The Complexity of Minimizing Receiver-Based and SINR Edge Interference

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***Abstract*—Topology control has been used to minimize interference or to reduce power consumption while maintaining connectivity in wireless ad hoc and sensor networks (WSNs) (which are represented as undirected graphs). In the graph model, the interference experienced by an edge in a WSN has been defined in at least two different ways: the sender-based and receiver-based interference models. These models have been extensively studied in the literature although the receiver-based model has received more attention. Recently, several researchers have started investigating interference in the more realistic physical model which is known as the Signal-to-Interference-Noise-Ratio (SINR) model. The SINR model better reflects the real environment than the receiver-based or sender-based graph models. In this paper, we study the problem of assigning power to nodes in the plane to yield a connected network of minimum edge interference in the receiver-based (RB-MEI) as well as the SINR models (SINR-MEI). We show that RB-MEI is NP-complete for geometric graphs. For SINR-MEI, NP-completeness holds for planar geometric graphs. We also propose some simple greedy heuristics based on the minimum spanning tree approach, and study their performance through simulation.**

***Index Terms*—Topology control, interference, connectivity, NP-Completeness, Signal-to-Interference-Noise-Ratio, receiver-based, heuristic, geometric graph.**

I. Introduction

Topology control in wireless ad hoc and sensor networks has been the focus of many researchers to conserve energy of the sensor nodes since they still have limited energy resources despite recent advances in sensing technology. One of the main approaches of using topology control is minimizing transmission powers assigned to the nodes while maintaining network connectivity, thereby decreasing overall energy consumption [1], [2], [3], [4], [5]. Another interesting approach is minimizing interference by assigning appropriate transmission powers to the nodes while maintaining connectivity because having low interference can reduce the number of retransmissions.

The wireless ad hoc and sensor networks are commonly modeled as undirected graphs in the literature. In the graph model, two approaches for measuring interference have been proposed. For the *sender-based interference model*, [6], [7], [8], [9], the *node interference* of a node v is defined as the number of other nodes residing within v's transmission range. [7] proved that finding a spanning tree that minimizes the maximum node interference in a grid network is NP-hard, and [9] showed that finding a connected planar graph by assigning powers to nodes that minimizes the maximum node interference is NP-hard as well. [8] studied a special 1-dimensional network which is called the *highway model*, and showed that $\sqrt{n}$ is a lower bound for the maximum node interference, where n is the number of nodes. Similarly, [10], [11], [12], [13] focused on the *interference load of an edge* $e = (u, v)$ which is defined as the number of other nodes that are affected by nodes u and v communicating with each other using their transmission powers. [10] observed that there exists a network having low degree but high interference, implying that networks with low node degree do not necessarily have low interference. [13] showed that assigning powers to nodes to produce a connected geometric graph with bounded edge interference is NP-complete.

Another model for measuring interference in wireless networks is the *receiver-based interference model* that is more realistic than the sender-based model. In the receiver-based interference model, the interference load of a node v is defined as the number of nodes that cover node v in their broadcasting disks. And the interference load of an edge (u, v) is defined as the number of nodes whose transmission ranges cover u or v. [14] showed that for a set of nodes in the plane, a graph can be constructed using computational geometric tools with $O(\sqrt{\Delta})$ node interference, where Δ is the maximum interference in the unit disk graph. [6] proved that minimizing the maximum node interference is hard to approximate, and [15] showed that the problem of assigning powers to get a connected graph minimizing the maximum node interference is NP-complete for the 2-dimensional case.

Recently, several researchers have started investigating interference in the more realistic physical model which is known as the Signal-to-Interference-Noise-Ratio (SINR) model. In the previous graph model, if a node v resides in the broadcasting disk of some other node u, then the node v can receive and decode the data sent by u as long as there is no collision or interference. However, in the physical model, the signal sent by node u fades, and background noise and signals sent by all other nodes concurrently transmitting can interfere with u's signal. Thus, the signal sent to node v may not be strong enough to be received and, hence, transmitted data is lost. In the SINR model, only if the SINR is beyond a certain threshold, the transmitted data can be successfully received and decoded at the receiver node. [16] studied maximizing network capacity by minimizing spatial reuse in the physical SINR model, and introduced a topology control algorithm called MaxSR. [17] investigated the problem of minimizing interference and defined a new interference model integrating SINR-based model into the graph-based model. The paper also introduced a heuristic algorithm called HIMTC.

