Chapter 8: New Keynesian Monetary Economics (Monetary Theory and Policy, 3rd ed.)*

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1 Introduction

In the 1970s, 1980s, and early 1990s, models used for monetary policy analysis combined the assumption of nominal rigidity with a simple structure that linked the quantity of money to aggregate spending. While the theoretical foundations of these models were weak, the approach proved remarkably useful in addressing a wide range of monetary policy topics.[1] Today, the standard approach in monetary economics and monetary policy analysis incorporates nominal wage and/or price rigidity into a dynamic, stochastic, general equilibrium (DSGE) framework that is based on optimizing behavior by the agents in the model.

These modern DSGE models with nominal frictions are commonly labelled "new Keynesian" models because, like older versions of models in the Keynesian tradition, aggregate demand plays a central role in determining output in the short run and there is a presumption that some fluctuations both can be and should be dampened by countercylical monetary and/or fiscal policy.[2] Early examples of models with these properties include those of Yun (1996), Goodfriend and King (1997), Rotemberg and Woodford (1995, 1997), and McCallum and Nelson (1999). Galí (2002) discusses the derivation of the model's equilibrium conditions, and book length treatments of the new Keynesian model are provided by Woodford (2003) and Galí (2008).

The first section of this chapter shows how a basic money-in-the-utility function (MIU) model, combined with the assumption of monopolistically competitive goods markets and price stickiness, can form the basis for a simple linear new Keynesian model.[3] The model is a consistent general equilibrium model in which all agents face well-defined decision problems and behave optimally, given the environment in which they find themselves. To obtain a canonical new Keynesian model, three key modifications will be made to the MIU model of chapter

[1] Chapter 10 provides a taste of the many interesting insights obtained from these models.

[2] Goodfriend and King (1997) proposed the name, "the new neoclassical synthesis," to emphasize the connection with the neoclassical, as opposed to Keynesian, traditions.

[3] See Chapter 2 for a discussion of money-in-the-utility function models.

2. First, endogenous variations in the capital stock are ignored. This follows McCallum and Nelson (1999), who show that, at least for the United States, there is little relationship between the capital stock and output at business-cycle frequencies. Endogenous capital stock dynamics play a key role in equilibrium business-cycle models in the real business-cycle tradition, but as Cogley and Nason (1995) show, the response of investment and the capital stock to productivity shocks actually contributes little to the dynamics implied by such models. For simplicity, then, the capital stock will be ignored.[4]

Second, the single final good in the MIU model is replaced by a continuum of differentiated goods produced by monopolistically competitive firms. These firms face constraints on their ability to adjust prices, thus introducing nominal price stickiness into the model. In the basic model, nominal wages will be allowed to fluctuate freely, although section 3.6 will explore the implications of assuming both prices and wages are sticky.

Third, monetary policy is represented by a rule for setting the nominal rate of interest. Most central banks today use a short-term nominal interest rate as their instrument for implementing monetary policy. The nominal quantity of money is then endogenously determined to achieve the desired nominal interest rate. There are important issues involved in choosing between money supply policy procedures and interest rate procedures, and some of these will be discussed in chapter 11.

These three modifications yield a new Keynesian framework that is consistent with optimizing behavior by private agents and incorporates nominal rigidities, yet is simple enough to use for exploring a number of policy issues. It can be linked directly to the more traditional aggregate supply-demand (AS-IS-LM) model that long served as one of the workhorses for monetary policy analysis and is still common in most undergraduate texts. Once the basic framework has been developed, section 4 considers optimal policy, as well as a variety of policy rules and policy frameworks, including inflation targeting.

2 The Basic Model

The model consists of households and firms. Households supply labor, purchase goods for consumption, and hold money and bonds, while firms hire labor and produce and sell differentiated products in monopolistically competitive goods markets. The basic model of monopolistic competition is drawn from Dixit and Stiglitz (1977). The model of price stickiness is taken from Calvo (1983).[5] Each firm sets the price of the good it produces, but not all firms reset their price in each period. Households and firms behave optimally; households maximize the expected present value of utility, and firms maximize profits. There is also

[4] However, Dotsey and King (2001) and Christiano, Eichenbaum, and Evans (2005) have emphasized the importance of variable capital utilization for understanding the behavior of inflation. Firm-specific capital in a new Keynesian framework is analyzed by Altig, Christiano, Eichenbaum, and Linde (2005).

[5] See section 5.4.4.

a central bank that controls the nominal rate of interest. Initially, the central bank, in contrast to households and firms, is not assumed to behave optimally; optimal policy is explored in section 4.

2.1 Households

The preferences of the representative household are defined over a composite consumption good C_t, real money balances M_t/P_t, and the time devoted to market employment N_t. Households maximize the expected present discounted value of utility:

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]. \tag{1}$$

The composite consumption good consists of differentiated products produced by monopolistically competitive final goods producers (firms). There is a continuum of such firms of measure 1, and firm j produces good c_j. The composite consumption good that enters the household's utility function is defined as

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \qquad \theta > 1. \tag{2}$$

The household's decision problem can be dealt with in two stages. First, regardless of the level of C_t the household decides on, it will always be optimal to purchase the combination of individual goods that minimizes the cost of achieving this level of the composite good. Second, given the cost of achieving any given level of C_t, the household chooses C_t, N_t, and M_t optimally.

Dealing first with the problem of minimizing the cost of buying C_t, the household's decision problem is to

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj$$

subject to

$$\left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t, \tag{3}$$

where p_{jt} is the price of good j. Letting ψ_t be the Lagrangian multiplier on the constraint, the first order condition for good j is

$$p_{jt} - \psi_t \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0.$$

Rearranging, $c_{jt} = (p_{jt}/\psi_t)^{-\theta} C_t$. From the definition of the composite level of consumption (2), this implies

$$C_t = \left[\int_0^1 \left[\left(\frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \right]^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} = \left(\frac{1}{\psi_t} \right)^{-\theta} \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}} C_t.$$

Solving for ψ_t,

$$\psi_t = \left[\int_0^1 p_{jt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \equiv P_t. \tag{4}$$

The Lagrangian multiplier is the appropriately aggregated price index for consumption. The demand for good j can then be written as

$$c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} C_t. \tag{5}$$

The price elasticity of demand for good j is equal to θ. As $\theta \to \infty$, the individual goods become closer and closer substitutes, and, as a consequence, individual firms will have less market power.

Given the definition of the aggregate price index in (4), the budget constraint of the household is, in real terms,

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t}\right) N_t + \frac{M_{t-1}}{P_t} + (1+i_{t-1})\left(\frac{B_{t-1}}{P_t}\right) + \Pi_t, \tag{6}$$

where M_t (B_t) is the household's nominal holdings of money (one-period bonds). Bonds pay a nominal rate of interest i_t. Real profits received from firms are equal to Π_t.

In the second stage of the household's decision problem, consumption, labor supply, money, and bond holdings are chosen to maximize (1) subject to (6). This leads to the following conditions, which, in addition to the budget constraint, must hold in equilibrium:

$$C_t^{-\sigma} = \beta(1+i_t)\mathrm{E}_t\left(\frac{P_t}{P_{t+1}}\right) C_{t+1}^{-\sigma}; \tag{7}$$

$$\frac{\gamma\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}} = \frac{i_t}{1+i_t}; \tag{8}$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \tag{9}$$

These conditions represent the Euler condition for the optimal intertemporal allocation of consumption, the intratemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money, and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage.[6]

[6] See chapter 2 for further discussion of these first order conditions in an MIU model.

2.2 Firms

Firms maximize profits, subject to three constraints. The first is the production function summarizing the available technology. For simplicity, we have ignored capital, so output is a function solely of labor input N_{jt} and an aggregate productivity disturbance Z_t:

$$c_{jt} = Z_t N_{jt}, \qquad \mathrm{E}(Z_t) = 1,$$

where constant returns to scale has been assumed. The second constraint on the firm is the demand curve each firm faces. This is given by (5). The third constraint is that each period some firms are not able to adjust their price. The specific model of price stickiness we will use is due to Calvo (1983). Each period, the firms that adjust their price are randomly selected, and a fraction $1-\omega$ of all firms adjust while the remaining ω fraction do not adjust. The parameter ω is a measure of the degree of nominal rigidity; a larger ω implies that fewer firms adjust each period and that the expected time between price changes is longer. Those firms that do adjust their price at time t do so to maximize the expected discounted value of current and future profits. Profits at some future date $t+s$ are affected by the choice of price at time t only if the firm has not received another opportunity to adjust between t and $t+s$. The probability of this is ω^s.[7]

Before analyzing the firm's pricing decision, consider its cost minimization problem, which involves minimizing $W_t N_{jt}$ subject to producing $c_{jt} = Z_t N_{jt}$. This problem can be written, in real terms, as

$$\min_{N_t} \left(\frac{W_t}{P_t}\right) N_t + \varphi_t \left(c_{jt} - Z_t N_{jt}\right).$$

where φ_t is equal to the firm's real marginal cost. The first order condition implies

$$\varphi_t = \frac{W_t/P_t}{Z_t}. \tag{10}$$

The firm's pricing decision problem then involves picking p_{jt} to maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}}\right) c_{jt+i} - \varphi_{t+i} c_{jt+i} \right],$$

where the discount factor $\Delta_{i,t+i}$ is given by $\beta^i (C_{t+i}/C_t)^{-\sigma}$. Using the demand curve (5) to eliminate c_{jt}, this objective function can be written as

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}}\right)^{1-\theta} - \varphi_{t+i} \left(\frac{p_{jt}}{P_{t+i}}\right)^{-\theta} \right] C_{t+i}.$$

[7] In this formulation, the degree of nominal rigidity, as measured by ω, is constant, and the probability that a firm has adjusted its price is a function of time but not of the current state. State-dependent pricing models have been developed by Dotsey, King, and Wolman (1999) based on fixed costs of adjustment and by Kiley (2000) based on information costs. See also Wolman (1999). Haubrich and King (1991) develop a model with endogenous price stickiness in which nominal contracts provide insurance in the presence of random monetary injections that are distributed unequally across agents.

While individual firms produce differentiated products, they all have the same production technology and face demand curves with constant and equal demand elasticities. In other words, they are essentially identical, except that they may have set their current price at different dates in the past. However, all firms adjusting in period t face the same problem, so all adjusting firms will set the same price. Let p_t^* be the optimal price chosen by all firms adjusting at time t. The first order condition for the optimal choice of p_t^* is

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta) \left(\frac{p_t^*}{P_{t+i}} \right) + \theta \varphi_{t+i} \right] \left(\frac{1}{p_t^*} \right) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta} C_{t+i} = 0. \quad (11)$$

Using the definition of $\Delta_{i,t+i}$, (11) can be rearranged to yield

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{\theta}{\theta - 1} \right) \frac{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta}}{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t} \right)^{\theta-1}}. \quad (12)$$

Consider the case in which all firms are able to adjust their prices every period ($\omega = 0$). When $\omega = 0$, (12) reduces to

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{\theta}{\theta - 1} \right) \varphi_t = \mu \varphi_t. \quad (13)$$

Each firm sets its price p_t^* equal to a markup $\mu > 1$ over its nominal marginal cost $P_t \varphi_t$. This is the standard result in a model of monopolistic competition. Because price exceeds marginal cost, output will be inefficiently low. When prices are flexible, all firms charge the same price. In this case, $p_t^* = P_t$ and $\varphi_t = 1/\mu$. Using the definition of real marginal cost, this means $W_t/P_t = Z_t/\mu < Z_t$ in a flexible-price equilibrium. However, the real wage must also equal the marginal rate of substitution between leisure and consumption to be consistent with household optimization. This condition implies, from (9), that

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t} = \frac{Z_t}{\mu}. \quad (14)$$

Goods market clearing and the production function imply that $C_t = Y_t$ and $N_t = Y_t/Z_t$. Using these conditions in (14), and letting Y_t^f denote equilibrium output under flexible prices, Y_t^f is given by

$$Y_t^f = \left(\frac{1}{\chi \mu} \right)^{\frac{1}{\sigma+\eta}} Z_t^{\frac{1+\eta}{\sigma+\eta}}. \quad (15)$$

When prices are flexible, output is a function of the aggregate productivity shock, reflecting the fact that, in the absence of sticky prices, the new Keynesian model reduces to a real business cycle model.

When prices are sticky ($\omega > 0$), output can differ from the flexible-price equilibrium level. Because a firm will not adjust its price every period, it must take into account expected future marginal cost as well as current marginal cost whenever it has an opportunity to adjust its price. Equation (**??**) shows how adjusting firms set their price, conditional on the current aggregate price level P_t. This aggregate price index is an average of the price charged by the fraction $1-\omega$ of firms setting their price in period t and the average of the remaining fraction ω of all firms that do not change their price in period t. However, because the adjusting firms were selected randomly from among all firms, the average price of the nonadjusters is just the average price of all firms that prevailed in period $t-1$. Thus, from (4), the average price in period t satisfies

$$P_t^{1-\theta} = (1-\omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}. \tag{16}$$

To summarize, (7)-(10), (12), (14), and (16) represent a system in C_t, N_t, M/P_t, Y_t, φ_t, P_t, p_t^*, W_t/P_t, and i_t that can be combined with the aggregate production function, $Y_t = Z_t N_t$, and a specification of monetary policy to determine the economy's equilibirum.

3 A Linearized New Keynesian Model

One reason for the popularity of the new Keynesian model is that it allows for a simple linear representation in terms of an inflation adjustment equation, or Phillips curve, and an output and real interest rate relationship that corresponds to the IS curve of undergraduate macroeconomics. To derive this linearized version of the model, let $\hat{x}_t$ denote the percentage deviation of a variable X_t around its steady state and let the superscript f denote the flexible-price equilibrium. The equilibrium conditions in the model will be linearized around a steady state in which the inflation rate is zero.

3.1 The Linearized Phillips Curve

Equations (**??**) and (16) can be approximated around a zero average inflation, steady-state equilibrium to obtain an expression for aggregate inflation (see section 6.1 of the chapter appendix for details) of the form

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\hat{\varphi}_t \tag{17}$$

where

$$\tilde{\kappa} = \frac{(1-\omega)(1-\beta\omega)}{\omega}$$

is an increasing function of the fraction of firms able to adjust each period and $\hat{\varphi}_t$ is real marginal cost, expressed as a percentage deviation around its steady-state value.[8]

Equation (17) is often referred to as the *new Keynesian Phillips curve*. Unlike more traditional Phillips curve equations, the new Keynesian Phillips curve

[8] Ascari (2004) shows that the behavior of inflation in the Calvo model can be significantly affected if steady-state inflation is not zero.

implies that real marginal cost is the correct driving variable for the inflation process. It also implies that the inflation process is forward-looking, with current inflation a function of expected future inflation. When a firm sets its price, it must be concerned with inflation in the future because it may be unable to adjust its price for several periods. Solving (17) forward,

$$\pi_t = \tilde{\kappa} \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t \hat{\varphi}_{t+i},$$

which shows that inflation is a function of the present discounted value of current and future real marginal costs.

The new Keynesian Phillips curve also differs from traditional Phillips curves in having been derived explicitly from a model of optimizing behavior on the part of price setters, conditional on the assumed economic environment (monopolistic competition, constant elasticity demand curves, and randomly arriving opportunities to adjust prices). This derivation reveals how $\tilde{\kappa}$, the impact of real marginal cost on inflation, depends on the structural parameters β and ω. An increase in β means that the firm gives more weight to future expected profits. As a consequence, $\tilde{\kappa}$ declines; inflation is less sensitive to current marginal costs. Increased price rigidity (a rise in ω) reduces $\tilde{\kappa}$; with opportunities to adjust arriving less frequently, the firm places less weight on current marginal cost (and more on expected future marginal costs) when it does adjust its price.

Equation (17) implies that inflation depends on real marginal cost and not directly on a measure of the gap between actual output and some measure of potential output or on a measure of unemployment relative to the natural rate, as is typical in traditional Phillips curves.[9] However, real marginal costs can be related to an output gap measure. The firm's real marginal cost is equal to the real wage it faces divided by the marginal product of labor (see 10). In a flexible price equilibrium, all firms set the same price, so (13) implies that real marginal cost will equal its steady-state value of $1/\mu$. Because nominal wages have been assumed to be completely flexible, the real wage must, according to (9), equal the marginal rate of substitution between leisure and consumption. Expressed in terms of percentage deviations around the steady state, (9) implies that $\hat{w}_t - \hat{p}_t = \eta \hat{n}_t + \sigma \hat{y}_t$. Recalling that $\hat{c}_t = \hat{y}_t$ and $\hat{y}_t = \hat{n}_t + \hat{z}_t$, the percentage deviation of real marginal cost around its steady-state value is

$$\begin{aligned} \hat{\varphi}_t &= (\hat{w}_t - \hat{p}_t) - (\hat{y}_t - \hat{n}_t) \\ &= (\sigma + \eta) \left[\hat{y}_t - \left(\frac{1+\eta}{\sigma+\eta} \right) \hat{z}_t \right]. \end{aligned}$$

To interpret the term involving $\hat{z}_t$, linearize (15) giving flexible-price output to obtain

$$\hat{y}_t^f = \left(\frac{1+\eta}{\sigma+\eta} \right) \hat{z}_t. \tag{18}$$

[9] See Ravenna and Walsh (2008) and Blanchard and Gali (2008) for models of labor market frictions that relate inflation to unemployment.

Thus, we can use (18) to express real marginal cost as

$$\hat{\varphi}_t = \gamma\left(\hat{y}_t - \hat{y}_t^f\right), \tag{19}$$

where $\gamma = \sigma + \eta$. Using this result, the inflation adjustment equation (17) becomes

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \tag{20}$$

where $\kappa = \gamma\tilde{\kappa} = \gamma(1-\omega)(1-\beta\omega)/\omega$ and $x_t \equiv \hat{y}_t - \hat{y}_t^f$ is the gap between actual output and flexible-price equilibrium output.

