

## GENERIC PROBLEMS AND ADVANCES IN THE COMPUTER AIDED DESIGN OF INSTRUMENTS AND INSTRUMENT ELEMENTS

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**Abstract:** The paper is based on a long-term programme of research on mathematical modelling and computer aided design of instruments and instrument elements in the Measurement and Instrumentation Centre of City University London. It considers the principal generic problems of computer aided design of instrument systems and aspects of the research agenda that arises from them. It describes recent advances in the computer aided design of sensors and actuators.

**Keywords:** design, models, instruments

### 1. INTRODUCTION

There continues to be rapid progress in the capability and affordability of computer software and hardware leading to the extensive applications of computer aided modelling and design in engineering. Instrumentation is a technology in which the application is particularly important. The nature of instrumentation is also progressing rapidly driven by the progress of information technology.

The Measurement and Instrumentation Centre of City University has been engaged in a long-term programme of research into the mathematical models of instruments and instrument elements, for the purposes of their analysis and design.

This paper reviews the generic issues that arise in the computer aided modelling and design of instruments and instrument elements and the advances that have been made in the field. It is intended to outline the state of the art and indicate an agenda for future action.

The paper reports recent advances that have been made in computer aided design of instrument elements.

### 2. ADVANCES IN INSTRUMENTATION

In modern instrumentation, information is generally carried by electrical signals. The analysis and design of these signals is generally performed by standard methods of signal theory. The signal and information processing components of modern instrumentation are generally standard components and are described by functional models. They are commonly implemented by standard information technology hardware and software and are

analysed and designed by the general methods of information technology.

However, the sensors and actuators of instrument systems require to be analysed and designed in terms of their physical embodiment and function. Their analysis and design thus requires special methods.

### 3. ADVANCES IN MODELS OF THE DESIGN PROCESS

The ultimate objective of developing computer-aided methods of design of instrument elements is the development of integrated computer environments in which the total design of instrument systems can be undertaken.

Such an environment would be based on a modern model of the design process, based on the concepts of knowledge processing and problem solving. A model of the design process, based on a blackboard architecture, has been proposed and discussed in [1].

There continue to be developments reported in the literature of knowledge engineering, and artificial intelligence, problem solving and design. Models of the design process based on these advances have significant conceptual value for measurement science and education in the field. However, there is considerable distance between these models and practical application. Advances in mathematical models of instrument elements have a higher priority.

### 4. ADVANCES IN MODELLING OF INSTRUMENT ELEMENTS

Computer aided design of engineering objects is based on their appropriate representation by models which can be handled by computer. The general concepts of instrument modelling have been considered in a previous paper to which the authors have contributed [2]. The present paper updates the previous one.

In the case of sensors and actuators two kinds of models are used: power flow models which represent the functional relation between physical inputs and physical outputs, and embodiment models that represent these relations in terms of the geometry and material properties of the embodiment.

Power flow models have seen substantial application. They are extensively used in the modelling and design of systems that consist of interacting components with diverse forms of energy. Mechatronics is an area in which such models are extensively and effectively used. In general instrumentation they provide a means of representing archetype models of sensors and actuators. They also are tools for modelling the interaction of sensors and the system being sensed, and that of actuators with the system upon which they act.

The main advances in these types of models have been in the development and application of computer software that automates model formulation and solution of system models.

Significant advances have been made in languages and computer packages for power flow models. In particular they are bondgraphs, Modelica, and the widely applied MATLAB [3-8].

The main requirement in the modelling of instrument system models is for embodiment models. It is in this area where the principal advances have been made for sensors and actuators. They will be reported later in the paper.

Qualitative, computer implemented, models have significant application potential in the description and in the design concept generation of complex instrumentation systems. While such models have made progress and provide useful insight they have not seen effective practical application. They are at the exploratory stage [9].

## **5. REQUIREMENTS ENGINEERING**

Design depends essentially upon the requirements specification. Advances have been made in the understanding and development of the processes of requirements capture and analysis in the design of systems [10].

The advances are in the development of tools and in applications. The latter have been in particular in the design of complex software systems.