Some other researchers have been concerned with scheduling algorithms in the SINR model. [18] suggested a distributed scheduling algorithm for nodes with non-uniform power levels. It also showed that a ρ-UDG, in which any two nodes are connected if and only if their distance is at most ρ, can be emulated satisfying the constraints of the SINR, precisely with $\rho = 1/\sqrt{n \ln n}$, where n is the number of nodes in a network. [19] suggested a scheduling algorithm that has a constant approximation guarantee for the problem of maximizing the number of links scheduled in one time slot. It also contained an $O(\log n)$ approximation for minimizing the number of time slots needed to schedule a given set of requests. Later, [20] studied the scheduling problem with uniform power, and showed that the number of time slots needed is constant in 1-dimensional as well as 2-dimensional grids.

In this paper, we study the problem of assigning power to nodes in the plane to yield a connected network of minimum edge interference in the receiver-based (RB-MEI) as well as the SINR models (SINR-MEI). We show that RB-MEI is NP-complete for geometric graphs. For SINR-MEI, NP-completeness holds for planar geometric graphs. To the best of our knowledge, these results have not been obtained before for geometric graphs. We also propose some simple greedy heuristics based on the minimum spanning tree approach, and study their performance through simulation.

This paper is organized as follows. In Section 2, we describe our network models and introduce the definitions used in this paper. Section 3 establishes the NP completeness results. In Section 4, greedy heuristics and their performance are studied. Section 5 contains some concluding remarks.

II. Preliminaries

A. Network Models

Consider a set V of sensor nodes in the plane. Each node v is assigned a transmission power $p(v)$. In the graph model, the data sent by a sender node i via a link (i, j) can be received by the receiver node j if $d(i, j) \leq p(i)$, where $d(i, j)$ denotes the distance between nodes i and j. We consider the bidirectional case, where two nodes i and j can communicate with each other via an undirected edge (i, j) if $d(i, j) \leq p(i)$ and $d(i, j) \leq p(j)$. However, in the SINR model, the signal power sent by a sender fades, and background noise and signals sent by all other nodes that concurrently transmit data can interfere with the sender's signal. Thus, the receiver j can successfully receive the data transmitted by the sender i only if the ratio of the received signal power at j to interference by all other nodes and background noise is beyond an SINR threshold β. Formally, the receiver j can receive and decode the data sent by the sender i via the link (i, j) only if

$$SINR_i(j) = \frac{p(i) \cdot d(i,j)^{-\alpha}}{N + I_j} \geq \beta$$

where α is the path loss exponent, N is the background noise, and I_j is the cumulative interference at j caused by all the other concurrently transmitting senders. From this we can see that the necessary transmission power for a sender to transmit data via a link whose distance is d is $\Omega(d)$ for the graph model, and $\Omega(d^\alpha)$ for the SINR model.

B. Minimum Edge Interference Problem

In this paper, we are concerned with two definitions of edge interference, namely the edge interference in the receiver-based graph model (RB-EI) and in the SINR model (SINR-EI).

