The Bene¯ts of Shallow Pockets¤

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Abstract

This paper argues that a commitment to `shallow pockets', namely by limiting the size of funds raised initially, may improve an investor's ability to deal with entrepreneurial agency problems. Shallow pockets allow the investor to create competition for continuation ¯nance between her portfolio entrepreneurs. Although this increases the investor's ex post bargaining power vis-à-vis entrepreneurs ¯nanced by her, we show that it can nevertheless improve ex ante e®ort incentives as well as allow sorting across entrepreneurial types, when neither of which can be achieved through contractual means.

As a result, our paper provides an endogenization of a widespread phenomenon in venture capital ¯nance, namely provisions that limit a venture capital fund's size and make it more di±cult to attract further investors after the fund has been raised.

Keywords: Venture Capital; Fund Size; Financial Contracting.

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1 Introduction

Venture capital finance frequently takes place in an environment in which informational problems are severe. Not only is it difficult for venture capitalists to assess the quality and potential of business plans submitted to them for funding, but once initial funding has taken place, entrepreneurs must be given adequate incentives and must be monitored (see, for instance, Gompers and Lerner (2000)). Much of the recent theoretical literature on venture capital finance has considered these issues as central to the provision and characteristics of venture capital funding. Contributions by Gompers (1995), Hellmann (1998) and Schmidt (2003) as well as Repullo and Suarez (1999), Inderst and Mueller (forthcoming) and Casamatta (forthcoming), for example, have concentrated on contractual means to mitigate agency problems.[1]

This paper departs from the existing literature by considering non-contractual instruments to control agency problems, namely the initial size of the investor's funds. Limiting the funds available to the investor creates competition between her portfolio entrepreneurs for cheap 'informed' (or 'inside') money at the refinancing stage. Although competition among entrepreneurs increases the investor's bargaining power during renegotiations, we find that competition may nevertheless enhance entrepreneurial effort incentives and allow the sorting of entrepreneurs according to their type. This, our main insight, arises from the fact that competition created by limited funds increases the responsiveness of the entrepreneur's payoff to the profits generated by his project.

We derive conditions under which such an increase in responsiveness can be achieved through the non-contractual means of limited funds. When this is the case, the benefit of improved incentives may outweigh the costs of inefficient refinancing by the initial investor, e.g. the increased cost of refinancing through uninformed outside investors or the failure of receiving funding for the second stage at all. We find that raising limited funds is an optimal strategy when the following two conditions hold: first, the probability that a given project fails at the interim stage is relatively high; second, the *incremental* returns from improving the project are relatively small compared to the absolute value of financial rewards if the project is a success. Intuitively, if the probability of failure is high this reduces the expected allocational inefficiency implied by limited funds. Similarly, when the incremental project payoff from an improvement

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[1]For an empirical analysis of venture capital contracts see Sahlman (1990) as well as Kaplan and Stromberg (2003).

is small compared to the overall payoff of a successful project, standard incentive contracts are relative weak in providing incentives. In this situation, creating competition through limited funds may provide more powerful incentives since even a small change in performance (due to the correct action) may be crucial in determining whether refinancing is obtained, and the (relatively large) financial rewards associated with it are reaped. These two conditions, i.e. the high failure rate and the high rewards in case of having the 'right idea' as opposed to some incremental improvements, may apply to some of the projects financed by venture capital.[2]

While the above discussion was clothed in terms of entrepreneurial moral hazard, we show that it is easily extended to an adverse selection setting. In particular, we show that separation between entrepreneurs is not possible when all entrepreneurs borrow from investors with deep pockets. When the two conditions described above are met and shallow pockets provide more responsiveness, limited funds can be used to separate entrepreneurs by type. Good entrepreneurs can separate themselves from bad entrepreneurs by exposing themselves to the risk of not receiving continuation finance when borrowing from an investor with shallow pockets - a risk that bad entrepreneurs are unwilling to take.

Venture capital funds and the partnership agreements governing them have several characteristics that suggest the relevance of a mechanism to create competition between a portfolio's projects. Most obviously, venture capital funds are generally close-ended so that, once raised, they cannot be easily augmented by taking on additional funds. This limits the size of the fund from an *ex ante* perspective. Moreover, partnership agreements very often contain covenants that reduce a venture capitalist's ability to raise further 'informed' funds and hence render the commitment to *ex post* competition more credible.[3]

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[2] The notion, for example, that venture capitalists are in the business of funding very risky projects with a high probability of failure is explicitly recognized in its alternative nomer 'risk capital'. Evidence of the low frequency with which truly successful projects are funded abounds and venture capitalists explicitly acknowledge that they 'go for the homerun' in order to offset the large number of failures in their portfolio. (See, for example, Bygrave and Timmons (1992) and Quindlen (2000).) Practitioners, in turn, frequently attest to the overruling importance of a project's 'fundamentals', i.e. what might be loosely described as having the 'right idea', a good business plan, and a sufficient target market size (See, e.g. Bygrave (1999) or Quindlen (2000)).

[3] See Bartlett (1995), Gompers and Lerner (2000), and Brooks (1999) for a more in-depth discussion of venture capital partnership agreements. These covenants include those that restrict the co-investment of different funds run by the same general partner in a particular project or the requirement that realized gains are immediately paid out to limited partners rather than re-invested into the fund. Such characteristics enable a venture capital

There is also substantial evidence that venture capitalists are forced to engage in portfolio management, i.e. in decisions about which project to concentrate funds on, which implies that portfolio projects are engaged in exactly the kind of competition that is at the centre of our paper. Silver (1985), for example, describes this paradox in detail and argues that *"... The need for greater amounts of venture capital, frequently not cited in the business plan, occurs sooner than expected. Because the Murphy's Law affliction attacks most venture capital portfolios, there arises a serious need for portfolio management."*

Related Literature

As noted above, existing papers on agency problems in venture capital financing have focused exclusively on contractual solutions. Instead, we focus on a non-contractual solution, i.e. the creation of competition by limiting the amount of initially raised funds. Instead of dealing with one entrepreneur in isolation, as is the case with the existing literature, this also requires considering the interaction between a venture capitalist's portfolio projects.

Gompers and Lerner (2000) and Lerner and Schoar (2002) provide an in-depth description of the characteristics of venture capital fund partnership agreements and argue that their purpose is to ameliorate agency problems between the general and limited partners of a venture capital fund. Our analysis is complementary as we point towards the implications of partnership agreements for the agency problems between the fund and portfolio entrepreneurs.[4]

Our paper also contributes to the literature on how competition between different projects, or divisions in a conglomerate, affects incentives. This literature has considered the effect of an internal capital market on incentives to gather information about investment opportunities (Stein (2002)), to create cash flows (Brusco and Panunzi (2002)), or to create new growth opportunities (Inderst and Laux (2001)).[5] These papers focus on whether a firm's divisions should be granted discretion over the use of their initially allocated or internally generated funds. In our setting, by contrast, investors compete to attract entrepreneurs and use the size of their initial funds as

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fund to credibly commit to a given size *ex ante.*

[4] The related issue of what determines the optimal number of projects in a venture capital fund's portfolio is addressed in Kanniainen and Keuschnigg (2003), where the limited management capacity of the venture capitalist determines the optimal span.

[5] In a related earlier contribution, Rotemberg and Saloner (1994) show how the joint incorporation of two projects can undermine the incentives from a promised (cash) payment that is made only if a generated research idea is realized.

the main instrument to overcome agency problems.

Finally, our use of an *ex ante* commitment to stop refinancing, or to force refinancing through more expensive outside capital when a project is relatively worse, shares a certain familiarity with the soft-budget constraint literature, e.g. Dewatripont and Maskin (1995). In this literature, an investor with few funds can commit to *ex post* inefficient project abandonment when the borrower undertakes an opportunistic action, something which an investor with more funds cannot credibly achieve. Crucially, this mechanism relies on the assumption that the opportunistic action requires additional *ex post* funding relative to the desired action. In our setting, by contrast, it is the 'good action' which is rewarded by additional funding once we introduce competition for inside money.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 analyzes when constrained finance increases the responsiveness of entrepreneurs' payoffs to the profitability of their project. Sections 4 and 5 embed this analysis in a moral hazard and adverse selection setting respectively. Section 6 considers the robustness of our results by discussing various changes to the model presented in Section 2. Section 7 concludes.

2 The Model

2.1 Project Technologies

There are two types of agents, entrepreneurs and investors, and three periods, which we denote by $t = 0, 1, 2$. For simplicity we assume that all parties are risk neutral and do not discount future cash flows. Furthermore, there are more potential investors than entrepreneurs so that, *ex ante,* there is competition for entrepreneurs.[6]

Entrepreneurs have zero wealth and are risk neutral. Each entrepreneur has an idea that is embedded in a project which requires an investment $I_1 > 0$ at $t = 0$ and can be refinanced at cost $I_2 > 0$ at $t = 1$.[7] Refinancing is best understood as an extension of the project. Projects that

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[6]We do not model subsequent product market competition between portfolio entrepreneurs. See Hellmann (2000) for an empirical description of the impact of venture capital finance on product market competition.

[7]Undertaking a project in stages has been justified on the basis of trading-off entrepreneurial moral hazard against investor monitoring costs (Gompers (1995)), limiting the entrepreneur's hold-up power in contract renegotiations (Neher (1999)), and allowing entrepreneurs to create a reputation for repayment (Egli, Ongena and Smith (2002)). Admati and Pfleiderer (1994) and Cornelli and Yosha (2003) trace out the implications of agency

are not refinanced continue on a smaller scale in a sense made precise below. The entrepreneur's project creates a verifiable return of either $R > 0$ or 0 at $t = 2$. The *ex ante* probability of either return is determined by two factors: the quality of the idea, i.e. the project's fundamentals, and the entrepreneur's type. Below we discuss two scenarios where the entrepreneur can either influence his type (moral hazard) or where his exogenous type is his private information (adverse selection).

Our specification of the project technology, to which we turn next, is meant to capture some of the salient features of risk capital financing. This comprises, in particular, the high failure rate and the importance of the right project 'fundamentals'. We have more to say on this when interpreting our main results in Sections 3 and 4.

We denote a project's interim type at $t = 1$ by $\psi \in \{n, l, h\}$. With probability $1 - \tau$, the idea turns out to be a failure at $t = 1$ and the project returns 0 with certainty. This state corresponds to interim type $\psi = n$. With probability τ, the idea turns out to be successful, in which case the project will create a positive return. This state corresponds to interim types $\psi \in \{l, h\}$. The size of the (positive) final return depends on whether the project receives additional funding and on how successful the idea was, i.e. whether the interim type is $\psi = l$ or $\psi = h$. Precisely, the probability of payoff R is given by p_ψ when refinancing takes place. Let $R_\psi := p_\psi R - I_2$ denote the expected net return from refinancing at $t = 1$. When refinancing does not take place, the success probability becomes p_0 irrespective of whether the interim type is l or h. Denote the expected return without refinancing by $R^0 := p_0 R$.[8] Hence, the incremental expected return to refinancing a project with interim type $\psi \in \{l, h\}$ is given by $r_\psi := R_\psi - R^0$.

Assumption 1: *Refinancing a project of interim type $\psi \in \{l, h\}$ is ex post efficient and creates positive incremental returns that are strictly higher for $\psi = h$: $r_h > r_l > 0$.*

Ex-ante, i.e. at $t = 0$, the entrepreneur's project can be of two types, $\theta \in \{b, g\}$. When the entrepreneur is 'good' ($\theta = g$), his project has a higher probability of being of interim type h and a lower probability of being of interim type l than when the entrepreneur is 'bad' ($\theta = b$).

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problems created by stage-financing on financial contract design. We make the realistic assumption that the project requires more than one injection of capital over time. We argue below that pre-committing $I_1 + I_2$ right at the outset is not optimal due to agency problems even under unconstrained financing, where sufficient funds are raised initially. In case of constrained financing, the aspect of stage-financing (without pre-committing funds) is a vital ingredient in the mechanism that creates competition.

[8]We argue in footnote 22 below that changes in this specification do not affect our results.

We denote by q_θ the conditional probability that a project is of type $\psi = h$ given that it was successful at the interim stage.

