Participants' Perceptions and Experiences with Worked-out Examples in Technical Calculus

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Introduction

Calculus is one of the most challenging courses in college. Much of the difficulty stems from the fact that during class they work diligently to copy all the notes, but have little time to digest the material. When the instructor is lecturing in class, the material seems understandable and easy. However, when students attempt the assigned work on their own, they find that the problems are more difficult and confusing than anticipated. Gunawardena states that "students who enter college are often under prepared and lack the background and motivation to succeed in college-level mathematics" (2002, p.108). Ainsworth (1994) argues that students who come to college without an adequate background in math will likely withdraw from the course or quit performing when a math class becomes difficult. Students who are under prepared --- often even those who are adequately prepared -- fail to be successful because the class becomes difficult in their eyes and they don't believe that they can succeed. Half of the battle of helping students become successful in a course is to get them to believe that they can succeed and that they have the ability to learn and to do mathematics. Worked-out examples are one way for students to build confidence in their mathematics ability and build mathematical schema.

Motivation

Many students struggle to comprehend calculus at the university level each year. Reasons for student difficulties can be contributed to a host of factors, some students who struggle with calculus do so because of the difficulty in applying theory to both procedural and conceptual problems. This difficulty results in a DFW rate ranging from forty to sixty percent at many institutions in the United States. The DFW rate at the university where this study was conducted has ranged between thirty and sixty-four percent and averaged nearly forty-three percent. This data was taken from course records for three years prior to the beginning of this study. To promote better success rates in the course, instructors offered voluntary discussion sessions using the worked-out method.

Background

Memory, Cognitive Load Theory, and Worked-out Examples

There are three types of memory: sensory, long-term, and working. Our senses -- sight, sound, smell, taste, and touch -- serve as stimuli for our sensory memory. Long-term memory, which is similar to a hard drive on a computer, is where the immense body of knowledge and skills is located. Finally working memory is where we think, solve problems, and are expressive. In general, everything that we “know” is stored in long-term memory and, through a query of working memory, activation occurs when needed. Miller (1956) says that working memory has a limited capacity that can deal with no more than about seven chucks of information simultaneously. One thing that helps to expand the capacity of working memory slightly is combining the senses to present information. Either some or all of the information will be lost during processing if the capacity of working memory is exceeded, unless information is recorded in a permanent form as it is being processed.

The discipline of cognitive science deals with the mental processes of learning, memory, and problem-solving. The total load on working memory at any moment in time is referred as the cognitive load. Miller's (1956) theory that most people can retain seven "chunks" of information in their working memory was the beginning of cognitive load theory. Simon and Chase's (1973) research, where they studied expert and novice chess players, showed that when expert chess players were presented with a game configuration that could occur during a regular chess game for a few moments and the configuration was then removed, they could reconstruct the same game configuration much better than novice chess players. However, when a configuration did not come from an actual chess game, expert and novice chess players showed no difference in their ability to reconstruct the game configurations. Just like the chess experts, problem-solving experts have an immense knowledge of problem situations and have constructed many mathematical schema, or "a cognitive structure that specifies both the category to which a problem belongs and the most appropriate moves for problems of that category" (Sweller and Owen, 1989) to activate when needed.

John Sweller (1988) developed cognitive load theory while studying problem-solving and has defined it to state that 'optimum learning occurs in humans when one minimizes the load on working memory which in turn facilitates changes in long term memory'. Cognitive load theory, which deals with the architecture of human cognition, has broad implications for instructional design (Sweller, 1999) and current research is focused on differentiating three types of cognitive load: intrinsic cognitive load, germane cognitive load, and extraneous cognitive load. For further information on cognitive load you can start by reading the following papers can be referenced (Ayres, 2006; Sweller, 1988; Sweller, 2006; and Sweller, van Merrienboer, Paas, 1998). We will focus our attention on "The Worked-Out Example" research, which falls under cognitive load theory.