The advances offer useful insights into the design process in instrumentation [11, 12]. The techniques have not seen application. They remain part of a research agenda.

## **6. DESIGN CONCEPT GENERATION**

The generation of design concepts is a central problem in design.

There have been significant advances in artificial intelligence concepts and techniques. The advances are enhancing the potential of the application of such computer aided methods in design. The paper will outline the advances and their application.

One of the principal methods of computer aided design concept generation is the reuse of concepts. This is implemented by searching organised knowledge bases. The internet provides a powerful tool for searching data bases of

design concepts, for the purpose of conceptual design of instrument systems.

However the main problem is the organisation of comprehensive and appropriate databases of instrument design concepts. It is an important aspect of the research agenda [13].

## **7. OPTIMAL FORM FINDING AND DIMENSIONING**

Computer implemented models enable the finding of optimal form and dimensions of a design concept. However in the design of physical objects such as sensors and actuators, there is, in general, an infinite number of variables. Some examples of the handling of problems in computer aided design, by model reduction and bounded optimisation, will be outlined later in the paper.

## **8. REALISTIC MATHEMATICAL MODELS OF SENSORS AND ACTUATORS**

The modelling of sensors and actuators and of the interaction between the sensor and the measured system are carried out considering the embodiment of those systems, that is their geometry, dimensions and material properties. These idealised embodiment models which give relations between an element physical embodiment and its function can be represented by idealised lumped parameter models. Such models have their origin in electrical circuit analysis. They are based on the relation between the input power flow to an element, or system, and the output flow.

While idealised models are useful in the representation and analysis of concepts and in conceptual design, detailed analysis and design requires realistic models which relate the detailed geometry, dimensions and material properties of the object modelled to its functional behaviour. Engineering objects are characterized by complex geometries and distributed properties. The physical laws governing their behaviour are represented by partial differential equations, which are often non-linear and transcendental.

There are a number ways of solving such realistic models – analytical, experimental and numerical. Analytical solutions are generally not feasible. They are normally applicable to problems with simple topology and linear materials. The experimental techniques, although applicable to many systems, are usually inaccurate, very time consuming and extremely expensive. In comparison, the numerical techniques based on, for example, the finite element (FEM) [14, 15], boundary element (BEM) [16, 17] and hybrid finite element-boundary element (FEM-BEM) [18] methods can tackle a wide variety of electrical, mechanical, thermal, structural and coupled problems. With the availability of powerful and affordable desktop computers, these techniques have revolutionised the formulation and solution of realistic models in the past few decades or so. This is especially true for the numerical finite element (FE) technique because of the relative ease of its computer implementation and the flexibility it provides in the definition of complex topology. FE models are fast, accurate and applicable to most physical systems. Finite

element techniques have made possible the formulation and analysis of realistic models. The underlying principle of this method lies in the fact that the problem domain is divided ('discretised') into a number of triangular or rectangular elements of finite size ('finite elements') and the solution is sought at the vertices ('nodes') of these elements. The size, shape and the density of the 2D/3D 'mesh' thus obtained affect the accuracy of the numerical solution obtained by FEM. Today, significant progress has been made in this area. Various 2D/3D finite element models are being routinely used for computer aided design, investigation and performance modelling of instrument transducers and sensors. Some examples of such FE modelling are given in the following Sections.

## 9. GENERIC MATHEMATICAL MODELS OF SENSORS AND ACTUATORS BASED ON ELECTRICAL PRINCIPLES

### 9.1 Capacitive sensors and actuators

Capacitive sensors and actuators are used in a wide variety of diversified industrial applications ranging from measuring displacement to moving micro-mirrors in MEMS-based video projection systems. In general, they are based on the well-known capacitive technique in which the capacitance in a system of electrodes is changed owing to the redistribution of electric field caused by changes in the dielectric properties and/or geometric parameters in the system. In most cases, for modelling, design and performance evaluation of these sensor/actuator sub-systems, the core activities focus on the accurate computation and characterisation of 2D/3D electrostatic fields in complex geometry [19-22]. This constitutes the main mathematical model of these sub-systems, the solution of which involves the solution of the following Laplace's or Poisson's equation governing the field distribution in the 2D/3D problem domain  $\Omega(x, y, z)$ :

$$\nabla \cdot \epsilon \nabla \Phi = 0 \quad (1)$$

$$\nabla \cdot \epsilon \nabla \Phi = -\rho \quad (2)$$

where  $\rho$  is the charge density and  $\epsilon = \epsilon(x, y, z)$  is the dielectric permittivity distribution in the problem domain.

