Structural Identification of Production Functions

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Abstract

This paper examines some of the recent literature on the empirical identification of production functions. We focus on structural techniques suggested in two recent papers, Olley and Pakes (1996), and Levinsohn and Petrin (2003). While there are some solid and intuitive indentification ideas in these papers, we argue that the techniques, particularly those of Levinsohn and Petrin, suffer from collinearity problems which we believe cast doubt on the methodology. We then suggest alternative methodologies which make use of the ideas in these papers, but do not suffer from these collinearity problems.

1 Introduction

Production functions are a fundamental component of all economics. As such, estimation of production functions has a long history in applied economics, starting in the early 1800's. Unfortunately, this history cannot be deemed an unqualified success, as many of the econometric problems that hampered early estimation are still an issue today.

Production functions relate productive inputs (e.g. capital, labor, materials) to outputs. Perhaps the major econometric issue confronting estimation of production functions is the possibility that some of these inputs are unobserved. If this is the case, and if the observed inputs are chosen as a function of these unobserved inputs (as will typically be the case for a profit-maximizing or cost-minimizing firm), then there is an endogeneity problem and OLS estimates of the coefficients on the observed inputs will be biased.

Much of the literature in the past half century has been devoted to solving this endogeneity problem. Two of the earliest solutions to the problem are instrumental variables (IV) and fixed-effects estimation (Mundlak (1961)). IV estimation requires finding variables that are correlated

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with observed input choices, but uncorrelated with the unobserved inputs. Fixed-effects estimation requires the assumption that the unobserved input or productivity is constant across time. Unfortunately, for a variety of reasons, these methodologies have not been particularly successful at solving these endogeneity problems. As such the search has continued for reliable methods for identifying production function parameters.

The past fifteen years has seen the introduction of a couple of new techniques for identification of production functions. One set of techniques follows the dynamic panel data literature, e.g. Chamberlain (1982), Arellano and Bover (1995), Blundell and Bond (1999). A second set of techniques, advocated by Olley and Pakes (1996) and Levinsohn and Petrin (2003), are somewhat more structural in nature - using observed input decisions to control for unobserved productivity shocks. This second set of techniques has been applied in a large number of recent empirical papers, including Pavcnik(2002), Sokoloff (2003), Sivadasan (2004), Fernandes (2003), Ozler and Yilmaz (2001), Criscuolo and Martin (2003), and Topalova (2003).

This paper starts by analyzing this second set of techniques. We first argue that there are potentially serious collinearity problems with these estimation methodologies.[1] We show that, particularly for the Levinsohn and Petrin approach, one needs to make what we feel are very strong and unintuitive assumptions for the model to remain correctly identified in the wake of this collinearity problem. To address this problem, we then suggest an alternative estimation approach. This approach builds upon the ideas in Olley and Pakes and Levinsohn and Petrin, e.g. using investment or intermediate inputs to "proxy" for productivity shocks, but does not suffer from these collinearity problems. As well as solving the above collinearity problem, another important benefit of our estimator is that it makes comparison to the aforementioned dynamic panel literature, e.g. Blundell and Bond, quite easy. This is important, as up to now, the two literatures have evolved separately. In particular, our estimator makes it quite easy to see the tradeoffs in assumptions needed by the two distinct literatures. We feel that this should help guide empirical researchers in choosing between the approaches. Lastly, using the same dataset as Levinsohn and Petrin, we examine how our estimator works in practice. Preliminary estimates using our methodology appear more stable across different potential proxy variables than the Levinsohn-Petrin methodology, consistent with our theoretical arguments.

2 Review of Olley/Pakes and Levinsohn/Petrin

We start with a brief review of the techniques of Olley/Pakes (henceforth OP) and Levinsohn/Petrin (henceforth LP). Consider the following Cobb-Douglas production function in logs:

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \quad (1)$$

[1] Susanto Basu made a less formal argument regarding this possible collinearity problem in 1999 as a discussant of an earlier version of the Levinsohn-Petrin paper.

y_{it} is the log of output, k_{it} is the log of capital input, and l_{it} is the log of labor input.[2] There are two terms in this equation that are unobservable to the econometrician, ω_{it} and ϵ_{it}. The distinction between the two is important. The ϵ_{it} are intended to represent shocks to production or productivity that are not observable (or predictable) by firms before making their input decisions at t. In contrast, the ω_{it} represent shocks that are potentially observed or predictable by firms when they make input decisions. Intuitively, ω_{it} might represent shocks such as the managerial ability of a firm, expected down-time due to machine breakdown, expected defect rates in a manufacturing process, or the expected rainfall at a farm's location. On the other hand, ϵ_{it} represents deviations from expected breakdown, defect, or rainfall amounts in a given year. ϵ_{it} can also represent measurement error in the output variable. We will often refer to ω_{it} as the "productivity shock" of firm i in period t.

The classic endogeneity problem estimating equation (1) is that the firm's optimal choice of inputs k_{it} and l_{it} will generally be correlated with the observed or predictable productivity shock ω_{it}. This renders OLS estimates of the β's biased and inconsistent. As mentioned in the introduction, perhaps the two most commonly used solutions to this endogeneity problem are fixed effects (Mundlak (1961), Hoch (1962)) and instrumental variables estimation techniques. In our context, fixed-effects estimation requires the additional assumption that $\omega_{it} = \omega_{it-1} \forall t$. This is a strong assumption and, perhaps as a result, the technique has not worked well in practice - often generating unrealistically low estimates of β_k. IV estimation requires instruments that are correlated with input choices k_{it} and l_{it} and uncorrelated with ω_{it}. On one hand, there do exist natural instrumental variables in this situation - input prices, as long as one is willing to assume firms operate in competitive input markets. On the other hand, this again has not worked well in practice. Too often these input prices are not observed, do not vary or vary enough across firms, or are suspected to pick up variables, e.g. input quality, that would invalidate their use as instruments. The review of this literature in Ackerberg, Benkard, Berry, and Pakes (2005) (ABBP) contains more discussion of the limitations of the fixed effects and IV approaches.

2.1 Olley and Pakes

The OP and LP methodologies take a more structural approach to identification of production functions. OP address the endogeneity problem as follows. They consider a firm operating through discrete time, making decisions to maximize the present discounted value of current and future profits. First, they assume that the productivity shock ω_{it} evolves exogenously following an first-order markov process, i.e.

$$p(\omega_{it+1}|I_{it}) = p(\omega_{it+1}|\omega_{it})$$

[2] These inputs and outputs are measured in various ways across studies depending on data availability. For example, labor inputs could be measured in man-hours, or in money spent on labor. Output could also be measured in either physical or monetary units, and in some cases is replaced with a value added measure.

where I_{it} is firm i's information set at t. Current and past realizations of ω, i.e. $(\omega_{it}, ..., \omega_{i0})$ are assumed to be part of I_{it}. Importantly, this is not just an *econometric* assumption on unobservables. It is also an *economic* assumption regarding what determines a firm's expectations about future productivity, i.e. that these expectations depend only on ω_{it}.

OP assume that labor is a non-dynamic input. A firm's choice of labor for period t has no impact on the future profits of the firm. In contrast, capital is assumed to be a dynamic input subject to an investment process. Specifically, in every period, the firm decides on an investment level i_{it}. This investment adds to future capital stock deterministically, i.e.

$$k_{it} = \kappa(k_{it-1}, i_{it-1})$$

Importantly, this formulation implies that the period t capital stock of the firm was actually determined at period $t-1$. The economics behind this is that it may take a full period for new capital to be ordered, delivered, and installed. Intuitively, one can see how this assumption regarding timing helps solve the endogeneity problem with respect to capital. Since k_{it} is actually decided upon at $t-1$ (and thus is in I_{it-1}), the above informational assumptions imply that it must be uncorrelated with the unexpected innovation in ω_{it} between $t-1$ and t, i.e. $\omega_{it} - E[\omega_{it}|I_{it-1}] = \omega_{it} - E[\omega_{it}|\omega_{it-1},]$. This orthogonality will be used to form a moment to identify β_k.[3] We explicitly show how this is done in a moment.

More challenging is solving the endogeneity problem with respect to the assumed variable input, l_{it}. This is because unlike capital, l_{it} is decided at t and thus potentially correlated with the innovation component of ω_{it}. To accomplish this, OP make use of the investment variable i_{it}. Considering the firm's dynamic decision of investment level i_{it}, OP state conditions under which a firm's optimal investment level is a *strictly increasing* function of their current productivity ω_{it}, i.e.

$$i_{it} = f_t(\omega_{it}, k_{it}) \tag{2}$$

Note that this investment function will in general contain all current state variables for the optimizing firm, e.g. its current level of capital and the current ω_{it}. Labor does not enter the state because it is a non-dynamic input, and values of ω_{it} prior to t do not enter because of the first order Markov assumption on the ω_{it} process. The reason f is indexed by t is that variables such as input prices, demand, etc. also may be part of the state space. OP simply treat these as part of f_t. The assumption here is that these variables are allowed to vary across time, but not across firms (i.e. firms operate in the same input markets).

[3] In the special case where ω_{it} is a random walk, i.e. $\omega_{it} = \omega_{it-1} + \eta_{it}$, one can easily see how this can be done - if we first-difference the production function, $(k_{it} - k_{it-1})$ is uncorrelated with the resulting unobserved term.

Given that this investment function is strictly monotonic in ω_{it}, it can be inverted to obtain

$$\omega_{it} = f_t^{-1}(i_{it}, k_{it}) \tag{3}$$

The essence of OP is to use this inverse function to control for ω_{it} in the production function. Substituting this into the production function, we get:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + f_t^{-1}(i_{it}, k_{it}) + \epsilon_{it} \tag{4}$$

The first stage of OP is to estimate this equation. Recall that f is the solution to a complicated dynamic programming problem. As such, solving for f (and thus f^{-1}) would not only require assuming all the primitives of the firm (e.g. demand conditions, evolution of environmental state variables), but also be computationally demanding. To avoid these extra assumptions and computations, OP simply treat f_t^{-1} non-parametrically. Given this non-parametric treatment, direct estimation of (4) does not identify β_k, as k_{it} is collinear with the non-parametric function.[4] However, one does obtain an estimate of the labor coefficient β_l, $\widehat{\beta}_l$. One also obtains an estimate of the composite term $\beta_1 k_{it} + f_t^{-1}(i_{it}, k_{it})$, which we denote $\widehat{\Phi}_{it}$.