Generally, mathematics classes, as well as other STEM courses, are taught by lecturing on the new topic, presenting or demonstrating the concepts through a few examples, and assigning homework practice problems so students will learn the material that has just been discussed. The good students practice the problems assigned within a short time of the lecture and begin to master the material, while other students procrastinate for long periods of time before they decide to work the assigned problems. When students procrastinate or simply cannot focus on the covered material until a later time, they have more difficulty remembering what was said during lecture and/or details of the instructor's examples. Most, if not all, instructors use examples in class to illustrate the content's key principles to their students. However, students have little or no time to absorb the examples before another example or more theory is covered when taking notes in class. Sweller and Owen (1989) state that "some views of mathematics and the way it should be taught owe more to tradition than to our current knowledge of cognitive processes" (pg. 322).The worked example theory would place emphasis on worked examples in class by coupling problems solved in class with active student participation by having students work similar problems. In fact, research studies (Cooper and Sweller, 1985; Ward and Sweller, 1990; Zhu and Simon, 1987; Carroll, 1994, Tarmizi and Sweller, 1988) present students with a worked example on paper and tell them to study the example. Once the students are done studying the worked example, the instructor asks the student to solve a similar problem without any help from the worked example. It has been suggested that worked examples reduce the cognitive load on a student and might optimize schema acquisition (Sweller and Owen, 1989; Sweller and Cooper, 1985).

Worked examples are focused on skill acquisition in a subject and Trafton & Reiser found that "the most efficient way to present material to acquire a skill is to present an example, then a similar problem to solve immediately following" (1993, p. 1022). Worked examples have been used in a many different disciplines. To mention just a few studies that have been done in STEM fields: mathematics (Cooper and Sweller, 1985) and (Zhu and Simon, 1987), engineering (Chi et al., 1989), physics (Ward and Sweller, 1990), computer science (Catrambone, Yuasa, 2006), and chemistry (Crippen, and Boyd, 2007). Furthermore, A. Renkl has done studies in education with worked examples. One such study is (Hilbert, Schworm, and Renkl, 2004). The questions guiding this study were:

1. What are student's perceptions and experience with worked-out examples in Calculus?
2. What are the student's perceptions on how the worked-out examples help them in the course?
3. In what ways, if any, do worked-examples build self-efficacy in student's ability to learn material and instill confidence that they will be successful in Calculus?

LITERATURE REVIEW

Sweller and Cooper (1985) conducted one of the first studies on worked-out examples. Through five experiments they examined the use of worked-out examples as a substitute for problem solving. The first experiment found that the more experienced students had a better cognitive representation of algebraic equations than less experienced students as measured by their ability to (i) recall equations, and (ii) distinguish between perceptually similar equations on the basis of solution mode. Sweller and Cooper (1985) concluded that there was "evidence that expertise in solving algebra manipulations problems is, at least in part, schema based." (p. 67) It should be noted that during this experiment students were only asked to read and to make sure they understood the worked-out examples. Experiments 2 through 5 integrated an alternating pattern between worked-out examples and conventional problems because it increases the motivation for students to read and to understand the worked-out example if they have to solve another conventional problem immediately after the worked-out example.

The second experiment established that the worked-out example group (experimental group) required significantly less time during the acquisition phase than the conventional problem group (control group) and that the control group had a greater number of test errors than the experimental group.

The third experiment differed from second experiment by adding a self-explanation step after the worked-out example group read and stated that they understood the examples. The results of the experiment showed that worked-out example group spent significantly less time during both the acquisition phase and the test phase. The reason for the reduction in time is two-fold: (1) the worked-out example group had significantly less mathematical errors than the conventional problem group during the test phase, and (2) the conventional problem group sometimes unnecessarily expanded expressions which cause less efficient solutions.

The fourth experiment differed from previous experiments by varying the problems in the test phase from problems similar to the worked-out examples to problems that were structured differently (transfer problems) from the worked-out examples and conventional problem. This experiment wanted to determine whether students could transfer their knowledge from the acquisition phase to these transfer problems. Again the conventional problem group took significantly more time during the acquisition and test phase when each group started with a similar problem and then worked a dissimilar problem. In contrast, the two groups showed no significant difference in the test phase when each group was presented with a dissimilar problem and then a similar problem. Sweller and Cooper concluded that while “worked examples are of assistance to students when faced with similar problems, the advantage does not extend to dissimilar problems” (p. 83).

The final experimental setup was identical to the setup of fourth experiment. However, this time the worked-out example group and the conventional group spent the same amount of time during the acquisition phase of the experiment. It was hypothesized that the worked-out example group would be able to work through many more problems than the conventional group and hence perform even better during the test phase than the conventional group. Although the worked-out example group worked through more problems in the acquisition phase, the results were nearly the same as in the forth experiment.

Zhu and Simon (1987) demonstrated the feasibility and effectiveness of teaching mathematical skills through chosen sequence of worked-out examples and problems in a

Chinese-middle school's algebra and geometry curriculum – and without lectures or other direct instruction.