Assumption 2. *A good entrepreneur has a higher probability of obtaining a project with high interim type: $q_g > q_b$.*

Before summarizing the specification of the project technology, our choices warrant some comments. Below we will show that creating competition by constraining finance is beneficial if τ, the probability of success, is not too high and if r_l, the low return under success, is not too small compared to the incremental return $r_h - r_l$. By the first condition, we attempt to capture the high failure rate of start-ups. By the second condition, we attempt to capture the notion that a 'good idea' is key and of relative more importance than achieving an incremental improvement.

Figure 1 summarizes the preceding discussion of the project technology.
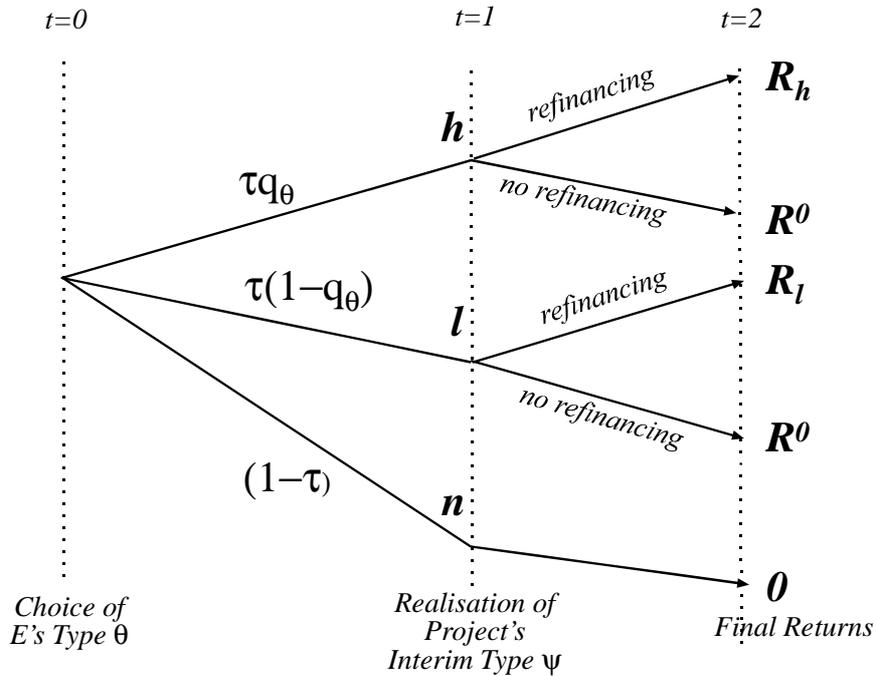

Figure 1: The Project Technology

Finally, we assume that providing start-up finance I_1 is *ex ante* efficient and feasible. In Section 4.2, we will investigate in detail when financing is feasible, i.e. when investors can break even. We postpone a statement of the precise conditions until then.

Note that under Assumptions 1 and 2, an entrepreneur of type g has a project with a higher *ex ante* net present value than that of entrepreneur b if refinancing takes place. In this paper, we consider two ways in which the entrepreneur's type θ is determined. In the *adverse selection* setting, the entrepreneur's type is chosen by nature prior to $t = 0$, and is known only to the entrepreneur. In the *moral hazard* setting, by contrast, the entrepreneur himself chooses his type after having received financing at $t = 0$. This choice is his private information. Choosing type θ confers private benefits B_θ on the entrepreneur at $t = 2$, where we assume that $B_b = B > B_g = 0$.[9] These benefits are only obtained if the project is a success, but they can not be enjoyed if $\psi = n$.[10] We also assume that $\theta = g$ is socially more efficient, i.e. that $(q_g - q_b)(r_h - r_b) > B$.

2.2 Financing

Investors compete at $t = 0$ to provide finance to entrepreneurs. They can raise finance at a fixed interest, which we normalize to zero. At $t = 1$, the initial investor observes (with the entrepreneur) the project's interim type ψ, while outside investors are not able to do so. This creates an informational advantage of the original investor over any outside investor that is crucial for our results.[11] Investors can initially raise sufficient funds such that it would not be necessary to raise additional funds at some later point of time. The central claim of this paper, however, is that the ratio of funds raised *ex ante* to projects financed at the first stage is important in addressing agency problems.

We specify that each investor optimally provides start-up finance to two entrepreneurs.[12] Our analysis then concentrates on the issue of how much funds a venture capitalist raises at $t = 0$. Importantly, we do not preclude the investor from raising additional funds at $t = 1$ or approaching other outside investors for funds at that date. However, the asymmetric information

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[9]Without qualitatively changing results, we could likewise choose a specification where choosing $\theta = g$ involves costly effort.

[10]While this specification allows us to streamline the exposition, our qualitative results do not hinge on it.

[11]The underlying assumption here is that the financing relationship between entrepreneur and investor allows the investor a superior insight into the quality of the project and creates an informational advantage *vis-à-vis* outside investors.

[12]By managing more than two projects, which represents the optimal span of the fund, the venture capitalist would spread himself to thin in the important start-up phase.

between the inside and outside investors at $t = 1$ will render such financing more expensive.

The investors' crucial choice is then between what we term 'unconstrained finance' (or deep pockets) and 'constrained finance' (or shallow pockets). This financing choice is observable by entrepreneurs. Under unconstrained finance, investors raise sufficient funds at $t = 0$ to be able to refinance both portfolio projects at $t = 1$, i.e. $2I_1 + 2I_2$. Under constrained finance, in contrast, investors only raise $2I_1 + I_2$ at $t = 0$ [13], so that they are not able to refinance both projects with certainty at $t = 1$. In this case, constrained finance will create competition (or a tournament) for inside money between the entrepreneurs.[14]

2.3 Contracts and Negotiations

In our setting, the initial contract specifies for entrepreneur i a share s_i of the final verifiable return R that accrues to the entrepreneur, with the investor receiving the remaining share $1 - s_i$. This sharing rule is renegotiated whenever refinancing takes place at $t = 1$.

This specification of contracts and (re-)negotiations rests on three restrictions. First, the investor cannot transfer any funds in excess of the initial investment I_1 for her share $1 - s_i$. (Or, in the words of contracting theory, up-front payments are not feasible.) Secondly, the entrepreneur cannot receive more than the cash flow realization of the project at $t = 2$. Finally, the refinancing decision is not part of the original contract.

It is the third of these restrictions that is truly important as it creates a bilateral hold-up problem that forces the entrepreneur and the investor to newly negotiate at $t = 1$. As we argue next, the restriction is quite realistic and, furthermore, follows from standard assumptions in the contracting literature. We then comment on the less important first and second restrictions.

To begin with, we assume that it is impossible for the agents to commit themselves not to renegotiate. This raises the question of where the agents' bargaining power emanates from in renegotiations. In case of the entrepreneur, his bargaining power simply stems from his ability to withdraw his inalienable and essential human capital from the project. That is, the entrepreneur is essential in order to continue the refinanced version of the project. The investor,

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[13]Here, we assume that the project is non-divisible, i.e. if refinanced, it must be refinanced in its entirety. Relaxing this assumption, e.g. introducing a scalable project, does not affect the main insights of our model.

[14]For an economic analysis of tournaments, beginning with the seminal work of Lazear and Rosen (1981), see McLaughlin (1988) and Prendergast (1999). To the best of our knowledge, the tournament literature has not addressed the issues raised in this paper.

by contrast, possesses bargaining power only to the degree that she has discretion over the refinancing decision. Such discretion arises as an endogenous characteristic of the contract when we allow for a large pool of fraudulent entrepreneurs that do not possess a real project (see Rajan (1992) for a similar use of this assumption). If the entrepreneur were to be given any say in the refinancing of the project, a fraudulent entrepreneur would be able to extract rents at the interim stage.

During the renegotiations in $t = 1$, both sides can therefore threaten to withhold their essential assets: financial and human capital, respectively.[15] In case of disagreement (or break-down), the project can only continue at the small size and the expected payoff is shared as initially agreed (s_i). Hence, while renegotiations take place at $t = 1$, the initial contract is not meaningless. The initial contract determines both the outside options in the renegotiation game at $t = 1$ and how returns are split if refinancing does not take place.[16]

We discuss next the other restrictions on contracts. The first restriction of not allowing up-front transfers can be similarly justified through the existence of fraudulent entrepreneurs. Any up-front transfers would attract fraudulent entrepreneurs and are thus not optimal for the investor. Finally, relaxing the assumption that the entrepreneur cannot be paid more than the realized cash flow of his project would not impact on our central results. Such additional payments would only affect the outside option in the renegotiation game and would not affect the incentive or sorting problem. They would, however, allow the entrepreneur to keep the investor to zero expected profits when the simple sharing rule would not be sufficient, e.g. when

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[15]Of course, this story could be easily enriched by assuming that the initial investor also has to contribute her human capital at the refinancing stage.

[16]It would still be possible to specify at $t = 0$ two sharing rules contingent on whether refinancing occurs or not, say s_i^F and s_i^N, respectively. As either side has the ability to block the successful continuation of the larger project, the sharing rule for refinancing, s_i^F, would always be renegotiated and only the sharing rule without refinancing, s_i^N, would be relevant as an outside option. Note that any 'penalties' that would render it costly for the investor to renegotiate s_i^F are ruled out by appeal to a pool of fraudulent entrepreneurs. Finally, it could be argued that s_i^F should still be relevant as the entrepreneur could simply refuse to renegotiate the contract, knowing that the investor would be better off providing finance under s_i^F than continuing only with the small project. But such an argument defies the whole principle of (re-)negotiations and is not supported by standard models of the bargaining game (as considered in detail in Appendix B). While the investor may be better off by implementing the original agreement instead of not refinancing at all, she is still better off by proposing another and more profitable offer, which the entrepreneur accepts to avoid risk of breakdown or delay.

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the initial investment is very small compared to the NPV of the project. In Section 4.2, we will specify parameters for which we can impose this realistic restriction without loss of generality.

To summarize, the timeline or our model is presented in Figure 2.
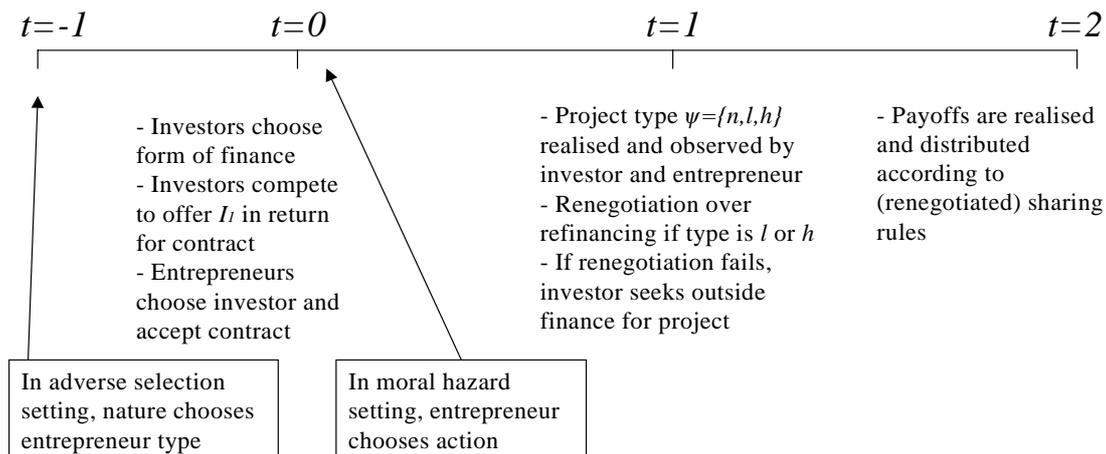
Figure 2: The Timeline

3 Refinancing and Renegotiation

3.1 Sources of Finance

At $t = 1$, the entrepreneur can obtain refinancing either from the informed inside investor or the uninformed outside investor. Under Assumption 1, refinancing is *ex post* efficient whenever the project has interim type l or h and is inefficient when the interim type is n.

Outside investors cannot infer the interim type of the entrepreneur. As projects of interim type n have zero success probability, investors and entrepreneurs do not strictly profit from luring an outside investor into refinancing an unsuccessful project. We assume, however, that

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this indifference is resolved in favor of seeking finance from outside investors.[17] As a result, outside investors are faced with a lemon's problem at $t = 1$ since they do not know the interim type of the project for which they are to provide funds. In particular, when τ, the probability of a project being of type l or h, is low, this lemon's problem is sufficiently strong to prevent outside investors from providing continuation finance to entrepreneurs.[18] In what follows, we assume that τ is sufficiently low such that outside finance is always too costly. In Section 6.2 we derive the precise conditions for when this is the case. There, we also relax this assumption and show that our qualitative results carry over to the case where projects are always refinanced, albeit sometimes with more costly outside finance.