The proceeding has assumed that firms face constant returns to scale. If, instead, each firm's production function is $c_{jt} = Z_t N_{jt}^a$, where $0 < a \leq 1$, then the results must be modified slightly. When $a < 1$, firms with different production levels will face different marginal costs, and real marginal cost for firm j will equal

$$\varphi_{jt} = \frac{W_t/P_t}{aZ_t N_{jt}^{a-1}} = \frac{W_t/P_t}{ac_{jt}/N_{jt}}.$$

Linearizing this expression for firm $j's$ real marginal cost and using the production function yields

$$\hat{\varphi}_{jt} = (\hat{w}_t - \hat{p}_t) - (\hat{c}_{jt} - \hat{n}_{jt}) = (\hat{w}_t - \hat{p}_t) - \left(\frac{a-1}{a}\right)\hat{c}_{jt} - \left(\frac{1}{a}\right)\hat{z}_t. \tag{21}$$

Marginal cost for the individual firm can be related to average marginal cost, $\varphi_t = (W_t/P_t)/(aC_t/N_t)$, where

$$N_t = \int_0^1 N_{jt}dj = \int_0^1 \left(\frac{c_{jt}}{Z_t}\right)^{\frac{1}{a}} dj = \left(\frac{C_t}{Z_t}\right)^{\frac{1}{a}} \int_0^1 \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\theta}{a}} dj.$$

When this last expression is linearized around a zero inflation steady state, the final term involving the dispersion of relative prices, turns out to be of second order,[10] so one obtains

$$\hat{n}_t = \left(\frac{1}{a}\right)(\hat{c}_t - \hat{z}_t),$$

and

$$\hat{\varphi}_t = (\hat{w}_t - \hat{p}_t) - (\hat{c}_t - \hat{n}_t) = (\hat{w}_t - \hat{p}_t) - \left(\frac{a-1}{a}\right)\hat{c}_t - \left(\frac{1}{a}\right)\hat{z}_t. \tag{22}$$

Subtracting (22) from (21) and gives

$$\hat{\varphi}_{jt} - \hat{\varphi}_t = -\left(\frac{a-1}{a}\right)(\hat{c}_{jt} - \hat{c}_t)$$

[10] When linearized, the last term becomes

$$-\left(\frac{\theta}{a}\right)\int (\hat{p}_{jt} - \hat{p}_t)\,dj,$$

but to a first order approximation, $\int \hat{p}_{jt}dj = \hat{p}_t$, so the price dispersion term is approximately equal to zero.

Finally, employing the demand relationship (5) to express $\hat{c}_{jt} - \hat{c}_t$ in terms of relative prices,

$$\hat{\varphi}_{jt} = \hat{\varphi}_t - \left[\frac{\theta(1-a)}{a}\right](\hat{p}_{jt} - \hat{p}_t).$$

Firms with relatively high prices (and therefore low output) will have relatively low real marginal costs. In the case of constant returns to scale ($a = 1$), all firms face the same marginal cost. When $a < 1$, Sbordone (2002) and Galí, Gertler, and López-Salido (2001) show that the new Keynesian inflation adjustment equation becomes[11]

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\left[\frac{a}{a+\theta(1-a)}\right]\hat{\varphi}_t.$$

In addition, the labor market equilibrium condition under flexible prices becomes

$$\frac{W_t}{P_t} = \frac{aZ_t N_t^{a-1}}{\mu} = \frac{\chi N_t^{\eta}}{C_t^{-\sigma}},$$

which implies flexible-price output is

$$\hat{y}_t^f = \left[\frac{1+\eta}{1+\eta+a(\sigma-1)}\right]\hat{z}_t.$$

When $a = 1$, this reduces to (18).

3.2 The Linearized IS Curve

Equation (20) relates output, in the form of the deviation around the level of output that would occur in the absence of nominal price rigidity, to inflation. It forms one of the two key components of an optimizing model that can be used for monetary policy analysis. The other component is a linearized version of the household's Euler condition, (7). Because consumption is equal to output in this model (there is no government or investment since capital has been ignored), (7) can be approximated around the zero-inflation steady state as[12]

$$\hat{y}_t = \mathrm{E}_t \hat{y}_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - \mathrm{E}_t \pi_{t+1}), \tag{23}$$

where $\hat{\imath}_t$ is the deviation of the nominal interest rate from its steady-state value. Expressing this in terms of the output gap $x_t = \hat{y}_t - \hat{y}_t^f$,

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - \mathrm{E}_t \pi_{t+1}) + u_t, \tag{24}$$

where $u_t \equiv \mathrm{E}_t \hat{y}_{t+1}^f - \hat{y}_t^f$ depends only on the exogenous productivity disturbance (see 18). Combining (24) with (20) gives a simple two-equation, forward-looking, rational-expectations model for inflation and the output gap measure

[11] See the appendix, section 5.7.3, for further details on the derivation.

[12] See the appendix to chapter 2 for details on linearizing the Euler condition.

x_t, once the behavior of the nominal rate of interest is specified.[13] This two-equation model consists of the equilibrium conditions for a well-specified general equilibrium model. The equations appear broadly similar, however, to the types of aggregate demand and aggregate supply equations commonly found in intermediate-level macroeconomics textbooks. Equation (24) represents the demand side of the economy (an expectational, forward-looking IS curve), while the new Keynesian Phillips curve (20) corresponds to the supply side. In fact, both equations are derived from optimization problems, with (24) based on the Euler condition for the representative household's decision problem and (20) derived from the optimal pricing decisions of individual firms.

There is a long tradition of using two equation, aggregate demand-aggregate supply (AD-AS) models in intermediate-level macroeconomic and monetary policy analysis. Models in the AD-AS tradition are often criticized as "starting from curves" rather than starting from the primitive tastes and technology from which behavioral relationships can be derived, given maximizing behavior and a market structure (Sargent 1982). This criticism does not apply to (24) and (20). The parameters appearing in these two equations are explicit functions of the underlying structural parameters of the production and utility functions and the assumed process for price adjustment.[14] And (24) and (20) contain expectations of future variables; the absence of this type of forward-looking behavior is a critical shortcoming of older AD-AS frameworks. The importance of incorporating a role for future income has been emphasized by Kerr and King (1996).

Equations (24) and (20) contain three variables: the output gap, inflation, and the nominal interest rate. The model can be closed by assuming that the central bank implements monetary policy through control of the nominal interest rate.[15] Alternatively, if the central bank implements monetary policy by setting a path for the nominal supply of money, (24) and (20), together with the linearized version of (8), determine x_t, π_t, and $\hat{\imath}_t$.[16]

3.3 Uniqueness of the Equilibrium

If a policy rule for the nominal interest rate is added to the model, this must be done with care to ensure that the policy rule does not render the system unstable or introduce multiple equilibria. For example, suppose monetary policy is represented by the following purely exogenous process for $\hat{\imath}_t$:

$$\hat{\imath}_t = v_t, \tag{25}$$

[13] With the nominal interest rate treated as the monetary policy instrument, (8) simply determines the real quantity of money in equilibrium.

[14] The process for price adjustment, however, has not been derived from the underlying structure of the economic environment.

[15] Important issues of price-level determinancy arise under interest-rate-setting policies, and these will be discussed in chapter 11.

[16] An alternative approach, discussed in section 6.4 specifies an objective function for the monetary authority and then derives the policy maker's decision rule for setting the nominal interest rate.

where v_t is a stationary, stochastic process. Combining (25) with (24) and (20), the resulting system of equations can be written as

$$\begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix}.$$

Premultiplying both sides by the inverse of the matrix on the left produces

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = M \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix}, \tag{26}$$

where

$$M = \begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Equation (26) has a unique, stationary solution for the output gap and inflation if and only if the number of eigenvalues of M outside the unit circle is equal to the number of forward-looking variables, in this case, two (see Blanchard and Kahn 1980). However, only the largest eigenvalue of this matrix is outside the unit circle, implying that multiple bounded equilibria exist and that the equilibrium is locally indeterminate. Stationary sunspot equilibria are possible.

This example illustrates that an exogenous policy rule – one that does not respond to the endogenous variables x and π – introduces the possibility of multiple equilibria. To understand why, consider what would happen if expected inflation were to rise. Since (25) does not allow for any endogenous feedback from this rise in expected inflation to the nominal interest rate, the real interest rate must fall. This decline in the real interest rate is expansionary, and the output gap increases. The rise in output increases actual inflation, according to (20). Thus, a change in expected inflation, even if due to factors unrelated to the fundamentals of inflation, can set off a self-fulfilling change in actual inflation.

This discussion suggests that a policy which raised the nominal interest rate when inflation rose, and raised $\hat{\imath}_t$ enough to increase the real interest rate so that the output gap fell, would be sufficient to ensure a unique equilibrium. For example, suppose the nominal interest rate responds to inflation according to the rule

$$\hat{\imath}_t = \delta \pi_t + v_t. \tag{27}$$

Combining (27) with (24) and (20), $\hat{\imath}_t$ can be eliminated and the resulting system written as

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = N \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix} \tag{28}$$

where

$$N = \begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & \frac{\beta\delta - 1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Bullard and Mitra (2002) show that a unique, stationary equilibrium exists as long as $\delta > 1$.[17] Setting $\delta > 1$ is referred to as the *Taylor principle*, because

[17] If the nominal interest rate is adjusted in response to expected future inflation (rather than current inflation), multiple solutions again become possible if $\hat{\imath}_t$ responds too strongly to $\mathrm{E}_t \hat{\pi}_{t+1}$. See Clarida, Galí, and Gertler (2000).

John Taylor was the first to stress the importance of interest-rate rules that called for responding more than one for one to changes in inflation.

Suppose that, instead of reacting solely to inflation, as in (27), the central bank responds to both inflation and the output gap according to

$$\hat{\imath}_t = \delta_\pi \pi_t + \delta_x x_t + v_t.$$

This type of policy rule is called a *Taylor rule* (Taylor 1993a), and variants of it have been shown to provide a reasonable empirical description of the policy behavior of many central banks (Clarida, Galí, and Gertler 2000).[18] With this policy rule, Bullard and Mitra (2002) show that the condition necessary to ensure that the economy has a unique stationary equilibrium becomes

$$\kappa(\delta_\pi - 1) + (1 - \beta)\delta_x > 0. \tag{29}$$

Determinacy now depends on both the policy parameters δ_π and δ_x. A policy that failed to raise the nominal interest rate sufficiently when inflation rose would lead to a rise in aggregate demand and output. This rise in x could produce a rise in the real interest rate that served to contract spending if δ_x were large. Thus, a policy rule with $\delta_\pi < 1$ could still be consistent with a unique, stationary equilibrium. At a quarterly frequency, however, β is about 0.99, so δ_x would need to be vary large to offset a value of δ_π much below one.

The Taylor principle is an important policy lesson that has emerged from the new Keynesian model. It has been argued that the failure of central banks such as the Federal Reserve to respond sufficiently strongly to inflation during the 1970s provides an explanation for the rise in inflation experiences at the time (see Lubik and Schorfheide 2004). Further, Orphanides (2001) has argued that estimated Taylor rules for the Federal Reserve are sensitive to whether or not real-time data is used, and he finds a much weaker response to inflation in the 1987-1999 period based on real-time data.[19] Because the Taylor principle is based on the mapping from policy response coefficients to eigenvalues in the state space representation of the model, one would expect that the exact restrictions the policy responses must satisfy to ensure determinacy will depend on the specification of the model. Two aspects of the model have been explored that lead to significant modifications of the Taylor principle.

First, Ascari and Ropele (2007), Kiley (2007b) find that the Taylor rule can be insufficient to ensure determinacy when trend inflation is positive rather than zero as assumed when obtaining the standard linearized new Keynesian inflation equation. For example, Coibion and Gorodnichenko (2008) show, in a calibrated model, that the central bank's response to inflation would need to

[18] Sometimes the term "Taylor rule" is reserved for the case in which $\delta_\pi = 1.5$ and $\delta_x = 0.5$ when inflation and the interest rate are expressed at annual rates. These are the values Taylor (1993a) found matched the behavior of the federal funds rates rate during the Greenspan period.

[19] Other paper employing real-time data to estimate policy rules include Rudebusch (2006) for the U.S. and and Papell, Molodtsova, and Nikolsko-Rzhevskyy (2008) for the U.S. and for Germany.

be over ten-to-one to ensure determinacy if steady-state inflation exceeded 6 percent. However, many models assume some form of indexation as discussed in chapter 6, and for these models, the Taylor principle would continue to hold even in the face of a positive steady-state rate of inflation.

Second, the Taylor principle can be significantly affected when interest rates have direct effects on real marginal cost. Such an effect, usually referred to as the cost channel of monetary policy, is common in models in which firms need to finance wage payments as in Christiano, Eichenbaum, and Evans (2005) or Ravenna and Walsh (2006) or in which search frictions in the labor market introduce an intertemporal aspect to the firm's labor demand condition (Ravenna and Walsh 2008). For example, Llosa and Tuesta (2006) for a model with a cost channel and Kurozumi and Van Zandwedge (2008) for a model with search and matching frictions in the labor market find that satisfying the standard Taylor principle of responding more than one-for-one to inflation need not ensure determinacy.

Finally, note that if v_t and u_t are zero for all t, the solution to (28) would be $\pi_t = x_t = 0$ for all t. In this case, the parameter δ in the policy rule (27) could not be identified. As Cochrane (2007) emphasizes, determinacy relies on assumptions about how the central bank would respond to movements of inflation out of equilibrium. Estimated Taylor rules may not reveal how policy would react in circumstances that are not observed.

3.4 The Monetary Transmission Mechanism

The model consisting of (24) and (20) assumes that the impact of monetary policy on output and inflation operates through the real rate of interest. As long as the central bank is able to affect the real interest rate through its control of the nominal interest rate, monetary policy can affect real output. Changes in the real interest rate alter the optimal time path of consumption. An increase in the real rate of interest, for instance, leads households to attempt to postpone consumption. Current consumption falls relative to future consumption.[20]

Figure 1 illustrates the impact of a monetary policy shock (an increase in the nominal interest rate) in the model consisting of (24), (20), and the policy rule (27). The parameter values used in constructing the figure are $\beta = 0.99$, $\sigma = \eta = 1$, $\delta = 1.5$, and $\omega = 0.8$. In addition, the policy shock v_t in the policy rule is assumed to follow an $AR(1)$ process given by $v_t = \rho_v v_{t-1} + \varepsilon_t$, with $\rho_v = 0.5$. The rise in the nominal rate causes inflation and the output gap to fall immediately. This reflects the forward-looking nature of both variables. In fact, all the persistence displayed by the responses arises from the serial correlation introduced into the process for the monetary shock v_t. If $\rho_v = 0$, all variables return to their steady-state values in the period after the shock.[21]

[20] Estrella and Fuhrer (2002) have noted that the forward-looking Euler equation implies counterfactual dynamics; (24) implies that $\mathrm{E}_t\hat{c}_{t+1} - \hat{c}_t = \sigma^{-1}(\hat{\imath}_t - \mathrm{E}_t\pi_{t+1})$, so that a rise in the real interest rate means that consumption must *increase* from t to $t+1$.

[21] See Galí (2002) for a discussion of the monetary transmission mechanism incorporated in the basic new Keynesian model.

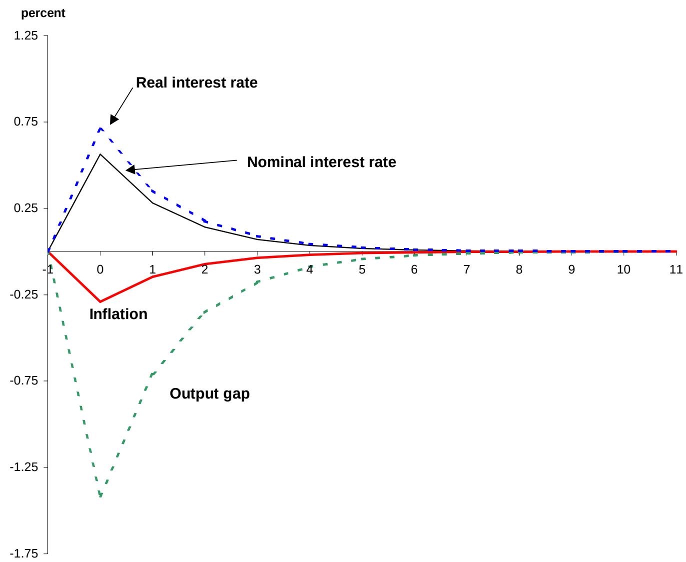

Figure 1: Output, Inflation, and Real Interest Rate Responses to a Policy Shock in the New Keynesian Model

To emphasize the interest rate as the primary channel through which monetary influences affect output, it is convenient to express the output gap as a function of an *interest rate gap*, the gap between the current interest rate and the interest rate consistent with the flexible-price equilibrium. For example, let $\hat{r}_t \equiv \hat{\imath}_t - \mathrm{E}_t \pi_{t+1}$ be the real interest rate and write (24) as

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{r}_t - \tilde{r}_t),$$

where $\tilde{r}_t \equiv \sigma u_t$. Woodford (2000) has labeled $\tilde{r}_t$ the *Wicksellian real interest rate*. It is the interest rate consistent with output equaling the flexible-price equilibrium level. If $\hat{r}_t = \tilde{r}_t$ for all t, then $x_t = 0$ and output is kept equal to the level that would arise in the absence of nominal rigidities. The interest rate gap $\hat{r}_t - \tilde{r}_t$ then summarizes the effects on the actual equilibrium that are due to nominal rigidities.[22]

The presence of expected future output in (24) implies that the future path of the one-period real interest rate matters for current demand. To see that this is the case, recursively solve (24) forward to yield

$$x_t = -\left(\frac{1}{\sigma}\right)\sum_{i=0}^{\infty} \mathrm{E}_t\left(\hat{r}_{t+i} - \tilde{r}_{t+i}\right).$$

[22] Neiss and Nelson (2001) use a structural model to estimate the real interest-rate gap $\hat{r}_t - \tilde{r}_t$ and find that it has value as a predictor of inflation.

Changes in the one-period rate that are persistent will influence expectations of future interest rates. Therefore, persistent changes should have stronger effects on x_t than more temporary changes in real interest rates.

The basic interest rate transmission mechanism for monetary policy could be extended to include effects on investment spending if capital were reintroduced into the model (Christiano, Eichenbaum, and Evans 2005, Dotsey and King 2001). Increases in the real interest rate would reduce the demand for capital and lead to a fall in investment spending. In the case of both investment and consumption, monetary policy effects are transmitted through interest rates.

