

Towards a multi-valued logic for argumentation

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Abstract. The objective of our paper is to investigate the possibilities of a logical representation for argumentation executed in multi-agent systems. We concentrate particularly on belief's uncertainty and revision that an agent deals with throughout the process of discussion. Moreover, we propose a formalism based on a multi-valued logic that allows for specification of properties of multi-agent systems concerned with argumentation process.

keywords: argumentation, multi-agent systems, a multi-valued logic

1 Introduction

In the last several years, the issue of argumentation has gained increasing attention in the study of multi-agent systems (MAS). As in human society, dialog is a crucial process in the interactions among individuals - it allows agents to revise their beliefs in a dynamic environment. Nevertheless, formal modeling of this process is very difficult and still waits for satisfactory representation. Our objective in this paper is to show a formalism in a multi-valued logic that describes one of the essential aspects of argumentation - how agents' beliefs and/or degrees of belief are influenced through discussion. In our work, we investigate the following questions: (1) when do we deal with argumentation in communication among agents, (2) what formal tools can be applied to describe the process of discussion, (3) how expressive will such formal theory of argumentation be?

(1) *When do we deal with argumentation in communication among agents?* As the starting point, we want to identify the integral elements that constitute argumentation. As defining conditions, they are intended to differentiate this particular kind of communication from other types of agents' interactions. On the informal level, let us understand argumentation as the process in which:

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- (i) there is a difference of opinion,
- (ii) there are arguments to defend one's opinion,
- (iii) there is an aim to convince somebody of something.

These characteristics indicate what need to be included in a theory which aims to describe the notion of argumentation. The first important point to be outlined is that the agents play different roles in the dialogue. Condition (i) requires that there are two parties which represent contrary opinions - one that plays the role of the proponent (the one that proposes the thesis), and second that plays the opponent (the one that opposes, disagrees with the thesis). Condition (ii) specifies that the parties introduce arguments in order to defend their opinions. The first party, that is obliged to defend his point of view, is the proponent (in rhetoric this obligation is called *onus probandi*). Simple argumentation occurs when only one party presents their arguments in a disagreement. When an opposing defense follows, complex argumentation occurs. Condition (iii) demands that in argumentation we must also have someone who constitutes the target of persuasion. He plays the role of the audience of argumentation. Let us note that when the proponent convinces someone other than the party that has opposite opinion to the proponent, then we may say that the opponent and audience constitute disjoint sets.

Secondly, we want agents to discuss their opinions (see - conditions (i), (ii)), i.e. their beliefs, not their knowledge. Our theory is to apply for the circumstances when agents do not have complete or certain information about the world. However, it is assumed in MAS that an agent's knowledge is true information. Thus, we consider beliefs in order to describe discussions between agents about things they believe are true, but are not necessarily true.

Third, let us note that the difference of opinion (condition (i)) arises even when the proponent believes the thesis seems to be true and the opponent believes that it seems to be false. Such an example of the disagreement cannot be expressed by means of non-graded beliefs. Similarly, the proponent may consider his argumentation as success (in the sense that he convinced his audience condition (iii)) even though the audience is not fully certain that the thesis is true. Argumentation is here perceived as the kind of "game" involving agents' levels of confidence about the structure of the world. For these and other reasons, we do not only consider agents' beliefs, but also take into account the degrees to which agents believe something is true.

Lastly, when condition (iii) is fulfilled (i.e. the audience becomes convinced that the thesis is true) then the argumentation leads to the change of agents' beliefs. Therefore, in this paper we consider this aspect of belief revision.

(2) *What formal tools have to be applied to describe the process of argumentation?* At this stage, we have to make a choice with regard to formal tools that we will use to construct the logic for argumentation. In order to guarantee that our choices are well-founded, we follow the characteristics highlighted above.

First, we decide how to represent the utterances which are the subject of disputation. As the starting point we take propositional logic. In the future we plan to investigate the possibility of formalization in more expressive logic.