The second stage of OP proceeds given these estimates of $\widehat{\beta}_l$ and $\widehat{\Phi}_{it}$. Recalling the above discussion regarding the timing of the choice of k_{it}. Note that we can write:

$$\omega_{it} = E[\omega_{it}|I_{it-1}] + \xi_{it} = E[\omega_{it}|\omega_{it-1}] + \xi_{it}$$

where again, by the timing assumptions regarding capital, ξ_{it} is orthogonal to k_{it}, i.e.

$$E[\xi_{it}|k_{it}] = 0$$

This is the moment which OP use to identify the capital coefficient. To operationalize this procedure in a GMM context, note that given a guess at the capital coefficient β_k, one can "invert" out the ω_{it}'s in all periods, i.e.

$$\omega_{it}(\beta_k) = \widehat{\Phi}_{it} - \beta_k k_{it}$$

Given these $\omega_{it}(\beta_k)$'s, one can compute ξ_{it}'s in all periods by non-parametrically regressing $\omega_{it}(\beta_k)$'s on $\omega_{it-1}(\beta_k)$'s (and a constant term) and taking the residual, i.e.

$$\xi_{it}(\beta_k) = \omega_{it}(\beta_k) - \widehat{\Psi}(\omega_{it-1}(\beta_k))$$

where $\widehat{\Psi}(\omega_{it-1}(\beta_k))$ are predicted values from the non-parametric regression. Note that treating the regression of ω_{it} on ω_{it-1} non-parametrically allows for ω_{it} to follow an arbitrary first-order

[4] Note that we have also merged the constant term β_0 into the non-parametric function.

Markov process. These $\xi_{it}(\beta_k)$'s can then be used to form an empirical analogue to the above moment, e.g.

$$\frac{1}{T}\frac{1}{N}\sum_t\sum_i \xi_{it}(\beta_k)\cdot k_{it}$$

In a GMM procedure, β_k would be estimated by setting this empirical analogue as close as possible to zero.[5] Quickly recapping the intuition behind identification in OP, β_l is identified by using the information in firms' investment decisions i_{it} to control for the productivity shock ω_{it} that is correlated with l_{it}. β_k is identified off the timing assumption that k_{it} is decided before the full realization of ω_{it}.

2.2 Levinsohn and Petrin

LP take a related approach to solving the production function endogeneity problem. The key difference is that rather than using an investment demand equation, they use an intermediate input demand function to "invert" out ω_{it}. Their motivation for this alternative inversion equation is very reasonable. For the straightforward OP procedure to work, recall one needs the investment function to be *strictly* monotonic in ω_{it}. However, in actual data, investment is often very lumpy, and one often sees zeros. In the Chilean data studied by LP, for example, more than 50% of firm-year observations have zero investment. This casts doubt on this strict monotonicity assumption regarding investment. While the OP procedure can actually work in this situation, it requires discarding the data with zero investment (see ABBP for discussion), an obvious efficiency loss.

LP avoid this efficiency loss by considering the following production function:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + \omega_{it} + \epsilon_{it}$$

where m_{it} is an intermediate input such as electricity, fuel, or materials. LP's basic idea is that since intermediate input demands are typically much less lumpy (and prone to zeros) than investment, the strict monotonicity condition is more likely to hold and these may be superior "proxies" to invert out the unobserved ω_{it}. LP consider the following intermediate input demand function:

(5) $$m_{it} = f_t(\omega_{it}, k_{it})$$

Again, f is indexed by t, implicitly allowing input prices (and/or market conditions) to vary across time (but not across firms). Note the timing assumptions implicit in this formulation. First, the intermediate input at t is chosen as a function of ω_{it}. This implies that the intermediate

[5] Wooldridge (2005) shows how one can perform both the first and second stages of OP (or LP) simultaneously. Not only will this be more efficient, but it also makes it easier to compute standard errors. We discuss the Wooldridge moments in more detail when we describe our suggested procedure. For details on standard errors for the OP 2-step process, see Pakes and Olley (1995).

input is essentially chosen at the time production takes place. We describe this as a "perfectly variable" input. Secondly, note that l_{it} does not enter (5). This implies that labor is also a "perfectly variable" input. If l_{it} was chosen at some point in time before m_{it}, then l_{it} would impact the firm's optimal choice of m_{it}.

Given this specification, LP proceed similarly to OP. Under the assumption that intermediate input demand (5) is monotonic in ω_{it}[6], we can invert:

$$\omega_{it} = f_t^{-1}(m_{it}, k_{it}) \tag{6}$$

Substituting this into the production function gives

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + f_t^{-1}(m_{it}, k_{it}) + \epsilon_{it} \tag{7}$$

The first step of the LP estimation procedure estimates β_l using the above equation, treating f_t^{-1} non-parametrically. Again, β_k and β_m are not identified as k_{it} and m_{it} are collinear with the non-parametric term. One also obtains an estimate of the composite term, in this case $\beta_k k_{it} + \beta_m m_{it} + f_t^{-1}(m_{it}, k_{it})$, which we again denote $\widehat{\Phi}_{it}$.

The second stage of the LP procedure proceeds as OP, the only difference being that there is one more parameter to estimate, β_m. LP use the same moment condition as OP to identify the capital coefficient, i.e. that the innovation component of ξ_{it}, ω_{it}, is orthogonal to k_{it}. $\xi_{it}(\beta_k, \beta_m)$ can again be contructed as the residual from a non-parametric regression of $(\omega_{it}(\beta_k, \beta_m) = \widehat{\Phi}_{it} - \beta_k k_{it} - \beta_m m_{it})$ on $(\omega_{it-1}(\beta_k, \beta_m) = \widehat{\Phi}_{it-1} - \beta_k k_{it-1} - \beta_m m_{it-1})$. They also add an additional moment to identify β_m, the condition that that $\xi_{it}(\beta_k, \beta_m)$ is orthogonal to m_{it-1}. This results in the following moment condition on which to base estimation:

$$E[\xi_{it}(\beta_k, \beta_m) | \begin{matrix} k_{it} \\ m_{it-1} \end{matrix}] = 0$$

Note that the innovation ξ_{it} is clearly *not* orthogonal to m_{it}. This is because ω_{it} is observed at the time that m_{it} is chosen. On the other hand, according to the model, ξ_{it} should be uncorrelated with m_{it-1}, as m_{it-1} was decided at $t-1$.

2.3 Key Assumptions of OP and LP

Note that both the OP and LP procedures rely on a number of key structural assumptions. While these assumptions are described in these papers (see also Griliches and Mairesse (1998) and ABBP), we summarize them here. A first key assumption is the strict monotonicity assumption - for OP investment must be strictly monotonic in ω_{it} (at least when it is non-zero), while for LP intermediate input demand must be strictly monotonic in ω_{it}. Monotonicity is required for the

[6] LP provide conditions on primitives such that this is the case.

non-parametric inversion because otherwise, one cannot perfectly invert out ω_{it} and completely remove the endogeneity problem in (4).

A second key assumption is that ω_{it} is the *only* unobservable entering the functions for investment (OP) or the intermediate input (LP). We refer to this sometimes as a "scalar unobservable" assumption. This rules out, e.g. measurement error or optimization error in these variables, or a model in which exogenous productivity is more than single dimensional. Again, the reason for this assumption is that if either of these were the case, one would not be able to perfectly invert out ω_{it}.[7]

A third key set of assumptions of the models regards the timing and dynamic implications of input choices. By timing, we refer to the point in the ω_{it} process at which inputs are chosen. First, k_{it} is assumed to have been decided exactly at (OP) or exactly at/prior to (LP) time period $t-1$. Any later than this would violate the moment condition, as k_{it} would likely not be orthogonal to some part of the innovation in ω_{it}. For OP, were i_{it-1} (and thus k_{it}) to be decided any earlier than $t-1$, then one could not use i_{it-1} to invert out ω_{it-1}, making first-stage estimation problematic.

Regarding the labor input, there are a couple of important assumptions. First, in OP, l_{it} must have no dynamic implications. Otherwise, l_{it} would enter the investment demand function and prevent identification of the labor coefficient in the first stage. In LP, labor can have dynamic implications, but one would need to adjust the procedure suggested by LP by allowing l_{it-1} into the intermediate input demand function. Note that in principle, this still allows one to identify a coefficient on labor in the first stage. Second, for LP it is important that l_{it} and m_{it} are assumed to be perfectly variable inputs. By this we mean that they are decided *when* ω_{it} is observed by the firm. If m_{it} were decided before learning ω_{it}, then m_{it} could not be used to invert out ω_{it} and control for it in the first stage. If l_{it} were chosen before learning ω_{it}, then l_{it} would also be chosen before m_{it}. In this case, a firm's choice of materials m_{it} would directly depend on l_{it} and l_{it} would enter the LP control function, preventing identification of the labor coefficient in the first stage.

3 Collinearity Issues

This paper argues that even if the above assumptions hold, there are potentially serious identification issues with these methodologies, particularly the LP approach. The problem is one of collinearity arising in the first stage of the respective estimation procedures, respectively:

$$y_{it} = \beta_l l_{it} + \Phi(i_{it}, k_{it}) + \epsilon_{it} \tag{8}$$

[7] ABBP discuss how this assumption can be relaxed in some particular dimensions (e.g. allowing ω_{it} to follow a higher than first order Markov process), but all these cases require the econometrician to observe and use additional control variables in the first stage.

and

$$y_{it} = \beta_l l_{it} + \Phi(m_{it}, k_{it}) + \epsilon_{it} \tag{9}$$

where the obviously non-identified terms ($\beta_k k_{it}$ in OP, $\beta_k k_{it}$ and $\beta_m m_{it}$ in LP) have been subsumed into the non-parametric functions. Recall that in the first stage, the main goal in both methods is to identify β_l, the coefficient on the labor input. What we now focus on is the question of whether even β_l can be identified from these regressions under the above assumptions. There is clearly no endogeneity problem - ϵ_{it} are either unanticipated shocks to production not known at t or purely measurement error in output, so they are by assumption are uncorrelated with all the right hand variables. Thus, the only real identification question here is whether l_{it} is collinear with the non-parametric terms in the respective regressions, i.e. whether l_{it} varies independently of the non-parametric function that is being estimated.

3.1 Levinsohn and Petrin

First, consider the LP technique. To think about whether l_{it} varies independently of $f_t^{-1}(m_{it}, k_{it})$, we need to think about the data generating process for l_{it}, i.e. how the firm choses l_{it}. Given that we have already assumed that l_{it} and m_{it} are chosen simultaneously and are both perfectly-variable, non-dynamic inputs, a natural assumption might be that they are decided in similar ways. Since m_{it} has been assumed to be chosen according to

$$m_{it} = f_t(\omega_{it}, k_{it}) \tag{10}$$

this suggests that l_{it} might be chosen according to

$$l_{it} = g_t(\omega_{it}, k_{it}) \tag{11}$$

While g_t will typically be a different function than f_t (e.g. because of different prices of the inputs), they both will generally depend on the same state variables, ω_{it} and k_{it}. Intuitively, this is just saying that the choice of both variable inputs at t depends on the predetermined value of the fixed input and the current productivity shock.

Substituting (6) into (11) results in

$$l_{it} = g_t(f_t^{-1}(m_{it}, k_{it}), k_{it}) = h_t(m_{it}, k_{it})$$

which states that l_{it} is some time-varying function of m_{it} and k_{it}. While this is a very simple result, it has some very strong implications on the LP first stage estimating equation (9). In particular, it says that the coefficient β_l is *not identified*. One simply cannot simultaneously estimate a fully non-parametric (time-varying) function of (ω_{it}, k_{it}) along with a coefficient on a variable that is only a (time-varying) function of those same variables (ω_{it}, k_{it}). Given this perfect collinearity between l_{it} and the non-parametric function, β_l should not be identified.