In addition to these interest rate channels, monetary policy is often thought to affect the economy either indirectly through credit channels or directly through the quantity of money. Real money holdings represent part of household wealth; an increase in real balances should induce an increase in consumption spending through a wealth effect. This channel is often called the *Pigou effect* and was viewed as generating a channel through which price-level declines during a depression would eventually increase real balances and household wealth sufficiently to restore consumption spending. During the Keynesian/monetarist debates of the 1960s and early 1970s, some monetarists argued for a direct wealth effect that linked changes in the money supply directly to aggregate demand (Patinkin 1965). The effect of money on aggregate demand operating through interest rate effects was viewed as a Keynesian interpretation of the transmission mechanism, while most monetarists argued that changes in monetary policy lead to substitution effects over a broader range of assets than Keynesians normally considered. Since wealth effects are likely to be small at business-cycle frequencies, most simple models used for policy analysis ignore them.[23]

Direct effects of the quantity of money are not present in the model we have been using as the quantity of money appears in neither (24) nor (20). The underlying model was derived from a MIU model, and the absence of money in (24) and (20) results from the assumption that the utility function is separable (see 1). If utility is not separable, then changes in the real quantity of money alter the marginal utility of consumption. This would affect the model specification in two ways. First, the real money stock would appear in the household's Euler condition and therefore in (24). Second, to replace real marginal cost with a measure of the output gap in (20), the real wage was equated to the marginal rate of substitution between leisure and consumption, and this would also involve real money balances if utility is nonseparable (see problem 10). Thus, the absence of money constitutes a special case. However, McCallum and Nelson (1999) and Woodford (2001a) have both argued that the effects arising with nonseparable utility are quite small, so that little is lost by assuming separability. Ireland (2001c) finds little evidence for nonseparable preferences in a model estimated on U.S. data.

The quantity of money is not totally absent from the underlying model, since (8) also must hold in equilibrium. Linearizing this equation around the steady

[23] For a recent analysis of the real balance effect, see Ireland (2001b).

state yields[24]

$$\hat{m}_t - \hat{p}_t = \left(\frac{1}{bi^{ss}}\right)(\sigma \hat{y}_t - \hat{\imath}_t). \tag{30}$$

Given the nominal interest rate chosen by the monetary policy authority, this equation determines the nominal quantity of money. Alternatively, if the policy maker sets the nominal quantity of money, then (20), (24), and (30) must all be used to solve jointly for x_t, π_t, and $\hat{\imath}_t$.

Chapter 10 discusses the role of credit channels in the monetary transmission process.

3.5 Adding Economic Disturbances

As the model consisting of (20) and (24)stands, there are no underlying non-policy disturbances that might generate movements in either the output gap or inflation other than the productivity disturbance that affect the flexible-price output level. It is common to include in these equations stochastic disturbances arising from other sources.

Suppose the representative household's utility from consumption is subject to random shocks that alter the marginal utility of consumption. Specifically, let the utility function in (1) be modified to include a taste shock ψ:

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{\left(\psi_{t+i} C_{t+i}\right)^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b}\left(\frac{M_{t+i}}{P_{t+i}}\right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]. \tag{31}$$

The Euler condition (7) becomes

$$\psi_t^{1-\sigma} C_t^{-\sigma} = \beta(1+i_t)\mathrm{E}_t \left(P_t/P_{t+1}\right)\left(\psi_{t+1}^{1-\sigma} C_{t+1}^{-\sigma}\right),$$

which, when linearized around the zero-inflation steady state yields

$$\hat{c}_t = \mathrm{E}_t \hat{c}_{t+1} - \left(\frac{1}{\sigma}\right)(\hat{\imath}_t - \mathrm{E}_t \pi_{t+1}) + \left(\frac{\sigma - 1}{\sigma}\right)\left(\mathrm{E}_t \psi_{t+1} - \psi_t\right). \tag{32}$$

If, in addition to consumption by households, the government purchases final output G_t, the goods market equilibrium condition becomes $Y_t = C_t + G_t$. When this is expressed in terms of percentage deviations around the steady state, one obtains

$$\hat{y}_t = \left(\frac{C}{Y}\right)^{ss} \hat{c}_t + \left(\frac{G}{Y}\right)^{ss} \hat{g}_t.$$

Using this equation to eliminate $\hat{c}_t$ from (32) and then replacing $\hat{y}_t$ with $x_t + \hat{y}_t^f$ yields an expression for the output gap ($\hat{y}_t - \hat{y}_t^f$),

$$x_t = \mathrm{E}_t x_{t+1} - \tilde{\sigma}^{-1}(\hat{\imath}_t - \mathrm{E}_t \pi_{t+1}) + \xi_t, \tag{33}$$

[24] See the appendix to chapter 2.

where $\tilde{\sigma}^{-1} = \sigma^{-1} (C/Y)^{ss}$ and

$$\xi_t \equiv \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{C}{Y} \right)^{ss} (\mathrm{E}_t \psi_{t+1} - \psi_t) - \left(\frac{G}{Y} \right)^{ss} (\mathrm{E}_t \hat{g}_{t+1} - \hat{g}_t) + \left(\mathrm{E}_t \hat{y}^f_{t+1} - \hat{y}^f_t \right).$$

Equation (33) represents the Euler condition consistent with the representative household's intertemporal optimality condition linking consumption levels over time. It is also consistent with the resource constraint $Y_t = C_t + G_t$. The disturbance term arises from taste shocks that alter the marginal utility of consumption, shifts in government purchases, and shifts in the flexible-price equilibrium output. In each case, it is expected changes in ψ, g, and $\hat{y}^f$ that matter. For example, an expected rise in government purchases implies that future consumption must fall. This reduces current consumption.

The source of a disturbance term in the inflation adjustment equation is both more critical for policy analysis and more controversial. While policy analysis will be taken up in section 4, it is easy to see why exogenous shifts in (20) can have important implications for policy. Two commonly assumed objectives of monetary policy are to maintain a low and stable average rate of inflation and to stabilize output around full employment. These two objectives are often viewed as presenting central banks with a trade-off. A supply shock, such as an increase in oil prices, increases inflation and reduces output. To keep inflation from rising calls for contractionary policies that would exacerbate the decline in output; stabilizing output calls for expansionary policies that would worsen inflation. However, if the output objective is interpreted as meaning that output should be stabilized around its flexible-price equilibrium level, then (20) implies that the central bank can always achieve a zero output gap (i.e., keep output at its flexible-price equilibrium level) and simultaneously keep inflation equal to zero. Solving (20) forward yields

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t x_{t+i}.$$

By keeping current and expected future output equal to the flexible-price equilibrium level, $\mathrm{E}_t \hat{x}_{t+i} = 0$ for all i and inflation remains equal to zero. Blanchard and Galí (2007) describe this as the "divine coincidence." However, if an error term is added to the inflation adjustment equation so that

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{34}$$

then

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t e_{t+i}.$$

As long as $\sum_{i=0}^{\infty} \beta^i \mathrm{E}_t e_{t+i} \neq 0$, maintaining $\sum_{i=0}^{\infty} \beta^i \mathrm{E}_t x_{t+i} = 0$ is not sufficient to ensure that inflation always remains equal to zero. A trade-off between stabilizing output and stabilizing inflation can arise. Disturbance terms in the inflation adjustment equation are often called *cost shocks* or *inflation shocks*.

Since these shocks ultimately affect only the price level, they are also called *price shocks*.

Clarida, Galí, and Gertler (2001) suggest one means of including a stochastic shock in the inflation adjustment equation. They add a stochastic *wage markup* to represent deviations between the marginal rate of substitution between leisure and consumption and the real wage. Thus, the labor-supply condition (9) becomes

$$\left(\frac{\chi N_t^\eta}{C_t^{-\sigma}}\right) e^{\mu_t^w} = \frac{W_t}{P_t},$$

where μ_t^w is a random disturbance.[25] This could arise from shifts in tastes that affect the marginal utility of leisure. Or, if labor markets are imperfectly competitive, it could arise from stochastic shifts in the markup of wages over the marginal rate of substitution (Clarida, Galí, and Gertler 2002). When linearized around the steady state, one obtains

$$\eta\hat{n}_t + \sigma\hat{c}_t + \mu_t^w = \hat{w}_t - \hat{p}_t. \tag{35}$$

The real marginal cost variable becomes

$$\varphi_t = (\eta\hat{n}_t + \sigma\hat{c}_t) - (\hat{y}_t - \hat{n}_t) + \mu_t^w,$$

and this suggests that the inflation adjustment equation becomes

$$\pi_t = \beta E_t\pi_{t+1} + \gamma\tilde{\kappa}x_t + \tilde{\kappa}\mu_t^w. \tag{36}$$

In this formulation, μ_t^w is the source of inflation shocks.

While this approach appears to provide an explanation for a disturbance term to appear in the inflation adjustment equation, if μ_t^w reflects taste shocks that alter the marginal rate of substitution between leisure and consumption, then μ_t^w also affects the flexible-price equilibrium level of output. The same would be true if μ_t^w is a markup due to imperfect competition in the labor market. Thus, if the output gap variable in the inflation adjustment equation is correctly measured as the deviation of output from the flexible-price equilibrium level, μ_t^w no longer has a separate, independent impact on π_t.

Benigno and Woodford (2005) have show that a cost shock arises in the presence of stochastic variation in the gap between the welfare maximizing level of output and the flexible-price equilibrium level of output. In the model developed so far, only two distortions were present – one due to monopolistic competition and one due to nominal price stickiness. The first distortion implies the flexible-price output level is below the efficient output level even when prices are flexible. However, this "wedge" is constant, so when the model is linearized, percent deviations of the flexible-price output and the efficient output

[25] With the utility function given in (31), this becomes

$$\left(\frac{\chi N_t^\eta}{C_t^{-\sigma}}\right)\left(\frac{e^{\mu_t^w}}{\psi_t^{1-\sigma}}\right) = \frac{W_t}{P_t},$$

showing that μ_t^w affects the labor-market condition in a manner similar to a taste shock.

around their respective steady-state values are equal. If there are time varying distortions such as would arise with stochastic variation in distortionary taxes, then fluctuations in the two output concepts will differ. In this case, if x_t^w is the percent deviation of the welfare-maximizing output level around its steady state (the welfare gap),

$$x_t = x_t^w + \delta_t,$$

where δ_t represents these stochastic distortions. Since policy makers would be concerned with stabilizing fluctuations in x_t^w, the relevant constraint the policy maker will face is obtained by rewriting the Phillips curve (20) as

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t^w + \kappa \delta_t. \tag{37}$$

In this formulation, δ_t acts as a cost shock; stabilizing inflation in the face of non-zero realizations of δ cannot be achieved without creating volatility in the welfare gap x_t^w. One implication of (37) is that the variance of the cost shock will depend on κ^2. Thus, if the degree of price rigidity is high, implying that κ is small, cost shocks will also be less volatile (see Walsh 2005a).

Recent models, particularly those designed to be taken to the data, introduce a disturbance in the inflation equation by assuming that individual firms face random variation in the price elasticity of demand, that is, θ_t becomes time varying (see 13). This modtification raises similar issues to the introduction of a stochastic wage markup.

3.6 Sticky Wages and Prices

Erceg, Henderson, and Levin (2000) have employed the Calvo specification to incorporate sticky wages *and* sticky prices into an optimizing framework.[26] The goods market side of their model is identical in structure to the one developed in section 3.2. In the labor market, however, they assume that individual households supply differentiated labor services; firms combine these labor services to produce output. Output is given by a standard production function, $F(N_t, K_t)$, but the labor aggregate is a composite function of the individual types of labor services:

$$N_t = \left[\int_0^1 n_{jt}^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{\gamma}{\gamma-1}}, \qquad \gamma > 1,$$

where n_{jt} is the labor from household j that the firm employs. With this specification, households face a demand for their labor services that depends on the wage they set relative to the aggregate wage rate. Erceg, Henderson, and Levin assume that a randomly drawn fraction of households optimally set their wage each period, just as the models of price stickiness assume that only a fraction of firms adjust their price each period (see also Christiano, Eichenbaum, and Evans 2005 and Sbordone 2001).

[26] Other models incorporating both wage and price stickiness include those of Guerrieri (2000), Ravenna (2000), Christiano, Eichenbaum, and Evans (2001), and Sbordone (2001, 2002). This is now standard in models being taken to the data.

The model of inflation adjustment based on the Calvo specification implies that inflation depends on real marginal cost. In terms of deviations from the flexible-price equilibrium, real marginal cost equals the gap between the real wage and the marginal product of labor (mpl). Similarly, wage inflation (when linearized around a zero inflation steady state) responds to a gap variable, but this time the appropriate gap depends on a comparison between the real wage and the household's marginal rate of substitution between leisure and consumption. With flexible wages, as in the earlier sections where only prices were assumed to be sticky, workers are always on their labor supply curves; nominal wages can adjust to ensure that the real wage equals the marginal rate of substitution between leisure and consumption (mrs). When nominal wages are also sticky, however, ω_t and mrs_t can differ. If $\omega_t < mrs_t$, workers will want to raise their nominal wage when the opportunity to adjust arises. Letting π_t^w denote the rate of nominal wage inflation, Erceg, Henderson, and Levin show that

$$\pi_t^w = \beta \mathrm{E}_t \pi_{t+1}^w + \kappa^w \left(mrs_t - \omega_t \right). \tag{38}$$

From the definition of the real wage,

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t. \tag{39}$$

Equations (38) and (39), when combined with teh new Keynesian Phillips curve in which inflation depends on $\omega_t - mpl_t$, constitute the inflation adjustment block of an optimizing model with both wage and price rigidities.

4 Monetary Policy Analysis in New Keynesian Models

During the ten years after its first intorduction, the new Keynesian model discussed in section 3 has become the standard framework for monetary policy analysis. Clarida, Galí, and Gertler (1999), Woodford (2003), McCallum and Nelson (1999), Woodford (2003), and Svensson and Woodford (1999, 2005), among others, have popularized this simple model for use in monetary policy analysis. Galí (2002), and Galí and Gertler (2007) discusses some of the model's implications for monetary policy, while Galí (2008) provides an excellent treatment of the model and its implications for policy.

As seen in section 3, basic new Keynesian model takes the form

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma} \right) \left(i_t - \mathrm{E}_t \pi_{t+1} \right) + u_t \tag{40}$$

and

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{41}$$

where x is the output gap, defined as output relative to the equilibrium level of output under flexible prices, i is the nominal rate of interest, and π is the inflation rate. All variables are expressed as percentage deviations around their steady-state values. The demand disturbance u can arise from taste shocks to the preferences of the representative household, fluctuations in the flexible-price equilibrium output level, or shocks to government purchases of goods and services. We will refer to the e shock as a *cost shock*. In this section, we will use (40) and (41) to addess issues of monetary policy design.

4.1 Policy Objectives

Given the economic environment that leads to (40) and (41), what are the appropriate objectives of the central bank? There is a long history in monetary policy analysis of assuming that the central bank is concerned with minimizing a quadratic loss function that depended on output and inflation.Models that make this assumption were discussed in chapter 7. While such an assumption is plausible it is ultimately ad hoc. In the new Keynesian model, the description of the economy is based on an approximation to a fully specified general equilibrium model. Can we therefore develop a policy objective function that can be interpreted as an approximation to the utility of the representative household? Put differently, can we draw insights from the general equilibrium foundations of (40) and (41) to determine the basic objectives central banks should pursue? Woodford (2003), building on the earlier work by Rotemberg and Woodford (1997), has provided the most detailed analysis of the link between a welfare criterion derived as an approximation to the utility of the representative agent and the types of quadratic loss functions common in the older literature.

Woodford assumes that there is a continuum of differentiated goods c_{it} defined on the interval $[0,1]$ and that the representative household derives utility from consuming a composite of these individual goods. The composite consumption good is defined as

$$Y_t = C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{42}$$

In addition, each household produces one of these individual goods and experiences disutility from production. Suppose labor effort is proportional to output. Woodford assumes the period utility of the representative agent is then assumed to be

$$V_t = U(Y_t, z_t) - \int_0^1 v(c_{jt}, z_t) dj, \tag{43}$$

where $v(c_{jt}, z_t)$ is the disutility of producing good c_{jt} and z_t is a vector of exogenous shocks.[27] Woodford demonstrates that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda \left(x_{t+i} - x^* \right)^2 \right] + t.i.p, \tag{44}$$

[27] Woodford considers a cashless economy, so real money balances do not appear in the utility function as they did in (1).

where *t.i.p.* indicates terms independent of policy. The detailed derivation of (44) and the values of Ω and λ are given in the appendix to this chapter. In (44), x_t is the gap between output and the output level that would arise under flexible prices, and x^* is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the actual steady-state level of output.

Equation (44) looks like the standard quadratic loss function employed in chapter 7 to represent the objectives of the monetary policy authority. There are, however, two critical differences. First, the output gap is measured relative to equilibrium output under flexible prices. It was more common in the traditional literature the output variable was more commonly interpreted as output relative to trend or output relative to the natural rate of output which in turn was often defined as output in the absence of price surprises (see chapter 6, section 2.1).

A second difference between (44) and a standard quadratic loss function arises from the reason inflation variability enters the loss function. When prices are sticky, and firms do not all adjust simultaneously, inflation results in an inefficient dispersion of relative prices and production among individual producers. The representative household's utility depends on its consumption of a composite good; faced with a dispersion of prices for the differentiated goods produced in the economy, the household buys more of the relatively cheaper goods and less of the relatively more expensive goods. Because of diminishing marginal utility, the increase in utility derived from consuming more of some goods is less than the loss in utility due to consuming less of the more expensive goods. Hence, price dispersion reduces utility. Similarly, if we had assumed diminishing returns to labor in the production process, rather than constant returns to scale, dispersion on the production side would also be costly. The increased cost of producing more of some goods is greater than the cost saving from reducing production of other goods. For these reasons, price dispersion reduces utility, and, when each firm does not adjust its price every period, price dispersion is caused by inflation. These welfare costs can be eliminated under a zero inflation policy.

In Chapter 7, the efficiency distortion represented by x^* was used to motivate an overly ambitious output target in the central bank's objective function. As shown in that chapter, the presence of $x^* > 0$ implies that a central bank acting under discretion to maximize (44) would produce a positive average inflation bias. However, with average rates of inflation in the major industrialized economies remaining low during the 1990s, many authors now simply assume that $x^* = 0$. In this case, the central bank is concerned with stabilizing the output gap x_t, and no average inflation bias arises.[28] If tax subsidies can be used to offset the distortions associated with monopolistic competition, then one could assign fiscal policy the task of ensuring that $x^* = 0$. In this case, the central bank has no incentive to create inflationary expansions, and average

[28] In addition, the inflation equation was derived by linearizing around a zero-inflation steady state. It would thus be inappropriate to use it to study situations in which average is positive.

inflation will be zero under discretion. Dixit and Lambertini (2002) show that when both the monetary and fiscal authorities are acting optimally, the fiscal authority will use its tax instruments to set $x^* = 0$ and the central bank then ensures that inflation remains equal to zero.