Second, considering the role of belief in argumentation, we use epistemic logic to describe cognitive attitudes of agents. We follow a convention accepted broadly in multi-agent systems and present epistemic logic in a modal manner, i.e. with the modal operator “belief”.

Third, in formal theory of argumentation we need to represent the degrees to which agents are certain of their beliefs. We need to have the ability to express e.g. that some arguments lead to the stronger conviction than other arguments or that the degree of an agent’s certainty of the truthfulness of given information has increased as a consequence of the discussion. In particular, we are interested in a formal description of operators that represent belief-attitudes of the type: “according to me it is certainly false”, “according to me it seems to be false”, “according to me it is neither true nor false”, “according to me it seems to be true”, “according to me it is certainly true”. In order to model graded beliefs, different logics (multi-valued, probabilistic, fuzzy) may be applied. In our talk we will present results with regard to multi-valued logic. In the future we plan to compare them with the results of the research project for the probabilistic and fuzzy logic in respect of their contribution to the description of argumentation process.

Lastly, we will apply the tools of algorithmic logic to express that an agent revised its beliefs as a consequence of argumentation.

(3) *How expressive will such formal theory of argumentation be?* On the last stage of our project we want to show what can be expressed by means of a formal system, what kinds of specific interaction-communication situations among agents may be modeled. In particular, we are able to express agents uncertainty, to define the process of argumentation, to consider successful convincing, belief revision and to describe specific instances of persuasion.

Thus in paragraph 2, we establish the framework of the logic for argumentation. In the paragraph 3, we propose an example of agents’ argumentation expressed in the terms of introduced formalism.

1.1 Related Work

Argumentation plays a crucial role in multi-agent negotiations and dialogues. It provides tools for modeling social commitments, resolving conflicts between self-interested agents, producing mutually acceptable compromises, etc. The process of argumentation finds application in many domains such as auctions [5], electronic commerce, exchanges for equities and commodities, transportation, as well as modeling multi-agent interactions in education-related applications [9].

In the literature, there are several propositions of argumentation-based approaches [7, 6, 10]. However none of them touches on the topic of graded beliefs and their applications. Moreover, most of the works concern the problem of how to persuade somebody to do something rather than to convince somebody of something (and, what follows, to change agents beliefs). Therefore we believe there is a need for an alternative framework. We take note of the fact that agents’ (as well as humans’) beliefs are characterized by some degree of certainty. For example, in medicine the greatest authorities carry on many polemics based on

hypotheses which are questionable and, what is worse, impossible to verify. Notice, that there is no way to establish exactly when a healthy man becomes ill. Thereby, the statement “There are no healthy people” can not be rejected but it can be recognized as doubtful at the most.

Consequently, to convince an agent of something means to increase its degree of certainty to “*it is certainly true*” or “*it seems to be true*”. Formally, we are going to achieve this by exploring a graded belief operator which is defined by means of a multi-valued logic and the adoption of a logic of programs to enable the revision of agents’ beliefs.

Our paper is inspired by work on issue of argumentation processes in human society [2, 1, 8] as well as investigations on a multi-valued version of the logic CTL* [3] and multi-valued logic of knowledge and time [4].

2 The logical model

In this section we specify the syntax and semantics of the main elements we use in our framework. The formal language we propose is based on the classical propositional logic extended with new operators that allow for reasoning about argumentation processes in multi-agent systems. In this paper we focus on modeling agents’ beliefs and changes of these beliefs as a result of accepting new arguments.

2.1 Syntax and semantics

Let $Agt = \{1, \dots, n\}$ be a set of agents, PV a set of propositional variables, Arg a finite set of **arguments**. We identify arguments with actions which agents can perform to convince the others of new facts or relationships. Therefore by argumentation we mean a process of presenting arguments, i.e., sequence of actions that can change worlds of agents and thereby their beliefs.

