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# Credit Rationing in Markets with Imperfect Information

By JOSEPH E. STIGLITZ AND ANDREW WEISS\*

Why is credit rationed? Perhaps the most basic tenet of economics is that market equilibrium entails supply equalling demand; that if demand should exceed supply, prices will rise, decreasing demand and/or increasing supply until demand and supply are equated at the new equilibrium price. So if prices do their job, rationing should not exist. However, credit rationing and unemployment do in fact exist. They seem to imply an excess demand for loanable funds or an excess supply of workers.

One method of "explaining" these conditions associates them with short- or long-term disequilibrium. In the short term they are viewed as *temporary disequilibrium* phenomena; that is, the economy has incurred an exogenous shock, and for reasons not fully explained, there is some stickiness in the prices of labor or capital (wages and interest rates) so that there is a transitional period during which rationing of jobs or credit occurs. On the other hand, long-term unemployment (above some "natural rate") or credit rationing is explained by governmental constraints such as usury laws or minimum wage legislation.<sup>1</sup>

The object of this paper is to show that in *equilibrium* a loan market may be characterized by credit rationing. Banks making loans are concerned about the interest rate

they receive on the loan, and the riskiness of the loan. However, the interest rate a bank charges may itself affect the riskiness of the pool of loans by either: 1) sorting potential borrowers (the adverse selection effect); or 2) affecting the actions of borrowers (the incentive effect). Both effects derive directly from the residual imperfect information which is present in loan markets after banks have evaluated loan applications. When the price (interest rate) affects the nature of the transaction, it may not also clear the market.

The adverse selection aspect of interest rates is a consequence of different borrowers having different probabilities of repaying their loan. The expected return to the bank obviously depends on the probability of repayment, so the bank would like to be able to identify borrowers who are more likely to repay. It is difficult to identify "good borrowers," and to do so requires the bank to use a variety of *screening devices*. The interest rate which an individual is willing to pay may act as one such screening device: those who are willing to pay high interest rates may, on average, be worse risks; they are willing to borrow at high interest rates because they perceive their probability of repaying the loan to be low. As the interest rate rises, the average "riskiness" of those who borrow increases, possibly lowering the bank's profits.

Similarly, as the interest rate and other terms of the contract change, the behavior of the borrower is likely to change. For instance, raising the interest rate decreases the return on projects which succeed. We will show that higher interest rates induce firms to undertake projects with lower probabilities of success but higher payoffs when successful.

In a world with perfect and costless information, the bank would stipulate precisely all the actions which the borrower could

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<sup>1</sup>Indeed, even if markets were not competitive one would not expect to find rationing; profit maximization would, for instance, lead a monopolistic bank to raise the interest rate it charges on loans to the point where excess demand for loans was eliminated.

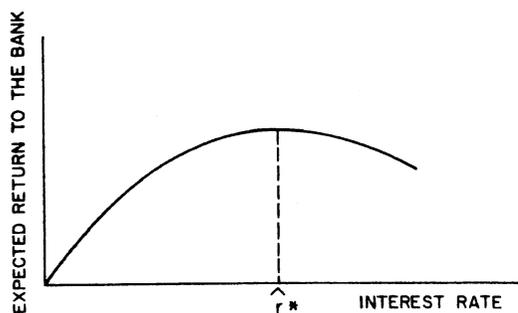


FIGURE 1. THERE EXISTS AN INTEREST RATE WHICH MAXIMIZES THE EXPECTED RETURN TO THE BANK

undertake (which might affect the return to the loan). However, the bank is not able to directly control all the actions of the borrower; therefore, it will formulate the terms of the loan contract in a manner designed to induce the borrower to take actions which are in the interest of the bank, as well as to attract low-risk borrowers.

For both these reasons, the expected return by the bank may increase less rapidly than the interest rate; and, beyond a point, may actually decrease, as depicted in Figure 1. The interest rate at which the expected return to the bank is maximized, we refer to as the "bank-optimal" rate,  $\hat{r}^*$ .

Both the demand for loans and the supply of funds are functions of the interest rate (the latter being determined by the expected return at  $\hat{r}^*$ ). Clearly, it is conceivable that at  $\hat{r}^*$  the demand for funds exceeds the supply of funds. Traditional analysis would argue that, in the presence of an excess demand for loans, unsatisfied borrowers would offer to pay a higher interest rate to the bank, bidding up the interest rate until demand equals supply. But although supply does not equal demand at  $\hat{r}^*$ , it is the equilibrium interest rate! The bank would not lend to an individual who offered to pay more than  $\hat{r}^*$ . In the bank's judgment, such a loan is likely to be a worse risk than the average loan at interest rate  $\hat{r}^*$ , and the expected return to a loan at an interest rate above  $\hat{r}^*$  is actually lower than the expected return to the loans the bank is presently making. Hence, there are

no competitive forces leading supply to equal demand, and credit is rationed.

But the interest rate is not the only term of the contract which is important. The amount of the loan, and the amount of collateral or equity the bank demands of loan applicants, will also affect both the behavior of borrowers and the distribution of borrowers. In Section III, we show that increasing the collateral requirements of lenders (beyond some point) may decrease the returns to the bank, by either decreasing the average degree of risk aversion of the pool of borrowers; or in a multiperiod model inducing individual investors to undertake riskier projects.

Consequently, it may not be profitable to raise the interest rate or collateral requirements when a bank has an excess demand for credit; instead, banks deny loans to borrowers who are observationally indistinguishable from those who receive loans.<sup>2</sup>

It is not our argument that credit rationing will always characterize capital markets, but rather that it may occur under not implausible assumptions concerning borrower and lender behavior.

This paper thus provides the first theoretical justification of true credit rationing. Previous studies have sought to explain why each individual faces an upward sloping interest rate schedule. The explanations offered are (a) the probability of default for any particular borrower increases as the amount borrowed increases (see Stiglitz 1970, 1972; Marshall Freimer and Myron Gordon; Dwight Jaffee; George Stigler), or (b) the mix of borrowers changes adversely (see Jaffee and Thomas Russell). In these circumstances we would not expect loans of different size to pay the same interest rate, any more than we would expect two borrowers, one of whom has a reputation for prudence and the other a reputation as a bad credit risk, to be able to borrow at the same interest rate.

We reserve the term credit rationing for circumstances in which either (a) among loan applicants who appear to be identical some

<sup>2</sup>After this paper was completed, our attention was drawn to W. Keeton's book. In chapter 3 he develops an incentive argument for credit rationing.

receive a loan and others do not, and the rejected applicants would not receive a loan even if they offered to pay a higher interest rate; or (b) there are identifiable groups of individuals in the population who, with a given supply of credit, are unable to obtain loans at any interest rate, even though with a larger supply of credit, they would.<sup>3</sup>

In our construction of an equilibrium model with credit rationing, we describe a market equilibrium in which there are many banks and many potential borrowers. Both borrowers and banks seek to maximize profits, the former through their choice of a project, the latter through the interest rate they charge borrowers and the collateral they require of borrowers (the interest rate received by depositors is determined by the zero-profit condition). Obviously, we are not discussing a "price-taking" equilibrium. Our equilibrium notion is competitive in that banks compete; one means by which they compete is by their choice of a price (interest rate) which maximizes their profits. The reader should notice that in the model presented below there are interest rates at which the demand for loanable funds equals the supply of loanable funds. However, these are not, in general, equilibrium interest rates. If, at those interest rates, banks could increase their profits by lowering the interest rate charged borrowers, they would do so.

Although these results are presented in the context of credit markets, we show in Section V that they are applicable to a wide class of principal-agent problems (including those describing the landlord-tenant or employer-employee relationship).

### I. Interest Rate as a Screening Device

In this section we focus on the role of interest rates as screening devices for distinguishing between good and bad risks. We assume that the bank has identified a group

<sup>3</sup>There is another form of rationing which is the subject of our 1980 paper: banks make the provision of credit in later periods contingent on performance in earlier period; banks may then refuse to lend even when these later period projects stochastically dominate earlier projects which are financed.

of projects; for each project  $\theta$  there is a probability distribution of (gross) returns  $R$ . We assume for the moment that this distribution cannot be altered by the borrower.

