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Coordinating Replenishment Cycles in Three-Stage Inventory-Distribution Supply Chains

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Abstract**—**There have been many studies on two-stage supply models. Because supply chains are increasingly more complicated, this study considers the three-stage supply chain configurations which involve multiple companies at each stage and each company at the upstream stages can supply two or more customers. The coordination mechanisms for the members along the chain are achieved in the following two aspects: (i) each company takes general-integer (GI), stationary and nested inventory replenishment policies; (ii) two differential transportation costs are incorporated into the ordering and inventory costs. This paper analyzes the cost properties such that we can develop an efficient heuristic method to deal with the three-stage inventory-distribution problems (TSIDP). The relative outcomes between our proposed heuristic approach and LINGO® software indicate that the former outperforms the latter.

Keywords**—**Three-stage inventory/production/distribution supply chains, General-Integer policies, Heuristics.

1. INTRODUCTION

Transportation costs are often neglected in the literature, as mentioned in the review by Drexl and Kimms (1997), and the issue of incorporating transportation costs into lot-sizing models is gradually growing. For instance, Swenseth and Godfrey (2002) pointed out that transportation costs can be almost 50% of the total logistics costs of a product. Furthermore, Burwell et al. (1997), Bertazzi and Speranza (1999) and Vroblefski et al. (2000) also emphasized the consideration of transportation costs in inventory decisions. To achieve efficient supply chain management, Sarmiento and Nagi (1999) claimed that much research is needed on the issues of integrating inventory and distribution, as well as the integration of inventory and production. However, there have been studies on the integration of inventory, production and distribution. Some researchers have focused on proposing coordination mechanisms among members of the supply chain. For example, Chen et al. (2001) addressed the coordination mechanisms on the franchise fees, quantity discounts, volume discounts, and frequency discounts in a system with one supplier and multiple retailers. Klastorin et al. (2002) evaluated the two-echelon decentralized supply chains and applied price discounts to coordinate the orders. Maxwell and Muckstadt (1985) and Jackson et al. (1988) assumed the general integers or the power-of-two integers multiple of the basic period of time as shipping frequencies for products. Many recent papers relating to a central warehouse and N retailers network have been concentrated on power-of-two and general-integer policies. For example, Roundy (1985), and Abdul-Jalbar et al. (2005) further developed a new heuristic to compare with that proposed by Roundy (1985). Khouja (2003) utilized three coordination mechanisms of equal cycle time, integer multipliers and integer power-of-two, for a three-stage supply chain configuration. However, Khouja (2003) did not consider transportation costs. The model in this study considers key ideas from two research streams of literature: (1) the multi-echelon production-inventory systems, and (2) the distribution systems. The key findings in this paper include: (i) total cost structure for all members in the chain comprises of piece-wise convex but discontinuous curve at some transportation-cost breakpoints; (ii) the theoretical results derived in this study can become the foundation for carrying on further investigations of the *TSIDP*.

The structure of this study is as follows. In Section 2, we define the problem scope of this study. Section 3 discusses the insights of the model so that we can develop the solution method in Section 4. In Section 5, we present an example to illustrate our solution procedure for the *TSIDP*. The computational and comparative results are shown in Section 6. Finally, we present conclusions in Section 7.

2. PROBLEM STATEMENT

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This study refers to Khouja (2003) in assuming that the distribution system has arborescence, i.e. a tree-like structure, as shown in Figure 1. The area circled by dashed lines in Figure 1 is our research scope. In addition to the original inventory and production concerns in Khouja (2003), we also consider the transportation factor in this decentralized network. We adopt the inventory and production elements considered in Khouja (2003) and then add other notations used in the aspect of the transportation cost such that notations we use are shown in Table 1.

-- Insert Figure 1 here --

-- Insert Table 1 here--

The assumptions in the *TSIDP* include:

(i) companies in the same stage share the same reorder interval,

(ii) the replenishment interval at each stage is an integer multiple of the basic reorder interval at the immediate and consecutive downstream stage,

(iii) the annual demand of firm *j* at stages 1 and 2 is calculated by $\lambda_{11}=\lambda_{21}+\lambda_{22}+\lambda_{23}$, $\lambda_{21}=\lambda_{31}+\lambda_{32}+\lambda_{33}$, $\lambda_{22}=\lambda_{34}+\lambda_{35}$, and $\lambda_{23}=\lambda_{36}+\lambda_{37}$,

(iv) each firm's replenishment order is in the amount of $\lambda_{ij}T_{ij}$ units (please refer to Table 1, and this assumption can be transformed into $q_{3j}=\lambda_{3j}B$, $q_{2j}=\lambda_{2j}K_2B$ and $q_{1j}=\lambda_{1j}K_1K_2B$) because we assume that the inventory level is down to zero when the next replenishment epoch begins,

(v) two differential transportation costs C_{i0} and C_{i1}, i.e. one price breakpoint, are considered in this problem,

(vi) the supplier's and manufacturers' production capacity are not overloaded.

The decision variables in the *TSIDP* are *B*, K_2, and K_1. Upon a general-integer-ratio policy, the ratio of T_{2j}/T_{3j} or that of T_{1j}/T_{2j} needs to be a positive integer such that K_1 and K_2 are restricted to be positive integers.

The annual total costs in the model *TSIDP* are the summation of the cost elements described below:

Problem *TSIDP*:

$$\begin{aligned} ATC = Minimize\{ & \sum_{j=1}^{J_3} a_3/B + \sum_{j=1}^{J_3} h_3(B\lambda_{3j}/2) + \sum_{j=1}^{J_3} \lambda_{3j}(C'_{31}s_{3j}+C_{30}) \\ & + \sum_{j=1}^{J_2} a_2/K_2B + \sum_{j=1}^{J_2} h_1(K_2B\lambda_{2j}^2/2R_{2j}) + \sum_{j=1}^{J_2}(h_2B\lambda_{2j}/2)[K_2\lambda_{2j}/R_{2j}+(K_2-1)] \\ & + \sum_{j=1}^{J_2} \lambda_{2j}(C'_{21}s_{2j}+C_{20}) \\ & + \sum_{j=1}^{J_1} a_1/K_1K_2B + \sum_{j=1}^{J_1} h_0(K_1K_2B\lambda_{1j}^2/2R_{1j}) + \sum_{j=1}^{J_1}(h_1K_2B\lambda_{1j}/2)[K_1\lambda_{1j}/R_{1j}+(K_1-1)] \\ & + \sum_{j=1}^{J_1} \lambda_{1j}(C'_{11}s_{1j}+C_{10})\} \end{aligned} \tag{1}$$

subject to

$$s_{3j} \in \{0,1\},\ j=1,\ 2,\ ...,\ J_3 \tag{2}$$

$$s_{3j}b - q_{3j} \le 0,\ j=1,\ 2,\ ...,\ J_3 \tag{3}$$

$$q_{3j} = \lambda_{3j}B,\ j=1,\ 2,\ ...,\ J_3 \tag{4}$$

$$s_{2j} \in \{0,1\},\ j=1,\ 2,\ ...,J_2 \tag{5}$$

$$s_{2j}b - q_{2j} \le 0,\ j=1,\ 2,\ ...,\ J_2 \tag{6}$$

$$q_{2j} = \lambda_{2j}K_2B,\ j=1,\ 2,\ ...,\ J_2 \tag{7}$$

$$s_{1j} \in \{0,1\},\ j=1,\ 2,\ ...,\ J_1 \tag{8}$$

$$s_{1j}b - q_{1j} \le 0,\ j=1,\ 2,\ ...,\ J_1 \tag{9}$$

$$q_{1j} = \lambda_{1j}K_1K_2B,\ j=1,\ 2,\ ...,\ J_1 \tag{10}$$

$$K_2 \ge 1,\ \text{integer} \tag{11}$$

$$K_1 \ge 1,\ \text{integer} \tag{12}$$

The objective function of the *TSIDP* is to minimize the summation of the ordering, the inventory holding, and the transportation costs of all firms. Under *GI* policy, the inventory replenishment multipliers K_2 and K_1 must be the positive integers as the constraints (11) and (12) demonstrate. The constraints (4), (7), and (10) represent that the firms need to meet the constraints of the *GI* ordering period policy and their ordering quantities are their own yearly demand multiple of the ordering period. After deciding how many quantities need to be replenished, each firm is limited to applying two different transportation costs. The constraints (3), (6), and (9) are used to judge the relationship between ordering quantities and transportation discount quantities. If our given transportation discount quantities are greater than ordering quantities, the variable s_{ij} is restricted to be 0 as shown in the Eq. (2), (5), and (8) such that the transportation cost C_{i0} is adopted. On the other hand, the transportation cost C_{i1} is to be adopted when $s_{ij}=1$.

3. THE COST-CURVE STRUCTURE ANALYSIS

In this section, we illustrate the cost-curve properties for the *TSIDP*. In order to solve the mathematical model, we plot curves to investigate cost functions. We denote those curves as "cost curves." The solution procedure has two steps: (i) to analyze the cost curves on mathematical models and (ii) to develop heuristic approaches from the findings in (i). Lee and Yao (2003) studied cost curves of *the joint replenishment problem*, and Lee and Wen (2007) discussed cost curves of *the serially distributed storage depots problem*. First of all, this study focuses on the cost structure for each stage before applying suitable methods for the problems. Cost curves are generated by referring to the parameters in parentheses demonstrated in Figure 1. The values with transportation costs are given by this study, while others are the same as those mentioned in Khouja (2003). Taking the transportation cost into account, we discover that cost functions separately reveal the convexity but discontinuity, the piece-wise convexity but discontinuity, and the piece-wise convexity but discontinuity and with some truncation points on the minimal cost curve, as shown in Figures 2, 3, and 4. Finally, we find that the cost structure of the model *TSIDP* is the piece-wise convexity but discontinuity and with some truncation points as Figure 5 demonstrates.