Chi et. al. (1989) showed that while students studied worked-out examples, "good" students generally monitored their own understanding and misunderstanding through self-explanations. Compare this to "poor" students who did not generate sufficient self-explanations or monitor their learning inaccurately. They found "poor" students relied heavily on examples.

Ward and Sweller (1990) established that students who used worked-out examples formatted to reduce the need for students to mentally integrate multiple sources of information achieved test performances superior to either those exposed to conventional problems or to those shown worked-out examples that required students to split their attention.

There is a gap in the literature on the perception and experiences of students with worked-out examples. Chi et. al. studied students self-regulation of their solutions but did not look at students perceptions and experiences with worked-out examples. This paper ties to fill in a small part of this gap by looking at students' perception and experience with worked-out examples in a technical calculus course.

METHODOLOGY

Participants and Setting

The participants in the study were twenty students out of over ninety students in a technical calculus class at a research university in the southwest part of the United States. Also one other student participated for a month before choosing to end participation and two other students who were non-participants interviewed at the end of the spring semester in which the study was conducted. The worked-out example method was conducted in the same computer classroom for every session.

Qualitative data was collected through surveys, interviews, and course documents. Quantitative data was collected through course grades, course attendance records, discussion session attendance, and pre- and post- algebra assessments. We will concentrate on the data collected through interviews to analyze students' perceptions and experiences with the worked-out examples.

DISCUSSION AND RESULTS

Students Perspectives on Worked-Out Examples

The stories of a few of the participants provide additional depth into students' mathematical background and into perceptions and experiences with worked-out examples. The following three students, Alex, Rachel, and Henry, are three of the twenty students who participated in the voluntary discussion sessions and used the worked-out example method. These three stories form two viewpoints: (1) how two under prepared algebra students used the worked-out example method to change from a state where they were not confident in mathematics and thought that they would not be successful in the course to a state where they were very confident with calculus and were successful in the course, and (2) how a more prepared algebra student who had little interest in mathematics due to past experiences and who has little confidence in his ability to earn a grade above a C became very confident in his ability, loved mathematics again, and was successful in the course. These viewpoints are examples of how the worked-out example method helped three students, although it is not implied that these results could be generalized to other students with similar beginning states.

The students in these three states varied in major, age, pre-assessment on algebra, math background, and previous college G.P.A. Pseudonyms are used instead of the participants' real names to ensure confidentiality.

The Case of Alex

Alex was a junior majoring in construction management technology. It had been five or six years since Alex graduate from high school and so he did not take the ACT to gain admittance to the university. He became frustrated with his previous college math courses including his many troubles with college algebra and trigonometry. As a result, he was quick to quit trying when he did not understand the material. In essence, Alex had the mindset that if he did not understand the mathematics right away, he would not be able to understand it. He did not understand that struggling to learn a concept is a very valuable process and this is where learning can occur. Alex's method of throwing up his hands when he did not comprehend the material was one of the reasons he struggled so much with college algebra and trigonometry. Alex failed at his first attempt at college algebra and decided to enroll into a general mathematics course. Although he successfully completed the general math course, Alex again withdrew the next time he took college algebra. This did not stop Alex from preserving: he enrolled in college algebra a third time only to result in additional failure. Alex's determination drove him to enroll for the forth time the following summer. This time, Alex passed with an A. The next semester he enrolled in trigonometry and completed the course with an F. During the semester of this study, Alex was enrolled in both trigonometry and technical calculus. His cumulative G.P.A. was 2.143 and he scored an eight out of twenty-five on the pre-algebra assessment and did not take the post-algebra assessment.

Before enrolling in technical calculus, Alex knew that calculus was one of the harder courses on campus. Alex enrolled in the course knowing that he was going to struggle and that he may not be successful. The first day that Alex showed up to the voluntary sessions, he did not know "which way to look or go about fitting into this … But once you explained overall what we were doing as far as looking at an [work-out] example, working through a [work-out] example with you, and then working on our own, it became more and more easy to fit in and feel comfortable." Alex attended most of the voluntary sessions and would work to understand the material of the previously covered lecture material by working many problems through the worked-out example method. He would use the confidence that he gained by solving problems and work on more problems in his individual study time. He stated that he immediately started seeing an impact on his performance on his homework by getting scores of 9's and 10's out of 10. This had an impact on his exam scores and Alex became very confident with his mathematics ability. Alex stated that without the worked-out examples, "I am not sure if I would even pass. I would either fail it or get a D. I would be really low." From Alex's past performances with college algebra and trigonometry and the heavy emphasis of these courses in calculus, Alex would have had a high probability of being unsuccessful in the course and continuing his past mathematical failures. The worked-out examples not only helped him understand and to work calculus problems, but through using the worked-out example method, Alex also became more and more confident in his knowledge of calculus. Alex became so confident that he became cocky with other classmates before exams. He stated that his classmates would be jealous when they found out he did not need a formula sheet like they did and they would ask "I guess you know how to work them out, don't you?" Alex would look them in the eye and say, "I sure do. I know how to work every single problem and that feels good."