Different firms have different probability distributions of returns. We initially assume that the bank is able to distinguish projects with different mean returns, so we will at first confine ourselves to the decision problem of a bank facing projects having the same mean return. However, the bank cannot ascertain the riskiness of a project. For simplicity, we write the distribution of returns<sup>4</sup> as  $F(R, \theta)$  and the density function as  $f(R, \theta)$ , and we assume that greater  $\theta$  corresponds to greater risk in the sense of mean preserving spreads<sup>5</sup> (see Rothschild-Stiglitz), i.e., for  $\theta_1 > \theta_2$ , if

$$(1) \quad \int_0^{\infty} Rf(R, \theta_1) dR = \int_0^{\infty} Rf(R, \theta_2) dR$$

then for  $y \geq 0$ ,

$$(2) \quad \int_0^y F(R, \theta_1) dR \geq \int_0^y F(R, \theta_2) dR$$

If the individual borrows the amount  $B$ , and the interest rate is  $\hat{r}$ , then we say the individual defaults on his loan if the return  $R$  plus the collateral  $C$  is insufficient to pay back the promised amount,<sup>6</sup> i.e., if

$$(3) \quad C + R \leq B(1 + \hat{r})$$

<sup>4</sup>These are subjective probability distributions; the perceptions on the part of the bank may differ from those of the firm.

<sup>5</sup>Michael Rothschild and Stiglitz show that conditions (1) and (2) imply that project 2 has a greater variance than project 1, although the converse is not true. That is, the mean preserving spread criterion for measuring risk is stronger than the increasing variance criterion. They also show that (1) and (2) can be interpreted equally well as: given two projects with equal means, every risk averter prefers project 1 to project 2.

<sup>6</sup>This is not the only possible definition. A firm might be said to be in default if  $R < B(1 + \hat{r})$ . Nothing critical depends on the precise definition. We assume, however, that if the firm defaults, the bank has first claim on  $R + C$ . The analysis may easily be generalized to include bankruptcy costs. However, to simplify the analysis, we usually shall ignore these costs. Throughout this section we assume that the project is the sole project

Thus the net return to the borrower  $\pi(R, \hat{r})$  can be written as

$$(4a) \quad \pi(R, \hat{r}) = \max(R - (1 + \hat{r})B; -C)$$

The return to the bank can be written as

$$(4b) \quad \rho(R, \hat{r}) = \min(R + C; B(1 + \hat{r}))$$

that is, the borrower must pay back either the promised amount or the maximum he can pay back ( $R + C$ ).

For simplicity, we shall assume that the borrower has a given amount of equity (which he cannot increase), that borrowers and lenders are risk neutral, that the supply of loanable funds available to a bank is unaffected by the interest rate it charges borrowers, that the cost of the project is fixed, and unless the individual can borrow the difference between his equity and the cost of the project, the project will not be undertaken, that is, projects are not divisible. For notational simplicity, we assume the amount borrowed for each project is identical, so that the distribution functions describing the number of loan applications are identical to those describing the monetary value of loan applications. (In a more general model, we would make the amount borrowed by each individual a function of the terms of the contract; the quality mix could change not only as a result of a change in the mix of applicants, but also because of a change in the relative size of applications of different groups.)

We shall now prove that the interest rate acts as a screening device; more precisely we establish

**THEOREM 1:** *For a given interest rate  $\hat{r}$ , there is a critical value  $\hat{\theta}$  such that a firm borrows from the bank if and only if  $\theta > \hat{\theta}$ .*

This follows immediately upon observing that profits are a convex function of  $R$ , as in Figure 2a. Hence expected profits increase with risk.

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undertaken by the firm (individual) and that there is limited liability. The equilibrium extent of liability is derived in Section III.

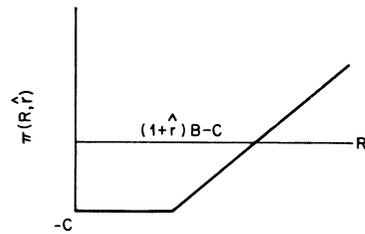


FIGURE 2a. FIRM PROFITS ARE A CONVEX FUNCTION OF THE RETURN ON THE PROJECT

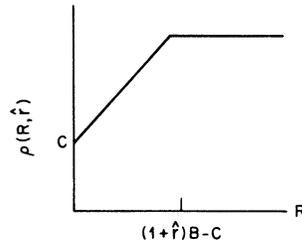


FIGURE 2b. THE RETURN TO THE BANK IS A CONCAVE FUNCTION OF THE RETURN ON THE PROJECT

The value of  $\hat{\theta}$  for which expected profits are zero satisfies

$$(5) \quad \Pi(\hat{r}, \hat{\theta}) \equiv \int_0^\infty \max[R - (\hat{r} + 1)B; -C] dF(R, \hat{\theta}) = 0$$

Our argument that the adverse selection of interest rates could cause the returns to the bank to decrease with increasing interest rates hinged on the conjecture that as the interest rate increased, the mix of applicants became worse; or

**THEOREM 2:** *As the interest rate increases, the critical value of  $\theta$ , below which individuals do not apply for loans, increases.*

This follows immediately upon differentiating (5):

$$(6) \quad \frac{d\hat{\theta}}{d\hat{r}} = \frac{B \int_{(1+\hat{r})B-C}^\infty dF(R, \hat{\theta})}{\partial \Pi / \partial \hat{\theta}} > 0$$

For each  $\theta$ , expected profits are decreased;

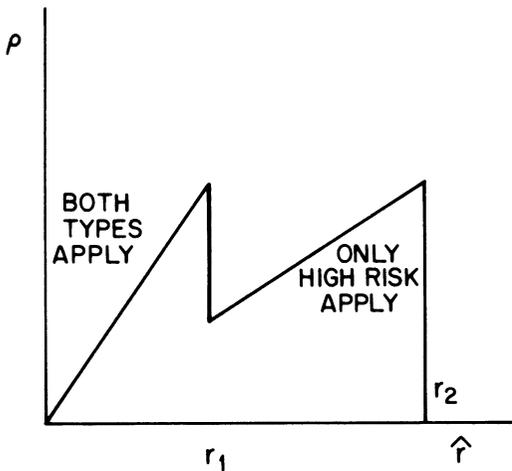


FIGURE 3. OPTIMAL INTEREST RATE  $\hat{r}_1$

hence using Theorem 1, the result is immediate.

We next show:

**THEOREM 3:** *The expected return on a loan to a bank is a decreasing function of the riskiness of the loan.*

**PROOF:**

From (4b) we see that  $\rho(R, \hat{r})$  is a concave function of  $R$ , hence the result is immediate. The concavity of  $\rho(R, \hat{r})$  is illustrated in Figure 2b.

Theorems 2 and 3 imply that, in addition to the usual direct effect of increases in the interest rate increasing a bank's return, there is an indirect, adverse-selection effect acting in the opposite direction. We now show that this adverse-selection effect may outweigh the direct effect.

To see this most simply, assume there are two groups; the "safe" group will borrow only at interest rates below  $r_1$ , the "risky" group below  $r_2$ , and  $r_1 < r_2$ . When the interest rate is raised slightly above  $r_1$ , the mix of applicants changes dramatically: all low risk applicants withdraw. (See Figure 3.) By the same argument we can establish

**THEOREM 4:** *If there are a discrete number of potential borrowers (or types of borrowers) each with a different  $\theta$ ,  $\bar{\rho}(\hat{r})$  will not be a monotonic function of  $\hat{r}$ , since as each succes-*

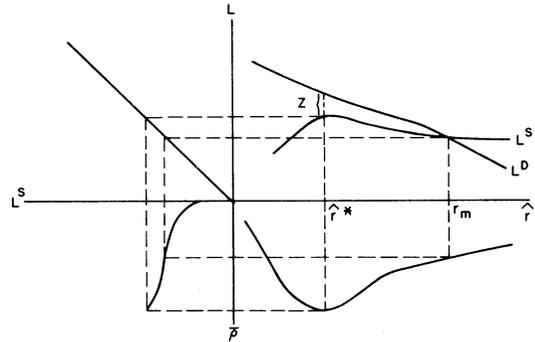


FIGURE 4. DETERMINATION OF THE MARKET EQUILIBRIUM

sive group drops out of the market, there is a discrete fall in  $\bar{\rho}$  (where  $\bar{\rho}(\hat{r})$  is the mean return to the bank from the set of applicants at the interest rate  $\hat{r}$ ).