-- Insert Figure 2 here--
-- Insert Figure 3 here--
--Insert Figure 4 here--
--Insert Figure 5 here--

By means of our foregoing discussion, it is clear that the model *TSIDP* has some features on its cost structure. In the next section, we develop the appropriate heuristic approach using the interesting findings on cost curves of the model *TSIDP*. The following theoretical analysis is used to indicate our observations on the cost curve properties. It is convenient to interpret that we place cost function equations into lemmas and properties.

Lemma 1. *The cost function $PTC_{3j}(B)$ is convex with respect to B.*

$$PTC_{3j}(B) = Minimize\{a_3 / B + b_3(B\lambda_{3j} / 2)\} \quad . \tag{13}$$

Proof. The cost function $PTC_{3j}(B)$ is a function that is differentiable, its second derivative $PTC''_{3j}(B) = a_3 / 2B^3 > 0$ if $B > 0$. Because the graph of $PTC_{3j}(B)$ is above all of its tangent lines, the graph is convex at all of its positive values of B. $PTC_{3j}(B)$ is convex with respect to B for each firm j (for $j = 1, 2, \ldots, J_3$) at stage 3. □

Lemma 2. *The cost function $ATC_{3j}(B)$ is convex but discontinuous at a breakpoint with respect to B.*

$$ATC_{3j}(B) = Minimize\{a_3 / B + b_3(B\lambda_{3j} / 2) + \lambda_{3j}(C'_{31}s_{3j} + C_{30})\} \tag{14}$$

subject to the constraints (2), (3), and (4).

Proof. By Lemma 1, the function $PTC_{3j}(B)$ is convex with respect to B. Therefore, $ATC_{3j}(B)$ is convex but discontinuous at a breakpoint with respect to B after considering the constraints of transportation-cost discounts into the convex function $PTC_{3j}(B)$. □

Proposition 1. *All the breakpoints of the function $ATC_{3j}(B)$ will be inherited by the $ATC_3(B)$. The cost function $ATC_3(B)$ is convex but discontinuous at some breakpoints with respect to B.*

$$ATC_3(B) = Minimize\{\sum_{j=1}^{J_3} a_3 / B + \sum_{j=1}^{J_3} b_3(B\lambda_{3j} / 2) + \sum_{j=1}^{J_3} \lambda_{3j}(C'_{31}s_{3j} + C_{30})\} \tag{15}$$

subject to the constraints (2), (3), and (4).

Proof. Since $PTC_{3j}(B)$ is a convex function, so is $PTC_3(B)$. Because $PTC_3(B)$ is the summation of J_3 convex functions, $PTC_3(B)$ is another convex function. $ATC_3(B)$ is a convex but discontinuous function at some breakpoints after imposing the constraints of transportation-cost discounts into the convex function $PTC_3(B)$.□

Lemma 3. *For a given value of K_2, the cost function $ATC_{2j}(K_2, B)$ is convex but discontinuous at a breakpoint with respect to B.*

$$\begin{aligned} ATC_{2j}(K_2,B) = Minimize\{ & a_2 / K_2B + h_1(K_2B\lambda_{2j}^2 / 2R_{2j}) \\ & + (h_2B\lambda_{2j}/2)[K_2\lambda_{2j}/R_{2j} + (K_2-1) \\ & + \lambda_{2j}(C_{21}^{'}s_{2j} + C_{20})\} \end{aligned} \tag{16}$$

subject to the constraints (5), (6), (7) and (11).

Proof. For a given value of K_2, the cost function $PTC_{2j}(K_2, B)$ is a convex function with respect to B. Transportation-cost constraints are added into the convex function $PTC_{2j}(K_2, B)$ so that $ATC_{2j}(K_2, B)$ is a convex but discontinuous at a breakpoint with respect to B. □

Proposition 2. *For a given value of K_2, all the breakpoints of the function $ATC_{2j}(K_2, B)$ will be inherited by the $ATC_2(K_2, B)$ and $ATC_2(K_2, B)$ is convex but discontinuous at some breakpoints with respect to B.*

$$\begin{aligned} ATC_2(K_2,B) = Minimize\{ & \textstyle\sum_{j=1}^{J_2} a_2 / K_2B + \sum_{j=1}^{J_2} h_1(K_2B\lambda_{2j}^2 / 2R_{2j}) \\ & + \textstyle\sum_{j=1}^{J_2} (h_2B\lambda_{2j}/2)[K_2\lambda_{2j}/R_{2j} + (K_2-1)] \\ & + \textstyle\sum_{j=1}^{J_2} \lambda_{2j}(C_{21}^{'}s_{2j} + C_{20})\} \end{aligned} \tag{17}$$

subject to the constraints (5), (6), (7) and (11).

Proof. By Lemma 3, for a given value of K_2, $ATC_2(K_2, B)$ is a convex but discontinuous function with respect to B because $ATC_2(K_2, B)$ is the summation of J_2 convex but discontinuous functions $ATC_{2j}(K_2, B)$. □

Proposition 3. *The minimal cost function of the function $ATC_2(K_2, B)$ is piece-wise convex but discontinuous at some breakpoints with respect to B.*

We define $\overline{ATC}_2(B)$ as the minimal cost function of the function $ATC_2(K_2, B)$ with respect to B, where

$$\overline{ATC}_2(B) = \min_{K_2}\{ATC_2(K_2,B)\} \quad . \tag{18}$$

Proof. Choosing one K_2 that results in the minimal cost of $ATC_2(K_2,B)$ for each positive value of B. For $B > 0$, $\overline{ATC}_2(B)$ is piece-wise convex but discontinuous at some breakpoints with respect to B. □

Definition 1. Let a junction point, a particular value of B where two consecutive curves concatenate, on the curve of $\overline{ATC}_2(B)$ be the truncation point for the curve of $ATC_1(K_1, B)$. Suppose that $K_2^{\min}(L)$ and $K_2^{\min}(R)$, respectively, are the K_2 multipliers that result in the minimal cost function $\overline{ATC}_2(B)$ of the left-side and right-side curves with regard to a junction point, where

$$K_2^{\min} = \arg\min_{K_2}\{ATC_2(K_2,\ B)\} \quad . \tag{19}$$

Lemma 4. *Let $K_2^{\min}$ and its corresponding JP also be decided while we are analyzing on the function $\overline{ATC}_2(B)$. For a given value of K_1, all breakpoints of the function $ATC_{1j}(K_1, B)$ will be inherited by the $ATC_1(K_1, B)$ and $ATC_1(K_1, B)$ is convex but discontinuous at some breakpoints with respect to B.*

Proof. The multiplier $K_2^{\min}$ and its corresponding JP are already given. For a given value of K_1, $ATC_1(K_1, B)$ is a convex but discontinuous function with respect to B because $ATC_1(K_1, B)$ is the summation of J_1 convex but discontinuous functions $ATC_{1j}(K_1, B)$. □

Lemma 5. *Let $K_2^{\min}$ and its corresponding JP also be decided while we are analyzing the function $\overline{ATC}_2(B)$. The minimal cost function of the function $ATC_1(K_1,B)$ is piece-wise convex but discontinuous with respect to B.*

Let $\overline{ATC}_1(B)$ be the minimal cost function of the function $ATC_1(K_1,B)$ with respect to B, where

$$\overline{ATC}_1(B) = \min_{K_1}\{ATC_1(K_1, K_2 = K_2^{\min}, B)\} \quad . \tag{20}$$

Proof. The multiplier $K_2^{\min}$ and its corresponding JP are already given. For $B>0$, $\overline{ATC}_1(B)$ is piece-wise convex but discontinuous at some breakpoints with respect to B. □

Proposition 4. *The junction points (JPs) and breakpoints (BPs) appearing on curves of $ATC_3(B)$, $\overline{ATC}_2(B)$, and $\overline{ATC}_1(B)$ will be inherited by the function ATC.*

Proof. Because the function *ATC* is the summation of $ATC_3(B)$, $\overline{ATC}_2(B)$, and $\overline{ATC}_1(B)$, *ATC* is another piece-wise convex but discontinuous curve with respect to *B*. That means *JPs* and *BPs* on curves of $ATC_3(B)$, $\overline{ATC}_2(B)$, and $\overline{ATC}_1(B)$ will be inherited by the function *ATC*. □

4. A PROPOSED HEURISTIC METHOD

In this section, we introduce the proposed search method to solve the *TSIDP*. Observing the cost curve structure disscussed in the previous section, we conclude that there are two kinds of points, called breakpoints (*BPs*) and junction points (*JPs*), shown on cost curves. By some insights into the cost-curve structure, this study develops the following search mechanisms.

4.1 Starting and termination conditions

Before introducing the proposed search method, we derive a lower bound and an upper bound to reduce the searched region. At first, we consider a relaxed problem of the *TSIDP*, defined by *RP*, as shown in Eq. (21):
Problem *RP*:

$$Minimize\{\sum_{j=1}^{J_3} a_3 / B + \sum_{j=1}^{J_3} h_3(B\lambda_{3j}/2) + \sum_{j=1}^{J_3} \lambda_{3j}C_{31} + \sum_{j=1}^{J_2} \lambda_{2j}C_{21} + \sum_{j=1}^{J_1} \lambda_{1j}C_{11}\} \tag{21}$$

B^* and B_{RP} are denoted as the value of *B* for the problems *TSIDP* and *RP*, respectively. The closed form for B_{RP} is given as follows:

$$B_{RP} = \sqrt{\frac{2\sum_{j=1}^{J_3} a_3}{h_3 \sum_{j=1}^{J_3} \lambda_{3j}}} \quad . \tag{22}$$

Let $ATC(B_{RP})$ be the objective function value of the *TSIDP* at B_{RP}. We will demonstrate that a lower bound and an upper bound on B^* are given by values of *B* where the objective function value of the problem *RP* equals $ATC(B_{RP})$. The derivation of the bounds is presented by the following proposition.

Proposition 5. *Let B_{LB} be the smallest and B_{UB} be the largest B for which the objective function value of the problem RP equals $ATC(B_{RP})$. Then,* $B_{LB} \le B^* \le B_{UB}$.