That said, while (11) might be the most natural specification for the data generation process (DGP) for l_{it}, it is not the only possibility. Our goal now is to search for an alternative DGP for l_{it} (and possibly for m_{it}) that will allow the LP first stage procedure to work. Not only must this alternative DGP move l_{it} around independently of the non-parametric function $f_t^{-1}(m_{it}, k_{it})$, but it must simultaneously be consistent with the basic assumptions of the LP procedure detailed in the last section.

First, consider adding firm-specific input prices to the above model of input choice, e.g. prices of labor (p_{il}) and materials (p_{im}). Obviously these firm-specific input prices will generally affect a firm's choices of l_{it} and m_{it}. A first note is that these input prices would have to be observed by the econometrician. Unobserved firm-specific input prices would enter (5) and violate the scalar unobservable assumption necessary for the first stage LP inversion. In other words, with unobserved firm-specific input prices, one can no longer invert out the firm's productivity shock as a function of the observables m_{it} and k_{it} and perform the first stage.

If the firm-specific input prices are observed, the inversion is not a problem - one simply can include the observed input prices in the non-parametric function. However, for the same reason, observed firm-specific input prices also do not solve the collinearity problem. Given that l_{it} and m_{it} are set at the same points in time, they will generally *both* be a function of *both* p_l and p_m. As such, we have the same problem as before - there are no variables that affect l_{it} but that do not affect m_{it} (and thus enter the non-parametric function). Our conclusion is that firm-specific input prices do not generally help matters. A related possibility is to allow labor to have dynamic effects. As discussed above, this is consistent with the LP assumptions as long as one adds l_{it-1} to the first stage non-parametric term. However, for the same reason, dynamic labor does not break the collinearity problem. As both l_{it} and m_{it} will generally depend on l_{it-1}, the term will not move l_{it} around independently of f_t^{-1}

In the basic model described above, l_{it} and m_{it} are chosen simultaneously at period t, i.e. after observing ω_{it}. A second alternative to try to break the collinearity problem is to perturb the model by changing these points in time at which l_{it} and m_{it} are set, i.e. allow l_{it} to be set before or after m_{it}. To formally analyze these situations, consider a point in time, $t-b$, between period $t-1$ and t (i.e. $0 < b < 1$). Assume that ω evolves through these "subperiods" $t-1$, $t-b$, and t according to a first order Markov process, i.e.

$$p(\omega_{it-b}|I_{it-1}) = p(\omega_{it-b}|\omega_{it-1}) \tag{12}$$

and

$$p(\omega_{it}|I_{it-b}) = p(\omega_{it}|\omega_{it-b}) \tag{13}$$

Note that we continue to assume that production occurs "on the period", i.e. at periods $t-1$ and t. The main point of introducing the subperiod $t-b$ is to allow the firm to have a different information set when chosing l_{it} than when choosing m_{it}. The hope is that these different information sets might generate independent variation in the two variables that could break the collinearity problem.

Given this setup, we can now consider perturbing the points in time at which l_{it} and m_{it} are set. First consider the situation where m_{it} is chosen at $t-b$ and l_{it} is chosen at t. Now a firm's optimal choice of m_{it} will depend on ω_{it-b}, while the choice of l_{it} will depend on ω_{it}. In this setup, l_{it} does have variance that is independent of m_{it}, because of the innovation in ω_{it} between ω_{it-b} and ω_{it}. However, this setup is also problematic for the first stage of the LP procedure. Since m_{it} is a function of ω_{it-b}, not ω_{it}, it cannot completely inform us regarding ω_{it}. In other words, the first stage non-parametric function will not be able to capture the entire productivity shock ω_{it}. Unfortunately, the part of ω_{it} that is not captured and left in the residual (which amounts to the unexpected innovation in ω_{it} given ω_{it-b}, i.e. $\omega_{it} - E[\omega_{it}|\omega_{it-b}]$) will be highly correlated with any independent variation in l_{it}. This creates an endogeneity problem whereby first stage estimates of β_l will be biased.[8]

Next, consider the situation where where l_{it} is chosen at $t-b$ and m_{it} is chosen at t. Again, in this case, the fact that m_{it} and l_{it} are chosen with different information sets generates independent variation. However, in this case, there is another problem. Since l_{it} is chosen before m_{it}, a profit maximizing (or cost-minimizing) firm's optimal choice of m_{it} will generally *directly* depend on l_{it}, i.e.

$$m_{it} = f_t(l_{it}, \omega_{it}, k_{it})$$

Given this, l_{it} should directly enter the first-stage non-parametric function and an LP first stage estimate of β_l is obviously not identified. In summary, neither of these timing stories appears to be able to justify the LP first stage procedure.

We next consider stories based on measurement error or optimization error on the part of firms. The difference between the two can be illustrated in the following model:

$$m_{it} = m_{it}^* + \lambda_{it}^m = f_t(\omega_{it}, k_{it}) + \lambda_{it}^m$$

When λ_{it}^m represents measurement error, m_{it}^* is the variable that actually enters the production function. When λ_{it}^m represents optimization error, m_{it} is the variable entering the production function. With optimization error, a firm should optimally be choosing input level $f_t(\omega_{it}, k_{it})$,

[8] Note that there is definitely not a sense in which one will "almost" get a correct estimate of estimate of β_l because m_{it} "almost" inverts out the correct ω_{it}. The reason is that all the variation in l_{it} that is independent of the non-parametric function is due to the innovation in ω_{it} between $t-b$ and t (e.g. if ω_{it} does not vary between $t-b$ and t we are back to the original collinearity problem). This innovation in ω_{it} between $t-b$ and t is also exactly what remains in the residual because of the incorrect inversion. Hence, any independent variation in l_{it} will be highly correlated with the residual, likely creating a big endogeneity problem.

but for some reason chooses $f_t(\omega_{it}, k_{it}) + \lambda_{it}^m$ instead.

A first observation is that neither measurement error or optimization error in m_{it} is a workable solution to the collinearity problem. Either measurement error or optimization error in m_{it} adds another unobservable to the m_{it} equation, which like unobserved input prices violates the scalar unobservable assumption and makes the first stage inversion impossible.

What if there is measurement error in l_{it}? In this case, l_{it} will vary independently of the non-parametric function, as the measurement error moves l_{it} around independently of m_{it}. However, while there is independent variation in l_{it}, this independent variation is just noise that does not affect output. All the meaningful variation in l_{it} is still collinear with the non-parametric function. Because the only independent variation in l_{it} is noise, the LP first stage estimate of β_l will converge to zero - certainly not a consistent estimate of the labor coefficient.

Lastly, consider optimization error in l_{it}. Like measurement error, this optimization error will move l_{it} around independently of the non-parametric function. However, unlike the measurement error situation, this independent variance does end up affecting output through β_l. Hence, LP first stage estimates should correctly identify the coefficient. While this does finally give us a DGP that validates the LP first stage procedure, we feel that it is not an identification argument that empirical researchers will generally feel comfortable applying. First, in this situation, the extent of identification is completely tied to the extent of optimization error. In many situations one might feel uncomfortable basing identification entirely on optimization error. Second, note that while one needs to assume that there is enough optimization error in l_{it} to identify β_l, one simultaneously needs to assume exactly no optimization error in m_{it}. Recall, that if there were optimization error in m_{it}, the inversion would not be valid. This sort of DGP assumption, i.e. that there is simultaneously lots of optimization error in one variable input yet no optimization error in the other variable input, strikes us as one that would be very hard to motivate or maintain.[9]

In addition to this optimization error story, there is one other DGP that can at least in theory rationalize the LP first stage procedure. Let us go back to moving around the points in time when inputs are chosen. Specifically, suppose that at time $t-b$, intermediate input m_{it} is chosen by the firm. Subsequently, at time t, labor input l_{it} is chosen. Recall from the above that this is problematic if ω varies between these two points in time. Therefore, consider a DGP where ω *does not* evolve between the points $t-b$ and t. What we do want to happen between the choice of m_{it} at $t-b$ and the choice of l_{it} at t is some *other* shock that affects a firm's choice of l_{it}. Consider, e.g., an unobserved shock to the price of labor that occurs between these two points in time. Call this shock $\varkappa_{it}$. Since $\varkappa_{it}$ is realized *after* the firm's choice of m_{it}, the firm's choice of m_{it} will not depend on the shock. Hence, the first stage inversion is still valid. Because the shock occurs *before* the choice of l_{it}, it does influence the firm's choice of l_{it} and hence moves l_{it} around independently from the non-parametric function. As such, the existence of $\varkappa_{it}$ will break the collinearity problem and

[9] Note that it is hard to motivate such an assumption by appealing to unions restricting the hiring and firing of labor. Such restrictions will generally affect m_{it} as well as l_{it}, invalidating the first stage inversion.

in theory will allow first stage identification of the labor coefficient. However, we again find this DGP to be one that would be very hard to motivate in real world examples. One needs to assume that 1) firms choose m_{it} before choosing l_{it}, 2) in the time period between these choices, ω_{it} *does not* evolve, and 3) in the time period between these choices, $\varkappa_{it}$ *does* evolve. One additionally needs to assume that 4) $\varkappa_{it}$ is i.i.d. over time - otherwise, a firm's optimal m_{it+1} would depend on the unobserved $\varkappa_{it}$, violating the first stage inversion, and 5) that the unobserved $\varkappa_{it}$ varies across firms - because the non-parametric function is indexed by t, variation $\varkappa_{it}$ across time is not helpful at moving around l_{it} independently. This strikes us as a very particular and unintuitive set of assumptions. Not only are they untuitive, but the assumptions also seem asymmetric in arbitrary ways - one unobservable is allowed to be correlated across time while the other is not, there must exist a period of time during which one unobservable evolves but the other doesn't, and some input prices must be constant across firms, while others must not be. It is hard for us to imagine a situation where this DGP would hold, even to an approximation.

To summarize, there appears to be only two potential DGPs that save the LP procedure from collinearity problems. One requires a significant amount of optimization error in l_{it}, yet no optimization error in m_{it}. The second requires a very unintuitive set of assumptions on timing and unobservables. Neither of these DGPs seem like sensible arguments for identification to us. An important note is that, in practice, one probably would not observe this collinearity problem. It is very likely that estimation of (7) would produce an actual numerical estimate. Our point is that unless one believes that one or both of the above two DGPs holds (and additionally that these are the *only* reasons why the first stage equation is not collinear), this is simply not a consistent estimator of β_l.[10] Another way to describe this result is that unless one believes in one of the above two DGPs, the extent to which the LP first stage is identified is also the extent to which is misspecified.

3.2 Olley and Pakes

Now consider the OP model. Given the above results regarding the LP procedure, a reasonable question is whether the OP model also suffers from a similar collinearity problem. While we show there are similar collinearity issues with the OP model, we argue that this collinearity can be broken under more reasonable assumptions than in LP.