3.2   Renegotiations at $t = 1$

This section characterizes the renegotiation game that arises out of the hold-up problem at $t = 1$ whenever a project is of interim type l or h. Importantly, we assume that renegotiations at $t = 1$ are unstructured in the sense that it is not possible to commit at $t = 0$ to a particular renegotiation game at $t = 1$. Hence, we do not consider the optimal *ex ante* design of the bargaining game at $t = 1$. Instead, the renegotiation game used here is simply one that we consider natural in such an unstructured environment.

Renegotiations at $t = 1$ proceed in the following way:

t$'$= 1 : The investor picks entrepreneur E_i and negotiates with him over refinancing by investing I_2. If negotiations with E_i are successful, E_i receives funds I_2 in return for a renegotiated share of final cash flows.

t$'$= 2 : The investor next negotiates with the other entrepreneur, E_j. Under unconstrained finance, the investor negotiates with E_j over refinancing irrespective of the outcome at $t' = 1$. Under constrained finance, the investor can only renegotiate over refinancing if negotiations with E_i were unsuccessful. If refinancing is feasible and renegotiations are successful, E_j receives funds I_2 in return for a renegotiated share of final cash flows.

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[17]Note that this indifference is turned into a strict preference as soon as we were to introduce small private benefits from continuation or having a project that is 'in operation', for example.

[18]The inside investor could solve the adverse selection problem faced by the outside investors by raising additional funds $\Delta < I_2$ at $t = 0$ and by investing them in interim projects of type l or h as a signal to outside investors. As will become more apparent below, raising *ex ante* $\Delta + I_1$ is not in the investor's interest as it undermines the use of constrained finance as an incentive or sorting device.

Bargaining outcomes are determined by Nash bargaining with equal bargaining powers. The robustness of our results to changes in the bargaining procedure is discussed in Section 6.1. There we also show how the outcome of our bargaining procedure can be generated as an equilibrium of a fully non-cooperative bargaining game with open time horizon.

3.3 Unconstrained Finance

Under unconstrained finance, the investor has raised sufficient funds *ex ante* to refinance all worthwhile projects. As a result, the investor cannot credibly threaten not to provide refinance to a project with interim type $\psi \in \{l, h\}$, irrespective of the type of the other portfolio project. As a result, the refinancing decision and the renegotiated payoffs for a particular entrepreneur are independent of the interim type of his competitor in the portfolio. In the renegotiations at $t = 1$, entrepreneur E_i and investor have outside options of $s_i R^0$ and $(1 - s_i) R^0$, respectively. Given interim type ψ_i, the surplus to be bargained over is r_{ψ_i}. As a result, the *ex post* payoff to entrepreneur i given his interim type ψ_i is $s_i R^0 + \frac{1}{2} r_{\psi_i}$, i.e. the sum of his outside option, $s_i R^0$, and half of the net surplus, r_{ψ_i}. The *ex ante* payoff resulting from these *ex post* returns is given in the following lemma.

Lemma 2. *Under unconstrained finance, the expected payoff to entrepreneur i at $t = 0$, given initial contract s_i and entrepreneurial type θ_i, is*

$$\tau \left\{ s_i R^0 + \frac{1}{2} \left[r_l + q_{\theta_i} (r_h - r_l) \right] \right\},$$

while the investor's expected payoff is

$$\tau \left\{ (1 - s_i) R^0 + \frac{1}{2} \left[r_l + q_{\theta_i} (r_h - r_l) \right] \right\} - I_1$$

per portfolio entrepreneur.[19]

Proof. At $t = 0$, the probability of interim type $\psi_i = l$ or $\psi_i = h$ is $\tau (1 - q_{\theta_i})$ and τq_{θ_i}, respectively. Hence, the entrepreneur's expected payoff is $\tau q_{\theta_i} (s_i R^0 + \frac{1}{2} r_h) + \tau (1 - q_{\theta_i})(s_i R^0 + \frac{1}{2} r_l)$ and the investor's expected payoff is $\tau q_{\theta_i} ((1 - s_i) R^0 + \frac{1}{2} r_h) + \tau (1 - q_{\theta_i})((1 - s_i) R^0 + \frac{1}{2} r_l) - I_1$, which transform to the respective payoffs. **Q.E.D.**

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[19] Without loss of generality, we concentrate on the investor's profit per portfolio entrepreneur for the remained of the paper.

3.4 Constrained Finance

In contrast to unconstrained finance, the investor can now credibly threaten to use all funds for the other portfolio project if current negotiations fail. As a result, the outcome of renegotiations between E_i and the investor at $t = 1$ under constrained finance are dependent on the interim type of the other portfolio company, namely for two reasons: firstly, the interim type of E_j (the other entrepreneur) determines which entrepreneur is picked first to be bargained with; secondly, it also determines the investor's outside option in the bargaining game and hence the surplus to be bargained over.

Suppose that $E'_j s$ interim type is n, i.e. there is no profitable refinancing opportunity. Then negotiations with E_i are identical to those under unconstrained finance. Consider next the case where both entrepreneurs are of interim type l or h. We will now derive the bargaining outcome by analyzing the sequential bargaining game backwards. Suppose that E_j with $\psi_j \neq n$ is the last entrepreneur to be bargained with. If the limited funds have already been used up for E_i, the entrepreneur will only realize $s_j R^0$ while the investor realizes $(1 - s_j) R^0$ from this project. Alternatively, if funds are still available, we can apply results from the case with unconstrained finance and obtain that E_j realizes $s_j R^0 + \frac{1}{2} r_{\psi_j}$ while the investor realizes $(1 - s_j) R^0 + \frac{1}{2} r_{\psi_j}$.

Turn next to negotiations with E_i, who is picked first. In case of a breakdown, E_i receives just $s_i R^0$. Using our previous calculations, we know that in this case the investor's payoff is the sum of $(1 - s_i) R^0$ and $(1 - s_j) R^0 + \frac{1}{2} r_{\psi_j}$.[20] In case they reach an agreement, their joint surplus is the sum of R_{ψ_i} and $(1 - s_j) R^0$. Given that their net surplus is therefore just $r_{\psi_i} - \frac{1}{2} r_{\psi_j}$ [21], which is then split equally, we obtain that E_i receives a payoff of $s_i R^0 + \frac{1}{2} \left[r_{\psi_i} - \frac{1}{2} r_{\psi_j} \right]$.

We can now sum up these results.

Lemma 3. *Take the case with constrained finance. If at least one project is of type n, then payoffs are as in Lemma 2. If both projects are successful, i.e. if $\psi_i \neq n$ and $\psi_j \neq n$, and if the*

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[20]Note that the Nash bargaining solution assumes that both sides have the same information on the values of the surplus and of their outside options. In Section 6.1 we analyze a simpler version of the bargaining game which does not require that one entrepreneur knows the profitability of the other project. There, we obtain qualitatively similar results.

[21]The outside options in the bargaining game between E_i and the investor are $s_i R^0$ and $(1 - s_i) R^0 + (1 - s_j) R^0 + \frac{1}{2} r_{\psi_j}$, respectively. Also recall that $r_{\psi_i} = R_{\psi_i} - R^0$. Hence, the total surplus to be bargained over between E_i and the investor equals $R_{\psi_i} + (1 - s_j) R^0 - \{ s_i R^0 + (1 - s_i) R^0 + (1 - s_j) R^0 + \frac{1}{2} r_{\psi_j} \}$, which is $r_{\psi_i} - \frac{1}{2} r_{\psi_j}$.

investor picks E_i to bargain with first, the payoffs are as follows:

i) $s_i R^0 + \frac{1}{2} \left[r_{\psi_i} - \frac{1}{2} r_{\psi_j} \right]$ *for E_i;*

ii) $s_j R^0$ *for E_j;*

iii) and $(1 - s_i) R^0 + (1 - s_j) R^0 + \frac{1}{2} \left[r_{\psi_i} + \frac{1}{2} r_{\psi_j} \right]$ *for the investor.*

Next, consider the issue of who is chosen first to be bargained with. First of all, note that the initial sharing rules, s_i, do not affect the investor's preference for either of the two projects. When interim types are not identical, the investor will then prefer to bargain first with the better interim type. When interim types are identical, we stipulate that the investor chooses each of the two entrepreneurs with equal probability. Lemma 4 summarizes the resulting *ex post* payoffs.

Lemma 4. *Under constrained finance, $E_i's$ payoff at $t = 1$ is*

i) $s_i R^0 + \frac{1}{8} r_h$ *if both projects are of interim type h;*

ii) $s_i R^0 + \frac{1}{2} \left[r_h - \frac{1}{2} r_l \right]$ *if $E_i's$ and $E_j's$ projects are of interim type h and l, respectively;*

iii) $s_i R^0$ *if $E_i's$ and $E_j's$ projects are of interim type l and h, respectively;*

iv) $s_i R^0 + \frac{1}{8} r_l$ *if both projects are of interim type l;*

v) 0 *if i's project is of interim type n, regardless of the interim type of $E_j's$ project;*

vi) $s_i R^0 + \frac{1}{2} r_{\psi_i}$ *if $E_j's$ project is of interim type n and $E_i's$ project is of interim type l or h.*

As the final step of this analysis, we now derive the *ex ante* payoff for E_i. Note that this will depend on both entrepreneurs' interim types as their realizations will determine who is picked first. Given the *ex post* payoffs in the preceding lemma, the *ex ante* payoffs are then easily calculated.[22]

Lemma 5. *Under constrained finance, $E_i's$ expected payoff at $t = 0$ given entrepreneurial types $\theta_i, \theta_j \in \{b, g\}$ and initial contract s_i is*

$$\tau \left\{ s_i R^0 + \frac{1}{2} \left[r_l + q_{\theta_i} (r_h - r_l) \right] \right\} - \frac{\tau^2}{8} \left\{ r_l \left(3 - q_{\theta_i} + q_{\theta_j} \right) + 3 q_{\theta_i} q_{\theta_j} (r_h - r_l) \right\}.$$

---

[22]Note that since R^0 is type-independent, the change in $E_i's$ payoff due to a change in his type is independent of s_i (and R^0). Importantly, this implies that the initial contract cannot be used for sorting or incentive purposes. This feature is robust to the introduction of a type-dependent payoff R^0_ψ, as long as $R^0_h \geq R^0_l$. In such an extended setting, the responsiveness of $E_i's$ payoff to his type ψ_i increases in s_i. As a result, the optimal contract would involve setting s_i as large as possible, subject to the investor's participation constraint. This is, however, exactly how s_i is determined in our simpler setting. Hence, although introducing type-dependency of R^0 would change the particular form of equation (1) it would not qualitatively affect results.

The investor earns

$$\tau \left\{ (1 - s_i) R^0 + \frac{1}{2} [r_l + q_{\theta_i} (r_h - r_l)] \right\} - \frac{\tau^2}{8} \left\{ \left(1 + 3q_{\theta_j} - 3q_{\theta_i} \right) r_l + q_{\theta_i} q_{\theta_j} (r_h - r_l) \right\} - I_1.$$

Proof. Recall that the *ex ante* probabilities of interim states n, l, and h are $1 - \tau$, $\tau (1 - q_\theta)$, and τq_θ, respectively. The asserted payoffs follow then immediately from substitution from Lemma 4. **Q.E.D.**

3.5 The Responsiveness Condition

The responsiveness of the entrepreneur's payoff to his type will be crucial in solving his agency problem, whether it be in a moral hazard or an adverse selection setting. The more responsive the entrepreneur's payoff is to his type, the easier it is to provide effort incentives or to induce self-selection. We now analyze how constrained and unconstrained finance compare in terms of the responsiveness they induce in the entrepreneur's payoff.

The responsiveness under unconstrained finance is easily derived from Lemma 2. Subtracting the entrepreneur's payoff for $\theta = b$ from that for $\theta = g$, we obtain

$$\frac{1}{2} \tau (q_g - q_b) (r_h - r_l). \tag{1}$$

Importantly, the responsiveness does not correspond to the full difference in project value as the hold-up problem at $t = 1$ allows the investor to extract half of the increase in value.