Proof. We have the range $B_{LB} \le B_{RP} \le B_{UB}$ shown on the *B*-axis because the objective function of the problem *RP* is strictly convex. For $B > B_{UB}$, the objective function value of the problem *RP* is definitely larger than $ATC(B_{RP})$. The objective function value of the *TSIDP* is also larger than $ATC(B_{RP})$ for $B > B_{UB}$ since the problem *RP* is a relaxation of the *TSIDP*. Consequently, B_{UB} is an upper bound on B^*. It is analogous that the proof is used for $B_{LB} \le B^*$.□

By Proposition 5, we show how to place the bounds B_{LB} and B_{UB} on B^* in Figure 6. The Bisection Method is applied to find B_{LB} and B_{UB}. We set the approximation, denoted as m_n, which converges to $ATC(B_{RP})$ at step *n* with an error E_n, where

$$|E_n| = |ATC(B_{RP}) - m_n| \le 10^{-4} \tag{23}$$

Therefore, we need to determine the objective function value of B_{LB} or B_{UB} to accuracy within 10^{-4}. The following search method is implemented in the interval $[B_{LB}, B_{UB}]$.

--Insert Figure 6 here—

4.2 A search mechanism for finding bps

For stage 3, the breakpoint, denoted as ρ_{3j} for $j = 1, 2, \ldots, J_3$, appears at the point of

$$\rho_{3j} = b / \lambda_{3j} \quad . \tag{24}$$

Define the breakpoint at stage 2 as $\rho_{2j}^{K_2}$ where K_2= 1, 2, … and j= 1, 2, …, J_2 and it is located on the point of

$$\rho_{2j}^{K_2} = b/(K_2\lambda_{2j}). \tag{25}$$

The breakpoint $\rho_{1j}^{K_1}$ at stage 1 we can obtain by using the equation below:

$$\rho_{1j}^{K_1} = b/(K_1 K_2^{\min}\lambda_{1j}), \tag{26}$$

where K_1 is a positive integer and j= 1, 2, …, J_1.

4.3 A search mechanism for finding jps

Before introducing *JPs* of cost curves at stages 1 and 2, we need to know that each K_2 curve has the following properties:

Each K_2 curve, for instance (K_2= v), was divided into (J_2+1) segments by J_2 different *BPs*, and segments of each K_2 curve can be shown as

$$ATC_2(K_2,\ B)$$
$$= \begin{cases} PTC_2(K_2,\ B) + (\lambda_{21}+\lambda_{22}+\lambda_{23}+\ldots+\lambda_{2J_2})C_{21}, & \text{for } B\in[\rho_{2t}^{K_2},\infty),\ t\in\{1,\ 2,\ \ldots,J_2\} \\ PTC_2(K_2,\ B) + \lambda_{2t}C_{20} + \sum_{j\neq t}\lambda_{2j}C_{21}, & \text{for } B\in[\rho_{2r}^{K_2},\rho_{2t}^{K_2}),\ r,t\in\{1,\ 2,\ \ldots,J_2\} \\ PTC_2(K_2,\ B) + (\lambda_{2t}+\lambda_{2r})C_{20} + \sum_{j\neq r, j\neq t}\lambda_{2j}C_{21}, & \text{for } B\in[\rho_{2u}^{K_2},\rho_{2r}^{K_2}),\ u,r\in\{1,\ 2,\ \ldots,J_2\} \\ \vdots & \\ PTC_2(K_2,\ B) + (\lambda_{21}+\lambda_{22}+\lambda_{23}+\ldots+\lambda_{2J_2})C_{20}, & \text{for } B\in(0,\rho_{2w}^{K_2}),\ w\in\{1,\ 2,\ \ldots,J_2\} \end{cases} \tag{27}$$

Taking (K_2=1) for an example, we illustrate the meaning of the above equation in Figure 7.

-- Insert Figure 7 here --

Two K_2 curves, say (K_2= v) and (K_2= v+1), intersect such that the junction points are generated. The *JP* of stage 2 is defined as δ_{2g}, where g= 1, 2, … is a counter for *JPs* and is obtained by the following procedure, called ***The JP-Stage 2 Finding Procedure***. Figure 8 demonstrates this procedure.

-- Insert Figure 8 here --

Remark 1 The unit transportation costs of all firms at stage 2 on the curve (K_2= v) are the same as those of all firms at stage 2 on the curve (K_2= v+1) in the interval $\Psi_{n_\tau}^{E2}$, where $\tau = 1,2,3,\ldots$ means that at stage 2, we number the intervals which are divided by *BPs* appearing on the curves (K_2= v) and (K_2= v+1). We summarize that *JPs* of curves (K_2= v) and (K_2= v+1) in the interval $\Psi_{n_\tau}^{E2}$ can be computed using the following equation:

$$\delta_{2g} = \left\{ \sqrt{\frac{\sum_{j=1}^{J_2} a_2}{\frac{v(v+1)}{2}[(h_1+h_2)\sum_{j=1}^{J_2}\frac{\lambda_{2j}^2}{R_{2j}} + h_2\sum_{j=1}^{J_2}\lambda_{2j}]}} \right\}. \tag{28}$$

Remark 2 Find the firm which has unit transportation costs C_{20} and C_{21} on the curves (K_2= v) and (K_2= v+1), respectively. The numerator of Eq. (21) consists of the firm whose unit transportation costs are not the same on curves (K_2= v) and (K_2= v+1). The *JPs* of curves (K_2= v) and (K_2= v+1) in the interval $\Psi_{n_\tau}^{E2}$ can be calculated as follows:

$$\delta_{2_g}=\left\{\begin{array}{l}\dfrac{(\lambda_{2t}+\lambda_{2r}+\lambda_{2u}+\ ...+\lambda_{2w})(C_{20}-C_{21})}{[(2h_1+h_2)\sum_{j=1}^{J_2}\frac{\lambda_{2j}^2}{R_{2j}}+h_2\sum_{j=1}^{J_2}\lambda_{2j}]}\\ +\dfrac{\sqrt{(\lambda_{2t}+\lambda_{2r}+\lambda_{2u}+\ ...+\lambda_{2w})^2(C_{20}-C_{21})^2+4[\frac{(2h_1+h_2)}{2}\sum_{j=1}^{J_2}\frac{\lambda_{2j}^2}{R_{2j}}+\frac{h_2}{2}\sum_{j=1}^{J_2}\lambda_{2j}][\frac{1}{v(v+1)}\sum_{j=1}^{J_2}a_2]}}{[(2h_1+h_2)\sum_{j=1}^{J_2}\frac{\lambda_{2j}^2}{R_{2j}}+h_2\sum_{j=1}^{J_2}\lambda_{2j}]}\end{array}\right\}, \tag{29}$$

where the firms $t,\ r,\ u,\ ...,\ w\in\{1,\ 2,\ 3,\ ...,J_2\}$.

The δ_{2_g} appearance represents that the K_2 curve changes from ($K_2= v$) to ($K_2= v+1$) and results in the truncation point generation for the K_1 curve. Each of K_1 curves is divided into (J_1+1) segments by J_1 different *BPs*.

And for each K_1 curve at stage 1, its cost function is

$$\begin{aligned}&ATC_1(K_1,\ B)=ATC_1(K_1,K_2^{\min},B)\\ &=\left\{\begin{array}{l}PTC_1(K_1,K_2^{\min},B)+(\lambda_{11}+\lambda_{12}+\lambda_{13}+...+\lambda_{1J_1})C_{11},\ \text{ for } B\in[\rho_{1t}^{K_1},\infty),t\in\{1,\ 2,\ ...,J_1\}\\ PTC_1(K_1,\ K_2^{\min},B)+\lambda_{1t}C_{10}+\sum_{j\neq t}\lambda_{1j}C_{11},\ \text{ for } B\in[\rho_{1r}^{K_1},\rho_{1t}^{K_1}),r,t\in\{1,\ 2,\ ...,J_1\}\\ PTC_1(K_1,K_2^{\min},B)+(\lambda_{1t}+\lambda_{1r})C_{10}+\sum_{j\neq r,j\neq t}\lambda_{1j}C_{11},\ \text{ for } B\in[\rho_{1u}^{K_1},\rho_{1r}^{K_1}),u,r\in\{1,\ 2,\ ...,J_1\}\\ \vdots\\ PTC_1(K_1,\ K_2^{\min},B)+(\lambda_{11}+\lambda_{12}+\lambda_{13}+...+\lambda_{1J_1})C_{10},\ \text{ for } B\in(0,\rho_{1w}^{K_1}),w\in\{1,\ 2,\ ...,J_2\}\end{array}\right.\end{aligned} \tag{30}$$

The *JP* of stage 1 is defined as δ_{1_g}, where g= 1, 2, … is a counter for *JPs* at stage 1 and is obtained by ***The JP-Stage 1 Finding Procedure***, as illustrated in Figure 9.