In OP, the question is whether l_{it} is collinear with the non-parametric function $f_t^{-1}(i_{it}, k_{it})$. Again, the most obvious formulation of labor input demand is that l_{it} is just a function of ω_{it} and k_{it}, i.e. $l_{it} = g_t(\omega_{it}, k_{it})$. If this is the case, then we again have a collinearity problem. To obtain identification, one again needs a DGP in which something moves l_{it} around independently of $f_t^{-1}(i_{it}, k_{it})$. Two possibilities are analagous to the DGPs we just described in the LP model -

[10] Analagously, one might regress l_{it} on k_{it} and m_{it} and not find a perfect fit. In the context, our point would be that according to the LP assumptions, there is no really believable DGP why one wouldn't get a perfect fit in such a regession.

i.e. either optimization error in l_{it} (with no optimization error in i_{it}), or i.i.d., firm-specific, shocks to the price of labor (or other relevant variables) that are realized between the points in time at which i_{it-1} is chosen and l_{it} is chosen. However, as we have just argued, these DGPs rely on very strong and unintuitive assumptions.

Fortunately, in the OP context, there is an alternative DGP which breaks the collinearity problem and is consistent with the assumptions of the model. We also feel that this is considerably more believable than the above two DGPs. Consider the OP context where l_{it} is actually not a perfectly variable input, and is chosen at some point in time between periods $t-1$ and t. Similar to above, denote this point in time as $t-b$, where $0 < b < 1$. Suppose that ω evolves between the subperiods $t-1$, $t-b$, and t according to a first-order markov process, as in eqs (12) and (13).

In this case, a firm's optimal labor input will not be a function of ω_{it}, but of ω_{it-b}, i.e.

$$l_{it} = f_t\left(\omega_{it-b}, k_{it}\right)$$

Since ω_{it-b} cannot generally be written as a function of k_{it}, and i_{it}, l_{it} will *not* generally be collinear with the non-parametric term in (4), allowing the equation to be identified. Note the intution behind this - the fact that labor is set before production means that labor is determined by ω_{it-b} rather than ω_{it}. The movement of ω between $t-b$ and t is what breaks the collinearity problem between l_{it} and the non-parametric function. This seems to us to be a much more reasonable identification story than those needed for the LP procedure to properly identify β_l. However, note that this DGP does rule out a firm's choice of l_{it} having dynamic implications. If labor did have dynamic effects, then l_{it} would directly impact a firm's choice of i_{it}. As a result, l_{it} would directly enter the first stage non-parametric function and prevent identification of β_l.

Lastly, note why this more reasonable DGP *does not work* in the context of the LP model. In the LP model, if l_{it} is chosen before m_{it}, then m_{it} will *directly* depend on l_{it}, making β_l unidentified in the first stage. In OP, even if l_{it} is chosen before i_{it}, i_{it} does not depend on l_{it} (as long as one maintains the assumption that labor has no dynamic implications). This is because i_{it}, unlike m_{it}, is not directly linked to period t outcomes, and thus l_{it} will not affect a firm's optimal choice of i_{it}. The fact that this type of DGP works for OP but does not work for LP is the reason that we describe the collinearity problem as being worse for the LP methodology.

4 Parametric Versions of LP?

The collinearity problem in LP is that in the first stage equation,

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + f_t^{-1}\left(m_{it}, k_{it}\right) + \epsilon_{it} \tag{14}$$

the non-parametric function $f_t^{-1}\left(m_{it}, k_{it}\right)$ will generally be collinear with l_{it} under the maintained assumptions of the model. One approach to solving this collinearity problem might be to treat

$f_t^{-1}(m_{it}, k_{it})$ parametrically. Note that even though l_{it} might again just be a function of m_{it} and k_{it}, if it is a *different* function of m_{it} and k_{it} than f_t^{-1} is, this parametric version is potentially identified. While using a parametric version makes more assumptions than the non-parametric approach, one might be willing to make such assumptions with relatively uncomplicated input choices such as materials.

Unfortunately, this parametric approach does not work, at least for some popular production functions. In the case of Cobb-Douglas, the first order condition for m_{it} (conditional on k_{it}, l_{it}, and ω_{it}) is:

$$\beta_m K_{it}^{\beta_k} L_{it}^{\beta_l} M_{it}^{\beta_m - 1} e^{\omega_{it}} = \frac{p_m}{p_y}$$

assuming firms are price takers in both input and output markets. Recall that capital letters represent levels (rather than logs) of the inputs. Inverting this out for ω_{it} gives:

$$\begin{aligned} e^{\omega_{it}} &= \frac{1}{\beta_m}\frac{p_m}{p_y} K_{it}^{-\beta_k} L_{it}^{-\beta_l} M_{it}^{1-\beta_m} \\ \omega_{it} &= \ln(\frac{1}{\beta_m}) + \ln(\frac{p_m}{p_y}) - \beta_1 k_{it} - \beta_2 l_{it} + (1-\beta_m) m_{it} \end{aligned}$$

and plugging this inversion into the production function results in:

$$y_{it} = \ln(\frac{1}{\beta_m}) + \ln(\frac{p_m}{p_y}) + m_{it} + \epsilon_{it} \tag{15}$$

The key point here is that β_l has dropped out of the estimating equation, making a moment condition in ϵ_{it} worthless for identifying β_l. As such, with a Cobb-Douglas production function, a parametric approach cannot generally be used as a first stage to identify β_l.

One gets a similar result with a production function that is Leontief in the material inputs. Consider, for example:

$$Y_{it} = \min\left[\gamma_0 + \gamma_1 M_{it}, K_{it}^{\beta_k} L_{it}^{\beta_l} e^{\omega_{it}}\right] + \epsilon_{it}$$

With this production function, the first order condition for M_{it} satisfies

$$\gamma_0 + \gamma_1 M_{it} = K_{it}^{\beta_k} L_{it}^{\beta_l} e^{\omega_{it}}$$

as long as $\gamma_1 p_y > p_m$. At this optimum, note that:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \tag{16}$$

which could form an estimating equation if not for endogeneity problems. Inverting out ω_{it} results in:

$$\begin{aligned} e^{\omega_{it}} &= \frac{\gamma_0 + \gamma_1 M_{it}}{K_{it}^{\beta_k} L_{it}^{\beta_l}} \\ \omega_{it} &= \ln(\gamma_0 + \gamma_1 M_{it}) - \beta_k k_{it} - \beta_l l_{it} \end{aligned}$$

and substituting into (16) results in

$$y_{it} = \ln(\gamma_0 + \gamma_1 M_{it}) + \epsilon_{it} \tag{17}$$

so again, this procedure is not helpful for identifying β_l.

In summary, even with parametric assumptions, there may be an identification problem in the first stage of the LP technique using intermediate inputs to control for unobserved factors of production. However, it is possible that as one moves away from Cobb-Douglas production functions (or Hicks neutral unobservables), a parametric approach might be identified (see Van Biesebroeck (2003) for a related example).

5 Our Alternative Procedure

We now suggest an alternative estimation procedure that avoids the collinearity problems discussed above. This procedure draws on aspects of both the OP and LP procedures and is able to use either the 'intermediate input as proxy' idea of LP, *or* the 'investment as proxy' idea of OP. The main difference between this new approach and OP and LP is that in the new approach, no coefficients will be estimated in the first stage of estimation. In contrast, the input coefficients are all estimated in the second stage. However, as we shall see, the first stage will still be important to net out the untransmitted error ϵ_{it} from the production function. We exhibit our approaches using value added production functions. They could also be used in the case of gross output production functions, although in this case one might need to consider issues brought up by Bond and Söderbom (2005).[11] We start by showing how our method works with the LP intermediate input proxy. We then show how our method is consistent even if labor has dynamic implications and how out procedure works using the OP investment proxy. Lastly, we compare our procedure to methods used in the dynamic panel data literature, e.g. Arellano and Bover (1995), and Blundell and Bond (1998, 1999). We feel that this is important because up to now, these two literatures (OP, LP vs. dynamic panel methods) have evolved somewhat separately. Our estimation procedure makes it quite easy to see the tradeoffs and different assumptions behind the two approaches.

[11] Bond and Söderbom (2005) argue that it may be hard (if not impossible) to identify coefficients on perfectly variable (and non-dynamic) inputs in a Cobb-Douglas framework. Note that this is also an critique of the original LP procedure's identification of the materials coefficient. On the other hand, value-added production functions have their own issues, see, e.g. Basu and Fernald (1997).

5.1 The Basic Procedure

Consider the following value added production function,

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \tag{18}$$

Our basic idea is quite simple - to give up on trying to estimate β_l in the first stage. However, we will still estimate a first stage - the goal of this first stage will be to separate ω_{it} from ϵ_{it}. As we will see momentarily, this will be a key issue in permitting us to treat the ω_{it} process non-parametrically.

Perhaps the most intuitive way to "give up" estimation of β_l in the first stage is to allow for labor inputs to be chosen *before* material inputs. More precisely, suppose that l_{it} is chosen by firms at time $t-b$ $(0 < b < 1)$, *after* k_{it} was chosen at (or before) $t-1$ but *prior* to m_{it} being chosen at t. Suppose that ω_{it} evolves according to a first order markov process between these subperiods $t-1$, $t-b$, and t, i.e.

$$p(\omega_{it}|I_{it-b}) = p(\omega_{it}|\omega_{it-b})$$

and

$$p(\omega_{it-b}|I_{it-1}) = p(\omega_{it-b}|\omega_{it-1})$$

Our feeling is that this assumption that labor is "less variable" than materials will make sense in many industries. For example, it is consistent with firms needing time to train new workers, or needing to give workers some time period of notice before firing. Given these timing assumptions, a firm's material input demand at t will now directly depend on the l_{it} chosen prior to it, i.e.

$$m_{it} = f_t\left(\omega_{it}, k_{it}, l_{it}\right) \tag{19}$$

Inverting this function for ω_{it} and substituting into the production function results in a first stage equation of the form:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + f_t^{-1}\left(m_{it}, k_{it}, l_{it}\right) + \epsilon_{it} \tag{20}$$

β_l is clearly not identified in this first stage. However, one does obtain an estimate, $\widehat{\Phi}_t$, of the composite term,

$$\Phi_t(m_{it}, k_{it}, l_{it}) = \beta_k k_{it} + \beta_l l_{it} + f_t^{-1}\left(m_{it}, k_{it}, l_{it}\right)$$

which represents output net of the untransmitted shock ϵ_{it}. Intuitively, by conditioning on a firm's choice of material inputs (or analagously in this case conditioning on the information set at

t), this procedure allows us to isolate and eliminate the portion of output determined by shocks unanticipated at t (e.g. unanticipated weather shocks, defect rates, or machine breakdown) or by measurement error.

However, with no coefficients obtained in the first stage, we still need to identify β_k and β_l. This now requires *two* independent moment conditions for identification in the second stage. Given the first-order Markov assumption on ω_{it}, we have

$$\omega_{it} = E[\omega_{it}|I_{it-1}] + \xi_{it} = E[\omega_{it}|\omega_{it-1}] + \xi_{it}$$

where ξ_{it} is mean independent of all information known at $t-1$. Given the OP/LP timing assumption that k_{it} was decided at $t-1$ (and hence $k_{it} \in I_{it-1}$), this leads to the second stage moment condition used by both OP and LP, namely that:

$$E[\xi_{it}|k_{it}] = 0 \qquad (21)$$

Of course, this moment would not hold if one replaced k_{it} with l_{it}. Since l_{it} is chosen *after* t, at time $t-b$, l_{it} will generally be correlated with at least part of ξ_{it}. On the other hand, lagged labor, l_{it-1}, was chosen at time $t-b-1$. Hence, it *is* in the information set I_{it} and will be uncorrelated with ξ_{it}. This is the second moment we suggest using in estimation, i.e. we suggest using the moments

$$E\left[\xi_{it}\middle| \begin{matrix} k_{it} \\ l_{it-1} \end{matrix}\right] = 0 \qquad (22)$$

to identify β_k and β_l.

