Information channels in labor markets. On the resilience of referral hiring.*

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Abstract

Economists and sociologists disagree over markets' potential to substitute for personal connections. We study a model of labor markets where social ties are stronger between similar individuals, and firms prefer to rely on personal referrals than to hire on the open market. Workers in the market can take a costly action that can signal their productivity. The paper asks whether signaling reduces the reliance on the network. We find that the network is remarkably resilient. Signaling is caught in two contradictory requirements: to be informative it must be expensive, but if it expensive it can be undercut by the network.

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1 Introduction

Personalized networks, systems of personal connections that function as privileged channels of information and trust, are part of daily experience. In situations where the reliability of information is particularly important (when applying for a job, when needing fresh capital for a new enterprise, when moving to a new country, when substituting for formal enforcement), their role becomes often crucial, either as means of entering formal markets, or indeed as substitutes for these markets. Hence the ethnic enclaves, both residential and professional, in New York City; the economic weight of the Overseas Chinese in their countries of residence; the success of Medieval networks of merchants, organized along ethnic or religious lines (the Armenians, the Italians, the Jews, the Dutch...). All of these phenomena have in common the essential role played by the personal network, with its rapidity and its freedom from the constraints of unwieldy procedures.[1]

As the function of the network is recognized, an important policy question is the extent to which market mechanisms can substitute for personal connections. The question is important both because the networks are often very successful, and thus it would be good to be able to copy them, and because they are by their nature exclusionary, and thus tend to generate resentment and opposition among those excluded. If the networks could be replicated, these delicate distributional problems could be faced.

Economists and sociologists usually disagree on the potential for artificial replication of the networks. Not surprisingly, economists tend to be more optimistic, believing that appropriate market mechanisms, encouraged and supported by appropriate policy where necessary, can substitute for the missing personal channels. Sociologists, on the other hand, see the personal, non-anonymous link as the essence of the relation, the fundamental, inescapable reason not only for the truthful transmission of information, but for the "trust"

[1]See, among many others, Braudel (1982), Greif (1993), Kotkin (1993), Rauch (2001), and Redding (1990). The classic studies of personal referrals in labor markets are Rees (1966) and Granovetter (1974). A large literature focussing on labor markets has emerged since, starting with Saloner (1985), Holzer (1988), Staiger (1990), Montgomery (1991), and Mortensen and Vishwanath (1994). Ioannides and Datcher Loury (2004) provides an exhaustive review.

that accompanies the exchange.[2]

In this paper, we approach the issue by focusing on one channel of information transmission that seems a plausible alternative to personalized exchanges taking place in a network: the possibility of signaling. Individuals who are not part of the network can take a costly action that signals high productivity, the only relevant information in our model. The question we ask is how effective the availability of the signal is in weakening the reliance on the network. We find that the network is extremely resilient; for the majority of our parameter space, reliance on the network continues and is profitable, even though signaling takes place in the market and typically leads to more precise information than is being revealed in the network. The reason is, ex post at least, obvious: when signaling is informative, it is caught in two contradictory requirements. On one hand, it must be effective in separating the different types: it must induce the more productive type to signal while the less productive type cannot profitably do so. To induce this separation, either the signal is extremely precise, or it must be costly. But at the same time, signaling is competing with reliance on the network; if signaling is costly enough to differentiate among types, then the network is likely to be profitable because it can undercut the cost of information in the market. Indeed, the savings can be enough to expand the network beyond what the acquisition of useful information would dictate, where firms prefer hiring through the network, even when that implies a more than average probability of hiring a less productive worker.

In the next section, we describe the model, in section 3 we describe the main properties of the network that will be exploited repeatedly in solving the model, section 4 characterizes the equilibria of the model, first without and then with signaling, section 5 discusses the resilience of the network and its causes, section 6 describes the main empirical predictions of the model, and section 7 concludes. Most proofs can be found in the Appendix available at (website version address).

[2]See, for example, Tienda and Raijman (2001), discussing Rauch.

2 The Model

A model allowing us to study the relative performance of personal connections versus signaling must be very flexible. It must include both a network and a market, which must differ in some substantive way, and it must capture the equilibrium effects linking the two; it must allow workers and firms to choose between networking and signaling, and it must be tractable enough to allow us to study the results' sensitivity to different costs of signaling and different precisions of the two mechanisms. Montgomery (1991) proposed a simple, beautiful model that satisfies all our requirements. We start from a streamlined version of Montgomery's set-up and add the possibility of signaling.

There is a potentially infinite number of identical infinitely-lived firms and, at any period in time, two overlapping generations of workers, each composed of an equal large number of individuals. Everything we write below will apply to the limit of this number becoming very large. Each firm employs at most one worker. Workers live two periods, and work in the second period of their lives. In each generation, half of the workers are productive and produce one unit of output when employed (H workers), and half are unproductive and produce no output (L workers). The two types of workers cannot be distinguished ex ante, and wages cannot be made conditional on production. We will call "H firms" firms whose current employee is of type H, and "L firms" firms whose current employee is of type L.

Young workers, who are not yet employed, can establish a connection to an older employed worker at no cost. Employed workers' types are not observable outside the firm, but following Montgomery and building on sociologists' concept of "in-breeding," we assume that a young individual will have a higher probability of establishing a link to an older worker of his own type. More precisely, each young individual will be connected to an employed worker of his own type with probability $\alpha > 1/2$, where α is common knowledge. The links are otherwise random.

Personal connections can be valuable because firms have the option of hiring their new workers through referrals from their current employees, whose productivity is known and

who, through α, are more likely than not to have connections to young workers of their own type. If a firm chooses to hire through referrals, its employee transmits the offer to one of the young workers he is connected to (choosing randomly if he has several connections). If the young worker accepts the offer, the contract is concluded and the worker is hired for the next period. Young workers who either reject the referral offer or do not receive any will need to find employment in the anonymous market. Before entering the market, however, each young worker has the option of engaging in a costly action that has the potential to announce publicly that he is of type H; for example a worker can attempt to be certified through an exam. Certification costs λ and the probability of success is $\beta > 1/2$ if the individual is of type H, and $(1 - \beta) < 1/2$ if he is of type L.[3] Failure at the exam is not observed, and the market cannot distinguish between workers who are not certified because they never attempted certification and those who tried but failed. We call this option "signaling," because it has the core properties of effective signaling strategies: a costly action whose expected return is higher for type H.

Finally, the markets for certified and uncertified workers open, and all young workers who have not been hired though referrals offer their labor. Firms can enter these markets freely, and expected profits from market hiring are brought down to zero. Once the new workers are hired, the old workers retire, and a new generation of young agents is born, not yet working but ready to network.

The solution of the model is straightforward, once the stochastic properties of the network have been characterized. We begin then by studying the network, in particular the density of connections at each node that determines the probabilities of contacts between young and old workers of the two types.

[3]Workers can borrow λ at no cost against their future labor earnings. More generally, λ is the amount that will need to be repaid out of future wages if a worker decides to signal. Hence it could include borrowing costs, and a decline in λ could be interpreted as, for example, more generous conditions on loans financing extra schooling.

3 The Network

Because establishing a personal connection entails no cost and does not prevent the option of signaling later, doing so is a weakly undominated strategy; if the young worker does not receive a referral offer, or if the offer is inferior to the market wage, he can access the market. Thus we will study scenarios where all young workers establish a personal connection.

The stochastic nature of the connections implies that in general some older workers will have several links, while others will have none. Suppose a young worker is connected to an old worker of type i ($i = H, L$), employed by a firm who has chosen to make a referral offer. What is the probability that the young worker will receive the offer? We begin our analysis of the network by deriving such a probability, which we call p_i. The details of the derivation are in the Appendix, available at (website version address), but the main steps are easily described. With a very large number of workers and firms, the density of connections at each old worker node can be studied independently of the rest of the network. Suppose the size of each generation of workers is $2N$, which then equals the number of active firms. Begin by concentrating on p_H: the probability that a young worker connected to an old worker of type H will receive an offer, conditional on the old worker's firm having decided to use referrals. Any young H worker has probability (α/N) of connecting to any specific old H worker, while the same probability equals $(1-\alpha)/N$ for any young L worker. For large N, the number of ties connecting any individual old worker to young workers of either type is described by a Poisson distribution: the probability that an old H worker has x ties to young H workers, for example, is given by $\frac{\alpha^x}{x!}e^{-\alpha}$. Denote by $\gamma_{k,H}$ the probability that the old worker to whom the young worker is connected has k additional connections to workers of either type. Then $\gamma_{k,H}$ can be written as

$$\gamma_{k,H} = \sum_{j=0}^{k}\left[\frac{\alpha^{k-j}}{(k-j)!}e^{-\alpha}\frac{(1-\alpha)^j}{j!}e^{-(1-a)}\right] = \sum_{j=0}^{k}\left[e^{-1}\frac{\alpha^{k-j}(1-\alpha)^j}{(k-j)!j!}\right] = \frac{e^{-1}}{k!} \tag{1}$$

where we have used[4]

$$\frac{1}{k!} = \sum_{j=0}^{k} \frac{\alpha^{(k-j)}(1-\alpha)^j}{j!(k-j)!}.$$

Therefore, taking into account that the old worker chooses randomly,

$$p_H = \lim_{N\to\infty}\left[1 - \left(\frac{2N-1}{2N}\gamma_{2N-1,H} + \frac{2N-2}{2N-1}\gamma_{2N-2,H} + ... + \frac{1}{2}\gamma_{1,H}\right)\right] =$$
$$= \lim_{N\to\infty}\left(1 - \sum_{k=1}^{2N-1}\frac{k}{k+1}\gamma_{k,H}\right) = 1 - e^{-1}\left[\lim_{N\to\infty}\left(\sum_{k=1}^{2N-1}\frac{k}{(k+1)!}\right)\right].$$

It is possible to show that

$$\lim_{N\to\infty}\left(\sum_{k=1}^{2N-1}\frac{k}{(k+1)!}\right) = 1,$$

and thus we can conclude

$$p_H = 1 - e^{-1}. \tag{2}$$

Notice that p_H does not depend on the parameter α, the in-breeding bias; as shown in (1), α affects the composition of the pool of young workers connected to a given old H worker, but not the size of the pool. Thus it is also the case that $p_H = p_L$, and in what follows we will identify both terms by $p \equiv 1 - e^{-1}$. Recall however that p is conditional on the firms using referrals and that H and L young workers will expect to be connected to firms of different types, firms generally making different decisions about their reliance on referrals. In equilibrium the unconditional probability of receiving a referral offer will not be equal for young workers of type H and L and will not be independent of α.