-- Insert Figure 9 here --

Remark 3 Assume that $K_2^{\min}=v$. Unit transportation costs of all firms at stage 1 on the curve ($K_1= x$) are the same as those of all firms at the stage 1 on the curve ($K_1= x+1$) in the interval $\Psi_{n_\tau}^{E1}$, where $\tau=1,2,3,...$. This means that at stage 1, we number the interval which divided by *BPs* appearing on the curves ($K_1= x$) and ($K_1= x+1$). *JPs* of curves ($K_1= x$) and ($K_1= x+1$) in the interval $\Psi_{n_\tau}^{E1}$ can be computed using the following equation:

$$\delta_{1_g}=\left\{\sqrt{\frac{2\sum_{j=1}^{J_1}a_1}{x(x+1)v^2[(h_0+h_1)\sum_{j=1}^{J_1}\frac{\lambda_{1j}^2}{R_{1j}}+h_1\sum_{j=1}^{J_2}\lambda_{1j}]}}\right\}. \tag{31}$$

Remark 4 Assume that $K_2^{\min}=v$. Find the firm which has unit transportation costs C_{10} and C_{11} on curves ($K_1= x$) and ($K_1= x+1$), respectively. The numerator of Eq. (32) consists of the firm whose unit transportation costs are not the same on curves ($K_1= x$) and ($K_1= x+1$). *JPs* of curves ($K_1= x$) and ($K_1= x+1$) in the interval $\Psi_{n_\tau}^{E1}$ can be calculated as follows:

$$\delta_{1_g}=\left\{\begin{array}{l}\dfrac{(\lambda_{1t}+\lambda_{1r}+\lambda_{1u}+\ ...+\lambda_{1w})(C_{10}-C_{11})}{[(h_0+h_1)v\sum_{j=1}^{J_1}\frac{\lambda_{1j}^2}{R_{1j}}+h_1v\sum_{j=1}^{J_1}\lambda_{1j}]}\\ +\dfrac{\sqrt{(\lambda_{1t}+\lambda_{1r}+\lambda_{1u}+\ ...+\lambda_{1w})^2(C_{10}-C_{11})^2+4[\frac{(h_0+h_1)}{2}\sum_{j=1}^{J_1}\frac{\lambda_{1j}^2}{R_{1j}}+\frac{h_1}{2}\sum_{j=1}^{J_1}\lambda_{1j}][\frac{1}{x(x+1)}\sum_{j=1}^{J_1}a_1]}}{[(h_0+h_1)v\sum_{j=1}^{J_1}\frac{\lambda_{1j}^2}{R_{1j}}+h_1v\sum_{j=1}^{J_1}\lambda_{1j}]},\end{array}\right\} \tag{32}$$

where the firms $t,\ r,\ u,\ ...,\ w\in\{1,\ 2,\ 3,\ ...,J_1\}$.

4.4 A search mechanism for solving the tsidp

We have discussed the cost structure of the *TSIDP* in the previous sections. Our search procedure for finding the searched points at stage 3 as depicted in Figure 10 is based on *JPs* and *BPs*. Following the same search mechanism, we can obtain searched points at stages 1 and 2.

-- Insert Figure 10 here --

5. AN ILLUSTRATIVE EXAMPLE

This study adopts the data for the inventory and production field in Khouja (2003) and adds new data for the transportation aspect, as revealed in parentheses in Figure 1. We introduce the proposed search mechanism in Section 4.4 to solve the demonstrated example. The computational tool we utilize to implement our proposed method is *MATLAB*® 6.5 with a computer *P4*-1.8 GHz, 512MB RAM. The search method starts at the point B_{UB}= 0.1666 years and terminates at the point B_{LB}= 0.0063 years. The search results are generated as shown in Table 2.

-- Insert Table 2 here --

For the *TSIDP*, we set the results obtained from our search method by the following equations:

$$ATC^* = \arg\min_m(ATC_m) \quad , \tag{33}$$

$$(K_2^*,\ K_1^*,\ B^*) = \arg\min_m(ATC^*((K_2)_m, (K_1)_m, B_m) \quad , \tag{34}$$

where m is a counter for the searched points.

Observe that the minimal cost of ATC is $ATC^* = \$104,818$, such that $(K_2^*,\ K_1^*,\ B^*) = (1,\ 2,\ 0.0625)$, and it takes 0.05 seconds instead of 3 seconds to find the same results by *LINGO*® 8.0. Recall that in Figure 5, we used small-step search method to enumerate a wide range of B using a small step size of $\Delta B = 0.0001$. This near-exhaustive search procedure is used to solve the *TSIDP* and the result is $ATC^* = \$103,613$, $(K_2^*,\ K_1^*,\ B^*) = (1,\ 2,\ 0.0556)$. The comparison of cost deviation between our proposed search method and the small-step search method is [(104,818－103,613)/103,613]×100%= 1.16%. This example demonstrates that our proposed search method could be an efficient method to obtain the solution to the *TSIDP*.

6. THE COMPUTATIONAL AND COMPARATIVE RESULTS

We will further test the performance of the proposed search method by the experiments shown in this section. First, we randomly generate parameters of solved problems from five settings, PS1, PS2, PS3, PS4, and PS5. For each setting, the data range for each parameter is defined in Table 3. There are 30 experimental problems in each setting. Therefore, we have a total of 150 experimental problems. We coded our search method in *MATLAB*® 6.5 and ran experiments on a computer with *P4*-1.8 GHz, 512MB RAM. Comparisons between our proposed search method and *LINGO*® software in the aspect of average run times and cost deviations are summarized in Table 4 and Table 5, respectively. Because some experiments ran over an hour using *LINGO*® software, we limited the longest run time to 3,600 seconds in *LINGO*® software. On average, the run time is within 0.057 sec. and the cost deviation is within 0.50%. These results demonstrate that our proposed heuristic method outperforms *LINGO*® software in solving the *TSIDP*.

-- Insert Table 3 here --
-- Insert Table 4 here --
-- Insert Table 5 here –

7. CONCLUSIONS

In this study, we focus on a three-stage inventory/production/distribution supply chain system with 11 members, including 1 supplier, 3 manufacturers and 7 retailers. This model was originally presented in Khouja (2003), who discussed only inventory and production activities in this system, without considering transportation costs. Therefore, applications could meet practical needs, we added the transportation cost into the inventory and production system. The objective of our research is to find the minimal cost of those activities conducted by members along the supply chain.

By analyzing the cost curve structure of the model *TSIDP*, we provide useful insights into the model *TSIDP* and develop a search method based on *JPs* and *BPs* appearing on the cost curve at each firm. The outcomes demonstrate that our search method outperforms *LINGO*® software by testing randomly generated experiments.

REFERENCES

1. Abdul-Jalbar, B., Gutiérrez, J., and Sicilia, J. (2005). Integer-ratio policies for distribution/inventory systems. *International*

Journal of Production Economics, 93-94, 407-415.

2. Bertazzi, L. and Speranza, M.G. (1999). Models and algorithms for the minimization of inventory and transportation costs: A survey. In: Speranza, M.G., and Stähly, P., *New Trends in Distribution Logistics*, Springer-Verlag, Amsterdam.
3. Burwell, T.H., Dave, D.S., Fitzpatrick, K.E., and Roy, M.R., (1997). Economic lot size model for price-dependent demand under quantity and freight discounts. *International Journal of Production Economics*, 48(2): 141-155.
4. Chen, F., Federgruen, A., and Zheng, Y.S., (2001). Coordination mechanisms for a distribution system with one supplier and multiple retailers. *Management Science*, 47(5): 693-708.
5. Drexl, A. and Kimms, A., (1997). Lot sizing and scheduling--- survey and extensions. *European Journal of Operational Research*, 99: 221-235.
6. Khouja, M., (2003). Optimizing inventory decisions in a multi-stage multi-customer supply chain. *Transportation Research Part E: Logistics and Transportation Review*, 39: 193-208.
7. Klastorin, T.D., Moinzadeh, K., and Son, J. (2002). Coordinating orders in supply chains through price discounts. *IIE Transactions*, 34(8): 679-689.
8. Lee, F.C. and Wen, U.P. (2007). A heuristic approach for solving serially distributed storage depots under general-integer policy. *Asia-Pacific Journal of Operational Research.*
9. Lee, F.C. and Yao, M.J. (2003). A global optimum search algorithm for the joint replenishment problem under power-of-two policy. *Computers and Operations Research*, 30(9): 1319-1333.
10. Maxwell, W.L. and Muckstadt, J.A. (1985). Establishing consistent and realistic reorder intervals in production-distribution systems. *Operations Research*, 33(6): 1316-1341.
11. Muckstadt, J. and Roundy, R.O. (1993). Analysis of multi-stage production systems. In: Graves, S.C. (Ed.), *Handbook in Operations Research and Management Science, Logistics of Production and Inventory*, 4, Elsevier Science Publishers, Amsterdam, 59-31.
12. Roundy, R.O. (1985). 98% Effective integer-ratio lot sizing for one-warehouse multi-retailer systems. *Management Science*, 31: 1416-1430.
13. Sarmiento, A.M. and Nagi, R., (1999). A review of integrated analysis of production-distribution systems. *IIE Transactions*, 31 (11): 1061-1074.
14. Swenseth, S.R. and Godfrey, M.R. (2002). Incorporating transportation costs into inventory replenishment decision. *International Journal of Production Economics*, 77(2): 113-130.
15. Vroblefski M., Ramesh, R., and Zionts, S, (2000). Efficient lot sizing under a differential transportation cost structure for serially distributed warehouses. *European Journal of Operational Research*, 127: 574-593.