Definition 1 (Argumentation program). *By an argumentation program K we shall understand a sequence of arguments, i.e., $K = (a_1 ; \dots ; a_k)$ where $a_1, \dots, a_k \in Arg$.*

The set of all argumentation programs will be denoted Π .

Since in the argumentation process it is important to distinguish proponent from other parties we need to be able to indicate who gives arguments. So, every program K is parameterized with an agent (which plays the role of the proponent), what will be denoted by $(K : i)$ for $K \in \Pi$ and $i \in Agt$.

The language of our formalism can be defined by the following syntactic rules.

Definition 2 (Syntax). *The set of formulas F is defined as follows:*

$$\alpha ::= p \mid \neg\alpha \mid \alpha \vee \alpha \mid \diamond(K : i)\alpha \mid B_i\alpha$$

where $p \in PV$, $K \in \Pi$, $i \in Agt$.

Other Boolean connectives are defined from \neg and \vee as usual.

Intuitively, a formula “ $\diamond(K : i)\alpha$ ” expresses “it is possible that after argumentation process K , which is leaded by an agent i , condition α holds”. Of course we are especially interested in formulas α that describe beliefs of agents. A formula “ $B_i\alpha$ ” expresses that the agent i is convinced of α .

Consider an algebra $\mathcal{A} = (L, \cup, \cap, \sim)$ with $L = \{\mathbf{0}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \mathbf{1}\}$ and operations \cup, \cap, \sim defined in Table 1. It is easy to observe that $\mathcal{L} = (L, \cup, \cap)$ is a 5-valued lattice. We use it as a logical domain of interpretation for the formulas of our multi-valued logic. Within this lattice we distinguish a set $\mathcal{D} = \{\frac{3}{4}, \mathbf{1}\}$ of designated values, which are the analogue of truth in two-valued logic. More precisely, a formula is said to be *truth* in a given world if its value in this world is in \mathcal{D} .

\sim	
$\mathbf{0}$	$\mathbf{1}$
$\frac{1}{4}$	$\frac{3}{4}$
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{1}{4}$
$\mathbf{1}$	$\mathbf{0}$

\cap	$\mathbf{0}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\mathbf{1}$
$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$\frac{1}{4}$	$\mathbf{0}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{2}$	$\mathbf{0}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{3}{4}$	$\mathbf{0}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$
$\mathbf{1}$	$\mathbf{0}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\mathbf{1}$

\cup	$\mathbf{0}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\mathbf{1}$
$\mathbf{0}$	$\mathbf{0}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\mathbf{1}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\mathbf{1}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\mathbf{1}$
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\mathbf{1}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

Table 1. Definition of operators \sim, \cap, \cup

The well-formed formulas of our framework are evaluated in the following model.

Definition 3 (Model). *Given a set of agents $Agt = \{1, \dots, n\}$ a model is a tuple $M = (W, T, R, V, \mathcal{L})$, where*

- W is a set, called the set of worlds,
- $T = \{T_a\}_{a \in Arg}$, where $T_a : W \times Agt \times W \longrightarrow \{\mathbf{0}, \mathbf{1}\}$ is a total function, called the interpretation of an argument a ,
- $R = \{R_i\}_{i \in Agt}$, where $R_i : W \times W \longrightarrow L$ is a total function, called the epistemic accessibility function for an agent i ,
- $V : W \times PV \longrightarrow L$ is a function, called the valuation function,
- $\mathcal{L} = (L, \cup, \cap)$ is the 5-valued lattice.

Every function T_a (for $a \in Arg$) indicates changes in a system if one of agents gives the argument a . Therefore $T_a(s, i, w)$ equals either $\mathbf{1}$ if the state w can be reached from the state s if the agent i gives the argument a or $\mathbf{0}$ otherwise.

The family of functions T can be extended to give an interpretation for any $K = (a_1; \dots; a_k) \in \Pi$. Let $T_K : W \times Agt \times W \longrightarrow \{\mathbf{0}, \mathbf{1}\}$ be a function such that $T_K(s, i, t) = \mathbf{1}$ iff there exists a sequence of worlds s_0, \dots, s_k such that $s_0 = s$, $s_k = t$ and for every $j = 1, \dots, k$, $T_{a_j}(s_{j-1}, i, s_j) = \mathbf{1}$. $T_K(s, i, t) = \mathbf{0}$ in all other cases.