The stochastic nature of the network determines two further probabilities that will be exploited repeatedly. It is convenient to derive them here. We observed above that some old workers will have no ties to any young worker. Thus, from the point of view of the firm, what is the probability that its current employee of type i is able to recommend at least one worker for possible hiring? In other words, what is the probability that the firm's current

[4]Since: $\sum_{j=0}^{k}\binom{k}{j}\alpha^{(k-j)}(1-\alpha)^j = \sum_{j=0}^{k}\frac{k!}{j!(k-j)!}\alpha^{(k-j)}(1-\alpha)^j = 1$.

employee has at least one connection? Call such a probability ϕ_i where i is the current employee's type. An old H worker is linked to a young H with probability α/N (since there are N old H workers). Thus the probability that he has no connection to any young H is given by $(1-\alpha/N)^N$, the exponent now reflecting the fact that there are N young H workers. The probability that he has no connection to either a young H or a young L is then $(1-\alpha/N)^N[1-(1-\alpha)/N]^N$, and thus ϕ_H, the probability that he has at least one connection, can be approximated by

$$\phi_H = \lim_{N\to\infty}[1-(1-\frac{\alpha}{N})^N(1-\frac{1-\alpha}{N})^N] = 1-e^{-1}. \tag{3}$$

Similarly $\phi_L = 1-e^{-1}$, and again we will identify both ϕ_H and ϕ_L by $\phi \equiv 1-e^{-1}$.[5]

Finally, conditional on having at least one connection, what is the probability that an old H worker making a random referral will choose a young worker of type H? Suppose that the old H worker has k connections, and call such a probability ζ^k_{HH}. Recall that the probability of having k connections can be approximated by γ_{kH} in (1). Hence, for very large N, we can write

$$\begin{aligned}\zeta^k_{HH} &= \left(\frac{1}{\gamma_{kH}}\right)\sum_{j=0}^{k}\left[\left(\frac{k-j}{k}\right)\frac{\alpha^{k-j}}{(k-j)!}e^{-\alpha}\frac{(1-\alpha)^j}{j!}e^{-(1-a)}\right] \\ &= \sum_{j=0}^{k}\left[\left(\frac{k-j}{k}\right)\frac{\alpha^{k-j}}{(k-j)!}\frac{(1-\alpha)^j}{j!}\right]k!.\end{aligned}$$

Using

$$\sum_{j=0}^{k}\left[(k-j)\frac{\alpha^{k-j}}{(k-j)!}\frac{(1-\alpha)^j}{j!}\right] = \frac{\alpha}{(k-1)!},$$

the expression simplifies to

$$\zeta^k_{HH} = \left(\frac{\alpha}{k(k-1)!}\right)k! = \alpha\ \forall k. \tag{4}$$

[5]Notice that there is no reason why p and ϕ should be equal in general, and indeed they would differ if either group of workers did not network with probability 1. (See Casella and Hanaki 2005).

If we define ζ_{ij}^k as the probability that an old worker of type i having k connections and making a random referral will choose a young worker of type j, the same procedure allows us to derive immediately that $\zeta_{HH}^k = \zeta_{LL}^k = \alpha$, and $\zeta_{HL}^k = \zeta_{LH}^k = 1 - \alpha$ for all k.

It is the simple characterization of these probabilities that makes the model tractable. The network, random but with a bias, combines the simplicity of purely random networks with the substantive concerns raised by in-breeding and selectivity.[6]

4 The Equilibria of the Model

Given our focus on equilibria where all workers establish personal connections, the only decision workers have to take is whether to attempt certification if they are not hired through referrals. The firms, on the other hand, have to decide whether or not to attempt to hire through referrals of their current employee, and if so, what wage to offer. Each worker i's strategy is the probability with which he chooses to signal, s_i, while each firm j's strategy is the probability with which the firm chooses to hire through referrals (conditional on its current employee being connected to at least one young worker) r_j and the referral wage w_{rj}. We focus on symmetrical equilibria where all workers of the same type and all firms employing the same type of workers follow the same strategy. In addition, given the stationarity of our set-up, we restrict attention to stationary strategies that remain unchanged over time.[7] If we use the terminology " $\forall i \in H$" to indicate all workers of type H, and "$\forall j \in H$" to indicate all firms employing workers of type H (and similarly for L), then: $s_{Hi} = s_H$ $\forall i \in H$, $s_{Li} = s_L$ $\forall i \in L$, $r_{Hj} = r_H$ and $w_{rHj} = w_{rH}$ $\forall j \in H$, $r_{Lj} = r_L$ and $w_{rLj} = w_{rL} \forall j \in L$. We neglect the time subscript to emphasize that these strategies hold for all times. Call w_C and w_U the wage for certified and uncertified workers in the anonymous market. An equilibrium is a set

[6] An interesting related literature studies the economic implications of different network architectures and thus different paths for the transmission of information. Calvo-Armengol and Jackson (2004) and Tassier and Menczer (2006) are examples focused on labor markets.

[7] All workers decide whether or not to signal only once, in the second and last period of their lives. Firms on the other hand are infinitely-lived and must decide whether or not to hire through referrals every period. By restricting attention to stationary strategies, we reduce the set of strategies for a firm of each type to a single probability, ignoring time.

of strategies $\{s_H,\ s_L,\ r_H,\ r_L,\ w_{rH},\ w_{rL}\}$, a pair of market clearing wages $\{w_C,\ w_U\}$, and a set of beliefs about the workers' types such that no worker and no firm can strictly gain from choosing a strategy different from that assigned to his or its type; the labor market clears, and all beliefs are rational.

Consider a scenario where firms extend referral offers with probabilities r_H and r_L, and workers who have not been hired through referrals attempt certification with probabilities s_H and s_L. For generic values of these probabilities, we can derive equilibrium wages and firms' profits. Different values of $\{s_H,\ s_L,\ r_H,\ r_L\}$ can then be posited, and the appropriate incentive compatibility constraints will identify the range of parameter values for which the positited strategies constitute an equilibrium.

Examine the problem first from the perspective of the firms. The workers they hire in any given period are valuable both because of their own productivity and because of their ties to younger workers in the future which will enable the firm to hire through referrals, if advantageous. Call V_H the value of hiring a H worker, and Π_H the firm's expected profits from referrals from a current H employee (and similarly for V_L and Π_L). Then

$$\begin{aligned} V_H &= 1 + \max\{0, \delta\Pi_H\} \\ V_L &= \max\{0, \delta\Pi_L\} \end{aligned} \tag{5}$$

where δ is the rate with which expected profits in the next hiring cycle are discounted. Keeping in mind that profits from hiring in the market must be zero, expected profits from referrals must equal the probability of hiring a worker whose value, combining productivity and future referrals, is larger than the referrals wage. If we call h_{LH} the probability of hiring a L worker through referrals from a current H employee (and similarly for the other types),

then expected profits from referrals are

$$\begin{aligned}\Pi_H &= h_{HH}(V_H - w_{rH}) + h_{LH}(V_L - w_{rH}) \\ \Pi_L &= h_{HL}(V_H - w_{rL}) + h_{LL}(V_L - w_{rL})\end{aligned} \tag{6}$$

where the results of the previous section imply

$$\begin{aligned}h_{HH} &= h_{LL} = \alpha\phi = \alpha(1 - e^{-1}) \\ h_{LH} &= h_{HL} = (1-\alpha)\phi = (1-\alpha)(1 - e^{-1}).\end{aligned} \tag{7}$$

Any firm that attempts to hire through referrals offers a wage that must be weakly smaller than the expected value of the referral. The wage must be acceptable to a H worker; indeed, it must be the lowest wage acceptable to a H worker and it must be identical for H and L firms: $w_{rH} = w_{rL} \equiv w_r$.[8] When some of the workers attempt certification, two different markets exist, a market for certified workers that clears at wage w_C, and a market for uncertified workers that clears at wage w_U. The extent to which the referral wage reflects the wage for uncertified or certified workers (net of certification costs) depends on the strategy followed by H workers left in the market. Since a H worker attempting certification is successful with probability β, the lowest referral wage he would accept must be

$$w_r = \begin{cases} \beta w_C + (1-\beta)w_U - \lambda & \text{if } s_H = 1 \\ w_U & \text{otherwise.} \end{cases} \tag{8}$$

The market wages reflect the probabilities and the values of hiring workers of either type.

[8]Because several young workers may be competing for the same referral offer, the model gives all bargaining power to the firms. However, allowing workers to share in the expected profits from referrals leaves the logic of the model unchanged, beyond the predictable shift in distribution See Casella and Hanaki.

Even when L firms choose to use referrals, and thus $V_L > 0$, they must be driven by the goal of hiring a H worker since L workers are not productive. If workers shared in the surplus generated by referral hiring, the wages offered by H and L firms would both be higher than the minimum acceptable wage for a H worker but they would differ, reflecting the different probabilities of hiring H workers and thus the different expected surplus.