Under appropriate boundary conditions, the solution of the above Laplace's (1) or Poisson's (2) equation gives the unknown electric potential distribution  $\Phi = \Phi(x, y, z)$  in the problem domain  $\Omega(x, y, z)$ . In most cases it is assumed that the dielectric materials in  $\Omega$  are linear, piece-wise homogeneous and isotropic. Following the solution of (1) or (2), the field intensity and flux density vectors  $\mathbf{E}$  and  $\mathbf{D}$ , and other quantities like capacitance are calculated. The capacitance  $C$  is calculated either from the electric field energy  $E_e$  for a given potential difference  $V$  or from charge  $Q$  using the following relationships:

$$E_e = \frac{1}{2} CV^2 \quad (3)$$

$$C = \frac{Q}{V} \quad (4)$$

Here charge  $Q$  is calculated by integrating the flux density vector  $\mathbf{D}$  over the appropriate electrode surfaces [20] using the Gauss's law:

$$Q = \oint_s \mathbf{D}_n ds = \oint_s D \cos \theta ds = \oint_s \mathbf{D} \cdot \mathbf{n} ds = \oint_s \mathbf{D} \cdot d\mathbf{s} \quad (5)$$

$$Q \approx \oint_s D ds \quad (6)$$

Although equations (1) and (2) are universal for sensors and actuators based on electrostatic principles, the specific formulation of their solution by FEM may vary depending on the material and geometric parameters, and the overall topology of the problem domain. An example of finite element models for capacitive sensors is given in Figs. 1-3 which show the FE models of a capacitive angular position sensor used in high-performance torque motor actuators.

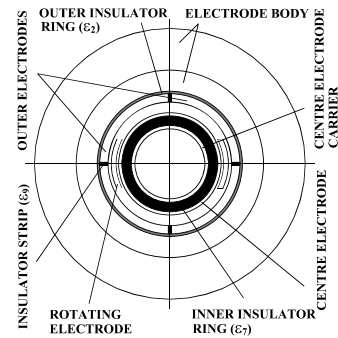


Fig. 1. Cross-section of a 4-electrode angular position sensor used in torque motor actuators.

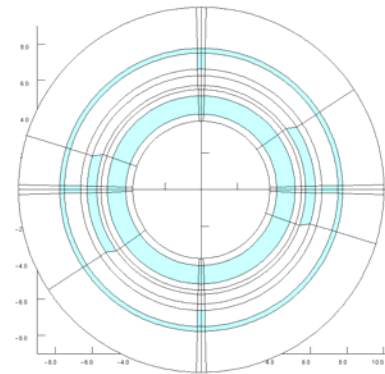


Fig. 2. 2D finite element model of the capacitive sensor shown in Fig. 1.

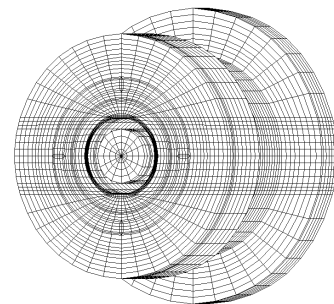


Fig. 3. 3D finite element model of the capacitive sensor in Fig. 1 showing 3D mesh consisting of about 75000 8-noded hexahedral elements.

## 9.2 Electromagnetic actuators

Electromagnetic (EM) actuators of wide variety of sizes, shapes, power outputs and technological realizations are used in many applications where discrete cyclic motions are required. Compared to other actuating mechanisms based on, for example, piezoelectric and hydraulic principles EM actuators are simpler, cheaper, easily repairable, robust, and easier to manufacture.