Operationalizing this moment is analagous to the second stage of the OP and LP procedures. We can recover the implied ξ_{it}'s for any value of the parameters (β_k, β_l) as follows. First, given a candidate value of (β_k, β_l), compute the implied $\omega_{it}(\beta_k, \beta_l)$'s $\forall t$ using the formula:

$$\omega_{it}(\beta_k, \beta_l) = \widehat{\Phi}_t - \beta_k k_{it} - \beta_l l_{it}$$

Second, non-parametrically regress $\omega_{it}(\beta_k, \beta_l)$ on $\omega_{it-1}(\beta_k, \beta_l)$ (and a constant term[12]) - the residuals from this regression are the implied $\xi_{it}(\beta_k, \beta_l)$'s.

[12] If one wanted to allow the constant in the production function to vary across time, one could either 1) add time dummies (or a time trend) to the regression of $\omega_{it}(\beta_k, \beta_l)$ on $\omega_{it-1}(\beta_k, \beta_l)$, or 2) estimate the constant term(s) directly with (23), i.e. compute $\omega_{it}(\beta_{0t,}\beta_k, \beta_l)$ and $\xi_{it}(\beta_{0t,}\beta_k, \beta_l)$ $\forall t$ and use time dummies as additional instruments in (23). In this second case, one can either a) include all but one of the time dummies in the production function and include a constant term in the regression of $\omega_{it}(\beta_{0t,}\beta_k, \beta_l)$ on $\omega_{it-1}(\beta_{0t,}\beta_k, \beta_l)$, or b) include a full set of time dummies and not include a constant term in the regression of $\omega_{it}(\beta_{0t,}\beta_k, \beta_l)$ on $\omega_{it-1}(\beta_{0t,}\beta_k, \beta_l)$.

Given these implied $\xi_{it}(\beta_k, \beta_l)$'s, one can form a sample analogue to the above moment, e.g.

$$\frac{1}{T}\frac{1}{N}\sum_t \sum_i \xi_{it}(\beta_k, \beta_l) \cdot \begin{pmatrix} k_{it} \\ l_{it-1} \end{pmatrix} \tag{23}$$

and estimate (β_k, β_l) by minimizing this sample analogue.

We end this section with several important observations regarding our suggested procedure. First, the moment condition we use to identify the labor coefficient, i.e. $E[\xi_{it}|l_{it-1}] = 0$ is actually used by LP (and OP in a more informal way) as an overidentifying restriction on the model in their second stage procedure. However, there is a fundamental difference between what we are doing and what OP/LP do - in OP/LP, the labor coefficient is estimated in the first stage *without* using any of the information from the second element of (23). In our procedure, the information in (23) is crucial in identifying the labor coefficient. Given the problem of the LP first stage identification of β_l described above, we prefer our method of identification.

Second, it is important to note that our procedure is completely consistent with labor choices having having dynamic implications. This would probably be the case if, e.g. there were firing, hiring or training costs of labor. Note that in this case, firms' optimal choices of l_{it} and k_{it} will depend on l_{it-1}, but the intermediate input demand function m_{it} will not. This is because choice of m_{it} already depends on l_{it} (since l_{it} was chosen before m_{it}), and because m_{it} is only relevant for period t production.[13] We feel that this is important not only for robustness reasons, but also because the additional variation in l_{it} generated by dynamics will likely improve identification (see Bond and Söderbom (2005)).[14]

Third, our procedure is also consistent with other unobservables, e.g. input price shocks or dynamic adjustment costs affecting firm's choices of l_{it} and k_{it}. Importantly, these other unobservables *can* be correlated across time - since 1) m_{it} depends directly on l_{it} and k_{it} and 2) because m_{it} only affects current production, even serially correlated such unobservables will not influence a firm's optimal choice of m_{it}. Again, such unobservables would likely actually be helpful for identification by generating extra exogenous variation in k_{it} and l_{it}.[15] Note that one cannot allow

[13] If one wanted to assume that $b = 0$, i.e. that l_{it} is chosen at the *same time* as m_{it}, then one would want to replace l_{it} with l_{it-1} in the first stage non-parametric function.

[14] While it may not be as empirically relevant, our procedure can also be extended to allow dynamics in m_{it} - this could be accomplished by adding m_{it-1} into the first stage non-parametric function. However, in this case, one would probably need to rule out possible unobservables discussed in the next paragraph.

[15] An interesting case is where there are no dynamic effects of labor and no other unobservables affecting labor and/or capital. Suppose also that firms are price takers, are risk-neutral and choose labor optimally given a Cobb-Douglas production function (given the risk neutrality, firms choose labor as a function of the expectation of ω_{it} at $t-b$, i.e. $E[\omega_{it}|\omega_{it-b}]$). In this case, one can show that (β_k, β_l) are in fact not *globally* identified. In particular, there is a point at the boundary of parameter space, $\hat{\beta}_k = 0$, $\hat{\beta}_l = \beta_k + \beta_l$, that necessarily set the expectation of our moment condition equal to zero. This result is related to, but distinctly different from, the complete non-identification result in Bond and Söderbom (2005), which assumes that $b = 0$. Monte-carlo results when $b > 0$ are at least suggestive that the model is identified away from the above boundary point. However, identification based on dyanamic effects of labor or other unobservables seems preferable.

other unobservables to *directly* affect a firm's optimal choice of m_{it} - this would violate the scalar unobservable assumption necessary for the inversion.[16]

Fourth, in some situations one might feel comfortable assuming that l_{it} was chosen at or prior to $t-1$. This might be the case if the time period in a particular dataset is short, or if, e.g. there is a significant amount of training required before workers can enter production. If this is the case, one can alternatively use the moment conditions

$$E[\xi_{it}| \begin{array}{c} k_{it} \\ l_{it} \end{array}] = 0 \tag{24}$$

This is likely to generate more efficient estimates than the moment condition using l_{it-1}, as l_{it} is more directly linked to current output. Note that one could add additional lags of capital and labor to either set of moments (22) or (24) to generate overidentifying restricitions, although it is unclear how much extra identifying power these additional moments add.

Lastly, note that with the above two-stage procedure, it is probably most straightforward to derive asymptotic standard errors as done in LP, by bootstrapping. As mentioned above, Wooldridge (2005) suggests an alternative implementation of OP/LP that involves estimating the first and second stages simultaneously. This can easily be extended to our methodology by simply adding l_{it} to the first stage non-parametric function. This leads to the following two moments:

$$E\left[\begin{array}{c} \epsilon_{it} \mid I_{it} \\ \xi_{it}+\epsilon_{it} \mid I_{it-1} \end{array}\right] = E\left[\begin{array}{c} y_{it}-\beta_k k_{it}-\beta_l l_{it}-f^{-1}\left(m_{it},k_{it},l_{it};\beta_{f,t}\right) \mid I_{it} \\ y_{it}-\beta_k k_{it}-\beta_l l_{it}-g(f^{-1}\left(m_{it-1},k_{it-1},l_{it-1};\beta_{f,t-1}\right);\beta_g)|I_{it-1} \end{array}\right] = 0$$

where f and g are, e.g., polynomial functions with parameters β_{f_t} and β_g. The two moments correspond, respectively, to our first and second stages. Note the different conditioning sets, as ξ_{it} will generally be correlated with I_{it}. In practice, one would likely want to use k_{it}, l_{it} and polynomial basis functions of the arguments of f^{-1} interacted with time dummies as instruments for the first moment (ϵ_{it}), and k_{it}, l_{it-1} (not l_{it}) and polynomial basis functions of the scalar $f^{-1}\left(m_{it-1},k_{it-1},l_{it-1};\beta_{f_t}\right)$ as instruments for the second moment ($\xi_{it}+\epsilon_{it}$). Note that the polynomial basis functions of $f^{-1}\left(m_{it-1},k_{it-1},l_{it-1};\beta_{f_t}\right)$ will depend on the parameters β_{f_t}. The sample analogue of this set of moments can be minimized w.r.t. $(\beta_k,\beta_l,\beta_{f_t},\beta_g)$ to generate consistent estimates.

An important advantage of applying the Wooldridge one-step approach to our estimating equations is that standard errors can be computed using standard GMM formulas. One limitation is that it requires a non-analytic search over a much larger set of parameters, $(\beta_k,\beta_l,\beta_{f_t},\beta_g)$. The dimension of β_g is the dimension of the polynomial used to represent g, and the dimension of β_{f_t} is the dimension of the polynomial used to represent f times the number of time periods. In the

[16] If one had multiple intermediate inputs and conditioned on all these inputs along with making an appropriate multivariate invertibility assumption, one might be able to allow a limited number of such unobservables. The key is whether one can still recover ω_{it} as a function of the intermediate inputs, l_{it}, and k_{it}.

two-stage approach, one only has to search over the two production function parameters β_k and β_l - the parameters of the polynomials are all analytically computable. To avoid optimization problems we suggest using parameters from the two-step procedure as starting values if using the one-step approach. An even more reliable alternative might be to take *one* Newton-Raphson step with the one-step objective function using two-step estimates as starting parameters. This requires no additional optimization, and based on a result by Pagan (1986) produces estimates asymptotically equivalent to maximizing the one-stage objective function. As such, one can use the simpler method to compute asymptotic standard errors from the one stage approach.

5.2 Investment Proxy

One can also use our methodology with the investment proxy variable of OP. Interestingly, moving all identification to the second stage simultaneously makes the procedure robust to dynamic effects of labor.[17] Suppose that, as in OP, i_{it-1} (and thus k_{it}) is chosen exactly at $t-1$ (unlike when using an intermediate input proxy, where k_{it} can be chosen at *or before* t, this assumption is necessary for i_{it} to "invert out" the correct ω_{it}). As above, suppose that l_{it} is chosen at time $t-b$, and allow there to be possible dynamic effects of labor. In this case, a firms optimal investment decision will generally take the form

$$i_{it} = f_t\left(\omega_{it}, k_{it}, l_{it}\right) \tag{25}$$

since l_{it} is chosen before i_{it} and because l_{it} has possible dynamic implications. Inverting this function and substituting into the production function results in:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + f_t^{-1}\left(i_{it}, k_{it}, l_{it}\right) + \epsilon_{it} \tag{26}$$

which again clearly does not identify any coefficients in the first stage. However, one can again use the first stage to estimate the composite term

$$\Phi_t(i_{it}, k_{it}, l_{it}) = \beta_k k_{it} + \beta_l l_{it} + f_t^{-1}\left(i_{it}, k_{it}, l_{it}\right)$$

and proceed exactly as above, using the estimated $\widehat{\Phi}_t$'s to infer $\omega_{it}(\beta_k, \beta_l)$'s, $\xi_{it}(\beta_k, \beta_l)$'s, and form the moment (23).