Other conditions for nonmonotonicity of  $\bar{\rho}(\hat{r})$  will be established later. Theorems 5 and 6 show why nonmonotonicity is so important:

**THEOREM 5:** *Whenever  $\bar{\rho}(\hat{r})$  has an interior mode, there exist supply functions of funds such that competitive equilibrium entails credit rationing.*

This will be the case whenever the "Walrasian equilibrium" interest rate—the one at which demand for funds equals supply—is such that there exists a lower interest rate for which  $\bar{\rho}$ , the return to the bank, is higher.

In Figure 4 we illustrate a credit rationing equilibrium. Because demand for funds depends on  $\hat{r}$ , the interest rate charged by banks, while the supply of funds depends on  $\rho$ , the mean return on loans, we cannot use a conventional demand/supply curve diagram. The demand for loans is a decreasing function of the interest rate charged borrowers; this relation  $L^D$  is drawn in the upper right quadrant. The nonmonotonic relation between the interest charged borrowers, and the expected return to the bank per dollar loaned  $\bar{\rho}$  is drawn in the lower right quadrant. In the lower left quadrant we depict the relation between  $\bar{\rho}$  and the supply of loanable funds  $L^S$ . (We have drawn  $L^S$  as if it

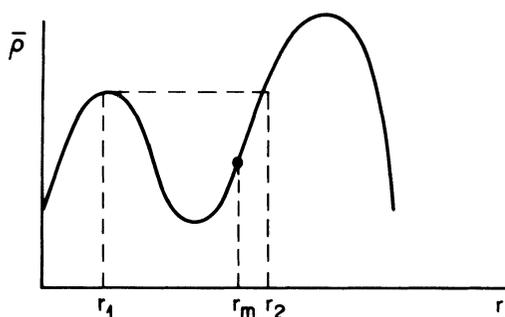


FIGURE 5. A TWO-INTEREST RATE EQUILIBRIUM

were an increasing function of  $\bar{\rho}$ . This is not necessary for our analysis.) If banks are free to compete for depositors, then  $\bar{\rho}$  will be the interest rate received by depositors. In the upper right quadrant we plot  $L^S$  as a function of  $\hat{r}$ , through the impact of  $\hat{r}$  on the return on each loan, and hence on the interest rate  $\bar{\rho}$  banks can offer to attract loanable funds.

A credit rationing equilibrium exists given the relations drawn in Figure 4; the demand for loanable funds at  $\hat{r}^*$  exceeds the supply of loanable funds at  $\hat{r}^*$  and any individual bank increasing its interest rate beyond  $\hat{r}^*$  would lower its return per dollar loaned. The excess demand for funds is measured by  $Z$ . Notice that there is an interest rate  $r_m$  at which the demand for loanable funds equals the supply of loanable funds; however,  $r_m$  is not an equilibrium interest rate. A bank could increase its profits by charging  $\hat{r}^*$  rather than  $r_m$ ; at the lower interest rate it would attract at least all the borrowers it attracted at  $r_m$  and would make larger profits from each loan (or dollar loaned).

Figure 4 can also be used to illustrate an important comparative statics property of our market equilibrium:

**COROLLARY 1.** *As the supply of funds increases, the excess demand for funds decreases, but the interest rate charged remains unchanged, so long as there is any credit rationing.*

Eventually, of course,  $Z$  will be reduced to zero; further increases in the supply of funds then reduce the market rate of interest.

Figure 5 illustrates a  $\bar{\rho}(\hat{r})$  function with multiple modes. The nature of the equilibrium for such cases is described by Theorem 6.

**THEOREM 6:** *If the  $\bar{\rho}(r)$  function has several modes, market equilibrium could either be characterized by a single interest rate at or below the market-clearing level, or by two interest rates, with an excess demand for credit at the lower one.*

#### PROOF:

Denote the lowest Walrasian equilibrium interest rate by  $r_m$  and denote by  $\hat{r}$  the interest rate which maximizes  $\rho(r)$ . If  $\hat{r} < r_m$ , the analysis for Theorem 5 is unaffected by the multiplicity of modes. There will be credit rationing at interest rate  $\hat{r}$ . The rationed borrowers will not be able to obtain credit by offering to pay a higher interest rate.

On the other hand, if  $\hat{r} > r_m$ , then loans may be made at two interest rates, denoted by  $r_1$  and  $r_2$ .  $r_1$  is the interest rate which maximizes  $\rho(r)$  conditional on  $r \leq r_m$ ;  $r_2$  is the lowest interest rate greater than  $r_m$  such that  $\rho(r_2) = \rho(r_1)$ . From the definition of  $r_m$ , and the downward slope of the loan demand function, there will be an excess demand for loanable funds at  $r_1$  (unless  $r_1 = r_m$ , in which case there is no credit rationing). Some rejected borrowers (with reservation interest rates greater than or equal to  $r_2$ ) will apply for loans at the higher interest rate. Since there would be an excess supply of loanable funds at  $r_2$  if no loans were made at  $r_1$ , and an aggregate excess demand for funds if no loans were made at  $r_2$ , there exists a distribution of loanable funds available to borrowers at  $r_1$  and  $r_2$  such that all applicants who are rejected at interest rate  $r_1$  and who apply for loans at  $r_2$  will get credit at the higher interest rate. Similarly, all the funds available at  $\rho(r_1)$  will be loaned at either  $r_1$  or  $r_2$ . (There is, of course, an excess demand for loanable funds at  $r_1$  since every borrower who eventually borrows at  $r_2$  will have first applied for credit at  $r_1$ .) There is clearly no incentive for small deviations from  $r_1$ , which is a local maximum of  $\rho(r)$ . A bank lending at an interest rate  $r_3$  such that  $\rho(r_3) < \rho(r_1)$  would not be able to obtain credit. Thus, no bank

would switch to a loan offer between  $r_1$  and  $r_2$ . A bank offering an interest rate  $r_4$  such that  $\rho(r_4) > \rho(r_1)$  would not be able to attract any borrowers since by definition  $r_4 > r_2$ , and there is no excess demand at interest rate  $r_2$ .

A. Alternative Sufficient Conditions for Credit Rationing

Theorem 4 provided a sufficient condition for adverse selection to lead to a nonmonotonic  $\bar{\rho}(\hat{r})$  function. In the remainder of this section, we investigate other circumstances under which for some levels of supply of funds there will be credit rationing.

1. Continuum of Projects

Let  $G(\theta)$  be the distribution of projects by riskiness  $\theta$ , and  $\rho(\theta, r)$  be the expected return to the bank of a loan of risk  $\theta$  and interest rate  $r$ . The mean return to the bank which lends at the interest rate  $\hat{r}$  is simply

$$(7) \quad \bar{\rho}(\hat{r}) = \frac{\int_{\theta(\hat{r})}^{\infty} \rho(\theta, \hat{r}) dG(\theta)}{1 - G(\hat{\theta})}$$

From Theorem 5 we know that  $d\bar{\rho}(\hat{r})/d\hat{r} < 0$  for some value of  $\hat{r}$  is a sufficient condition for credit rationing. Let  $\rho(\hat{\theta}, \hat{r}) = \bar{\rho}$  so that

$$(8) \quad \frac{d\bar{\rho}}{d\hat{r}} = - \frac{g(\hat{\theta})}{[1 - G(\hat{\theta})]} (\hat{\rho} - \bar{\rho}) \frac{d\hat{\theta}}{d\hat{r}} + \frac{\int_{\hat{\theta}}^{\infty} [1 - F((1 + \hat{r})B - C, \theta)] dG(\theta)}{1 - G(\hat{\theta})}$$

From Theorems 1 and 3, the first term is negative (representing the change in the mix of applicants), while the second term (the increase in returns, holding the applicant pool fixed, from raising the interest charges) is positive. The first term is large, in absolute value, if there is a large difference between the mean return on loans made at interest rate  $\hat{r}$  and the return to the bank from the project making zero returns to the firm at interest rate  $\hat{r}$  (its "safest" loan). It is also

large if  $(g(\hat{\theta})/[1 - G(\hat{\theta})]) (d\hat{\theta}/d\hat{r})$  is large, that is, a small change in the nominal interest rate induces a large change in the applicant pool.