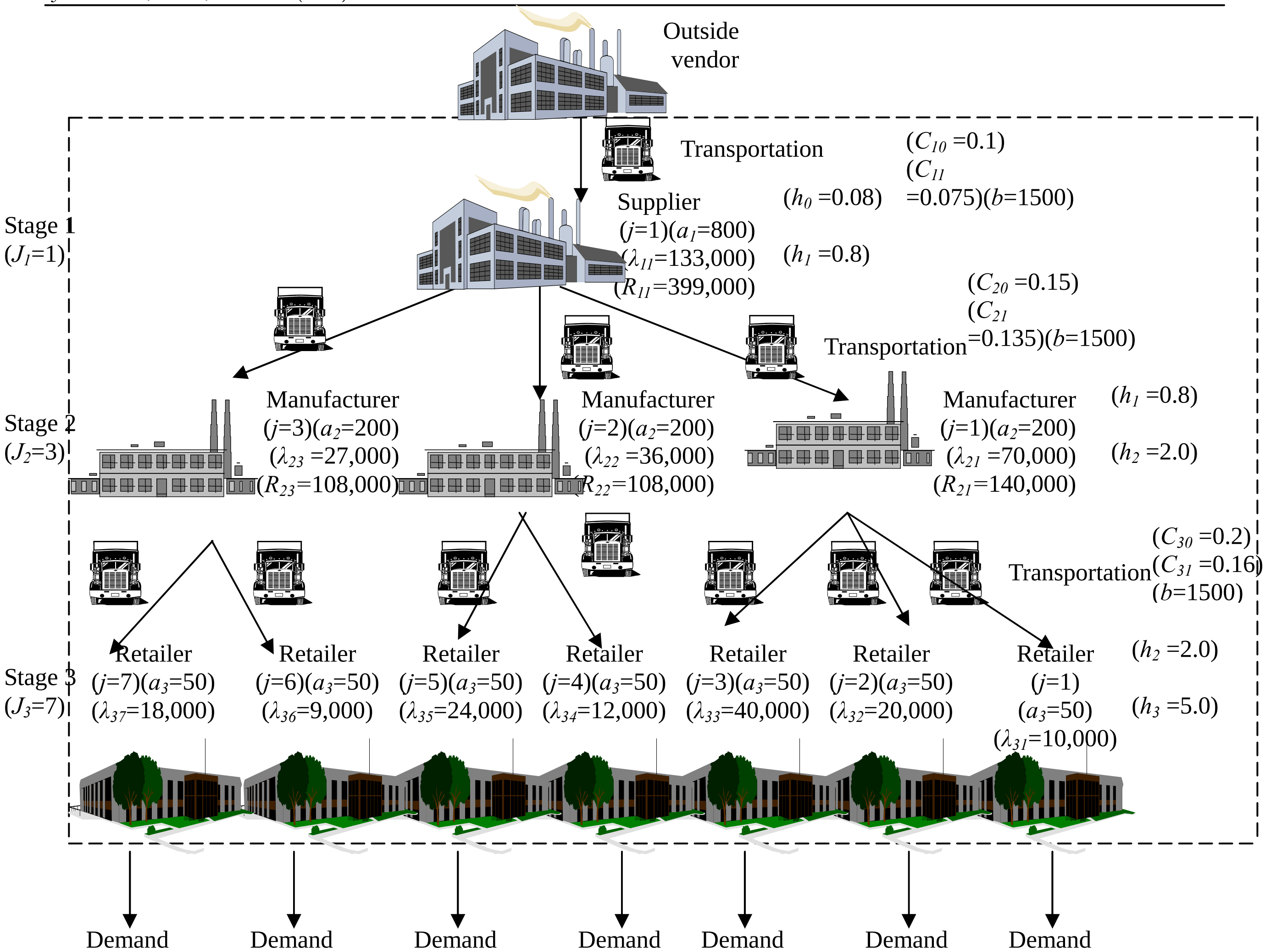

Figure 1. A three-stage inventory/production/distribution supply chain configuration.

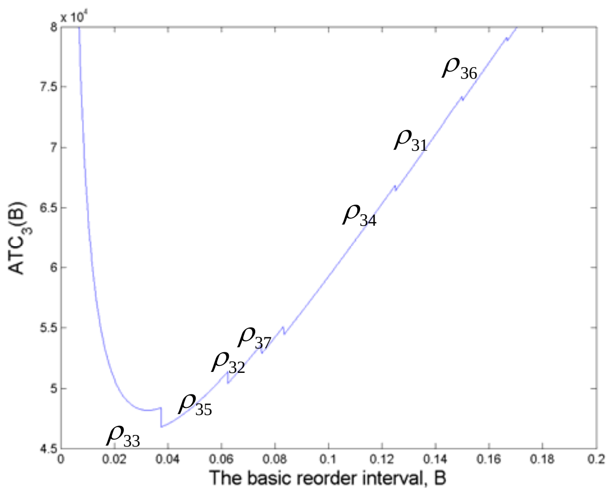

Figure 2. The total annual costs of all firms at stage 3.

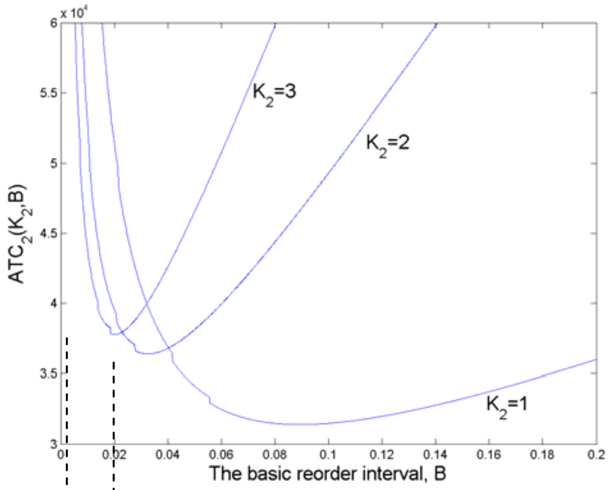

Figure 3. The total annual costs of all firms at stage 2.

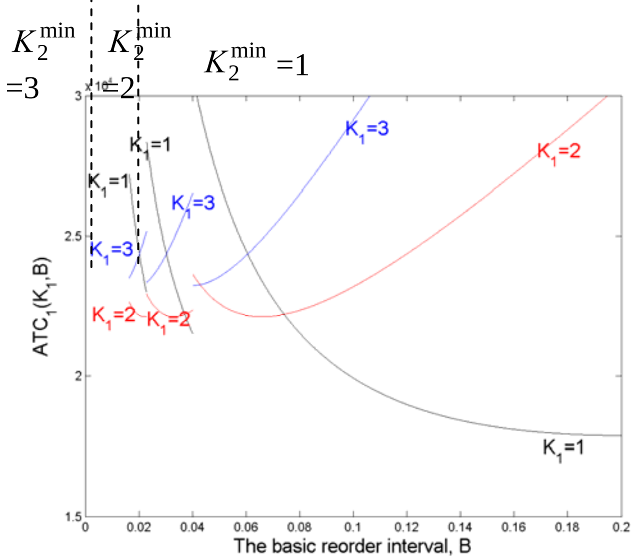

Figure 4. The total annual costs of all firms at stage 1.

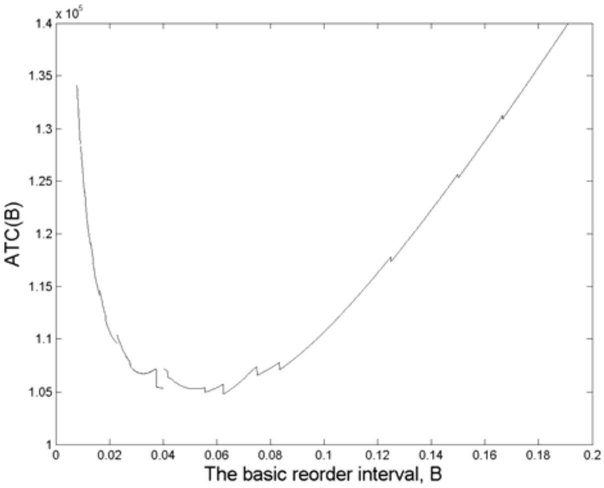

Figure 5. The total annual costs of all firms at all stages.

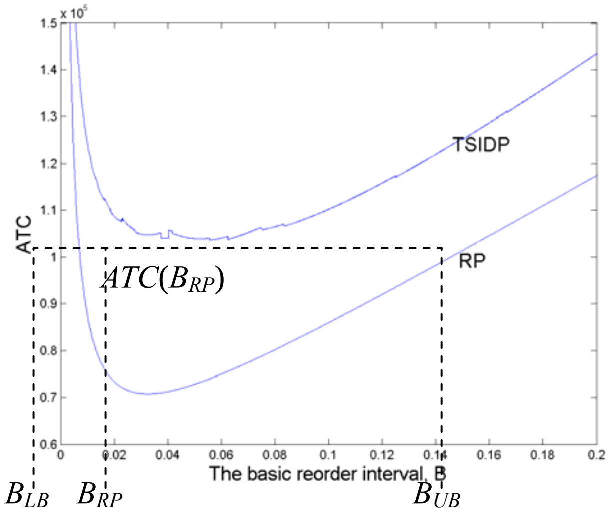

Figure 6. Starting and termination conditions on B^*.

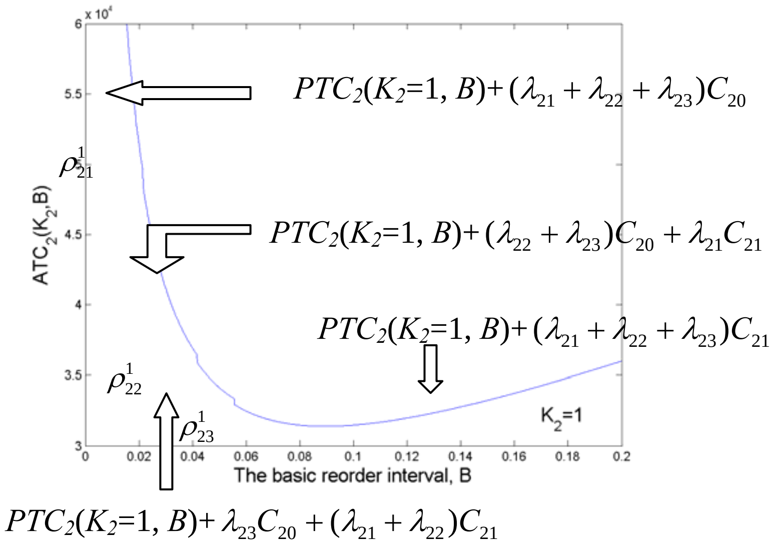

Figure 7. To illustrate the cost properties of each K_2 curve (Take K_2=1 for an example).