Like our procedure using the intermediate input proxy, this procedure is consistent with labor having dynamic effects. However, unlike the above, it is not generally consistent with other, serially correlated unobservables entering either the i_{it} or l_{it} decisions. Another unobservable affecting the i_{it} equation is clearly problematic for the inversion. Less obviously, another serially correlated unobservable that affects the l_{it} decision will generally also affect the i_{it} decision directly

[17] Buettner (2005) also makes this suggestion for extending OP to allow dynamic effects.

since i_{it} is a dynamic decision variable. As a result, the inversion is problematic. The reason the intermediate input proxy is more robust to these addtional serially correlated unobservables is because it is only relevant for current output.

5.3 Relation to Dynamic Panel Models

Interestingly, the form of our suggested estimators make them fairly easy to compare it to estimators used in an alternative literature, the dynamic panel literature. This is important because up to now, researchers interested in estimating production functions have essentially been choosing between the OP/LP general approach versus the dynamic panel approach without a clear description of the similarities and differences in identifying assumptions used in the two methods. We start with a brief discussion of dynamic panel methods before comparing them to our estimator. To briefly summarize, there are distinct advantages and disadvantages of both approaches.

As developed by work such as Chamberlain (1982), Anderson and Hsiao (1982), Arellano and Bond (1991), Arellano and Bover (1995), and Blundell and Bond (1998, 1999), the dynamic panel literature essentially extends the fixed effects literature to allow for more sophisticated error structures. Consider the following production function model:

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + \alpha_{it} + \epsilon_{it} \tag{27}$$

Whereas the standard fixed effects estimator necessarily assumes that α_{it} is constant over time, the dynamic panel literature can allow more complex error structures. For example, suppose that α_{it} is composed of both a fixed effect (α_i) and a serially correlated unobservable (ω_{it}), i.e.

$$\begin{aligned} y_{it} &= \beta_1 k_{it} + \beta_2 l_{it} + \alpha_i + \omega_{it} + \epsilon_{it} && (28) \\ &= \beta_1 k_{it} + \beta_2 l_{it} + \psi_{it} && (29) \end{aligned}$$

Notationally, the composite error term ψ_{it} represents the sum of all three error term components.

The dynamic panel literature proceeds by first making assumptions on 1) the evolution of the error components α_i, ω_{it}, and ϵ_{it}, and 2) possible correlations between these error components and the explanatory variables k_{it} and l_{it}. Given these assumptions, the key is then to find functions of the aggregate error terms ψ_{it} (often these functions involve differencing the ψ_{it}'s) that are uncorrelated with past, present, or future values of the explanatory variables. Since the ψ_{it}'s are "observable" given particular values of the parameters (unlike the individual components of ψ_{it}), one can easily set up sample analogues of these moment conditions.

Continuing with the above production function example, a reasonable set of assumptions on the error components might be as follows. First, one might allow for potential correlation between the time-invariant error component α_i and k_{it} and l_{it}. Second, one could assume that ϵ_{it} is i.i.d. over time and uncorrelated with k_{it} and l_{it} (e.g. ϵ_{it} might represent measurement error or unanticipated shocks to y_{it}). Lastly, one could assume that ω_{it} follows an AR(1) process, i.e. $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$. Regarding correlation between ω_{it} and the inputs, one might allow that ω_{it} is correlated with k_{it} and l_{it} $\forall t$ but assume that the *innovation* in ω_{it} between $t-1$ and t, i.e. ξ_{it}, is uncorrelated with all input choices *prior* to t. Note that the intuition behind this assumption is similar to that behind the second stage moments in our procedure (and OP/LP). This idea is that since the innovation in ω_{it}, ξ_{it} occurs *after* time $t-1$, it should not be correlated with inputs dated $t-1$ and earlier.[18]

Given these particular assumptions, estimation can proceed as follows. Consider the following function of ψ_{it},

$$(\psi_{it} - \rho\psi_{it-1}) - (\psi_{it-1} - \rho\psi_{it-2}) = \xi_{it} - \xi_{it-1} + (\epsilon_{it} - \rho\epsilon_{it-1}) - (\epsilon_{it-1} - \rho\epsilon_{it-2})$$

The equality follows from the definitions of ψ_{it} and ω_{it}. Note that only the innovations in the AR(1) process enter this expression, and that all terms containing α_i have been differenced out. Now, since the innovations ξ_{it} and ξ_{it-1} have been assumed uncorrelated with all input choices prior to $t-1$ (and ϵ_{it} have been assumed uncorrelated with all input choices), we can easily form a method of moments estimator for β and ρ. By assumption the moment

$$E\left[(\psi_{it} - \rho\psi_{it-1}) - (\psi_{it-1} - \rho\psi_{it-2}) \mid \left\{ \begin{array}{c} k_{i\tau} \\ l_{i\tau} \end{array} \right\}_{\tau=1}^{t-2}\right] \tag{30}$$

is equal to zero. Again, the sample analogue of this moment is trivial to construct, since given values of the parameters, all ψ_{it}'s (and thus all $(\psi_{it} - \rho\psi_{it-1}) - (\psi_{it-1} - \rho\psi_{it-2})$'s) are "observed".

Before continuing, note that this estimation procedure can be adapted in various dimensions. For example, suppose one is unwilling to assume that the ϵ_{it} are uncorrelated with all inputs in all time periods, but prefers making the weaker assumption that ϵ_{it} is sequentially exogenous, i.e. uncorrelated with all input choices dated prior to t. In this case, the above moment is not equal to zero, as there is potentially correlation between ϵ_{it-2} and (k_{it-2}, l_{it-2}). However, the moment still holds for lagged inputs prior to $t-2$, so the alternative moment

$$E\left[(\psi_{it} - \rho\psi_{it-1}) - (\psi_{it-1} - \rho\psi_{it-2}) \mid \left\{ \begin{array}{c} k_{i\tau} \\ l_{i\tau} \end{array} \right\}_{\tau=1}^{t-3}\right]$$

could be used for estimation.

[18] As with the analagous assumption in the OP/LP/ACF models, this assumption is not just an assumption on the time series properties of ν_{it} - it is also an assumption on the information sets of firms (i.e. that firms do not observe ξ_{it}'s until they occur).

As another example, suppose we remove the fixed effect from the model, i.e.

$$y_{it} = \beta_1 k_{it} + \beta_2 l_{it} + \omega_{it} + \epsilon_{it} = \beta_1 k_{it} + \beta_2 l_{it} + \psi_{it} \tag{31}$$

but keep the same assumptions on ω_{it} and ϵ_{it}. In this case, one only needs to difference once to form a usable moment. More specifically, since

$$\psi_{it} - \rho\psi_{it-1} = \xi_{it} + (\epsilon_{it} - \rho\epsilon_{it-1})$$

we can use[19]

$$E\left[\psi_{it} - \rho\psi_{it-1} \mid \left\{ \begin{array}{c} k_{i\tau} \\ l_{i\tau} \end{array} \right\}_{\tau=1}^{t-1}\right] \tag{32}$$

as a moment for estimation if the ϵ_{it} are strictly exogenous (k_{it-1} and l_{it-1} could not be used as instruments if the ϵ_{it} were assumed sequentially exogneous).

This last example is particularly relevant for our current goals since the model (31) is very similar to our OP/LP style model and makes comparison quite easy. Note the differences in the construction of the second stage moments in the dynamic panel model versus those used in the second stage of our suggested procedure. In our procedure, the first stage serves to net out the ϵ_{it}. After this is done, we can compute ω_{it} $\forall t$ (conditional on parameters) and form moments in the innovations in ω_{it}. This contrasts with the dynamic panel approach, where conditional on the parameters, one *cannot compute* the individual ω_{it}'s, but can instead only compute the sums $\psi_{it} = \omega_{it} + \epsilon_{it}$ $\forall t$. While in both these cases one can form moments for estimation to consistently estimate the parameters, the difference between being able to "observe" ω_{it} versus being able to only "observe" the sum $\omega_{it} + \epsilon_{it}$ (conditional on parameters) has a number of important implications.

First, recall that in our model, ω_{it} can follow an arbitrary first order Markov process. This is not the case in the dynamic panel model. Not only must the Markov process generating ω_{it} be parametric, but it must also have a linear form. In the above example, it is the linearity of the AR(1) process that allows us to construct a useable moment using the sums $\omega_{it} + \epsilon_{it}$. To see this, suppose that instead of the Markov process being $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$, it is $\omega_{it} = \rho\omega_{it-1}^3 + \xi_{it}$. In this case, it is not clear how one could manipulate the sums ψ_{it} to form a useable moment. One

[19] This would not be a valid moment in the model with α_i because α_i would enter this particular moment and is potentially correlated with all values of the inputs.

can construct the difference $\psi_{it} - \rho\psi^3_{it-1}$, i.e.

$$\begin{aligned}\psi_{it} - \rho\psi^3_{it-1} &= \omega_{it} + \epsilon_{it} - \rho(\omega_{it-1} + \epsilon_{it-1})^3 \\ &= \omega_{it} + \epsilon_{it} - \rho(\omega^3_{it-1} + \epsilon_{it-1}{}^3 + 2\omega_{it-1}\epsilon_{it-1}{}^2 + 2\omega^2_{it-1}\epsilon_{it-1}) \\ &= \xi_{it} + \epsilon_{it} - \rho(\epsilon_{it-1}{}^3 + 2\omega_{it-1}\epsilon_{it-1}{}^2 + 2\omega^2_{it-1}\epsilon_{it-1})\end{aligned}$$

but while one term in this expression is the innovation term ξ_{it}, the expression also contains numerous other terms that are very likely correlated with the inputs.[20] More generally, it appears that with a non-linear Markov process, it will not be possible to cleanly construct a valid moment in the innovation term. In contrast, in our procedure, because we are able to recover the individual ω_{it}'s, it is trivial to deal with non-linear first order Markov processes (in this example, just regress ω_{it} on ω^3_{it-1} and form moments with the residual). Not only can our procedure deal with such non-linearities, but it also easily permits non-parametric estimation of these processes. Again, the crucial step here is the first stage estimation, which nets out the ϵ_{it} and allows us to observe ω_{it} conditional on the parameters. This flexibility in modelling of the ω_{it} process is a clear advantage of our procedure over dynamic panel methods.[21]

A second difference concerns the relative efficiency of the two estimators. The variance of a GMM estimator is proportional to the variance of the moment condition being used. Suppose, for example, that we know that ω_{it} follows an AR(1) process. In this case, our second stage would involve regressing ω_{it} on just ω_{it-1} (conditional on parameters) and setting the residual orthogonal to appropriately lagged instruments. This residual is equal to the innovation in ω_{it}, i.e. ξ_{it}, at the true parameters. In contrast, the dynamic panel approach sets the residual $\psi_{it} - \rho\psi_{it-1}$ orthogonal to instruments. This residual is equal to the innovation in ω_{it} *plus* some additional terms, i.e. $\xi_{it} + (\epsilon_{it} - \rho\epsilon_{it-1})$. Since these additional terms only add variance to the moment condition (for a given set of instruments), we suspect that our estimator will generally have lower variance than the dynamic panel estimator.[22]

There are also significant advantages of the dynamic panel estimator over our estimator. We feel that the most important one concerns possible fixed effects. For example, the start of this section showed how dynamic panel methods can allow for a fixed effect α_i *in addition* to the serially correlated process ω_{it}. The resulting estimator is consistent even for fixed T. This, to our knowledge, cannot be done with our estimator.[23] On the other hand, allowing for fixed effects in the dynamic panel literature requires an additional differencing and further lagging of instruments (compare the moment in (30) to that in (32)) - this likely puts considerably greater demands on

[20] In particular, given that ω_{it} is correlated with input choices (in all periods), it is highly likely that $2\omega_{it-1}\epsilon_{it}{}^2$ will be correlated with the inputs as well.