2. Two Outcome Projects

Here we consider the simplest kinds of projects (from an analytical point of view), those which either succeed and yield a return  $R$ , or fail and yield a return  $D$ . We normalize to let  $B = 1$ . All the projects have the same unsuccessful value (which could be the value of the plant and equipment) while  $R$  ranges between  $S$  and  $K$  (where  $K > S$ ). We also assume that projects have been screened so that all projects within a loan category have the same expected yield,  $T$ , and there is no collateral required, that is,  $C = 0$ , and if  $p(R)$  represents the probability that a project with a successful return of  $R$  succeeds, then

$$(9) \quad p(R)R + [1 - p(R)]D = T$$

In addition, the bank suffers a cost of  $X$  per dollar loaned upon loans that default, which could be interpreted as the difference between the value of plant and equipment to the firm and the value of the plant and equipment to the bank. Again the density of project values is denoted by  $g(R)$ , the distribution function by  $G(R)$ .

Therefore, the expected return per dollar lent at an interest rate  $\hat{r}$ , if we let  $J = \hat{r} + 1$ , is (since individuals will borrow if and only if  $R > J$ ):

$$(10) \quad \rho(J) = \frac{1}{\int_J^K g(R) dR} \left[ J \int_J^K p(R) g(R) dR + \int_J^K [1 - p(R)] [D - X] g(R) dR \right]$$

Using l'Hopital's rule and (1), we can establish sufficient conditions for  $\lim_{J \rightarrow K} (\partial \rho(J) / \partial J) < 0$  (and hence for the nonmonotonicity of  $\rho$ ):<sup>7</sup>

<sup>7</sup>The proofs of these propositions are slightly complicated. Consider 1. Since  $p(R) = T - D / R - D$ , the

(a) if  $\lim_{R \rightarrow K} g(R) \neq 0, \infty$  then a sufficient condition is  $X > K - D$ , or equivalently,  $\lim_{R \rightarrow K} p(R) + p'(R)X < 0$

(b) if  $g(K) = 0, g'(K) \neq 0, \infty$  then a sufficient condition is  $2X > K - D$ , or equivalently,  $\lim_{R \rightarrow K} p(R) + 2p'(R)X < 0$

(c) if  $g(K) = 0, g'(K) = 0, g''(K) \neq 0$ , then a sufficient condition is  $3X > K - K - D$ , or equivalently,  $\lim_{R \rightarrow K} p(R) + 3p'(R)X < 0$

Condition (a) implies that if, as  $1 + \hat{r} \rightarrow K$ , the probability of an increase in the interest rate being repaid is outweighed by the deadweight loss of riskier loans, the bank will maximize its return per dollar loaned at an interest rate below the maximum rate at which it can loan funds ( $K - 1$ ). The conditions for an interior bank optimal interest rate are significantly less stringent when  $g(K) = 0$ .

### 3. Differences in Attitudes Towards Risk

Some loan applicants are clearly more risk averse than others. These differences will be reflected in project choices, and thus affect

expected profit per dollar loaned may be rewritten as

$$\rho(J) = [J - D + X][T - D] \frac{\int_J^K \frac{g(R)}{R - D} dR}{\int_J^K g(R) dR} + D - X$$

Differentiating, and collecting terms

$$\begin{aligned} \frac{1}{T - D} \frac{\partial \rho}{\partial J} &= \frac{\int_J^K \frac{g(R)}{R - D} dR}{\int_J^K g(R) dR} + [J - D + X] \\ &\times \left[ \frac{-\frac{g(J)}{J - D} \int_J^K g(R) dR + g(J) \int_J^K \frac{g(R)}{R - D} dR}{\left[ \int_J^K g(R) dR \right]^2} \right] \end{aligned}$$

Using l'Hopital's rule and the assumption that  $g(K) \neq 0, \infty$

$$\lim_{J \rightarrow K} \left( \frac{1}{T - D} \frac{\partial \rho}{\partial J} \right) = \left( \frac{1}{K - D} - \frac{K - D + X}{2(K - D)^2} \right);$$

or 
$$\text{sign} \left( \lim_{J \rightarrow K} \frac{1}{T - D} \frac{\partial \rho}{\partial J} \right) = \text{sign}(K - D - X)$$

Conditions 2 and 3 follow in a similar manner.

the bank-optimal interest rate. High interest rates may make projects with low mean returns—the projects undertaken by risk averse individuals—unfeasible, but leave relatively unaffected the risky projects. The mean return to the bank, however, is lower on the riskier projects than on the safe projects. In the following example, it is systematic differences in risk aversion which results in there being an optimal interest rate.

Assume a fraction  $\lambda$  of the population is infinitely risk averse; each such individual undertakes the best perfectly safe project which is available to him. Within that group, the distribution of returns is  $G(R)$  where  $G(K) = 1$ . The other group is risk neutral. For simplicity we shall assume that they all face the same risky project with probability of success  $p$  and a return, if successful, of  $R^* > K$ ; if not their return is zero. Letting  $\hat{R} = (1 + \hat{r})B$  the (expected) return to the bank is

$$\begin{aligned} (11) \quad \bar{\rho}(\hat{r}) &= \frac{\{\lambda(1 - G(\hat{R})) + (1 - \lambda)p\}}{\lambda(1 - G(\hat{R})) + (1 - \lambda)} (1 + \hat{r}) \\ &= \left[ 1 - \frac{(1 - p)(1 - \lambda)}{\lambda(1 - G(\hat{R})) + (1 - \lambda)} \right] \frac{\hat{R}}{B} \end{aligned}$$

Hence for  $R < K$ , the upper bound on returns from the safe project

$$(12) \quad \frac{d \ln \bar{\rho}}{d \ln(1 + \hat{r})} = 1 - \frac{(1 - \lambda)(1 - p)\lambda g(\hat{R})\hat{R}}{(1 - \lambda G(\hat{R}))(\lambda(1 - G(\hat{R})) + p(1 - \lambda))}$$

A sufficient condition for the existence of an interior bank optimal interest rate is again that  $\lim_{R \rightarrow K} \frac{\partial \bar{\rho}}{\partial \hat{r}} < 0$ , or from (12),  $\lambda / (1 - \lambda \lim_{R \rightarrow K} g(\hat{R})\hat{R}) > p / (1 - p)$ . The greater is the riskiness of the risky project (the lower is  $p$ ), the more likely is an interior bank optimal interest rate. Similarly, the higher is the relative proportion of the risk averse individuals affected by increases in the interest rate to risk neutral borrowers, the more important is

the self-selection effect, and the more likely is an interior bank optimal interest rate.

**II. Interest Rate as an Incentive Mechanism**

*A. Sufficient Conditions*

The second way in which the interest rate affects the bank's expected return from a loan is by changing the behavior of the borrower. The interests of the lender and the borrower do not coincide. The borrower is only concerned with returns on the investment when the firm does not go bankrupt; the lender is concerned with the actions of the firm only to the extent that they affect the probability of bankruptcy, and the returns in those states of nature in which the firm *does* go bankrupt. Because of this, and because the behavior of a borrower cannot be perfectly and costlessly monitored by the lender, banks will take into account the effect of the interest rate on the behavior of borrowers.

In this section, we show that increasing the rate of interest increases the relative attractiveness of riskier projects, for which the return to the bank may be lower. Hence, raising the rate of interest may lead borrowers to take actions which are contrary to the interests of the lender, providing another incentive for banks to ration credit rather than raise the interest rate when there is an excess demand for loanable funds.