Alex had a very different disposition about mathematics at the end of the course than in previous mathematics classes. First, Alex changed his study habits and his mentality about the homework. This is revealed with the statement, "The mentality that I think a lot of students, and myself… I catch myself doing it, you will see something in class, like an example, and they think that it is so easy… no problem I really don't have to study much. Do my homework, bam boom, it won't take long. But I have found that you can't really start your homework too early, there is no … you can start it too late but you can't start it too early … and you will find that you think one way … you will perceive it one way … you will think another way when you start your homework. Just because you get your homework done does not mean that you understand. It means your homework is done. In order to understand something you need to go back two or three times and do the problems again." When studying, Alex became very conscientious about making sure that he wrote the problems completely correctly. If he thought "there was one mistake with it (a written solution of a problem), I would erase it until I am complete happy." His change in mentality about mathematics was also revealed very plainly when comparing his experience in college algebra to calculus. He stated "I think what has made the difference between difficulty in college algebra and not as much difficulty in calculus, is just me sitting down and not thinking okay I have to do this homework as fast as I can, but me sitting down and saying that I have to do this right. I just kind of opened my mind up recently and I am not fighting it."

The Case of Rachel

Rachel is a junior majoring in biomedical science. Rachel was an adult student (around twenty-six years old) who came back to school and who did not take the ACT exam. Her cumulative G.P.A was 1.742 and she scored a nine out of twenty-five on the pre algebra assessment and a thirteen out of twenty-five, on the post-algebra assessment. She stated that she "was not a very dedicated high school student. I never did any homework, ever. I would just go in and take my tests. I would average C's and B's, but did not retain it." After being out of school for years, Rachel "was actually really worried about it (the course)." She knew that she would struggle with the course because of her weak algebra skills and the course's difficulty level. During lecture she believed that she understood the material, but would struggle when she tried to do the work on her own. Rachel entered the course with a deflated attitude towards mathematics and a lot of doubt about her probability of succeeding. She thought immediately when she learned about the worked-out example method and the discussion sessions that she was saved. With the help she believed that she might be successful. The worked-out example method helped her to understand the material and gradually she gained confidence. At one point during the semester, after being successful on an examination, she called her dad and said, "Wow, I can do this, calculus, so when I can do one after you said do one on your own and I got it right, I was like, 'Wow'. It was a good feeling. I did not think I could even pass this class." In her mind the worked-out example method was so helpful because "I like to see one and well, I kind of got it, and then when we talk through it (another example), you hear it from other students, and for some reason it clicks in your mind. And then doing it on your own … I can do this."

She went on to express how in high school she would come home crying because she could not understand algebra or geometry. Rachel said the worked-out examples were great because, "for me if I see it, hear it, and write it, then I remember it." Rachel, if she got stuck on a problem while working problems outside of the discussion sessions, would go back and review the worked-out examples before reexamining to the problem she was stuck on. Usually she could successfully complete the problem through this procedure. She stated that the worked-out example method contributed greatly to her understanding and she believed it saved her from failing or dropping the class. Rachel's attitude changed dramatically during the semester. She realized that she could not just show up, not do homework, not study, and pass the course. She realized that with hard work and a foundation that was laid through the worked-out example method; she could be successful in the course and obtain a good understanding of the course material. Rachel said that the worked-out example method had a "direct correlation to my grade."

The Case of Henry

Henry is a freshman majoring in fire protection and safety. Henry decided to go to school here because this university has one of the best programs in fire protection and safety. He took the SAT instead of the ACT and scored a 510 on the math portion, with an overall score of a 980. Henry's high school mathematics experience consisted of taking geometry his freshman year, algebra II his sophomore year, and pre-calculus his senior year. He stated that he did not take any mathematics his junior year because he did not like math anymore. He was so disinterested and bored that he did not pay attention in algebra II and had no clue what was going on in the course. He thought that the assignments were ridiculous and he did not like to work on problems.