For RB-EI, the interference load of an edge (i, j) is defined as

$$\begin{aligned} RB\text{-}EI(i,j) :=& |\{u \in V | u \notin \{i,j\} \text{ and} \\ & (i \in D(u, p(u)) \text{ or } j \in D(u, p(u)))\}| \end{aligned}$$

where $D(u, p(u))$ is the covered area of node u with power $p(u)$. Informally, $RB\text{-}EI(i, j)$ is the number of other nodes that cover nodes i or j in their broadcasting ranges. The edge interference of a graph $G(V, E)$ is defined as

$$RB\text{-}EI(G) := \max_{(i,j) \in E} \{RB\text{-}EI(i,j)\}$$

For SINR-EI, we observe that a receiver j cannot receive data sent by a sender i with the transmission power $p(i)$ via link (i, j) if there is a node u that is transmitting its data simultaneously with power $p(u)$ satisfying the following condition:

$$\frac{p(i) \cdot d(i,j)^{-\alpha}}{N + p(u) \cdot d(u,j)^{-\alpha}} < \beta$$

We say that node u interferes with node i. Following this observation, we can define the edge interference of an edge $e(i, j)$ as in [16]

$$SINR\text{-}EI(i,j) := |\{I(i_j) \cup I(j_i)\}|$$

where

$$I(i_j) := \left\{ u \in V | u \notin \{i,j\} \text{ and } \frac{p(i) \cdot d(i,j)^{-\alpha}}{N + p(u) \cdot d(u,j)^{-\alpha}} < \beta \right\}$$

and the edge interference of a graph $G(V, E)$ is defined as

$$SINR\text{-}EI(G) := \max_{(i,j) \in E} \{SINR\text{-}EI(i,j)\}$$

The Minimum Edge Interference problem in the receiver-based graph model (RB-MEI) (or in the SINR model (SINR-MEI)) is defined as follows:

Input. A set V of nodes in the plane, a set of M power levels $P = \{p_1, p_2, ..., p_M\}$, and a positive number R.

Question. Is there a power assignment to all nodes which induces a connected geometric graph $G(V, E)$ such that $RB\text{-}EI(G) \leq R$ (or $SINR\text{-}EI(G) \leq R$)?

III. NP-Completeness of Minimum Edge Interference

In this section, we prove the NP-completeness of minimum edge interference for the receiver-based model as well as the SINR model.

A. Receiver-Based Minimum Edge Interference

This subsection is devoted to the NP completeness of the minimum edge interference problem in the receiver-based model (RB-MEI).

Theorem 1. *RB-MEI is NP-complete for connected geometric graphs.*

Proof: RB-MEI is obviously in NP. Given a set V of nodes in the plane, a set P of power levels and a positive integer R, we can nondeterministically assign power levels to the nodes, and verify in polynomial time that (1) the power assignment yields a connected graph $G(V, E)$, and (2) the receiver-based interference load of each edge $(i, j) \in E$ is $\leq R$.

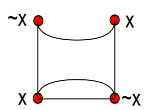

Fig. 1. A variable gadget

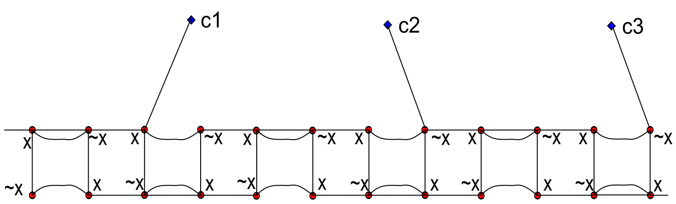

Fig. 2. A chain of variable gadgets

To prove the NP-hardness of RB-MEI, we construct a polynomial time reduction from the planar 3-SAT problem (P3SAT) which was proven NP-complete in [21]. Consider an instance ϕ of P3SAT, and the planar instance graph G of ϕ, where $G = (X \cup C, E \cup E')$ with edge sets $E = \{(x, c)|x \in C \vee \neg x \in C\}$ and $E' = \{(x_i, x_{i+1})|1 \leq i \leq n-1\}$.