Under constrained finance, by contrast, entrepreneurs compete for scarce inside money. As a result, for a given sharing rule, the investor will extract more from a funded entrepreneur whenever the other type has a profitable refinancing opportunity (i.e. of interim type l or h). In other words, constrained finance embodies the investor with more bargaining power when entrepreneurs are forced to compete. Our key insight is that constrained finance can nevertheless increase the responsiveness of the entrepreneur's payoff to his type. Although the entrepreneur's total payoff is reduced (for a given type θ and sharing rule s) the difference in payoff across types b and g can be increased.

Under constrained finance, $E_i's$ payoff depends on the type of E_j as well as on his own. In what follows, we are interested in the case where, under constrained finance, both types will be $\theta = g$. Hence, set $\theta_j = g$. The responsiveness for constrained finance, i.e. the difference in payoffs for E_i if $\theta_i = g$ rather than $\theta_i = b$, is then given by

$$\frac{1}{2} (q_g - q_b) \tau \left\{ (r_h - r_l) + \frac{\tau}{4} [r_l - 3q_g (r_h - r_l)] \right\} \tag{2}$$

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according to Lemma 5. Subtracting expression (1) from expression (2) then allows us to establish the following proposition.

Proposition 1. *The responsiveness of the entrepreneur's ex ante payoff is higher under constrained finance if and only if*

$$r_h - r_l < \frac{r_l}{3q_g}. \tag{3}$$

We will subsequently refer to condition (3) as the 'Responsiveness Condition'. It is at the core of our analysis and describes the circumstances under which constrained finance is more adept at dealing with agency problems than unconstrained finance. In Section 6 we show how the responsiveness condition extends to alternative bargaining procedures.

The responsiveness condition captures the trade-off between two effects of competition for inside money that is introduced through constrained finance:

Competition Effect: Under constrained finance, not being picked first to be bargained with implies that the entrepreneur will not receive refinancing in equilibrium, in contrast to unconstrained finance. As a result, competition introduces an additional incremental return to being first to be bargained with, making the payoff more responsive to the entrepreneur's type.

Bargaining Power Effect: Under constrained finance, entrepreneurs compete for inside money and an investor can threaten to refinance the other entrepreneur when bargaining with her first pick. This creates additional bargaining power for the investor. As a result, the return to being refinanced is reduced and the responsiveness is lowered.

Unconstrained finance provides responsiveness through the difference in final project payoffs: $r_h - r_l$. Constrained finance, by contrast, creates responsiveness through an artificial 'jump' in payoffs induced by competition for refinancing funds. As a result, the above trade-off can be summarized as follows. If $r_h - r_l$ is high, then incentives under constrained finance are already substantial and competition for inside money mainly allows the investor to extract more surplus. Unconstrained finance provides more responsiveness. In contrast, if $r_h - r_l$ is low, the jump in payoffs created through the threat of no refinancing implies that unconstrained finance dominates constrained finance in responsiveness terms.

This interpretation is illustrated in Figures 3 and 4. (For simplicity, we deviate somewhat from our previous set-up and notation in these figures.)
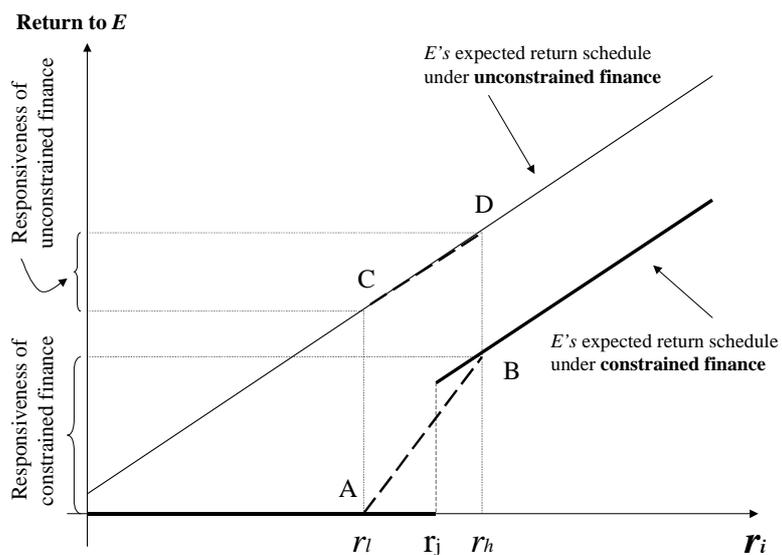
17

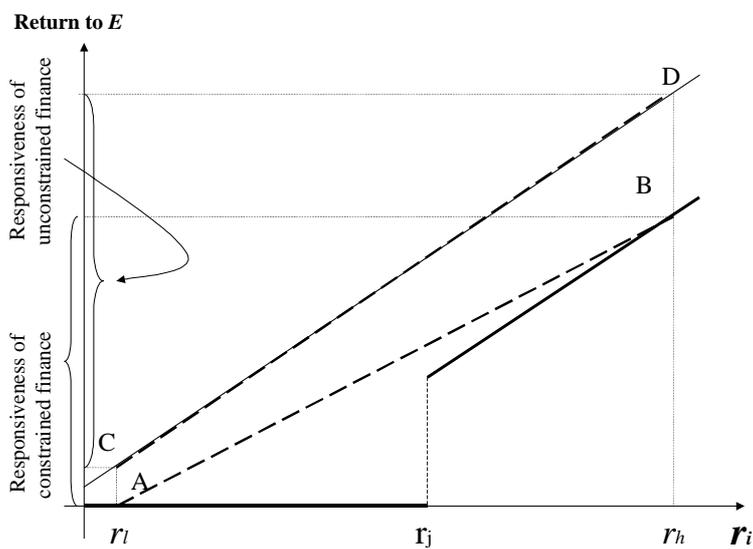Figure 3: Situation in which the responsiveness condition holds.

Figure 4: Situation in which the responsiveness condition does not hold.

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In Figure 3, the line through points C and D represents the *ex ante* payoff under unconstrained finance as a function of r_i. The line through points A and B, by contrast, represents expected payoffs under constrained finance. Importantly, this payoff is not continuous in r_i but jumps as soon as it exceeds a 'threshold' r_j, i.e. the rival's prospects. This discontinuity captures the competition effect. It is very strong relative to the responsiveness provided by unconstrained finance when $r_h - r_l$ is very small and these payoffs are grouped around the threshold, i.e. when r_l is large. Furthermore, to the right of the discontinuity, the payoff profile under constrained finance is below that of unconstrained finance, albeit with the same slope. This shift downwards in payoffs captures the bargaining power effect. When this effect is small, the responsiveness condition is more likely to hold.

Figure 4, by contrast, exhibits a setting in which the responsiveness of unconstrained finance outweighs that of constrained finance. Here, $r_h - r_l$ is too large relative to the competition effect and constrained finance cannot provide sufficient responsiveness.

Below, when completing the analysis of the model for the cases of moral hazard and adverse selection, we will describe in more detail the implications of Proposition 1 for the case of venture capital financing.

4   Moral Hazard

In the previous section, we established when constrained finance dominates unconstrained finance in terms of responsiveness. In this section, we will investigate when this increased responsiveness outweighs the negative impact of increased investor bargaining power and inefficient project continuation so that constrained finance dominates unconstrained finance from an *ex ante* point of view.

Recall that, in the moral hazard setting, the entrepreneur chooses her type $\theta \in \{b, g\}$ at $t = 0$. The choice of $\theta = b$ will result in private benefits B at $t = 2$ in case the project has success. Furthermore, we assumed that the good action $\theta = g$ is socially desirable.

4.1   Unconstrained vs. Constrained Finance

Because of the hold-up problem at $t = 1$, the entrepreneur is not a residual claimant to the entire incremental surplus of his choice of action. As a result, he will not choose the good action

if

$$\frac{1}{2}\left(q_g - q_b\right)\left(r_h - r_l\right) < B \tag{4}$$

and the hold-up problem will lead to a suboptimal effort choice when this inequality is fulfilled. In what follows, we assume that the hold-up problem is sufficiently strong for this to be the case, so that the entrepreneur chooses the bad action under unconstrained finance.

We now assume that the project is financially viable. Hence, its expected NPV at $t = 0$ is positive and, what is more, there exists a sharing rule s for $i = 1, 2$ such that the investor can break even. In Section 4.2, we discuss in more detail the issue of financial viability and how this can be influenced by the choice between constrained and unconstrained financing. Under unconstrained financing, the project is financially viable if and only if

$$I_1 \leq \tau R^0 + \frac{1}{2}\tau\left(r_l + q_b\left(r_h - r_l\right)\right). \tag{5}$$

As is easily seen, the right-hand side of (5) represents the investor's expected payoff if $s_i = 0$ and if the entrepreneur chooses the bad action. (Recall that the choice of s_i does not affect the choice of θ_i.) Furthermore, we assume that adjusting s_i is sufficient to extract all profits from the investor, i.e. that investors compete themselves down to zero profits at $t = 0$. In analogy to (5) this is the case if

$$I_1 \geq \frac{1}{2}\tau\left(r_l + q_b\left(r_h - r_l\right)\right), \tag{6}$$

where the right-hand side of (6) now represents the investor's expected payoff if $s_i = 1$.[23] We will now take (5) and (6) as given.

The equilibrium contract for each project $i = 1, 2$ specifies now the sharing rule s_i which allows investors to just break even.

Lemma 6. *With moral hazard, the equilibrium contract under unconstrained financing specifies the sharing rule*

$$s_i = 1 + \frac{1}{2}\left(\frac{r_l + q_b\left(r_h - r_l\right)}{R^0}\right) - \frac{I_1}{\tau R^0},$$

---

[23] In case (6) does not hold, investors would have to make an up-front transfer to the entrepreneur in order to receive only zero profits. Applying standard arguments from the contracting literature, we ruled out the feasibility of such transfers. However, even if investors made positive profits our results would be qualitatively unchanged. The only difference this would make is that we would have to introduce an additional case distinction when determining the equilibrium contract.

which leaves the investor with zero profits and the entrepreneur with the expected payoff

$$\tau \left[R^0 + r_l + q_b \left(r_h - r_l \right) + B \right] - I_1.$$

Proof. Condition (4) implies that $\theta_i = b$. From Lemma 2, the investor's expected payoff is zero if

$$\tau \left\{ (1 - s_i) R^0 + \frac{1}{2} \left[r_l + q_b \left(r_h - r_l \right) \right] \right\} - I_1 = 0,$$

which we can solve for s_i. As the investor just breaks even, the payoff of E_i is just the sum of net profits and B. **Q.E.D.**

When unconstrained finance cannot provide sufficient incentives for the good action but the responsiveness condition does not hold, constrained finance clearly cannot induce the good action, either. When both forms of finance lead to the same action, however, unconstrained finance always dominates as it does not suffer from the *allocational inefficiency* created by constrained finance - namely, its failure to provide refinancing to all projects of interim type l or h.

We assume now that the responsiveness condition (3) holds. If the additional incentives provided through constrained finance are sufficient, in particular when

$$\frac{1}{2} \left(q_g - q_b \right) \left(r_h - r_l \right) + \frac{1}{8} \tau \left[r_l - 3 q_g \left(r_h - r_l \right) \right] > B, \tag{7}$$

then both entrepreneurs will find it strictly optimal to choose action $\theta_i = g$. In other words, inequality (7) ensures that there exists a unique, symmetric pure-strategy Nash equilibrium where both projects are of the good type.[24] In what follows, we assume that inequality (7) holds.

Regarding financial viability, we must now assume in analogy to (5) that the original investment is not too large. It is easily established that this is the case if

$$I_1 \leq \tau R^0 + \frac{1}{2} \tau \left(r_l + q_b \left(r_h - r_l \right) \right) - \frac{\tau^2}{8} \left(r_l + q_g^2 \left(r_h - r_l \right) \right). \tag{8}$$

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[24] This follows immediately from Lemma 5, which reveals that it was also strictly optimal for E_i to choose $\theta_i = g$ if the E_j chose $\theta_j = b$. For

$$B \in \left[\frac{1}{2} \tau \left(q_g - q_b \right) \left(r_h - r_l \right) - \frac{1}{8} \tau^2 \left[r_l - 3 q_g \left(r_h - r_l \right) \right], \frac{1}{2} \tau \left(q_g - q_b \right) \left(r_h - r_l \right) - \frac{1}{8} \tau^2 \left[r_l - 3 q_b \left(r_h - r_l \right) \right] \right]$$

there exist two equilibria in asymmetric pure strategies and one equilibrium in mixed strategies. For reasons of brevity, we do not consider this region.