The semantics of formulas is given by the function $[\cdot]^M(s)$, which, for each formula α , a model M , and a world s in M , returns the value of α at world s of the model M , defined as follows:

- $[p]^M(s) = V(s, p)$ for $p \in PV$
- $[\neg\alpha]^M(s) = \sim [\alpha]^M(s)$
- $[\alpha \vee \beta]^M(s) = [\alpha]^M(s) \cup [\beta]^M(s)$
- $[\diamond(K : i)\alpha]^M(s) = \bigcup_{w \in W} (T_k(s, i, w) \cap [\alpha]^M(w))$
- $[B_i\alpha]^M(s) = \bigcap_{w \in D_i(s)} (R_i(s, w) \cap [\alpha]^M(w))$,
where $D_i(s) = \{w \in W : R_i(s, w) \neq \mathbf{0}\}$

As we have mentioned above, the aim of argumentation is to change agents' beliefs. However, we assume that every belief has its degree. In particular, we are interested in a formal description of operators that represent belief with the degree of $\mathbf{0}$ and $\mathbf{1}$ (the belief-attitudes of the type "according to me it is certainly false", "according to me it is certainly true", respectively), belief with the degree of $\frac{1}{2}$ (the neutral belief, i.e. one accepts that it is neither true nor false), and the degrees of certainty belonging to the open intervals: $(0, \frac{1}{2})$ - the belief-attitude of the type "according to me it seems to be false", which will be symbolized as $\frac{1}{4}$, and $(\frac{1}{2}, 1)$ - the belief-attitude: "according to me it seems to be true", which will be labelled with the numeric representation of $\frac{3}{4}$. The main reason, for which we do not consider particular values from the intervals $(0, \frac{1}{2})$ and $(\frac{1}{2}, 1)$, is that in this theory of argumentation we do not see the necessity to compare two beliefs that have values from the same interval e.g. $\frac{1}{20}$ and $\frac{1}{10}$ (at least at this stage of consideration). Here, it is only important that both of these beliefs express the same attitude of the agent i.e. the attitude of the type: "I am not sure if it is false, however I bet more on its falsity than on its truthfulness". Summarizing - we can say that if $[B_i\alpha]^M(s) = v$ then if v equals $\mathbf{0}$ then the agent is certain that α is not true at s . If v equals $\frac{1}{4}$ then the agent believes α seems to be false. If v equals $\frac{1}{2}$ then the agent does not have any opinion about α . If v equals $\frac{3}{4}$ then the agent believes α seems to be true. If v equals $\mathbf{1}$ then the agent is fully convinced of α .

For example we can say that at world s an agent i believes "every red cat is aggressive" in the degree $\mathbf{1}$, i.e., $[B_i(\text{every red cat is aggressive})]^M(s) = \mathbf{1}$. It means that the agent i is sure of this fact. On the other hand, the same agent can believe that "every bird can fly" in the degree $\mathbf{0}$ since it knows that a penguin is a bird and it does not fly. We can also say that this agent believes that "every woman is a bad driver" in the degree $\frac{1}{2}$ what means that it does not know any pros and cons.

The issue of modeling agents' beliefs is undoubtedly a key problem in argumentation theory. There are many ways of defining the value of operator B . For example we can assume that

$$[B_i\alpha]^M(s) = \mathcal{I}\left(\frac{\sum_{w \in D_i(s)} (R_i(s, w) \cdot [\alpha]^M(w))}{|D_i(s)|}\right)$$

where \sum and \cdot are usual operations on reals, $|D_i(s)|$ denotes cardinality of $D_i(s)$, and $\mathcal{I}(1) = \mathbf{1}$, $\mathcal{I}(t) = \frac{3}{4}$ if $t \in (\frac{1}{2}, 1)$, $\mathcal{I}(\frac{1}{2}) = \frac{1}{2}$, $\mathcal{I}(t) = \frac{1}{4}$ if $t \in (0, \frac{1}{2})$, and $\mathcal{I}(0) = \mathbf{0}$.