Define h_{HU} as the probability of hiring a H worker in the market for uncertified workers (and similarly h_{LU}), and h_{HC} and h_{LC} as the corresponding probabilities in the market for certified workers. Then, keeping in mind that expected profits when hiring in the market are zero,

$$w_U = h_{HU}V_H + h_{LU}V_L = h_{HU}V_H + (1 - h_{HU})V_L = h_{HU}(V_H - V_L) + V_L$$
$$w_C = h_{HC}V_H + h_{LC}V_L = h_{HC}V_H + (1 - h_{HC})V_L = h_{HC}(V_H - V_L) + V_L.$$

The probability of hiring workers of either type in the two markets is given by the relative frequencies of such workers. For example, if we define $\mu_{C|H}$ ($\mu_{C|L}$) as the probability of being in the market for certified workers conditional on being a H type (a L type), then

$$h_{HC} = \frac{\mu_{C|H}(1/2)}{\mu_{C|H}(1/2) + \mu_{C|L}(1/2)}$$

where 1/2 is the unconditional frequency of H and L types in the population of young workers. Note that $\mu_{C|H}$ is the joint probability of not having been hired through referrals and being certified, conditional on being a H worker (and correspondingly for $\mu_{C|L}$). Thus,

$$\mu_{C|H} = [1 - r_H\alpha p - r_L(1-\alpha)p](s_H\beta)$$
$$\mu_{C|L} = [1 - r_H(1-\alpha)p - r_L\alpha p]s_L(1-\beta).$$

Therefore,

$$w_C = \frac{[1 - r_H\alpha p - r_L(1-\alpha)p](s_H\beta)(V_H - V_L)}{[1 - r_H\alpha p - r_L(1-\alpha)p](s_H\beta) + [1 - r_H(1-\alpha)p - r_L\alpha p]s_L(1-\beta)} + V_L. \quad (9)$$

Similarly,

$$h_{HU} = \frac{\mu_{U|H}(1/2)}{\mu_{U|H}(1/2) + \mu_{U|L}(1/2)}$$

and

$$\mu_{U|H} = [1 - r_H \alpha p - r_L(1-\alpha)p](1 - s_H \beta)$$
$$\mu_{U|L} = [1 - r_H(1-\alpha)p - r_L \alpha p][1 - s_L(1-\beta)].$$

Therefore,

$$w_U = \frac{[1 - r_H \alpha p - r_L(1-\alpha)p](1 - s_H\beta)(V_H - V_L)}{[1 - r_H \alpha p - r_L(1-\alpha)p](1 - s_H\beta) + [1 - r_H(1-\alpha)p - r_L\alpha p][1 - s_L(1-\beta)]} + V_L. \quad (10)$$

(Recall that $p = 1 - e^{-1}$).

We can now write the incentive compatibility constraints for firms and workers. Firms will use referrals only if it profitable to do so, or, taking (6) and (7) into account,

$$\begin{aligned} r_H > 0 \Leftrightarrow \Pi_H = (1 - e^{-1})[\alpha V_H + (1-\alpha)V_L - w_r] \geq 0 \\ r_L > 0 \Leftrightarrow \Pi_L = (1 - e^{-1})[(1-\alpha)V_H + \alpha V_L - w_r] \geq 0. \end{aligned} \quad (11)$$

Workers will attempt certification if the cost of doing so is compensated by the difference in the wages, or, given the different probabilities of success for H and L workers,

$$\begin{aligned} s_H > 0 \Leftrightarrow \beta w_C + (1-\beta)w_U - \lambda \geq w_U \\ s_L > 0 \Leftrightarrow (1-\beta)w_C + \beta w_U - \lambda \geq w_U. \end{aligned} \quad (12)$$

The characterization of the economy is complete: the three different wages are given by (8), (9) and (10), the firms' profits from referrals by (6), the value to the firm of hiring a worker of either type by (5), and finally the incentive compatibility constraints by (11) and (12). If all incentive compatibility constraints hold with strict inequality, the equilibrium is in pure strategies, and the probabilities $\{r_H, r_L, s_H, s_L\}$ assume only 0 or 1 values; otherwise mixed strategies are possible.

As described earlier, once a candidate set of equilibrium strategies is posited, the incentive compatibility constraints, evaluated at the correct wages, identify the range of parameter values for which the strategies are indeed an equilibrium, if one such range exists. Different equilibria can exist for different parameter values, or indeed multiple equilibria can occur over the same range of parameters. The number of candidate equilibria is large, but some combinations of strategies can be ruled out ex ante. We can state,

Lemma 1. *(i) If* $s_H = s_L = 0$*, then* $r_H = 1$ *and* $r_L = 0$*; (ii) If* $r_L > 0$*, then* $r_H = 1$*; (iii) If* $s_L > 0$*, then* $s_H = 1$*; (iv) if* $s_H \in (0, 1)$*, then* $r_H = 1$.

The lemma is proved in the Appendix,[9] but the logic behind the four observations is not difficult to see. The first point states that in the absence of signaling H firms strictly benefit from hiring through referrals, while L firms do not. The reason is that, absent signaling, H firms have a higher probability of hiring a productive worker through referrals than in the market, while the opposite is true for L firms.[10] The second point follows immediately from the fact that it is always better to hire a H worker than a L worker; since the probability of doing so through referrals is always higher for H firms, the incentive to use referrals must always be strictly higher for H firms. Similarly, if any worker incurs the positive costs of attempting certification, the market wage for certified workers must always be strictly higher than the wage for uncertified workers, and since the probability of success, upon attempting certification, is always higher for H workers, the incentive to signal must be strictly higher for H workers. Finally, we know from (i) that H firms always rely on referrals in the absence of signaling. If $s_H \in (0, 1)$, then $s_L = 0$ (by (iii)), and firms can offer as referral wage the market wage for uncertified workers. Because some H workers, and only some H workers, exit the uncertified workers pool, the referral wage is lower than in the case of no signaling, while the expected productivity of a referral hire for H firms remains constant at α. If referrals were

[9] Available at (website version address)

[10] The hiring decisions of H firms are strategic complements. Given that other H firms hire through referrals, and L firms do not, $h_{HU} < 1/2 < \alpha$. Thus a H firm benefits from using referrals. For L firms, it is not difficult to verify that in this scenario $h_{HU} > (1 - \alpha)$ (see the Appendix available at (website version address)). Thus it is preferable to hire in the market.

profitable in the absence of signaling, they must be profitable when $s_H \in (0, 1)$.

All remaining combinations of pure and mixed strategies could be analyzed systematically, and the range of parameter values supporting them identified, but given the equations derived above, the procedure is mechanical and rather tedious. We limit ourselves to two examples.

4.1 Example 1. Equilibrium with referrals and no signaling.

The workers' choice not to signal can be supported in equilibrium for all parameter values: given $\lambda > 0$, off-equilibrium beliefs on the part of the firms according to which anybody deviating must be an L type are sufficient to induce rational workers not to signal, independently of their type. Lemma 1 states that in such a scenario, if an equilibrium exists it must be such that H firms use referrals while L firms do not. The question then is the identification of the range of parameter values, if any, for which these strategies are an equilibrium. We can state,

Lemma 2. *For all $\alpha > 1/2$, $\lambda > 0$, and $\delta \in [0, 1]$, there exists an equilibrium where $s_H = s_L = 0$, $r_H = 1$ and $r_L = 0$.*

Proof. In principle, a firm can envision any deviating path, extending into the future: a once-for-all deviation where the firm takes as given its own future strategy, a permanent deviation where the firm envisions changing its strategy forever, or indeed any repeated deviation over any subset of future periods, and this for either of the firm's future types. In our stationary environment, however (where all other agents repeat the same strategy and a single deviating firm is negligible), the gain from repeated deviations is only the appropriately discounted sum of the gain from a once-for-all deviation. Ruling out the latter is thus sufficient to rule out any other pattern of deviation.[11]

[11] The question arises because a firm's profits from deviation depend on the firm's own future expected strategies; the value of making referrals, which affect today's profit from referrals, depends on the firm's future strategies. This leads to questioning whether a repeated or a permanent deviation could be more advantageous that a once-for-all deviation. It is possible to show, however, that this is never the case, and the conclusion in the text holds.

Consider first the scope for deviation by a H firm (the gain from foregoing referrals). The firm will deviate if current expected profits from referrals are negative, taking as given its future strategy. Along the candidate equilibrium path, $V_L = 0$ and the individual firm's deviation has no effect on the market wage, and thus on the referral wage. Deviation is profitable if $\Pi_H = (1 - e^{-1})(\alpha V_H - w_r) < 0$, or substituting $s_H = s_L = r_L = 0$, $r_H = 1$ and $p = 1 - e^{-1}$ in (10), if

$$(1 - e^{-1}) \left(\alpha - \frac{e - \alpha(e-1)}{e+1} \right) V_H < 0.$$

But $V_H \geq 1$ and the second parenthesis is positive for all $\alpha > 1/2$. Deviation cannot be profitable.

Consider now the gain from deviation for a L firm (the gain from switching to using referrals). Again, given $V_L = 0$ along the candidate equilibrium path, deviation is profitable if

$$(1 - e^{-1}) \left((1 - \alpha) - \frac{e - \alpha(e-1)}{e+1} \right) V_H > 0,$$

a condition that is violated for all $\alpha > 1/2$. Deviation by either type of firm is then ruled out.□

This is the equilibrium discussed by Montgomery in his original paper. In the absence of signaling, it is the unique equilibrium of the model (as implied by Lemma 1): H firms always attempt to hire through referrals and L firms never do. Expected profits are zero for L firms, who hire on the open market, but are positive for H firms and equal to the expected profits from referral hiring. From (6),

$$\Pi_H = \frac{(e-1)(2\alpha - 1)}{(1+e) - \delta(e-1)(2\alpha - 1)}.$$

The value of hiring a H worker is given by

$$V_H = \frac{1+e}{(1+e) - \delta(e-1)(2\alpha - 1)},$$

and the wage by

$$w_U = w_r = \frac{e - \alpha(e-1)}{(1+e) - \delta(e-1)(2\alpha - 1)}.$$

As expected, the value of a H worker is higher than his personal productivity ($V_H > 1$) for all $\alpha > 1/2$ because employing a H worker leads to a higher than random chance of hiring a H worker in the following period. This effect is more important the higher is α (the higher the probability that connected agents are of the same type) and the higher is δ (the less the future is discounted). The profit from referral hiring accrues entirely to the firm and is given by Π_H, an expression increasing in α and δ, like V_H and for the same reasons. The wage, on the other hand, is declining in α; it would equal 1/2 if α equalled 1/2 because referral hiring by H firms would then not affect the average productivity of the market pool, but its value falls monotonically at higher α values, reflecting the increased selection of young H workers out of the market. Because the wage reflects not only the probability but also the value of hiring a H worker in the market, it is increasing in δ; it is higher the less the future is discounted. Note that the expected wage is identical for H and L workers, a consequence of the model granting all bargaining power to the firm when hiring through referrals and a conclusion that would be easily reversed in favor of a referral premium if referred workers shared in the surplus.[12]

4.2 Example 2. Equilibrium with signaling and no referrals.

Can signaling support an equilibrium where referrals are not advantageous? Lemma 3 provides the answer:

Lemma 3. *For all $\alpha > 1/2$, $\delta \in [0,1]$, there exists an equilibrium where $s_H = 1$, $s_L = 0$, $r_H = 0$ and $r_L = 0$ if $\beta \geq 2\alpha/(1+\alpha)$ and $(1-\beta)/(2-\beta) \leq \lambda \leq \beta/(2-\beta)$.*

[12] If workers hired through referrals had more bargaining power than our model grants them, equilibrium strategies would be unchanged, but their own wage would be higher than the market wage. The market wage would be lower, reflecting the lower value to the firm of hiring a H worker, but would again depend negatively on α.