Moreover, to ensure that giving investors a sufficiently low initial share of cash flow rights is sufficient to bring them down to zero profits, we must assume in analogy to (6) that

$$I_1 \leq \frac{1}{2}\tau \left(r_l + q_b \left(r_h - r_l \right) \right) - \frac{\tau^2}{8} \left(r_l + q_g^2 \left(r_h - r_l \right) \right). \tag{9}$$

Again we take (8) and (9) as given. We then have the following result.

Lemma 7. *With moral hazard, the equilibrium contract under constrained financing specifies the sharing rule*

$$s_i = 1 + \frac{1}{2} \left(\frac{r_l + q_g \left(r_h - r_l \right)}{R^0} \right) - \frac{I_1}{\tau R^0} - \frac{\tau}{8R^0} \left[r_l + q_g^2 \left(r_h - r_l \right) \right],$$

which leaves the investor with zero profits and the entrepreneur with the expected payoff

$$\tau \left[R^0 + r_l + q_g \left(r_h - r_l \right) \right] - I_1 - \frac{\tau^2}{2} \left[r_l + q_g^2 \left(r_h - r_l \right) \right].$$

Proof. Given conditions (3) and (7), $\theta_i = \theta_j = g$ holds under constrained finance. From Lemma 5, the investor's expected payoff is zero when

$$\tau \left\{ \left(1 - s_i \right) R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\} - \frac{\tau^2}{8} \left\{ r_l + q_g^2 \left(r_h - r_l \right) \right\} - I_1 = 0,$$

which we can solve for s_i. From Lemma 5, $E_i's$ expected payoff under constrained finance and with $\theta_i = \theta_j = g$ is

$$\tau \left\{ s_i R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\} - \frac{\tau^2}{8} \left\{ 3r_l + 3q_g^2 \left(r_h - r_l \right) \right\}$$
$$= \tau \left[R^0 + r_l + q_g \left(r_h - r_l \right) \right] - I_1 - \frac{\tau^2}{2} \left[r_l + q_g^2 \left(r_h - r_l \right) \right].$$

Q.E.D.

Comparing the payoffs under unconstrained finance and constrained finance, we can finally establish the following result.

Proposition 2. *Suppose the following conditions hold in the case with moral hazard. First, the responsiveness condition (3) is fulfilled, implying that constrained financing is better at creating incentives. Second, (4) and (7) hold, implying that constrained financing is also necessary to create sufficient incentives. Then constrained finance will dominate unconstrained finance ex ante, i.e. it will be the equilibrium choice of all investors, if and only if*

$$\tau < 2\frac{(r_h - r_l)(q_g - q_b) - B}{r_l + (r_h - r_l)q_g^2}. \tag{10}$$

Proof. Under conditions (3), (4) and (7), constrained finance will dominate unconstrained finance if the entrepreneur's payoff from Lemma 7 is larger than that from Lemma 6:

$$\tau \left[R^0 + r_l + q_g \left(r_h - r_l \right) \right] - I_1 - \frac{\tau^2}{2} \left[r_l + q_g^2 \left(r_h - r_l \right) \right] > \tau \left[R^0 + r_l + q_b \left(r_h - r_l \right) + B \right] - I_1,$$

which transforms to (10). **Q.E.D.**

Condition (10) is intuitive as constrained finance, *ceteris paribus*, is more likely to dominate if the allocational inefficiency it induces is relatively small in expected terms. This is the case when τ, the probability of having a successful idea, is small.[25] As we have argued in the Introduction, this condition may well apply to the venture capital industry, where the likelihood that a project returns positive profits is widely acknowledged to be small.

In determining the optimality of constrained financing, Proposition 2 is complemented by the responsiveness condition of Proposition 1. This condition requires that the incremental return $r_h - r_l$ is relatively small compared to r_l. If this were not the case, incentives based on the differential in project returns across actions would be sufficient to elicit the correct effort. When $r_h - r_l$ is small, however, these incentives are small. Here, i.e. where the potential of incremental improvements is relatively low compared to the absolute level of returns when successful, constrained finance increases responsiveness by ensuring that the loser of a competition for limited funds is not refinanced. Our results may go some way towards explaining some of the features related to financial constraints in venture capital funds, which we mentioned in the introduction.

4.2 Financial Viability

Until now we have assumed that the investor can break even. Clearly, this need not be true, in which case the project would not be financed *ex ante*. Recall now that the investor's return depends on the initial contract, the project's interim type, and her bargaining power in the negotiations for refinancing. Importantly, as we have seen in the previous sections, the investor's *ex post* bargaining power may be increased under constrained finance relative to unconstrained

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[25]This effect is further accentuated by the presence of the term q_g^2 in the denominator of the right-hand side, which captures the likelihood with which constrained finance will create this allocational inefficiency once both projects are successful at $t = 1$. Clearly, the lower q_g^2, the less restrictive the condition in Proposition 2.

finance. When this increase in bargaining power is sufficient to outweigh the loss of *ex ante* returns due to inefficient refinancing, constrained finance will render projects financially viable that would not have been financed under unconstrained finance.

To make this intuition more precise, we compare the respective conditions when projects are financially viable under the two financing regimes, i.e. conditions (5) and (8). We obtain the following result.

Proposition 3. *Constrained finance allows the investor to appropriate a larger absolute amount of expected returns during renegotiations than unconstrained finance when*

$$\tau < 4 \frac{(q_g - q_b)(r_h - r_l)}{r_l + (r_h - r_l) q_g^2}. \tag{11}$$

Proof. From conditions (5) and (8), the investor's expected payoff is greater under constrained finance than unconstrained finance if

$$\tau \left[R^0 + \frac{1}{2} (r_l + q_b (r_h - r_l)) \right] - \frac{\tau^2}{8} (r_l + q_g^2 (r_h - r_l)) - I_1 > \tau \left[R^0 + \frac{1}{2} (r_l + q_b (r_h - r_l)) \right] - I_1,$$

which transforms to (11).

   Q.E.D.

As in Proposition 2, this condition is more likely to be fulfilled if the probability of incurring the allocational inefficiency generated by constrained finance is low, i.e. if both τ and q_g^2 are low. While constrained finance certainly implies that the investor appropriates a larger share of a given amount of *ex post* returns, its impact on the size of *ex post* returns is ambiguous - inefficient project continuation lowers expected returns for a given level of effort while improved effort incentives increase expected returns.

In case condition (11) holds, there is a strictly positive wedge between the two conditions (5) and (8). Then, for intermediate values of I_1, it is only possible to finance the projects with constrained financing. For lower values of I_1, where both conditions hold, both modes of financing are feasible, while for higher values, where condition (8) no longer holds either, it is not possible at all to finance the projects.

Note, furthermore, that the condition on τ in Proposition 3 is less strict than that on τ in Proposition 2. This implies that, over a range of values, the entrepreneur would have preferred unconstrained finance if financial viability had not been an issue. When unconstrained financing

is, however, not feasible, the entrepreneur has no choice but to seek funding from a constrained investor.

5 Adverse Selection and Constrained Finance

In this section, the entrepreneur's type θ is chosen by nature just prior to $t = 0$. With probability α, nature chooses the good type $\theta = g$ while the likelihood of $\theta = b$ is $(1 - \alpha)$. As investors do not observe individual types, investors face an adverse selection problem at $t = 0$. It is now helpful to briefly recall the game played by investors in $t = 0$. We consider a large pool of potential investors, i.e. it is (the entrepreneurs') ideas that are in short supply. Investors raise funds of value $2I_1 + 2I_2$ or $2I_1 + I_2$ and try to attract entrepreneurs by offering sharing rules, s. Each successful investor finances exactly two projects.[26]

Suppose first that all entrepreneurs are financed by financially unconstrained investors. Then the following result is immediate.[27]

Lemma 8. *Suppose all investors are financially unconstrained. Then the unique equilibrium is a pooling equilibrium where all entrepreneurs, regardless of their type, receive the same contract s. Entrepreneurs of types $\theta = g$ and $\theta = b$ are randomly matched in the investors' funds.*

We now argue that allowing investors to choose between constrained and unconstrained finance may enable them to separate type $\theta = g$ from type $\theta = b$ entrepreneurs.

Consider thus the case of constrained finance. As stated above, initial sharing rules s_i do not affect the investor's preferences regarding which project she would like to refinance. Consequently, separation between types $\theta = g$ and $\theta = b$ can not be achieved by offering a menu of initial sharing rules. Each type will strictly prefer the highest sharing rule. Recall next from Proposition 1 that when responsiveness condition (3) holds, the payoff differential across types is larger under constrained than unconstrained finance. This establishes that condition (3) is a necessary - albeit not yet sufficient - condition for separation across entrepreneurial types. To allow for separation, the difference in responsiveness across financing regimes must be large enough so that separation can be achieved at sufficiently favourable terms for $\theta = g$.

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[26] Formally, we could imagine that entrepreneurs can shop around between different investors whose offers, s_i, are made conditional on the fact that they can finance two entrepreneurs.

[27] Note that we assume throughout this section that projects are always financially viable under both forms of financing.

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Additionally, the allocational inefficiency created by constrained finance cannot be too large. Otherwise the constrained investor will be unable to offer a mutually profitable contract that achieves separation.[28]

In addition to these conditions, we have the standard condition arising in screening models of this type that the *ex ante* probability α of being $\theta = g$ cannot be too high. Otherwise, a pooling equilibrium offers financial terms that are not sufficiently worse than those in a separating equilibrium and would be preferred by $\theta = g$.

The following proposition establishes conditions under which these requirements are jointly met and separation is achieved.

Proposition 4. *Consider the following separating equilibrium: type $\theta = b$ receives unconstrained finance, while $\theta = g$ receives constrained finance. Suppose that the responsiveness condition (3) holds. Then this separating equilibrium exists and is the unique pure-strategy equilibrium if*

$$\tau \leq \frac{(q_g - q_b)(r_h - r_l)}{r_l + q_g^2(r_h - r_l)}$$

and if

$$\alpha \leq \min\left[\frac{\tau}{8}\frac{r_l - 3q_g(r_h - r_l)}{r_h - r_l}, \frac{1}{2}\left(1 - \tau\frac{r_l + q_g^2(r_h - r_l)}{(q_g - q_b)(r_h - r_l)}\right)\right].$$

Proof. See Appendix A.

6 Robustness of the Responsiveness Condition

6.1 Alternative Bargaining Procedures

In Section 3, we found that responsiveness can increase under constrained finance. Even though constrained finance accords more bargaining power to the investor, which, at first glance, only seems to worsen the hold-up problem, what matters in terms of responsiveness are not absolute returns but the difference in returns across entrepreneurial types.

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[28]Note that in this model we do not have a continuous and unbounded sorting variable that would allow separation at some point while ensuring that the constrained investor's participation constraint always holds with equality.

We now argue that these results are robust to changes in the bargaining procedure. We proceed as follows. We first present a much simplified bargaining procedure. Subsequently, we set up a fully non-cooperative framework with an open time horizon.

A simpler bargaining procedure

Consider the following bargaining procedure:

$t\prime = \mathbf{1}$: The investor picks entrepreneur E_i for refinancing.

$t\prime = \mathbf{2}$: The investor negotiates separately with each entrepreneur, E_i and E_j. If the investor has constrained funds and if negotiations with E_i fail, it is too late to use the funds to refinance E_j instead.

In this bargaining game neither of the two entrepreneurs needs information about the profitability of the other entrepreneur. In the case of unconstrained finance it is immediate that the outcome is identical to that of our previous bargaining game. If the investor's funds are constrained, we can also use previous results and obtain that the chosen entrepreneur E_i realizes payoff $s_i R^0 + \frac{1}{2} r_{\psi_i}$ for $\psi_i \neq n$. The other entrepreneur just realizes $s_j R^0$ in this case, provided of course that $\psi_j \neq n$. Consequently, the responsiveness condition becomes now

$$r_h - r_l < \frac{r_l}{q_g}. \tag{12}$$

Note that condition (12) is less strict that the original condition (3).