The choice of a definition may depend on individual applications. In our presentation we are going to show different definitions of semantics of operator B and discuss desirable and undesirable properties of operator B that follow from these definitions.

For instance, since we want agents to maximize certainty, one of such desirable properties of the operator B is the following: “if $[B_i\alpha]^M(s) = v$ then $[B_i\alpha]^M(s) \neq w$ ” for $w \in L \setminus \{v\}$. If the agent believes α in the degree $\mathbf{1}$, then we do not want it to believe α in any other degree. In particular, we expect the agent not to believe α in the degree $\frac{3}{4}$. We want to understand that if the agent is absolutely sure that “ $2+2=4$ ” (i.e. with the degree of $\mathbf{1}$), then it will not assume that “ $2+2$ ” probably gives 4 (i.e. with the degree of $\frac{3}{4}$). Other desirable properties are: “if $[B_i\alpha]^M(s) = \mathbf{1}$ then $[B_i\neg\alpha]^M(s) = \mathbf{0}$ ” or “if $[B_i\alpha]^M(s) \in \mathcal{D}$ then $[B_i\neg\alpha]^M(s) \notin \mathcal{D}$ ”. On the contrary, the property of graded operator B , described by the next condition, is not desired in our logical model for argumentation: “if $[B_i\alpha]^M(s) = v$ then it is not true that $[B_i\neg\alpha]^M(s) = v$ ”. One could consider such property, since the formula is a “direct translation” for the axiom of the epistemic logic with not graded belief operator. However, such property does not hold for graded beliefs, as for the degree of value $\frac{1}{2}$, for example.

In some cases there is a need to use a belief operator syntax of which indicates the degree of agents’ beliefs, e.g., $B_i^v\alpha$ with $v \in L$. The logical value of this operator could be defined as follows:

$$[B_i^v\alpha]^M(s) = \mathbf{1} \text{ iff } [B_i\alpha]^M(s) = v \text{ and } [B_i\alpha]^M(s) = \mathbf{0} \text{ otherwise.}$$

The above listed extensions of our formalism we intend to study in next papers.

2.2 Argumentation

According to the integral elements that we indicated for argumentation (conditions (i)-(iii) in the introduction of this paper), we propose to understand argumentation as follows:

Let *prop*, *op*, *aud* symbolize proponent, opponent and audience, respectively. The sequence: a_1, \dots, a_k is argumentation for *prop*, *op*, *aud* in support of α iff:

1. *prop* believes α and *op* does not believe α ,
2. *prop* presents arguments a_1, \dots, a_k in order to make *aud* believe α .

A difference of opinion (required by the condition (i) from the introduction) is guaranteed by point (1). When the proponent believes thesis α is true and at the same time the opponent is not convinced that α is true, then here we have disagreement. Note that in human society, the disagreement may be ostensible - the argumentation will start when the proponent thinks that someone opposes

him, even though actually he may not oppose. However, here we do not consider such situations. We want agents to discuss only when there is a real disagreement between them, because only such argumentation may actually revise agents' beliefs.

The point (2) expresses that the proponent introduces arguments a_1, \dots, a_k (what fulfils condition (ii) from the introduction) in order to convince somebody of something (condition (iii)), i.e. the audience of the thesis α .

Above we have introduced the notion of **simple argumentation**. The sequence of such simple argumentations will be called **complex argumentation**.

When formulating properties of multi-agent systems in L we use the following derived operator:

$$SuccessSimpleArg(i, j, \alpha) = B_i\alpha \wedge \neg B_j\alpha \wedge \bigvee_{K \in \Pi} \diamond(K : i)B_j\alpha$$

where $i=prop$ and $j=op=aud$.