Proof. In this scenario, all certified workers must necessarily be H types and firms can, if they so choose, guarantee themselves a new H worker.[13] In the candidate equilibrium, no profits are available to firms though referral hirings, and $V_H = 1$ and $V_L = 0$. The wages for certified and uncertified workers are given by (9) and (10) above. Because all certified workers are of type H, their value is fully reflected in the market wage, $w_C = 1$, and $w_U = (1-\beta)/(2-\beta)$. The incentive compatibility constraints for the workers (12) impose a first set of constraints on parameters:

$$\begin{aligned} s_H = 1 &\Rightarrow \frac{\beta}{2-\beta} \geq \lambda \\ s_L = 0 &\Rightarrow \frac{1-\beta}{2-\beta} \leq \lambda. \end{aligned} \tag{13}$$

With $\beta > 1/2$, there is a non-empty range of λ values for which the two constraints can both be satisfied.

Consider the potential for deviation by the firms. Any firm willing to hire through referrals would need to offer a wage acceptable to H workers, who prefer to attempt certification. Hence, from (8), $w_r = \beta + (1-\beta)^2/(2-\beta) - \lambda$. Because the incentive to use referrals is always larger for H firms (see Lemma 1), ruling out deviation by a H firm is sufficient to rule out deviation by a L firm. In addition, it is sufficient to focus on once-for-all deviations. A H firm will switch to using referrals if $\Pi_H = (1-e^{-1})[\alpha - w_r] > 0$. Thus equilibrium requires

$$r_H = 0 \Rightarrow \lambda \leq \beta + \frac{(1-\beta)^2}{2-\beta} - \alpha = \frac{1-2\alpha+\beta\alpha}{2-\beta}. \tag{14}$$

Because (13) and (14) must both be satisfied, this equilibrium exists if $(1-\beta) \leq (1-2\alpha+\beta\alpha)$, or $\beta \geq 2\alpha/(1+\alpha)$.□

Thus for a suitable range of parameter values, the candidate equilibrium indeed exists; signaling provides separation of the workers' types, in the limited sense that all certified

[13] The only other candidate equilibrium where all certified workers are guaranteed to be of H type has strategies $s_H \in (0,1)$ and $s_L = 0$. But then we know by Lemma 1 that $r_H = 1$, if an equilibrium exists, referrals must take place.

workers must be of type H, and firms refrain from using referrals and prefer to hire in the market, an option they would not take in the absence of signaling. Signaling provides information and supplants the use of personal referrals. Notice that hiring in either market, for certified or uncertified workers, leads to zero firm profits, and thus firms are indifferent between paying the premium attached to certification and guaranteeing themselves a H worker, or hiring a cheaper uncertified worker with a lower but positive probability of being a H type. But hiring an uncertified worker in this equilibrium is not equivalent to hiring an uncertified worker when no signaling takes place; the more precise information reflected in the wage makes this option superior to referrals in this equilibrium but inferior when no signaling takes place.

Notice however that the range of parameter values for which the equilibrium exists is surprisingly limited. Since $\alpha > 1/2$, β cannot be smaller than 2/3, and λ must fall in a correspondingly narrow interval. Interestingly, the tight range of acceptable parameter values is mostly dictated by the incentive compatibility constraints ensuring that firms refrain from making referrals. In their absence, all values of $\beta > 1/2$ could sustain the equilibrium (although the range of acceptable λ values would remain quite small). This suggests two conjectures: first it seems likely that other equilibria exist, for ranges of parameter values that may differ from or overlap the range identified here, where signaling provides separation between the workers' types but firms still choose to make referral offers,[14] and second, that the firms' incentive compatibility constraints select values of λ close to the lower bound of the interval satisfying the workers' incentive constraints. A higher λ would induce firms to deviate. This is an interesting point, beginning to suggest the resilience of referrals in our model; when β is not very high, signaling can be informative only if λ is high enough to induce separation of workers' types, but if λ is high, signaling is informative but also expensive since referral hiring can undercut it and be less precise but preferable. We make

[14]The existence of such equilibria does not follow immediately from a violation of the firms' incentive compatibility constraints in the candidate equilibrium studied. The composition of the markets, the wages and thus all constraints would reflect the different firms' strategies.

this conjecture more precise in the next section.

5 Reliance on Personal Connections when Signaling is Informative.

Having verified that signaling can come to eliminate the recourse to personal referrals, we now focus on the opposite question. Under what conditions is the availability of a signaling mechanism compatible with continued reliance on referrals? And if reliance on referrals continues to exist, does signaling affect referrals indirectly, by influencing the quality of referral hiring (i.e. the percentage of referrals falling on workers of different type)?

Some scenarios are trivial. If the cost of signaling λ is very low, and the probability of success β not very different across types (β close to 1/2), then all workers may choose to signal, but certification is uninformative and H firms continue to prefer hiring through referrals (and the more so the higher is α). The question becomes interesting if we restrict attention to equilibria where signaling is informative, and we use the strict criterion that only H workers attempt to signal ($s_H > 0$, $s_L = 0$) because in these equilibria signaling is informationally superior to referrals. The first result is summarized in the following proposition:

Proposition 1. *For all $\alpha \in (1/2, 1)$, there exist equilibria where signaling is perfectly informative and firms strictly prefer to hire through referrals.*

All the proofs of the results in this section can be found in the Appendix available at (website version address). In the text we summarize the intuition behind the results. The proof of Proposition 1 amounts to showing the existence of one equilibrium where signaling is perfectly informative and firms strictly prefer to hire through referrals. In particular, we focus on the case $\beta = \alpha$ and on strategies $\{s_H = 1, s_L = 0, r_H = 1, r_L = 1\}$ (H workers in the market prefer to signal, L workers do not, and all firms prefer to hire through referrals) and show that for all $\alpha \in (1/2, 1)$ there exists a non-empty range of λ values for which such strategies are indeed equilibrium strategies. The case $\beta = \alpha$ is chosen primarily because

it simplifies the algebra and the presentation of the results, but is also a natural reference point: the exogenous precision of the personal connections equals the exogenous precision of the signal. The assumption does not imply that personal connections and signaling are constrained to be equally informative because the signaling mechanism has one additional element, the cost λ, that leads to self-selection in the decision to engage in signaling, hence the possibility that signaling be perfectly informative even for β very close to 1/2, as in the equilibrium described here. Because we are evaluating equilibria over the whole range of possible λ values, the restriction $\beta = \alpha$ is compatible with a large number of scenarios while leaving the model ex ante unbiased. We will use it repeatedly as our reference case.

Although the proof selects one particular example, in fact there are several equilibrium regimes where signaling is informative and firms prefer to use referrals: equilibria where only H firms use referrals, or where a share of them do so while L firms hire in the market, or where all H firms and a share of L use referrals, or equilibria where only a share of H workers signal while L workers do not. The range of parameter values for which referral hiring takes place and is imprecise, while firms could guarantee themselves H workers, is far from limited or special. Figure 1 shows equilibrium strategies in $\lambda - \alpha$ space in the case $\beta = \alpha$ and $\delta = 0.90$. The model is obviously very stylized, but we can read parameter values keeping in mind that the unit of time is the hiring cycle, or more precisely the length of employment of a worker at a single firm. The difference in productivity between a productive and unproductive worker over that cycle is normalized to 1. Thus if we think of the time horizon as about five years, $\delta = 0.90$ corresponds to a yearly discount rate of 2 percent.[15] The parameter λ, the fixed cost of certification, can be thought of as the cost of further education, for example college, and should be read relative to 1; $\lambda = 1$ in our model represents the case where the cost of college education equals the total difference in productivity between a productive and a non-productive worker over five years. As we saw, in the presence of referrals the value of a worker to the firm includes the value of future referral hiring, and thus the premium that

[15] Our qualitative results are effectively insensitive to the specific value we assign to δ, for all $\delta \in [0, 1]$.

firms may be willing to pay to college educated workers may be well above the one-cycle difference in productivity, implying in turn that acceptable values of λ in our model may well be above 1.

Figure 1a depicts workers' strategies, and figure 1b firms' strategies. As mentioned earlier, there is always an equilibrium without signaling, supported by firms' negative off-equilibrium beliefs. In Figure 1a, we have allowed signaling to take place whenever it can be supported in equilibrium with rational beliefs.[16]

The figures show that there is a large area of the parameter space for which signaling takes place and is fully informative. Workers signal when the cost λ is not too large. If the precision of the signal (β, which in the figure equals α) is high, only H workers signal, and they do so for a large range of λ values; if on the contrary β is low, then signaling can occur only if λ is low, and for most of these values signaling is not informative because the incentives to signal are very similar for H and L workers.[17] Unless β is high, informative signaling requires intermediate values for λ, low enough to be affordable by H workers, but high enough to discourage L workers. As for referrals, the immediate observation from Figure 1b is that there is no equilibrium where firms do not use referrals: over the entire parameters range, referrals never cease to be profitable for firms, whether signaling is informative or not, whether λ is high or low, whether α and β are high or low. The figure relies on $\alpha = \beta$, and thus, as made clear by the equilibrium with no referrals characterized in the previous section, the result is not general. However given our emphasis on the $\alpha = \beta$ case as plausible unbiased reference, we emphasize this conclusion in a separate proposition:

Proposition 2. *Suppose $\alpha = \beta$. Then there exist equilibria where signaling is perfectly informative, but there are no equilibria where referrals are not used.*

[16] No signaling is the unique equilibrium in the area left white: a worker of either type would not want to attempt certification even if firms, off-equilibrium, expected any certified worker to be of type H. A second equilibrium, with $s_H \in (0, 1)$ and $s_L = 0$, exists in the region bordering the upward sloping frontier between the white and gray areas in Figure 1a. The figure omits it for ease of reading.