[21] Note that in the special case where ϵ_{it} is assumed zero for all i and t, the dynamic panel methodology can allow a non-linear (or non-parametric) Markov process. This is because in this case (i.e. $\epsilon_{it} = 0$), the ω_{it}'s *are* recoverable given parameters.

[22] Formally proving this would need to account for the first stage estimation error in netting out the ϵ_{it}'s.

[23] If $T \to \infty$, we could simply estimate the fixed effects in our model, but this is a much weaker result.

the data.[24] Perhaps this is one reason why these estimators have tended not to work particularly well in practice.[25] Regardless, the ability to allow for α_i's is definitely an advantage of the dynamic panel approach.

Another advantage of the dynamic panel literature is that it requires fewer assumptions regarding input demand equations. Recall that our procedure requires both a strict monotonicity and a scalar unobservable assumption on one of the input demand equations, e.g. on either investment or materials.[26] The dynamic panel literature does not require such assumptions. Of course, it is these assumptions that allow us to form the first stage equation, net out the ϵ_{it}'s, observe the ω_{it}'s conditional on the parameters, and thus treat the ω_{it} process non-parametrically.

The dynamic panel literature also permits one to make weaker assumptions on the ϵ_{it}'s. As described above, dynamic panel procedures can proceed either under a strict exogeneity assumption (ϵ_{it} uncorrelated with input choices at all t) or a weaker sequential exogeneity assumption (ϵ_{it} uncorrelated with input choices prior to t). Our procedure pretty much depends on the strict exogeneity assumption. The problem here does not regard the second stage moments (since ϵ_{it} does not enter the second stage moments) - it is with the first stage. The problem is that sequential exogeneity allows ϵ_{it} to affect future input choices. This will tend to violate the scalar unobservable assumption necessary for the first stage of our procedure.[27] Note that both types of procedures can allow ϵ_{it} to be correlated over time, at least in some cases. The key assumption in both is that ϵ_{it} is not in any way predictable by firms. For example, ϵ_{it} could contain measurement error in y_{it} that is serially correlated over time. Another seeming advantage of the dynamic panel literature is that it can allow for a higher than first order Markov process for ω_{it}, as long as this process is linear (e.g. an AR(2) process - note that this would require further differencing to construct a valid moment). However, ABBP show that our methods can be extended to non-parametric higher order Markov process if one observes a set of control variables equal to the order of the Markov process.

Lastly, there are some differences in how these two types of estimators have been used in practice, but that are less fundamental than the differences above. For example, a frequent assumption in the literature applying OP/LP methods has been that k_{it} was decided at period $t-1$. This generates orthogonality between ξ_{it} and k_{it}, which is likely a more informative moment than orthogonality between ξ_{it} and k_{it-1}. This assumption has typically not been made in the dynamic panel literature, but it easily could be. One can simply add k_{it-1} to the conditioning set

[24] Both the additional differencing and the further lagging of the instruments are likely to reduce the information in the moment condition (see, e.g. Griliches and Hausman (1981)).

[25] Talk about Blundell Bond additional instruments for this.

[26] It is possible to relax the scalar unobservable assumption in some cases, but this requires multiple proxy variables (e.g. investment choice and advertising choice) and a multidimensional strict monotonicity assumption (see ABBP).

[27] Formally, our procedure can allow ϵ_{it} to affect future choices of inputs not used for the first stage inversion (e.g. labor), but allowing ϵ_{it} to affect future labor choices but not, e.g. future material or investment choices, seems somewhat arbitrary.

in (30) or k_{it} to the conditioning set in (32) (under strict exogeneity). Presumably, this would increase the efficiency of dynamic panel estimates. The same idea could be applied to other "fixed" inputs as well. Another difference between how these estimators have been applied in practice is that while the dynamic panel literature has typically utilized orthogonality between differenced residuals and *all* inputs suitably lagged (i.e. from $\tau = 1$ to $\tau = t - 2$ or $t - 3$), applications using OP/LP methodology have often only used the latest dated valid observation for each input as instruments (the application in LP is a notable exception). Of course, all further lagged inputs are also valid instruments in our methodology (or OP/LP) and could also be used, analagous to the dynamic panel methodology. The tradeoff is as often the case - more moments generate more efficiency and result in overidentification (which is useful for testing purposes), but they often can also generate significant small sample biases.

In summary, while our procedure and dynamic panel methods for estimating production functions are related, there are fundamental differences between the two. While our procedure has more flexibility regarding the serially correlated transmitted error ω_{it}, it is less flexible regarding the non-transmitted error ϵ_{it} and in allowing fixed effects α_i. In some cases, data considerations and/or a-priori beliefs about a particular production process may guide choices between the two approaches. In other cases, one may want to try both techniques. Finding that production function parameters are consistent across multiple techniques with different assumptions is surely more robust than only using one technique.

6 Empirical Example (still preliminary)

Lastly, we briefly show how our estimator works in practice. We utilize the same Chilean plant level data as do LP and look at the same four industries. These industries are food products (ISIC code 311), Textiles (321), Wood Products (331), and Metals (381). They were chosen by LP because they contain the most plant-year observations. 311 in particular has a little more than 5000 plant-year observations over the period 1979-1986.

One key difference between our results and those exhibited in LP is that we estimate value-added production functions rather than gross-revenue production functions. There are two reasons for this. First, as noted previously, the aforementioned work by Bond and Söderbom (2005) casts some doubt on being able to reliably identify coefficients on perfectly variable inputs in Cobb-Douglas production functions. Second, estimating a gross-revenue production function requires estimating coefficients on *all* intermediate inputs. In this dataset this includes materials, electricity, and fuels. These variables are highly colinear with each other (and with capital and labor), and we have found it hard using any of the available techniques to generate particularly stable estimates for parameters on all these inputs.

Table 1 contains estimates using both LP and our (ACF) methods with various proxies. With the LP method, we use k_{it} as the second stage instrument. For ACF, we use k_{it} and l_{it} as second stage instruments, i.e. we use the moment (24). In this particular dataset, using l_{it-1} as a second stage instrument did not provide seem to provide enough identification power and led to high standard errors. As such we make the timing assumption that l_{it} was decided before (or without knowledge of) the realization of ξ_{it}, allowing us to use l_{it} as an instrument. This is similar to what LP assume regarding the i_{it} variable (that it is decided without knowledge of ξ_{it}). Obviously, the appropriability of such an assumption will depend on the industry of study. We not only try each of three possible intermediate input proxies (materials, electricity, and fuel) seperately, we also try using all the proxies at once. This might help if in some regions of $(k_{it}, l_{it}, \omega_{it})$ space, one of the intermediate input demands is not monotonic in ω_{it} but another one is. As described earlier, it also could help if there were a limited number other unobservables entering the intermediate input demand equations.

Looking at the table suggests a number of patterns. First, our procedure tends to move the coefficients in intuitive directions from the OLS estimates. In particular, the labor coefficient clearly moves down going from the OLS to the ACF results, except in ISIC code 331, where it doesn't change much. This decrease is to be expected, as one would expect l_{it} to be more correlated with ω_{it} than k_{it}. On the other hand, the coefficient on k_{it} tends to increase - this is not necessarily an intuitive result, but a possible one if ω_{it} is more correlated with l_{it} than k_{it}. Overall returns to scale also tend to decrease relative to OLS - again an intuitive result.

Comparing the ACF results to the LP results, one interesting observation is that the ACF labor coefficient estimates are generally higher than those in LP. That is, the ACF labor coefficient estimates are generally between the OLS and LP estimates. This is suggestive that the LP first stage labor coefficient estimates may be biased downward. One explanation why this might be the case is that given possible collinearity problems between l_{it} and the first stage non-parametric function, the only left over independent variation in l_{it} may primarily be due to measurement error. This would lead to the coefficient being biased towards zero. Of course, measurement error will also affect the ACF estimates, but probably not as much, since the signal-to-noise ratio would likely be higher (in LP, if the "signal" is highly correlated with the first stage non-parameteric function, the signal to noise ratio in the independent variation in l_{it} will be very low). Also of interest in comparing the LP and ACF estimates is how the estimates change with the various proxy variables. It seems like the ACF procedure is more robust to different intermediate input proxies than is the LP procedure - for example, in ISIC 311, over the 4 different proxy specifications, the LP labor coefficient varies between 0.301 and 0.516. In contrast, the ACF labor coefficient varies between 0.563 and 0.592. This seems supportive of the ACF methodology. Another worrisome point regarding the LP estimates is that in all 4 industries, when one conditions on all 3 of the intermediate inputs, one gets a very low estimate of the labor coefficient. This is consistent with a story in which as one is better able to condition on ω_{it} using the non-parametric term, the lower the signal-to-noise ratio in the independent variation in l_{it} becomes.

7 Conclusions

This paper has examined some of the recent literature on identification of production functions (Olley and Pakes (1996) and Levinsohn and Petrin (2003)) and argues that there may be significant collinearity problems in the first stages of these methods. Given these potential collinearity problems, we search for possible data generating processes that *simultaneously* 1) break this collinearity problem, and 2) are consistent with the LP/OP assumptions. For LP, we conclude that there are only two such DGP's, and that both rely on very strong and untintuitive assumptions - one involves a story where one variable input choice has a large amount of optimization error, while another variable input choice has exactly no optimization error. The second DGP involves a story where 1) intermediate inputs are chosen *prior* to labor, 2) that between the points in time when intermediate inputs are chosen and when labor is chosen, the firm's productivity level does not change, 3) that between these points in time, the firm is exposed to a price or demand shock that influences its choice of labor, and 4) that this price or demand shock varies across firms and is not correlated across time. Neither of these DGP assumptions seem realistic enough (even to an approximation) to rely on in practice. For OP, there is an additional DGP that breaks the collinearity and is consistent with the model - this involves labor being chosen prior to production and relies on the evolution of productivity between the time when labor is chosen and when production takes place to break the collinearity. This DGP seems more realistic to us than those needed validate the LP procedure.