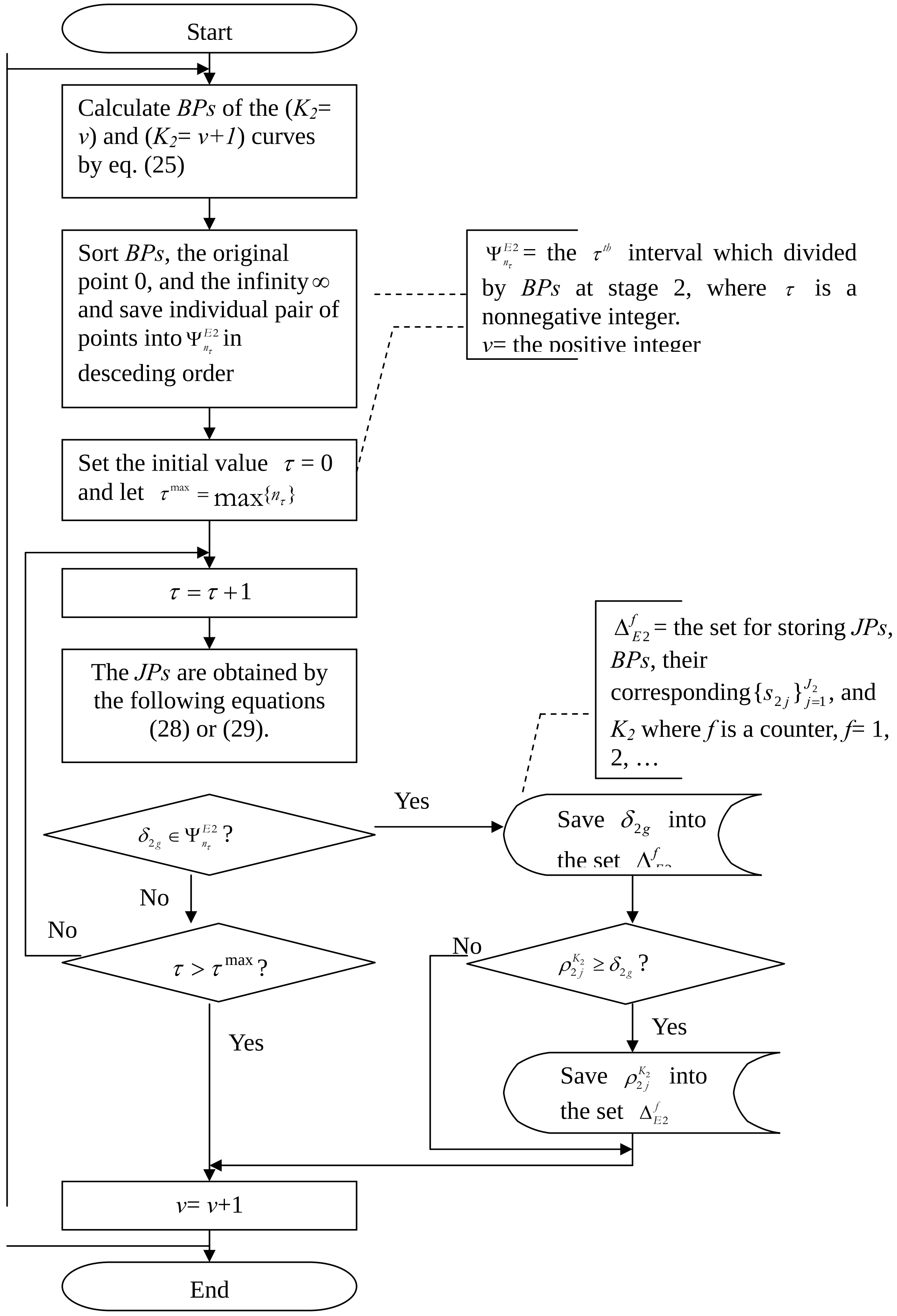

Figure 8. *The JP-Stage 2 Finding Procedure.*

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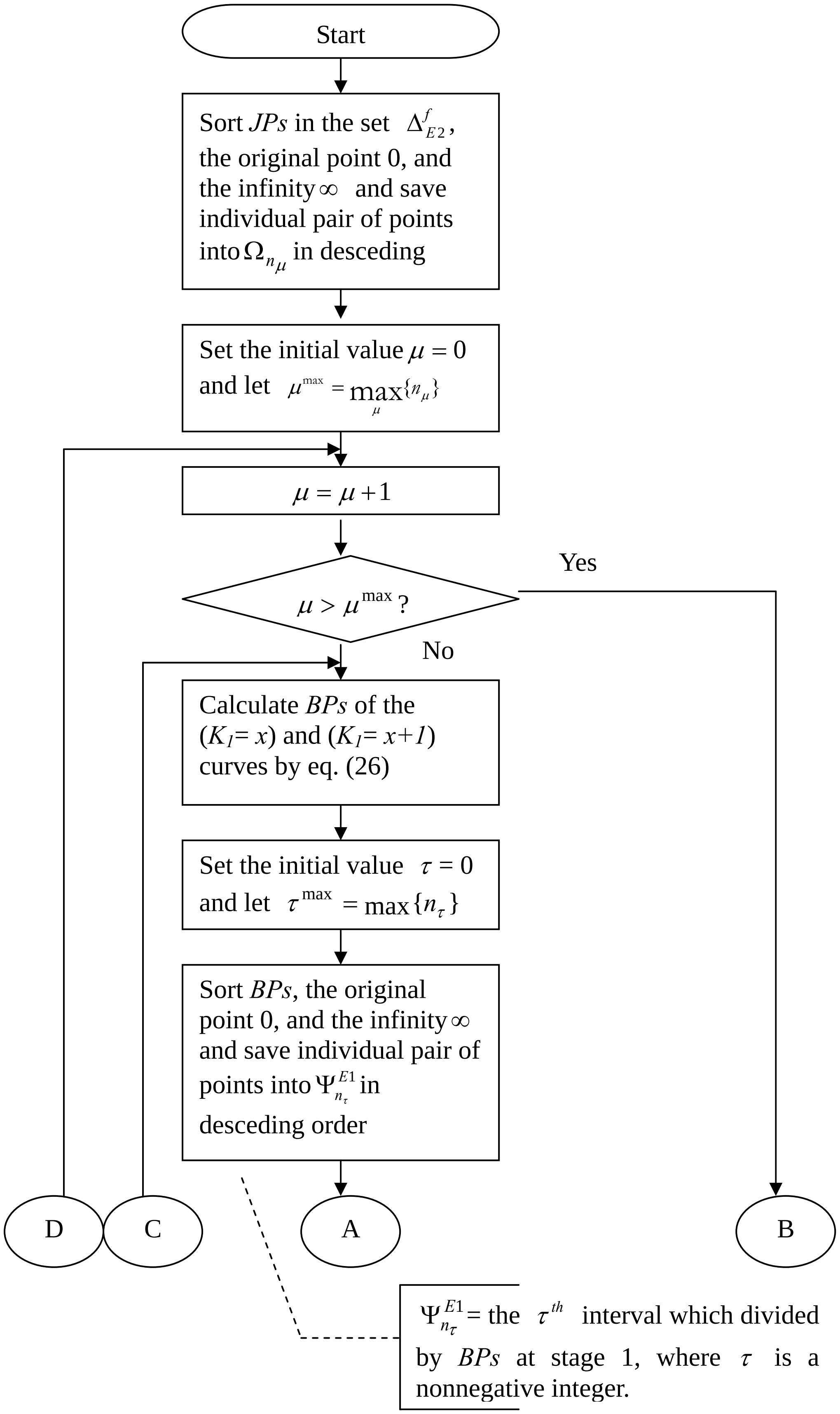

Figure 9. *The JP-Stage 1 Finding Procedure.*

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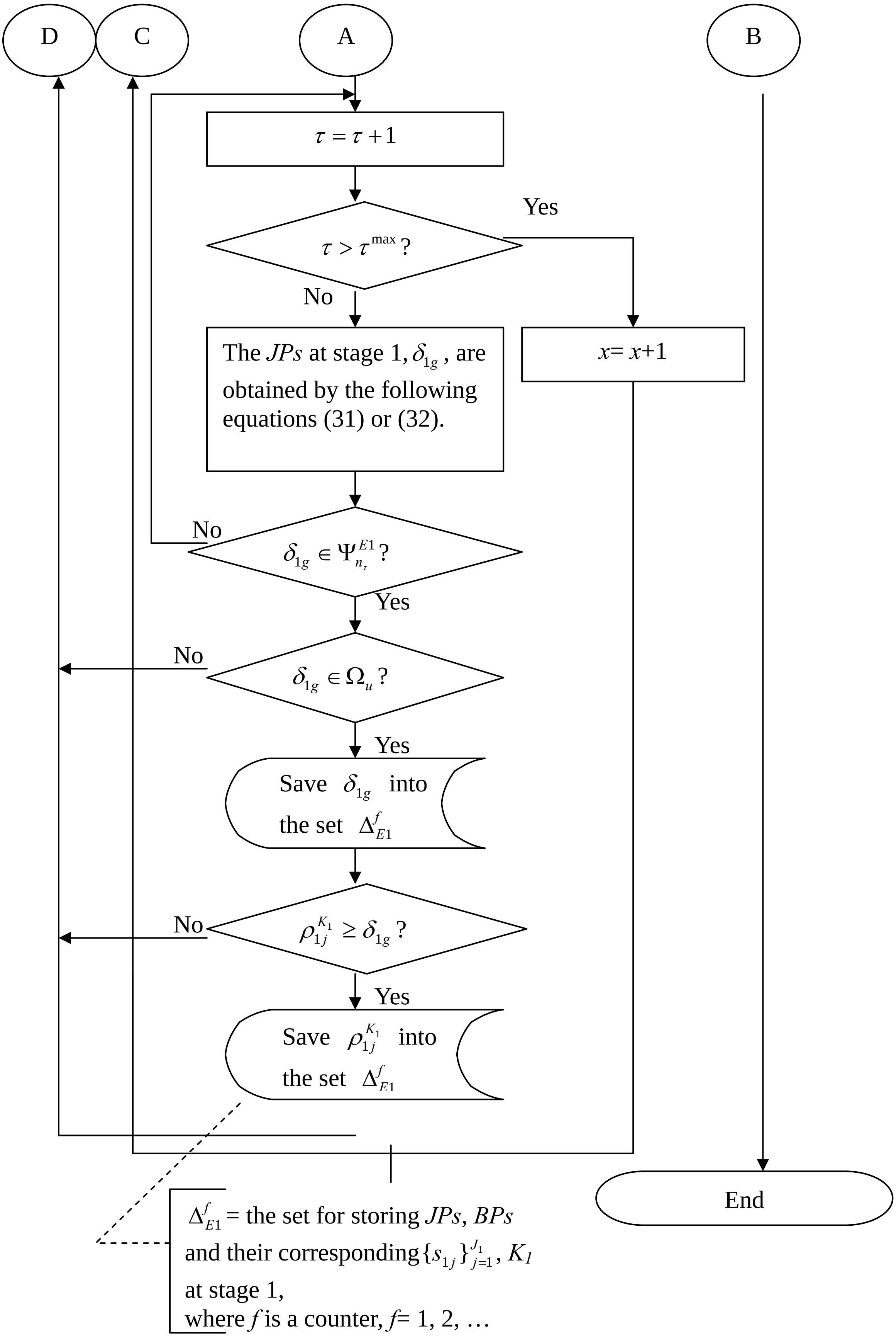

Figure 9. *The JP-Stage 1 Finding Procedure* (*Continued*).