A bargaining procedure with open time horizon

The renegotiation procedures that we presented so far are relatively simple but not open-ended in the sense that the investor cannot infinitely 'shuttle' back and forth between entrepreneurs and play them against each other *ad infinitum*. Clearly, merely positing such a game is a short-cut. Suppose instead that we have the following bargaining game:

$t\prime = \mathbf{1}$: The investor picks an entrepreneur, say E_i, and makes him an offer of I_2 in return for a share of the final payoff.

$t\prime = \mathbf{2}$: It is up to E_i to decide on the offer. If E_i accepts, then further negotiations proceed only between the investor and the other entrepreneur, E_j. In this case, we return to $t' = 1$ in the next period, with the restriction that the investor must then choose E_j. If E_i rejects, then he can make a counter-offer. In this case we proceed to $t' = 3$.

$t\prime = 3$: It is now up to the investor to respond to the offer of E_i. If she accepts, then further negotiations proceed only between the investor and E_j. In this case, we return to $t' = 1$ in the next period, with the restriction that the investor must then choose E_j. If the investor rejects, we return to period $t' = 1$, where the investor can now still choose between both entrepreneurs.

To make coming to an immediate agreement attractive in this setting, we introduce some frictions. We suppose that there is an exogenous risk of negotiations breaking down. Let $1 - \delta$ denote the probability with which negotiations will break down when either of the parties rejects an offer. (The break-down only affects the two parties which are currently negotiating.) We show in Appendix B that this bargaining game has an equilibrium in which we obtain immediate agreement and where, as $\delta \to 1$, equilibrium payoffs are the same as those derived in the previous sections.

This may, at first, seem surprising. In particular, in case both entrepreneurs are of the same type, say $\psi_1 = \psi_2 = h$, one may ask why the investor does not deviate and go to the other entrepreneur who would be eager to be financed even under less favorable conditions. However, the other entrepreneur, say E_j, would optimally not accept any offer that makes him just a little better off than without refinancing. Instead, he would reject such an offer and use the opportunity to propose to the investor a new contract that makes the investor just as well off as when going back to E_i.[29]

Determinants of responsiveness

If the investor starts to bargain with E_i under constrained financing, the other entrepreneur's project still represents a valuable outside option in our basic bargaining model studied in Section 3. This limits the payoff that can be extracted by the 'winner', E_i. In contrast, in the simple bargaining procedure introduced at the beginning of this section the alternative project represents no longer a viable opportunity once the investor has picked the winning project. Intuitively, in the latter case payoffs are more sensitive to entrepreneurs' types, which is captured

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[29]The investor's outside option is even worse in the simple procedure studied at the beginning of this section, where we assumed that the investor can not (re-)allocate her limited funds to the other entrepreneur once she has failed to come to an agreement with her first choice. It can be shown that an equilibrium of the game with open time horizon generates the same responsiveness (12) if frictions are due to impatience, i.e., if δ is the discount factor used by all parties. (However, in general we do not recoup the exact payoffs as in the one-shot game.) This extreme result is a reflection of the 'outside option principle' in alternating-offer games with discounting (Binmore, Rubinstein, and Wolinsky (1986)).

by a less stringent responsiveness condition. The opposite extreme would be the case where the investor could 'line up' the two entrepreneurs and engage them in open competition for the limited funds. It is straightforward that this would sharply reduce the responsiveness.[30]

In total, the responsiveness under constrained financing is higher the less the investor can manage to play entrepreneurs against each other in order to extract a better deal from the entrepreneur who finally receives refinancing. The bargaining game analyzed in Section 3 represents a 'middle ground' between the extreme case of 'Bertrand competition' between entrepreneurs and the case where the presence of the other entrepreneur does not improve at all the investor's bargaining position, which was the case with the procedure introduced at the beginning of this section.[31]

6.2 Outside Finance

In this section, we relax the condition in Lemma 1, which ensures that the lemon's problem for outside finance at $t = 1$ was sufficiently strong such that projects could only be profitably refinanced by the inside investor. We now show that our main insights extend to the case where outside financing is feasible. Intuitively, what now drives our results is no longer the impossibility to refinance the project but, instead, the higher costs of doing so by drawing on fresh outside funds. Recall that outside funds are more expensive as new investors are uninformed about the interim type of the project, ψ. As unsuccessful projects ($\psi = n$) will also seek fresh funds, successful projects ($\psi = l$ or $\psi = h$) will be pooled with these inferior projects, leading to a dilution of the insiders' ownership stakes.

Suppose then that, under constrained finance, a project with interim type $\psi = l$ or $\psi = h$ did not receive refinancing from the inside investor, although this would be *ex post* profitable. In contrast to our previous analysis, we now assume that the lemon's problem is sufficiently small so that an equilibrium exists in which both interim types $\psi = l$ and $\psi = h$ find it profitable to be refinanced by outside investors. This is the case if projects are sufficiently likely to succeed, i.e. if

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[30]Formally, this could be captured by allowing the investor to make an offer simultaneously to the two entrepreneurs.

[31]While it would be useful to have some guidance which bargaining procedure was more realistic in certain circumstances, this must ultimately remain a futile task. In most economic situations people simply do not move according to a well-specified game form. The strength of the postulated competition effect must, therefore, remain an empirical issue.

τ is sufficiently large. (Precise conditions are derived in the proof of the following proposition).[32] The detailed derivation of this threshold is delegated to Proposition 5.

Outside investors will now charge all three interim types the same conditions for receiving I_2. This is merely a reflection of the fact that outside investors cannot observe the interim type. We denote the repayment requirement by outside investors by D. As outside investors compete themselves down to zero profits, D is determined by their requirement to break even.[33] A formal derivation of D is relegated to the proof of the following Proposition.

The repayment that must be made to the outside investor, D, reduces what the original investor and the entrepreneur can share. Again, we assume that the two 'insiders' will equally share the gains from refinancing.[34] The net surplus to inside investors from securing outside refinancing is then simply

$$\lambda_\psi := p_\psi(R - D) - R^0.$$

Recall that the net surplus from refinancing with inside funds is $r_\psi := p_\psi R - R^0 - I_2$. As is immediate, the high interim type will always face a lemon's premium, i.e. $r_h > \lambda_h$. For the low interim type this depends on the relation between τ, the overall probability of success, and q_g, the conditional probability of $\psi = h$. It is reasonable to assume that the first effect still dominates. As we show in the proof of the following Proposition, this assumption is compatible with assuming that both successful interim types, l and h, prefer outside finance to not refinancing at all. The following Proposition then establishes the equivalent of condition (3) in the presence of outside refinancing.

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[32] But even in this case there still exists an equilibrium where no outside financing is raised. This is the case if any attempt to raise fresh funds is interpreted by outside investors as a signal that $\psi = n$. Similarly, there may exist an equilibrium in which only $\psi = n$ and - provided that both projects are successful - $\psi = h$, who has more to gain from refinancing, will receive outside financing. Such a multiplicity of equilibria is standard in games of signaling. For the purpose of our robustness analysis, we can conveniently ignore these issues.

[33] The precise game we have in mind for the refinancing stage with outside investors at $t = 1$ is one where projects first express their need for outside funding and where there is, subsequently, competition by investors for providing the required funds, I_2.

[34] In other words, the investor and the entrepreneur have joint ownership and control rights over the project. This represents a realistic specification for the case of venture capital finance. Our insights on the robustness of the responsiveness condition extend, however, to the case where the entrepreneur could decide unilaterally to obtain outside finance in order to expand the project. In this case, the entrepreneur would have to leave the investor only with claims of value $(1 - s) R^0$.

Proposition 5. *In an equilibrium where outside financing is preferred by all interim types to not refinancing at all, the responsiveness is higher under constrained finance if and only if*

$$(r_h - \lambda_h) - (r_l - \lambda_l) < \frac{r_l - \lambda_l}{3q_g}. \tag{13}$$

Proof. See Appendix A.

The responsiveness condition is now expressed in terms of the lemon premium $r_\psi - \lambda_\psi$ since the entrepreneur and inside investor bargain over the cost differential between inside and outside refinancing in this setting. However, it retains its fundamental structure from Proposition 1. This reveals the crucial driver behind the responsiveness condition: not obtaining refinancing from the insider is costly. Whether this cost is incurred because of a loss in surplus due to a lemon's premium in the outside market or due to non-continuation is irrelevant for the central mechanism of our model.

7 Conclusion

Much of the literature on the financing of entrepreneurial firms is concerned with the design of contracts to mitigate entrepreneurial agency problems. This paper, by contrast, concentrates on a non-contractual solution. When an investor funds a portfolio of entrepreneurs, we show that this investor can use the depth of his financial pockets to overcome entrepreneurial agency problems. Limiting the amount of funds raised *ex ante* credibly commits the investor to induce a tournament among portfolio projects for (cheaper) inside funds. Interestingly, although this tournament increases the investor's *ex post* bargaining power, we show that it can also increase entrepreneurial effort incentives or allow investors to screen out bad entrepreneurs when contracts cannot achieve either. As a result, shallow pockets may improve on the second-best outcome despite creating an allocational inefficiency in the form of inefficient project continuation.

The main mechanism at work in this result is the fact that shallow pockets ('constrained finance') are able to render the entrepreneur's expected returns more sensitive to the payoff from his project than deep pockets ('unconstrained finance'). When the increase in effort incentives induced by this superior 'responsiveness' is sufficient to outweigh the allocational inefficiency created by constrained finance, investors will choose shallow pockets.

In an adverse selection setting, the increased responsiveness of entrepreneurial payoff to type under shallow pockets is shown to create a single-crossing property that is absent when the investor has deep pockets. As a result, shallow pockets allow good entrepreneurs to separate themselves from bad entrepreneurs by exposing themselves to competition for refinancing.

Anecdotal evidence suggests that the mechanism presented in this paper plays a role in the design of venture capital funds. These are typically close-ended and governed by covenants that render additional fund-raising at a later life-cycle stage of a fund difficult. So-called 'down-rounds', in which the worst-performing start-ups must raise additional funds at reduced valuations from outside investors are a testimony to the fact that venture capitalists 'manage' their portfolio and distribute funds according to performance.

8 Appendix A: Proofs

Proof of Proposition 4.

Let s_C and s_U denote the equilibrium contracts offered by unconstrained and constrained investors, respectively. A separating equilibrium in which type $\theta = g$ borrows from constrained investors and type $\theta = b$ borrows from unconstrained investors will exist if, firstly, s_C and s_U are incentive compatible, secondly, participation constraints for investors and entrepreneurs are fulfilled, and thirdly, there are no other contracts with which investors could make more (i.e. strictly positive) profits. We will now consider each of these constraints in turn.

First, consider incentive compatibility. In equilibrium, the unconstrained investor attracts $\theta = b$ only and her participation constraint must be binding so that, proceeding as for Lemma 6, we obtain

$$s_U = 1 + \frac{1}{2} \frac{r_l + q_b (r_h - r_l)}{R^0} - \frac{I_1}{\tau R^0}. \tag{14}$$

Now consider s_C. Incentive compatibility for type $\theta = b$ implies that the constrained investor offers s_C such that type $\theta = b$ weakly prefers unconstrained finance:

$$\tau \left\{ s_U R^0 + \frac{1}{2} \left[r_l + q_b (r_h - r_l) \right] \right\}$$

$$\geq \tau \left\{ s_C R^0 + \frac{1}{2} \left[r_l + q_b (r_h - r_l) \right] \right\} - \frac{\tau^2}{8} \left\{ (3 - q_b + q_g) r_l + 3 q_g q_b (r_h - r_l) \right\},$$

which transforms to

$$s_C \leq s_U + \frac{\tau}{8} \frac{(3 + q_g - q_b) r_l + 3 q_g q_b (r_h - r_l)}{R^0}. \tag{15}$$

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where s_U is defined in equation (14).

Incentive compatibility for type $\theta = g$, in turn, implies that type g weakly prefers s_C:

$$\tau \left\{ s_C R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\} - \frac{\tau^2}{8} \left\{ 3r_l + 3q_g^2 \left(r_h - r_l \right) \right\}$$

$$\geq \tau \left\{ s_U R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\},$$

which transforms to

$$s_C \geq s_U + \frac{\tau}{8} \frac{3r_l + 3q_g^2 \left(r_h - r_l \right)}{R^0}. \tag{16}$$

Note that these incentive compatibility conditions (15) and (16) are compatible with each other if and only if $\frac{r_l}{3q_g} > r_h - r_l$, i.e. if and only if the responsiveness condition holds.