We then suggest a new approach for estimating production functions. This approach builds upon the ideas in OP and LP, e.g. using investment or intermediate inputs to "proxy" for productivity shocks, but does not suffer from the above collinearity problems. The key difference is that unlike the OP and LP procedures, which estimate the labor coefficient in the first stage (where the collinearity issue arises), our estimator involves estimating the labor coefficient in the second stage. Even though no parameters are identified in our first stage, we still use the first stage to net out the non-transmitted production function error ϵ_{it}. This is what allows us to treat the evolution of the transmitted error ω_{it} non-parameterically. We show that our estimator is robust to a number of alternative (and seemingly reasonable) DGPs. As well as addressing the above collinearity problem, another important benefit of our estimator is that it makes comparison to the dynamic panel literature, e.g. Blundell and Bond (1999), quite easy. We are able to highlight the advantages and disadvantages of our estimator in relation to this dynamic panel literature. Lastly, using the same dataset as Levinsohn and Petrin, we examine how our estimator works in practice. Preliminary estimates using our methodology appear more stable across different potential proxy variables than the Levinsohn-Petrin methodology, consistent with our theoretical arguments.

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8 Appendix 1 - An Alternative Procedure

This section examines an alternative procedure to estimate production function coefficients. While it also breaks the potential collinearity problems of OP/LP, it does rely on some additional assumptions, specifically an additional monotonicity assumption and some independence assumptions on the innovations in ω_{it}. It is also a bit more complicated than the procedure we suggest above. On the other hand, this procedure does allow one to learn something about the timing of the choice of inputs. In particular, it can help us understand the actual timing of input choice, e.g. whether labor is more of a flexible or fixed input.

The intuition behind identification in this second approach follows directly from the intuition of identification of the coefficient on capital in OP (and LP). We make heavy use of the fact that if an input is determined prior to production, the *innovation* in productivity *between* the time of the input choice and the time of production should be orthogonal to that input choice. Again, this is not only an econometric assumption, but an assumption on the information set of the firm at various points in time. More formally, if ω_i is the productivity level of the firm at the time input i is chosen, and ω_p is the productivity level at the time of production, then:

$$(\omega_p - E[\omega_p|\omega_i]) \perp \quad i$$

This type of moment identifies the capital coefficient in OP and LP. Our approach simply extends this intuition to identification of parameters on labor inputs, combining this with non-parametrics to "invert out" values of the productivity shock at various decision times.

Consider a production model with 3 inputs, capital, labor, and an intermediate input, e.g. materials. We make the following timing assumptions regarding when k, l, and m are chosen. Suppose between periods $t-1$ and t, the following occurs, where $0 < b < 1$:

Time	Action
$t-1$	ω_{it-1} is observed, m_{it-1} is chosen, k_{it} is chosen, period $t-1$ production occurs
$t-b$	ω_{it-b} is observed, l_{it} is chosen
t	ω_{it} is observed, m_{it} is chosen, k_{it+1} is chosen, period t production occurs

Like OP/LP, we assume that k_{it} is determined at time $t-1$. Actually, like LP (but not OP), we only really need to assume that k_{it} is determined at either $t-1$ or earlier. For the more variable inputs, we assume that l_{it} is chosen at some time between $t-1$ and t, and that m_{it} is perfectly flexible and chosen at time t.

Note that we assume ω evolves between $t-1$, $t-t_b$, and t. As in our "story" behind OP, this movement is needed to alleviate possible collinearity problems between labor and other inputs. We assume that ω evolves as a first-order markov process between these stages, i.e.:

$$\begin{aligned} \omega_{it-b} &= g_1(\omega_{it-1}, \eta_{it}^b) \\ \omega_{it} &= g_2(\omega_{it-1}, \eta_{it}) \end{aligned} \tag{33}$$

where the η's are independent of the ω's (as well as all other variables that are chosen before their realizations). Note that this is a slightly stronger assumption than that of OP and LP, which assume only a first-order markov process. On the other hand, the fact that the g's are arbitrary functions allows some forms of heteroskedasticity. While our "staggered" input choice process might initially seem somewhat *ad-hoc*, we feel that it does capture some interesting aspects of reality.[28]

Given the above timing assumptions and assuming that labor is a static input, a firm's choice of labor will be a function of ω_{it-b}, i.e.

$$l_{it} = f_{1t}(\omega_{it-b}, k_{it}) \tag{34}$$

Since the firm's choice of labor in a given period is made before its choice of materials, the labor term must be taken into account when choosing the level of materials, i.e.

$$m_{it} = f_{2t}(\omega_{it}, k_{it}, l_{it}) \tag{35}$$

[28] Though this is clearly a stylized model of what is likely a more continuous decision process.

Once again, we will assume monotonicity of this equation in ω_{it}, allowing us to invert this function and obtain:

$$\omega_{it} = f_{2t}^{-1}(m_{it}, k_{it}, l_{it})$$

This term can be substituted into the production function from (**??**) to get:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + f_{2t}^{-1}(m_{it}, k_{it}, l_{it}) + \epsilon_{it}$$

and collecting terms results in the first stage equation:

$$y_{it} = \Phi_t(m_{it}, k_{it}, l_{it}) + \epsilon_{it} \tag{36}$$

This is exactly the same first stage as section 5.1, and the Φ function can be estimated in the same way. Similarly, we can construct the same moment condition for capital:

$$E[\xi_{it}(\beta_k, \beta_l)|k_{it}] = 0 \tag{37}$$

where $\xi_{it} = \omega_{it} - E[\omega_{it}|\omega_{it-1}]$, and $\xi_{it}(\beta_k, \beta_l)$ can be constructed in the usual way, i.e. by non-parametrically regressing $(\omega_{it}(\beta_k, \beta_l) = \widehat{\Phi}_t(m_{it}, k_{it}, l_{it}) - \beta_k k_{it} - \beta_l l_{it})$ on $(\omega_{it-1}(\beta_k, \beta_l) = \widehat{\Phi}_t(m_{it-1}, k_{it-1}, l_{it-1}) - \beta_k k_{it-1} - \beta_l l_{it-1})$.

What differs between this and the above procedures is the moment condition intended to identify the labor coefficient. Define ξ_{it}^b as the unexpected innovation in ω between time $t-b$ and t, i.e.

$$\xi_{it}^b = \omega_{it} - E[\omega_{it}|\omega_{it-b}]$$

Given that labor is chosen at $t-b$, it should be orthogonal to this innovation

$$E[\xi_{it}^b|l_{it}] = 0 \tag{38}$$

This is the moment condition we will use - what remains to be shown is how we can construct this moment given a value of the parameter vector. To do this, first note that the first stage estimates of (36) allow us to compute, conditional on the parameters, ω_{it} for all t. Call these terms $\omega_{it}(\beta_k, \beta_l)$. Now consider the firm's labor demand function (34). Substituting in (33) results in

$$\begin{aligned} l_{it} &= f_{1t}(g_1(\omega_{it-1}, \eta_{it}^b), k_{it}) \\ &= \widetilde{f}_{1t}(\omega_{it-1}(\beta_k, \beta_l), \eta_{it}^b, k_{it}) \end{aligned}$$

Note that conditional on (β_k, β_l), the only unobservable in this equation is η_{it}^b. Thus, assuming that the equation is strictly monotonic in η_{it}^b, one can use the methods of Matzkin (2003) to non-parametrically invert out η_{it}^b up to a normalization. Call this function $\tau(\eta_{it}^b; \beta_k, \beta_l)$. Again, the dependence on β_k and β_l comes from the fact that the ω_{it} are inferred conditional on β_k and β_l. This non-parametric inversion relies on the assumption that η_{it}^b is independent of ω_{it-1} and k_{it}. The basic intuition is that for a given ω_{it-1} and k_{it}, one can form a distribution of l_{it}. $\tau(\eta_{it}^b; \beta_k, \beta_l)$ for a given i is simply the quantile of l_{it} in that distribution.

Next, note that since ω_{it-b} is a function of η_{it}^b and ω_{it-1}, we can also write it as a function of

$\tau(\eta_{it}^b;\beta_k,\beta_l)$ and ω_{it-1}, i.e.

$$\begin{aligned}\omega_{it-b}(\beta_k,\beta_l) &= g_1(\omega_{it-1}(\beta_k,\beta_l),\eta_{it}^b) \\ &= \widetilde{g}_1(\omega_{it-1}(\beta_k,\beta_l),\tau(\eta_{it}^b;\beta_k,\beta_l))\end{aligned}$$

As a result, to construct $\xi_{it}^b = \omega_{it} - E[\omega_{it}|\omega_{it-b}]$, we can form the necessary conditional expectation by non-parametrically regressing $\omega_{it}(\beta_k,\beta_l)$ on $\omega_{it-1}(\beta_k,\beta_l)$ and $\tau(\eta_{it}^b;\beta_k,\beta_l)$ (as an alternative to non-parametrically regressing $\omega_{it}(\beta_k,\beta_l)$ on $\omega_{it-b}(\beta_k,\beta_l)$). Denoting the residual from this regression by $\xi_{it}^b(\beta_k,\beta_l)$, we can form the moment

$$E[\xi_{it}^b(\beta_k,\beta_l)|l_{it}] = 0 \tag{39}$$

to be used for estimation. Note that this procedure can easily be adjusted to allow for labor to have dynamic implications. One simply needs to include l_{it-1} in both the material and labor demand functions.

Table 1

	OLS	LP				ACF			
Proxy	None	Materials	Electricity	Fuel	All	Materials	Electricity	Fuel	All
Industry 311									
Capital	0.544	0.622	0.612	0.612	0.670	0.522	0.543	0.568	0.550
	(0.014)	(0.162)	(0.110)	(0.069)	(0.149)	(0.143)	(0.122)	(0.104)	(0.133)
Labor	0.656	0.370	0.447	0.516	0.301	0.564	0.592	0.563	0.563
	(0.024)	(0.034)	(0.031)	(0.030)	(0.041)	(0.064)	(0.063)	(0.039)	(0.063)
Industry 321									
Capital	0.411	0.529	0.485	0.467	0.567	0.455	0.457	0.437	0.445
	(0.026)	(0.117)	(0.079)	(0.056)	(0.135)	(0.111)	(0.093)	(0.086)	(0.134)
Labor	0.670	0.451	0.566	0.609	0.445	0.611	0.634	0.632	0.642
	(0.042)	(0.056)	(0.054)	(0.051)	(0.072)	(0.066)	(0.056)	(0.060)	(0.077)
Industry 331									
Capital	0.404	0.504	0.505	0.401	0.485	0.417	0.424	0.387	0.408
	(0.022)	(0.185)	(0.167)	(0.049)	(0.133)	(0.134)	(0.104)	(0.092)	(0.068)
Labor	0.701	0.501	0.503	0.694	0.484	0.712	0.656	0.717	0.711
	(0.039)	(0.049)	(0.050)	(0.045)	(0.068)	(0.062)	(0.067)	(0.048)	(0.065)
Industry 381									
Capital	0.377	0.537	0.481	0.452	0.549	0.462	0.422	0.428	0.453
	(0.025)	(0.160)	(0.100)	(0.039)	(0.155)	(0.123)	(0.090)	(0.077)	(0.125)
Labor	0.891	0.578	0.677	0.821	0.505	0.777	0.815	0.825	0.802
	(0.045)	(0.064)	(0.057)	(0.052)	(0.081)	(0.062)	(0.047)	(0.041)	(0.060)