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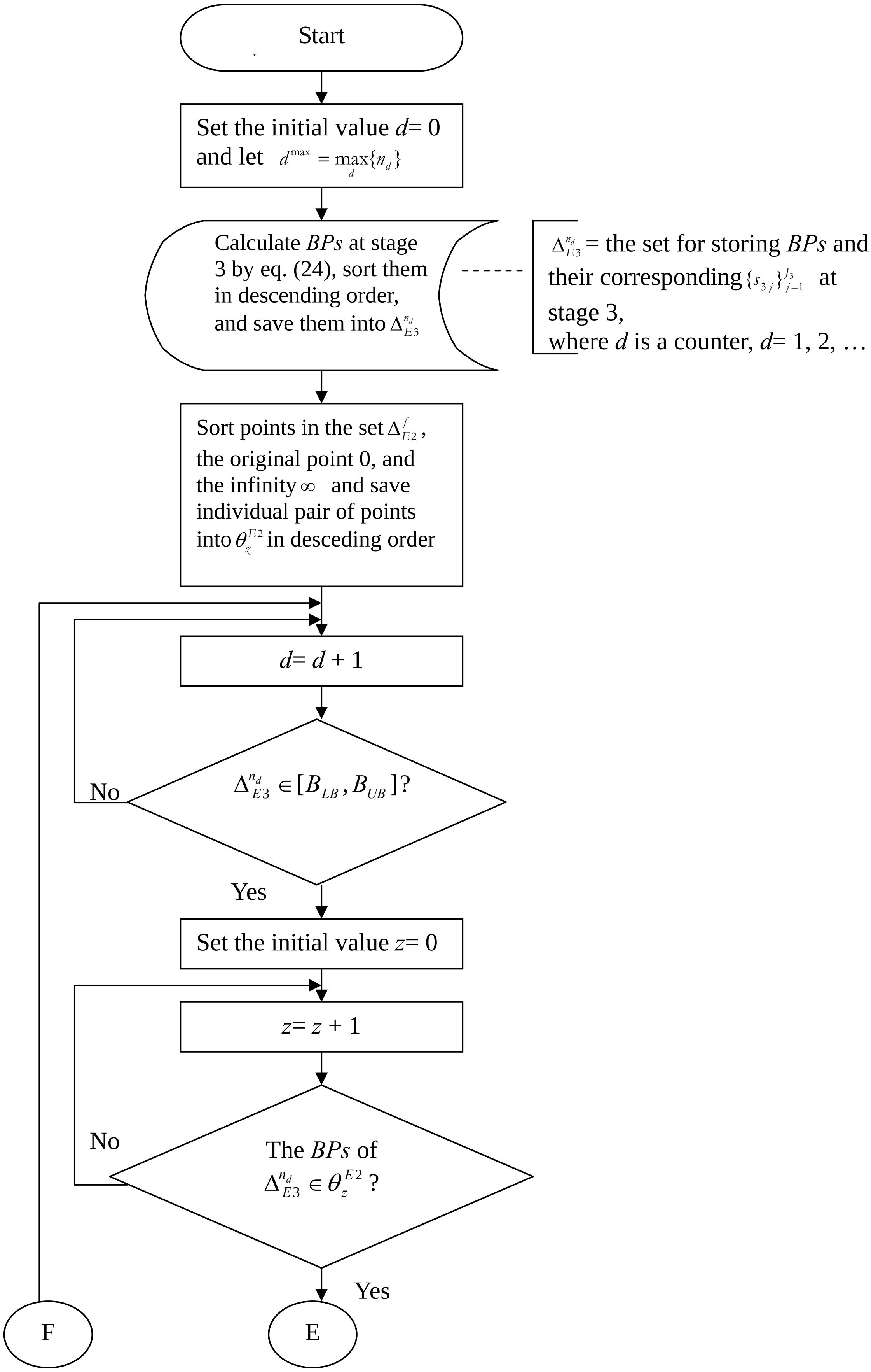

Figure 10. A search mechanism for solving searched points at stage. 3

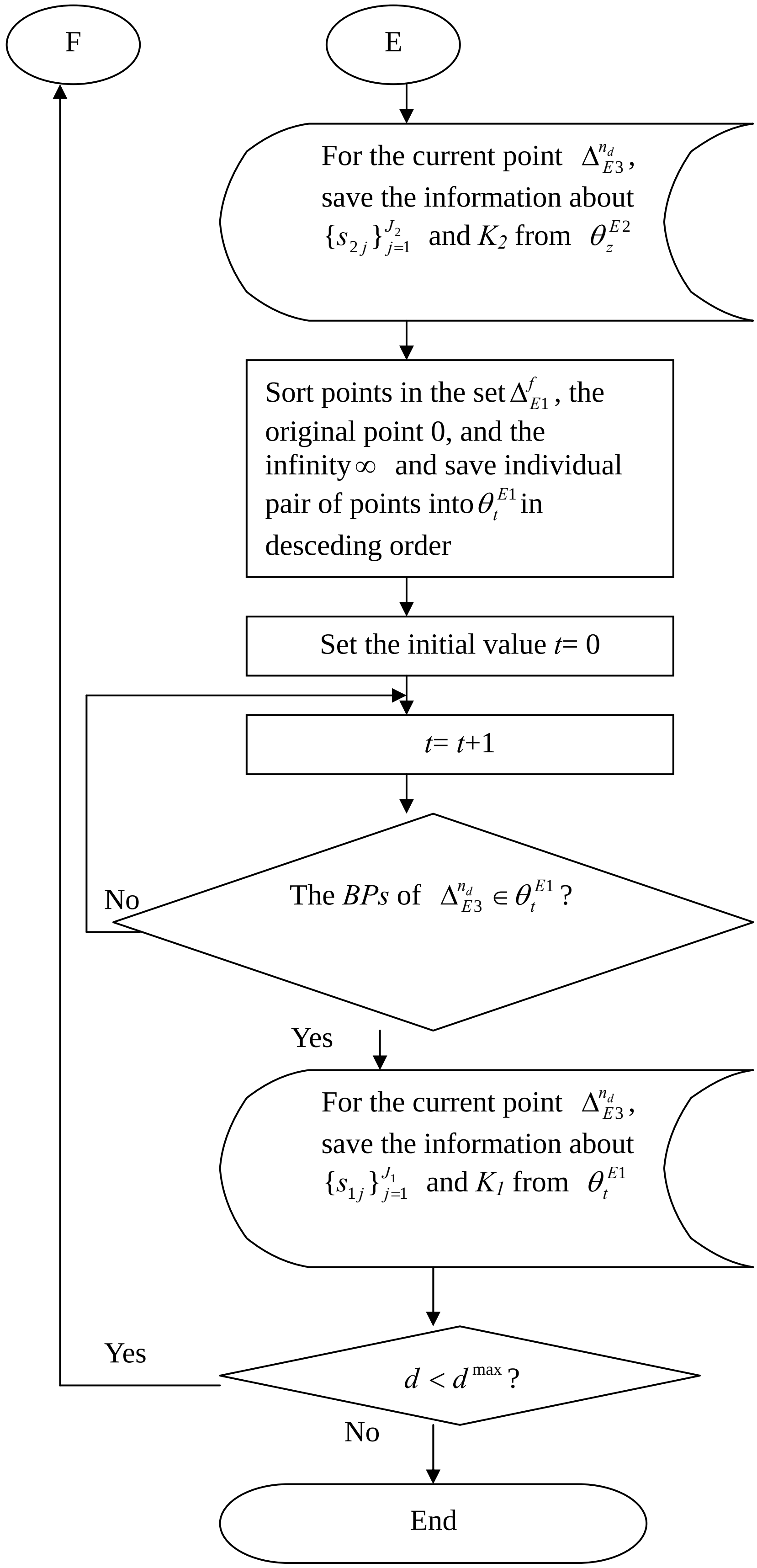

Figure 10. A search mechanism for solving searched points at stage 3 (Continued).

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Table 1. A glossary of notations.

Notations	Definition
i	an index of the echelon in the supply chain, i= 1, 2, 3.
J_i	the total number of firms at each stage i.
j	an index represents each firm. For each stage i, each firm is numbered from 1 to J_i.
a_i	setup/ordering cost of each company at each stage i. (dollars/per year)
h_{i-1}	the inventory carrying cost of incoming raw material at stage i. (dollars/per unit/per year)
h_i	the inventory carrying cost of outgoing finished goods at stage i. (dollars/per unit/per year)
λ_{ij}	annual demand of each firm j at stage i. (units/per year)
R_{ij}	the annual production rate of firms at stages 1 and 2. (units/per year)
B	the basic reorder interval. (years)
K_2	the ordering frequency multiplier at stage 2.
K_1	the ordering frequency multiplier at stage 1.
T_{ij}	the ordering frequency of each firm j at stage i. (years) $T_{3j} = B$, for j= 1, 2, … J_3. $T_{2j} = K_2 B$, for j= 1, 2, … J_2. $T_{1j} = K_1 K_2 B$, for j= 1, 2, … J_1.
q_{ij}	the ordering quantity of firm j at stage i. (units)
PTC_{ij}	The annual setup/ordering cost and inventory holding cost of each firm j at stage i. (dollars/per year)
PTC_i	The annual setup/ordering cost and inventory holding cost of all firms at stage i. (dollars/per year)
PTC	The annual setup/ordering cost and inventory holding cost of all firms at all stages. (dollars/per year)
ATC_{ij}	the total annual cost of each firm j at stage i. (dollars/per year)
ATC_i	the total annual cost of all firms at stage i. (dollars/per year)
ATC	the total annual cost of all firms at all stages. (dollars/per year)
b	the transportation cost breakpoint. (units)
s_{ij}	the 0/1 variable to judge the relationship between q_{ij} and b.
C_{i0}	the unit transportation cost for dispatching a single resource from the firm at the stage i (i−1) to that in the next successive stage i while the reorder or shipped quantity q_{ij} is less than a certain transportation volume, b. (dollars/per unit/per year)
C_{i1}	the unit transportation cost when q_{ij} is greater than or equal to b. C_{i0} and C_{i1} would result in s_{ij}= 0 and s_{ij} = 1, respectively. (dollars/per unit/per year)
$C_{i1}^{'}$	$= (C_{i1} - C_{i0}) < 0$ and denote it as the transportation cost difference. (dollars/per unit/per year)

Table 2. The results of the illustrative example.