[17] When β is close to 1/2, the probability of success upon signaling is very similar for H and L workers, but the wage premium for certified workers is small. At high β, the premium is high but the probability of success for a L worker is low. The incentive to signal for L workers is maximal for intermediate values of β.

A moment of reflection shows that the reason is obvious: unless β is high, when signaling is informative it is also rather expensive (λ is high), expensive enough to allow H workers to differentiate themselves. Market wages for certified workers compensate them for the cost of certification, and thus tend to be high. Firms may well prefer the less informative but cheaper reliance on referrals. In the trade-off between information and cost, cost becomes the deciding factor. The conclusion always holds when $\alpha = \beta$ because informative signaling can be cheap only when its precision is high, but if the precision of referral hirings is also correspondingly high, then the cost advantage, small at it may be, remains the dominant consideration.

The firms' equilibrium strategies are particularly interesting when β (and α) are low. There is then a range of parameter values where, if signaling occurs in equilibrium, *all* firms hire through referrals (the black area in figure 1b), including L firms, which never use referrals in the absence of signaling because referrals give them a more than even chance of hiring an unproductive worker. The observation deserves emphasizing in a separate remark.

Remark. *Signaling, including informative signaling, may induce L firms to hire through referrals.*

Once again, the reason is the market premium for certified workers. Indeed, Figure 1b makes clear that such an equilibrium occurs when, for each α and β, λ approaches its highest acceptable value. Because certified workers must be compensated for the cost of signaling, the areas where L firms use referrals correspond to those ranges of parameter values where the certification premium is highest. When $\alpha = \beta$, L firms' incentive to use referrals is maximal when α is close to $1/2$: the bias inherent in referral hiring is not too strong, while referrals allow firms to avoid the premium for certified workers and still hire workers of higher the average productivity than in the uncertified market.

What is remarkable about this equilibrium is that the existence of signaling, and in particular of informative signaling, leads to an *increase* in firms' reliance on referrals. In fact, if $\alpha = \beta$ signaling always weakly increases the expected proportion of workers hired

through referrals; either referral hiring is used by H firms alone, as in the absence of signaling, or it is used by both H firms and some positive share of L firms (possibly all).

We can now address a question we raised at the beginning of this section: how does signaling affect the quality of referral hiring? The answer is an immediate implication of our previous remark.

Implication. *Signaling always weakly lowers the quality of referral hirings.*

Since only H firms recur to referrals in the absence of signaling, any expansion of referral hiring to L firms necessarily lowers the average productivity of workers hired through referrals. Paradoxically then, signaling not only need not eliminate referrals, but in fact may lead to increased reliance on personal connections, and lower expected productivity of referral hires.

We have phrased our discussion mostly in terms of the $\alpha = \beta$ case, but the results generalize predictably when we loosen this constraint. To generate transparent figures, we can specialize the model in a different direction. Figure 2 describes equilibrium strategies for workers (figure 2a) and firms (figure 2b) for arbitrary values of β and λ when we fix α at 0.75 (and δ at 0.90, as earlier).[18] Figure 2a is very similar to figure 1a; signaling can occur in equilibrium only if the cost λ is not too high, where the highest acceptable λ increases with β, but if λ is low, both types of workers have an incentive to signal.[19] The difference is in figure 2b. First, of all, there are now equilibria without referrals, when β is high (higher than α) and λ low, the area left white in the lower right corner of the figure. This is the equilibrium with no referrals and informative signaling discussed in section 4.2, and a second equilibrium with no referrals and all workers signaling when λ is particularly low. The low

[18] As in Figure 1a, a second equilibrium, with $s_H \in (0, 1)$ and $s_L = 0$, exists in the region bordering the upward sloping frontier between the white and gray areas in Figure 2a. Again the figure omits it for ease of reading.

[19] The sensitivity of the acceptable λ range to β is reduced, relative to the case where $\alpha = \beta$. At low β but constant α (and H firms only using referrals), equilibrium market wages are lower than they would be if α equalled β, keeping certification valuable and raising the highest acceptable λ. At high β, the effect is reversed: with constant α, smaller than β, market wages for uncertified workers are now higher and the incentive to signal is reduced, reducing in turn the highest acceptable λ.

λ makes the firms' savings from referral hirings negligible, and the high β makes signaling a good channel of information, even in the absence of full separation.[20] It is clear from the figure though that even in the area where $\beta > \alpha$ ($\beta > 0.75$), the absence of referrals occurs in a small subset of the parameter space. Second, the equilibrium where both H and L firms rely on referrals now is concentrated mostly in an area of high β and λ values. With $\alpha = 0.75$, L firms are discouraged from using referrals unless the savings from doing so are substantial: a range of high λ and β where signaling still occurs, the certification premium is high, and the composition of the uncertified labor pool worse than when hiring through referrals.

We can now provide a summary answer to our original question: does the availability of signaling lead firms to forego referrals? More precisely, how large is the region of the parameters space where equilibria without referrals can be supported? Figure 3 shows the full three-dimensional picture, for α and β between 1/2 and 1, and λ between 0 and 2.5, with $\delta = 0.90$.[21] On the left-hand side, in grey, is the space where referrals do not take place, as a function of the three parameters. Notice the two requirements $\beta > \alpha$ and, simultaneously, λ small. For comparison, the right-hand side shows the parameters space where signaling takes place in equilibrium, a much larger fraction of the total space. The darker shade of grey shows, in both cases, regions where signaling is exclusive to H workers; in the majority of this space referrals continue to be employed.

A final but important note of caution. Throughout our discussion we have emphasized how small is the range of parameters for which referrals cannot occur in equilibrium relative to the full admissible space, but it is true of course that not all combinations of parameters need be equally likely in reality. A more precise and more constructive reading of our results then is as an invitation to empirical scholars, emphasizing the very specific requirements on

[20] When only H workers signal, all certified workers must be H types, but the composition of the uncertified labor pool and the difference in wages still depend on β. Similarly, when all workers signal, some L workers are expected to become certified, but the composition of the two labor pools, and the wages, remain significantly different if β is high enough.

[21] As mentioned earlier, results show little sensitivity to δ.

parameters that are needed for signaling to supplant reliance on personal connections.

6 Discussion

We can ask other questions. For example, what is the effect of making certification less expensive (reducing λ)? Will personal connections become less important in labor markets? According to our model, the answer is not immediate because it depends both on the precision of the information inherent in the personal network and on the standards used in certification. Lowering certification costs reduces the self-selection that limits L workers' attempts to signal, and thus, ceteris paribus, reduces the information conveyed by being certified. Only when the signal is in itself precise enough, relative to the network, to limit the importance of this effect (when $\beta >> \alpha$) does the cheap availability of public information reduce the reliance on personal connections. The opposite option of making certification *more* expensive (increasing λ), relying then on the information provided by self-selection, never reduces the use of personal referrals. On the contrary, it tends to increase the use of referrals, even when they are known to lead to a less than average probability of hiring productive workers. Similarly, what is the effect of increasing the standards for certification (increasing β)? Again, the answer depends on other factors too. Only when certification is both rigorous *and* cheap does it become a preferred option to personal referrals. We can also interpret the parameter α as correlated to the openness of a society; the more open and diverse, the lower the "in-breeding bias" and the less predictable the "quality" of a personal referral. Not surprisingly, the model tells that at high α, in a predictable and segregated society, it is very difficult to substitute personal referrals with signaling; the information transmitted through personal connections is just too cheap and too accurate. But as the society becomes more open, diverse and mixed, that value of social contacts falls. With appropriate institutions (with high β and low λ) signaling can emerge.

A more ambitious task is linking the results of our model to the regularities that have

emerged from the empirical literature on the use of social contacts in labor markets.[22] The least controversial observation is the negative correlation between the use of social ties in finding employment and the workers' education level (for example Staiger for the U.S., and Pellizzari 2003 for European countries), an observation that fits well the story told in this model: incomplete information prevents the firms' full knowledge of workers' characteristics, and referrals and education are two alternative means of partly revealing such characteristics. Referrals and education are substitutes, as they are in our model where workers hired through referrals can forego the step of obtaining certification. In a particularly detailed data set on Italian college graduates, a more specific finding is the reduced reliance on social ties in professions like engineering and medicine, relative to social sciences or literature.[23] According to our model, the difference can be read in terms of β: either because the standards are tougher or because proficiency is more precisely defined, in medicine and engineering certification is a more precise measure of skills (β is higher) than in social sciences or literature. At low λ (at low cost of public education in Italy), the difference has a direct impact on the reliance on social ties.

The empirical finding that has received most attention and has been most disputed is the existence of a systematic premium paid to workers hired through referrals. A positive premium was found first by the U.S. studies (for example, Granovetter 1974, Corcoran et al. 1980, Staiger 1990), but has since been disputed. Bentolilla et al. (2003) and Pistaferri (1999) report a negative premium, while Pellizzari (2003) and Santamaria-Garcia (2003) find no systematic relation once workers' and jobs' characteristics are controlled for.[24] Figure 4 depicts the referral wage and the market wages in our model. The top row (figures 4a) reports the referral wage and the average market wage as function of β, on the left, and as function of λ, on the right; the central row (figures 4b) distinguishes between the two market

[22] Again Ioannides and Datcher Loury provide a careful survey.