Consider next participation constraints. Entrepreneurs always realize non-negative payoffs, while the construction of s_U ensures that unconstrained investors break even. Finally, the constrained investor's expected payoff is non-negative if

$$\tau \left\{ \left(1 - s_C \right) R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\} - \frac{\tau^2}{8} \left\{ r_l + q_g^2 \left(r_h - r_l \right) \right\} - I_1 \geq 0,$$

which transforms to

$$s_C \leq s_U + \frac{1}{2} \frac{\left(q_g - q_b \right) \left(r_h - r_l \right)}{R^0} - \frac{\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l \right)}{R_0}. \tag{17}$$

Condition (17) is compatible with the incentive compatibility constraint for $\theta = g$ (16) if

$$s_U + \frac{1}{2} \frac{\left(q_g - q_b \right) \left(r_h - r_l \right)}{R^0} - \frac{\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l \right)}{R_0} \geq s_U + \frac{\tau}{8} \frac{3r_l + 3q_g^2 \left(r_h - r_l \right)}{R^0},$$

which transforms to

$$\tau \leq \frac{\left(q_g - q_b \right) \left(r_h - r_l \right)}{r_l + q_g^2 \left(r_h - r_l \right)}. \tag{18}$$

Finally, the existence of the separating equilibrium requires that investors cannot make strictly positive profits from another offer. As we have argued that a self-selecting menu of contracts is not feasible, the other feasible offer is a pooling contract to all entrepreneurs. The zero-profit pooling offer is given by

$$s_P = 1 + \frac{1}{2} \frac{r_l + \left(\alpha q_g + \left(1 - \alpha \right) q_b \right) \left(r_h - r_l \right)}{R^0} - \frac{I_1}{\tau R^0}.$$

For type g to prefer s_C (and constrained finance) to s_P (and unconstrained finance) it must hold that

$$\tau \left\{ s_C R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\} - \frac{\tau^2}{8} \left\{ 3r_l + 3q_g^2 \left(r_h - r_l \right) \right\} \geq \tau \left\{ s_P R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\},$$

which transforms to

$$s_C \geq s_P + \frac{\tau}{8} \frac{3r_l + 3q_g^2 \left(r_h - r_l\right)}{R^0}.$$

This condition is compatible with (15) if

$$s_U + \frac{\tau}{8} \frac{\left(3 + q_g - q_b\right) r_l + 3q_g q_b \left(r_h - r_l\right)}{R^0} \geq s_P + \frac{\tau}{8} \frac{3r_l + 3q_g^2 \left(r_h - r_l\right)}{R^0},$$

which transforms to

$$\alpha \leq \frac{\tau}{8} \frac{r_l - 3q_g \left(r_h - r_l\right)}{r_h - r_l}, \tag{19}$$

and with the zero-profit constraint (17) if

$$s_U + \frac{1}{2} \frac{\left(q_g - q_b\right) \left(r_h - r_l\right)}{R^0} - \frac{\tau}{8} \frac{r_l + q_g^2 \left(r_h - r_l\right)}{R_0} \geq s_P + \frac{\tau}{8} \frac{3r_l + 3q_g^2 \left(r_h - r_l\right)}{R^0},$$

which transforms to

$$\alpha \leq \frac{1}{2} \left[1 - \tau \frac{r_l + q_g^2 \left(r_h - r_l\right)}{\left(q_g - q_b\right) \left(r_h - r_l\right)} \right]. \tag{20}$$

We can now sum up conditions. Firstly, if the responsiveness condition together with (18) hold, there exists an offer made by constrained investors such that (i) only type g prefers this offer to that made by unconstrained investors to type b and (ii) constrained investors break even. Secondly, if (19) and (20) jointly hold, the separating equilibrium cannot be destabilized by a (more profitable) pooling offer.

Proof of Proposition 5.

In this proof we also state the exact conditions such that any equilibrium prescribes that both successful interim types will *not* seek outside finance in case of constrained finance. This was the case analyzed in Section 3.

We first describe more formally the operation of the market for outside finance. Projects, represented by the respective entrepreneurs and the original investor, first express their willingness to seek outside financing and investors subsequently compete to provide funds I_2 in return for being paid $D \leq R$ in case of success. Because of their lack of information, outside investors cannot condition D on the project's interim type. Our equilibrium concept is that of a perfect Bayesian equilibrium, where investors rationally anticipate which projects seek outside finance. Given these shared beliefs, investors compete themselves down to zero profits.

We further assume that ownership of the project is jointly held by the entrepreneur and the inside investor, so that surplus net of the repayment to outside investors is shared equally

between them. This specification is not crucial to our model, although it seems characteristic of venture capital financing.[35]

We already know that unsuccessful types, $\psi = n$, will always seek outside finance. As outside investors have rational beliefs, it is then immediate that in equilibrium successful types, i.e. $\psi = l$ and $\psi = h$, have a strict preference for using inside funds in case of unconstrained financing.

Turn now to the case of constrained finance. Suppose that some entrepreneur has not received inside money. (We turn later to the question which entrepreneur is chosen.) Recall that without refinancing both $\psi = l$ and $\psi = h$ realize $R^0 > 0$. It is only weakly optimal to obtain outside finance in case

$$p_\psi [R - D] - R^0 \geq 0. \tag{21}$$

A necessary condition for (21) to hold is $D < R$. Given $R - D > 0$ and $p_h > p_l$, it is then immediate that $\psi = h$ prefers outside finance strictly whenever $\psi = l$ only prefers it weakly. This implies that we only have the following equilibrium candidates: (i) no successful type is refinanced with outside money; (ii) only $\psi = h$ and $\psi = n$ are refinanced with outside funds; and (iii) both successful types - and also $\psi = n$ - are refinanced with outside funds.[36]

Note next that by $p_h > p_l$ and as outside investors must break even, $\psi = h$ is sure to pay a lemon's premium under outside finance. If $\psi = l$ also pays a lemon's premium as τ is sufficiently low, this premium is nevertheless strictly smaller than that for $\psi = h$. This strengthens the argument in Section 3 by which the investor will choose $\psi = h$ over $\psi = l$ if both projects are successful. We are now in a position to consider the various equilibrium candidates.

Equilibrium where no successful project is refinanced by outside investors

This is trivially always an equilibrium. If outside investors believe that no successful types seek outside finance the market for outside finance shuts down completely.

Equilibrium where all successful projects are refinanced by outside investors

We next characterize the equilibrium beliefs of outside investors, which we denote by $\pi(\psi)$. With probability $\tau^2 q_{\theta_i} q_{\theta_j}$, both projects are of type h, in which case also the project seeking outside finance is h: $\pi(h) = \tau^2 q_{\theta_i} q_{\theta_j}$. The project seeking outside finance is of type n whenever

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[35] In an alternative setting, in which the entrepreneur retains all ownership, he must leave the inside investor with $(1 - s_i) R^0$. Our qualitative results extend also to this setting.

[36] We restrict attention to pure-strategy equilibria.

at least one project is unsuccessful such that $\pi(n) = 1 - \tau^2$.[37] With the residual probability $\pi(l) = \tau^2(1 - q_{\theta_i}q_{\theta_j})$, outside investors believe that the project is of type l. To break even D must satisfy

$$D[\tau^2 q_{\theta_i} q_{\theta_j} p_h + \tau^2(1 - q_{\theta_i}q_{\theta_j})p_l] = I_2. \tag{22}$$

To validate that we have characterized an equilibrium, we must verify that both successful types weakly prefer outside finance and that (22) generates a value $D \leq R$. By our previous arguments we know that out of these three conditions the incentives for $\psi = l$ generate the most stringent requirements. From (21) and (22) it must hold that

$$p_l \left[R - \frac{I_2}{\tau^2 q_{\theta_i} q_{\theta_j} p_h + \tau^2(1 - q_{\theta_i}q_{\theta_j})p_l} \right] \geq R^0,$$

which transforms to

$$\tau^2 \geq \left[\frac{I_2}{p_l R - R^0} \right] \left[\frac{1}{q_{\theta_i} q_{\theta_j} p_h + (1 - q_{\theta_i}q_{\theta_j})p_l} \right]. \tag{23}$$

Before proceeding to the analysis of the remaining equilibrium candidate, we turn to the assertion that, for low values of τ that are still compatible with (23), $\psi = l$ also has to pay a lemon's premium. This is the case if $p_l R - I_2$ exceeds $p_l[R - D]$, where we have to substitute D from (22). Reorganizing terms, we obtain from

$$p_l \left[R - \frac{I_2}{\tau^2 q_{\theta_i} q_{\theta_j} p_h + \tau^2(1 - q_{\theta_i}q_{\theta_j})p_l} \right] < p_l R - I_2$$

the requirement

$$\tau^2 < \frac{p_l}{q_{\theta_i} q_{\theta_j} p_h + (1 - q_{\theta_i}q_{\theta_j})p_l}. \tag{24}$$

As $I_2 < p_l R - R^0$, which must hold by $r_l > 0$, (23) and (24) are jointly satisfied for intermediate values of τ.

Equilibrium where only $\psi = n$ and $\psi = h$ are refinanced by outside investors

The outside investors' beliefs are now given by $\pi(l) = 0$, $\pi(n) = \frac{1-\tau^2}{(1-\tau^2)+\tau^2 q_{\theta_i} q_{\theta_j}}$, and $\pi(h) = \frac{\tau^2 q_{\theta_i} q_{\theta_j}}{(1-\tau^2)+\tau^2 q_{\theta_i} q_{\theta_j}}$. To break even, D must solve

$$D \frac{\tau^2 q_{\theta_i} q_{\theta_j}}{(1 - \tau^2) + \tau^2 q_{\theta_i} q_{\theta_j}} = I_2. \tag{25}$$

---

[37]In case $\psi_i = \psi_j = n$ we specify that the inside investor proposes only one project to outside investors as they could otherwise immediately conclude that both projects are unsuccessful. (Note that in this case the inside investor would ask for outside finance even though she has still I_2 in her coffers.)

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Substituting (25) into (21), type h weakly prefers outside finance at these conditions if

$$p_h \left[R - I_2 \frac{(1 - \tau^2) + \tau^2 q_{\theta_i} q_{\theta_j}}{\tau^2 q_{\theta_i} q_{\theta_j}} \right] \geq R^0,$$

which transforms to

$$\tau^2 \geq \left[\frac{I_2}{p_h R - R^0} \right] \left[\frac{1}{q_{\theta_i} q_{\theta_j} + (1 - q_{\theta_i} q_{\theta_j}) \frac{I_2}{p_h R - R^0}} \right]. \tag{26}$$

For our purposes, i.e. as we merely want to rule out this equilibrium candidate, it is sufficient to stop here.

Having characterized all candidate equilibria, it remains to show that for all sufficiently low values of τ the only equilibrium is that without outside finance. This follows from conditions (23) and (26).

The responsiveness condition with outside finance

Suppose now that condition (23) holds, so that all interim types $\psi \in \{n, l, h\}$ can seek refinancing from outside investors if inside funds are not available. We now derive responsiveness condition (13).

First, we turn to the entrepreneur's payoff from negotiating refinancing with the inside investor. If $E'_j s$ *interim* type is n, negotiations with E_i are identical to those under unconstrained finance.[38] Suppose instead that both entrepreneurs are of *interim* type l or h and that E_j with $\psi_j \neq n$ is the last entrepreneur to be bargained with. If the limited funds have already been used for E_i, the entrepreneur will only realize $s_j R^0 + \frac{1}{2} \lambda_{\psi_j}$ while the investor realizes $(1 - s_j) R^0 + \frac{1}{2} \lambda_{\psi_j}$ from this project.