The searched points, B	s_{31}	s_{32}	s_{33}	s_{34}	s_{35}	s_{36}	s_{37}	s_{21}	s_{22}	s_{23}	s_{11}	K_2	K_1	ATC
0.1500	1	1	1	1	1	0	1	1	1	1	1	1	1	125,325
0.1250	0	1	1	1	1	0	1	1	1	1	1	1	1	117,377
0.0833	0	1	1	0	1	0	1	1	1	1	1	1	1	107,055
0.0750	0	1	1	0	1	0	0	1	1	1	1	1	1	106,548
0.0625	0	0	1	0	1	0	0	1	1	1	1	1	2	104,818*
0.0375	0	0	1	0	0	0	0	1	1	1	1	2	1	105,493
0.0556	0	0	1	0	0	0	0	1	1	1	1	1	2	105,006
0.0417	0	0	1	0	0	0	0	1	1	0	1	1	1	113,137
0.0383	0	0	1	0	0	0	0	1	0	0	1	1	1	115,969
0.0278	0	0	0	0	0	0	0	1	1	1	1	2	1	110,525
0.0218	0	0	0	0	0	0	0	1	1	0	1	2	1	117,437
0.0185	0	0	0	0	0	0	0	1	1	1	1	3	1	114,978
0.0157	0	0	0	0	0	0	0	1	1	0	1	3	1	120,909
0.0139	0	0	0	0	0	0	0	1	1	1	1	4	1	120,354
0.0123	0	0	0	0	0	0	0	1	1	0	1	4	1	125,357
0.0111	0	0	0	0	0	0	0	1	1	1	1	5	1	126,100
0.0103	0	0	0	0	0	0	0	1	1	0	1	5	1	130,102
0.0093	0	0	0	0	0	0	0	1	1	1	1	6	1	132,030
0.0088	0	0	0	0	0	0	0	1	1	0	1	6	1	134,909
0.0079	0	0	0	0	0	0	0	1	1	1	1	7	1	138,066
0.0078	0	0	0	0	0	0	0	1	1	0	1	7	1	139,680
0.0742	0	0	1	0	1	0	0	1	1	1	1	1	1	107,254
0.0428	0	0	1	0	0	0	0	1	1	0	1	1	2	106,270
0.0371	0	0	0	0	0	0	0	1	1	1	1	2	1	107,134
0.0247	0	0	0	0	0	0	0	1	1	0	1	2	1	113,598
0.0185	0	0	0	0	0	0	0	1	1	1	1	3	1	114,941
0.0148	0	0	0	0	0	0	0	1	1	1	1	4	1	118,191
0.0124	0	0	0	0	0	0	0	1	1	0	1	4	1	125,288
0.0106	0	0	0	0	0	0	0	1	1	0	1	5	1	128,572

Table 3. The parameter settings for the random experiments.

PS1	$(C_{10} \sim U[0.05,\ 0.1])$ $(C_{11} \sim U[0.05,\ 0.1])$$(b \sim U[900, 3000])$ $(b_0 \sim U[0.05,\ 0.2])$ $(b_1 \sim U[0.6,\ 0.9])$$(a_1 \sim U[600, 1000])$ $(R_{11} \sim U[460000, 480000])$$(C_{20} \sim U[0.12,\ 0.16])$$(C_{21} \sim U[0.12,\ 0.16])$$(b_2 \sim U[1.5,\ 2.5])$$(a_2 \sim U[100,\ 300])$ $(R_{21},\ R_{22},\ R_{23} \sim U[130000,\ 150000])$ $(C_{30} \sim U[0.17,\ 0.25])$ $(C_{31} \sim U[0.17,\ 0.25])$$(b_3 \sim U[4.0,\ 7.0])$ $(a_3 \sim U[20,\ 80])$$(\lambda_{31},\ \lambda_{32},\ \lambda_{33}, \lambda_{34},\ \lambda_{35},\ \lambda_{36},\ \lambda_{37} \sim U[8000,\ 40000])$
PS2	$(C_{10} \sim U[0.1,\ 0.16])$ $(C_{11} \sim U[0.1,\ 0.16])$$(b \sim U[2000, 4000])$ $(b_0 \sim U[0.15,\ 0.3])$ $(b_1 \sim U[1.2,\ 1.6])$$(a_1 \sim U[1000, 1500])$ $(R_{11} \sim U[600000, 700000])$$(C_{20} \sim U[0.2,\ 0.24])$ $(C_{21} \sim U[0.2,\ 0.24])$$(b_2 \sim U[3.0,\ 4.5])$$(a_2 \sim U[300,\ 600])$ $(R_{21},\ R_{22},\ R_{23} \sim U[160000,\ 220000])$$(C_{30} \sim U[0.25,\ 0.3])$ $(C_{31} \sim U[0.25,\ 0.3])$$(b_3 \sim U[6.5,\ 8.5])$ $(a_3 \sim U[70,\ 100])$$(\lambda_{31},\ \lambda_{32},\ \lambda_{33}, \lambda_{34},\ \lambda_{35},\ \lambda_{36},\ \lambda_{37} \sim U[10000,\ 600000])$
PS3	$(C_{10} \sim U[0.15,\ 0.2])$ $(C_{11} \sim U[0.15,\ 0.2])$ $(b \sim U[2500, 4500])$ $(b_0 \sim U[0.3,\ 0.45])$ $(b_1 \sim U[1.5,\ 3])$$(a_1 \sim U[2000, 2500])$ $(R_{11} \sim U[650000, 750000])$$(C_{20} \sim U[0.25,\ 0.3])$ $(C_{21} \sim U[0.25,\ 0.3])$$(b_2 \sim U[3.5,\ 5.5])$ $(a_2 \sim U[600,\ 1500])$ $(R_{21},\ R_{22},\ R_{23} \sim U[250000,\ 300000])$$(C_{30} \sim U[0.35,\ 0.45])$ $(C_{31} \sim U[0.35,\ 0.45])$$(b_3 \sim U[6,\ 7.5])$$(a_3 \sim U[40,\ 150])$$(\lambda_{31},\ \lambda_{32},\ \lambda_{33}, \lambda_{34},\ \lambda_{35},\ \lambda_{36},\ \lambda_{37} \sim U[15000,\ 80000])$
PS4	$(C_{10} \sim U[0.25,\ 0.3])$ $(C_{11} \sim U[0.25,\ 0.3])$$(b \sim U[3500, 4500])$ $(b_0 \sim U[0.3,\ 0.75])$ $(b_1 \sim U[2.0,\ 3.0])$$(a_1 \sim U[2000, 4500])$ $(R_{11} \sim U[650000, 750000])$$(C_{20} \sim U[0.35,\ 0.45])$ $(C_{21} \sim U[0.35,\ 0.45])$$(b_2 \sim U[3.5,\ 5.5])$$(a_2 \sim U[800,\ 1500])$ $(R_{21},\ R_{22},\ R_{23} \sim U[250000,\ 300000])$$(C_{30} \sim U[0.55,\ 0.65])$ $(C_{31} \sim U[0.55,\ 0.65])$ $(b_3 \sim U[7.0,\ 8.5])$$(a_3 \sim U[150,\ 450])$$(\lambda_{31},\ \lambda_{32},\ \lambda_{33}, \lambda_{34},\ \lambda_{35},\ \lambda_{36},\ \lambda_{37} \sim U[55000,\ 80000])$
PS5	$(C_{10} \sim U[0.15,\ 0.35])$ $(C_{11} \sim U[0.15,\ 0.35])$$(b \sim U[2500, 5500])$ $(b_0 \sim U[0.5,\ 0.95])$ $(b_1 \sim U[2.5,\ 3.5])$$(a_1 \sim U[1300, 2000])$ $(R_{11} \sim U[500000, 550000])$$(C_{20} \sim U[0.45,\ 0.50])$ $(C_{21} \sim U[0.45,\ 0.50])$$(b_2 \sim U[4.5,\ 6.5])$$(a_2 \sim U[600,\ 1200])$ $(R_{21},\ R_{22},\ R_{23} \sim U[150000,\ 300000])$$(C_{30} \sim U[0.65,\ 0.75])$ $(C_{31} \sim U[0.65,\ 0.75])$ $(b_3 \sim U[7.0,\ 8.5])$ $(a_3 \sim U[200,\ 500])$$(\lambda_{31},\ \lambda_{32},\ \lambda_{33}, \lambda_{34},\ \lambda_{35},\ \lambda_{36},\ \lambda_{37} \sim U[25000,\ 90000])$

Table 4. Average run time between our proposed method and *LINGO*® software.

Parameter Settings	Average Run Time	
	(Heuristic)	(*LINGO*®)
PS1	0.056 sec.	2.462 sec.
PS2	0.052 sec.	3343 sec.
PS3	0.057 sec.	3480 sec.
PS4	0.051 sec.	3000 sec.
PS5	0.055 sec.	3600 sec.

Table 5. Average cost deviation between our proposed method and *LINGO*® software.

Parameter Settings	Average Cost Deviation
	$(ATC_{\text{Heuristic}} - ATC_{LINGO®}) / ATC_{LINGO®}$
PS1	0.50%
PS2	0.33%
PS3	0.06%
PS4	0.02%
PS5	0.01%