We return to the general model presented above, but now we assume that each firm has a choice of projects. Consider any two projects, denoted by superscripts *j* and *k*. We first establish:

**THEOREM 7:** *If, at a given nominal interest rate  $r$ , a risk-neutral firm is indifferent between two projects, an increase in the interest rate results in the firm preferring the project with the higher probability of bankruptcy.*

**PROOF:**

The expected return to the *i*th project is given by

$$(13) \quad \pi^i = E \left[ \max \left( R^i - (1 + \hat{r})B, -C \right) \right]$$

so

$$(14) \quad \frac{d\pi^i}{d\hat{r}} = -B(1 - F_i((1 + \hat{r})B - C))$$

Thus, if at some  $\hat{r}$ ,  $\pi^j = \pi^k$ , the increase in  $\hat{r}$  lowers the expected return to the borrower from the project with the higher probability of paying back the loan by more than it lowers the expected return from the project with the lower probability of the loan being repaid.

On the other hand, if the firm is indifferent between two projects with the same mean, we know from Theorem 2 that the bank prefers to lend to the safer project. Hence raising the interest rate above  $\hat{r}$  could so increase the riskiness of loans as to lower the expected return to the bank.

**THEOREM 8:** *The expected return to the bank is lowered by an increase in the interest rate at  $\hat{r}$  if, at  $\hat{r}$ , the firm is indifferent between two projects *j* and *k* with distributions  $F_j(R)$  and  $F_k(R)$ , *j* having a higher probability of bankruptcy than *k*, and there exists a distribution  $F_l(R)$  such that*

(a)  $F_j(R)$  represents a mean preserving spread of the distribution  $F_l(R)$ , and

(b)  $F_k(R)$  satisfies a first-order dominance relation with  $F_l(R)$ ; i.e.,  $F_l(R) > F_k(R)$  for all  $R$ .

**PROOF:**

Since *j* has a higher probability of bankruptcy than does *k*, from Theorem 7 and the initial indifference of borrowers between *j* and *k*, an increase in the interest rate  $\hat{r}$  leads firms to prefer project *j* to *k*. Because of (a) and Theorem 3, the return to the bank on a project whose return is distributed as  $F_l(R)$  is higher than on project *j*, and because of (b) the return to the bank on project *k* is higher than the return on a project distributed as  $F_l(R)$ .

*B. An Example*

To illustrate the implications of Theorem 8, assume all firms are identical, and have a choice of two projects, yielding, if successful, returns  $R^a$  and  $R^b$ , respectively (and nothing

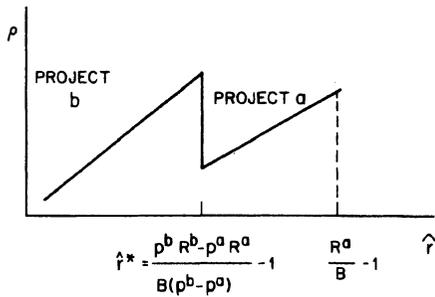


FIGURE 6. AT INTEREST RATES ABOVE  $\hat{r}^*$ , THE RISKY PROJECT IS UNDERTAKEN AND THE RETURN TO THE BANK IS LOWERED

otherwise) where  $R^a > R^b$ , and with probabilities of success of  $p^a$  and  $p^b$ ,  $p^a < p^b$ . For simplicity assume that  $C=0$ . If the firm is indifferent between the projects at interest rate  $\hat{r}$ , then

$$(15) [R^a - (1 + \hat{r})B] p^a = [R^b - (1 + \hat{r})B] p^b$$

i.e.,

$$(16) B(1 + \hat{r}) = \frac{p^b R^b - p^a R^a}{p^b - p^a} \equiv (1 + \hat{r}^*)B$$

Thus, the expected return to the bank as a function of  $r$  appears as in Figure 6.

For interest rates below  $\hat{r}^*$ , firms choose the safe project, while for interest rates between  $\hat{r}^*$  and  $(R^a/B) - 1$ , firms choose the risky project. The maximum interest rate the bank could charge and still induce investments in project  $b$  is  $\hat{r}^*$ . The highest interest rate which attracts borrowers is  $(R^a/B) - 1$ , which would induce investment only in project  $a$ . Therefore the maximum expected return to a bank occurs when the bank charges an interest rate  $\hat{r}^*$  if and only if

$$p^a R^a < \frac{p^b(p^b R^b - p^a R^a)}{p^b - p^a}$$

Whenever  $p^b R^b > p^a R^a$ ,  $1 + \hat{r}^* > 0$ , and  $\rho$  is not monotonic in  $\hat{r}$ , so there may be credit rationing.

### III. The Theory of Collateral and Limited Liability

An obvious objection to the analysis presented thus far is: When there is an excess demand for funds, would not the bank increase its collateral requirements (increasing the liability of the borrower in the event that the project fails); reducing the demand for funds, reducing the risk of default (or losses to the bank in the event of default) and increasing the return to the bank?

This objection will not in general hold. In this section we will discuss various reasons why banks will not decrease the debt-equity ratio of borrowers (increasing collateral requirements)<sup>8</sup> as a means of allocating credit.

A clear case in which reductions in the debt-equity ratio of borrowers are not optimal for the bank is when smaller projects have a higher probability of "failure," and all potential borrowers have the same amount of equity. In those circumstances, increasing the collateral requirements (or the required proportion of equity finance) of loans will imply financing smaller projects. If projects either succeed or fail, and yield a zero return when they fail, then the increase in the collateral requirement of loans will increase the riskiness of those loans.

Another obvious case where increasing collateral requirements may increase the riskiness of loans is if potential borrowers have different equity, and all projects require the same investment. Wealthy borrowers may be those who, in the past, have succeeded at risky endeavors. In that case they are likely to be less risk averse than the more conservative individuals who have in the past invested in relatively safe securities, and are consequently less able to furnish large amounts of collateral.

In both these examples collateral requirements have adverse selection effects. However, we will present a stronger result. We

<sup>8</sup>Increasing the fraction of the project financed by equity and increasing the collateral requirements both increase the expected return to the bank from any particular project. They have similar but identical risk and incentive effects. Although the analysis below focuses on collateral requirements, similar arguments apply to debt-equity ratios.

will show that even if there are no increasing returns to scale in production and all individuals have the same utility function, the sorting effect of collateral requirements can still lead to an interior bank-optimal level of collateral requirements similar to the interior bank-optimal interest rate derived in Sections I and II. In particular, since wealthier individuals are likely to be less risk averse, we would expect that those who could put up the most capital would also be willing to take the greatest risk. We show that this latter effect is sufficiently strong that increasing collateral requirements will, under plausible conditions, lower the bank's return.

To see this most clearly, we assume all borrowers are risk averse with the same utility function  $U(W)$ ,  $U' > 0$ ,  $U'' < 0$ . Individuals differ, however, with respect to their initial wealth,  $W_0$ . Each "entrepreneur" has a set of projects which he can undertake; each project has a probability of success  $p(R)$ , where  $R$  is the return if successful. If the project is unsuccessful, the return is zero;  $p'(R) < 0$ . Each individual has an alternative safe investment opportunity yielding the return  $\rho^*$ . The bank cannot observe either the individual's wealth or the project undertaken. It offers the same contract, defined by  $C$ , the amount of collateral, and  $\hat{r}$ , the interest rate, to all customers. The analysis proceeds as earlier; we first establish:

**THEOREM 9:** *The contract  $\{C, \hat{r}\}$  acts as a screening mechanism: there exist two critical values of  $W_0$ ,  $\hat{W}_0$ , and  $\check{W}_0$ , such that if there is decreasing absolute risk aversion all individuals with wealth  $\hat{W}_0 < W_0 < \check{W}_0$  apply for loans.*

**PROOF:**

As before, we normalize so that all projects cost a dollar. If the individual does not borrow, he either does not undertake the project, obtaining a utility of  $U(W_0\rho^*)$ , or he finances it all himself, obtaining an expected utility of (assuming  $W_0 \geq 1$ )

$$(17) \max_R \{ U((W_0 - 1)\rho^* + R)p(R) + U((W_0 - 1)\rho^*)(1 - p(R)) \} \equiv \hat{V}(W_0)$$

Define

$$(18) V_0(W_0) = \max \{ U(W_0\rho^*), \hat{V}(W_0) \}$$

We note that

$$(19) \frac{dU(W_0\rho^*)}{dW_0} = U'\rho^*$$

$$(20) \frac{d\hat{V}(W_0)}{dW_0} = [U_1'p + U_2'(1-p)]\rho^*$$

(where the subscript 1 refers to the state "success" and the subscript 2 to the state "failure"). We can establish that if there is decreasing absolute risk aversion,<sup>9</sup>

$$\frac{dU(W_0\rho^*)}{dW_0} < \frac{d\hat{V}(W_0)}{dW_0}$$

Hence, there exists a critical value of  $W_0$ ,  $\hat{W}_0$ , such that if  $W_0 > \hat{W}_0$  individuals who do not borrow undertake the project.