EM actuators rarely operate in the steady state and various operational factors like start-stop duty, operating frequency, response time and damping have a significant influence on their design. The EM part of the system is represented by electric and magnetic circuits with self-inductance, resistance and reluctance which are subject to variations, in general, due to eddy currents, saturation conditions, motional electromotive force (e.m.f.), demagnetisation and hysteresis. The mechanical part is represented by friction, damping, elasticity and inertia as well as external forces. The nonlinear and transient EM, thermal, and motional problems that need to be solved in high speed actuators pose substantial challenges because of their high frequency of operation and the requirement, in many cases, for a continuous and fail-safe multibillion cycle operational regimes.

In general, the mathematical model of an EM actuator can be adequately represented by the following four differential equations shown below: (7) an electrical circuit equation for the excitation coil and control circuitry, (8) a nonlinear magnetic field equation (Poisson's equation) for the flux, the change of which changes the EM energy storage in the system and produces the magnetic force, (9) a mechanical equation for this force, load (e.g. pneumatic force), friction, inertia, acceleration, speed and displacement, and (10) a nonlinear thermal diffusion equation for the conduction of heat produced by electrical power losses:

$$u(t) = iR + N \frac{d\Psi(i, z)}{dt} \quad (7)$$

$$\text{curl}(\nu \text{curl } \mathbf{A}) = \mathbf{J} - \sigma \frac{\partial \mathbf{A}}{\partial t} + \sigma \mathbf{V} \times (\text{curl } \mathbf{A}) \quad (8)$$

$$F_m(i, z) = m \frac{d^2 z}{dt^2} + B \frac{dz}{dt} + Kz + F_e \quad (9)$$

$$\rho C \frac{\partial T}{\partial t} - \nabla \cdot [k(T) \nabla T] = q^B \quad (10)$$

In the above equations  $u(t)$ ,  $i$  and  $\Psi(i, z)$ , and  $z$  are the applied voltage, coil current, flux linkage with the coil, and the displacement of moving part (plunger) respectively,  $R$  and  $N$  are the coil resistance and the number of turns in the coil,  $\mathbf{J}$ ,  $\mathbf{A}$ ,  $\mathbf{V}$  are the coil current density, magnetic vector potential, and the plunger velocity;  $m$ ,  $B$ ,  $K$ ,  $F_m$  and  $F_e$  are the mass of the plunger, viscous damping coefficient, spring constant, magnetic force and the load force respectively; and  $T$ , and  $q^B$  are the temperature and the internal rate of heat

generated per unit volume respectively. The material parameters  $\nu$ ,  $\sigma$ ,  $\rho$ ,  $C$  and  $k$  denote the magnetic reluctivity ( $\nu=1/\mu$ ,  $\mu$  is the permeability), the electric conductivity, density, specific heat and the thermal conductivity respectively. In general, the above equations are nonlinear and inseparable. The current produced by (7) creates the magnetic field given by (8) and produces the magnetic force which causes the displacement, speed and acceleration of the actuator obtained from (9). The current also generates the heat and the resulting temperature distribution given by (10). There are two main approaches to the coupled solution of these equations: the direct coupled approach and the indirect coupled approach, neither of which alone is suitable to incorporate the whole array of factors which are expected to be encountered in the practical exploitation of high-speed EM actuators.

The thermal modelling involves the development of 2D/3D thermal models and the FE solution of the steady-state and/or transient heat transfer equation given by equation (10) above. The heat sources needed for this are given by the various losses mentioned above. The coupling of the magnetic field and the thermal equations (owing to the dependence of the power density on the magnetic vector potential and the temperature dependence of the magnetic permeability and electric conductivity) may be realised either by indirect coupling (in which the equations are solved separately and coupled by means of power density and an iterative process is used to compute the power density and the temperature distribution) or by direct coupling in which the equations are solved simultaneously. The prime aim here is to obtain a vital insight into the thermal behaviour of actuators and to enable quantification of the effects of various factors that affect such behaviour. The FE models also enable the simulation of possible modes of thermal failure and create an essential basis for the design of a predictable, thermally stable and reliable actuator sub-system.

