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AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH TIME DEPENDENT DEMAND UNDER PARTIAL BACKLOGGING

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ABSTRACT- In this work, we study the inventory replenishment policy over a fixed planning period for a deteriorating item having a deterministic demand pattern with a linear trend and shortages. The model is solved analytically by minimizing the total inventory cost. The model can be applied to optimize the total inventory cost for the business enterprises where both the holding cost and deterioration rate are constant.

Keywords: Inventory model, Deteriorating items, Time-dependent demand

1. INTRODUCTION

In recent years, many researchers have studied inventory models for perishable items such as electronic components, food items, drugs and fashion goods. In many real life situations such as failure of batteries as they age, spoilage of foodstuffs, and evaporation of volatile liquids, the effect of determination on the replenishment policies should not be neglected. In fact the stock level of the inventoried item is continuously depleting due to the combined effects of its demand and deterioration. In the last few years, considerable attention has been given to inventory lot-sizing models with deterioration.

Inventory problems involving time variable demand patterns have received the attention of several researchers in recent years. Silver and Meal [1] constructed an approximate solution procedure for the general case of a deterministic, time varying demand pattern. The classical no-shortage inventory problem for a linear trend in demand over a finite time horizon was analytically solved by Donaldson [2]. However, Donaldson’s solution procedure was computationally complicated. Silver [3] derived a heuristic for the special case of positive, linear trend in demand and applied it to the problem Donaldson. Ritchie [4] obtained an exact solution, having the simplicity of the EOQ formula, for Donaldson’s problem for linear, increasing demand. Mitra et al. [5] presented a simple procedure for adjusting the economic order quantity model for the cases of increasing or decreasing linear trend in demand. In all these models, the possibilities of shortages and deterioration in inventory were left out of consideration.

Harris [6] developed the first inventory model, Economic Order Quantity, which was generalized by Wilson [7] who introduced a formula to obtain EOQ. Witin [8] considered the deterioration of the fashion goods at the end of prescribed shortage period. Dave and Patel [9] studied a deteriorating inventory with linear increasing demand when shortages are not allowed. Ghare and Schrader [10] addressed the inventory lot-sizing problem with constant demand and deterioration rate. With the help of some mathematical approximations, they developed a simple Economic Order Quantity, EOQ, model. Then, Covert and Philip [11] and Tadikamalla [12] extended Ghare and Schrader’s work by considering variable rate of deterioration. Shah [13] provided a further generalization of all these models by allowing shortages and using a general distribution for the deterioration

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rate. Other authors [14-18] readjusting Ghare and Schrader's model by relaxing the assumption of infinite replenishment rate.

All these inventory models were formulated in a static environment where the demand is assumed to be constant and steady over a finite planning horizon. However, in a realistic product life cycle, demand is increasing with time during the growth phase. Naddor [19] assumed a demand function that increasing in linear proportion with time during the growth phase and analyzed the cost performances of three inventory policies. Mandal [20] studied a EOQ model for Weibull distributed deteriorating items under ramp-ype demand and shortages. Mishra and Singh [21, 22] constructed an inventory model for ramp-type demand, time dependent deteriorating items with salvage value and shortages and deteriorating inventory model for time dependent demand and holding cost and with partial back logging. Hung [23] investigated an inventory model with generalized type demand, deterioration and back order rates. In this paper, we made the work of Mishra et al. [24] more realistic by considering time dependent demand and developed an inventory model for deteriorating items where deterioration rate and holding cost are constants. Shortages are allowed and partially backlogged.

2. NOTATION AND ASSUMPTION

The fundamental assumption and notation used in this paper are given as below:

a. The demand rate is time dependent and linear, i. e. $D(t)=a+bt$; a, $b>0$ and are constant.

b. The replenishment arte is infinite, thus replenishment is instantaneous.

c. $I(t)$ is the level of inventory at time t, $0 \le t \le T$.

d. T is the length of the cycle.

e. θ is the constant deteriorating rate, $0 < \theta < 1$.

f. t_1 is the time when the inventory level reaches zero.

g. t_1^* is the optimal point.

h. Q is the ordering quantity per cycle.

i. A_0 is the fixed ordering cost per order.

j. C_1 is the cost of each deteriorated item.

k. C_2 is the inventory holding cost per unit per unit of time.

l. C_3 is the shortage cost per unit per unit of time.

m. S is the maximum inventory level for the ordering cycle, such that $S=I(0)$.

n. $C_1(t_1)$ is the average total cost per unit time under the condition $t_1 \le T$.