For the rest of the analysis we confine ourselves to the case of decreasing absolute risk aversion and wealth less than  $\hat{W}_0$ .

If the individual borrows, he attains a utility level<sup>10</sup>

$$(21) \left\{ \max_R U(W_0\rho^* - (1 + \hat{r}) + R)p + U((W_0 - C)\rho^*)(1 - p) \right\} \equiv V_B(W_0)$$

The individual borrows if and only if

$$(22) V_B(W_0) \geq V_0(W_0)$$

<sup>9</sup>To prove this, we define  $\hat{W}_0$  as the wealth where undertaking the risky project is a mean-utility preserving spread (compare Peter Diamond-Stiglitz) of the safe project. But writing  $U'(W(U))$ , where  $W(U)$  is the value of terminal wealth corresponding to utility level  $U$ ,

$$\frac{dU'}{dU} = \frac{U''}{U'} = -A; \quad \frac{d^2U'}{dU^2} = -\frac{A'}{U'} \geq 0 \text{ as } A' \leq 0$$

Hence with decreasing absolute risk aversion,  $U'$  is a convex function of  $U$  and therefore  $EU'$  for the risky investment exceeds  $U'(\rho^*W_0)$ .

<sup>10</sup>In this formulation, the collateral earns a return  $\rho^*$ .

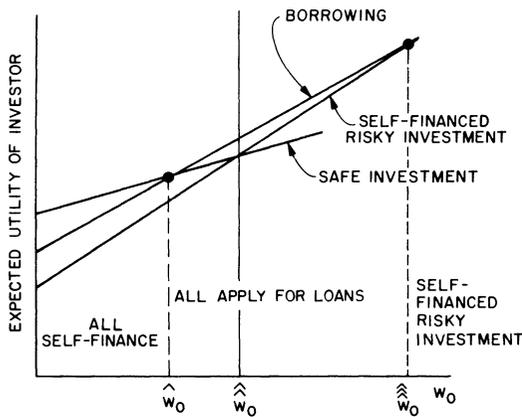


FIGURE 7. COLLATERAL SERVES AS A SCREENING DEVICE

But

$$(23) \quad \frac{dV_B}{dW_0} = (U_1'p + U_2'(1-p))\rho^*$$

Clearly, only those with  $W_0 > C$  can borrow. We assume there exists a value of  $W_0 > 0$ , denoted  $\hat{W}_0$ , such that  $V_B(\hat{W}_0) = U(\rho^*\hat{W}_0)$ . (This will be true for some values of  $\rho^*$ .) By the same kind of argument used earlier, it is clear that at  $\hat{W}_0$ , borrowing with collateral is a mean-utility preserving spread of terminal wealth in comparison to not borrowing and not undertaking the project. Thus using (20) and (23),  $dV_B/dW_0 > dV_0(W_0)/dW_0$  at  $\hat{W}_0$ . Hence, for  $\hat{W}_0 < W_0 < \hat{W}_0$  all individuals apply for loans, as depicted in Figure 7. Thus, restricting ourselves to  $W_0 < \hat{W}_0$ , we have established that if there is any borrowing, it is the wealthiest in that interval who borrow. (The restriction  $W_0 < \hat{W}_0$  is weaker than the restriction that the scale of projects exceeds the wealth of any individual.)

Next, we show:

**THEOREM 10:** *If there is decreasing absolute risk aversion, wealthier individuals undertake riskier projects:  $dR/dW_0 > 0$ .*

**PROOF:**

From (21), we obtain the first-order condition for the choice of  $R$ :

$$(24) \quad U_1'p + (U_1 - U_2)p' = 0$$

so, using the second-order conditions for a maximum, and (24),

$$(25) \quad \frac{dR}{dW_0} \geq 0 \text{ as } \frac{U_1''p + (U_1' - U_2')p'}{U_1'p} = -A_1 - \frac{(U_1' - U_2')}{U_1 - U_2} \geq 0$$

But

$$\lim_{w_1 \rightarrow w_2} -\frac{U_1' - U_2'}{U_1 - U_2} = -\frac{U_1''}{U_1'} = A_1$$

implying that, if  $W_1 = W_2$ ,  $dR/dW_0 = 0$ . However,

$$\begin{aligned} & \frac{\partial \left( -A_1 - \frac{U_1' - U_2'}{U_1 - U_2} \right)}{\partial W_1} \Bigg|_{A_1 = -\frac{U_2' - U_1'}{U_1 - U_2}} \\ &= -A_1' - \frac{U_1''}{U_1 - U_2} + \frac{U_1' - U_2'}{U_1 - U_2} \frac{U_1'}{U_1 - U_2} \\ &= -A_1' \geq 0 \text{ as } A_1' \leq 0 \end{aligned}$$

Hence  $dR/dW_0 > 0$  if  $A' < 0$ .

Next we show

**THEOREM 11:** *Collateral increases the bank's return from any given borrower:*

$$dp/dC > 0$$

**PROOF:**

This follows directly from the first-order condition (24):

$$\text{sign } \frac{dR}{dC} = \text{sign } U_2'\rho^*p' < 0$$

and thus  $dp/dC > 0$ . But

**THEOREM 12:** *There is an adverse selection effect from increasing the collateral requirement, i.e., both the average and the marginal borrower who borrows is riskier,<sup>11</sup>  $d\hat{W}_0/dC > 0$ .*

<sup>11</sup>At a sufficiently high collateral, the wealthy individual will not borrow at all.

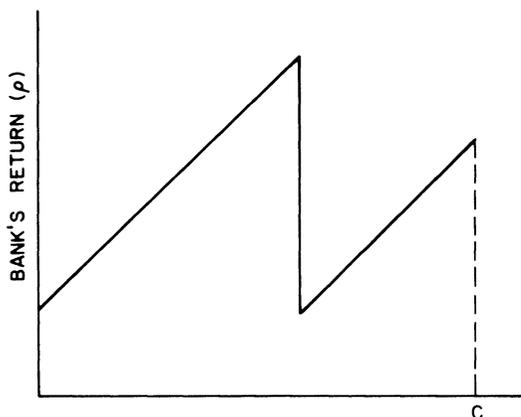


FIGURE 8. INCREASING COLLATERAL REQUIREMENT LOWERS BANK'S RETURNS

PROOF:

This follows immediately upon differentiation of (21)

$$dV_B/dC = -U'_2 \rho^* (1-p) < 0$$

It is easy to show now that this adverse selection effect *may* more than offset the positive direct effect. Assume there are two groups; for low wealth levels, increasing  $C$  has no adverse selection effect, so returns are unambiguously increased; but there is a critical level of  $C$  such that requiring further investments select against the low wealth-low risk individuals, and the bank's return is lowered.<sup>12</sup> (See Figure 8.)

This simple example has demonstrated<sup>13</sup> that although collateral may have beneficial incentive effects, it may also have countervailing adverse selection effects.

A. Adverse Incentive Effects

Although in the model presented above, increasing collateral has a beneficial incen-

<sup>12</sup>If we had not imposed the restriction  $W_0 < \hat{W}_0$ , then there may exist a value of  $W_0$ ,  $\hat{W}_0 > \hat{W}_0$ , such that for  $W_0 > \hat{W}_0$ , individuals self-finance. It is easy to show that  $\partial \hat{W}_0 / \partial C < 0$ , so there is a countervailing positive selection effect. However if the density distribution of wealth is decreasing fast enough, then the adverse selection effect outweighs the positive selection effect.