The methodologies for modelling and design of EM actuators are normally based on modelling and computation of 2D/3D nonlinear magnetic field distribution using the numerical FE technique. This involves the steady-state and transient solutions of nonlinear Poisson's equation (8). The results are used for design optimisation and for investigating the effects of various geometric, material, EM and mechanical parameters on the output performance of actuators. As mentioned above the thermal modelling involves the development of 2D/3D thermal models and the FE solution of the heat transfer equation given by (10). The coupling of magnetic field and thermal equations may be realised either by indirect coupling or by direct coupling in which the equations are solved simultaneously.

### 9.3 Force calculation

Following the field computation, the magnetic force  $\mathbf{F}$  (a major design parameter) is calculated using the second Maxwell stress tensor  $\mathbf{T}$  due to magnetic field.  $\mathbf{T}$  is calculated from the tangential component of magnetic field intensity,  $H_\tau$  and the normal component of the flux density,  $B_n$  on a closed surface  $S$  in air surrounding the moving part.

The surface density  $F_\rho$  of the force  $F$  is defined as the divergence of Maxwell's stress tensor  $T$ :

$$F_\rho = \text{div}T \quad (11)$$

In 3D Cartesian coordinates Maxwell stress tensor:

$$T = \begin{bmatrix} \frac{1}{2\mu_0}B_x^2 - \frac{1}{2}\mu_0(H_y^2 + H_z^2) & H_yB_x & H_zB_x \\ H_xB_y & \frac{1}{2\mu_0}B_y^2 - \frac{1}{2}\mu_0(H_x^2 + H_z^2) & H_zB_y \\ H_xB_z & H_yB_z & \frac{1}{2\mu_0}B_z^2 - \frac{1}{2}\mu_0(H_x^2 + H_y^2) \end{bmatrix} \quad (12)$$

The surface force density  $F_\rho$  can be defined as the sum of its normal  $F_{\rho n}$  and tangential  $F_{\rho\tau}$  components:

$$F_{\rho n} = \frac{1}{2} \left( \frac{1}{\mu_0} B_n^2 - \mu_0 H_\tau^2 \right) = \frac{1}{2\mu_0} (B_n^2 - B_\tau^2) \quad (13)$$

$$F_{\rho\tau} = H_\tau B_n = \frac{1}{\mu_0} B_n B_\tau \quad (14)$$

For the closed surface  $S$  surrounding the moving part, the total force  $F$  is calculated by integrating the Maxwell stress tensor  $T$  on this surface:

$$F = \oint_S T \cdot n \, ds \Rightarrow \oint_S F_\rho \, ds = \oint_S \left[ \frac{1}{2\mu_0} (B_n^2 - B_\tau^2) n + \frac{1}{\mu_0} (B_n B_\tau) \tau \right] ds \quad (15)$$

This means the normal  $F_n$  and tangential  $F_\tau$  components of the force  $F$ :

$$F_n = \frac{1}{2} \oint_S \left( \frac{1}{\mu_0} B_n^2 - \mu_0 H_\tau^2 \right) ds = \frac{1}{2\mu_0} \oint_S (B_n^2 - B_\tau^2) ds \quad (16)$$

$$F_\tau = \oint_S H_\tau B_n \, ds = \frac{1}{\mu_0} \oint_S B_n B_\tau \, ds \quad (17)$$

Besides Maxwell stress tensor, there exist two other methods for the calculation of magnetic force – the virtual work method (based on the rate of change of magnetic co-energy) and the magnetizing current method (involving the integral of  $\mathbf{J} \times \mathbf{B}$ ). Although in general these three methods give results of comparable accuracy, the stress tensor method is often used for calculating the force because of the relative ease of its implementation. In order to increase the accuracy of force calculation by Maxwell stress tensor method, at least two layers of finer elements are used in the airgap and the surface of integration is chosen in such a way as to completely surround the movable part and lie completely in air. This increases accuracy by reducing the effects of the tangential components of field vectors at the boundary of high and low-permeability FE regions.