To construct an instance $< V', P, R >$ of RB-MEI, we first create a gadget for each variable of ϕ. As shown in Figure 1, the gadget for variable x contains 4 nodes, which are 2 pairs of x and $\neg x$. These nodes are connected through *straight* and *curved* lines. To have sufficient nodes for x to connect x to some clause nodes, we construct for variable node x in G a chain of the above gadgets in such a way that every second gadget has its nodes and edges rearranged as depicted in Figure 2. Note that consecutive gadgets are connected by two parallel *straight* lines. If the degree of node x is d, then the number of $x's$ gadgets in this chain is $2 * d$ (see Figure 2 for the case x has degree 3). From each chain of gadgets representing a variable node in G, we only use the nodes with degree 3 to connect to a clause node. Furthermore, the variable links connecting all variables should enter each chain of gadgets via a degree 3 node at one end, and exit via another degree 3 node at the other end. Thus, the new graph (also denoted G in the following for convenience) obtained from the original graph G has a maximum degree of 4 while the planarity of the graph is still preserved.

Next, we use Valiant's result [22] to embed the graph G with maximum degree 4 into the Euclidean plane:

> *A planar graph with maximum degree 4 can be embedded in the plane using $O(|V|)$ area in such a way that its vertices are at integer coordinates and its edges are drawn so that they are made up of line segments of form x = i or y = j, for integers i and j.*

Moreover, this embedding process can easily be designed to satisfy the additional requirement that each edge be represented by at least 3 line segments.

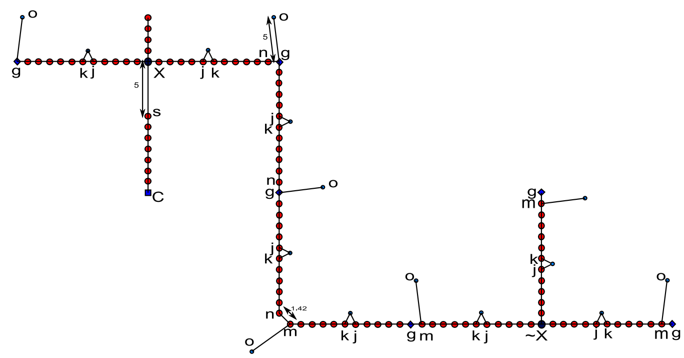

Fig. 3. Nodes added to graph G

Letting d be the unit on the plane, we define three radii r_1, r_2 and r_3 as follows: $r_1 := \frac{d}{12}$, $r_2 := \frac{1.5d}{12}$ and $r_3 := \frac{5d}{12}$. For the sake of convenience let us call the variable and clause nodes of the P3SAT instance G embedded in the plane the *variable* and *clause* nodes, respectively. The line segments in the embedded graph G are further modified by placing additional new nodes, called *intermediate* nodes, to create the instance $< V', P, R >$ of RB-MEI as follows:

1) At each grid point on a line segment, add a new node called *grid* node (see node g's in Figure 3.)
2) On the line segments representing a *curved* line in a variable gadget and on the line segments connecting a *variable* node (x or $\neg x$) to a *clause* node, add 11 *intermediate* nodes on each unit segment to devide each unit into 12 smaller pieces of equal length r_1 except for the unit segment that interfaces with a variable node. On this unit segment, only 7 *intermediate* nodes are added starting at distance r_3 from the *variable* node. The first *intermediate* node at distance r_3 to the *variable* node is called an *interfacing* node (see node s in Figure 3.) These 7 new *intermediate* nodes are placed at distance r_1 from one node to the next.
3) On the line segments representing a *straight* line connecting a variable node x and its negated node $\neg x$ in a chain

of gadgets, perform the following steps starting from both *variable* nodes x and $\neg x$:

a) Among the corners, which are the intersection of a horizontal segment and a vertical segment, of this set of line segments, pick one corner to be the *switch* corner. (If this set of line segments does not have a corner, i.e., all are vertical or horizontal segments, then pick one segment and move it away one unit (See Figure 3) to create corners.)
b) On every unit segment, add 11 *intermediate* nodes to divide it into 12 smaller pieces of equal length r_1.
c) Starting from the *variable* nodes x and $\neg x$, add one *auxiliary* node between the 5th *intermediate* node and the 6th *intermediate* node on each unit segment until before we reach the *switch* corner. Each *auxiliary* node is placed at distance r_1 from the 5th and 6th *intermediate* nodes (see nodes j and k in Figure 3).
d) Starting from the *variable* node x and $\neg x$, add a *control* node between the *grid* node and its preceding neighbor *intermediate* node (nodes n and m in Figure 3.) This *control* node is placed at distance r_3 from both the *grid* node and its neighbor *intermediate* node (see node o in Figure 3.) Continue to do this until we reach the *switch* corner.
e) At the *switch* corner, replace the *grid* node by a *control* node placed at distance r_3 to its nearest *intermediate* nodes (see nodes o, n and m in Figure 3).