[23] See Sylos Labini (2004), based on the 1998 ISTAT Survey of Italian college graduates.

[24] Sylos Labini makes the interesting point that not all social ties are equivalent: family ties and professional ties, in particular, may transmit quite different information. In his analysis of Italian data on college graduates, the negative premium is limited to jobs found through family ties.

wages for certified and for uncertified workers, and the bottom row (figures 4c) compares the referral wage to the average market wage of H workers and L workers separately.[25] The discontinuity points correspond to changes in equilibrium strategies, which can be read from Figure 2; at high values of λ multiple equilibria are possible, as shown in all figures on the right-hand side of Figure 4. All the figures support the difficulties found by researchers in identifying a systematic bias. The first row of figures suggests that if the referral wage is simply compared to the average market wage (a not unreasonable comparison in our model where there is only one type of job), the results are ambiguous; the direction of the bias depends on parameters. If these parameters plausibly differ across sectors and/or across countries, the conclusion confirms the heterogeneity across sectors and countries found by the literature. Controlling for the workers' characteristics first appears to yield cleaner results; all referral offers are made to uncertified workers, and the premium, relative to the market wage for uncertified workers only, is unambiguously positive (figures 4b). But certification in our model is an imperfect signal of workers' productivity, and the fact that all referrals are made to uncertified workers is an artifact of the timing of the model. A better analysis would try to control for workers' unobservable characteristics, here their productivity. Figures 4c show that the referral wage has a positive premium for L workers for most parameter values (i.e. it is higher than the market wage they would expect to receive) but has a discount for H workers. Again, as in the literature, the results are not consistent across different groups of workers, with discounts more probable in the case of more productive hires (as reported by Pellizzari, for example). Of course we are aware that the model is extremely stylized and these results, although suggestive, should be taken with care.

[25]The figures should be read keeping in mind that the referral premium would be higher if workers hired through referrals shared in the surplus.

7 Conclusions

This paper has studied the extent to which firms' and workers' reliance on networks of personal connections as channels of information about jobs is weakened by the availability of a signaling mechanism. The signaling mechanism is available to all but is costly, and its outcome is an imperfect signal of the worker's true productivity. The network on the other hand is free and connects a young worker to someone who is currently employed and whose productivity is known by the firm. Because connections are more likely between similar agents, the existence of a tie provides information to the firm about the productivity of the young worker. We have found that although signaling can transmit information more precisely because the cost induces workers to self-select, reliance on the network is very robust; only for a very small fraction of the parameters space of the model do referrals disappear.

The analysis was motivated by the desire to compare the network, possibly successful but exclusionary, to a market mechanism, open to all and anonymous. In fact, to the extent that they convey any information at all, both mechanisms must differentiate between different types of workers, a bias summarized in the model by the two parameters α and β. In what sense then is signaling in our model a "market mechanism" while the network is not? There are two main differences. First, the cost involved in the decision to signal plays a central role in the revelation of information, contrary to the case of free personal connections. Second, the information revealed by the network is local, whereas the information transmitted by the signal is global, a sentence that could be rephrased more precisely by plausibly defining the information as "verifiable" in the signaling case, and "non-verifiable" in the network case. In our model, the implication is that in the market firms compete for the surplus from the revelation of information, but they do not in the network. It is the firms' open entry into the market that at the end fundamentally distinguishes the two mechanisms.

Our analysis relies on the cost advantage enjoyed by personal referrals. The advantage is realistic; "free" connections, emerging naturally from social and family life, are exactly those

singled out by sociologists as irreplaceable by more formal mechanisms. But in societies where "networking" has become an intentional and costly activity, it remains important to see how our conclusions are affected by the possibility of establishing connections at a cost. We pursue this direction in a companion paper (see Casella and Hanaki).

We have not addressed the efficiency of two mechanisms. In the model, referrals affect distribution (distribution of profits between H and L firms, between incumbent and potential entrants, and of wages between different workers) but do not affect efficiency. All workers are always employed and production always equals the total product of all H workers: there is no efficiency rationale to recommend shifting from one mechanism to another.[26] We have chosen this approach because real world objections to preferential networks are rarely based on efficiency; for the most part they stem from the belief that the network is advantageous but exclusionary, and thus plays essentially an "unfair" redistributive purpose. It is this role that is highlighted in the current analysis, although comparing the efficiency of two mechanisms is an interesting direction for future research.

Finally, other market-type mechanisms seem possible and interesting. The most obvious one, closest in spirit to Rauch and to our knowledge neglected by the formal literature on referrals in labor market, is a profit-driven intermediary: an employment agency. How would such an intermediary function in this model? Could it take over successfully the functions fulfilled by the network? Here, too, we leave the answer to future research.

[26]Indeed, strictly speaking, signaling is inferior to referrals because of the lost costs λ. In a full model, costs λ would be paid to an education sector.

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A Appendix

Derivation of p_H in Section 3.

The exact probability that an old worker of type H (of which N exist) has k connections in addition to i's is given by:

$$\widetilde{\gamma}_{k,H} = \sum_{j=0}^{k} \left[\binom{N-1}{c} \left(\frac{\alpha}{N}\right)^c \left(1-\frac{\alpha}{N}\right)^{N-1-c} \binom{N}{d} \left(\frac{1-\alpha}{N}\right)^d \left(1-\frac{1-\alpha}{N}\right)^{N-d} \right] \tag{A1}$$

where

$$c \equiv \min(k-j, N-1)$$
$$d \equiv \min(j, N).$$

The Poisson approximation $\gamma_{k,H}$

$$\gamma_{k,H} = \sum_{j=0}^{k} \left[\frac{\alpha^{k-j}}{(k-j)!} e^{-\alpha} \frac{(1-\alpha)^j}{j!} e^{-(1-a)} \right] = \sum_{j=0}^{k} \left[e^{-1} \frac{\alpha^{k-j}(1-\alpha)^j}{(k-j)!j!} \right] = \frac{e^{-1}}{k!} \tag{A2}$$

is the limit of (A1) as N tends to infinity and has well established bounds of error. It can be shown (Feller 1968, Ch.6) that

$$e^{-\alpha}\frac{\alpha^c}{c!} e^{-\frac{c^2}{N-c}-\frac{\alpha^2}{N-\alpha}} < \binom{N}{c}\left(\frac{\alpha}{N}\right)^c \left(1-\frac{\alpha}{N}\right)^{N-c} < e^{-\alpha}\frac{\alpha^c}{c!} e^{c\frac{\alpha}{N}},$$

implying that

$$\lim_{N\to\infty} \widetilde{\gamma}_{k,H} = \lim_{N\to\infty} \gamma_{k,H}$$

and therefore:

$$\lim_{N\to\infty} \left(1 - \sum_{k=1}^{2N-1} \frac{k}{k+1} \widetilde{\gamma}_{k,H}\right) = \lim_{N\to\infty} \left(1 - \sum_{k=1}^{2N-1} \frac{k}{k+1} \gamma_{k,H}\right)$$

(i.e. the error introduced by the approximation does not distort the sum).

The last part of the derivation in the text states without proof that

$$\lim_{N\to\infty}\left(\sum_{k=1}^{2N-1}\frac{k}{(k+1)!}\right)=1.$$

To see this, begin by expanding e^x:[27]

$$e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}=1+x+\sum_{k=2}^{\infty}\frac{x^k}{k!}=1+x+x\sum_{k=1}^{\infty}\frac{x^k}{(k+1)!}.$$

Differentiating both sides with respect to x,

$$(e^x)'=e^x=1+\sum_{k=1}^{\infty}\frac{x^k}{(k+1)!}+x\sum_{k=1}^{\infty}\frac{kx^k}{(k+1)!},$$

or

$$e^x=1+(e^x-x-1)/x+x\sum_{k=1}^{\infty}\frac{kx^k}{(k+1)!}.$$

Setting $x=1$,

$$e=1+(e-1-1)/1+\sum_{k=1}^{\infty}\frac{k}{(k+1)!},$$

or

$$\sum_{k=1}^{\infty}\frac{k}{(k+1)!}=1.$$

□

Proof of Lemma 1. (i) When $s_H=s_L=0$,we can write

$$\begin{aligned}\Pi_H&=\phi(V_H-V_L)(\alpha-h_{HU})\\ \Pi_L&=\phi(V_H-V_L)(1-\alpha-h_{HU})\end{aligned}\tag{A3}$$

[27]We thank Peter S. Dodds for this proof.

and

$$h_{HU} = \frac{1 - r_H \alpha p - r_L(1-\alpha)p}{2 - r_H p - r_L p}. \tag{A4}$$

We proceed in two stages. First we show that $(V_H - V_L) > 0$ always. Then we show that $\Pi_H > 0$ and $\Pi_L < 0$ always. (1) If $r_L < 1$, $V_L = 0$ and $V_H \geq 1$, hence $(V_H - V_L) > 0$. If $r_L = 1$, there are two possibilities: (a) if $r_H < 1$, then $\Pi_H = 0$ and $V_H = 1$. In this case, $V_L = \delta\phi(1 - \alpha - h_{HU})/(1 - \delta\alpha) < 1$ for all $\delta \leq 1$. Hence $(V_H - V_L) > 0$. (b) If $r_H = 1$ and $\Pi_H > 0$, then, by (5) and (A3) above, $(V_H - V_L) = 1 + \delta\phi(V_H - V_L)(2\alpha - 1) > 0$. Thus we can conclude that in all cases, $(V_H - V_L) > 0$. (2) From (A4), $\partial h_{HU}/\partial r_H < 0$, and $\partial h_{HU}/\partial r_L > 0$. Thus h_{HU} is maximal at $\{r_H = 0,\ r_L = 1\}$ and minimal at $\{r_H = 1,\ r_L = 0\}$: $h_{HU} \in [\underline{h_{HU}}, \overline{h_{HU}}]$, where $\underline{h_{HU}} \equiv (1-\alpha p)/(2-p)$ and $\overline{h_{HU}} \equiv [1-(1-\alpha)p]/(2-p)$. But $\alpha > \overline{h_{HU}}$ and $(1-\alpha) < \underline{h_{HU}}$ for all $\alpha > 1/2$. Since $(V_H - V_L) > 0$, we can then conclude that $\Pi_H > 0$ and $\Pi_L < 0$ always. If an equilibrium exists, it must have $r_H = 1$ and $r_L = 0$.