Consequently, the agent i can *convince* the agent j of α if the agent i believes α and the agent j does not believe α and there exists a program after execution of which the agent j believes α .

We assume here that the discussion takes place between exactly two agents (agent i plays the role of proponent and agent j plays both the role of opponent and audience). We do not want to describe one agent's inner-dialogue (i.e. when $prop=op=aud$) or many agents' social-discourse, where two parties fight in order to convince the third party (i.e. when $prop \neq op \neq aud$, like e.g. in political debates). Instead, we are interested in such situations where we have two agents: first agent ($prop$), who defends its thesis with arguments, and second agent ($op=aud$), who opposes to $prop$ and, at the same time, is to be convinced by $prop$.

According to how we understand operator B, the formula $B_i\alpha$ expresses that proponent-agent i believes the thesis α at least in the degree of $\frac{3}{4}$. Further, the formula $\neg B_j\alpha$ expresses that the opponent-agent j does not believe the thesis at that degree, what means that he believes the thesis at the degree $\frac{1}{2}$ at the most. Another words, we may say that the opponent believes that α is certainly false (degree $\mathbf{0}$) or rather false (degree $\frac{1}{4}$) or it believes that α is neither false nor true (degree $\frac{1}{2}$). Each of these degrees of the opponent's confidence generates disagreement - even the degree of $\frac{1}{2}$. When the opponent has a neutral belief of the thesis, it does not agree with proponent belief-attitude, i.e. $\frac{3}{4}$ or $\mathbf{1}$ (intuitively - the proponent bets rather for truthfulness than for falsity of the thesis). For this reason the condition for the opponent should not be formulated as follows: $B_i\neg\alpha$. In such a case we would not cover the possibility of disagreement resulted from neutral belief-attitude of the opponent.

The above operator describes successful simple argumentation. The argumentation is successful when there exists a program, which is processed by proponent-agent i and after execution of which the audience-agent j believes α in the degree at least $\frac{3}{4}$. From our point of view, this notion is very important, because only successful argumentation revises agent's beliefs. Thus, we concen-

trate especially on conditions under which one agent is able to convince another through processing the argumentation and not on conditions under which agent is able to formulate argumentation.

When we consider complex argumentation, the condition, under which it is successful, will alter. Next derived operator intuitively says that after “n”-step quarrel the agent i can convince the agent j of α :

$$\begin{aligned} \text{SuccessComplexArg}(i, j, n, \alpha) = & B_i\alpha \wedge \neg B_j\alpha \wedge \\ (& \bigvee_{K_1, \dots, K_n \in \Pi} \bigvee_{t_1, \dots, t_n \in \{i, j\}} \diamond(K_1 : t_1) \dots \diamond(K_n : t_n) B_j\alpha) \end{aligned}$$

Note that in such a case the agent i and the agent j may change the roles of proponent and opponent with one another and then they constitute an audience for each other.

3 Example

Next we show an example which justify the use of multiple logical values for modeling beliefs and show application of our formalism.

Consider two scientists: A_1 and A_2 representing three medical clinics C_1 , C_2 and C_3 . They carry on conversation in which scientist A_1 tries to convince scientist A_2 that vitamins A i E can prevent cancer. We assume that both scientists are trustworthy.

A_1 : I advance a hypothesis that vitamins A i E can prevent cancer.

A_2 : I did some research into this subject in my medical center (C_1) and affirmed that these vitamins have no influence on cancer.

A_1 : I'll try to sell you on this hypothesis. Note that you should consider results obtained in all medical centers in our district.

A_2 : You are right, but it does not change my mind now since I do not know these results.

A_1 : There are four medical centers C_1, C_2, C_3, C_4 in our district. I work in two of them (C_2, C_3) and I know that results of the research confirm the hypothesis there.

A_2 : Assume it is true. It still does not convince me since we do not know results from the medical center C_4 .