4) On the segments replacing the line connecting variables, add 11 *intermediate* nodes on each unit segment to divide it into 12 small pieces of length r_1 each.

Let $P = \{p_1, p_2, p_3\}$ where $p_1 := r_1$, $p_2 := r_2$, and $p_3 := r_3$, and define $R := 4$. To show the correctness of the above polynomial reduction we prove that the instance ϕ of P3SAT is satisfiable if and only if the RB-MEI instance $< V', P, R >$ has a power assignment that yields a connected geometric graph such that the interference load $EI(e)$ of each edge e is $\leq R$.

For the "only-if" direction, suppose that ϕ has a satisfying Boolean assignment. We assign power levels to nodes in V as follows:

1) Assign power level p_3 to *variable* node $x \in V'$ if variable $x \in \phi$ has the value $true$; otherwise, assign power level p_1.
2) Assign power level p_1 to all *intermediate*, *grid* and *clause* nodes on the line segments connecting a *variable* node and a *clause* node. Assign power level p_3 to an *interfacing* node if its adjacent *variable* node is also assigned power level p_3; otherwise, assign power level p_1. Similarly, assign power levels to nodes on the line segments representing the *curved* line including the *interfacing* node s residing on these segments.
3) Assign power level p_3 to all *control* nodes o's on the line segments representing a *straight* line. On these line segments, if a *variable* node has power level p_3, then assign all *grid* nodes power level p_3, starting from the *variable* node leading to a *switch* corner (see Figure 3 where x has power p_3.) Otherwise, if a *variable* node has power level p_1, then assign power level p_3 to all *intermediate* nodes at distance r_3 to the *control* node, starting from the *variable* node until the *switch* corner (see nodes m in Figure 3.) At this switch corner, the *intermediate* node m is assigned power level p_3, whereas the *intermediate* node n is assigned p_2 instead.
4) Assign power level p_1 to all remaining nodes.

Clearly, this power assignment produces a connected geometric graph. In the receiver-based model, the edges (j, k) have the heaviest interference load $RB\text{-}EI(j, k)$ that is contributed by its two neighboring *intermediate* nodes, its neighboring *auxiliary* node, and either a 5-hop away *grid* node or a 5-hop away *intermediate* node which is assigned power level p_3. Given this power assignment, it is straightforward to verify that the interference load $RB\text{-}EI(i, j)$ is $\leq R = 4$.

For the "if" direction, suppose the instance $< V', P, R >$ has a power assignment that yields a connected geometric graph $G'(V', E')$ in which each edge $e \in E'$ has an interference load $RB\text{-}EI(e) \leq R = 4$. We construct a satisfying Boolean assignment for the P3SAT instance ϕ based on the following observations:

1) On the line segments leading to a *clause* node, at least one *interfacing* node s must use power level p_3 to connect to at least one *variable* node for the graph $G'(V', E')$ to be connected.
2) Of the nodes on the line segments of $G(V', E')$ representing a *curved* line of $G(V, E)$, at least one of two *variable* nodes (representing a variable or its negation) must have power level p_3 for the graph to be connected.
3) Since the distance between every pair of consecutive *intermediate* nodes u and v is r_1, there is always an edge $(u, v) \in E'$. Now for the "middle" edge (j, k) consider the *variable* or *grid* node 5-hop away from j, and the *intermediate* node 5-hop away from k (node m in Figure 3). Because $RB\text{-}EI(j, k)$ is ≤ 4, either the *variable* (or *grid*) node has power level p_3 or the *intermediate* node m has power level p_3, but not both. Therefore, either *variable* nodes x have power level p_3 or *variable* nodes $\neg x$ have power level p_3, but not both.