Before Henry arrived on campus, he successfully completed intermediate algebra at a college in a state near where he resided with his parents. He successfully passed college algebra and trigonometry when he arrived on campus, earning a C in both during the spring and fall of 2004. He admits that he hated math in high school and college other than the second semester of his senior year in high school, when he frequently worked on his math skills with his pre-calculus teacher. Henry said that he liked math that semester because his teacher helped him understand math and got him to think about math more than during any other time in his high school career.

Unlike Alex, Henry's college G.P.A. was fairly strong with a cumulative G.P.A. of 3.067, but he knew that calculus was going to be hard. He scored a thirteen out of twenty-five on the pre-algebra assessment and a sixteen out of twenty-five on the post-algebra assessment. Henry through the process of the worked-out examples developed pride in his mathematical ability and restored a favorable attitude towards mathematics. His pride is a product of being more successful in the course which resulted from building a better understanding of the concepts in the course through worked-out examples and individual work. In high school mathematics classes, Henry became frustrated and turned off on mathematics. He determined that he wasn't as good in mathematics as his he thought he was during elementary and middle school. The worked-out examples allowed him to see that through hard work in groups and individually study, he could "do" mathematics once again. The mathematics light was switched on once again. At the beginning of the semester, Henry thought the worked-out examples helped him keep from withdrawing, and later during the semester when his mathematics confidence was back and he was doing well on course assessments, Henry thought the worked-out examples were helping him minimally. Henry would use the worked-out examples, when he studied by himself, to review himself on problems so that he could work other problems. Anytime that he was stuck on a problem he would go back and review a similar worked-out example. As the semester progressed, he did not have to review worked-out examples as extensively as he did during the beginning of the semester. He even went to the extent at the end of the semester to say that students should explain the steps of the problem as they solve it; even go to the board and explain their solutions to others.

CONCLUSION

We will summarize results by including data from all the participants as we answer the research questions. While we discuss each of the questions and viewpoints from the participants, we will analyze the case studies to bring out trends from the viewpoint of the weaker to the stronger algebra students.

Question 1: What are student's perceptions and experience with worked-out examples in Calculus?

Pretty much every participant was very positive about the worked-out examples. The majority of students taking calculus across the country expect to see examples that illustrate the concepts and theory in calculus. Many would say the more the better. Students build understanding of concepts and the theory in calculus through examples. Sometimes a few examples will be enough for a student to get a clear understanding of a concept in calculus. Other times it takes many different examples before a student gets a clear understanding. Furthermore, understanding builds from students working examples. It is no surprise that students are very receptive of the worked-out example method. Participants made it very clear that just because they see problems worked in lecture and things seem clear, this does not mean that they understand. It is not until they work problems that they understand. The worked-out example method is so valuable because it allows students to build confidence and understanding by transitioning from seeing examples to working examples. Participants stressed many times that although they thought they understood the concepts and examples worked in class, things were much harder when they tried to work things by themselves. This would lead to frustration and an attitude of surrender. The worked-out example method helped participants build up confidence that they could understand and work other problems by themselves, especially the weaker students. Most of the participants with weaker backgrounds thrived in this environment. Although, the worked example method tends to emphasize building of procedural schema, it does build a foundation of schema that can be used to build higher order thinking skills.

Question 2: What are the participants' perceptions on how the worked-out examples help them in the course?

Participants' perceptions on how the worked-out examples help them in the course varied from responses "minimally" to responses "I would have dropped or withdrew from the course." Out fifteen participants that talked more extensively about how the worked-out examples helped them in the course, six stated that they believed they would have earned a F or would have withdrew from the course with out the worked-out example method. Three of them were Alex, Henry, and Rachel. Three other participants believed that they would only earn a D without the worked-out example method and the other six believed that they would have earned a C or better.

Question 3: In what ways, if any, do worked-out examples build self-efficacy in participant's ability to learn material and instill confidence that they will be successful in Calculus?

We have seen that the worked-out example method increased Alex, Rachel, and Henry's confidence in their ability to "do" mathematics and be successful in the course. The case studies showed the dramatic change in self-efficacy of all three case studies. These three were extreme cases because both Alex and Rachel started technical calculus with a weak background and had dramatic improvements and Henry's dramatic change in mathematics attitude where he enjoyed mathematics again. Overall, most all participants confidence rose throughout the semester and participants overall confidence with the material showed how the worked-out examples helped students build confidence in their abilities to "do" mathematics. Furthermore, the change in the way participants viewed their ability to do mathematics, were very dramatic.

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