In the context of our linear-quadratic model, (44) represents a second order approximation to the welfare of the representative agent around the steady state. Expanding the period loss function,

$$\pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 = \pi_{t+i}^2 + \lambda x_{t+i}^2 - 2\lambda x^* x_{t+i} + \lambda (x^*)^2 .$$

Employing first order approximation for the structural equations will be adequate for evaluating the π_{t+i}^2 and x_{t+i}^2 terms, since any higher order terms in the structural equations would become of order greater than 2 when squared. However, this is not the case for the $2\lambda x^* x_{t+i}$ term, which is linear in x_{t+i}. Hence, to approximate this correctly to the required degree of accuracy would require second order approximations to the structural equations, rather the the linear approximations we have derived in (40) and (41). Thus, we will assume the fiscal authority employs an subsidy to undo the distortion arising from imperfect competition so that $x^* = 0$. In this case, the linear approximations to the structural equations will allow us to correctly evaluate the second-order approximation to welfare. See Benigno and Woodford (2005) for discussion of optimal policy in the presence of a distorted steady state.

4.2 Policy Trade-offs

The basic new Keynesian inflation-adjustment equation derived in chapter 6, took the form

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

That is, no additional disturbance term, such as the e_t that was added to (41), appears. The absence of e implies that there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero. If $x_{t+i} = 0$ for all $i \geq 0$, then $\pi_{t+i} = 0$. In this case, a central bank that wants to maximize the expected utility of the representative household will ensure that output is kept equal to the flexible-price equilibrium level of output. This also guarantees that inflation is equal to zero, thereby eliminating the costly dispersion of relative prices that arises with inflation. When firms do not need to adjust their prices, the fact that prices are sticky is no longer relevant. Thus, a key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.[29]

The optimality of zero inflation conflicts with the Friedman rule for optimal inflation. M. Friedman (1969) concluded that the optimal inflation rate must be negative to make the nominal rate of interest zero (see chapter 4). The reason we reach a different conclusion now is due to the absence of any explicit role for money when the utility approximation given by (44) is derived. In general, zero

[29] Notice that the conclusion that price stability is optimal is independent of the degree of nominal rigidity (see Adao, Correia and Teles 1999).

inflation still generates a monetary distortion. With zero inflation, the nominal rate of interest will be positive and the private opportunity cost of holding money will exceed the social cost of producing it. Khan, King, and Wolman (2000) and Adao, Correia, and Telles (2001) consider models that integrate nominal rigidities and the Friedman distortion. Khan, King and Wolman introduce money into a sticky-price model by assuming the presence of cash and credit goods, with money required to purchase cash goods. If prices are flexible, it is optimal to have a rate of deflation such that the nominal interest rate is zero. If prices are sticky, price stability would be optimal in the absence of the cash-in-advance (CIA) constraint. With both sticky prices and the monetary inefficiency associated with a positive nominal interest rate, the optimal rate of inflation is less than zero but greater than the rate that yields a zero nominal interest rate. Khan, King, and Wolman conduct simulations in a calibrated version of their model and find that the relative price distortion dominates the Friedman monetary inefficiency. Thus, the optimal policy is close to the policy that maintains price stability.

In the baseline model with no monetary distortion and with $x^* = 0$, the optimality of price stability is a reflection of the presence of only one nominal rigidity. The welfare costs of a single nominal rigidity can be eliminated using the single instrument provided by monetary policy. As discussed in section 3.6, Erceg, Henderson, and Levin (2000) introduced nominal wage stickiness into the basic new Keynesian framework as a second nominal rigidity. Nominal wage inflation with staggered adjustment of wages causes a distortions of relative wages and reduces welfare. In this case, Erceg, Henderson, and Levin show that the approximation to the welfare of the representative agent will include a term in wage inflation as well as the inflation and output gap terms appearing in (44). Wage stability is desirable because it eliminates dispersion of hours worked across households. With two distortions – sticky prices and sticky wages – the single instrument of monetary policy cannot simultaneously offset both distortions. With sticky prices but flexible wages, the real wage can adjust efficiently in the face of productivity shocks and monetary policy should maintain price stability. With sticky wages and flexible prices, the real wage can still adjust efficiently to ensure that labor-market equilibrium is maintained in the face of productivity shocks and monetary policy should maintain nominal wage stability. If both wages and prices are sticky, a policy that stabilizes either prices or wages will not allow the real wage to move so as to keep output equal to the flexible-price output. Productivity shock will lead to movements in the output gap, and the monetary authority will be forced to trade off stabilizing inflation, wage inflation, and the output gap.

When wages are sticky, they adjust to the gap between the real wage and the marginal rate of substitution between leisure and consumption. When prices are sticky, they adjust to the gap between the marginal product of labor and the real wage. Galí, Gertler, and López-Salido (2002) define the *inefficiency gap* as the sum of these two gaps, the gap between the household's marginal rate of substitution between leisure and consumption (mrs_t) and the marginal product of labor (mpl_t). This inefficiency gap can be divided into its two parts, the wedge between the real wage and the marginal rate of substitution, which they label the *wage markup*, and the wedge between the real wage and the marginal product of labor, labeled the *price markup.* Based on U.S. data, they conclude that the wage markup accounts for most of the time-series variation in the inefficiency gap. Levin, Onatski, Williams, and Williams (2005) estimate a new Keynesian general equilibrium model with both price and wage stickiness. They find that the welfare costs of nominal rigidity is primarily generated by wage stickiness rather than by price stickiness. This finding is consistent with Christiano, Eichenbaum, and Evans (2005) who conclude that a model with flexible prices and sticky wages does better at fitting impulse responses estimated on U.S. data than a sticky price-flexible wage version of their model. Sbordone (2001) also suggests that nominal wage rigidity is more important empirically than price rigidity. Huang and Liu (2002) also argue that wage stickiness is more important than price stickiness for generating output persistence.

In contrast, Goodfriend and King (2001) argue that the long-term nature of employment relationships reduces the effects of nominal wage rigidity on real resource allocations. Models that incorporate the intertemporal nature of employment relationships based on search and matching models of unemployment include Walsh (2003b, 2005b), Trigari (2004), Krause, López-Salido, and Lubik (2007), Thomas (2008), Ravenna and Walsh (2008), and Sala, Söderström, and Trigari (2008).

4.3 Optimal Commitment and Discretion

Suppose the central bank attempts to minimize a quadratic loss function such as (44), defined in terms of inflation and output relative to the flexible-price equilibrium.[30] Assume also that the steady-state gap between output and its efficient value is zero (i.e., $x^* = 0$). In this case, the central bank's loss function takes the form

$$L_t = \left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right). \tag{45}$$

We can consider two alternative policy regimes. In a discretionary regime, the central bank behaves optimally in each period, taking as given the current state of the economy and private sector expectations. Given that the public know the central bank optimizes each period, any promises the central bank makes about future inflation will not be credible – the public knows that whatever may have been promised in the past, the central bank will do what is optimal at the time it sets policy. The alternative regime is one of commitment. In a commitment regime, the central bank can make credible promises about what it will do in the future. By promising to take certain actions in the future, the central bank can influence the public's expectations about future inflation.

[30] Svensson (1999b, 1999c) argues that there is widespread agreement among policy makers and academics that inflation stability and output gap stability are the appropriate objectives of monetary policy.

In this section, we analyze the case of commitment first and then turn to optimal policy under discretion. When forward-looking expectations play a role, as in (41), discretion will lead to what is known as a *stabilization bias.*

4.3.1 Commitment

A central bank able to precommit chooses a path for current and future inflation and the output gap to minimize the loss function (45) subject to the expectational IS curve (40) and the inflation-adjustment equation (41). Let θ_{t+i} and ψ_{t+i} denote the Lagrangian multipliers associated with the period $t+i$ IS curve and the inflation-adjustment equation. The central bank's objective is to pick i_{t+i}, π_{t+i} and x_{t+i} to minimize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \left(\frac{1}{2}\right) \left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right) + \theta_{t+i} \left[x_{t+i} - x_{t+i+1} + \sigma^{-1} \left(i_{t+i} - \pi_{t+i+1}\right) - u_{t+i}\right] \right.$$
$$\left. + \psi_{t+i} \left(\pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}\right) \right\}.$$

The first order conditions for i_{t+i} take the form

$$\sigma^{-1} \mathrm{E}_t \left(\theta_{t+i}\right) = 0 \qquad i \geq 0.$$

Hence, $\mathrm{E}_t \theta_{t+i} = 0$ for all $i \geq 0$. This result implies that (40) imposes no real constraint on the central bank as long as there are no restrictions on, or costs associated with, varying the nominal interest rate. Given the central bank's optimal choices for the output gap and inflation, (40) will simply determine the setting for i_t necessary to achieve the desired value of x_t. For that reason, it is often more convenient to treat x_t as if it were the central bank's policy instrument.

Setting $\mathrm{E}_t \theta_{t+i} = 0$, the remaining first order conditions for π_{t+i} and x_{t+i} can be written as

$$\pi_t + \psi_t = 0 \tag{46}$$

$$\mathrm{E}_t \left(\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}\right) = 0 \qquad i \geq 1 \tag{47}$$

$$\mathrm{E}_t \left(\lambda x_{t+i} - \kappa \psi_{t+i}\right) = 0 \qquad i \geq 0. \tag{48}$$

Equations (46) and (47) reveal the dynamic inconsistency that characterizes the optimal precommitment policy. At time t, the central bank sets $\pi_t = -\psi_t$ and promises to set $\pi_{t+1} = -\left(\psi_{t+1} - \psi_t\right)$. But when period $t+1$ arrives, a central bank that reoptimizes will again obtain $\pi_{t+1} = -\psi_{t+1}$ as its optimal setting for inflation. That is, the first order condition (46) updated to $t+1$ will reappear.

An alternative definition of an optimal precommitment policy requires that the central bank implement conditions (47) and (48) for all periods, including the current period. Woodford (1999a) has labeled this the *timeless perspective* approach to precommitment. One can think of such a policy as having been chosen in the distant past, and the current values of the inflation rate and output gap are the values chosen from that earlier perspective to satisfy the two conditions (47) and (48). McCallum and Nelson (2000a) provide further discussion of the timeless perspective and argue that this approach agrees with the one commonly used in many studies of precommitment policies.

Combining (47) and (48), under the timeless perspective optimal commitment policy inflation and the output gap satisfy

$$\pi_{t+i} = -\left(\frac{\lambda}{\kappa}\right)(x_{t+i} - x_{t+i-1}) \tag{49}$$

for all $i \geq 0$. Using this equation to eliminate inflation from (41) and rearranging, one obtains

$$\left(1 + \beta + \frac{\kappa^2}{\lambda}\right) x_t = \beta \mathrm{E}_t x_{t+1} + x_{t-1} - \frac{\kappa}{\lambda} e_t. \tag{50}$$

The solution to this expectational difference equation for x_t will be of the form $x_t = a_x x_{t-1} + b_x e_t$. To determine the coefficients a_x and b_x, note that if $e_t = \rho e_{t-1} + \varepsilon_t$, the proposed solution implies $\mathrm{E}_t x_{t+1} = a_x x_t + b_x \rho e_t = a_x^2 x_{t-1} + (a_x + \rho) b_x e_t$. Substituting this into (50) and equating coefficients, the parameter a_x is the solution less than 1 of the quadratic equation

$$\beta a_x^2 - \left(1 + \beta + \frac{\kappa^2}{\lambda}\right) a_x + 1 = 0$$

and b_x is given by

$$b_x = -\left\{\frac{\kappa}{\lambda\left[1 + \beta\left(1 - \rho - a_x\right)\right] + \kappa^2}\right\}.$$

From (49), equilibrium inflation under the timeless perspective policy is

$$\pi_t = \left(\frac{\lambda}{\kappa}\right)(1 - a_x) x_{t-1} + \left[\frac{\lambda}{\lambda\left[1 + \beta\left(1 - \rho - a_x\right)\right] + \kappa^2}\right] e_t. \tag{51}$$

Woodford (1999a) has stressed that, even if $\rho = 0$, so that there is no natural source of persistence in the model itself, $a_x > 0$ and the precommitment policy introduces inertia into the output gap and inflation processes. Because the central bank responds to the lagged output gap (see 49), past movements in the gap continue to affect current inflation. This commitment to inertia implies that the central bank's actions at date t allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap. Equation (41) implies that the inflation impact of a positive cost shock, for example, can be stabilized at a lower output cost if the central bank can induce a fall in expected future inflation. Such a fall in expected inflation is achieved when the central bank follows (49).

A condition for policy such as (49) that is derived from the central bank's first order conditions and only involves variables that appear in the objective function (in this case, inflaiton and the output gap), is generally called a *targeting rule* (e.g., Svensson and Woodford 2005). It represents a relationship among the targeted variables that the central bank should maintain, as doing so is consistent with the first order conditions from its policy problem.

Because the timeless perspective commitment policy is not the solution to the policy problem under optimal commitment, the policy rule given by (49) may be dominated by other policy rules. For instance, it may be dominated by the optimal discretion policy discussed below. Under the timeless perspective, inflation as given by (49) is the same function each period of the current and lagged output gap; the policy displays the property of continuation in the sense that the policy implemented in any period continues the plan it was optimal to commit to in an earlier period. Blake (2001), Damjanovic, Damjanovic, and Nolan (2008) and Jensen and McCallum (forthcoming) have considered *optimal* continuation policies that require that the policy instrument, in this case x_t, be a time-invariant function as under the timeless perspective, but rather than ignore the first period conditions as is done under the timeless perspective, they focus on the optimal unconditional continuation policy to which the central bank should commit. This policy minimizes the unconditional expectation of the objective function so that the Lagrangian for the policy problem becomes

$$\begin{aligned}\tilde{\mathrm{E}}\mathcal{L} &= \tilde{\mathrm{E}}\left\{\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[\frac{1}{2}\left(\pi_{t+i}^2+\lambda x_{t+i}^2\right)\right.\right.\\ &\left.\left.+\theta_{t+i}\left(\pi_{t+i}-\beta\mathrm{E}_t\pi_{t+i+1}-\kappa x_{t+i}-e_{t+i}\right)\right]\right\},\end{aligned}$$

where $\tilde{E}$ denotes the unconditional expectations operator. Because

$$\tilde{\mathrm{E}}\mathrm{E}_t\theta_{t+i}\pi_{t+i+1}=\tilde{\mathrm{E}}\theta_{t-1}\pi_t$$

the unconditional Lagrangian can be expressed as

$$\tilde{\mathrm{E}}\mathcal{L}=\left(\frac{1}{1-\beta}\right)\tilde{\mathrm{E}}\left\{\left[\frac{1}{2}\left(\pi_t^2+\lambda x_t^2\right)+\theta_t\pi_t-\beta\theta_{t-1}\pi_t-\kappa\theta_t x_t-\theta_t e_t\right]\right\}.$$

The first-order conditions then become

$$\pi_t+\theta_t-\beta\theta_{t-1}=0 \tag{52}$$

and

$$\lambda x_t-\kappa\theta_t=0.$$

Combining these to eliminate the Lagrangian multiplier yields the optimal unconditional continuation policy:

$$\pi_t=-\left(\frac{\lambda}{\kappa}\right)\left(x_{t+i}-\beta x_{t+i-1}\right). \tag{53}$$

Comparing this to (49) shows that rather than give full weight to past output gaps, the optimal unconditional continuation policy discounts the past slightly (recall $\beta\approx 0.99$).

4.3.2 Discretion

When the central bank operates with discretion, it acts each period to minimize the loss function (45) subject to the inflation-adjustment equation (41). Because the decisions of the central bank at date t do not bind it at any future dates, the central bank is unable to affect the private sector's expectations about future inflation. Thus, the decision problem of the central bank becomes the single-period problem of minimizing $\pi_t^2 + \lambda x_t^2$ subject to the inflation-adjustment equation (41).

The first order condition for this problem is

$$\kappa \pi_t + \lambda x_t = 0. \tag{54}$$

Equation (54) is the optimal targeting rule under discretion. Notice that by combining (46) with (48) evaluated at time t, one obtains (54); thus, the central bank's first order condition relating inflation and the output gap at time t is the same under discretion or under the fully optimal precommitment policy (but not under the timeless perspective policy). The differences appear in subsequent periods. For $t+1$, under discretion $\kappa \pi_{t+1} + \lambda x_{t+1} = 0$, while under precommitment (from 47 and 48), $\kappa \pi_{t+1} + \lambda (x_{t+1} - x_t) = 0$.

The equilibrium expressions for inflation and the output gap under discretion can be obtained by using (54) to eliminate inflation from the inflation-adjustment equation. This yields

$$\left(1 + \frac{\kappa^2}{\lambda}\right) x_t = \beta \mathrm{E}_t x_{t+1} - \left(\frac{\kappa}{\lambda}\right) e_t. \tag{55}$$

Guessing a solution of the form $x_t = \delta e_t$, so that $\mathrm{E}_t x_{t+1} = \delta \rho e_t$, one obtains

$$\delta = -\left[\frac{\kappa}{\lambda(1 - \beta\rho) + \kappa^2}\right].$$

Equation (54) implies that equilibrium inflation under optimal discretion is

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right) x_t = \left[\frac{\lambda}{\lambda(1 - \beta\rho) + \kappa^2}\right] e_t. \tag{56}$$

According to (56) the unconditional expected value of inflation is zero; there is no average inflation bias under discretion. However, there is a stabilization bias in that the response of inflation to a cost shock under discretion differs from the response under commitment. This can be seen by comparing (56) to (51).

4.3.3 Discretion versus Commitment

The impact of a cost shock on inflation and the output gap under the timeless perspective optimal precommitment policy and optimal discretionary policy can be obtained by calibrating (41) and (49) and solving them numerically. Four unknown parameters appear in the model: β, κ, λ and ρ. The discount factor,

β, is set equal to 0.99, appropriate for interpreting the time interval as one quarter. A weight on output fluctuations of $\lambda = 0.25$ is used. This value is also used by Jensen (2002) and McCallum and Nelson (2000a).[31] The parameter κ captures both the impact of a change in real marginal cost on inflation and the comovement of real marginal cost and the output gap and is set equal to 0.05. McCallum and Nelson (2000) report that the empirical evidence is consistent with a value of κ in the range [0.01, 0.05]. Roberts (1995) reports higher values; his estimate of the coefficient on the output gap is about 0.3 when inflation is measured at an annual rate, so this translates into a value for κ of 0.075 for inflation at quarterly rates. Jensen (2002) uses a baseline value of $\kappa = 0.1$, while Walsh (2003) uses 0.05.

The solid lines in figures 2 and 3) show the response of the output gap and inflation to a transitory, one standard deviation cost push shock under the optimal precommitment policy.[32] Despite the fact that the shock itself has no persistence, the output gap displays strong positive serial correlation. By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in $\mathrm{E}_t\pi_{t+1}$ at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.