A_1 : Yes, but this medical center looks after workers who have permanent contact with asbestos and we can suppose that most of them had been ill before they started to eat vitamins A i E. Therefore results obtained in this center are unreliable and we should take no notice of them.

A_2 : Now, I can agree with you about the hypothesis. However I am not one hundred percent sure because of my own research.

Let's formalize this discussion. Assume two agents $Agt = \{A_1, A_2\}$, a set of worlds $W = \{s_1, \dots, s_{14}\}$, one proposition $p =$ “Vitamins A i E can prevent

cancer”, and three arguments a_1 = “we need to consider all medical clinics”, a_2 = “in clinics C_2 and C_3 the hypothesis is confirmed”, a_3 = “results from the clinic C_4 are unreliable”. Functions T, R_1, R_2 , and V of a model M are shown in Figure 1.

Worlds s_1, \dots, s_4 represent changes which follow arguments a_1, a_2, a_3 given by the agent A_1 . Worlds s_5, \dots, s_{14} indicate what results of the research on the hypothesis p are. So, for every $s_m \in \{s_5, \dots, s_{14}\}$, $s_m = (s_m[1], s_m[2], s_m[3], s_m[4])$ with $s_m[n] \in \{N, Y, 0\}$ ($n = 1, 2, 3, 4$). If $s_m[n] = N$ it means that results in medical center C_n are negative, if $s_m[n] = Y$ it means that results in medical center C_n are positive, if $s_m[n] = 0$ it means that we do not take results from medical center C_n into consideration. According to this $s_5 = (N, 0, 0, 0)$, $s_6 = (N, N, N, N)$, $s_7 = (N, N, N, Y)$, $s_8 = (N, N, Y, N)$, $s_9 = (N, Y, N, N)$, $s_{10} = (N, N, Y, Y)$, $s_{11} = (N, Y, N, Y)$, $s_{12} = (N, Y, Y, Y)$, $s_{13} = (N, Y, Y, N)$, $s_{14} = (N, Y, Y, 0)$.

Notice (cf. Figure 1), that at world s_1 the agent A_2 considers as his epistemic alternative only world s_5 since he is interested in results of the research only in medical center C_1 . At world s_2 (after receiving the argument a_1) the agent A_2 considers as his epistemic alternative eight worlds (s_6, \dots, s_{13}) since he knows that in C_1 results are negative and takes into account all possible results in all other medical centers. At world s_3 (after receiving the argument a_2) the agent A_2 eliminates these eight worlds to two of them in which results of the research in medical centers C_2, C_3 are positive. Finally, at world s_4 (after receiving the argument a_3) the agent A_2 considers as his epistemic alternative only world s_{14} in which he disregards results of the research from C_4 .

Observe that: $[B_{A_1}p]^M(s_1) = \mathbf{1}$ and $[B_{A_2}p]^M(s_1) = \frac{1}{4}$. Consequently, $[\neg B_{A_2}p]^M(s_1) = \frac{3}{4}$. Moreover, $[B_{A_2}p]^M(s_4) = \frac{3}{4}$ and $[\diamond(a_1, A_1)\diamond(a_2, A_1)\diamond(a_3, A_1)B_{A_2}p]^M(s_1) = \frac{3}{4}$. Therefore, $[SuccessSimpleArg(A_1, A_2, p)]^M(s_1) = \frac{3}{4}$. As a result the agent A_1 can convince the agent A_2 of the hypothesis p .

4 Conclusions and future work

This paper discusses some initial work on the subject of the application of a multi-valued propositional logic for modeling agents’ beliefs in argumentation process. We have introduced a framework which allows the capturing of important features of agents’ beliefs: uncertainty in reasoning about facts and relationships.

In future work we plan to compare different interpretations of the graded belief modality and analyze how logical properties of the modality vary with these interpretations. It is a key issue in our research since multi-agent argumentation, negotiation, dialogue, and agents’ beliefs must satisfy specific requirements which are compatible with the intuition.

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