From the above observations, we can construct the Boolean assignment for the P3SAT instance ϕ using the following rules:

- If a *variable* node in G' has the power level p_3, then assign value $true$ to that variable in ϕ; otherwise, assign value $false$.

From Observation 1 it follows that each clause in ϕ is satisfied by at least 1 literal having value *true*. Moreover, from Observations 2 and 3, the Boolean assignment for ϕ is consistent. This concludes the proof of Theorem 1. ■

B. Minimum SINR Edge Interference

In this subsection, we prove the NP completeness of SINR-MEI for planar connected geometric graphs. A geometric graph is said to be planar if no edge crosses another.

Theorem 2. *SINR-MEI is NP-complete for planar connected geometric graphs.*

Proof: We only need to prove the NP-hardness. To this end, we use the reduction in the proof of Theorem 1 which produces the instance $< V', P, R >$ from the given P3SAT instance. To obtain an instance of SINR-MEI, we further modify $< V', P, R >$ as follows. Consider the edge (j, k) and the two edges that are 1-hop away from (j, k) denoted by (p, q) in Figure 4. For each edge (p, q) we add an *auxiliary* node at distance r_1 from p and q (see Figure 4). Let the set of power levels be $P = \{p_1, p_2, p_3\}$ where $p_1 := r_1^\alpha$, $p_2 := r_2^\alpha$ and $p_3 := r_3^\alpha$. Given $\alpha \in \{2..5\}$ and $N \geq 0$, β is defined as $\beta := \frac{r_2^\alpha}{N+(1-N)} = r_2^\alpha = (\frac{1.5d}{12})^\alpha$. Observe that β can be set at any desired value since the unit distance d (and hence r_2) can be adjusted accordingly. Letting $R = 4$ we obtain an instance of SINR-MEI. To show the correctness of this polynomial-time reduction, we prove that the P3SAT instance is satisfiable if and only if the SINR-MEI instance has a power assignment that yields a planar geometric graph with bounded SINR edge interference.

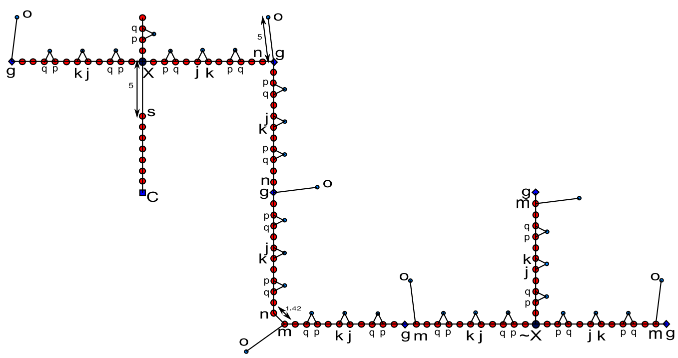

Fig. 4. Nodes added to graph G

For the "only-if" direction, let the P3SAT instance ϕ be satisfied by a Boolean assignment. In the SINR-MEI instance, the nodes are assigned power levels in a similar manner as done in the proof of Theorem 1. We calculate the SINR edge interference load of (j, k) as follows. The interference values of the *intermediate* nodes that are 1 to 4 hops away from i (or j) are: $\frac{1}{N+1}$, $\frac{2^\alpha}{2^\alpha N+1}$, $\frac{3^\alpha}{3^\alpha N+1}$, $\frac{4^\alpha}{4^\alpha N+1}$. For an *intermediate* node that is 5 hop away from j or k, its interference value is $\frac{1}{N+1}$ if it has power level p_3, or $\frac{5^\alpha}{5^\alpha N+1}$ if its power level is p_1.