Alternatively, if the funds are still available, E_j and the investor bargain over the cost savings from internal finance $r_{\psi_j} - \lambda_{\psi_j}$ so that E_j realizes the sum of $s_j R^0 + \frac{1}{2} \lambda_{\psi_j}$ and $\frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})$, which transforms to $s_j R^0 + \frac{1}{2} r_{\psi_j}$. The investor, on the other side, realizes the sum of $(1 - s_j) R^0 + \frac{1}{2} \lambda_{\psi_j}$ and $\frac{1}{2} \left(r_{\psi_j} - \lambda_{\psi_j} \right)$, which transforms to $(1 - s_j) R^0 + \frac{1}{2} r_{\psi_j}$. Turn next to negotiations with E_i who was picked first. In case of breakdown of the negotiations for inside funds, E_i receives $s_i R^0 + \frac{1}{2} \lambda_{\psi_i}$, while the investor's payoff is equal to the sum of $(1 - s_i) R^0 + \frac{1}{2} \lambda_{\psi_i}$ and $(1 - s_j) R^0 + \frac{1}{2} r_{\psi_j}$.[39]

---

[38] Recall that the success probability of n is zero. We assumed that continuing the project (with outside finance) generates arbitrarily small private benefits, implying that owners of a type-n project are no longer indifferent between seeking outside finance or not.

[39] An alternative scenario would be where the break-down of negotiations is 'complete', implying that the two

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In case they reach agreement, the joint surplus is $R_{\psi_i} + (1 - s_j) R^0 + \frac{1}{2}\lambda_{\psi_j}$. Subtracting the outside options of the two sides yields the net surplus $r_{\psi_i} - \lambda_{\psi_i} - \frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})$. As E_i gets half of it, his payoff is the sum of $s_i R^0 + \frac{1}{2}\lambda_{\psi_i}$ and $\frac{1}{2}[r_{\psi_i} - \lambda_{\psi_i} - \frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})]$, which becomes $s_i R^0 + \frac{1}{2}[r_{\psi_i} - \frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})]$.

Using this and the payoff $s_j R^0 + \frac{1}{2}\lambda_{\psi_j}$ for E_j, we can next calculate the resulting *ex post* payoffs. When both entrepreneurs are of type h, E_i is picked first with probability $\frac{1}{2}$, implying that his payoff equals $s_i R^0 + \frac{1}{2}[r_h - \frac{1}{2}(r_h - \lambda_h)]$ with probability $\frac{1}{2}$ and $s_i R^0 + \frac{1}{2}\lambda_h$ with probability $\frac{1}{2}$, which adds up to $s_i R^0 + \frac{1}{8}(r_h + 3\lambda_h)$. When $\psi_i = h$ and $\psi_j = l$, then E_i receives inside refinancing for sure and earns $s_i R^0 + \frac{1}{2}[r_h - \frac{1}{2}(r_l - \lambda_l)]$. When $\psi_i = l$ and $\psi_j = h$, by contrast, E_i has to go to the outside market where he will earn $s_i R^0 + \frac{1}{2}\lambda_l$. Finally, when both entrepreneurs are of type l, E_i is again picked first with probability $\frac{1}{2}$ and we obtain for his payoff $s_i R^0 + \frac{1}{8}(r_l + 3\lambda_l)$.

Finally, we turn to *ex ante* payoffs. Substitution of *ex post* payoffs yields $E_i's$ *ex ante* payoff:

$$
\tau \left[s_i R^0 + \frac{1}{2} \left[r_l + q_{\theta_i}(r_h - r_l) \right] \right]
$$
$$
- \frac{\tau^2}{8} \left[(r_l - \lambda_l)\left(3 - q_{\theta_i} + q_{\theta_j}\right) + 3 q_{\theta_i} q_{\theta_j} [r_h - \lambda_h - (r_l - \lambda_l)] \right]
$$

Now, let $\theta_i = g$. The responsiveness of $E_i's$ payoff to his project's profitability under constrained finance is then

$$
\frac{1}{2}\tau (q_g - q_b)(r_h - r_l) - \frac{\tau^2}{8}(q_g - q_b)\left[3 q_g (r_h - \lambda_h - (r_l - \lambda_l)) - (r_l - \lambda_l) \right].
$$

This exceeds the responsiveness under unconstrained finance if and only if condition (13) holds. **Q.E.D.**

9   Appendix B: Bargaining Procedures with Open Time Horizon

In this section, we formulate and solve the bargaining game with an open time horizon. To save space we focus on the case with constrained finance. The case with unconstrained finance is similar but more immediate. It also convenient to introduce the following additional notation. If there is no refinancing of project i, denote the respective payoffs by y_i for the investor and by z_i for E_i, i.e. $y_i := (1 - s_i)R^0$ and $z_i := s_i R^0$. If there is refinancing their joint surplus

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sides also cannot agree to obtain outside finance. However, this does not affect our results as the entrepreneur's payoff from negotiating with the inside investor would remain unchanged at $s_i R^0 + \frac{1}{2}[r_{\psi_i} - \frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})]$.

from project i is denoted by v_i, i.e. $v_i := R_{\psi_i} - I_2$. The respective net surplus is denoted by $w_i := v_i - (y_i + z_i)$.

Recall next the bargaining procedure. First, the investor picks an entrepreneur, E_i, and makes an offer x. In the next period E_i can respond. If E_i accepts, the bargaining game ends as there are no funds available for E_j. If E_i rejects he can make a counter-offer, to which the investor can respond in the following period. If the investor rejects the counter-offer, she can pick again one of the two entrepreneurs and make a (new) offer etc..

Recall also that we capture bargaining frictions as follows. Suppose the investor currently negotiates with E_i. If E_i rejects the offer and makes a counter-offer, this offer will only be received by the investor with probability δ. With probability δ negotiations between the two parties break down, in which case project i will not be expanded. Note that a break-down of negotiations with E_i does not affect the possibility of negotiating with E_j. Also, the same risk of break-down exists if the investor reacts to the offer of E_i.[40]

We now analyze the bargaining game. Our aim is to characterize a (subgame perfect) equilibrium that generates the payoffs used in the main text as $\delta \to 1$. We start with histories where there has been breakdown with one entrepreneur, say entrepreneur j. As a consequence, only E_i remains and there are still funds for refinancing. The characterization of equilibrium strategies is standard. For the sake of completeness, we specify strategies in some detail. We characterize an offer by the payoff X it leaves to E_i. If it is the investor's turn, she always makes the offer X_i^I. If she has to respond, she accepts any X satisfying

$$v_i - X \geq (1 - \delta)y_i + \delta(v_i - X_i^I). \tag{27}$$

E_i always makes the offer X_i^E and accepts any X satisfying

$$X \geq (1 - \delta)z_i + \delta X_i^E. \tag{28}$$

X_i^I and X_i^E are determined by using equalities in (27) and (28) and substituting $X = X_i^E$ into (27) and $X = X_i^I$ into (28). In other words, the proposer extracts the maximum feasible payoff

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[40]Modelling bargaining frictions by a risk of break-down is standard. In contrast to the case with delay, it ensures that the two sides' outside options are always of relevance. That bargaining with the risk of break-down, but not bargaining with delay, can support the axiomatic Nash bargaining solution with threatpoints has been shown by Binmore, Rubinstein, and Wolinsky (1986).

while still ensuring acceptance. We then obtain that the investor offers

$$X_i^I = \frac{z_i + v_i\delta - \delta y_i}{1 + \delta}, \text{ with } \lim_{\delta \to 1} X_i^I = z_i + \frac{w_i}{2},$$

which is accepted immediately by E_i.

Turn now to histories where there has not yet been any break-down. We specify that the investor always makes an offer to the entrepreneur with the highest net surplus, w_i. In case of indifference, we make the following specification. In the very first period, $t' = 1$, the invesor randomizes with equal probability. Later, he will always pick with probability one this particular entrepreneur E_i.[41] The investor now makes the offer x_i^I and accepts any counter-offer x by E_i satisfying

$$(v_i - x) + y_j \geq (1 - \delta)\left[y_i + (v_j - X_j^I)\right] + \delta(v_i - x_i^I + y_j).$$

Observe that after break-down, which happens with probability $1 - \delta$, the investor realizes the outside option y_i with E_i and continues to bargain with E_j. In this case we know that she has to leave E_j with X_j^I and, thereby, realizes $v_j - X_j^I$. If no break-up occurs, we work again with the assumption that the investor's next offer to E_i, i.e. x_i^I, will be accepted for sure. We will check below that it is optimal for the investor to again approach E_i and for E_i to accept.

E_i now always makes the offer x_i^E and accepts any x satisfying[42]

$$x \geq (1 - \delta)z_i + \delta x_i^E.$$

Proceeding in the standard way, i.e. by choosing x_i^I and x_i^E to make the respective responder just indifferent, we obtain

$$
\begin{aligned}
(v_i - x_i^E) + y_j &= (1 - \delta)\left[y_i + (v_j - X_j^I)\right] + \delta(v_i - x_i^I + y_j), \\
x_i^I &= (1 - \delta)z_i + \delta x_i^E,
\end{aligned}
$$

and after substitution

$$x_i^I = \frac{z_i + \delta v_i + \delta y_j - \delta(y_i + v_j - X_j^I)}{1 + \delta}, \text{ with } \lim_{\delta \to 1} x_i^I = \frac{z_i + v_i + y_j - y_i - v_j + X_j^F}{2}.$$

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[41]This specification for resolving the investor's indifference is only for convenience. Letting the investor randomize with equal probability in all future rounds would not change results as $\delta \to 1$.

[42]Note that this is identical to the condition that we employed for analyzing histories with break-down. This follows as the entrepreneur rightly expects that the investor will approach him again next time, provided there was no break-down. We have more to say on the investor's option to approach a different entrepreneur below.

Hence, for $\delta \to 1$ the entrepreneur who is chosen first, E_i, realizes

$$x_i^I = z_i + \frac{1}{2}\left[(v_i - z_i - y_i) - (v_j - X_j^I - y_j)\right].$$

Substituting

$$v_j - X_j^I - y_j = v_j - \frac{v_j + z_j - y_j}{2} - y_j = \frac{1}{2}(v_j - (z_j + y_j)),$$

we obtain

$$x_i^I = z_i + \frac{1}{2}\left[w_i - \frac{1}{2}w_j\right].$$

This matches our result in the main text for the 'reduced' bargaining procedure.

By construction of the offers, to confirm that we have characterized an equilibrium we only have to consider histories where (i) both entrepreneurs are still around and (ii) where the investor deviates and makes an offer to the non-preferred entrepreneur j. For these histories, we make the following specifications. The investor offers x_j^I and accepts from E_j any x satisfying

$$(v_j - x) + y_i \geq (1-\delta)(y_j + (v_i - X_i^I)) + \delta(v_i - x_i^I + y_j).$$

Note that this inequality takes into account that the investor will switch back to E_i in case she rejects the offer of E_j. E_j makes the offer x_j^E and accepts any x satisfying

$$x \geq (1-\delta)z_j + \delta x_j^E.$$

Making the responders again indifferent, we obtain

$$\begin{aligned}(v_j - x_j^E) + y_i &= (1-\delta)(y_j + (v_i - X_i^I)) + \delta(v_i - x_i^I + y_j), \\ x_j^I &= (1-\delta)z_j + \delta x_j^E,\end{aligned}$$

yielding

$$x_j^I = (1-\delta)z_j + \delta\left[v_j + y_i - (1-\delta)(y_j + (v_i - X_i^I)) - \delta(v_i - x_i^I + y_j)\right].$$

Recall that we want to confirm that, given these payoffs, the investor does not find it profitable to deviate and make an offer to j if no break-down has yet occurred with E_i. If the investor does not deviate she obtains $(v_i - x_i^I) + y_j$. If she does deviate the payoff is $(v_j - x_j^I) + y_i$. Hence we need to show that

$$(v_i - x_i^I) + y_j \geq (v_j - x_j^I) + y_i. \tag{29}$$

41

Substituting for x_i^I and x_j^I, requirement (29) transforms to $(w_i - w_j)(1 - \delta) \geq 0$, which holds by assumption.

There is an indirect and more intuitive way to see that (29) must hold. Recall first that x_i^I and x_j^I are determined just to ensure acceptance by the respective entrepreneur, i.e., we have $x_i^I = (1 - \delta)z_i + \delta x_i^E$ and $x_j^I = (1 - \delta)z_j + \delta x_j^E$. Likewise, recall that x_j^E and x_i^E are chosen such that the investor is just indifferent between accepting and rejecting. As, following rejection, the investor returns to bargaining with E_i in both cases, we have that $v_i - x_i^E + y_j = v_j - x_j^E + y_i$, implying $x_i^I - x_j^I = (z_i - z_j) + \delta(w_i - w_j)$. Substituting this into (29) yields again the requirement $(w_i - w_j)(1 - \delta) \geq 0$.

10   References

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