Outcomes under optimal discretion are shown by the dashed lines in the figures. There is no inertia under discretion; both the output gap and inflation return to their steady-state values in the period after the shock occurs. The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion. The intuition behind the suboptimality of discretion can be see by considering the inflation adjustment equation given by (41). Under discretion, the central bank's only tool for offsetting the effects on inflation of a cost shock is the output gap. In the face of a positive realization of e_t, x_t must fall to help stabilize inflation. Under commitment, however, the central bank has two instruments as it can affect both x_t and $\mathrm{E}_t\pi_{t+1}$. By creating expectations of a deflation at $t+1$, the reduction in the output gap does not need to be as large. Of course, under commitment a promise of future deflation must be honored, so actually inflation falls below the baseline beginning in period $t+1$, as seen in figure 3. Consistent with producing a deflation, the output gap remains negative for several periods.

The analysis so far has focused on the goal variables, inflation and the output gap. Using (40), the associated setting for the interest rate can be derived. For example, under optimal discretion, the output gap is given by

$$x_t = -\left[\frac{\kappa}{\lambda(1-\beta\rho)+\kappa^2}\right]e_t,$$

[31] If we interpret (45) as an approximation to the welfare of the representative agent, the implied value of λ would be much smaller.

[32] The programs of Paul Söderlind (1999) were used to obtain these figures. These programs are available at http://home.datacomm.ch/paulsoderlind/.

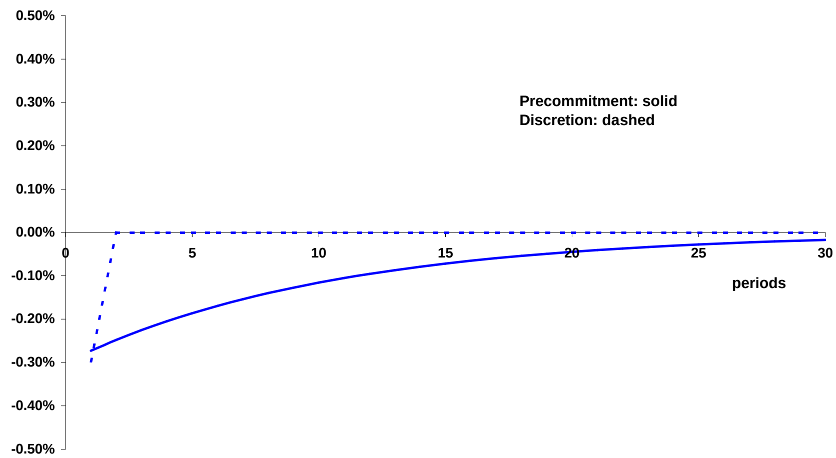

Figure 2: Output Gap Response to a Cost Shock: Timeless Precommitment and Pure Discretion

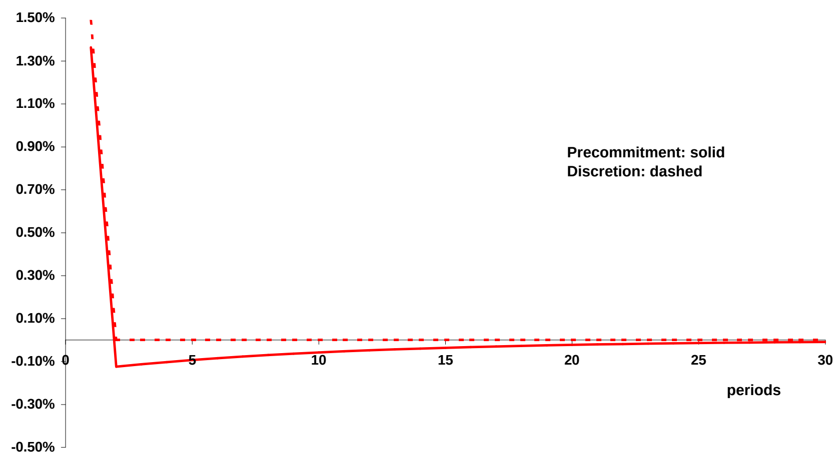

Figure 3: Response of Inflation to a Cost Shock: Timeless Precommitment and Pure Discretion

while inflation is given by (56). Using these to evaluate $\mathrm{E}_t x_{t+1}$ and $\mathrm{E}_t \pi_{t+1}$ and then solving for i_t from (40) yields

$$\begin{aligned} i_t &= \mathrm{E}_t \pi_{t+1} + \sigma \left(\mathrm{E}_t x_{t+1} - x_t + u_t \right) \\ &= \left[\frac{\lambda\rho + (1-\rho)\sigma\kappa}{\lambda(1-\beta\rho) + \kappa^2} \right] e_t + \sigma u_t. \end{aligned} \tag{57}$$

Equation (57) is the reduced-form solution for the nominal rate of interest. The nominal interest rate is adjusted to offset completely the impact of the demand disturbance u_t on the output gap. As a result, it affects neither inflation nor the output gap. Section 3.3 illustrated how a policy that commits to a rule that calls for responding to the exogenous shocks renders the new Keynesian model's equilibrium indeterminate. Thus, it is important to recognize that (57) describes the *equilibrium* behavior of the nominal interest rate under optimal discretion; (57) is not an instrument rule (see Svensson and Woodford 1999).

4.4 Commitment to a Rule

In the Barro-Gordon model popular in the 1980s and 1990s, and examined in chapter 7, optimal commitment was interpreted as commitment to a policy that was a (linear) function of the state variables. In the present model consisting of (40) and (41), the only state variable is the current realization of the cost shock e_t. Suppose then that the central bank can commit to a rule of the form[33]

$$x_t = b_x e_t. \tag{58}$$

What is the optimal value of b_x? With x_t given by (58), inflation satisfies

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa b_x e_t + e_t,$$

and the solution to this expectational difference equation is[34]

$$\pi_t = b_\pi e_t, \qquad b_\pi = \frac{1+\kappa b_x}{1-\beta\rho}. \tag{59}$$

Using (58) and (59), the loss function can now be written as

$$\left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2 \right) = \left(\frac{1}{2}\right) \sum_{i=0}^{\infty} \beta^i \left[\left(\frac{1+\kappa b_x}{1-\beta\rho} \right)^2 + \lambda b_x^2 \right] e_t^2.$$

[33] This commitment does not raise the same uniqueness of equilibrium problem that would arise under a commitment to an instrument rule of the form $i_t = b_i e_t$. See problem 2

[34] To verify this is the solution, note that

$$\begin{aligned} \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa b_x e_t + e_t = \beta b_\pi \rho e_t + \kappa b_x e_t + e_t \\ &= \left[\beta b_\pi \rho + \kappa b_x + 1 \right] e_t, \end{aligned}$$

so that $b_\pi = \beta b_\pi \rho + \kappa b_x + 1 = (\kappa b_x + 1)/(1-\beta\rho)$.

This is minimized when

$$b_x = -\left[\frac{\kappa}{\lambda(1-\beta\rho)^2+\kappa^2}\right].$$

Using this solution for b_x in (59), equilibrium inflation is given by

$$\pi_t = \left(\frac{1+\kappa b_x}{1-\beta\rho}\right)e_t = \left[\frac{\lambda(1-\beta\rho)}{\lambda(1-\beta\rho)^2+\kappa^2}\right]e_t. \tag{60}$$

Comparing the solution for inflation under optimal discretion, given by (56), and the solution under commitment to a simple rule, given by (60), note that they are identical if the cost shock is serially uncorrelated ($\rho = 0$). If $0 < \rho < 1$, there is a stabilization bias under discretion relative to the case of committing to a simple rule.

Clarida, Galí, and Gertler (1999) have argued that this stabilization bias provides a rationale for appointing a Rogoff-conservative central banker – a central bank who puts more weight on inflation objectives that is reflected in the social loss function – when $\rho > 0$, even though in the present context there is no average inflation bias.[35] A Rogoff-conservative central banker places a weight $\hat{\lambda} < \lambda$ on gap fluctuations (see section 9.3.2). In a discretionary environment with such a central banker, (56) implies that inflation will equal

$$\pi_t = \left[\frac{\hat{\lambda}}{\hat{\lambda}(1-\beta\rho)+\kappa^2}\right]e_t.$$

Comparing this with (60) reveals that if a central banker is appointed for whom $\hat{\lambda} = \lambda(1-\beta\rho) < \lambda$, the discretionary solution will coincide with the outcome under commitment to the optimal simple rule. Such a central banker stabilizes inflation more under discretion than would be the case if the relative weight placed on output gap and inflation stability were equal to the weight in the social loss function, λ. Because the public knows inflation will respond less to a cost shock, future expected inflation rises less in the face of a positive e_t shock. As a consequence, current inflation can be stabilized with a smaller fall in the output gap. The inflation-output trade-off is improved.

Recall, however, that the notion of commitment used here is actually suboptimal. As we saw earlier, fully optimal commitment leads to inertial behavior in that future inflation depends not on the output gap but on the change in the gap.

4.5 Endogenous Persistence

The empirical research on inflation discussed in section 5.5 has generally found that when lagged inflation is added to (41), its coefficient is statistically and

[35] Rogoff (1985) proposed appointing a conservative central banker as a way to solve the average inflition bias that can arise under discretionary policies, an issue discussed in chapter 7. There is no average inflation bias in the present model because we have assumed that $x^* = 0$, ensuring that the central bank's loss function depends on output only through the gap between actual output and flexible-price equilibrium output.

economically significant. If lagged inflation affects current inflation, then even under discretion the central bank faces a dynamic optimization problem; decisions that affect current inflation also affect future inflation, and this intertemporal link must be taken into account by the central bank when setting current policy. Svensson (1999b) and Vestin (2006) illustrate how the linear-quadratic structure of the problem allows one to solve for the optimal discretionary policy in the face of endogenous persistence.

To analyze the effects introduced when inflation depends on both expected future inflation and lagged inflation, suppose (41) is replaced by

$$\pi_t = (1-\phi)\beta \mathrm{E}_t \pi_{t+1} + \phi \pi_{t-1} + \kappa x_t + e_t. \tag{61}$$

The coefficient ϕ measures the degree of backward-looking behavior exhibited by inflation.[36] If the central bank's objective is to minimize the loss function given by (45), the policy problem under discretion can be written in terms of the value function defined by

$$\begin{aligned} V(\pi_{t-1}, e_t) \quad = \quad & \min_{\pi_t, x_t} \left\{ \left(\frac{1}{2}\right) \left(\pi_t^2 + \lambda x_t^2\right) + \beta \mathrm{E}_t V(\pi_t, e_{t+1}) \right. \\ & \left. + \theta_t \left[\pi_t - (1-\phi)\beta \mathrm{E}_t \pi_{t+1} - \phi \pi_{t-1} - \kappa x_t - e_t\right] \right\}. \end{aligned} \tag{62}$$

The value function depends on lagged inflation because it is an endogenous state variable.

Because the objective function is quadratic and the constraints are linear, the value function will be quadratic, and we can hypothesize that it takes the form

$$V(\pi_{t-1}, e_t) = a_0 + a_1 e_t + \frac{1}{2} a_2 e_t^2 + a_3 e_t \pi_{t-1} + a_4 \pi_{t-1} + \frac{1}{2} a_5 \pi_{t-1}^2. \tag{63}$$

As Vestin demonstrates, this guess is only needed to evaluate $\mathrm{E}_t V_\pi(\pi_t, e_{t+1})$, and $\mathrm{E}_t V_\pi(\pi_t, e_{t+1}) = a_3 \mathrm{E}_t e_{t+1} + a_4 + a_5 \pi_t$. If we assume that the cost shock is serially uncorrelated, $\mathrm{E}_t e_{t+1} = 0$ and, as a consequence, the only unknown coefficients in (63) that will play a role are a_4 and a_5.

The solution for inflation will take the form

$$\pi_t = b_1 e_t + b_2 \pi_{t-1}. \tag{64}$$

Using this proposed solution, one obtains $\mathrm{E}_t \pi_{t+1} = b_2 \pi_t$. This expression for expected future inflation can be substituted into (61) to yield

$$\pi_t = \frac{\kappa x_t + \phi \pi_{t-1} + e_t}{1 - (1-\phi)\beta b_2}, \tag{65}$$

which implies $\partial \pi_t / \partial x_t = \kappa / \left[1 - (1-\phi)\beta b_2\right]$.

[36] Galí and Gertler (1999), Woodford (2003), and Christiano, Eichenbaum, and Evans (2005) develop inflation-adjustment equations in which lagged inflation appears by assuming that some fraction of firms do not reset their prices optimally (see section 5.5.2).

Collecting these results, the first order condition for the optimal choice of x_t by a central bank whose decision problem is given by (62) is

$$\left[\frac{\kappa}{1-(1-\phi)\beta b_2}\right]\left[\pi_t+\beta \mathrm{E}_t V_\pi(\pi_t, e_{t+1})\right]+\lambda x_t=0. \tag{66}$$

By using (65) to eliminate x_t from (66) and recalling that $\mathrm{E}_t V_\pi(\pi_t, e_{t+1}) = a_4 + a_5\pi_t$, we obtain

$$\pi_t=\left[\frac{\Psi}{\kappa^2(1+\beta a_5)+\lambda\Psi^2}\right]\left[\lambda\phi\pi_{t-1}+\lambda e_t-\left(\frac{\beta\kappa^2}{\Psi}\right)a_4\right], \tag{67}$$

where $\Psi \equiv 1-(1-\phi)\beta b_2$.

From the envelope theorem and (66),

$$\begin{aligned} V_\pi(\pi_{t-1}, e_t) &= a_3 e_t + a_4 + a_5\pi_{t-1} \\ &= \left[\frac{\phi}{1-(1-\phi)\beta b_2}\right]\left[\pi_t+\mathrm{E}_t V_\pi(\pi_t, e_{t+1})\right] = -\left(\frac{\lambda\phi}{\kappa}\right)x_t. \end{aligned}$$

Again using (65) to eliminate x_t,

$$\begin{aligned} V_\pi(\pi_{t-1}, e_t) &= -\left(\frac{\lambda\phi}{\kappa}\right)\left[\frac{\Psi\pi_t-\phi\pi_{t-1}-e_t}{\kappa}\right] \\ &= -\left(\frac{\lambda\phi}{\kappa}\right)\left[\frac{(\Psi b_2-\phi)\,\pi_{t-1}+(\Psi b_1-1)\,e_t}{\kappa}\right]. \end{aligned} \tag{68}$$

However, (63) implies that

$$V_\pi(\pi_{t-1}, e_t) = a_3 e_t + a_4 + a_5\pi_{t-1}.$$

Comparing this with (68) reveals that $a_4 = 0$,

$$a_3=\lambda\phi\left(\frac{1-\Psi b_1}{\kappa^2}\right),$$

and

$$a_5=\lambda\phi\left(\frac{\phi-\Psi b_2}{\kappa^2}\right).$$

Finally, substitute these results into (67) to obtain

$$\pi_t=\left[\frac{\Psi}{\kappa^2+\beta\lambda\phi\,(\phi-\Psi b_2)+\lambda\Psi^2}\right]\left[\lambda\phi\pi_{t-1}+\lambda e_t\right].$$

Equating coefficients with (64),

$$b_1=\left[\frac{\lambda\Psi}{\kappa^2+\beta\lambda\phi\,(\phi-\Psi b_2)+\lambda\Psi^2}\right]$$

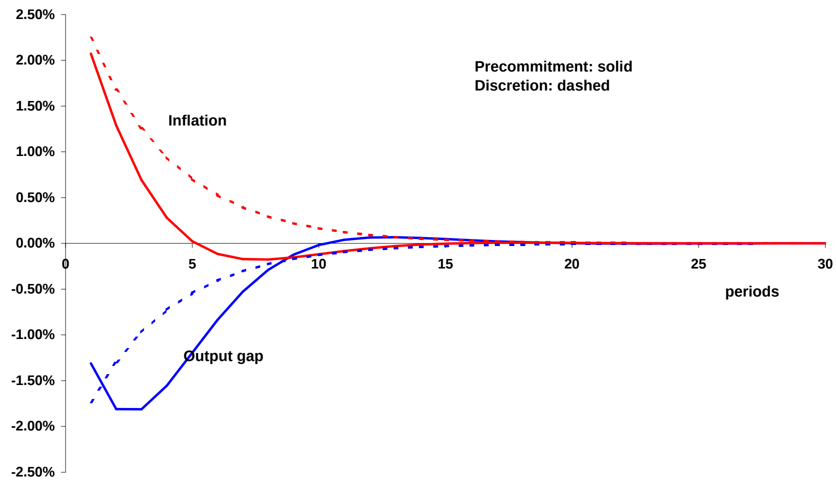

Figure 4: Responses to a Cost Shock with Endogenous Persistence ($\phi = 0.5$)

and

$$b_2 = \left[\frac{\lambda \Psi \phi}{\kappa^2 + \beta \lambda \phi \left(\phi - \Psi b_2 \right) + \lambda \Psi^2} \right]. \tag{69}$$

Because Ψ also depends on the unknown parameter b_2, (69) does not yield a convenient analytic solution. To gain insights into the effects of backward-looking aspects of inflation, it is useful to employ numerical techniques. This is done to generate figure 4, which shows the response of the output gap and inflation under optimal discretion when $\phi = 0.5$. Also shown for comparison are the responses under the optimal commitment policy. Both the output gap and inflation display more persistence than when $\phi = 0$ (see figures 2 and 3), and inflation returns to zero more slowly under discretion.

While figure 4 illustrates the responses of inflation and the output gap under commitment, it is insightful to consider explicitly the first order conditions for the optimal policy problem under commitment. Adopting the timeless perspective, maximizing (45) subject to (61) leads to the following first order conditions:

$$\pi_t = (1 - \phi)\beta \mathrm{E}_t \pi_{t+1} + \phi \pi_{t-1} + \kappa x_t + e_t$$

$$\pi_t + \psi_t - (1 - \phi)\psi_{t-1} - \beta \phi \mathrm{E}_t \psi_{t+1} = 0$$

$$\lambda x_t - \kappa \psi_t = 0,$$

where ψ_t is the Lagrangian multiplier associated with (61). Eliminating this multiplier, the optimal targeting rule becomes

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right)\left[x_t - (1-\phi)x_{t-1} - \beta\phi \mathrm{E}_t x_{t+1}\right]. \tag{70}$$

As we saw earlier, the presence of forward-looking expectations in the new Keynesian Phillips curve led optimal policy to be backward looking by introducing inertia through the appearance of x_{t-1} in the optimal targeting rule. The presence of lagged inflation in the inflation adjustment equation when $\phi > 0$ leads policy to be forward looking through the role of $\mathrm{E}_t x_{t+1}$ in the targeting rule. This illustrates a key aspect of policy design; when policy affects the economy with a lag, policy makers must be forward looking.