(ii). Suppose $r_L \in (0,1)$ and $\Pi_L = 0$. Then, by (6), $V_L = 0$, but since $V_H \geq 1$, $V_H > V_L$. By (6) then $\Pi_H > 0$ and $r_H = 1$. Suppose $r_L = 1$ and $\Pi_L > 0$. Could it be that $\Pi_H \leq 0$? If so, $V_H = 1$ and $V_L = \delta(1-e^{-1})(1-\alpha+aV_L-w_r)$, or $V_L = \delta(1-e^{-1})(1-\alpha-w_r)/(1-\delta\alpha) < 1$ for all $\delta \leq 1$ and $w_r \geq 0$. But if $V_H > V_L$ it must be that $\Pi_H > \Pi_L$ by (6), a contradiction. Hence if $r_L = 1$ and $\Pi_L > 0$, $\Pi_H > 0$ and $r_H = 1$.

(iii) By constraints (12), signaling by either type of workers requires $w_C > w_U$ for all $\lambda > 0$. But then, since $\beta > 1/2$, the incentive to signal is always strictly higher for a H worker than a L worker, and the conclusion follows.

(iv) If $s_H \in (0,1)$, then $w_r = w_U = h_{HU}(V_H - V_L) + V_L$ and $\Pi_H = \phi(V_H - V_L)(\alpha - h_{HU})$. By result (iii) in the Lemma, if $s_H \in (0,1)$, then $s_L = 0$, hence

$$h_{HU} = \frac{(1 - r_H \alpha p - r_L(1-\alpha)p)\,(1 - s_H\beta)}{(1 - r_H \alpha p - r_L(1-\alpha)p)\,(1 - s_H\beta) + (1 - r_H(1-\alpha)p - r_L \alpha p)}. \tag{A5}$$

It is then easy to verify that $\partial h_{HU}/\partial s_H < 0$, and since $\alpha > h_{HU}(s_H = 0)$, a fortiori $\alpha > h_{HU}(s_H > 0)$, $\Pi_H > 0$ and $r_H = 1$. $\square$

Proof of Proposition 1. To prove the proposition, it is sufficient to show the existence of some such equilibrium. Consider the candidate scenario $\{s_H = 1,\ s_L = 0,\ r_H = 1,\ r_L = 1\}$, where H workers in the market always signal but all firms always prefer hiring through referrals. Suppose $\beta = \alpha$. (5), (6), (8), (9) and (9) yield V_H, V_L, Π_H, Π_L, w_r, w_U, and w_C. The incentive compatibility constraints (11) and (12) allow us to conclude that such scenario is an equilibrium if and only if $\lambda \in [\underline{\lambda}, \overline{\lambda}]$ where

$$\underline{\lambda} = \max\{\frac{(1-\alpha)(1-h_{UH})}{1-\delta\phi(2\alpha-1)}, \frac{2\alpha-1+(1-\alpha)h_{UH}}{1-\delta\phi(2\alpha-1)}\}$$
$$\overline{\lambda} = \frac{\alpha(1-h_{UH})}{1-\delta\phi(2\alpha-1)}$$

and, by (A5),

$$h_{UH} = \frac{1-\alpha}{2-\alpha}.$$

The unique upper bound is given by the requirement that λ should be low enough for H workers to be willing to signal and the lower bounds by the incentive compatibility constraints on the L workers and the L firms (λ should be high enough for L workers not to signal and L firms to prefer referrals.)[28] It is not difficult to verify that for all $\alpha \in (1/2, 1)$, $\overline{\lambda}$ is larger than either of the two lower bounds (the relevant lower bound is the first for $\alpha \leq 2 - \sqrt{2}$ and the second for higher α). The incentive compatibility constraints are satisfied and the scenario is an equilibrium for all $\alpha \in (1/2, 1)$. □

Proof of Proposition 2. The existence of equilibria where signaling is perfectly informative has been established in Proposition 1. Consider now possible equilibria where referrals are not used. By Lemma 1, if $s_H = s_L = 0$, then $r_H = 1$; if $s_H \in (0, 1)$, then $r_H = 1$, and if $s_L > 0$, then $s_H = 1$. Thus the only workers' strategies compatible with the absence of referrals are $\{s_H = 1,\ s_L = 0\}$, $\{s_H = 1,\ s_L \in (0, 1)\}$, and $\{s_H = 1,\ s_L = 1\}$. The first case is the only one of the three possibilities where signaling is perfectly informative and referrals do

[28] If the incentive compatibility constraint ensuring that the L firms' strategy is a best response is satisfied, the equivalent constraint on H firms is satisfied automatically.

not take place. It was studied in detail in the text, and we showed there that the equilibrium requires $\beta \geq 2\alpha/(1+\alpha) > \alpha \ \forall \alpha \in (1/2, 1)$, and thus is ruled out if $\alpha = \beta$. Proving that the latter two candidate scenarios cannot be equilibria amounts once again to showing that the incentive compatibility constraints must be violated. Deriving such constraints requires working through the appropriate wage and profits equations for each scenario, but given (5), (6), (8), (9) and (9) the derivation of the constraints is trivial, and we leave the details to the reader. In the second scenario, $\{s_H = 1,\ s_L \in (0,1),\ r_H = r_L = 0\}$, the two binding constraints are that L workers must be indifferent about signaling, while H firms must prefer not to use referrals, or

$$\lambda = \frac{(1-\beta)(h_{HC} - h_{HU})}{1 - \delta(1-e^{-1})[\alpha - (2\beta - 1)h_{HC} - 2(1-\beta)h_{UC}]} \tag{A6}$$

$$\lambda < \beta(h_{HC} - h_{HU}) + h_{HU} - \alpha \equiv \overline{\lambda} \tag{A7}$$

where

$$h_{HC} = \frac{\beta}{\beta + (1-\beta)s_L}$$
$$h_{HU} = \frac{1-\beta}{1 + (1-\beta)(1-s_L)}.$$

For any given λ, (A6) identifies the equilibrium s_L, as long as (A7) is satisfied. Substituting $\alpha = \beta$ and (A7) in (A6), we can write

$$\lambda = \frac{(1-\beta)(h_{HC} - h_{HU})}{1 + \delta(1-e^{-1})[(1-\beta)(h_{HC} - h_{HU}) - \overline{\lambda}]}$$

or

$$\overline{\lambda} > \frac{(1-\beta)(h_{HC} - h_{HU})}{1 + \delta(1-e^{-1})[(1-\beta)(h_{HC} - h_{HU}) - \overline{\lambda}]}. \tag{A8}$$

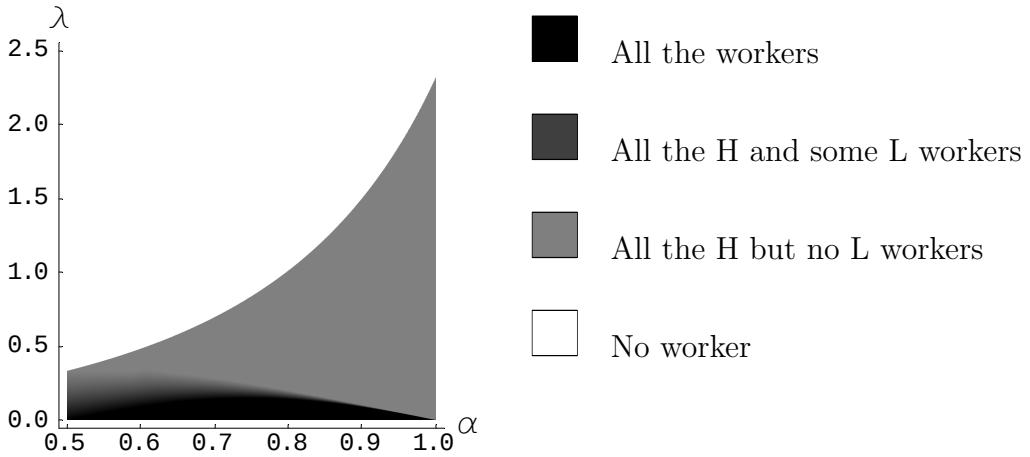
It is not difficult to verify that for all $s_L \in (0,1)$, $(1-\beta)(h_{HC} - h_{HU}) > \overline{\lambda}$. Hence (A8)

implies

$$\delta(1-e^{-1})[(1-\beta)(h_{HC}-h_{HU})-\overline{\lambda}]\overline{\lambda} > [(1-\beta)(h_{HC}-h_{HU})-\overline{\lambda}],$$

a condition that can only be satisfied for $\overline{\lambda} > 1$. But for all $s_L \in (0,1)$, $h_{HC} > h_{HU}$, implying $\overline{\lambda} < h_{HC} - \beta < 1$. The scenario cannot be an equilibrium when $\alpha = \beta$. In the third scenario, $\{s_H = 1,\ s_L = 1,\ r_H = r_L = 0\}$, if H firms do not use referrals it must be that $\lambda < \beta(2\beta - 1) + 1 - \beta - \alpha$. If $\alpha = \beta$ the constraint becomes $\lambda < (\beta - 1)(2\beta - 1) < 0$, which is violated for all $\lambda > 0$, $\beta \in (1/2, 1)$.