From the interference values of the neighboring *intermediate* nodes, observe that only the *intermediate* nodes that are adjacent to j (or k) or those that are 5 hops away and have power level p_3 interfere with the edge (j, k). Now, since (j, k) has 3 *intermediate* nodes that are 1-hop neighbors causing interference, and since $R = 4$, each edge (j, k) only allows exactly one more node that is 5 hops away from j (or k) to have power level p_3. Consequently, exactly one of the following two cases occurs: either (1) the *variable* node x and all nodes m's have power level p_3, or (2) the variable node $\neg x$ and all nodes n's have the power p_3. As observed in the proof of Theorem 1, this power assignment yields a planar connected geometric graph. Moreover, the SINR edge interference load is $\leq R$.

For the "if" direction, suppose that the SINR-MEI instance has a power assignment that yields a planar connected geometric graph whose SINR edge inteferece is bounded by R. We show how to obtain a satifying Boolean assignment for ϕ. The argument is based on the observations in the proof of the "if" direction in Theorem 1. Notice, however, that there is a significant difference between the receiver-based model and the SINR model in that the SINR interference load of an edge can be decreased by increasing the power levels of its incident nodes. We show that the planarity of the resulting geometric graph ensures that this cannot happen. To this end, let us assume that the *variable* node x in Figure 4 is assigned power level p_3 to connect to the *interfacing* node s. Consider an edge (j, k) that is closest to the *variable* node x. We have the following observations:

1) If nodes j and k each has the power level p_1 or p_2, then the SINR interference load of (j, k) is 4 which is $\geq R$. This forces the *grid* node g to have the power level p_3 to connect to the *control* node o. Thus, if all nodes j and k have power level $\leq p_2$, then, as shown before, either the *variable* nodes x's or the *variable* nodes $\neg x'$s have power level p_3.
2) Next we show that if either node j or k uses power level p_3, then the resulting geometric graph is no longer planar. In fact, suppose that either j or k has power level p_3, then on the edges (p, q) on both sides of (j, k), either p or q must have the power level p_3 in order for (p, q) to have an interference load $\leq R$. This is due to the fact that the *variable* node x, and either node n or the *grid* node g have power level p_3. A similar argument shows that all *intermediate*, *auxiliary*, *control* and *grid* nodes from the *variable* node x to the first *grid* node g must have power level p_3 in order to keep the SINR edge interference bounded by R. If these nodes have power level p_3, then new edges would be created which destroys the planarity requirement. Therefore, neither node j nor k has the power level p_3.

The above observations show that the high power usages of the *variable* nodes x and the variable nodes $\neg x$ are mutual exclusive. Thus, we can assign literal x or $\neg x$ in ϕ the value $true$ if it is represented by a *variable* node that has power level p_3; otherwise it is assigned the value $false$. Since the resulting geometric graph is connected, each clause of ϕ is satified by at least one literal having value $true$. This concludes the proof of Theorem 2. ■

IV. Heuristics and Performance

A. Heuristics

We now propose a simple greedy heuristic that finds a power assignment yielding a connected geometric graph with low edge interference. The basic idea is to find a minimum spanning tree based on edge weights that are defined as interference load on the edges for an initial graph. After obtaining a spanning tree, we assign power levels to the nodes based on that tree, i.e., each node is assigned a power level that is minimally required to maintain the tree. With the new power assignment, we compute $RB\text{-}EI(G)$ and $SINR\text{-}EI(G)$.

1) Initial power level: [10] shows that there exists a special network consisting of two *exponential node chains* in which if each node is assigned the maximum power level instead of the minimum power level required for connectivity, then the optimal power assignment with constant edge interference can be computed using the minimum spanning tree approach. Following this observation, we build the initial graphs using three different initial uniform powers, P_{min}, P_{avg} and P_{max}. P_{min} is set as the minimum power level that is required for connectivity whereas P_{max} is the power needed to connect the two nodes whose distance is largest in the network. P_{avg} is simply the average of P_{min} and P_{max}.