4.6 Targeting Regimes and Instrument Rules

The analysis of optimal policy contained in section 4.3 specified an objective function for the central bank. The central bank was then assumed to behave optimally, given its objective function and the constraints imposed on its choices by the structure of the economy. A policy regime in which the central bank is assigned an objective is commonly described as a *targeting regime*. A targeting regime is defined by 1) the variables in the central bank's loss function (the objectives) and 2) the weights assigned to these objectives, with policy implemented under discretion to minimize the expected discounted value of the loss function.[37] Targeting rules were also discussed in chapter 7, section 3.5, in the context of solving the inflation bias that can arise under discretion.

Perhaps the most widely discussed targeting regime is inflation targeting (Bernanke and Mishkin 1997, Svensson 1997a, 1997b, 1999b, 1999c, 1999d, Svensson and Woodford 1999). Experiences with inflation targeting are analyzed by Ammer and Freeman (1995), Bernanke, Laubach, Mishkin, and Posen (1998), Mishkin and Schmidt-Hebbel (2001), Amato and Gerlach (2002), and the papers in Leiderman and Svensson (1995). Mishkin and Schmidt-Hebbel identify 19 countries as inflation targeters as of 2001, with New Zealand, in 1990, being the first country to have adopted formal targets for inflation. Some of the lessons from the experiences with inflation targeting are discussed in Walsh (2009).

This section also briefly discusses instrument rules. These constitute an alternative approach to policy that assumes the central bank can commit to a simple feedback rule for its policy instrument. The most famous such rule is the Taylor rule (Taylor 1993).

[37] This definition of a targeting regime is consistent with that of Svensson (1999c), who states, "By a targeting rule, I mean, at the most general level, the assignment of a particular loss function to be minimized" (p. 617). An alternative interpretation of a targeting regime is that it is a rule for adjusting the policy instrument in the face of deviations between the current (or expected) value of the targeted variable and its target level (see, for example, McCallum 1990a and the references he cites). Jensen (2002) and Rudebusch (2002a) illustrate these two alternative interpretations of targeting.

4.6.1 Inflation Targeting

Inflation targeting has been characterized in a variety of ways in the academic literature, and it has been implemented in different ways in the countries that have adopted inflation targeting as a framework for monetary policy. In general, the announcement of a formal target for inflation is a key component, and this is often accompanied by publication of the central bank's inflation forecasts. An inflation targeting regime can be viewed as the assignment to the central bank of an objective function of the form

$$L_t^{IT} = \left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\left(\pi_{t+i} - \pi^T\right)^2 + \lambda_{IT} x_{t+i}^2 \right], \tag{71}$$

where π^T is the target inflation rate and λ_{IT} is the weight assigned to achieving the output gap objective relative to the inflation objective. λ_{IT} may differ from the weight placed on output gap stabilization in the social loss function (45). As long as $\lambda_{IT} > 0$, specifying inflation targeting in terms of the loss function (71) assumes that the central bank is concerned with output stabilization as well as inflation stabilization.[38] An inflation targeting regime in which $\lambda_{IT} > 0$ is described as a flexible inflation targeting regime.

In the policy problems we have been analyzing, the central bank's choice of its instrument i_t allows it to affect both output and inflation immediately. This absence of any lag between the time a policy action is taken and the time it affects output and inflation is unrealistic. If policy decisions taken in period t only affect future output and inflation, then the central bank must rely on forecasts of future output and inflation when making its policy choices. In analyzing the case of such policy lags, Svensson (1997a) and Svensson and Woodford (1999) emphasize the role of inflation-forecast targeting. To illustrate the role of forecasts in the policy process in a very simple manner, suppose the central bank must set i_t prior to observing any time-t information. This assumption implies that the central bank cannot respond to time-t shocks contemporaneously; information about shocks occurring in period t will affect the central bank's choice of i_{t+1} and, as a consequence, x_{t+1} and π_{t+1} can be affected. The model is otherwise given by (40) and (41) as before, with the additional assumption that the cost shock follows an $AR(1)$ process: $e_t = \rho e_{t-1} + \varepsilon_t$. Assume that the demand shock in (40) is serially uncorrelated. The central bank's objective is to choose i_t to minimize

$$\left(\frac{1}{2}\right) \mathrm{E}_{t-1} \sum_{i=0}^{\infty} \beta^i \left[\left(\pi_{t+i} - \pi^T\right)^2 + \lambda_{IT} x_{t+i}^2 \right],$$

where the subscript on the expectations operator is now $t-1$ to reflect the information available to the central bank when it sets policy. The choice of i_t is subject to the constraints represented by (40) and (41). Taking expectations based on the central bank's information, these two equations can be written as

[38] In the terminology of section 8.3.5, inflation targeting with the loss function (71) corresponds to a flexible targeting regime.

$$\mathrm{E}_{t-1}x_t = \mathrm{E}_{t-1}x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_{t-1}\pi_{t+1}) \tag{72}$$

and

$$\mathrm{E}_{t-1}\pi_t = \beta \mathrm{E}_{t-1}\pi_{t+1} + \kappa \mathrm{E}_{t-1}x_t + \rho e_{t-1}. \tag{73}$$

Under discretion, the first order condition for the central bank's choice of i_t implies that

$$\mathrm{E}_{t-1}\left[\kappa\left(\pi_t - \pi^T\right) + \lambda x_t\right] = 0. \tag{74}$$

Rearranging this first order condition yields

$$\mathrm{E}_{t-1}x_t = -\left(\frac{\kappa}{\lambda}\right)\mathrm{E}_{t-1}\left(\pi_t - \pi^T\right).$$

Thus, if the central bank forecasts that period-t inflation will exceed its target rate of inflation, it should adjust policy to ensure that the forecast of the output gap is negative.

Svensson and Woodford (1999) provide a detailed discussion of inflation-forecast targeting, and they focus on the implications for the determinacy of equilibrium under different specifications of the policy decision process. The possibility of multiple equilibria becomes particularly relevant if the central bank bases its own forecasts on private sector forecasts which are, in turn, based on expectations about the central bank's actions.

4.6.2 Other Targeting Regimes

Inflation targeting is just one example of a policy targeting regime. A number of alternative targeting regimes have been analyzed in the literature. These include price level targeting (Dittmar, Gavin, and Kydland 1999, Svensson 1999d, Vestin 2006), nominal income growth targeting (Jensen 2002), hybrid price level-inflation targeting (Batini and Yates 2001), average inflation targeting (Nessén and Vestin 2000), and regimes based on the change in the output gap or its quasi-difference (Jensen and McCallum 2002, Walsh 2002b). In each case, it is assumed that, given the assigned loss function, the central bank chooses policy under discretion. The optimal values for the parameters in the assigned loss function, for example, the value of λ_{IT} in (71), are chosen to minimize the unconditional expectation of the social loss function (45).

The importance of forward-looking expectations in affecting policy choice is well illustrated by work on price-level targeting. The traditional view argued that attempts to stabilize the price level, as opposed to the inflation rate, would generate undesirable levels of output variability. A positive cost shock that raised the price level would require a deflation to bring the price level back on target, and this deflation would be costly. However, as figure 3 shows, an optimal commitment policy that focuses on output and inflation stability also induces a deflation after a positive cost shock. By reducing $\mathrm{E}_t\pi_{t+1}$, such a policy achieves a better trade-off between inflation variability and output variability. The deflation generated under a discretionary policy concerned with output and price-level stability might actually come closer to the commitment policy outcomes than discretionary inflation targeting would. Using a basic new Keynesian model, Vestin (2006) shows that this intuition is correct. In fact, when inflation is given by (41) and the cost shock is serially uncorrelated, price level targeting can replicate the timeless precommitment solution exactly if the central bank is assigned the loss function $p_t^2 + \lambda_{PL} x_t^2$, where λ_{PL} differs from the weight λ in the social loss function.

Jensen (2002) shows that a nominal income growth targeting regime can also dominate inflation targeting. Walsh (2002b) adds lagged inflation to the inflation-adjustment equation and shows that the advantages of price-level targeting over inflation targeting decline as the weight on lagged inflation increases. Walsh analyzes discretionary outcomes when the central bank targets inflation and the change in the output gap (a *speed limit* policy). Introducing the change in the gap induces inertial behavior similar to that obtained under precommitment. For empirically relevant values of the weight on lagged inflation (ϕ in the range 0.3 to 0.7), speed limit policies dominate price-level targeting, inflation targeting, and nominal income growth targeting. For ϕ below 0.3, price-level targeting does best. Svensson and Woodford (1999) have considered interest-rate-smoothing objectives as a means of introducing into discretionary policy the inertia that is optimal under commitment.

4.6.3 Instrument Rules

The approach to policy analysis adopted in the preceding sections starts with a specification of the central bank's objective function and then derives the optimal setting for the policy instrument. An alternative approach specifies an instrument rule directly. The most famous of such instrument rules is the Taylor rule (Taylor 1993a). Taylor showed that the behavior of the federal funds interest rate in the United States from the mid-1980s through 1992 (when Taylor was writing) could be fairly well matched by a simple rule of the form

$$i_t = \pi_t + 0.5 x_t + 0.5 \left(\pi_t - \pi^T\right) + r^*,$$

where π^T was the target level of average inflation (Taylor assumed it to be 2%) and r^* was the equilibrium real rate of interest (Taylor assumed that this too was equal to 2%). The Taylor rule for general coefficients is often written

$$i_t = r^* + \pi^T + \alpha_x x_t + \alpha_\pi \left(\pi_t - \pi^T\right). \tag{75}$$

The nominal interest rate deviates from the level consistent with the economy's equilibrium real rate and the target inflation rate if the output gap is nonzero or if inflation deviates from target. A positive output gap leads to a rise in the nominal rate, as does a deviation of actual inflation above target. With Taylor's original coefficients, $\alpha_\pi = 1.5$, so that the nominal rate is changed more than one-for-one with deviations of inflation from target. Thus, the rule satisfies the Taylor principle (see section 3.3); a greater than one for one reaction of i_t ensures that the economy has a unique, stationary, rational expectations equilibrium. Lansing and Trehan (2001) explore conditions under which the Taylor rule emerges as the fully optimal instrument rule under discretionary policy.

A large literature has now developed that has estimated Taylor rules, or similar simple rules, for a variety of countries and time periods. For example, Clarida, Galí, and Gertler (2000) do so for the Federal Reserve, the Bundesbank, and the Bank of Japan. In their specification, however, actual inflation is replaced by expected future inflation so that the central bank is assumed to be forward-looking in setting policy. Estimates for the United States under different Federal Reserve chairmen are reported by Judd and Rudebusch (1997). In general, the basic Taylor rule, when supplemented by the addition of the lagged nominal interest rate, does quite well in matching the actual behavior of the policy interest rate. However, Orphanides (2000) finds that when estimated using the data on the output gap and inflation actually available at the time policy actions were taken (i.e., using real-time data), the Taylor rule does much more poorly in matching the U.S. funds rate. Clarida, Galí, and Gertler (2000) find that the Fed moved the funds rate less than one for one during the period 1960-1979, thereby violating the Taylor principle. In a further example of the importance of using real-time data, however, Perez (2001) finds that when the Fed's reaction function is reestimated for this earlier period using real-time data, the coefficient on inflation is greater than 1.

When a policy interest rate such as the federal funds rate in the United Sates is regressed on inflation and output gap variables, the lagged value of the interest rate normally enters with a statistically significant and large coefficient. The interpretation of this coefficient on the lagged interest rate has been the subject of debate. One interpretation is that it reflects inertial behavior of the sort we saw in section 6.4.3 that would arise under an optimal precommitment policy. It has also been interpreted to mean that central banks adjust gradually toward a desired interest-rate level. For example, suppose that i_t^* is the central bank's desired value for its policy instrument. Suppose, however, that it wants to avoid large changes in interest rates. Such an interest-smoothing objective might arise from a desire for financial market stability. If the central bank adjusts i_t gradually toward i_t^*, then the behavior of i_t may be captured by a partial adjustment model of the form

$$i_t = i_{t-1} + \theta\left(i_t^* - i_{t-1}\right) = (1-\theta)i_{t-1} + \theta i_t^*. \qquad (76)$$

The estimated coefficient on i_{t-1} provides an estimate of $1-\theta$. Values close to 1 imply that θ is small; each period the central bank closes only a small fraction of the gap between its policy rate and its desired value.

The view that central banks adjust slowly has been criticized. Sack (2000) and Rudebusch (2002b) argue that the presence of a lagged interest rate in estimated instrument rules is not evidence that the Fed acts gradually. Sack attributes the Fed's behavior to parameter uncertainty that leads the Fed to adjust the funds rate less aggressively than would be optimal in the absence of parameter uncertainty. Rudebusch argues that imperfect information about the degree of persistence in economic disturbances induces behavior by the Fed that appears to reflect gradual adjustment. He notes that if the Fed followed a rule such as (76), future changes in the funds rate would be predictable, but evidence from forward interest rates does not support the presence of predictable changes. Similarly, Lansing (2002) shows that the appearance of interest-rate smoothing can arise if the Fed uses real-time data to update its estimate of trend output each period. When final data are used to estimate a policy instrument rule, the serial correlation present in the Fed's real-time errors in measuring trend output will be correlated with lagged interest rates, creating the illusion of interest-rate-smoothing behavior by the Fed.

4.7 Model Uncertainty

Up to this point, the analysis has assumed that the central bank knows the true model of the economy with certainty. Fluctuations in output and inflation arose only from disturbances that took the form of additive errors. In this case, the linear-quadratic framework results in certainty equivalence holding; the central bank's actions depend on its expectations of future variables but not on the uncertainty associated with those expectations. When error terms enter multiplicatively, as occurs, for example, when the model's parameters are not known with certainty, equivalence will not hold. Brainard (1967) provided the classic analysis of multiplicative uncertainty. He showed that when there is uncertainty about the impact a policy instrument has on the economy, it will be optimal to respond more cautiously than would be the case in the absence of uncertainty.

Brainard's basic conclusion can be illustrated with a simple example. Suppose the inflation-adjustment equation given by (41) is modified to take the following form:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa_t x_t + e_t, \tag{77}$$

where $\kappa_t = \bar{\kappa} + v_t$ and v_t is a white noise stochastic process. In this formulation, the central bank is uncertain about the true impact of the gap x_t on inflation. For example, the central bank may have an estimate of the coefficient on x_t in the inflation equation, but there is some uncertainty associated with this estimate. The central bank's best guess of this coefficient is $\bar{\kappa}$, while its actual realization is κ_t. The central bank must choose its policy before observing the realization of v_t.

To analyze the impact uncertainty about the coefficient has on optimal policy, assume that the central bank's loss function is

$$L = \frac{1}{2} \mathrm{E}_t \left(\pi_t^2 + \lambda x_t^2 \right)$$

and assume that policy is conducted with discretion. In addition, assume that the cost shock e_t is serially uncorrelated.

Under discretion, the central bank takes $\mathrm{E}_t \pi_{t+1}$ as given, and the first order condition for the optimal choice of x_t is

$$\mathrm{E}_t \left(\pi_t \kappa_t + \lambda x_t \right) = 0.$$

Since all stochastic disturbances have been assumed to be serially uncorrelated, expected inflation will be zero, so we can use (77) to rewrite the first order condition as

$$\mathrm{E}_t\left[\left(\kappa_t x_t + e_t\right)\kappa_t + \lambda x_t\right] = \left(\bar{\kappa}^2 + \sigma_v^2\right)x_t + \bar{\kappa}e_t + \lambda x_t = 0.$$

Solving for x_t, one obtains

$$x_t = -\left(\frac{\bar{\kappa}}{\lambda + \bar{\kappa}^2 + \sigma_v^2}\right)e_t. \tag{78}$$

Equation (78) can be compared to the optimal discretionary response to the cost shock when there is no parameter uncertainty. In this case, $\sigma_v^2 = 0$ and

$$x_t = -\left(\frac{\bar{\kappa}}{\lambda + \bar{\kappa}^2}\right)e_t.$$

The presence of multiplicative parameter uncertainty ($\sigma_v^2 > 0$) reduces the impact of e_t on x_t. As uncertainty increases, it becomes optimal to respond less to e_t, that is, to behave more cautiously in setting policy.

Using (78) in the inflation-adjustment equation (77),

$$\pi_t = \kappa_t x_t + e_t = \left(\frac{\lambda + \sigma_v^2 - \bar{\kappa}\left(\kappa_t - \bar{\kappa}\right)}{\lambda + \bar{\kappa}^2 + \sigma_v^2}\right)e_t = \left(\frac{\lambda + \sigma_v^2 - \bar{\kappa}v_t}{\lambda + \bar{\kappa}^2 + \sigma_v^2}\right)e_t.$$

Since we have assumed the two disturbances v_t and e_t are uncorrelated, the unconditional variance of inflation is increasing in σ_v^2. In the presence of multiplicative uncertainty of the type modeled here, equilibrium output is stabilized more and inflation less in the face of cost shocks. The reason for this result is straightforward. With a quadratic loss function, the additional inflation variability induced by the variance in κ_t is proportional to x_t. Reducing the variability of x_t helps to offset the impact of v_t on the variance of inflation. It is optimal to respond more cautiously, thereby reducing the variance of x_t but at the cost of greater inflation variability.

Brainard's basic result–multiplicative uncertainty leads to caution–is intuitively appealing, but it is not a general result. For example, Söderström (2002) examines a model in which there are lagged variables whose coefficients are subject to random shocks. He shows that in this case, optimal policy reacts more aggressively. For example, suppose current inflation depends on lagged inflation, but the impact of π_{t-1} on π_t is uncertain. The effect this coefficient uncertainty has on the variance of π_t depends on the variability of π_{t-1}. If the central bank fails to stabilize current inflation, it increases the variance of inflation in the following period. It can be optimal to respond more aggressively to stabilize inflation, thereby reducing the impact the coefficient uncertainty has on the unconditional variance of inflation.

A recent literature has combined the notion of parameter uncertainty with models of learning to examine the implications for monetary policy (Sargent 1999). Wieland (2000a, 2000b) examines the trade-off between control and estimation that can arise under model uncertainty. A central bank may find it optimal to experiment, changing policy to generate observations that can help it learn about the true structure of the economy.

Another aspect of model uncertainty is measurement error or the inability to observe some relevant variables. For example, the flexible-price equilibrium level of output is needed to measure the gap variable x_t, but it is not directly observable. Svensson and Woodford (2003, 2004) provide a general treatment of optimal policy when the central bank's problem involves both an estimation problem (determining the true state of the economy such as the value of the output gap) and a control policy (setting the nominal interest rate to affect the output gap and inflation). In a linear-quadratic framework in which private agents and the central bank have the same information, these two problems can be dealt with separately. Orphanides (2000) has emphasized the role the productivity slowdown played during the 1970s in causing the Fed to overestimate potential output.[39] Svensson and Williams (2008) develop a general approach for dealing with a variety of sources of model and data uncertainty.