(a) Equilibrium Signalling



(b) Equilibrium Referral

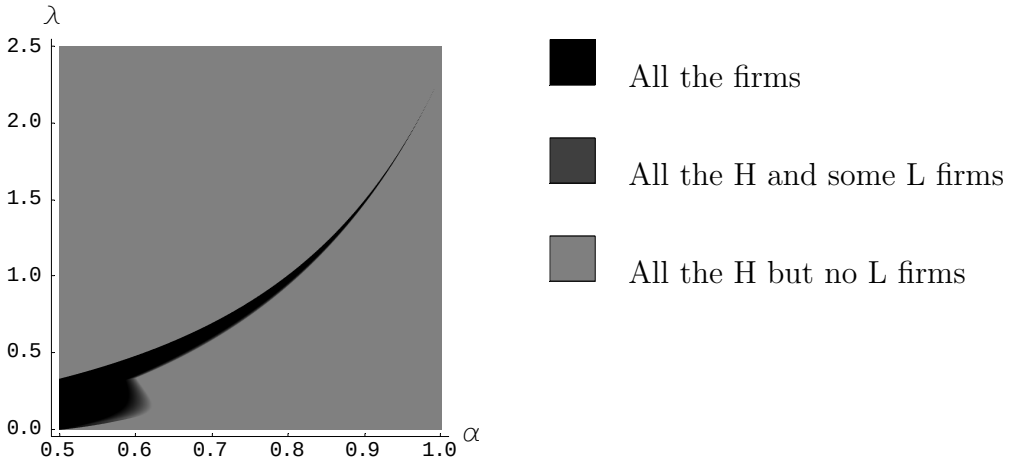
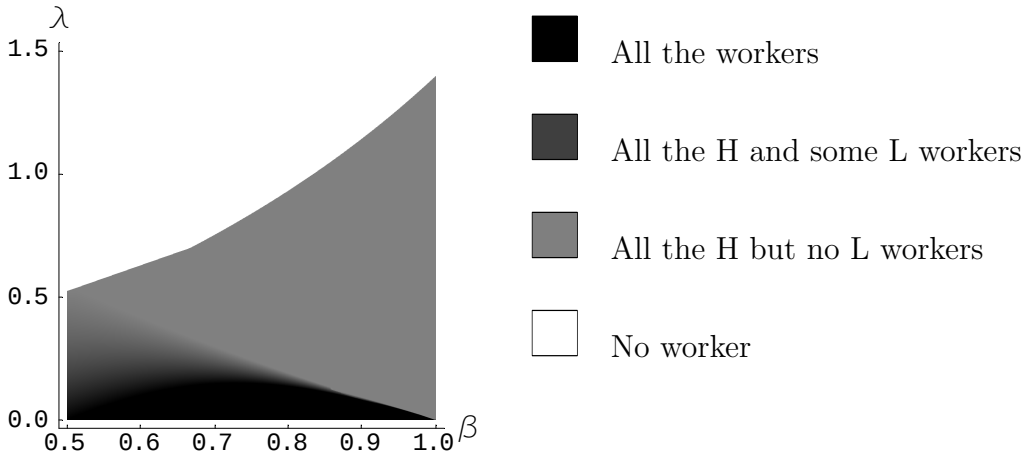


Figure 1: (a) Equilibrium signaling and (b) referral in $\alpha - \lambda$ space for $\alpha = \beta$ and $\delta = 0.90$.

(a) Equilibrium Signalling



(b) Equilibrium Referral

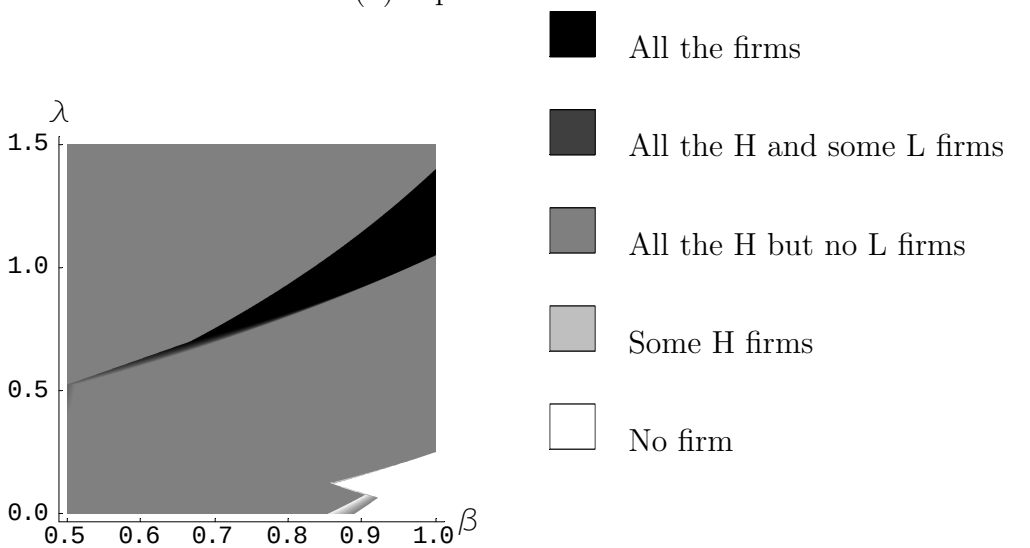


Figure 2: (a) Equilibrium signaling and (b) referral in $\beta - \lambda$ space for $\alpha = 0.75$ and $\delta = 0.90$.

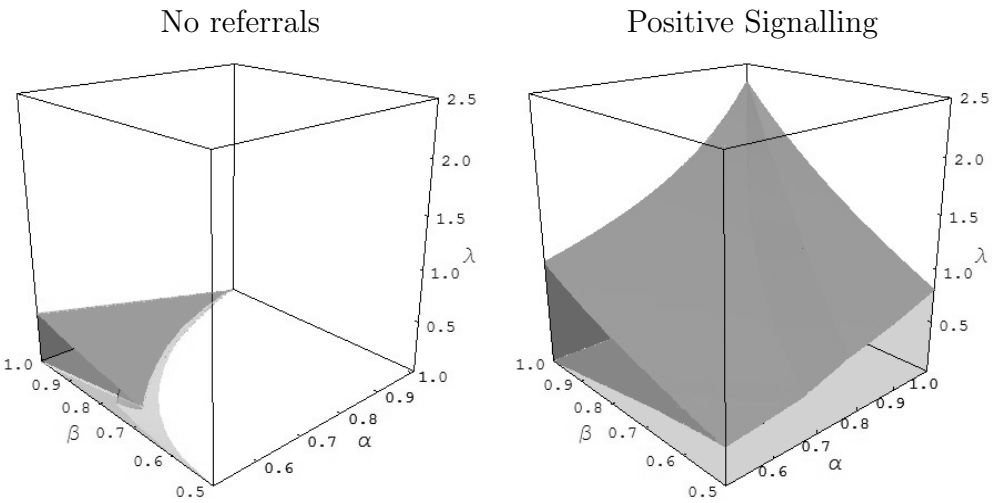
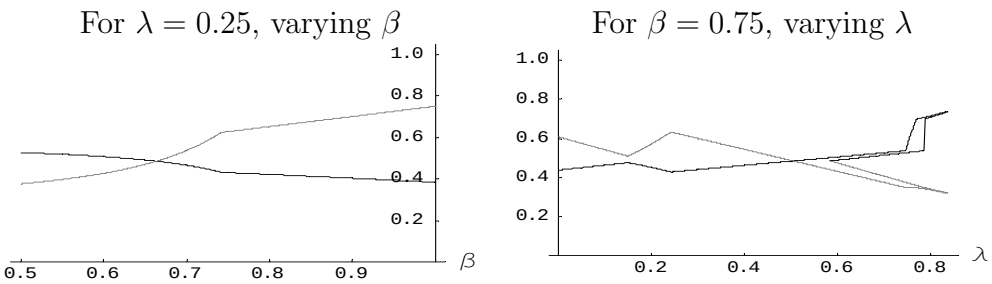
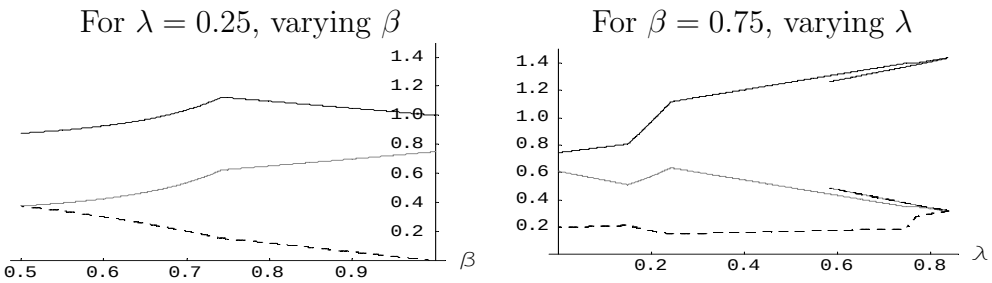


Figure 3: The space of parameters where equilibria without referrals exist (left) and where equilibria with signaling exist (right). Darker grey correspond to informative signaling ($s_L = 0$). $\delta = 0.90$

(a) Average market wage and referral wage



(b) Market wage for certified and uncertified workers and referral wage



(c) Average market wage for H and L workers and referral wage

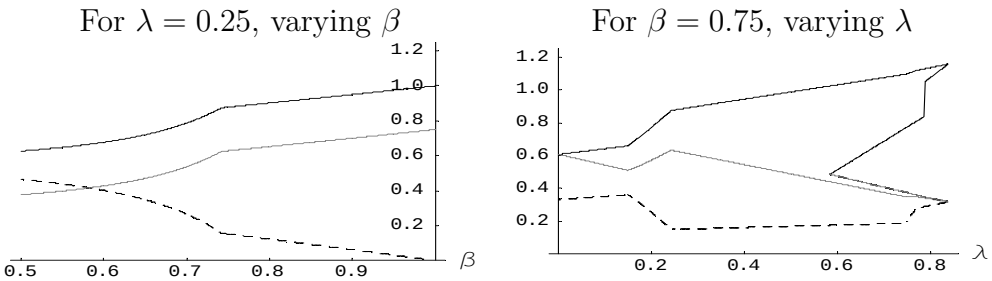


Figure 4: (a) Average market wage (black) and referral wage (gray). (b) Market wage for certified workers (black, solid), market wage for uncertified workers (black, dashed), and referral wage (gray). (c) Average market wage for H workers (black, solid), average market wage for L workers (black, dashed), and referral wage (gray). Notice the multiple equilibria in the $\beta = 0.75$ figures at high values of λ. For all figures $\alpha = 0.75$ and $\delta = 0.90$.