3. MATHEMATICAL FORMULATION

Here we consider the deteriorating inventory model with linearly time dependent demand rate. Replenishment occurs at time t=0 when the inventory level attains its maximum. From t=0 to t_1, the inventory level reduces due to demand and deterioration. At t_1, the inventory level achieves zero, then shortage is allowed to occur during the time interval (t_1, T) is completely backlogged. The total number of backlogged items is replaced by the next replenishment. According to the notations and assumptions mentioned above, the behavior of inventory system at any time can be described by the following differential equations:

$$\frac{dI(t)}{dt} = -D(t) - \theta I(t)\,,\; 0 \le t \le t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -D(t)\,,\quad t_1 \le t \le T \tag{2}$$

With boundary conditions $I(0)=S$, $I(t_1)=0$

The solutions of equations (1) and (2) with boundary conditions are as follows.

$$I(t) = \left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)e^{\theta(t_1-t)} - \frac{a+bt}{\theta} + \frac{b}{\theta^2}\,,\quad 0 \le t \le t_1 \tag{3}$$

$$I(t) = a(t_1 - T) + \frac{b}{2}(t_1^2 - T^2)\,,\quad t_1 \le t \le T \tag{4}$$

The beginning inventory level can be computed as

$$S = I(0) = \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta t_1} - 1) + \frac{b}{\theta}t_1 e^{\theta t_1} \tag{5}$$

The total number of items which perish in the interval $[0, t_1]$, say D_T, is

$$D_T = S - \int_0^{t_1} D(t)dt = S - \int_0^{t_1} (a+bt)dt$$

$$= \left(\frac{a}{\theta} - \frac{b}{\theta^2}\right)(e^{\theta t_1} - 1) + \frac{b}{\theta}t_1 e^{\theta t_1} - at_1 - \frac{1}{2}bt_1^2 \tag{6}$$

The total number of inventory carried during the interval $[0, t_1]$, say H_T, is

$$H_T = \int_0^{t_1} I(t)dt$$

$$= \int_0^{t_1}\left[\left(\frac{a+bt_1}{\theta} - \frac{b}{\theta^2}\right)e^{\theta(t_1-t)} - \frac{a+bt}{\theta} + \frac{b}{\theta^2}\right]dt$$

$$= \left(\frac{a+bt_1}{\theta^2} - \frac{b}{\theta^3}\right)(e^{\theta t_1} - 1) - \frac{a\theta - b}{\theta^2}t_1 - \frac{b}{2\theta}t_1^2 \tag{7}$$

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The total shortage quantity during the interval [t_1, T], say B_T, is

$$B_T = -\int_{t_1}^{T} I(t)dt$$

$$= -\int_{t_1}^{T} \left[a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) \right] dt$$

$$= \frac{a}{2}(T^2 + \frac{3}{2}t_1^2 - 2t_1T) + \frac{b}{6}(T^3 + 2t_1^3 - 3t_1^2T) \quad (8)$$

Then, the average total cost per unit time under the condition $t_1 \le T$ can be given by

$$C_1(t_1) = \frac{1}{T}[A_0 + C_1D_T + C_2H_T + C_3B_T] \quad (9)$$

The first order derivative of $C_1(t_1)$ with respect to t_1 is as follows:

$$\frac{dC_1(t_1)}{dt_1} = \frac{1}{T}\left[\left(C_1 + \frac{C_2}{\theta}\right)\left(e^{\theta t_1} - 1\right) + C_3(t_1 - T)\right](a + bt_1) \quad (10)$$

The necessary condition for $C_1(t_1)$ in (9) to be minimized is

$\frac{dC_1(t_1)}{dt_1} = 0$, that is

$$\left[\left(C_1 + \frac{C_2}{\theta}\right)\left(e^{\theta t_1} - 1\right) + C_3(t_1 - T)\right](a + bt_1) = 0 \quad (11)$$

Let $g(t_1) = \left[\left(C_1 + \frac{C_2}{\theta}\right)\left(e^{\theta t_1} - 1\right) + C_3(t_1 - T)\right]$

Since $g(0)=-C_3T<0$, $g(T) = (C_1 + \frac{C_2}{\theta})(e^{\theta T} - 1) > 0$ for $e^{\theta T} > 1$

and $g'(t_1) = (\theta C_1 + C_2)e^{\theta t_1} + C_3 > 0$, it implies that $g(t_1)$ is a strictly monotonic increasing function and equation (11) has unique solution at t_1^*, for $t^*_1 \epsilon (0, T)$

Therefore, we have

Property-1

The deteriorating inventory model under the condition $0 < t_1 \le T$, $C_1(t_1)$ obtains its minimum at $t_1=t^*_1$, where $g(t^*_1)=0$ if $t^*_1<T$.

4. CONCLUSION

In this paper, we study the inventory model for deteriorating items with linear time dependent demand rate. We proposed an inventory replenishment policy for this type of inventory model. Of course, the paper provides an interesting topic for the further study of such kind of important inventory models, the following two problems can be considered in our future research. (1) There is no set up cost in this inventory model. What will happen, if we add set up cost in to this inventory model? (2) How about the inventory model starting with shortages?

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