<sup>13</sup>It also shows that the results of earlier sections can be extended to the risk averse entrepreneur.

tive effect, this is not necessarily the case. The bank has limited control over the actions of the borrowers, as we noted earlier. Thus, the response of the borrower to the increase in lending may be to take actions which, in certain contingencies, will require the bank to lend more in the future. (This argument seems implicit in many discussions of the importance of adequate initial funding for projects.) Consider, for instance, the following simplified multiperiod model. In the first period,  $\theta$  occurs with probability  $p_1$ ; if it does, the return to the project (realized the second period) is  $R_1$ . If it does not, either an additional amount  $M$  must be invested, or the project fails completely (has a zero return). If the bank charges an interest rate  $r_2 \leq \hat{r}_2$  on these additional funds, they will invest them in "safe" ways; if  $r_2 > \hat{r}_2$  those funds will be invested in risky ways. Following the analysis in Section II, we assume that the risk differences are sufficiently strong that the bank charges  $\hat{r}_2$  for additional funds. Assume that there is also a set of projects (actions) which the firm can undertake in the first period, but among which the bank cannot discriminate. The individual has an equity of a dollar, which he cannot raise further, so the effect of a decrease in the loan is to affect the actions which the individual takes, that is, it affects the parameters of the projects,  $R_1$ ,  $R_2$ , and  $M$ , where  $M$  is the amount of second-period financing needed if the project fails in the first period. For simplicity, we take  $R_2$  as given, and let  $L$  be the size of the first-period loan. Thus the expected return to the firm is simply (if the additional loan  $M$  is made when needed)

$$p_1 (R_1 - (1 + \hat{r}_1)^2 L) + \hat{p} (R_2 - [(1 + \hat{r}_1)^2 L + (1 + \hat{r}_2) M])$$

where  $\hat{p} = p_2(1 - p_1)$ ,  $(1 + \hat{r}_1)^2$  is the amount paid back (per dollar borrowed) at the end of the second period on the initial loan and  $\hat{r}_2$  is the interest on the additional loan  $M$ ; thus the firm chooses  $R_1$  so that

$$p_1 = \hat{p} (1 + \hat{r}_2) \frac{dM}{dR_1}$$

Assume that the opportunity cost of capital to the bank per period is  $\rho^*$ . Then its net expected return to the loan is

$$p_1(1+\hat{r}_1)^2L + \hat{p}[(1+\hat{r}_1)^2L + (1+\hat{r}_2)M] - \rho^*[\rho^*L + (1-p_1)M]$$

We can show that under certain circumstances, it will pay the bank to extend the line of credit  $M$ . Thus, although the bank controls  $L$ , it does not control directly the total (expected value) of its loans per customer,  $L + (1-p_1)M$ .

But more to the point is the fact that the expected return to the bank may not be monotonically decreasing in the size of the first-period loans. For instance, under the hypothesis that  $\hat{r}_1$  and  $\hat{r}_2$  are optimally chosen and at the optimum  $\rho^* > p_2(1+\hat{r}_2)$ , the return to the bank is a decreasing function of  $M/L$ . Thus, if the optimal response of the firm to a decrease in  $L$  is an increase in  $M$  (or a decrease in  $M$  so long as the percentage decrease in  $M$  is less than the percentage decrease in  $L$ ), a decrease in  $L$  actually lowers the bank's profits.<sup>14</sup>

**IV. Observationally Distinguishable Borrowers**

Thus far we have confined ourselves to situations where all borrowers appear to be identical. Let us now extend the analysis to the case where there are  $n$  observationally distinguishable groups each with an interior bank optimal interest rate denoted by  $\hat{r}_i^*$ .<sup>15</sup> The function  $\rho_i(r_i)$  denote the gross return to a bank charging a type  $i$  borrower interest  $r_i$ . We can order the groups so that for  $i > j$ ,  $\max \rho_i(\hat{r}_i) > \max \rho_j(\hat{r}_j)$ .

<sup>14</sup>For instance, if some of the initial investment is for "back-up" systems in case of various kinds of failure, if the reduction in initial funding leads to a reduction in investment in these back-up systems, when a failure does occur, large amounts of additional funding may be required.

<sup>15</sup>The analysis in this section parallels Weiss (1980) in which it was demonstrated that market equilibrium could result in the exclusion of some groups of workers from the labor market.

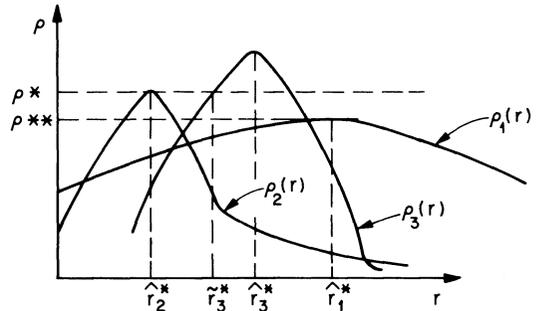


FIGURE 9. IF GROUPS DIFFER, THERE WILL EXIST RED LINING

**THEOREM 13:** For  $i > j$ , type  $j$  borrowers will only receive loans if credit is not rationed to type  $i$  borrowers.

**PROOF:**

Assume not. Since the maximum return on the loan to  $j$  is less than that to  $i$ , the bank could clearly increase its return by substituting a loan to  $i$  for a loan to  $j$ ; hence the original situation could not have been profit maximizing.

We now show

**THEOREM 14:** The equilibrium interest rates are such that for all  $i, j$  receiving loans,  $\rho_i(\hat{r}_i) = \rho_j(\hat{r}_j)$ .

**PROOF:**

Again the proof is by contradiction. Let us assume that  $\rho_i(\hat{r}_i) > \rho_j(\hat{r}_j)$ ; then a bank lending to type  $j$  borrowers would prefer to bid type  $i$  borrowers away from other banks. If  $\rho^*$  is the equilibrium return to the banks per dollar loaned, equal to the cost of loanable funds if banks compete freely for borrowers, then for all  $i, j$  receiving loans  $\rho_i(r_i) = \rho_j(r_j) = \rho^*$ . These results are illustrated for three types of borrowers in Figure 9.

If banks have a cost of loanable funds  $\rho^*$  then no type 1 borrower will obtain a loan; all type 3 borrowers wishing to borrow at interest rate  $\tilde{r}_3$  (which is less than  $\hat{r}_3^*$ , the rate which maximizes the bank's return) will obtain loans—competition for those borrowers drives their interest rate down; while some, but not necessarily all, type 2 borrowers re-

ceive a loan at  $\hat{r}_2^*$ . If the interest rate were to fall to  $\rho^{**}$ , then all types 2 and 3 would receive loans; and some (but not all) type 1 borrowers would be extended credit.

Groups such as type 1 which are excluded from the credit market may be termed "red-lined" since there is no interest rate at which they would get loans if the cost of funds is above  $\rho^{**}$ . It is possible that the investments of type 1 borrowers are especially risky so that, although  $\rho_1(\hat{r}_1^*) < \rho_3(\hat{r}_3^*)$ , the total expected return to type 1 investments (the return to the bank plus the return to the borrower) exceeds the expected return to type 3 investments. It may also be true that type 1 loans are unprofitable to the bank because they find it difficult to filter out risky type 1 investments. In that case it is possible that the return to the bank to an investment by a type 1 borrower would be greater than the return to a type 3 investment if the bank could exercise the same control (judgment) over each group of investors.

Another reason for  $\rho_1(\hat{r}_1^*) < \rho_3(\hat{r}_3^*)$  may be that type 1 investors have a broader range of available projects. They can invest in all the projects available to type 3 borrowers, but can also invest in high-risk projects unavailable to type 3. Either because of the convexity of the profit function of borrowers, or because riskier investments have higher expected returns type 1 borrowers will choose to invest in these risky projects.