Finally, the approach adopted in section 4.1 derived welfare-based policy objectives from an approximation to the welfare of the representative agent. The nature of this approximation, however, will depend on the underlying model structure. For example, Steinsson (2000) shows that in the Galí and Gertler (1999) hybrid inflation model, in which lagged inflation appears in the inflation-adjustment equation, the loss function also includes a term in the squared change in inflation. If price adjustment is characterized by partial indexation to lagged inflation so that the inflation adjustment equation involves $\pi_t - \gamma\pi_{t-1}$ and $\mathrm{E}_t\left(\pi_{t+1} - \gamma\pi_t\right)$ as discussed in chapter 6, section 3.2, Woodford (2003) finds that the period loss function includes $\left(\pi_t - \gamma\pi_{t-1}\right)^2$ rather than π_t^2. Thus, uncertainty about the underlying model will also translate into uncertainty about the appropriate objectives of monetary policy, as policy objectives cannot be defined independently of the model that defines the costs of economic fluctuations (see Walsh 2005).

5 Summary

This chapter has reviewed the basic new Keynesian model that has increasing come to dominate modern macroeconomics, particularly for addressing monetary policy issues. The basic model is a dynamic, stochastic general equilibrium model based on optimizing households, with firms operating in an environment of monopolistic competition and facing limited ability to adjust their prices. The staggered overlapping process of price adjustment apparent in the micro evidence discussed in chapter 6 is captured through the use of the Calvo-mechanism. While the details would differ slightly if alternative models of price stickiness were employed, the basic model structure would not change. This structure consists of two basic parts. The first is an expectational IS curve

[39] See also Levin, Wieland, and Williams (1999), Ehrmann and Smets (2001), and Orphanides and Williams (2002).

derived from the Euler condition describing the first order condition implied by intertemporal optimization on the part of the representative household. The second is a Phillips curve relationship linking inflation to an output gap measure.

The model provides insights into the costs of inflation in generating an inefficient dispersion of relative prices. A model consistent objective function for policy, derived as a second-order approximation to the welfare of the representative agent, calls for stabilizing inflation volatility and volatility in the gap between output and the output level that would arise under flexible prices.

The new Keynesian approach emphasizes the role of forward-looking expectations. The presence of forward-looking expectations in new Keynesian Phillips curve implies that expectations about future policy actions play an important role, and a central bank that can influence these expectations, as assumed under a policy regime of commitment, can do better than one that sets policy in a discretionary manner.

6 Appendix

This appendix provides details on the derivation of the linear new Keynesian Phillips curve and on the approximation to the welfare of the representative household.

6.1 The New Keynesian Phillips Curve

In this section, (**??**) and (16) are used to obtain an expression for the deviations of the inflation rate around its steady-state level. We will assume that the steady state involves a zero rate of inflation. Let $Q_t = p_t^*/P_t$ be the relative price chosen by all firms that adjust their price in period t. The steady-state value of Q_t is $Q = 1$; this is also the value Q_t equals when all firms are able to adjust every period. Dividing (16) by P_t, one obtains $1 = (1-\omega)Q_t^{1-\theta} + \omega\left(P_{t-1}/P_t\right)^{1-\theta}$. Expressed in terms of percentage deviations around the zero- inflation steady state, this becomes

$$0 = (1-\omega)\hat{q}_t - \omega\pi_t \Rightarrow \hat{q}_t = \left(\frac{\omega}{1-\omega}\right)\pi_t. \tag{79}$$

To obtain an approximation to (**??**), note that it can be written as

$$\left[\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}\right] Q_t = \mu \left[\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}\right]. \tag{80}$$

In the flexible-price equilibrium with zero inflation, $Q_t = \mu\varphi_t = 1$. The left side of (80) is approximated by

$$\left(\frac{C^{1-\sigma}}{1-\omega\beta}\right) + \left(\frac{C^{1-\sigma}}{1-\omega\beta}\right)\hat{q}_t + C^{1-\sigma}\sum_{i=0}^{\infty}\omega^i\beta^i\left[(1-\sigma)\,\mathrm{E}_t\hat{c}_{t+i} + (\theta-1)\left(\mathrm{E}_t\hat{p}_{t+i} - \hat{p}_t\right)\right].$$

The right side is approximated by

$$\mu\left\{\left(\frac{C^{1-\sigma}}{1-\omega\beta}\right)\varphi+\varphi C^{1-\sigma}\sum_{i=0}^{\infty}\omega^i\beta^i\left[\mathrm{E}_t\hat{\varphi}_{t+i}+(1-\sigma)\,\mathrm{E}_t\hat{c}_{t+i}+\theta\left(\mathrm{E}_t\hat{p}_{t+i}-\hat{p}_t\right)\right]\right\}.$$

Setting these two expressions equal and noting that $\mu\varphi=1$ yields

$$\begin{aligned}&\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t+\sum_{i=0}^{\infty}\omega^i\beta^i\left[(1-\sigma)\,\mathrm{E}_t\hat{c}_{t+i}+(\theta-1)\left(\mathrm{E}_t\hat{p}_{t+i}-\hat{p}_t\right)\right]\\ =\ &\sum_{i=0}^{\infty}\omega^i\beta^i\left[\mathrm{E}_t\hat{\varphi}_{t+i}+(1-\sigma)\,\mathrm{E}_t\hat{c}_{t+i}+\theta\left(\mathrm{E}_t\hat{p}_{t+i}-\hat{p}_t\right)\right].\end{aligned}$$

Canceling the terms that appear on both sides of this equation leaves

$$\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t=\sum_{i=0}^{\infty}\omega^i\beta^i\left(\mathrm{E}_t\hat{\varphi}_{t+i}+\mathrm{E}_t\hat{p}_{t+i}-\hat{p}_t\right),$$

or

$$\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t=\sum_{i=0}^{\infty}\omega^i\beta^i\left(\mathrm{E}_t\hat{\varphi}_{t+i}+\mathrm{E}_t\hat{p}_{t+i}\right)-\left(\frac{1}{1-\omega\beta}\right)\hat{p}_t.$$

Multiplying by $1-\omega\beta$ and adding $\hat{p}_t$ to both sides yields

$$\hat{q}_t+\hat{p}_t=(1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i\left(\mathrm{E}_t\hat{\varphi}_{t+i}+\mathrm{E}_t\hat{p}_{t+i}\right).$$

The left side is the optimal nominal price $\hat{p}_t^*=\hat{q}_t+\hat{p}_t$, and this is set equal to the expected discounted value of future nominal marginal costs. This equation can be rewritten as $\hat{q}_t+\hat{p}_t=(1-\omega\beta)\left(\hat{\varphi}_t+\hat{p}_t\right)+\omega\beta\left(\mathrm{E}_t\hat{q}_{t+1}+\mathrm{E}_t\hat{p}_{t+1}\right)$. Rearranging this expression yields

$$\begin{aligned}\hat{q}_t&=(1-\omega\beta)\hat{\varphi}_t+\omega\beta\left(\mathrm{E}_t\hat{q}_{t+1}+\mathrm{E}_t\hat{p}_{t+1}-\hat{p}_t\right)\\&=(1-\omega\beta)\hat{\varphi}_t+\omega\beta\left(\mathrm{E}_t\hat{q}_{t+1}+\mathrm{E}_t\pi_{t+1}\right).\end{aligned}$$

Now using (79) to eliminate $\hat{q}_t$, one obtains

$$\begin{aligned}\left(\frac{\omega}{1-\omega}\right)\pi_t&=(1-\omega\beta)\hat{\varphi}_t+\omega\beta\left[\left(\frac{\omega}{1-\omega}\right)\mathrm{E}_t\pi_{t+1}+\mathrm{E}_t\pi_{t+1}\right]\\&=(1-\omega\beta)\hat{\varphi}_t+\omega\beta\left(\frac{1}{1-\omega}\right)\mathrm{E}_t\pi_{t+1}.\end{aligned}$$

Multiplying both sides by $(1-\omega)/\omega$ produces the forward-looking new Keynesian Phillips curve:

$$\pi_t=\tilde{\kappa}\hat{\varphi}_t+\beta\mathrm{E}_t\pi_{t+1},$$

where

$$\tilde{\kappa}=\frac{(1-\omega)(1-\omega\beta)}{\omega}.$$

When production is subject to diminishing returns to scale, firm specific marginal cost may differ from average marginal cost. Let $A = \theta(1-a)/a$. All firms adjusting at time t set their relative price such that

$$
\begin{aligned}
\hat{q}_t + \hat{p}_t &= (1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i\left(\mathrm{E}_t\hat{\varphi}_{jt+i} + \mathrm{E}_t\hat{p}_{t+i}\right) \\
&= (1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i\left[\mathrm{E}_t\hat{\varphi}_{t+i} - A\left(\hat{q}_t + \hat{p}_t - \mathrm{E}_t\hat{p}_{t+i}\right) + \mathrm{E}_t\hat{p}_{t+i}\right].
\end{aligned}
$$

This equation can be rewritten as

$$
\begin{aligned}
\hat{q}_t + \hat{p}_t &= (1-\omega\beta)\left(\hat{\varphi}_t - A\hat{q}_t + \hat{p}_t\right) \\
&\quad \omega\beta(1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i\left[\mathrm{E}_t\hat{\varphi}_{t+1+i} - A\left(\hat{q}_t + \hat{p}_t - \mathrm{E}_t\hat{p}_{t+1+i}\right) + \mathrm{E}_t\hat{p}_{t+1+i}\right].
\end{aligned}
$$

By rearranging this equation, and recalling that $\hat{q}_t = \omega\pi_t/(1-\omega)$, one obtains

$$
\begin{aligned}
\left(\frac{\omega}{1-\omega}\right)(1+A)\,\pi_t &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta(1+A)\left[\left(\frac{\omega}{1-\omega}\right)\mathrm{E}_t\pi_{t+1} + \mathrm{E}_t\pi_{t+1}\right] \\
&= (1-\omega\beta)\hat{\varphi}_t + \omega\beta(1+A)\left(\frac{1}{1-\omega}\right)\mathrm{E}_t\pi_{t+1}
\end{aligned}
$$

Multiplying both sides by $(1-\omega)/\omega(1+A)$ produces

$$
\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \left(\frac{\tilde{\kappa}}{1+A}\right)\hat{\varphi}_t
$$

6.2 Approximating Utility

In this section, details on the derivation of (44) are provided. The analysis is based on Woodford (2003). To derive an approximation to the representative agent's utility, it is necessary to first introduce some additional notation. For any variable X_t, let $\bar{X}$ be its steady-state value, let X_t^* be its efficient level (if relevant), and let $\tilde{X}_t = X_t - \bar{X}$ be the deviation of X_t around the steady state. Let $\hat{X}_t = \log\left(X_t/\bar{X}\right)$ be the log deviation of X_t around its steady-state value. Using a second order Taylor approximation, the variables $\tilde{X}_t$ and $\hat{X}_t$ can be related as

$$
\tilde{X}_t = X_t - \bar{X} = \bar{X}\left(\frac{X_t}{\bar{X}} - 1\right) \approx \bar{X}\left(\hat{X}_t + \frac{1}{2}\hat{X}_t^2\right). \tag{81}
$$

Employing this notation, we can develop a second order approximation to the utility of the representative household.

The first term on the right of (43) is the utility from consumption. This can be approximated around the steady state as

$$
U(Y_t, z_t) \approx U(\bar{Y}, 0) + U_c\tilde{Y}_t + U_z z_t + \frac{1}{2}U_{cc}\tilde{Y}_t^2 + U_{c,z}z_t\tilde{Y}_t + \frac{1}{2}z_t'U_{z,z}z_t. \tag{82}
$$

Using (81), and ignoring terms involving of order three or higher, such as $\hat{Y}_t^i$ for $i > 2$ and $z_t \hat{Y}_t^2$, (82) becomes

$$\begin{aligned} U(Y_t, z_t) &\approx U(\bar{Y}, 0) + U_c \bar{Y} \left(\hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right) + U_z z_t + \frac{1}{2} U_{cc} \bar{Y}^2 \hat{Y}_t^2 \\ & + U_{c,z} z_t \bar{Y} \hat{Y}_t + \frac{1}{2} z_t' U_{z,z} z_t \\ &= \bar{Y} U_c \left\{ \hat{Y}_t + \frac{1}{2} \left[1 + \frac{U_{cc}}{U_c} \bar{Y} \right] \hat{Y}_t^2 + \frac{U_{c,z}}{U_c} z_t \hat{Y}_t \right\} + t.i.p., \end{aligned}$$

where *t.i.p.* are terms independent of policy. The choice of terms to include in *t.i.p.* is based on the implication of the new Keynesian model that the steady state is independent of monetary policy. To simplify the approximation, define

$$\sigma = -\frac{\bar{Y} U_{cc}}{U_c}$$

as the coefficient of relative risk aversion, and let

$$\phi_t = -\frac{U_{c,z}}{\bar{Y} U_{cc}} z_t.$$

Then the approximation for $U(Y, z)$ becomes

$$U(Y_t, z_t) \approx \bar{Y} U_c \left\{ \hat{Y}_t + \frac{1}{2} (1 - \sigma) \hat{Y}_t^2 + \sigma \phi_t \hat{Y}_t \right\} + t.i.p.$$

We next need to analyze the second term on the right in (43), the term arising from the disutility of work. Expanding this around the steady state yields

$$v(y_t(i), z_t) \approx v(\bar{y}, 0) + v_y \tilde{y}_t(i) + v_z z_t + \frac{1}{2} v_{yy} \tilde{y}_t(i)^2 + v_{y,z} z_t \tilde{y}_t(i) + \frac{1}{2} z_t' v_{z,z} z_t.$$

By approximating $\tilde{c}_{jt}$ with $\bar{y} \left(\hat{c}_{jt} + \frac{1}{2} \hat{c}_{jt}^2 \right)$, one obtains

$$v(c_{jt}, z_t) \approx v_y \bar{y} \left\{ \hat{c}_{jt} + \frac{1}{2} \hat{c}_{jt}^2 + \frac{1}{2} \frac{v_{yy} \bar{y}}{v_y} \hat{c}_{jt}^2 + \frac{v_{y,z}}{v_y} z_t \hat{c}_{jt} \right\} + t.i.p.$$

This last equation can be written as

$$\begin{aligned} v(y_t(i), z_t) &\approx v_y \bar{y} \left\{ \hat{c}_{jt} + \frac{1}{2} \left(1 + \frac{v_{yy} \bar{y}}{v_y} \right) \hat{c}_{jt}^2 + \frac{v_{y,z}}{v_y} z_t \hat{c}_{jt} \right\} + t.i.p. \\ &= v_y \bar{y} \left\{ \hat{c}_{jt} + \frac{1}{2} (1 + \eta) \hat{c}_{jt}^2 - \eta q_t \hat{c}_{jt} \right\} + t.i.p. \end{aligned}$$

where

$$\eta = \frac{v_{yy} \bar{y}}{v_y}$$

and

$$q_t = -\frac{v_{y,z} z_t}{v_{yy} \bar{y}}.$$

To proceed further, we need to recall the model of monopolistic competition underlying the new Keynesian framework. In a model of perfect competition, the household producer of good i would equate the marginal rate of substitution between leisure and consumption to the real wage, or $v_y/U_c = 1$, since the implicit production function is $y_t(i) = n_t(i)$. In the presence of monopolistic competition, $v_y/U_c = (\theta - 1)/\theta$ is 1 over the markup. Define $\Phi = 1/\theta$. Then $v_y/U_c = 1 - \Phi$. If the distortion created by monopolistic competition is small, terms such as $\Phi \hat{y}_t^2$ and $\Phi q_t \hat{y}_t$ will be of third order, and we obtain

$$v(\hat{c}_{jt}, z_t) \approx U_c \bar{Y} \left[(1 - \Phi) \hat{c}_{jt} + \frac{1}{2} (1 + \eta) \hat{c}_{jt}^2 - \eta q_t \hat{c}_{jt} \right] + t.i.p.$$

Integrating over all goods, and using the relationship $\hat{Y} \approx \mathrm{E}_j \hat{c}_{jt} + \frac{1}{2}(1 - \theta^{-1}) var_j \hat{c}_{jt}$,

$$\begin{aligned} \int_0^1 v(\hat{c}_{jt}, z_t) dj &\approx U_c \bar{Y} \left\{ (1 - \Phi) \mathrm{E}_j \hat{c}_{jt} + \frac{1}{2} (1 + \eta) \left[(\mathrm{E}_j \hat{c}_{jt})^2 + var_j \hat{c}_{jt} \right] - \eta q_t \mathrm{E}_j \hat{c}_{jt} \right\} + t.i.p. \\ &= U_c \bar{Y} \left\{ (1 - \Phi - \eta q_t) \hat{Y}_t + \frac{1}{2} (1 + \eta) \hat{Y}_t^2 + \frac{1}{2} \left(\theta^{-1} + \eta \right) var_j \hat{c}_{jt} \right\} + t.i.p. \end{aligned}$$

where terms such as $var_j \hat{c}_{jt}^4$ and $\hat{c}_{jt} var_j \hat{c}_{jt}$ are set equal to zero.

