta := g_{\mathrm{PSK}}/\sin^2\theta$ and $\alpha := [1 - (1/N)^{1/(N_t-1)}]$ for notational convenience. Substituting (37), (32) and (41) into (53), we need to prove that:

$$(1 + \beta\overline{\gamma})^{-N_t} \geq \left(1 + \beta\overline{\gamma}\alpha^{\frac{1}{N_t}-1}\right)^{-1} \left(1 + \beta\overline{\gamma}\alpha^{\frac{1}{N_t}}\right)^{1-N_t}, \tag{54}$$

for each θ. Taking logarithm on (54), we shall prove:

$$\begin{aligned} &\ln(1 + \beta\overline{\gamma}) \\ &\leq \frac{1}{N_t} \ln\left(1 + \beta\overline{\gamma}\alpha^{\frac{1-N_t}{N_t}}\right) + \frac{N_t - 1}{N_t} \ln\left(1 + \beta\overline{\gamma}\alpha^{\frac{1}{N_t}}\right). \end{aligned} \tag{55}$$

It is easy to verify that the function $f(x) = \ln(1 + \beta\overline{\gamma}e^{x \ln \alpha})$ is convex, since $f''(x) \geq 0$ regardless of the values of β, α and $\overline{\gamma}$. Eq. (55) holds true due to

$$f(0) \leq \frac{1}{N_t} f\left(-\frac{N_t - 1}{N_t}\right) + \frac{N_t - 1}{N_t} f\left(\frac{1}{N_t}\right). \tag{56}$$

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