Thus, *there is no presumption that the market equilibrium allocates credit to those for whom the expected return on their investments is highest.*

**IV. Debt vs. Equity Finance, Another View of the Principal-Agent Problem**

Although we have phrased this paper in the context of credit markets, the analysis could apply equally well to any one of a number of principal-agent problems. For example, in agriculture the bank (principal) corresponds to the landlord and the borrower (agent) to the tenant while the loan contract corresponds to a rental agreement. The return function for the landlord and tenant appears in Figures 10a and 10b. The central concern in those principal-agent

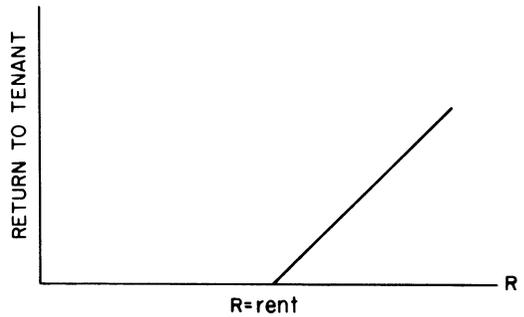


FIGURE 10a

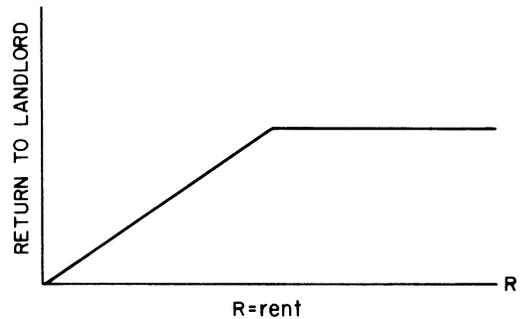


FIGURE 10b

problems is how to provide the proper incentives for the agent. In general, revenue sharing arrangements such as equity finance, or sharecropping are inefficient. Under those schemes the managers of a firm or the tenant will equate their marginal disutility of effort with their share of their marginal product rather than with their total marginal product. Therefore, too little effort will be forthcoming from agents.

Fixed-fee contracts (for example, rental agreements in agriculture, loan contracts in credit markets) have the disadvantage that they impose a heavy risk on the agent, and thus if agents are risk averse, they may not be desirable. But it has long been thought that they have a significant advantage in not distorting incentives and thus if the agent is risk neutral, fixed-fee contracts will be employed.<sup>16</sup> These discussions have not consid-

<sup>16</sup>See, for instance, Stiglitz (1974). For a recent formalization of the principal-agent problem, see Steven Shavell.

ered the possibility that the agent will fail to pay the fixed fee. In the particular context of the bank-borrower relationship, the assumption that the loan will always be repaid (with interest) seems most peculiar. A borrower can repay the loan in all states of nature only if the risky project's returns plus the value of the equilibrium level of collateral exceeds the safe rate of interest in all states of nature.

The consequences of this are important. Since the agent can by his actions affect the probability of bankruptcy, fixed-fee contracts do not eliminate the incentive problem.

Moreover, they do not necessarily lead to optimal resource allocations. For example, in the two-project case discussed above (Section II, Part B), if expected returns to the safe project exceed that to the risky ( $p^s R^s > p^r R^r$ ) but the highest rate which the bank can charge consistent with the safe project being chosen ( $r^*$ ) is too low (i.e.,  $p^s(1+r^*) > p^r R^r$ ) then the bank chooses an interest rate which causes all its loans to be for risky projects, although the expected total (social) returns on these projects are less than on the safe projects. In this case a usury law forbidding interest rates in excess of  $r^*$  will increase net national output. Our 1980 paper and Janusz Ordovery and Weiss show that government interventions of various forms lead to Pareto improvements in the allocation of credit.

Because neither equity finance nor debt finance lead to efficient resource allocations, we would not expect to see the exclusive use of either method of financing (even with risk-neutral agents and principals). Similarly, in agriculture, we would not expect to see the exclusive use of rental or sharecropping tenancy arrangements. In general, where feasible, the payoff will be a non-linear function of output (profits). The terms of these contracts will depend on the risk preferences of the principal and agent, the extent to which their actions (both the level of effort and riskiness of outcomes) can affect the probability of bankruptcy, and actions can be specified within the contract or controlled directly by the principal.

One possible criticism of this paper is that the single period analysis presented above artificially limits the strategy space of lenders.

In a multiperiod context, for instance, banks could reward "good" borrowers by offering to lend to them at lower interest rates, and this would induce firms to undertake safer projects (just as in the labor market, the promise of promotion and pay increases is an important part of the incentive and sorting structure of firms, see Stiglitz, 1975, J. L. Guasch and Weiss, 1980, 1981). In our 1980 paper, we analyze the nature of equilibrium contracts in a dynamic context. We show that such contingency contracts may characterize the dynamic equilibrium. Indeed, we establish that the bank may want to use quantity constraints—the availability of credit—as an additional incentive device; thus, in the dynamic context there is a further argument for the existence of rationing in a competitive economy.

Even after introducing all of these additional instruments (collateral, equity, non-linear payment schedules, contingency contracts) there may exist a contract which is optimal from the point of view of the principal; he will not respond, then, to an excess supply of agents by altering the terms of that contract; and there may then be rationing of the form discussed in this paper, that is, an excess demand for loans (capital, land) at the "competitive" contract.

## VI. Conclusions

We have presented a model of credit rationing in which among observationally identical borrowers some receive loans and others do not. Potential borrowers who are denied loans would not be able to borrow even if they indicated a willingness to pay more than the market interest rate, or to put up more collateral than is demanded of recipients of loans. Increasing interest rates or increasing collateral requirements could increase the riskiness of the bank's loan portfolio, either by discouraging safer investors, or by inducing borrowers to invest in riskier projects, and therefore could decrease the bank's profits. Hence neither instrument will necessarily be used to equate the supply of loanable funds with the demand for loanable funds. Under those circumstances credit restrictions take the form of limiting the num-

ber of loans the bank will make, rather than limiting the size of each loan, or making the interest rate charged an increasing function of the magnitude of the loan, as in most previous discussions of credit rationing.

Note that in a rationing equilibrium, to the extent that monetary policy succeeds in shifting the supply of funds, it will affect the level of investment, not through the interest rate mechanism, but rather through the availability of credit. Although this is a "monetarist" result, it should be apparent that the mechanism is different from that usually put forth in the monetarist literature.

Although we have focused on analyzing the existence of excess demand equilibria in credit markets, imperfect information can lead to excess supply equilibria as well. We will sketch an outline of an argument here (a fuller discussion of the issue and of the macro-economic implications of this paper will appear in future work by the authors in conjunction with Bruce Greenwald).<sup>17</sup> Let us assume that banks make higher expected returns on some of their borrowers than on others: they know who their most credit worthy customers are, but competing banks do not. If a bank tries to attract the customers of its competitors by offering a lower interest rate, it will find that its offer is countered by an equally low interest rate when the customer being competed for is a "good" credit risk, and will not be matched if the borrower is not a profitable customer of the bank. Consequently, banks will seldom seek to steal the customers of their competitors, since they will only succeed in attracting the least profitable of those customers (introducing some noise in the system enables the development of an equilibrium). A bank with an excess supply of loanable funds must assess the profitability of the loans a lower interest rate would attract. In equilibrium each bank may have an excess supply of loanable funds, but no bank will lower its interest rate.

The reason we have been able to model excess demand and excess supply equilibria in credit markets is that the interest rate

directly affects the quality of the loan in a manner which matters to the bank. Other models in which prices are set competitively and non-market-clearing equilibria exist share the property that the expected quality of a commodity is a function of its price (see Weiss, 1976, 1980, or Stiglitz, 1976a, b for the labor market and C. Wilson for the used car market).

In any of these models in which, for instance, the wage affects the quality of labor, if there is an excess supply of workers at the wage which minimizes labor costs, there is not necessarily an inducement for firms to lower wages.

The Law of Supply and Demand is not in fact a law, nor should it be viewed as an assumption needed for competitive analysis. It is rather a result generated by the underlying assumptions that prices have neither sorting nor incentive effects. The usual result of economic theorizing: that prices clear markets, is model specific and is not a general property of markets—unemployment and credit rationing are not phantasms.

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<sup>17</sup>A similar argument to that presented here appears in Greenwald in the context of labor markets.

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