Bringing together the results for the utility of consumption and the disutility of work,

$$\begin{aligned} V &\approx U_c \bar{Y} \left\{ \hat{Y}_t + \frac{1}{2} (1 - \sigma) \hat{Y}_t^2 + \sigma \phi_t \hat{Y}_t \right\} \\ &\quad - U_c \bar{Y} \left\{ (1 - \Phi - \eta q_t) \hat{Y}_t + \frac{1}{2} (1 + \eta) \hat{Y}_t^2 + \frac{1}{2} \left(\theta^{-1} + \eta \right) var_j \hat{c}_{jt} \right\} + t.i.p. \\ &= U_c \bar{Y} \left\{ [\Phi + \sigma \phi_t + \eta q_t] \hat{Y}_t - \frac{1}{2} (\sigma + \eta) \hat{Y}_t^2 - \frac{1}{2} \left(\theta^{-1} + \eta \right) var_j \hat{c}_{jt} \right\} + t.i.p. \end{aligned}$$

To gain insight into this expression for utility, it will be useful to derive the equilibrium output level under flexible prices. In a flexible-price equilibrium, the marginal product of labor equals the markup arising from monopolistic competition times the marginal rate of substitution between leisure and consumption. Given the specification of the composite consumption good in (42), the markup equals $\theta/(\theta - 1)$. Thus, in the flexible-price equilibrium,

$$\left(\frac{\theta}{\theta - 1} \right) \frac{v_y}{U_c} = 1.$$

Multiply both sides of this expression by U_c and then log-linearizing the result reveals that the flexible-price output level $\hat{Y}_t^f$ satisfies

$$\left(\frac{\theta}{\theta - 1} \right) \left[v_y(\bar{y}, 0) + v_{yy} \bar{Y} \hat{Y}_t^f + v_{y,z} \hat{z}_t \right] = U_c + U_{cc} \bar{Y} \hat{Y}_t^f + U_{c,z} \hat{z}_t.$$

Dividing both sides by $U_c = \theta v_y(\bar{y}, 0)/(\theta - 1)$,

$$\frac{v_{yy}\bar{Y}\hat{Y}_t^f + v_{y,z}\hat{z}_t}{v_y(\bar{y}, 0)} = \frac{U_{cc}\bar{Y}\hat{Y}_t^f + U_{c,z}\hat{z}_t}{U_c}$$

or

$$\eta\hat{Y}_t^f - \eta q_t = -\sigma\hat{Y}_t^f + \sigma\phi_t.$$

Solving for $\hat{Y}_t^f$,

$$\hat{Y}_t^f = \left(\frac{\sigma\phi_t + \eta q_t}{\sigma + \eta}\right).$$

The utility approximation can now be written as

$$\begin{aligned} V &\approx -\left(\frac{1}{2}\right)(\sigma + \eta) U_c\bar{Y}\left\{\hat{Y}_t^2 - 2\left[\frac{\Phi + \sigma\phi_t + \eta q_t}{\sigma + \eta}\right]\hat{Y}_t + \left(\frac{\theta^{-1} + \eta}{\sigma + \eta}\right) var_i\hat{y}_t(i)\right\} + t.i.p. \\ &= -\left(\frac{1}{2}\right)(\sigma + \eta) U_c\bar{Y}\left\{(x_t - x^*)^2 + \left(\frac{\theta^{-1} + \eta}{\sigma + \eta}\right) var_j\hat{c}_{jt}\right\} + t.i.p., \end{aligned}$$

where

$$x_t \equiv \hat{Y}_t - \hat{Y}_t^f$$

is the gap between output and the flexible-price equilibrium output, and

$$x^* \equiv \frac{\Phi}{\sigma + \eta}.$$

Letting $\bar{Y}^*$ be the steady-state, efficient level of output, x^* is equal to $\log(\bar{Y}^*/\bar{Y})$ and is a measure of the distortion created by the presence of monopolistic competition.

The next step in obtaining an approximation to the utility of the representative agent involves expressing the variance of $\hat{c}_{jt}$ in terms of the dispersion of prices across individual firms.

With the assumed utility function, the demand for good i satisfies $\hat{c}_{jt} = [p_{jt}/P_t]^{-\theta} Y_t$. Taking logs,

$$\log\hat{c}_{jt} = \log Y_t - \theta(\log p_{jt} - \log P_t),$$

so

$$var_i \log\hat{c}_{jt} = \theta^2 var_i \log p_{jt}.$$

Hence, we can evaluate alternative policies using as our welfare criterion

$$-\frac{1}{2}\bar{Y}U_c\left[(\sigma + \eta)(x_t - x^*)^2 + \left(\theta^{-1} + \eta\right)\theta^2 var_j \log p_{jt}\right]. \quad (83)$$

The last step in the approximation process is to relate $var_j \log p_{jt}$ to the average inflation rate across all firms. To do so, recall that the price-adjustment mechanism involves a randomly chosen fraction $1 - \omega$ of all firms optimally adjusting price each period. Define $\bar{P}_t \equiv \mathrm{E}_j \log p_{jt}$ and $\Delta_t \equiv var_j \log p_{jt}$. Then, since $var_j \bar{P}_{t-1} = 0$, we can write

$$\begin{aligned}
\Delta_t &= var_j \left[\log p_{jt} - \bar{P}_{t-1}\right] \\
&= \mathrm{E}_j \left[\log p_{jt} - \bar{P}_{t-1}\right]^2 - \left[\mathrm{E}_j \log p_{jt} - \bar{P}_{t-1}\right]^2 \\
&= \omega \mathrm{E}_j \left[\log p_{jt-1} - \bar{P}_{t-1}\right]^2 + (1-\omega)\left(\log p_t^* - \bar{P}_{t-1}\right)^2 \\
&\quad - \left(\bar{P}_t - \bar{P}_{t-1}\right)^2 .
\end{aligned}$$

where p_t^* is the price set at time t by the fraction $1-\omega$ of firms that reset their price. Given that $\bar{P}_t = (1-\omega)\log p_t^* + \omega \bar{P}_{t-1}$,

$$\log p_t^* - \bar{P}_{t-1} = \left(\frac{1}{1-\omega}\right)\left(\bar{P}_t - \bar{P}_{t-1}\right).$$

Using this result,

$$\begin{aligned}
\Delta_t &= \omega \Delta_{t-1} + \left(\frac{\omega}{1-\omega}\right)\left(\bar{P}_t - \bar{P}_{t-1}\right)^2 \\
&\approx \omega \Delta_{t-1} + \left(\frac{\omega}{1-\omega}\right)\pi_t^2.
\end{aligned}$$

This implies

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \Delta_{t+i} = \left[\frac{\omega}{(1-\omega)(1-\omega\beta)}\right] \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2 + t.i.p.,$$

where the terms independent of policy also include the initial degree of price dispersion.

Combining this with (83), the present discounted value of the utility of the representative household can be approximated by

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda\left(x_{t+i} - x^*\right)^2\right],$$

where

$$\Omega = \frac{1}{2}\bar{Y} U_c \left[\frac{\omega}{(1-\omega)(1-\omega\beta)}\right]\left(\theta^{-1} + \eta\right)\theta^2$$

and

$$\lambda = \left[\frac{(1-\omega)(1-\omega\beta)}{\omega}\right]\frac{(\sigma+\eta)}{(1+\eta\theta)\,\theta}.$$

7 Problems

1. Consider a simple forward-looking model of the form

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + u_t,$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.$$

Suppose policy reacts to the output gap:

$$i_t = \delta x_t.$$

Write this system in the form given by (26). Are there values of δ that ensure a unique stationary equilibrium? Are there values that do not?

2. Consider the model given by

$$x_t = E_t x_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1})$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

Suppose policy sets the nominal interest rate according to a policy rule of the form

$$i_t = \phi_1 E_t \pi_{t+1}$$

for the nominal rate of interest.

(a) Write this system in the form $E_t z_{t+1} = M z_t + \eta_t$, where $z_t = [x_t, \pi_t]'$.

(b) For $\beta = 0.99$, $\kappa = 0.05$, and $\sigma = 1.5$, plot the absolute values of the two eigenvalues of M as a function of $\phi_1 > 0$.

(c) Are there values of ϕ_1 for which the economy does not have a unique stationary equilibrium?

3. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\xi_t N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is $Y_t = Z_t N_t$. The notation is: C is consumption, ξ is a stochastic shock to "tastes," N is time spent working, Y output, and Z an aggregate productivity disturbance; σ and η are constants. The stochastic variable ξ has a mean of 1.

(a) Derive the household's first order condition for labor supply Show how labor supply depends on the taste shock and explain how a positive realization of ξ would affect labor supply.

(b) Derive an expression for the flexible-price equilibrium output $\hat{y}_t^f$ for this economy.

(c) Does the taste shock affect the flexible-price equilibrium? If it does, explain how and why.

(d) The household's Euler condition for optimal consumption choice (expressed in terms of the output gap and in percent deviations around the steady-state) can be written as

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - E_t\pi_{t+1} - r_t^n\right).$$

How does r^n depend on the behavior of the flexible price equilibrium output? Does it depend on the taste shock ξ? Explain intuitively whether a positive realisation of ξ raises, lowers, or leaves unchanged the flex-price equilibrium real interest rate.

4. Suppose the economy is characterized by) is

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - E_t\pi_{t+1} - r_t^n\right)$$

and.

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

What problems might arise if the central bank decides to set its interest rate instrument according to the rule $i_t = r_t^n$?

5. Suppose the economy is described by the basic new Keynesian model consisting of

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(i_t - \mathrm{E}_t\pi_{t+1}\right) \\ \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t \\ i_t &= \phi_\pi \pi_t + \phi_x x_t. \end{aligned}$$

(a) If $\phi_x = 0$, explain intuitively why $\phi_\pi > 1$ is needed to ensure that the equilibrium will be unique.

(b) If both ϕ_π and ϕ_x are nonnegative, the condition given by (29) implies that the economy can still have unique, stable equilibrium even when

$$1 - \frac{(1-\beta)\phi_x}{\kappa} < \phi_\pi < 1.$$

Explain intuitively why some values of $\phi_\pi < 1$ are still consistent with uniqueness when $\phi_x > 0$.

6. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(1+\xi_t)N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is $Y_t = Z_t N_t$. The notation is: C is consumption, ξ is a stochastic shock to "tastes," N is time spent working, Y is output, and $Z_t = (1 + z_t)$ is a stochastic aggregate productivity disturbance; σ and η are constants. Both ξ and z have zero means. Assume a standard model of monopolistic competition with Calvo pricing.

(a) Assuming a zero steady-state rate of inflation, the inflation adjustment equation can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \mu_t,$$

where μ_t is real marginal cost (expressed as a percent deviation around the steady-state). Derive an expression for μ_t in terms of an output gap.

(b) Does the taste shock affect the output gap? Does it affect inflation? Explain.

7. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma}\left(\frac{M_t}{P_t}\right)^{1-b}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is $Y_t = Z_t N_t^a$.

(a) Show that the household's first order condition for labor supply takes the form

$$\eta \hat{n}_t + \sigma \hat{c}_t - \mu_t^w = \hat{w}_t - \hat{p}_t,$$

where $\mu_t^w = (1-b)(\hat{m}_t - \hat{p}_t)$.

(b) Derive an expression for the flexible-price equilibrium output $\hat{y}_t^f$ and the output gap $x_t = \hat{y}_t - \hat{y}_t^f$.

(c) Does money affect the flexible-price equilibrium? Does the nominal interest rate? Explain.

8. Suppose the economy is characterized by (40) and (41), and let the cost shock be given by $e_t = \rho e_{t-1} + \varepsilon_t$. The central bank's loss function is (45). Assume that the central bank can commit to a policy rule of the form $\pi_t = \gamma e_t$.

(a) What is the optimal value of γ?

(b) Find the expression for equilibrium output gap under this policy.

9. In section 4.4, the case of commitment to a rule of the form $x_t = b_x e_t$ was analyzed. Does a unique, stationary, rational expectations equilibrium exist under such a commitment? Suppose instead that the central bank commits to the rule $i_t = b_i e_t$ for some constant b_i. Does a unique, stationary, rational expectations equilibrium exist under such a commitment? Explain why the two cases differ.

10. Suppose the economy's inflation rate is described by the following equation (all variables expressed as percentage deviations around a zero-inflation steady state:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{84}$$

where x_t is the gap between output and the flexible price equilibrium output level, and e_t is a cost shock. Assume that

$$e_t = \rho_e e_{t-1} + \varepsilon_t,$$

where ψ and ε are white noise processes. The central bank sets the nominal interest rate i_t to minimize

$$\frac{1}{2}\mathrm{E}_t \left[\sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2 \right) \right].$$

(a) Derive the first order conditions linking inflation and the output gap for the *fully* optimal commitment policy.

(b) Explain why the first order conditions for time t differs from the first order conditions for $t+i$ for $i > 0$.

(c) What is meant by a commitment policy that is optimal from a timeless perspective? (Explain in words.)

(d) What is the first order condition linking inflation and the output gap that the central bank follows under an optimal commitment policy from a timeless perspective?

(e) Explain why, under commitment, the central bank promises a deflation in the period after a positive cost shock (assume the cost shock is serially uncorrelated).

11. Explain why inflation is costly in a new Keynesian model.

12. Suppose the economy is described by the following log-linearized system:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma} \right) (i_t - \mathrm{E}_t \pi_{t+1}) + E_t (z_{t+1} - z_t) + u_t$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t,$$

where u_t is a demand shock, z_t is a productivity shock, and e_t is a cost shock. Assume that

$$u_t = \rho_u u_{t-1} + \xi_t$$
$$z_t = \rho_z z_{t-1} + \psi_t$$
$$e_t = \rho_e e_{t-1} + \varepsilon_t,$$

where ξ, ψ, and ε are white noise processes. The central bank sets the nominal interest rate i_t to minimize

$$\left(\frac{1}{2} \right) \mathrm{E}_t \left[\sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2 \right) \right].$$

(a) Derive the optimal time-consistent policy for the discretionary central banker. Write down the first order conditions and the reduced-form solutions for x_t and π_t.

(b) Derive the interest-rate feedback rule implied by the optimal discretionary policy.

(c) Show that under the optimal policy, nominal interest rates are increased enough to raise the real interest rate in response to a rise in expected inflation.

(d) How will x_t and π_t move in response to a demand shock? To a productivity shock?

13. Suppose the central bank cares about inflation variability, output gap, variablility and interest rate variability. The objective of the central bank is to minimize

$$\left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_i \left(i_{t+i} - i^*\right)^2\right].$$

The structure of the economy is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \tag{85}$$

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - E_t \pi_{t+1} - r_t\right), \tag{86}$$

where e and r are exogenopus stochastic shocks. Let ψ_t denote the Lagrangian multiplier on constraint (85) and let θ_t be the multipler on (86).

(a) Derive the first order conditions for the optimal policy of the central bank under discretion.

(b) Show that θ is non-zero if $\lambda_i > 0$. Explain the economics behind this result.

(c) Derive the first order conditions for the fully optimal optimal commitment policy. How do these differ from the conditions you found in (a)?

(d) Derive the first order conditions for the optimal optimal commitment policy from a timeless perspective. How do these differ from the conditions you found in (c)?

14. Consider a basic new Keynesian model with Calvo adjustment of prices and flexible nominal wages.

(a) In this model, inflation volatility reduces the welfare of the representative agent. Explain why.

(b) In the absence of cost shocks, optimal policy would ensure inflation and the output gap both remain equal to zero. What does this imply for the behavior of output? Why can output fluctuate efficiently despite sticky prices?

(c) Suppose both prices and nominal wages are sticky (assume a Calvo model for wages). Will volatility in the rate of wage inflation be welfare reducing? Explain.

(d) Is zero inflation and a zero output gap still feasible? Explain.A key

issue in the analysis of policy trade offs is the source of the stochastic shocks in the model. Consider these two examples. 1) The utility function takes the form

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta_t}}{1+\eta_t}$$

where η_t is stochastic. 2) There is a labor tax τ_t such that the after-tax wage is $(1-\tau_t)W_t$. Assume a standard model of monopolistic competition as in the lectures.

(e) Derive the condition for labor market equilibrium under flexible prices for each of the two cases.

(f) Linearize the conditions found in part (a) and, for each case, derive the flexible-price equilibrium output in terms of percent deviations from the steady state. Clearly state any assumptions you need to make on the η and τ processes or about other aspects of the model.

(g) Assume sticky prices ala Calvo. Express real marginal cost in terms of an output gap.

(h) Does either η_t or τ_t appear as a cost shock?

(i) Do you think either η_t or τ_t causes a wedge between the flexible-price output level and the efficient output level?

15. Suppose inflation-adjustment is given by (61). The central bank's objective is to minimize

$$\left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right)$$

subject to (61). Use the programs available from Paul Söderlind's web site (http://www.hhs.se/personal/Psoderlind/Software/Software.htm) to answer this question.

(a) Calculate the response of the output gap and inflation to a serially uncorrelated, positive cost shock for $\phi = 0$, 0.25, 0.5, 0.75, and 1 under the optimal discretionary policy.

(b) Now do the same for the optimal commitment policy.

(c) Discuss how the differences between commitment and discretion depend on ϕ, the weight on lagged inflation in the inflation-adjustment equation.

16. Suppose

$$\pi_t - \gamma\pi_{t-1} = \beta\left(E_t\pi_{t+1} - \gamma\pi_t\right) + \kappa x_t + e_t$$
$$e_t = 0.25e_{t-1} + \varepsilon_t$$

and the period loss function is

$$L = \left(\pi_t - \gamma\pi_{t-1}\right)^2 + 0.25x_t^2.$$

(a) Analytically find the optimal targeting rule under discretion.

(b) Analytically find the optimal targeting rule under commitment (timeless perspective).

(c) Assume $\beta = 0.99$, $\kappa = 0.0603$, $\rho = 0.25$, and $\lambda = 0.25$. Set $\sigma_\varepsilon^2 = 1$. Under the targeting rules found in (a) and (b), plot the loss L as a function of $\gamma = [0\ 1]$.

17. Suppose the inflation equation contains lagged inflation:

$$\pi_t = (1-\phi)\beta E_t\pi_{t+1} + \phi\pi_{t-1} + \kappa x_t + e_t.$$

(a) Show that the optimal commitment policy from a timeless perspective is

$$\pi_t + (\lambda/\kappa)\left[x_t - (1-\phi)x_{t-1} - \beta\phi E_t x_{t+1}\right] = 0.$$

(b) Show that the unconditional optimal commitment policy takes the form

$$\pi_t + (\lambda/\kappa)\left[x_t - \beta(1-\phi)x_{t-1} - \phi E_t x_{t+1}\right] = 0.$$

18. The following model has been estimated by Lindé (2002), though the values here are from Svensson and Williams (2005):

$$\pi_t = 0.4908E_t\pi_{t+1} + (1-0.4908)\pi_{t-1} + 0.0081y_t + \varepsilon_t^\pi$$

$$\begin{aligned} y_t \quad = \quad & 0.4408E_t y_{t+1} + (1-0.4408)\left[1.1778y_{t-1} + (1-1.1778)y_{t-2}\right] \\ & -0.0048\left(i_t - E_t\pi_{t+1}\right) + \varepsilon_t^y \end{aligned}$$

$$\begin{aligned} i_t \quad = \quad & (1-0.9557+0.0673)\left(1.3474\pi_t + 0.7948y_t\right) \\ & +0.9557i_{t-1} - 0.0673i_{t-2} + \varepsilon_t^i \end{aligned}$$

with $\sigma_\pi = 0.5923$, $\sigma_y = 0.4126$, and $\sigma_i = 0.9918$.

(a) Write this system in the form $\mathrm{E}_t z_{t+1} = M z_t + \eta_t$ for appropriately defined vectors z and η.

(b) Plot the impulse response functions showing how inflation and the output gap response to each of the three shocks.

(c) How are the impulse responses affected if the coefficient on inflation in the policy rule is reduced from 1.3474 to 1.1?

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