2) Edge weights: We use Kruskal's algorithm [23] to find the minimum spanning tree using different edge weights defined as $Cov(e)$, $RB\text{-}EI(e)$, and $SINR\text{-}EI(e)$. The first edge weight we use is the *coverage* of an edge $e = (i, j)$, defined as

$$Cov(e) :=|\{u \in V | u \notin \{i, j\} \text{ and } (u \in D\,(i, p\,(j)) \text{ or } u \in D\,(j, p\,(i)))\}|$$

that is the number of nodes covered by the disks centered at nodes i and j with radius $p(i)$ and $p(j)$, respectively [10]. The other edge weights are defined by $RB\text{-}EI(e)$ and $SINR\text{-}EI(e)$. The pseudocode is given in Figure 5 where we use the well-known subroutines FIND-SET and UNION of Kruskal's algorithm [23].

Input: a set V of nodes in the plane and initial uniform power level
Output: A power assignment P that generates a connected graph with low edge interference

```
E = set of edges between all nodes generated with the
  initial uniform power level
E_T = ∅
w(e) = weight of e for all e ∈ E
Sort the edges of E into nondecreasing order by weights
for each edge e = (u, v) ∈ E do
   if FIND-SET(u) ≠ FIND-SET(v) then
      E_T = E_T ∪ {e}
      UNION(u, v)
      E = E \ {e}
   end if
end for
Compute new power assignment P by assigning power
  levels to nodes that are minimally required to maintain
  the tree E_T.
return P
```

Fig. 5. Weight-Based Minimum Spanning Tree

B. Experimental Results

In our simulation, networks with 100 nodes are generated randomly in an area of dimension 400×400. The initial graphs are built by assigning three different uniform power levels, P_{max}, P_{avg} and P_{min}. In order to compare the performance of three different greedy approaches we run them on the same networks.

TABLE I
EDGE INTERFERENCE WITH $\alpha = 3$ AND $\beta = 1$

Edge Weights	Initial Power	$RB\text{-}EI(G)$	$SINR\text{-}EI(G)$
$Cov(e)$	P_{max}	5.71	4.53
	P_{avg}	5.71	4.53
	P_{min}	5.79	4.60
$RB\text{-}EI(e)$	P_{max}	98	78.99
	P_{avg}	72.03	53.17
	P_{min}	9.18	7.10
$SINR\text{-}EI(e)$	P_{max}	5.70	4.53
	P_{avg}	5.70	4.53
	P_{min}	5.78	4.60

Table I shows the edge interference computed by the different greedy approaches. For each case, we generate 100 different networks, and average the edge interference computed by the algorithms over the networks generated. As depicted in Table I, there is no significant difference in the cases of P_{max}, P_{avg} or P_{min} when we use $Cov(e)$ and $SINR\text{-}EI(e)$ as edge weights (although the latter gives slightly better results). However, when $RB\text{-}EI(G)$ is used as edge weight, the edge interference in the cases of P_{max} and P_{avg} is significantly higher than in the case of P_{min}. This is because as nodes are assigned a higher power level, the network becomes very dense. In our experiments, the graphs become nearly complete graphs when P_{max} is used. Therefore the end points of most edges are covered by a large number of nodes.

On the contrary to the observation of [10] that there exists a special network which has minimum edge interference when the maximum power level is used, we observe that starting with P_{min} gives a much better performance than starting with P_{max} or P_{avg} when $RB\text{-}EI(e)$ is used as edge weights in randomly generated graphs.

V. CONCLUSION

In this paper, we have studied the problem of minimizing maximum edge interference in the receiver-based graph model (RB-MEI) as well as the SINR model (SINR-MEI). We have shown that RB-MEI is NP-complete for geometric graphs, and so is SINR-MEI for planar geometric graphs. We also have compared the performance of some greedy heuristics through simulation. As to future work, we plan to design approximation algorithms for the MEI problem. We also plan to study the problem of minimizing interference of an edge (i, j) in the SINR model where the interference load of an edge is calculated by considering all other concurrently transmitting senders.

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