Examples of FE modelling of EM actuators are shown in Figs. 4-6. The actuator in this case represents a high-speed and long-lifetime ejector valve used in optical sorting machines for bulk food sorting [23]. Operating under continuous duty cycles at 150-300 Hz, they produce a large

force (8-15 N). Combined with a stroke length of 0.05-0.1 mm, 'on' and 'off' times of 0.2 ms and 0.46 ms respectively, and a requirement for multibillion cycle operation (in excess of 5 billion cycles) without maintenance, these actuators operate at the limit of what can be achieved by solenoid-based EM actuator technology. It is essentially an on/off valve actuator whose active components comprise an excitation coil wound around a magnetic core that attracts or releases a movable valve plate depending on the excitation state of the coil (Fig. 4). The EM force produced by the actuator needs to overcome the pneumatic force which the valve plate is subjected to as a result of high-pressure air (200-550 kPa) flowing through the valve.

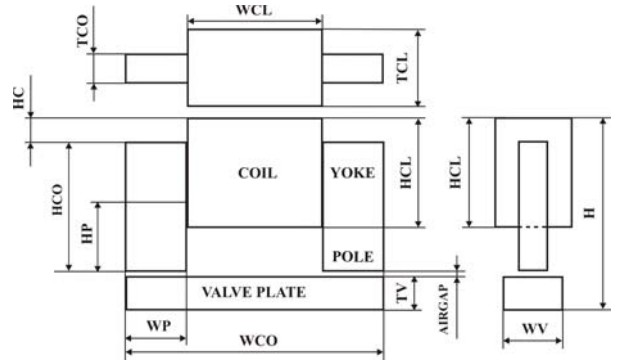


Fig. 4. Main constructive features of a long lifetime C-core EM actuator used as ejector valves in high-speed optical sorting machines.

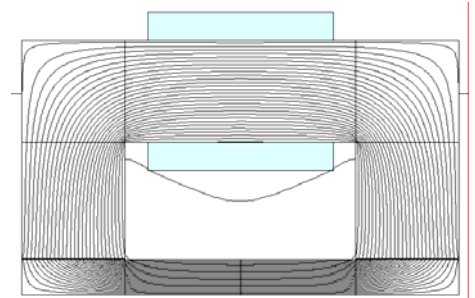


Fig. 5. Magnetic field distribution in the magnetic circuit of the above EM actuator obtained by 2D FE modelling.

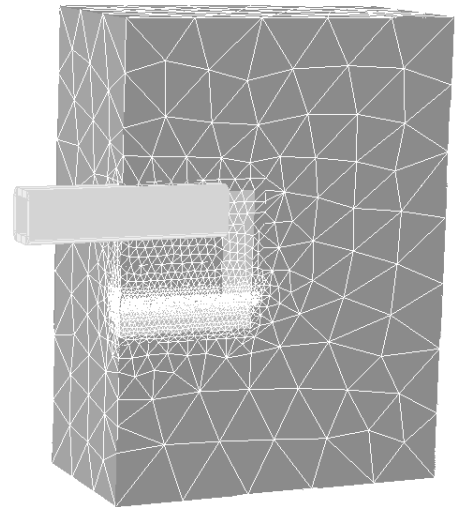


Fig. 6. 3D finite element model of the EM actuator shown in Fig. 4 (1/4 of the full model shown).

Following an extensive 2D modeling study of the nonlinear magnetic field in the EM valve actuator [Fig. 5], 3D FE models had been used to investigate the effects of 'z-direction' magnetic field distribution on various performance parameters, especially on the magnetic force produced for a given excitation current. A typical 3D FE model is shown in Fig. 6 which consists of approximately 250k tetrahedral elements, most of which are concentrated in the regions of the airgap, pole tips and the valve plate. For all modeling purposes commercial software package Opera-3d on Sun Blade 2000 workstations operating under Unix was used.

## 8. CONCLUSIONS

Computer aided design is advancing in capability and spread of application. The concepts and principles resulting from these advances are enhancing the core of measurement and instrumentation science.

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